Report no. 7
Local buckling of slender aluminium sections exposed to fire

Design model for local buckling

Date December 2007
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Number of pages 99
Number of Annexes 5
Sponsor NIMR
Project name PhD Local buckling of slender aluminium sections exposed to fire
Summary

This report gives a design model for local buckling of slender aluminium outstands and internal plates exposed to fire. Also the border between class 4 and class 3 sections for aluminium members in compression is given.

A number of design models for local buckling of aluminium and steel at room temperature are available. Literature also provides some proposals for design models for local buckling of steel in fire. These models are used as starting point for a newly developed design model for local buckling of aluminium sections in fire.

The new method is based on the determination of the inelastic critical stress, in an iterative way, and to apply an appropriate buckling curve. Buckling curves are given for both internal plates and outstands.

The method yields a simple classification border for sections in compression that fail before or after obtaining the plastic capacity (0.2 % proof stress times gross area). Simulations show that the ratio b/t at the classification border of fire exposed plates increases with approximately 30 to 90 %, compared to room temperature. The rule in EN 1999-1-2, that the classification in fire should be taken equal to room temperature, is thus very conservative.

An example of the analysis of a simple frame shows the application of the design model for members with realistic dimensions.
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1 Introduction

This report is a background document to a PhD research on local buckling of slender aluminium sections exposed to fire.

In [16], finite element models were developed for local buckling of simply supported outstands and internal plates at elevated temperature. The models were validated with tests.

This report gives a design model for local buckling of slender aluminium outstands and internal plates. The report also gives the border between class 4 and class 3 sections for aluminium members in compression (i.e. plastic cross-sectional capacity can be obtained, or resistance is limited by local buckling).

A number of design models for local buckling of aluminium and steel at room temperature are available. Literature also provides some proposals for design models for local buckling of steel in fire. These models are used as starting point for a design model for local buckling of aluminium sections in fire.

Chapter 2 of this document gives the reasons why local buckling of aluminium sections at elevated temperature is different from local buckling at room temperature. Chapter 3 gives the design models at room and at elevated temperature, as given in literature, and based on this, it gives a selection of possible basic principles for a design model. An investigation into the geometrical properties and boundary conditions that are important for local buckling are given in chapter 4. Chapter 5 gives the results of an investigation into the physical properties that are important for local buckling. The results of finite element calculations of plates subjected to local buckling are compared to existing calculation models in chapters 6 and 7. Chapter 8 gives a new calculation model, which is based on the existing models. This newly developed calculation model also results in a cross-sectional classification system for fire exposed aluminium compression members, which is the subject of chapter 9. An example of the analysis of a fire exposed frame, showing the applicability of the new design model for practice, is given in chapter 10. Conclusions are given in chapter 11.

Background knowledge on the principles of (failure due to) local buckling is required in order to understand the report. The required knowledge is given in background report [13] and in Mennink [20].
2 Differences between local buckling at room and at elevated temperature

There exist a couple of reasons why local buckling of aluminium alloy elements at elevated temperature is different from local buckling at room temperature. The reasons are listed in this chapter. Most reasons are due to differences in the stress-strain relationship.

2.1 Changing ratio between \(E\) and \(f_{0.2}\)

At elevated temperature, the ratio between \(E\) and \(f_{0.2}\) changes. For most alloys and tempers, \(f_{0.2}\) reduces faster than \(E\) at increasing temperature. This has a favourable influence on the load bearing resistance in case local buckling is the decisive failure mechanism.

2.2 Shape of the stress-strain relationship

At room temperature, the shape of the stress-strain relationship depends on the alloy. Some alloys have a stress-strain relationship which is substantially curved (grey curve in Figure 2.1), with a ratio between the proportional limit \(f_p\) and the 0.2 % proof stress \(f_{0.2}\) which is substantially lower than 1.0. Other alloys show a more elastic-plastic behaviour (black curve in Figure 2.1), with a ratio \(f_p / f_{0.2}\) which is close to 1.0.

Landolfo and Mazzolani [7] noted that the constitutive relations for strains up to 0.2 % plastic strain are important for the ultimate load bearing capacity of slender (class 4) sections subjected to local buckling (hereafter called ultimate buckling resistance). This paragraph therefore focuses on the shape of the stress-strain relation for strains up to 0.2 % plastic strain.

At elevated temperature, the ratio \(f_{p,θ} / f_{0.2,θ}\) is considerably lower than at room temperature for the alloys investigated (5083-H111 and 6060-T66). This is due to the influence of creep. As creep is observed for all aluminium alloys (see data Kaufman [10]), it is expected that the ratio \(f_{p,θ} / f_{0.2,θ}\) at elevated temperature is lower than at room temperature for all aluminium alloys.

If the design model is based on \(f_{0.2}\) (which is the case for aluminium), a lower ratio \(f_{p,θ} / f_{0.2,θ}\) has an unfavourable influence on the load bearing resistance.

![Figure 2.1 – Curvature of stress-strain relationships](image-url)
2.3 Load history dependent stress-strain relationships

At room temperature, the mechanical properties are not dependent on the load history of the structure.

Due to the large influence of creep, the stress-strain relations depend on the load history in case of fire. For example: if the material is subjected to a high stress level, say $\sigma_1$, during the first period in fire, and after this the stress is reduced to a certain level, say $\sigma_2$ (with $\sigma_2 < \sigma_1$), the strain developed at the end is larger than in case the stress is constant at $\sigma_2$ during the entire period.

