Invited Review

MRP and inventories

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1. Introduction

In this paper, multi-stage production-inventory systems will be considered. Such systems can be described as networks of production stages, separated by inventory points. Inventories may emerge for several well-known reasons, such as lot-sizing, buffering against uncertainty, or anticipating future demand-increases. In this paper we will focus on the function of inventories to buffer against uncertainty, especially against demand uncertainty.

The strict distinction between production stages and inventory points is useful for many purposes, but it hides an important fact: The fact that production stages are mainly built up by queues. These queues could be considered as points of inventories as well. The main difference between queues and inventory points is the following. The material in queues is waiting and competing for capacity, and further processing of this material should start as soon as capacity is available. The material in inventory is not yet released to the shop floor, and further processing is not allowed, even if capacity is available. Thus, the release decision is quite important in multi-stage production-inventory control systems.

In order to emphasize the similarity of queues and inventories, we shall call both forms of materials waiting by the same term: slack. However, queues are called intrastage slack, whereas inventories are called interstage slack.

Operations Research models have since long been used to set norms for both intrastage slack and interstage slack, for the purpose of dealing with uncertainties and short-term fluctuations in volume. A danger of setting such norms and creating slack is that it removes the pressure to reduce the uncertainties and fluctuations. This danger is rightly stressed by the Just-in-Time movement, but also by the Manufacturing Resources Planning (MRP II) approach. The following elements of the MRP approach fit in this anti-slack philosophy:

- the emphasis on a realistic Master Production Schedule (MPS) to coordinate the production stages in terms of volume of production; this prevents the creation of extraordinary intrastage slack (queues);
- the emphasis on calculating dependent demand (this prevents the creation of superfluous interstage slack, due to unbalanced lotsizing of matched sets of parts);
- the control of leadtimes by input–output control (this prevents again the creation of extraordinary intrastage slack);
- rescheduling of work-in-process to earlier or later due-dates (this is meant to cope with disturbances and to prevent the necessity of safety stocks). N.B. rescheduling is often called priority control in MRP circles (cf. Plossl and Welch [13]).

The only safety stocks which are allowed in
such an approach are stocks at the MPS level. For make-to-stock situations this level is the end-item level; for assemble-to-order situations this level corresponds to the highest sub-assemblies produced without known customer orders. In the MRP literature there are only two exceptions to this rule. The first exception is that some safety stock is allowed for lower level items with independent demand (e.g. spare parts). The second exception is that some safety stock is allowed for uncertainty with respect to external supply.

This approach has not led to software with much flexibility to use safety stocks. The software is mainly focussed on the above mentioned elements (realistic MPS, dependent demand, lead-time control, rescheduling). The software users, however, in many cases like to have stocks at more levels than only the MPS level. Such stocks can be attractive because of lower added value or a higher commonality, or the possibility of a smoother adjustment of subsequent production stages or a more effective input–output control.

This last point will be considered in more detail. Rescheduling is based on the idea that it is possible to compress or extend individual leadtimes without increasing the average leadtime. This is possible because actual leadtimes are largely built up from queueing times, so from slack. The total amount of slack and the average leadtimes are controlled by the input–output control function. More effective input–output control implies more reliable leadtimes. But effective input–output control requires some stock around the controlled stage. Effective input–output control makes the slack necessary at the MPS level or within the stages smaller.

Summarizing, the MRP approach foresees only slack at MPS level and within the production stages, while in many situations there are good arguments to have also slack (stock) between the stages. The subject of this paper is the possibility to use inventories (interstage slack) to absorb uncertainty. In the next section we come back to these assumptions. It has to be mentioned, however, that this is the approach used by most authors on ‘MRP and safety stocks’ (see Lambrecht et al. [6], Meal [8], Whybark and Williams [15]).

2. Interstage slack in multi-stage production systems

In this section we consider multi-stage production systems and the possibility to absorb uncertainty by using inventory buffers between the stages. The production throughputs are assumed to be given.

We use MRP terminology, but the approach in this section is not completely consistent with the MRP framework since we assume that there is no rescheduling (to influence through-put-times) and that MPS and demand forecast are identical. These assumptions are made here to show first the possibility to use inventories (interstage slack) to absorb uncertainty. In the next section we come back to these assumptions. It has to be mentioned, however, that this is the approach used by most authors on ‘MRP and safety stocks’ (see Lambrecht et al. [6], Meal [8], Whybark and Williams [15]).

2.1. One-stage case

First, the most simple case, the one-stage case, is considered (see Figure 1).

