Some comments on the transfer function of the cutting process
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SOME COMMENTS ON THE TRANSFER FUNCTION OF THE CUTTING PROCESS

H.J.J. KALS

Eindhoven University of Technology

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Presented to the C.I.R.P., Technical Committee Ma
SOME COMMENTS ON THE TRANSFER FUNCTION OF THE CUTTING PROCESS

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>b</td>
<td>Width of cut</td>
<td>m</td>
</tr>
<tr>
<td>b_g</td>
<td>Limit value of width of cut</td>
<td>m</td>
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<tr>
<td>F_{li}</td>
<td>Amplitude of the force component caused by the inner modulation of the chip</td>
<td>N</td>
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<tr>
<td>F_{lo}</td>
<td>Amplitude of the force component caused by the outer modulation of the chip</td>
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<tr>
<td>ΔF_j</td>
<td>Component of the resulting dynamic cutting force corresponding with the direction j</td>
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</tr>
<tr>
<td>ΔF_f</td>
<td>Dynamic component of the feed force</td>
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<tr>
<td>ΔF_v</td>
<td>Dynamic component of the main cutting force</td>
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<td>h_o</td>
<td>Nominal undeformed chip thickness</td>
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<tr>
<td>Δh_o</td>
<td>Amplitude of chip thickness modulation</td>
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<tr>
<td>l_1, l_o</td>
<td>Quadrature components of the dynamic stiffness per unit of width of cut caused by the inner chip modulation and the outer chip modulation respectively</td>
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<td>k</td>
<td>Stiffness of the machine tool structure</td>
<td>N/m</td>
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<tr>
<td>k_i</td>
<td>Specific process stiffness</td>
<td>N/m²</td>
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<tr>
<td>k_{1f}</td>
<td>Chip thickness coefficient of the feed force</td>
<td>N/m</td>
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</table>
\(k_{lv}\)  \(\text{Chip thickness coefficient of the main cutting force}\)  \(\text{N/m}\)

\(N_{2j}\)  \(\text{Inner modulation coefficient of the component of the specific dynamic cutting force in the direction } j\)  \(\text{Ns/m}^2\)

\(R_i, R_o\)  \(\text{In-phase component of the dynamic stiffness per unit of width of cut caused by the inner chip modulation and the outer chip modulation respectively}\)  \(\text{N/m}^2\)

\(R_{ij}\)  \(\text{Coefficient of the component of the specific dynamic cutting force in the direction } j\)  \(\text{N/m}^2\)

\(s\)  \(\text{Feed}\)  \(\text{mm/rev}\)

\(v\)  \(\text{Cutting speed}\)  \(\text{m/s}\)

\(Y\)  \(\text{Amplitude of the inner chip thickness modulation}\)  \(\text{m}\)

\(Y^*\)  \(\text{Amplitude of the outer chip thickness modulation}\)  \(\text{m}\)

\(\delta\)  \(\text{Phase shift (see eq. (6.1.))}\)  \(\text{rad}\)

\(\xi\)  \(\text{Phase shift (see eq. (6.2.))}\)  \(\text{rad}\)

\(\phi\)  \(\text{Phase shift between the inner and outer chip thickness modulations}\)  \(\text{rad}\)

\(\phi_m\)  \(\text{Mean shear angle}\)  \(\text{o}\)

\(\zeta\)  \(\text{Damping ratio of the machine tool structure}\)  \(\text{-}\)

\(\omega\)  \(\text{Angular frequency}\)  \(\text{rad/s}\)
1. Introduction

Over the last number of years there are investigators who advocate a more detailed approach of the dynamic cutting process. The introduction to this is found in the work of Das and Tobias (1). Starting from a shear plane model of the cutting process, these investigators present a pure geometrical analysis of the wave-on wave-cutting process that occurs when the tool vibrates. They consider separately the influence of the inner and the outer modulation of the undeformed chip and derive the relations on the dynamic cutting forces on the basis of static parameters only. From this it follows that a phase shift is introduced by both the dynamic force component of the inner modulation and the force component of the outer modulation.

In this way of thinking, Polacek (2) developed a method to measure the various dynamic components of the cutting force applying a dynamometer.

Van Brussel and Vanherck (3), (4) carried out experiments on the same subject. They propose a method yielding the dynamic stiffness of the cutting process. As a matter of fact, this method is basically analogous to the one described in a previous paper by the author (5). The observations of Van Brussel and Vanherck confirm the theoretical results of Das and Tobias concerning the inner and outer modulation forces behaving independently of each other.