It is known that, for slender sections, the ultimate load bearing resistance $F_u$ is larger than the critical load (or Euler load) $F_{cr}$ for local buckling. Up to $F_{cr}$, the stress distribution over the cross-section is uniform in case of a compressed member, whereas the distribution is non-uniform for loads in between $F_{cr}$ and $F_u$. (Figure 2.2).
In a fire, the strength and stiffness reduce in time. As a result, even for a constant normal force in time, the critical load of the plate is reached after some time: \( F_{cr} \). From this moment onwards, the stress distribution over the cross-section is not uniform. The temperature is able to increase further, until the strength and stiffness are reduced to such extend that \( F_u \) is reached. Hence, the load history for the stiffest parts of the section or plate (i.e. the intermediate corners) is different from that of the least stiff parts of the section or plate (i.e. the middle of the plate).

It is not yet known whether this has a significant influence on the local buckling behaviour. It is possible (and expected) that schematisation of the stress-strain relations with a constant stress in time gives an almost equal critical temperature as in case of the actual material properties including creep. Finite element simulations will be carried out in order to check this (chapter 3).

2.4 Restrained thermal expansion

Restrained thermal expansion leads to internal stresses. In case of class 4 sections (i.e. sections failing through local buckling before the 0.2 % proof stress is reached in the average section), the ultimate buckling resistance is reached at a small strain level. Restrained thermal expansion may influence the buckling resistance in this case. It is possible that the section buckles due to only thermal strain.

The effect of restrained thermal expansion is not investigated in this report. The report focuses on the differences in changing ratio between \( f_{0.2} \) and \( E \), the shape of the stress-strain relationship and the load history dependent material properties.
3 Influence of load history on critical temperature

Due to creep influence, the stress-strain relationship in fire depends on the load history, as indicated in paragraph 2.3. However, if this has to be accounted for in a design model on local buckling, the design model will inevitably become complicated and unfriendly for the designer.

In this chapter, it is investigated whether it is possible to approximate the material properties according to the Dorn-Harmathy model with stress-strain relations that are independent of the load history.

Paragraph 3.1 gives the proposed approximation of the stress-strain relationship. Paragraph 3.2 gives the influence of this approximation on the local buckling strength of plates.

3.1 Approximation of real stress-strain relationship

In case thermal expansion is not restrained, or in case axial thermal expansion need not be considered in the design (i.e. in case of the component approach in EN 1999-1-2), a simple approximation of the real stress-strain relationship is to assume a constant stress in time during exposure to elevated temperature.

For a constant load and a certain (constant) heating rate in time, the stress-strain relationship to be used in the design can be constructed from the Dorn Harmathy model, by carrying out simulations with various stress levels (but constant in time). Each simulation results in a certain strain at the end of the exposure period. In this way, the relation between stresses and strains can be determined for the heating rate and exposure period considered. The procedure is explained in Figure 3.1. A similar procedure was carried out for steel (Witteveen and Twilt, [25])

In this way, stress-strain relations according to the continuous curves in Figure 3.2 and Figure 3.3 are obtained (derived in background report [14]).
Figure 3.1 – Simulations of transient state tensile tests of alloy 5083-H111 at a heating rate of 6 °C / min

Figure 3.2 – Stress strain relationships for constant stress in time for alloy 5083-H111 (continuous curves: behaviour for constant stress in time, dotted curves: approximation with Ramberg-Osgood relationship)
These stress-strain relations are subsequently described by the Ramberg-Osgood relationship, using 3 material parameters to describe each curve: $f_{0.2}$, $E$ and $n$ (equation (3.1)).

\[
\varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{f_{0.2}} \right)^n \tag{3.1}
\]

In the description, $E$ and $f_{0.2}$ are determined for each graph, and $n$ is chosen such as to give the best agreement with the directly derived stress-strain relationships for constant stress in time.

Especially in case of alloy 6060-T66, it appeared to be not possible to determine $n$ such as to give a good approximation for strains smaller than 0.2 % plastic strain and strains much larger than 0.2 % plastic strain. The exponent was chosen such as to give a good agreement for small strains and a worse agreement for large strains (larger than 1 %), based on the following considerations:

- For local buckling of class 4 sections, strains smaller than 0.2 % plastic strain are usually more important than large strains;
- In case of restrained thermal expansion, strains larger than 0.2 % plastic strain may also become important. The thermal strain at 200 °C is 0.0045 and at 350 °C it is 0.0087. Assuming that thermal expansion is only partially restrained, this means that the relevant strain will still be smaller than 1 % in most cases;

With the chosen values for $n$ (given below), the stress at 1 % strain is overestimated with 4 % maximum. Assuming linear heating from the start of the fire onwards, this error results in an overestimation of the critical temperature with 2 %. Such an error is considered irrelevant.

