A lower upper-bound solution of cold strip-rolling

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A lower upper-bound solution of cold strip-rolling.
E. Mot

Summary
In the plastic zone of a strip, a kinematically admissible velocity field is formulated as a 4th-degree polynomial $v_x(x, y)$ and $v_y(x, y)$. All coefficients except one are found from the boundary conditions and the continuity requirement. Then the last coefficient is determined by a generalised version of the lower upper-bound theorem as formulated by Prager and Hodge [1].
This could be accomplished by introducing the formulation of the external load as calculated by Bland and Ford [2]. Throughout the analysis, the material was considered to be strain-hardening according to $\dot{\sigma} = c_0^{\kappa}$. It was found that in this way the extremum could be determined very accurately. Next, the velocity field was integrated along streamlines in such a way that the deformation of an initially straight line could be determined. The form of this line was compared with experimental results found by Tarnovskii, Pozdereyev and Lyashkov [3], and the results matched reasonably well.

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List of symbols.

a \quad \text{coefficients for } v_x
b \quad \text{coefficients for } v_y
c \quad \text{effective stress if } \bar{\sigma} = 1
h \quad \text{local thickness of strip}
k \quad \text{maximum shear stress}
m \quad \text{strain hardening exponent}
p \quad \text{normal pressure on the roll}
t \quad \text{time}
v \quad \text{velocity}

x,y,z \quad \text{cartesian coordinates (also suffixes)}

R \quad \text{radius of roll}

(\delta) \quad \bar{\delta} \quad \text{logarithmic strains}

(\varepsilon) \quad \varepsilon_{ij} \quad \text{quadratic strain rate tensor}

(\mu) \quad \mu \quad \text{coefficient of Coulomb friction}

(\sigma) \quad \sigma \quad \text{normal stresses}

(\tau) \quad \tau \quad \text{shear stress}

(\phi) \quad \phi \quad \text{polar coordinate}

(\omega) \quad \omega R \quad \text{surface velocity of rolls}

suffixes.

0 \quad \text{denotes} \quad "\text{entry}"
1 \quad \text{denotes} \quad "\text{exit}"
n \quad \text{denotes} \quad "\text{neutral point}"
s \quad \text{denotes} \quad "\text{streamline}"
- \quad \text{denotes} \quad "\text{between } 0 \text{ and } n"
+ \quad \text{denotes} \quad "\text{between } n \text{ and } 1"
\star \quad \text{denotes} \quad "\text{kinematically admissible}"
Basic assumptions.
(1) The broadening of the strip is negligible with respect to the other deformations.
(2) The rolls are considered to be rigid bodies (For the cases presented, the error due to roll deformation was calculated and appeared to be of the order of magnitude of 0.1 %).
(3) The area ABCD in Fig. 1 becomes fully plastic, all other material remains rigidly elastic.
(4) The friction between rolls and strip can sufficiently accurately be represented by a constant coefficient of friction $\mu$.
(5) Inertia forces can be neglected
(6) $\varphi_0 \approx \sin \varphi_0 \approx \tan \varphi_0$
(7) The flow condition can be described by Von Mises' law
$$\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \sigma_y - \sigma_y \sigma_z - \sigma_z \sigma_x$$
if $x, y$ and $z$ are principal directions.
(8) Strain hardening is given by
$$\bar{\sigma} = \frac{\sigma}{\bar{\sigma}_0}$$
in which
$$\bar{\sigma} = \frac{2}{3} (\delta_x^2 + \delta_y^2 + \delta_z^2),$$
where
$$\delta_x = -\ln \frac{h_0}{h_x}; \quad \delta_y = +\ln \frac{h_0}{h_y}; \quad \delta_z = 0$$
Bland and Ford [2] make use of three additional assumptions which we will not maintain but only mention since the results of their calculations are used as boundary conditions:
(9) $\tau_{xy} = 0$
(10) $\sigma_y$ is independent of the $X$-coordinate.
(11) Plane sections remain plane during the entire process.
Finally, we assume that
(12) the loads round the plastic zone as calculated by Bland and Ford are sufficiently accurate to be used as boundary conditions for a two-dimensional analysis of the process, in which assumptions 9 to 11 are no longer maintained.
Fig. 1. The velocity field $\mathbf{v} = \mathbf{v}(x,y)$ in the strip-rolling process.