Assume that ordering is ‘lot-for-lot’. In the MRP-scheme in Table 1 we see how demand forecast (gross requirement) determines planned order releases in case of no safety stock.

$$\text{Let } D(t, t + k) \text{ be the demand in period } t + k \text{ as forecasted at the start of period } t. \text{ Let } C(t) \text{ be the sum of inventory and work in process at the start of period } t \text{ (so } C(1) = 30 + 80 + 60 + 70). \text{ Then the general relationship between the } P(t) \text{ the planned order release in period } t, \text{ the inventory position } C(t) \text{ and the demand forecasts } D(t, t + k) \text{ is}$$

$$P(t) = \sum_{i=0}^{t} D(t, t + i) - C(t). \quad (1)$$

For sake of convenience we assume that

$$C(t) \leq \sum_{i=0}^{t} D(t, t + i).$$

If that is not the case the planned order release is equal to 0 (and not negative of course). For a good understanding of the role of safety stocks it is not necessary to consider in detail this difficulty. It may lead, however, to some more stock than suggested by relation (1).

In case of uncertainty and no rescheduling possibilities it may be necessary to release more material. This can be effectuated in three ways.

1. Use a fixed safety stock norm $S$. In this case
$$P(t) = \sum_{i=0}^{l} D(t, t + i) + S - C(t).$$

2. Use a fixed safety time norm $e$. In this case
$$P(t) = \sum_{i=0}^{l+e} D(t, t + i) - C(t).$$

3. Overestimate the future requirement (hedging):
$$P(t) = \sum_{i=0}^{l} \hat{D}(t, t + i) - C(t).$$

The amount to release extra in these cases is respectively: $S$, $\sum_{i=0}^{l} D(t, t + i)$ and $\sum_{i=0}^{l} \{ \hat{D}(t, t + i) - D(t, t + i) \}$. In case of relatively stable requirements the methods are basically equivalent. We come back to these three methods in the next two sections. In this section we restrict the attention to the use of safety stock norms.

The size of the safety stock has to depend on the amount of uncertainty. We distinguish three types of uncertainty: uncertainty with respect to demand, uncertainty with respect to leadtime and uncertainty with respect to yield.

Let $\sigma^2(i)$ be the variance of the forecast error with respect to the demand $i$ periods ahead. Let $\sigma^2_t$ be the variance of the leadtime, and let $\sigma^2_y$ be the variance of the yield. For sake of convenience we assume that this variance is independent of the period, which is only approximately the case if the total work in process is rather constant.

The safety stock has to cover all uncertainty over $l+1$ periods. That means that $S$ may be chosen equal to
$$S = k \sqrt{\sum_{i=0}^{l} \sigma^2(i) + \sigma^2_t + \sigma^2_y \mu^2}$$
where $\mu$ is the average demand. The factor $k$ can be used to control the stock-out risk. If the uncertainty is about normally distributed, the stock-out risk corresponding to a factor $k$ is equal to $1 - F(k)$, where $F(\cdot)$ is the standard normal distribution function. This is in fact an upperbound for the stock-out risk because negative releases are not possible which implies that it is not always possible to reset the content of the system at the right level. But this plays only a role in cases with very dynamic demand and demand forecasts.

To estimate the variance it may be better not to register each of these sources of uncertainty separately, but to register combinations, for instance the demand during the leadtime. We refer to Brown [4, Ch.8] for a more fundamental discussion with respect to this point.

In case of an effective input–output control part of the leadtime variance may be due to the fact that work orders have to wait before being released.

It has to be mentioned here that a stock-out in the model will not always correspond to a stock-out in reality. Even in the classical inventory control approach, without formal rescheduling possibilities, there will actually be some rescheduling. In such an approach safety stocks are used to protect against rescheduling, while in the MRP approach.

Table 1

<table>
<thead>
<tr>
<th>Gross requirement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheduled receipts</td>
<td>80</td>
<td>60</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Available balance</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>Net requirement</td>
<td></td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planned order</td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
rescheduling tends to protect safety stocks of being used (see next section).

2.2. The multi-stage case

In this section the two-stage linear case is considered (see Figure 2). But the results can be transferred to the general multi-stage linear case. Ordering is assumed to be 'lot-for-lot' again. In Table 2 an example of an MRP scheme is given. It shows how planned order releases at both release points (1 and 2) are determined by demand forecasts (gross requirements), inventories and work in process.

The general expressions for the planned order releases at point 1 \(P_1(t)\) and point 2 \(P_2(t)\) are

\[
P_2(t) = \sum_{i=0}^{l_2} D(t, t+i) - C_2(t),
\]

\[
P_1(t) = \sum_{i=0}^{l_1+l_2} D(t, t+i) - (C_1(t) + C_2(t)),
\]

where \(C_2(t)\) is the content of stage 2 at the start of period \(t\) (work in process plus final inventory) and \(C_1(t)\) is the content of stage 1 (work in process plus intermediate inventory). Note that the expression for \(P_1(t)\) would get much more complicated if we would include the possibility that \(C_2(t) < \sum_{i=0}^{l_2} D(t, t+i)\).