The values for $E$ as a function of temperature are given with equations (3.2) and (3.3). The values for $f_{0.2}$ are given in Table 3.1 and Table 3.2. The values for $n$ could be
chosen such as to be independent of the heating rate. $n$ decreases approximately linear with temperature. The relationships are according to equations (3.4) and (3.5).

\[
E = 72000 - 10 \cdot \theta - 0,21 \cdot \theta^2 \quad \text{(Alloy 5083-H111)} \quad (3.2)
\]

\[
E = 69000 - 10 \cdot \theta - 0,21 \cdot \theta^2 \quad \text{(Alloy 6060-T66)} \quad (3.3)
\]

\[
n = 8,8 - 0,016 \cdot \theta \quad \text{(Alloy 5083-H111)} \quad (3.4)
\]

\[
n = 19 - 0,04 \cdot \theta \quad \text{(Alloy 6060-T66)} \quad (3.5)
\]

Table 3.1 – $f_{0.2}$ [N/mm²] for alloy 5083-H111

<table>
<thead>
<tr>
<th>Temp [ºC]</th>
<th>t = 30 min</th>
<th>t = 60 min</th>
<th>t = 90 min</th>
<th>t = 120 min</th>
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<tr>
<td>175</td>
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</table>

Table 3.2 – $f_{0.2}$ [N/mm²] for alloy 6060-T66

<table>
<thead>
<tr>
<th>Temp [ºC]</th>
<th>t = 30 min</th>
<th>t = 60 min</th>
<th>t = 90 min</th>
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<td>25</td>
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</tbody>
</table>

Using these quantities for $E$, $f_{0.2}$ and $n$, the stress-strain relations according to the dotted curves in Figure 3.2 and Figure 3.3 (large strains) and Figure 3.4 and Figure 3.5 (small strains) are obtained.
Figure 3.4 – Stress strain relationships for constant stress in time for alloy 5083-H111, small strains only (continuous curves: behaviour for constant stress in time, dotted curves: approximation with Ramberg-Osgood relationship)

Figure 3.5 – Stress strain relationships for constant stress in time for alloy 6060-T66, small strains only (continuous curves: behaviour for constant stress in time, dotted curves: approximation with Ramberg-Osgood relationship)

In a similar way, if desired, the stress-strain relation can be derived for deviant heating rates or for a non-constant load in time.
3.2 Influence on ultimate buckling resistance of approximated stress-strain relationship

The constitutive material model was implemented in the finite element program DIANA release 9.2. For a number of plate dimensions and heating rates, the ultimate buckling resistance was determined with the material model, and also with the approximated stress-strain relationships according to Ramberg-Osgood.

The width of the plates was 100 mm and the length was 600 mm. The plate thickness was chosen such as to result in relative slenderness values (using the elastic critical stress) of 0.67, 1.00, 1.75, 2.30 and for a limited number of plates also 2.60, 3.20 and 4.60. The initial geometrical imperfection applied on the models had shape according to the first buckling mode with an amplitude of 0.2 mm (1/500 times plate width). The reasons for taking this amplitude are elaborated in Annex B. The plates were subjected to a constant heating rate and constant force, the heating rate and force being such that failure occurred after 30 minutes at temperatures of approximately 200, 250, 300, and 350 ºC. Annex A gives the dimensions, loads and resulting critical temperatures for all plates investigated.

In the post buckling range, the stress distribution in a plate is not uniform. The stress at the supported plate edges increases and the stress in the middle of the plate decreases. Consequently, the stress is not constant in time, although the total load on the plate remains constant. As an example, Figure 3.6 gives the stress as a function of the temperature at the supported edge of one of the plates, analysed with the Dorn Harmathy model.

![Stress distribution in a plate](image)

Figure 3.6 – Maximum Von Mises stress in a plate of alloy 5083-H111 with b/t = 74

The Ramberg Osgood stress-strain relationships in paragraph 3.1 are derived for a constant stress in time. This would mean that the high stress in the plate edges at the end of the analysis is assumed to be present during the entire analysis (grey dotted curve in Figure 3.6). This is accompanied by a much larger strain than according to the Dorn Harmathy model.

If the external load or heating rate changes during the analysis, it is possible to derive a stress-strain relationship for these conditions with the same procedure as described in paragraph 3.1. However, it is not possible to take the changing stress distribution in the
plate into account in the derivation of the stress-strain relationship, as it is a priori not known how the stress distribution changes in the plate. As a result, if the same plate is simulated with the Ramberg-Osgood relationship derived in paragraph 3.1 for a constant stress in time (and with the same heating rate), the critical temperature is somewhat different from the model with the Dorn-Harmathy model (compare the grey and black curves in Figure 3.7).

Figure 3.7 shows that the deformation of the plate with the derived stress-strain relationship is approximately equal to that of the plate with the Dorn Harmathy material model up to a strain of 0.1%. At this strain, the critical temperature of the plate with derived stress-strain relationship is reached, at 300 °C. Contrary, the plate with the Dorn-Harmathy model is able to withstand higher temperatures, up to 315 °C.

For reasons of comparison, Figure 3.7 also gives the strain-temperature curve of the section when it is restrained against local buckling (lb) (light grey curve).

![Figure 3.7 – Axial strain as a function of temperature for an internal plate of alloy 5083-H111 with b/t = 74](image)

Investigations showed that the stresses in the two plates are approximately equal up to failure of the plate with Ramberg-Osgood stress-strain relationship. At higher temperatures, the maximum Von Mises stress in the plate with Dorn Harmathy model is higher than the maximum strength of the derived (Ramberg Osgood) stress-strain relationship.

To summarise: the stress of the supported plate ends, which carry the majority of the load in the postbuckling range, increases during the fire exposure. The derived stress-strain relationship assumes that this high stress level is present during the entire fire exposure. This overestimates the strains that accompany the stress. Using the derived stress-strain relationship thus results in conservative values for the fire resistance of sections subjected to local buckling.