The velocity field.

We represent the velocities by

$$v_x = a_{00} + a_{10}x + a_{11}y + a_{20}x^2 + a_{21}xy + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 + a_{40}x^4 + a_{31}x^3y + a_{22}x^2y^2 + a_{13}xy^3 + a_{04}y^4$$  \hspace{1cm} (4)

$$v_y = b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{30}x^3 + b_{21}x^2y + b_{12}xy^2 + b_{03}y^3 + b_{40}x^4 + b_{31}x^3y + b_{22}x^2y^2 + b_{13}xy^3 + b_{04}y^4$$  \hspace{1cm} (5)

From symmetry, $v_x$ is an even function of $y$.

Therefore,

$$a_{01} = a_{11} = a_{21} = a_{03} = a_{31} = a_{13} = 0$$  \hspace{1cm} (6)

$v_y$ is an odd function of $y$.

Therefore,

$$b_{00} = b_{10} = b_{20} = b_{02} = b_{30} = b_{12} = b_{40} = b_{22} = b_{04} = 0$$  \hspace{1cm} (7)

Further, $v_x$ does not depend on $y$ if $x = 0$.

Hence,

$$a_{02} = a_{04} = 0,$$  \hspace{1cm} (8)

while $v_y = 0$ if $x = 0$.

Or,

$$b_{01} = b_{03} = 0$$  \hspace{1cm} (9)

With (6), (7), (8) and (9) we find for (4) and (5):

$$v_x = a_{00} + a_{10}x + a_{20}x^2 + a_{30}x^3 + a_{12}xy^2 + a_{40}x^4 + a_{22}x^2y^2$$  \hspace{1cm} (10)

$$v_y = b_{11}xy + b_{21}x^2y + b_{31}x^3y + b_{13}xy^3$$  \hspace{1cm} (11)
Since the material is incompressible, we must satisfy the continuity equation \( v_{1,1} = 0 \) for all \( x \) and \( y \).

Herefrom the following relations can be derived:

\[
\begin{align*}
a_{10} &= 0; \quad b_{11} = -2a_{20}; \quad b_{21} = -3a_{30}; \quad a_{12} = 0; \\
b_{31} &= -4a_{40}; \quad b_{13} = -\frac{2}{3}a_{22} \\
v_x &= a_{00} + a_{20}x^2 + a_{30}x^3 + a_{40}x^4 + a_{22}x^2y^2 \\
v_y &= -2a_{20}xy - 3a_{30}x^2y - 4a_{40}x^3y - \frac{2}{3}a_{22}xy^3
\end{align*}
\]

(12)

(13)

(14)

Now we introduce the first boundary condition, viz. that the velocity of the strip material which makes contact with the rolls, is tangent to the rolls. For reasons of simplicity, we approximate the shape of a roll by a parabola. (This assumption has only little influence on the calculated results, since \( \varphi_0 \) is small).

\[
y = \frac{x^2}{2R} + \frac{h_1}{2},
\]

(15)

\[
\frac{dy}{dx} = \frac{x}{R}
\]

(16)

Hence

\[
v_{y}^{(BC)} = v_{x}^{(BC)} \cdot \frac{x}{R}
\]

(17)

We substitute (15) in (13) and (14) and find

\[
v_{y}^{(BC)} = -(a_{20}h_1 + \frac{a_{22}h_1^3}{12})x - \frac{3a_{30}h_1}{2}x^2 - (\frac{a_{20}}{R} + 2a_{40}h_1 + \frac{a_{22}h_1^2}{4R})x^3 + \ldots
\]

(18)

\[
v_{x}^{(BC)} = \frac{x}{R} + \frac{a_{20}}{R} \cdot x + \left(\frac{a_{20}}{R} + \frac{a_{22}h_1}{4R}\right)x^3 + \ldots
\]

(19)

It appears that (17) cannot be entirely satisfied by substituting (18) and (19). Therefore, we make only the first two terms of the same power of (18) equal those of (19). This way of doing will be justified later.