As in the one-stage case it is possible to use safety stocks to absorb uncertainty with respect to demand, leadtimes and yield. Let \(S_1\) and \(S_2\) be the safety stock norms at levels 1 and 2 then

\[
P_2(t) = \sum_{i=0}^{l_2} D(t, t+i) - C_2(t) + S_2,
\]

\[
P_1(t) = \sum_{i=0}^{l_1+l_2} D(t, t+i) - (C_1(t) + C_2(t)) + S_1 + S_2.
\]

The next point is to determine \(S_1\) and \(S_2\). To simplify the notation we assume from now on that there is only demand uncertainty. At point 2 one has to take into account uncertainty over \(l_2 + 1\) periods (leadtime plus review period), at point 1 one has to take into account uncertainty over \(l_1 + l_2 + 1\) periods. That implies that it is reasonable to choose \(S_1\) and \(S_2\) so that

\[
S_1 + S_2 = k\left(\sum_{i=0}^{l_1} \sigma^2(i)\right),
\]

\[
S_1 \leq k\left(\sum_{i=0}^{l_2} \sigma^2(i)\right),
\]

where \(\sigma^2(i)\) is the variance of demand over period \(i\) for stage \(i\).

It is also possible of course to make the safety factors for \(S_2\) and \(S_1 + S_2\) different (i.e. different \(k_2\) and \(k_1\)).

In this two-stage case it is less clear how the stock-out risk depends on \(k\). In the one-stage case the stock-out risk is \(1 - F(k)\) (see the previous subsection). In this case the stock-out risk is between \(1 - F(k)\) and \(2(1 - F(k))\), because there are

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Table 2

<table>
<thead>
<tr>
<th>Stage 2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirement</td>
<td>100</td>
<td>50</td>
<td>80</td>
<td>40</td>
<td>130</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td>80</td>
<td>60</td>
<td>70</td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Available balance</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>30</td>
<td>160</td>
<td>210</td>
</tr>
<tr>
<td>Net requirement</td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planned order release</td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirement</td>
<td>30</td>
<td>130</td>
<td>50</td>
<td>40</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>40</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Available balance</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>40</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Net requirement</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planned order release</td>
<td>40</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Figure 2. One product, two stages
two release points, that is, two points where the demand can be misestimated. The stock-out risk is investigated for the case where \( \sigma(i) = \sigma \).

In Table 3 the stock-out risk is given for two values of \( I_1 \) and \( I_2 \), and \( k = 2 \). If the demand forecast is not too dynamic a reasonable possibility is to delete all time dependence in \( D(t, t + i) \), so \( D(t, t + i) = D \). In that case the system described here is equivalent to a Base Stock Control System (see Magee [7], Peterson-Silver [12]). The release quantities are

\[
P_2(t) = (l_2 + 1)D + S_2 - C_2(t),
\]
\[
P_1(t) = (l_1 + l_2 + 1)D + S_1 + S_2 - (C_1(t) + C_2(t)).
\]

Note that in the completely stationery case it will never occur that

\[C_2(t) > (l_2 + 1)D.\]

Base Stock Control is directed to an effective use of inventories (inter-stage slack) in case of a situation with not too divergent product structures and rather stable demand forecasts. For a comparison of Base Stock Control and MRP, see Timmer et al. [14]. See Clark/Scarf [5] for a proof of the optimality of Base Stock Control in case of a linear product structure with inflexible leadtimes and no capacity problems.

### 2.3. The convergent case

The case with a purely convergent product structure can be treated in precisely the same way as the linear case. See Figure 3 for a simple example. It is assumed that for each unit of the final product one needs one unit of each of the components. The planned order release at point 2, according to MRP calculations, is equal to

\[
P_2(t) = \sum_{i=0}^{l_1+t} D(t, t + i) - (C_3(t) + C_2(t))
\]

where \( C_3(t) \) is the content of stage 3 (plus final inventory) and \( C_2(t) \) is the content of stage 2 (plus intermediate inventory). The safety stocks necessary in this convergent case can be determined as the safety stocks in the linear case.