Figure 3.8 up to Figure 3.11 give a graphical comparison of the critical temperatures of all plates, analysed with the Dorn Harmathy material model and with the approximated Ramberg Osgood stress-strain relationships. The critical temperature is lower when using the approximated stress-strain relationship in comparison with using the Dorn Harmathy material model. The average value of the ratio between the fire resistance determined with the derived stress-strain relationship and the fire resistance determined with the Dorn Harmathy material model is 0.94 for internal plates (when assuming a constant heating rate during fire exposure). The standard deviation of this ratio is 0.02.
for internal plates. For outstands, the average value and standard deviation of this ratio are 0.96 and 0.02.

**Figure 3.8 – Critical temperatures of internal plates of alloy 5083-H111**

**Figure 3.9 – Critical temperatures of internal plates of alloy 6060-T66**
3.3 Chapter conclusion

It is concluded that it is, in general, a safe approach to use the approximated stress-strain relationships instead of the more complicated Dorn Harmathy material model in the design of slender plates subjected to local buckling. The design model in the following paragraphs will be based on the derived stress-strain relationship.
4 Important physical and geometrical properties for local buckling

4.1 Physical properties

It is known that not only the 0.2 % proof stress and the modulus of elasticity, but the entire stress-strain relationship determines the ultimate buckling resistance (e.g. Mazzolani and co-workers [18], [19], [7], Hopperstad et al. [4], [5]). In order to determine which parts of the stress-strain relationships dominate the buckling behaviour, the buckling resistance is determined for various schematisations of the stress-strain relationships:

- The entire stress-strain relationship, i.e. according to the Ramberg-Osgood relation in chapter 3;
- Stress strain relation up to \( f_{0.2} \), i.e. according to schematisation 1 for strains smaller than 0.2 % plastic strain, while the stress is equal to the 0.2 % proof stress for larger strains than 0.2 % plastic strain;
- Elastic-perfectly plastic relationship, with the 0.2 % proof stress as plastic limit;
- Linear elastic relationship.

The four schematisations of the stress-strain curves are indicated in Figure 4.1 for alloy 5083-H111 at a temperature of 300 ºC. The figure gives the normalised stress (ratio \( \sigma / f_{0.2} \)) as a function of the normalised strain (ratio \( \epsilon / (f_{0.2} / E) \)).

![Figure 4.1 – Schematisations of the stress-strain relationships](image)

4.1.1 Internal plates

FEM simulations of internal plates with the four schematisations of the stress-strain relationships are made of plates with various slenderness and at various temperatures with the material properties for alloys 6060-T66 and 5083-H111. Figure 4.2 up to Figure 4.4 give some examples. The three figures are the results of simulations of plates with different slenderness ratios (i.e. different ratios \( b/t \)). The vertical axis gives the relative stress and the horizontal axis gives the relative axial strain.
Figure 4.2 – Normalised stress vs normalised axial strain of an internal plate with $\lambda_{rel} = 0.71$ with stress strain relations according to Figure 4.1.

Figure 4.3 – Normalised stress vs normalised axial strain of an internal plate with $\lambda_{rel} = 1.05$ with stress strain relations according to Figure 4.1.

Figure 4.4 – Normalised stress vs normalised axial strain of an internal plate with $\lambda_{rel} = 1.84$ with stress strain relations according to Figure 4.1.
The figures show that the ultimate buckling resistance obtained with elastic-perfectly plastic material is significantly larger than the ultimate buckling resistance obtained with the entire stress-strain relation. The ultimate buckling resistance obtained with the stress-strain relationship up to the 0.2 % proof stress is approximately equal to the ultimate buckling resistance obtained with the entire stress-strain relationship (although the load-displacement trajectories for axial strains larger than that corresponding to the ultimate buckling resistance differs). Based on this, it is concluded that the stress-strain relation up to the 0.2 % proof stress is important for the ultimate buckling resistance of internal plates, while the strain hardening behaviour for larger strains than 0.2 % plastic strain is not important.

4.1.2 Outstands

A complication occurs for outstands. In some cases, two peaks are detected in the resistance determined with finite element calculations (Figure 4.5 a.). A test carried out by Hopperstadt et al. [4] shows a similar load-displacement trajectory (test with b/t = 20.5 in Figure 4.5 b.), which indicates that this is not due to instability in the finite element analyses, but that it also occurs in real structures.

![Figure 4.5 – Outstands with two peaks in the resistance](image)

Figure 4.5 – Outstands with two peaks in the resistance

a. Results of FEM simulations with $\lambda_{rel} = 1.02$ on alloy 5083-H111

b. Results of tests by Hopperstadt on alloy 6082-T4 at room temperature

The occurrence of the second peak is linked with a combination of material nonlinearity and second order deformations. It is therefore not possible to simply describe the phenomenon and predict in which cases it will occur. However, some patterns were observed:

- The second peak occurs only for material with large strain hardening. For such material properties, the strength increases for large axial strains. Therefore, after having reached the first peak and the drop in strength following this peak, the strength is able to increase again;

- The second peak occurs only for a limited range of plate slenderness, namely for plates with a critical stress just larger than the proportional limit. For more stocky plates, the first peak already occurs in the range of high stresses, where strain hardening occurs. Extra deformation does not result in a significant increase of the stress in this case. Contrarily, when very slender plates have reached their critical stress, a small increase in axial strain is attended by a large increase in stress. In the first part of the post-buckling range of this plate, the material behaviour is thus still
almost linear elastic, resulting initially in a significant increase of the postbuckling strength, as observed for linear elastic material (Figure 4.7). For intermediate plate slenderness, an intermediate failure mechanism occurs. This is illustrated in Figure 4.6:

- The second peak was detected only for outstands, not for internal plates. This is attributed to the fact that, in case of internal plates, the deformation localises in one buckle in the post-buckling range. The remaining part of the section is then unloaded. Contrarily, the stress distribution in the section in the post buckling range of outstands is more uniform than in case of internal plates. Therefore, at increasing deformation, the stress in a large part of the outstand increases, resulting in an increase of the resulting normal force \( N = \int_A \sigma dA \). In case of internal plates, only the stress near the edges increases while the part of the plate with these high stresses decreases. The summation of the stress, i.e. the resulting normal force, decreases;
- The second peak did not occur for outstands with a very short length \( l \) (\( l / b \) in order of 1). This is again attributed to the fact that the stress distribution over the cross-section in case of long outstands is more uniform than that of short outstands.

![Figure 4.6 - Relative resistance of outstands with various relative slendernesses of alloy 6060-T66 at 350 °C](image)

![Figure 4.7 – Stiffness of a plate of linear elastic material](image)

The question that rises is which of the peaks can be considered as the relative resistance. There are several reasons to consider the maximum of the two peaks as the ultimate buckling resistance:
- As the geometrical imperfection increases, the first peak reduces. For a certain geometrical imperfection, only the second peak remains (Figure 4.8, with $A_{\text{imp}} = \text{amplitude of the imperfection pattern, with a shape of the first buckling mode}$);
- Considering the first peak as the ultimate buckling resistance would result in the relative ultimate buckling resistance of this plate to be smaller than the relative resistance of a plate with higher slenderness (Figure 4.9). This is not in agreement with the usual theory for buckling (higher slenderness means lower relative resistance);
- The strain at which the second peak occurs is large, especially in case of a very low ratios $f_p / f_{0,2}$ (as an example, this strain is approximately 1.5 % for a plate of $\lambda_{\text{rel}} = 0,67$ of alloy 5083-H111 at $\theta = 350 ^\circ \text{C}$). In the ultimate limit state, and especially in case of fire, such large strains are considered to be acceptable (note that the thermal strain at 350 °C is equal to 0,87 %).

Figure 4.8 – Simulations of outstands with various values for the imperfection
  a. Results shown for large strains
  b. Results shown for small strains

Figure 4.9 – Relative resistance of outstands with various relative slenderness of alloy 5083-H11 at 350 °C

A selection of results of simulations with the four stress-strain curves according to Figure 4.1 is given in Figure 4.10 up to Figure 4.12.
Especially Figure 4.11 shows that, contrarily to internal plates, also the part of the stress-strain relationship for larger strains than 0.2 % plastic strain has an important influence on the buckling resistance of outstands with $\lambda_{rel} = \text{appr. 1.0}$. The entire stress strain relation is thus important for the buckling resistance of outstands. (Note that there is no or only a slight influence for smaller and larger values for $\lambda_{rel}$ (Figure 4.10 and Figure 4.12).)

![Figure 4.10 – Normalised stress vs normalised axial strain of an outstand with $\lambda_{rel} = 0.69$ with stress strain relations according to Figure 4.1](image1)

![Figure 4.11 – Normalised stress vs normalised axial strain of an outstand with $\lambda_{rel} = 1.02$ with stress strain relations according to Figure 4.1](image2)
4.2 Geometrical properties

Due to the large strains accompanying the ultimate buckling resistance of especially outstands, the boundary conditions of a section at the ultimate buckling resistance may be different from that of a simply supported plate. Three changes on support conditions are distinguished as the out-of-plane deformations increase:

- Due to the larger deformation of the part of the plate near the unsupported edge, the axial deformation of this part of the plate may be larger than the axial deformation of the part of the plate near the supported edge. To illustrate this, the enlarged part of Figure 4.13 shows the deformations in case the plate ends are allowed to distort. The other ultimate support condition is that the plate edge is completely fixed. In that case, a tensile stresses may occur in the part of the plate at the supported ends near the unsupported edge.

- At the supported edge, the plate may deform in-plane so that the stresses perpendicular to the loading direction reduce when the plate deforms out of plane (difference between dash-dot line and supported plate edge in Figure 4.13);

- Due to the in-plane deformation of a plate noted above, the connecting nodes of a section consisting of plates may shift as the out-of-plane deformation increases, so that an eccentricity $e$ occurs (Figure 4.14). As a result, the individual plates of the section may be subjected to a combination of flexural and local buckling.
To determine the influence of these changing boundary conditions on the ultimate buckling resistance, simulations of simply supported outstands (SS out) and simply supported angles (SS angle) with various boundary conditions have been carried out. Some results are indicated in Figure 4.15 and Figure 4.16. The figures show that the boundary conditions influence the buckling resistance. However, only in case of outstands in which the in-plane deflection of the supported plate edge is restrained, the load-displacement trajectory and the ultimate buckling resistance differ significantly from that of the other cases. The difference in ultimate buckling resistance of the other plates
and sections is 15% maximum. Based on this, it is concluded that the results of one type of plate or section is representative for all outstands and sections consisting of outstands.