From this we find

\[
a_{30} = 0
\]

(20)
As a second boundary condition we postulate that the elasto-plastic boundary at the entry is a parabola which cuts the rolls at right angles.

Such a parabola is formulated by

\[ x = \varphi_0 \left( -\frac{y^2}{h_0^2} + R \right), \quad \varphi_0 = \sqrt{\frac{h_0 - h_1}{R}} \]  

(23)

This parabola is considered to be a cylindrical surface on which the velocity is discontinuous in such a way that the velocity shock \( \Delta \vec{v} \) is tangent to it.

Hence,

\[ v_y^{(CD)} = v_x^{(CD)} \cdot \frac{2y\varphi_0}{t_0} \]  

(24)

Using (13), (14), (20), (21) and (23) we find that

\[ v_y^{(CD)} = -(2a_{20}\varphi_0^3 + 4a_{40}\varphi_0^3 R^3) y + O(y^3) \]  

(25)

\[ v_x^{(CD)} \cdot \frac{2y\varphi_0}{t_0} = \left( \frac{2a_{00} \varphi_0^2}{t_0} - \frac{2a_{20} \varphi_0^3 R}{t_0} + \frac{2a_{40} \varphi_0^5 R^3}{t_0} \right) y + O(y^3) \]  

(26)

Using (24) for the first coefficients of (25) and (26) only, we find (with the help of \( \varphi_0 = \sqrt{(h_0 - h_1)/R} \))

\[ a_{40} = -\frac{a_{00} + a_{20} R (2h_0 - h_1)}{R^2 (3h_0 - h_1) (h_0 - h_1)} \]  

(27)

If \( x = x_n \), we have \( v_x^{(BC)} \) \( x = x_n \) = \( \omega R \)

Substitution of this requirement in (22) yields, using (27) and (21),

\[ \omega R + a_{20} x_n^2 \left\{ \frac{(2h_0 - h_1) x_n^2}{R (3h_0 - h_1) (h_0 - h_1)} + \frac{6x_n^2}{h_1 R} + \frac{3x_n^4}{h_1 R^2} \right\} \]

\[ a_{00} = \frac{\frac{3x_n^2}{h_1 R} - \frac{x_n^4}{h_1 R^2} - \frac{6x_n^4}{h_1 R^2} - \frac{3x_n^6}{h_1 R^3}}{1 - \frac{x_n^2}{h_1 R} - \frac{x_n^4}{R (3h_0 - h_1) (h_0 - h_1)} - \frac{6x_n^4}{h_1 R^2} - \frac{3x_n^6}{h_1 R^3}} \]  

(28)
Summarising, we now have

\[ v_x = a_{00} + a_{20} x^2 + a_{40} x^4 + a_{22} x^2 y^2 \]  
\[ v_y = b_{11} x y + b_{31} x^3 y + b_{13} x y^3 \]

in which all coefficients can be explicitly given as functions of \( a_{20} \) with the help of (28), (27), (21) and (12). The value of \( a_{20} \) will be determined by means of the lower upper-bound theorem.

**Application of the lower upper-bound theorem.**

The formulation of this theorem for a strain-hardening material and a discontinuous velocity distribution is as follows [1], [4].

Of all kinematically admissible velocity fields, the actual one minimises the expression

\[ J^* = \sqrt{2} \int_V k \sqrt{\dot{c}_{ij} \dot{c}_{ij}} \, dV - \int_{F_T} T \, dF + \int_{F_D} \tau \Delta v^* \, dF \]  

In which \( \dot{c}_{ij} = \frac{1}{2} (\dot{v}_{i,j}^* + \dot{v}_{j,i}^*) \), \( V \) = volume, \( F_T \) is the area on which the stresses are prescribed, \( T \) are the stresses on \( F_T \); \( F_D \) is the area on which a discontinuity of speed \( \Delta v^* \) exists, \( \tau \) are the shear stresses along \( F_D \).