\[
S_3 = k_{l_3} \frac{\sum_{i=0}^{l_5} \sigma^2(i)}{\sum_{i=0}^{l_3} \sigma^2(i)},
\]
\[
S_1 = k_{l_1} \frac{\sum_{i=0}^{l_3 + l_5} \sigma^2(i)}{\sum_{i=0}^{l_1} \sigma^2(i)} - k_{l_3} \frac{\sum_{i=0}^{l_5} \sigma^2(i)}{\sum_{i=0}^{l_3} \sigma^2(i)},
\]
\[
S_2 = k_{l_2} \frac{\sum_{i=0}^{l_3 + l_5} \sigma^2(i)}{\sum_{i=0}^{l_2} \sigma^2(i)} - k_{l_3} \frac{\sum_{i=0}^{l_5} \sigma^2(i)}{\sum_{i=0}^{l_3} \sigma^2(i)}.
\]

As in the linear case it is possible to use different safety factors for the \( S_3 \), \( S_1 + S_3 \) and \( S_2 + S_3 \). Since both components have to be available to make one final product it seems to be reasonable to keep the safety factors for \( S_1 + S_3 \) (\( k_1 \)) and \( S_2 + S_3 \) (\( k_2 \)) equal.

### 2.4. The divergent case

The divergent case (see Figure 4) is less simple because of the possibility of inbalanced final inventories. However, for a wide range of situations it is still so that the planned order releases at the lower level only depend on the content of the

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**Table 3**

<table>
<thead>
<tr>
<th>( I_1 ) and ( I_2 ), ( k = 2 )</th>
<th>Stock-out risk</th>
<th>( 1 - F(2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 = 4, I_2 = 1 )</td>
<td>0.0391</td>
<td>0.0227</td>
</tr>
<tr>
<td>( I_1 = 2, I_2 = 3 )</td>
<td>0.0312</td>
<td>0.0227</td>
</tr>
</tbody>
</table>
system at that level and at higher levels. In such cases it is possible to use the same type of analysis to get good heuristics to determine safety stocks (see Wijngaard [17]). See also Zipkin [18] for a special way to treat imbalanced final inventories in the multi-location inventory problem. Related problems are investigated by Bemelmans [1].

2.5. Lot sizing

Consider first the linear two-stage case with fixed lots \( q_1 \) and \( q_2 \). Suppose the safety stocks are \( S_1 \) and \( S_2 \). That means that whether or not an order release at point 1 is necessary, depends on whether the following inequality is satisfied:

\[
S_1 + S_2 + \sum_{i=0}^{t} D(t, t+1) \leq C_1(t) + C_2(t).
\]

Neglecting for the moment the possibility that components are temporarily arrested in the intermediate inventory, the stock-out risk is the same as in the lot-for-lot case. But now it is a risk per order instead of a risk per period. The influence of arresting components in the intermediate inventory is not very important as long as the safety factor used at release point 2 is about equal to the safety factor used at release point 1. Essential is that the number of components arrested before point 2 is sufficient to form a reasonable lot to release. In the case of fixed lots this implies

\[ q_1 = n q_2, \quad n \text{ integer}. \]

In case of more flexibility with respect to the lot sizes the condition is much less severe.

It has to be mentioned here that in case of large lots in stage 2 the requirement of components becomes very irregular. That suggests that using a safety time norm with respect to stage 1 would work better than a safety stock norm. However, it can be shown that the slack generated by using a safety stock norm works at the component level also as safety time (see Timmer et al. [14] for an example).

3. Stocks in MRP-I software

In this section we discuss the possibility to generate and use stocks via MRP-I software. As in the previous section the MPS is considered to be identical to the demand forecast. In the next section we consider other types of MPS and the relationship of MPS and stocks. Section 4 will be more in the MRP-II spirit (MRP-II = Manufacturing Resources Planning).

There are three possibilities to generate stocks (interstage slack):

- safety stock norms,
- safety time norms,
- hedging.

These possibilities will be considered in the next three subsections. Special attention is given to the interference of rescheduling and the use of stocks.

3.1. Safety stock norms

The first possibility is to introduce safety stock norms per level. In the computation of the time-phased available balance of an arbitrary item, the very first step is to deduct the safety stock norm from the on-hand stock (see Orlicky [11]).
The situation where the on-hand stock is less than the safety stock norm is treated like a physical shortage: it does create an immediate rescheduling message, a rush-order to fill the gap. That means that within MRP-software packages safety stock is implemented as 'dead stock'. The MRP system does not use safety stocks, but rather tries to prevent safety stocks from being used. Only if rescheduling is not successful, the safety stock is used. That means that the effectiveness of safety stocks for absorbing uncertainties and fluctuations depends also on the length of the review period. The shorter the review period the more rescheduling opportunities to prevent the stock from being used. In case of a throughput time equal to the review period there are no rescheduling opportunities.

For dependent demand items, preventing the employment of safety stocks is in concordance with the MRP philosophy outlined in Section 1. For independent demand items, for MPS-items and for components supplied from outside, it is not in concordance with the MRP philosophy.