2. Discussion of the results

With respect to the different edges of the modulated chip, Van Brussel applies the following force equations:

\[ F_{li} e^{i(\omega t + \delta)} = Ye^{i\omega t} (R_i + iI_i) b \]
\[ F_{lo} e^{i(\omega t - \phi + \varepsilon)} = Ye^{i(\omega t - \phi)} (R_o + iI_o) b \]
where

- index $i$ refers to the direct chip modulation
- index $o$ refers to the delayed chip modulation
- $R$ is the in-phase component of the dynamic cutting stiffness
- $I$ is the quadrature component of the dynamic cutting stiffness
- $\phi$ is the phase shift between the two chip thickness modulations
- $\epsilon$ and $\lambda$ are the phase shifts of the dynamic cutting force components with respect to the direct and the delayed chip thickness modulation respectively.

The results of the various parameters obtained by Van Brussel (3) are shown in Table 1. The data show different values of $R_i$ and $R_o$. However, from a physical point of view, it may be expected that for any cutting condition the values of $R_i$ and $R_o$ are equal. The outer modulation will only change the resulting depth of cut. This has been proved by Van Brussel and Vanherck to have no influence on the overall dynamic cutting force (4). Thus, one may conclude that the values of $k_i$ obtained by the present author can stand for $R_i$ as well as for $R_o$.

In this way of thinking, differences between the inner modulation stiffness and the outer modulation stiffness can exist only on account of the respective quadrature components $I_i$ and $I_o$.

According to the theory of Das, Van Brussel explains the existence of the leading quadrature component $I_o$ with the aid of a shear plane model (3). Das derived the next equation of the phase shift $\epsilon$ between the outer modulation and the inner modulation force component:

$$
\epsilon = \frac{\omega}{v} h_0 \cot \phi_m
$$

where $\phi_m$ represents the average value of the shear angle.
Eq. III shows that for small values of the chip thickness and for high values of the cutting speed the influence of $c$ can be ignored.

The stability criterion as applied by Van Brussel and Vanherck yields the limit value $b$ when the dynamic stiffness of the machine tool equals the negative value of the resulting stiffness of the cutting process.

Fig. 1 shows the graphical solution method based on the stability criterion mentioned. A straight line approximates the dynamic stiffness of a low-damped machine tool. The intersection of this line with the out-of-phase axis of the polar diagram represents the dynamic stiffness $2\zeta k$ at natural frequency.

The inclination at the point of intersection approximates $2\zeta$ rad. The components $R_i$ and $I_i$ are plotted with reversed sign, whilst a circle, having $0'$ as centre and $\sqrt{(R_o^2 + I_o^2)}$ as the radius, gives all the loci of the stiffness of the cutting process per unit of width of cut. In order to obtain the limit value $b_g$, straight lines are drawn passing through the origin $0$, intersecting the circle and the machine tool stiffness locus. The minimum of the ratios $AO/BO$, $A'O/B'O$, etc. brings in $b_g$. The different lines correspond to different relative phase shifts between the inner and the outer modulation as indicated by the angle $\phi$.

With the aid of this method, the influence of the quadrature components $I_o$ and $I_1$ on the threshold of stability has been calculated. Fig. 2 shows the influence of the ratios $I_o/R_o$, $I_1/R_1$ on the limit value for the generalized situation that $R_o = R_1$, $R_o/\zeta k$ being constant, and $\zeta = 0.025$. From the figure it can be concluded that the influence of $I_1$ on $b_g$ is considerably stronger than the influence of $I_o$ on $b_g$. It should be noticed that in general Van Brussel's results lead to about the same values of $I_o/R_o$ and $I_1/R_1$.

With respect to the parameters of the outer modulation, another important remark can be made. Contrary to what may be expected from the
foregoing, the results of Table 1 show values of $R_o$ which are considerably smaller than those of $R_i$. However, it is remarkable that for all the different cutting speeds applied, the values of $R_i$ and $\sqrt{(R_o^2 + I_o^2)}$ are approximately the same. This is diagrammatically shown in Fig. 3. With the aid of the graphical solution method of Fig. 1 it can be seen that a substitution of $R_o$ and $I_o$ by the real component $\sqrt{(R_o^2 + I_o^2)}$ does not affect the limit value. The introduction of $I_o$ will only increase the phase shift between $R_o$ and $R_i$.

Van Brussel and Vanherck (4) computed stability charts for their special tool holder. Comparing the theoretical and the experimental values of $\phi$, it follows that the discrepancy between the values of both series of results generally is of the same magnitude as the influence of $I_o$ on $\phi$ (5%).