We rewrite (31) as

\[ J^* = I_1 - I_2 + I_3 \]  

Since \( k = \frac{\sigma}{\sqrt{3}} \), we find, using (2) and (3):

\[ k = \frac{c}{\sqrt{3}} \left( \frac{h}{2} \right)^2 \left( \ln \frac{h_0}{h} \right)^m \]  

in which

\[ h \sim h_1 + x^2/R \]  

Using (29) and (30) we find

\[ \sqrt{\dot{c}_{ij} \dot{c}_{ij}} = [2x^2(2a_{20} + 4a_{40} x^2 + 2a_{22} y^2)^2 + b_{11} + (2a_{22} + 3b_{31}) x^2 + b_{13} y^2]^2 \]  

\[ \frac{1}{2} \int_{F_T} T \, dF + \int_{F_D} \tau \Delta v^* \, dF \]
Now, if we replace the parabola CD by a straight line (in doing so, we make only a small error, since the enclosed area is small) we find

\[ I_1 = \sqrt{2} \int_0^h \left\{ \int_0 h \left[ k \sqrt{\epsilon_{ij} \epsilon_{ij}} dy \right] dx \right\} \]

(36)

As for the calculation of \( I_2 \), we will assume \( \sigma_{x_0} = 0 \). For the ultimate speed of the strip we have \( \dot{v}_1 = a_{00} \). Hence,

\[ I_2 = -\frac{1}{2} \sigma_{x_1} v_1 t_1 \]

(37)

The initial speed of the strip follows from \( \dot{v}_0 = t_1 \dot{v}_1 / t_0 \).

The velocity shock at CD is \( \Delta v = v_0 \sin \gamma - v_0 \gamma, 0 \leq \gamma \leq \phi_0 \).

Assuming a shear stress distribution \( \tau = \gamma k_0 / \phi_0 \) along CD, we find, if \( I_3 = I_{31} + I_{32} \), that

\[ I_{31} = -\int \tau \Delta v F = -\frac{k_0 v_0}{\phi_0} \cdot \frac{t_0}{2 \phi_0} \int_0^\phi \gamma^2 d \gamma = \frac{1}{6} k_0 v_0 t_0 \phi_0 \]

(38)

For \( k_0 \) we arbitrarily choose the maximum shear stress as caused by a plastic deformation of 1 %

\[ k_0 = \frac{c}{\sqrt{3}} \left( \frac{h}{3} \right)^2 0.01 \%
\]

(39)

Since \( v_y \ll v_x \), the velocity of the strip along the rolls is approximately equal to \( v_{(BC)}^x \).

Hence, for the rolls the shock is

\[ |\Delta v| = \left| v_{(BC)}^x - \omega_R \right| \]

(40)

For the shear stress, Bland and Ford found

\[ \tau = \mu \rho_+ \quad 0 \leq \gamma \leq \phi_n \]

(41)

\[ \tau = \mu \rho_- \quad \phi_n \leq \gamma \leq \phi_0 \]

(42)

in which

\[ p_+ = 2k(1 - \frac{\sigma_{x_1}}{2k_0} h_1) e^{\mu H} \]

\[ p_- = 2k(1 - \frac{\sigma_{x_0}}{2k_0} h_0) e^{\mu (H_0 - H)} \]

(43)

(44)
where
\[ H = 2 \sqrt{\frac{R}{h_1}} \arctan \left( \frac{\sqrt{\frac{R}{h_1}} \cdot \varphi}{2} \right) \] (45)