Of course the MRP software gives only release and reschedule suggestions, but it may be difficult for the system user to diagnose whether a certain rescheduling suggestion is only generated by a stock being less than the safety stock norm or by a real shortage. This is especially so in case of multi-stage structures.

3.2. Safety quantity and safety time

In the previous subsection, we discussed the implementation of safety stocks within MRP systems by means of a safety stock norm per item; such a norm results in maintaining a certain quantity of safety stock at each point in time. A second way of implementing safety stocks is by the usage of safety time, i.e. by adding some extra amount of time in the MRP-I offsetting procedure; employment of safety time results in creating time-varying safety stocks, as shown in Figure 5.

If this principle is to be implemented in software, a clear distinction should be made between the need-date of a certain amount of material, and the due-date of the corresponding scheduled receipt. The suppliers' delivery performance should be measured against the due-date. If the distinction between need-date and due-date is not supported adequately by the system, standard MRP-I logic will delay all scheduled receipts up to the need-date and the results will not be visible in the form of a time-varying safety stock, but in the form of an unplanned increase in work-in-process. On the other hand if need-date and due date are distinguished safety time norms will, just as safety stock norms, easily lead to 'dead stock' since rescheduling leads to not using the slack. It is difficult for the user of the system to decide whether or not it is necessary to react on exception messages, especially in multi-stage cases.

An obvious advantage of employing safety time norms as compared to safety stock norms is the following. Safety time norms do not generate any safety stock unless there exists a real demand in the near future; safety stock norms generate safety stock regardless of the existence of a real demand. An obvious disadvantage of employing safety time is caused by the requirement to extend the planning horizon at the MPS-level (at least if the item concerned is somewhere in the 'critical path' of an MPS-item, cf. New [10]).

Several writers investigate the question of

\[
\text{stock level} \\
\text{safety time} \\
\text{due-date} \\
\text{need-date} \\
\text{safety time} \\
\text{due-date} \\
\text{need-date} \\
\text{time}
\]

Figure 5. Safety stock pattern generated by adding safety time in offsetting. Infrequent ordering.
whether safety quantity is to be preferred to safety time. Whybark and Williams [15] relate this question to the source of uncertainty. They distinguish between timing uncertainty and quantity uncertainty. Timing uncertainty may occur both with respect to the demand pattern and the supply pattern of an item. As far as the demand pattern is concerned, timing uncertainty occurs when it is certain that a demand will occur, but when the exact period shows a random deviation from the expected value. As far as the supply pattern is concerned, timing uncertainty occurs similarly when it is certain that a scheduled receipt will arrive, but when the exact period shows a random deviation from the expected value. An analogous definition can be given for quantity uncertainty. The supply pattern may show some random scrap, the demand pattern may have a stochastic nature. Based on a number of simulation experiments for one item, Whybark and Williams [15] conclude that timing uncertainty should be met by safety time, whereas quantity uncertainty should be met by safety quantity.

Although these conclusions provide better understanding of the problem at hand, they do not provide an adequate answer to all problems faced in practice. This is due to the fact, that timing uncertainty and quantity uncertainty can be defined precisely in simulation experiments, but not in real life situations. For example, consider demand quantity uncertainty. In Table 4 a parent item is shown with average independent demand of 10 per period, with some small variance. Depending upon independent demand actual realization the planned order of period 4 will shift to period 3. Thus, for the component of this item, timing uncertainty is created. However, the magnitude of this timing uncertainty is revealed gradually over the three-period offsetting time. From the position of the component item, measuring the timing uncertainty becomes a highly complex job, because many scheduled receipts of the parent item face continuously changing due dates. It remains doubtful, whether safety time in offsetting the parent planned order is to be preferred to safety quantity at the component level. Recall that in the previous section it has been shown that in case of lot sizing the safety quantity at the component level has the same effect as safety time.

3.3. Hedging

A third possibility to create safety stocks within MRP systems consists of deliberately overplanning the master schedule (cf. Wight/Landvater, appendix D [16], New [10]).

In order to illustrate the technique, consider an assembly A, that is built in one period from a subassembly B; in turn, B is manufactured in one period from component C, which is fabricated in one period from raw material D (with, again 1 period leadtime). If all safety stock for demand fluctuations of A is stocked at the MPS level (i.e. item A) this safety stock has to cover the maximum reasonable demand over 5 periods (leadtime +1 (review period)). If the expected demand is 50 units per period and if the maximum reasonable demand per period is 70 units, the safety stock (at MPS level) should be set at $5 \times 20 = 100$ units.

The hedging technique employs a gross requirement pattern as shown in Table 5. If actual demand for a number of periods (at least 5) is equal to 50 the resulting inventory situation is depicted in Figure 6. The stock at level A is higher than the stocks at the other levels because of the review period. If actual demand increases suddenly to 70, all stocked items will be moved forward, creating after 5 periods the situation depicted in Figure 7.