The facts mentioned do not support the relevancy of $I_o$, the more so as the present author's results, which have been obtained excluding the influence of a quadrature outer modulation component, show a very good agreement between various series of practical and theoretical results. With respect to this, it is mentioned that in some cases the $c$-values can increase up to 0.7 rad.

Table 2 shows the results obtained by Polacek. The equation of the dynamic cutting force derived by the latter can be written as

$$\Delta F_j = -b \left[ (R_{1j} + i R_{2j} + i \omega N_{2j}) Y - (R_{3j} + i R_{4j}) Y^w \right]$$

(IV)

Polacek shows this equation to be similar to Van Brussel's equation with the only difference that the parameters in eq. IV relate to a particular direction $j$, whilst the parameters of the eqs. I and II stand for the resulting dynamic force. The analogy is

$$R_1 \rightarrow R_1 \quad R_3 \rightarrow R_0$$

$$R_2 + N_2 \omega \rightarrow I_1 \quad R_4 \rightarrow I_o$$
A direct comparison of all the results mentioned with the author's findings is not possible, since Polacek used a different work material and Van Brussel did not mention any materials specification.

With respect to Polacek's results it draws attention that positive as well as negative values of $R_4$ are obtained. It is obvious that the negative results do not fit the theory of Das. Moreover, according to Das' theory, the values of $R_4/R_3$ should show an increase with respect to an increasing feed. This, however, is also not confirmed by the results of Table 2.

Resuming, one can conclude that, at least for feed values up to 0.2mm/rev, the physical meaning of $I_o$ in relation to the shear plane theory is very doubtful. At this stage, one can make objections against the assumption made by Das that the orientation of the shear plane will remain unaffected by the vibration. Physical considerations suggest that the direction in which the shearing zone propagates will be controlled by the stress conditions close to the tip of the tool. Thus, the variation of the cutting force will be strongly affected by a dynamically changing shearing process.

In reference to the inner modulation damping, Polacek's results as well as the present author's results (6) show that in the directions of both the feed and the cutting speed, the damping can be positive and negative as well. Das' results only permit a negative damping with respect to the dynamic component of the force in the direction of the main cutting force, and a positive damping related to the component in the direction of the feed force, according to

\begin{align*}
\Delta F_v &= k_{1v} \Delta h_o \sin \omega t - k_{1f} h_o \frac{\omega}{v} \Delta h_o \cos \omega t \\
\Delta F_f &= k_{1f} \Delta h_o \sin \omega t + k_{1v} h_o \frac{\omega}{v} \Delta h_o \cos \omega t
\end{align*}

where $k_{1v}$ is the chip thickness coefficient of the main cutting force

$k_{1f}$ is the chip thickness coefficient of the feed force

$\Delta h_o$ is the amplitude of chip thickness modulation
Finally, it should be mentioned that the assumption, that the component of a vibration in the direction of the main cutting force has no influence on the dynamic force (5) (6), is confirmed up to a great extent by experiments carried out by Polacek (2).

References


(6) Kals, H.J.J., Fertigung 5 (1971) 165
SUBSCRIPTION OF THE FIGURES

Fig. 1 The graphical solution method for the limit width of cut by Van Brussel and Vanherck

Fig. 2 The influence of the ratios $I_o/R_o$ and $I_i/R_i$ on the limit value $b_g$ for a generalized situation.

Fig. 3 The agreement between the resulting dynamic stiffness of the outer modulation $\sqrt{(R_o^2 + I_o^2)}$ and $R_i$, after Van Brussel's results.

Table 1 Results after Van Brussel and Vanherck

Table 2 Results after Polacek

Table 3 The author's results
Fig. 1.

Dynamic stiffness of the cutting process

Dynamic stiffness of the machine tool

Out-of-phase axis

In-phase axis

Approximately 2ζ

2ζk
\[
R_o = R_i \\
\frac{R_o}{k} = \text{const.} \\
\zeta = 0.025 \\
b_g = \frac{b_g}{[b_g]}_{I_i=I_o=0} \\
b_g' = \left[ \phi \left( \frac{I_i}{R_i} \right) \right]_{I_o=0} \\
b_g' = \left[ \phi \left( \frac{I_o}{R_o} \right) \right]_{I_i=0} \\
b_g' = \left[ \phi \left( \frac{I_o}{R_o} \right) \right]_{I_i=0.25} \\
\]

Fig. 2.
Fig. 3.
### Table 1

$s = 0.22 \text{ m/s}$

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<th>$R_1$ (10^9 N/m²)</th>
<th>$R_o$ (10^9 N/m²)</th>
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### Table 2

$s = 0.05 \text{ m/s}$

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Table 3

SKF 1550