The neutral point can be found from
\[ h_n = H - \frac{1}{2 \mu} \ln \left( \frac{t_0 (1 - \frac{x_1}{2k_1})}{t_1 (1 - \frac{x_0}{2k_0})} \right) \] (46)
\[ \varphi_n = \sqrt{\frac{h_1}{R}} \tan \left( \frac{H}{2} \sqrt{\frac{h_1}{R}} \right) \] (47)
\[ x_n = \varphi_n R \] (48)

Now we can find
\[ I_{32} = \int_{\varphi_0}^{\varphi_n} \mu_+ (\omega R - v_x^{(BC)}) \text{Rd} \varphi + \int_{\varphi_0}^{\varphi_n} \mu_- (v_x^{(BC)} - \omega R) \text{Rd} \varphi \] (49)

With the help of foregoing formulas, the value of \(a_{20}\) for which \(J^*\) becomes minimum can be determined.

An estimation of \(a_{20}\) is found by
\[ v_x = \frac{v_1 h_1}{h} \approx \frac{v_1}{h_1} \approx v_1 - \frac{v_1}{h_1 R} x^2 + \ldots \] (50)
\[ a_{20} \approx -\frac{\omega R}{h_1 R} (a_{20})_0, \text{ since } \omega R < 0 \] (51)

Actually, (50) indicates that the polynomial representation of \(V\) in (4) and (5) is admissible. If (51) held exactly, (21) would yield \(a_{22} = 0\). This means that the present theory implies the one-dimensional "plane sections remain plane"-theory, since \(a_{22}\) is the curvature-parameter.

In Fig. 2 a solution for one specific case is presented.

Fig. 2. A lower upper-bound solution for one specific case.
Integration of the velocity field.

For the velocity field calculated in the former section, we will now calculate streamlines. These are the trajectories of the material elements in the deformed area. It is clear that each streamline $y_s(x)$ must satisfy the equation

$$\frac{dy_s}{dx} = \frac{v_y}{v_x}$$

(52)

Fig. 3. Streamlines $y_s(x)$

With the coefficients $a_{ij}$ and $b_{kl}$ calculated we can solve this differential equation numerically. As a boundary condition, we have

$$y_s(0) = \zeta h/2; \quad 0 \leq \zeta \leq 1$$

(53)

If $\zeta = 0$, the solution is $y_s = 0$. Hence, a material element on the $x$-coordinate moves with a velocity $v_x(x, 0)$ and remains on it.

If $\zeta = 1$, the solution should be given by (15).

Calculations show an error in the order of magnitude of 5%. This is an indication that the inaccuracies in determining the velocity field were reasonably admissible.

An infinitesimal element of streamline has a length

$$ds = \sqrt{1 + \left(\frac{dy_s}{dx}\right)^2} \, dx$$

and is covered in a time

$$dt = \frac{ds}{\sqrt{v_x^2 + v_y^2}} = \frac{dx}{\sqrt{v_x^2 + v_y^2}} \left( = \frac{dy}{v_y} \right)$$

(54)

Now we will calculate the shape of the initially straight line $R'S'$ (Fig. 3). The time, needed to cover the distance $\overline{PQ}$ is the same as the time needed to cover the distance $\overline{HGFE}$.

Or

$$t_0 = t_1 + t_2 + t_3$$

(55)
\[ t_0 = \int_0^R \frac{dx}{v_x(x_0, y_0)} \]  \hspace{1cm} (56)

If the point \( G \), with coordinates \((x_0(\zeta), y_0(\zeta))\) is the point of intersection between a streamline and the entrance parabola, we have

\[ t_1 = \frac{\zeta_0 y_0^2(\zeta)}{v_0 h_0} \]  \hspace{1cm} (57)

\[ t_2 = \int_0^R \frac{dx}{v_x(x, y_0)} \]  \hspace{1cm} (58)

\[ t_3 = -\frac{EF}{v_1} \]  \hspace{1cm} (59)

From formulae (56) to (60), \( \bar{EF} \) can be calculated. If this is done for a range of values of \( \zeta \) between 0 and 1, we find the shape of the deformed line \( R'S' \) which was initially the undeformed straight line \( RS \) (Fig. 3).