If, instead, the demand in the first period is 20 the inventory becomes 70 and the scheduled receipt of 50 for the next period will be ‘rescheduled

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Quantity uncertainty of parent results in timing uncertainty of component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>1</td>
</tr>
<tr>
<td>Dependent demand</td>
<td>50</td>
</tr>
<tr>
<td>Independent demand</td>
<td>10</td>
</tr>
<tr>
<td>Scheduled receipt</td>
<td>100</td>
</tr>
<tr>
<td>Available balance</td>
<td>60</td>
</tr>
<tr>
<td>Net requirements</td>
<td>70</td>
</tr>
<tr>
<td>Planned orders (due)</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Overplanning for item A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>1</td>
</tr>
<tr>
<td>Demand</td>
<td>50</td>
</tr>
<tr>
<td>Overplanning</td>
<td>20</td>
</tr>
<tr>
<td>Gross requirement</td>
<td>70</td>
</tr>
</tbody>
</table>
out’. For further analytical work, see Miller [9], who analyzes the situation where the demand per period is identically independent normally distributed.

Compared to using safety stock norms, demand hedging has the advantage that it is possible to choose the hedges so that the overplanning is used indeed and not enfeebled by rescheduling to ‘dead stock’. This is considered in more detail in subsection 3.4.

3.4. Static hedging

In this subsection we consider in more detail one specific way of hedging, namely static hedging. In case of static hedging the hedges (amount of overplanning) only depend on the horizon. We give special attention to the interference of static hedging and rescheduling. Demand uncertainty is assumed to be the only uncertainty.

3.4.1. One stage

In case of hedging the planned order release in period $t$, $P(t)$, is equal to (see Section 2)

$$P(t) = \sum_{i=0}^{l} \hat{D}(t, t+i) - C(t)$$

with

$$\hat{D}(t, t+i) = D(t, t+i) + h(t,t+i).$$

In these expressions $D(t, t+i)$ is the demand forecast and $h(t,t+i)$ the amount of overplanning of $D(t,t+i)$. The total amount of overplanning

$$\sum_{i=0}^{l} h(t,t+i)$$

has to cover all forecast errors over the leadtime plus review period.

Suppose the forecast error only depends on the horizon and let $\sigma^2(i)$ be the variance of the error of the forecast with horizon $i$ (see Section 2), then, one may choose $h(t, t+i) = \hat{h}(i)$ (thus independent of $t$) and one has to determine the $\hat{h}(i)$ such that

$$\sum_{i=0}^{l} \hat{h}(i) = k \sqrt{\sum_{i=0}^{l} \sigma^2(i)}.$$

The case with $h(t, t+i)$ independent of $t$ is called static hedging. Here we deviate from the terminology of Miller [9] who calls the way of hedging where $h(t, t+i)$ is independent of $t+i$ static.

One of the advantages of hedging is that it is possible to change $\hat{h}(i)$ without changing the planned order release, as long as the sum $\sum_{i=0}^{l} h(i)$ is kept constant. This can be used to reduce the number of rescheduling-in and rescheduling-out messages. Extreme are the cases

$$h(0) = k \sqrt{\sum_{i=0}^{l} \sigma^2(i)}, \quad h(j) = 0 \text{ for } j > 0,$$

and

$$h(0) = 0 \text{ for } j = 0, \ldots, l-1,$$

$$h(l) = k \sqrt{\sum_{i=1}^{l} \sigma^2(i)}.$$

The first case is equivalent to introducing a safety-stock norm equal to

$$k \sqrt{\sum_{i=0}^{l} \sigma^2(i)}.$$

In this case one may expect many rescheduling messages.
Table 6
Static hedging and rescheduling, \( h(1) = 20 \)

<table>
<thead>
<tr>
<th>Period</th>
<th>Gross requirement</th>
<th>Scheduled receipts</th>
<th>Available balance</th>
<th>Net requirement</th>
<th>Planned release</th>
</tr>
</thead>
<tbody>
<tr>
<td>First period</td>
<td>70 50 50 50 50</td>
<td>50 50</td>
<td>20 0 0 -50 -100 -150</td>
<td>50 50 50 50 50</td>
<td>50 50 50 50 50</td>
</tr>
<tr>
<td>Second period, demand in period 1 was 51</td>
<td>70 50 50 50</td>
<td>50 50</td>
<td>19 -1 -1 -51 -101</td>
<td>1 50 50</td>
<td>51 50</td>
</tr>
</tbody>
</table>

The last way of hedging follows from interpreting the one-stage case with leadtime \( l \) as an \( l \)-stage case, all stages having leadtime 1 (see the next subsection).