Figure 4.15 – Normalised stress vs normalised axial strain of outstands and angles with $\lambda_{rel} = 1.0$ and $b/l = 1/6$ and with various boundary conditions

Figure 4.16 – Normalised stress vs normalised axial strain of outstands and angles with $\lambda_{rel} = 1.8$ and $b/l = 1/6$ and with various boundary conditions

4.3 Chapter conclusions

In case of internal plates, only the part of the stress-strain relation for smaller strains than 0.2% plastic strain is important for the ultimate buckling resistance. In case of outstands, the entire stress-strain relation is important.

Outstands may, depending on the relative slenderness, exhibit two local maximums (peaks) in the resistance versus displacement diagram. The maximum value of these two peaks is considered as the ultimate buckling resistance of the fire exposed plate.

As the out-of-plane deformation of outstands increases, the boundary conditions of the plate change. This influences the ultimate buckling resistance, with a maximum influence of is 15%. The most unfavourable boundary conditions are applied in the
following chapters in the parameter study for the design model, so that the results are a safe approximation for outstands and sections consisting of outstands with various boundary conditions.
5 Existing work on a design model for local buckling

The proposals for design models for local buckling of steel and aluminium sections and for steel sections exposed to fire as found in literature are used as possible strategies for a design model for local buckling of aluminium sections exposed to fire. These strategies are listed in this chapter.

5.1 Incorporation of the shape of the stress-strain curve

Based on the method of virtual work, Stowell [22] proposed an equation for the so-called inelastic critical load for local buckling $\sigma_{cr}$, equation (5.1). This inelastic critical load gives the critical load for materials with a curved stress-strain relationship. It is originally derived for plates at room temperature. In the derivation, the assumption is applied that the tangent modulus of elasticity $E_t$ is effective in direction of the load and that the initial modulus of elasticity $E$ is valid in the direction perpendicular to the load.

$$\sigma_{cr,\text{inel}} = k_{cr} \frac{\pi^2 \eta E}{12(1-\nu^2)} \left( \frac{t}{b} \right)^2$$ (5.1)

The factor $\eta$ depends on the boundary conditions. In case of simply supported internal parts, equation (5.2) was derived, while equation (5.3) was given for simply supported outstands.

$$\eta = \frac{E_s}{E} \left( \frac{1}{2} + \frac{1}{4} \sqrt{\frac{1}{4} + \frac{3}{4} \frac{E_t}{E_s}} \right)$$ (5.2)

$$\eta = \frac{E_s}{E}$$ (5.3)

In which $E_i (\sigma) = \sigma / \varepsilon$ (secant modulus of elasticity) and $E_t (\sigma) = d\sigma / d\varepsilon$ (tangential modulus of elasticity), as indicated in Figure 5.1. The inelastic critical stress has to be calculated with an iterative procedure, in which $\sigma_{cr}$ is equal to the stress belonging to $E_t$ and $E_s$.

Figure 5.1 – Modulus of elasticity and secant and tangent mod. of el. at 250 N/mm² for alloy 5083 H34
In his equations for the inelastic critical stress, Stowell applied a Poisson ratio of 0.5, related to the incompressibility condition of the theory of plasticity.

Based on an extensive test program, Mazzolani et al. proposed a calculation model to determine the ultimate buckling resistance, in which Stowell’s equation (5.1) was applied to determine the critical stress.

In this method, the effective width of the plate is determined using equation (5.4) for internal plates and (5.5) for outstands. Equation (5.4) is equal to Winter’s equation [24] for elastic-perfectly plastic material, but using the inelastic critical stress and an imperfection parameter of 0.11 instead of 0.22. Equation (5.5) is equal to Kalyanaraman’s equation [8] for elastic-perfectly plastic material, but using the inelastic critical stress and an imperfection parameter of 0.149 instead of 0.298. By increasing step-by-step the axial strain, the corresponding values of \( \sigma_{cr,inel} \) and \( b_{eff} \) are computed. The ultimate buckling resistance is evaluated as the maximum load obtained in the loading process, while the buckling strain is the corresponding strain value.

Hopperstad [5] gave a design model for aluminium outstands. His method is similar to that of Mazzolani, but using the Von Karman equation for the effective width (equation (5.6)). Contrarily to Mazzolani’s equations, which require iteration to determine the ultimate buckling resistance, Hopperstad showed that it is possible to directly determine the ultimate buckling resistance with equation (5.6) if the stress-strain relationship is schematised with the Ramberg Osgood relation. At the ultimate buckling resistance, the derivative of equation (5.6) is equal to zero. Solving this equation gives an expression for the plastic strain at ultimate buckling resistance (\( \varepsilon_p^u \)) according to equation (5.7).

The effective width is obtained by determining the stress accompanying the plastic strain of equation (5.7), and applying this stress in equation (5.6). Note that the plastic strain at ultimate buckling resistance according to equation (5.7) is a function of the stress-strain relationship only, and not of the plate slenderness.

\[
\frac{b_{eff}}{b} = \sqrt{\frac{\sigma_{cr,inel}}{\sigma}} \left( 1 - 0.11 \cdot \sqrt{\frac{\sigma_{cr,inel}}{\sigma}} \right) \tag{5.4}
\]

\[
\frac{b_{eff}}{b} = 1.19 \sqrt{\frac{\sigma_{cr,inel}}{\sigma}} \left( 1 - 0.149 \cdot \sqrt{\frac{\sigma_{cr,inel}}{\sigma}} \right) \tag{5.5}
\]

\[
\frac{b_{eff}}{b} = \sqrt{\frac{\sigma_{cr,inel}}{\sigma}} \tag{5.6}
\]

\[
\varepsilon_p^u = \left[ \frac{(n-2)E \cdot 0.002\nu_n}{f_{0.2}} \left( \frac{n}{1-n} \right) \right] \tag{5.7}
\]