In ref. [3], the authors show a number of photographs of deformed grit patterns (pp. 68-70) which were applied both on the side surface and along the centre of the width of the specimen.

Since this model assumes plane strain, only the latter grits were considered significant. The material was aluminium, heated at a temperature of 450\(^\circ\)C. For the cases referred to, no friction coefficients were measured. For similar conditions, a separate table (p. 274), however, mentioned values of \( \mu \) between 0.171 and 0.396. Figs. 4, 5 and 6 give a comparison between the average shape of a grit-line as measured from photographs and the shape as calculated for \( \mu = 0.3 \) and 0.4.

The agreement seems to be reasonably good.

Differences may be caused by the following facts.

(a) The theory of Bland and Ford [2] is an approximate one.
(b) The coefficient of friction is rather an arbitrary quantity. In fact, near the neutral point, the material will probably stick to the roll and plastic friction with constant shear stress will occur.
(c) The assumption of plane strain does not hold exactly.
(d) The elastic recovery has been neglected.

Figs. 4, 5 and 6. Verification of the strain distribution in the rolled strip with the help of photographs of deformed grits.

Conclusion and prospects.
It has been shown that the lower upper-bound theorem can successfully be used for the calculation of strain distributions in plastically deformed materials. From the strain distribution, which might also be verified by means of micro-hardness measurements [5], it may be possible to calculate residual stresses, which may be verified by Röntgen-diffraction measurements.
Generally speaking, however, an "elementary" (e.g. one-dimensional) theory must be available to produce the boundary conditions for the stresses.
When the velocity-field is found, it can also be used to calculate a stress-distribution, satisfying the equilibrium condition, the Levy-Von Mises equations and the flow condition. Thus, improved expressions for roll pressure and torque could be found.
References.


Fig. 1
\[ J^* \]  \( (N/s) \)

- \( \omega R = -1000 \text{ mm/s} \)
- \( c = 700 \text{ N/mm}^2 \)
- \( m = 0.2 \)
- \( h_o = 5 \text{ mm} \)
- \( h_1 = 3 \text{ mm} \)
- \( R = 50 \text{ mm} \)
- \( \sigma_{x_1} = 240 \text{ N/mm}^2 \)
- \( \sigma_{x_0} = 0 \)

- \( a_{20} = 7.200 \)
- \( a_{00} = -1062.0 \text{ mm/s} \)
- \( a_{22} = -0.16034 \)
- \( a_{40} = -0.02301 \)

**Fig. 2**
\[ \omega R = -88 \text{ mm/s} \]
\[ h_0 = 9.65 \text{ mm} \]
\[ h_1 = 7.15 \text{ mm} \]
\[ R = 210 \text{ mm} \]
\[ m = 0 \]

\[
\begin{array}{|c|c|c|}
\hline
\mu & 0.3 & 0.4 \\
\hline
a_{00} & -91.958 & -91.996 \\
\hline
a_{20} & 0.06415 & 0.06600 \\
\hline
a_{40} & -0.00002884 & -0.000031789 \\
\hline
a_{22} & -0.00068213 & -0.0011040 \\
\hline
\end{array}
\]

Fig. 4
\( \omega R = -88 \text{ mm/s} \)
\( h_o = 14.5 \text{ mm} \)
\( h_i = 9.5 \text{ mm} \)
\( R = 210 \text{ mm} \)
\( m = 0 \)

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Fig. 5
$\omega R = \text{-}88 \text{ mm/s}$
$h_0 = 24.81 \text{ mm}$
$h_f = 16.76 \text{ mm}$
$R = 210 \text{ mm}$
$m = 0$

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<tr>
<td>$a_{20}$</td>
<td>$0.02749$</td>
<td>$0.02825$</td>
</tr>
<tr>
<td>$a_{40}$</td>
<td>$-0.0000047957$</td>
<td>$-0.000005027$</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>$-0.0000063596$</td>
<td>$-0.0000089773$</td>
</tr>
</tbody>
</table>

Fig. 6