3.4.2. More complex structures

Of the linear cases we consider only the two-stage case (see Figure 2). The multi-stage linear case can be treated similarly. The planned order releases in period \( t \) at points 1 and 2 are equal to (see Section 2)

\[
P_1(t) = \sum_{i=0}^{l_1 + l_2} \hat{D}(t, t + i) - (C_1(t) + C_2(t)),
\]

\[
P_2(t) = \sum_{i=0}^{l_2} \hat{D}(t, t + i) - C_2(t),
\]

with

\[
\hat{D}(t, t + i) = D(t, t + i) + h(t, t + i)
\]

and \( D(t, t + i) \) and \( h(t, t + i) \) as in the previous subsection. As in the one-stage case we assume \( h(t, t + i) \) being independent of \( t \), so \( h(i) \).

If all uncertainties have to be absorbed by hedging one may choose the hedges so that

\[
\sum_{i=0}^{l_2} h(i) = k \sqrt{\sum_{i=0}^{l_2} \sigma^2(i)},
\]

\[
\sum_{i=0}^{l_1 + l_2} h(i) = k \sqrt{\sum_{i=0}^{l_1 + l_2} \sigma^2(i)}.
\]

The way the total amount of overplanning is distributed influences again the number of reschedul-
ing messages. Relation (2) is an extra restriction on this distribution, which decreases the possibility to choose the hedges so that only few rescheduling messages are generated.

The divergent case is not considered in this paper (see also Section 2). It has to be mentioned however that once the total required safety stocks are determined, the problem to realize these stocks and to make them work effectively by static hedging is analogous to the corresponding problem in the linear case.

In the convergent case hedging is less flexible. Compare the case considered in Section 2.3. Suppose one wants to realize and use safety stocks as suggested there. This can be done by a pattern of hedging satisfying

\[ \sum_{i=0}^{l_1} h(i) = k \sqrt{\sum_{i=0}^{l_1} \sigma^2(i)}, \]

\[ \sum_{i=0}^{l_1+1} h(i) = k \sqrt{\sum_{i=0}^{l_1+1} \sigma^2(i)}, \]

\[ \sum_{i=0}^{l_2} h(i) = k \sqrt{\sum_{i=0}^{l_2} \sigma^2(i)}. \]

This shows that in case of convergent networks the distribution of the hedges is heavier restricted than in the linear case. In case of many components the most straightforward way of hedging is as follows:

\[ h(j) = k \left\{ \sqrt{\sum_{i=0}^{j-1} \sigma^2(i)} - \sqrt{\sum_{i=0}^{j-1} \sigma^2(i)} \right\}. \]

Such a choice of hedges reduces also the frequency of rescheduling-out messages.

4. The role of the MPS

The MPS is essential in the MRP-II approach. It is necessary to distinguish two aspects with respect to the MPS.

- The MPS is the result of coordination of sales and manufacturing
- The MPS is the basis for coordinating the various production stages.

This latter coordination is executed by the Material Coordination function (see Bertrand–Wijngaard [3]).

These two aspects of the MPS are also expressed by “The MPS represents a statement of production and not a forecast of market demand” (see Berry–Vollmann–Whybark [2, page 6]). In the previous two sections we have assumed that the MPS is identical to demand forecast. In this section we investigate how the results of the previous sections fit in the complete MRP-II approach (where the MPS is not identical to the demand forecast).

It is important to determine to what extent demand variations have to be dealt with by the MPS and to what extent by Material Coordination. We consider two possibilities.

The first possibility is that demand variations have to be dealt with by the MPS only. This leads to two types of MPS: the MPS that is frozen during the cumulative leadtime and the MPS that overplans the demand. These two types of MPS are considered in detail in Sections 4.1 and 4.2.

The second possibility is that part of the demand variations, especially the short term variations, are transmitted via the MPS to Material Coordination. This leads to an MPS which is rather similar to demand forecast. This type of MPS is considered in Section 4.3.

4.1. Frozen MPS

In this section we assume that demand variations have to be dealt with by the MPS function and that the MPS is frozen during the cumulative leadtime. That means that demand uncertainty over the leadtime has to be met by stocks at or beyond the MPS level. In our opinion, this type of MPS is in fact presupposed by the advocates of the MRP philosophy. With such an MPS, it can indeed be required that the MPS is ‘realistic’ (Plossl and Welch [13, ch. 6]).