The Poisson ratio is expected to rise in for stresses larger than the proportional limit, because the fully plastic value for incompressible, isotropic materials is equal to 0.5. Hopperstad used a value of 0.5 for the Poisson ratio. Mazzolani replaced the value of the Poisson ratio in Stowell’s equation by the relation of Gerhard and Becker [3]:

\[
v_i = \nu_p - \frac{E_i}{E} \left( \nu_p - \nu \right) \tag{5.8}
\]
In which $\nu$ and $\nu_p$ are the elastic and fully plastic Poisson ratios, respectively, with values 0.3 and 0.5 and $\nu_i$ is the actual Poisson ratio.

It has been shown that the ultimate load and the load-displacement diagrams determined with the method described in this paragraph corresponds reasonable with that determined in tests at room temperature (both models, Mazzolani and Hopperstad).

This method is able to incorporate the actual values for $f_{0.2}$ and $E$ and the shape of the stress-strain relationship (paragraphs 2.1 and 2.2). The fact that the stress-strain relationship for the stiffer parts is different from that of the less stiff parts (paragraph 2.3) is not incorporated. Iteration is required to determine the ultimate buckling resistance for Mazzolani’s equations.

### 5.2 Alternative material properties

The strength value used in simple calculation models for fire exposed steel is the 2 % proof stress $f_2$. Ranby [21] and Zhao et al. [26] showed that the calculation model for local buckling at room temperature could be used in fire with one modification. This modification consists of using $f_{0.2}$ instead of $f_{2.0}$ for the strength. In this case, the ultimate buckling resistance of the model corresponds to that of finite element calculations in which the entire stress-strain relation was modelled.

Based on calculations with the finite strip method, Uy and Bradford [23] proposed to distinguish between very slender sections with an elastic critical stress ($\sigma_{cr,\theta}$) smaller than the proportional limit ($f_{p,\theta}$) and sections with $\sigma_{cr,\theta}$ larger than $f_{p,\theta}$. Equations (5.9) and (5.10) were proposed.

\[
\begin{align*}
\sigma_{cr,\theta} &= \sigma_{cr,room} \frac{E_{\theta}}{E} \quad \text{for} \quad \sigma_{cr,\theta} \leq f_{p,\theta} \\
\sigma_{cr,\theta} &= \sigma_{cr,room} \frac{f_{p,\theta}}{f_p} \quad \text{for} \quad \sigma_{cr,\theta} > f_{p,\theta}
\end{align*}
\]  

These methods have in common that the buckling resistance is calculated with the existing models for local buckling at room temperature, but using different material properties than the strength and stiffness used at room temperature.

Based on a parameter study with finite element calculations, it is possible to determine whether the design model for aluminium or even steel at room temperature could be used, with applying alternative values for the strength and $I$ or stiffness (e.g. $f_{0.1}$ or $f_p$ instead of $f_{0.2}$).

In this way, the entire stress strain curve is characterised by two (or a limited number of) parameters. Thus, with this model, it may be possible to incorporate the stress-strain relationship at elevated temperature (paragraphs 2.1 and 2.2). The fact that the stress-strain relationship for the stiffer parts is different from that of the less stiff parts (paragraph 2.3) is not incorporated.

### 5.3 Alternative buckling curves

The design model for local buckling of aluminium at room temperature in EN 1999-1-1, uses the same parameters as the model for local buckling of steel at room temperature,
being the 0.2 % proof stress (equivalent to the yield strength of steel) and the elastic critical buckling load $\sigma_{cr,el}$, determining the elastic slenderness $\lambda_{rel,el}$.

The fact that the ratio $f_p / f_{0.2}$ varies among alloys, is incorporated in proposing alternative buckling curves for alloys with a ratio $f_p / f_{0.2}$ close to 1.0 (almost elastic-plastic) and alloys with a ratio $f_p / f_{0.2}$ substantially lower than 1.0. Thus, the shape of the stress-strain relationship is incorporated in the buckling curve.

The ratio $f_{p,0} / f_{0.2,0}$ in fire is lower than at room temperature. Based on finite element calculations, alternative buckling curves could be proposed for stress-strain relationships depending on the shape of the stress-strain relationship. A measure for the amount of curvature of the stress-strain relationship is the exponent in the Ramberg-Osgood relationship. Different buckling curves could be determined for different ranges of the exponent in the Ramberg-Osgood relationship.

This method is able to incorporate the actual values for $f_{0.2}$ and $E$ and the shape of the stress-strain relationship (paragraphs 2.1 and 2.2). The fact that the stress-strain relationship for the stiffer parts is different from that of the less stiff parts (paragraph 2.3) can be incorporated by curve fitting of the buckling curves.

5.4 Evaluation of possible strategies

This chapter showed a number of possible strategies that could be used as a basis for a design model of aluminium in fire.

1. The first method, in which the inelastic critical buckling load is determined, and subsequently an appropriate buckling curve has to be found, is an elegant method. However, the method requires iteration to find the inelastic buckling resistance. It is not known whether this method works for stress-strain relations with a very low value $f_{p,0} / f_{0.2,0}$, such as shown for aluminium alloys in fire.