Material coordination is directed to realizing the MPS. Rescheduling may be necessary to deal with supply disturbances (production and purchase delays). It may also be useful to apply stocks at the Material Coordination level, for instance safety time to tackle purchase disturbances or safety quantity to tackle yield uncertainty. But then there are the same problems with respect to the interference of rescheduling and the use of inter-stage slack as described in Section 3. Also, if stocks are used by both the MPS function and the Material Coordination function it makes sense to control these stocks integratively especially if stocks are implemented at both levels as safety quantity. The
most straightforward option therefore is to combine a frozen MPS with using no slack at the Material Coordination level (or only safety time).

The computation of safety stock norms at the MPS level can be based on classical inventory control theory (fixed leadtime, known demand distribution, etc.).

4.2. MPS as overplanned demand

It is not in all situations attractive to have the stocks necessary to deal with demand uncertainty at the MPS level only. This is especially far from optimal in cases with a high commonality (so a partly divergent structure).

It is possible to have these stocks at lower levels without transferring the control of these stocks to Material Coordination. But that requires that the MPS overplans the demand. The MPS is not frozen in that case of course, because overplanning combined with a frozen MPS would lead to cumulation of stocks.

Overplanning the demand at MPS level is a form of hedging. In Section 3 we considered a way of hedging where the hedges were put on the requirement following from the MPS, so within control of the Material Coordination function. Here the hedges are within control of the MPS function. The effect of the hedges on planned order releases and rescheduling messages, however, is the same and hedges can be determined in the same way therefore. As in the case of a frozen MPS, stocks at the lower levels (not generated by the MPS) do not fit well in this approach. Material Coordination has to rely on rescheduling (and maybe safety time).

In case of a frozen MPS the objective for Material Coordination is to realize the MPS. In this case where the MPS overplans the demand the objective for Material Coordination is in fact to maintain a certain inventory state or delivery potency.

A well-known form of overplanning is the so-called option overplanning. Suppose it is possible to produce two types of a final product, type A and type B. Let the total demand be rather predictable, but the distribution over the two types unpredictable. In that case it may be useful to structure the bill of material as in Figure 8 and to formulate the MPS at the lower level. To tackle distribution uncertainty it is possible to overplan the options.

Suppose the average demand of option A is 50 per period and the maximum demand is 70. Then the MPS for option A should be 70. Let the total cumulative leadtime until the MPS stage be \( l \). Then this option overplanning generates an average cumulative stock of \((l + 1)\frac{20}{20}\) which is distributed evenly over the total leadtime (see also Section 3.3 and the example mentioned there). This stock is necessary to maintain the potency to satisfy the maximum demand of 70 per period.

4.3. Demand variations partly transmitted to Material Coordination

A good cybernetic principle is to try not to have information processing delays. That implies that changes in the demand should be transmitted as soon as possible to Material Coordination. In that respect it can be severely sub-optimal to deal with demand variations via the MPS only. On the other hand it makes no sense to generate unrealistic requirements, because what to produce in such a situation may not only depend on the priorities of Material Coordination, but sales information is also important. It is indeed important that the MPS is realistic.

A reasonable possibility is to use the MPS function to adjust resource requirement and resource availability in an aggregate way. Changes in the demand not affecting this aggregate equilibrium can be transmitted immediately to Material Coordination. In this case the MPS may be almost equivalent to demand forecast, especially if all critical resources are shared by most final products. The results of the previous two sections (Sections 2 and 3) apply in this case.

5. Conclusions

The three most important levels of control in the MRP-II approach are Master Production
Scheduling, Material Coordination (or MRP-I) and Shop Floor Control. The MRP-II approach stresses the necessity to reduce the uncertainty. Nevertheless some uncertainty will be left in general. Slack in the form of inventories (interstage slack) and work in process (intraslack) may be an adequate mean to deal with uncertainty. In the standard MRP-II approach the use of slack is concentrated at the MPS level (final inventories) and at the SFC level (work in process). This work in process represents intrastage slack (queues). The possibility of rescheduling is implicitly based on the existence of intrastage slack. Rescheduling is supported well be standard MRP-I systems. These systems, however, do not provide many facilities for creating and using slack at the Material Coordination level (interstage slack) in the form of inventories.

These aspects of MRP have been analyzed here. It has also been shown how slack at the Material Coordination level can contribute to the overall performance of production just as well.

We did not derive general rules to distribute slack over the three levels of control. In our opinion such a distribution has to depend on flexibility and uncertainty with respect to manufacturing, purchasing and sales. That makes it hardly possible to derive such general rules. Flexibility and uncertainty vary from situation to situation and it is not even possible in general to formalize all types of flexibility.

However, deciding about this distribution of slack in a specific situation requires insight in the effectivity of slack at the Material Coordination level. That is the main topic of this paper. Effective usage of interstage slack depends on the available software. The now available software packages to support Material Coordination (i.e. current MRP-I software modules) are shown to be rather poor in this respect.

References