2. The second method, in which the entire stress-strain curve is characterised by two values (such as for example $E$ and $f_{0.1}$) is expected to give only a rough indication of the buckling strength, because the shape of stress-strain relations at different alloys and different temperatures are so different that it is expected that they cannot be schematised accurately by one value for the stiffness and one value for the strength. However, this rough method may be accurately enough for fire exposure.

3. The third method, in which alternative buckling curves are proposed depending on the exponent in the Ramberg-Osgood relationship, is a straightforward and easy-to-apply method for the buckling strength. It is not so sophisticated as the first two methods, because the strength is based on curve fitting of results of finite element simulations.

All methods given here are able to incorporate the values for the strength and stiffness, and the fact that the stress-strain relation is curved (paragraphs 2.1 and 2.2). It is not yet clear whether it is possible to incorporate the fact that the stress-strain relationship for the stiffer parts is different from that of the less stiff parts (paragraph 2.3). It will first be checked whether this last phenomenon has an important influence on the buckling resistance.
The first and third methods described are investigated for applicability of slender aluminium sections exposed to fire in the following chapters.
6 EC9-1-1 approach: using alternative buckling curves

In this chapter, it is investigated whether it is possible to determine the ultimate buckling resistance with the same approach as currently applied in EN 1999-1-1, but giving alternative buckling curves for very low ratios $f_{p,\theta}/f_{0.2,\theta}$.

6.1 Internal plates

6.1.1 Plates of alloy 5083-H111 and 6060-T66

Internal plates with various dimensions were analysed with the finite element method using approximated stress-strain relationships. Dimensions and results are given in Annex A. For each plate, the relative slenderness $\lambda_{rel}$ and the relative ultimate buckling resistance $\rho$ were calculated using equations (6.1) and (6.2).

The results are graphically shown in Figure 6.1 and Figure 6.2, together with the buckling curves in EN 1999-1-1.

\[
\lambda_{rel,\theta} = \frac{f_{0.2,\theta}}{\sigma_{cr,\theta}} \quad (6.1)
\]

\[
\rho = \frac{F_{u,\theta}}{F_{pl,\theta}} = \frac{F_{u,\theta}}{f_{0.2,\theta} \cdot b \cdot t} \quad (6.2)
\]

In which $\sigma_{cr,\theta}$ is the elastic critical stress, $F_{u,\theta}$ is the ultimate buckling resistance according to the finite element analysis and $F_{pl,\theta}$ is the plastic capacity of the plate at temperature $\theta$.

For each plate, the relative resistance $\rho$ is plotted against the relative slenderness $\lambda_{rel,\theta}$ in Figure 6.1 and Figure 6.2 for alloys 5083-H111 and 6060-T66, respectively.

Figure 6.1 – Relative resistance as a function of relative slenderness of internal plates of alloy 5083-H111 with approximated stress-strain relationships
Figure 6.2 – Relative resistance as a function of relative slenderness of internal plates of alloy 6060-T66 with approximated stress-strain relationships

The figures show that the relation between $\lambda_{rel}$ and $\rho$ depends on the temperature and alloy. Especially for plates of alloy 5083-H111 at high temperatures, the existing buckling curves in EN 1999-1-1 give unsafe values for the ultimate buckling resistance. This is attributed to the very low ratio $f_{p,0}/f_{o,2}$. Alternative buckling curves could be determined, depending on the alloy and on the temperature.

6.1.2 Plates with arbitrary material properties

A design model to be used in the design should not only work for the alloys considered, but also for other alloys for which the stress-strain relationships in fire are not yet known. The buckling curves to be used in fire design should therefore not depend on the alloy and/or temperature, but on the stress-strain relationship itself. An obvious choice is to provide buckling curves depending on the exponent of the Ramberg-Osgood relationship $n$.
- For high values of $n$, meaning a stress-strain relationship close to elastic-perfectly plastic behaviour, a favourable buckling curve could be given, e.g. equal to the curve of class ‘A’ alloys in EN 1999-1-1 and equal to steel at room temperature;
- For lower values of $n$, mere unfavourable buckling curves could be given.

In order to check this possibility of giving alternative buckling curves depending on $n$, some plates were investigated using the finite element method, with the following characteristics of the stress-strain relationships:
- $n = 3.3$ or $n = 15$ (i.e. more and less curved stress-strain relationship)
- $f_{0.2} = 20$ N/mm$^2$ or $f_{0.2} = 150$ N/mm$^2$ (i.e. weak and strong)

With $E = 50000$ N/mm$^2$. The results of these four analyses are given in Figure 6.3. The figure shows that the relative resistance of plates with equal values for $\lambda_{rel}$ and $n$, but different values for $f_{0.2}$, give a different relative resistance $\rho$. This is related to the fact that the shape of the stress-strain relationship not only depends on $n$, but also on $f_{0.2}$. To explain this, Figure 6.4 gives the normalised stress-strain relationships for properties $f_{0.2} = 20$ N/mm$^2$ and $f_{0.2} = 150$ N/mm$^2$ with $n = 3.3$. The vertical axis gives the ratio $\sigma/f_{o,2}$ and the horizontal axis gives the ratio $\varepsilon/\varepsilon_{el}$, with $\varepsilon_{el} = f_{o,2}/E$. The normalised stress-
strain relationships are different. Consequently also $E_s(\sigma)$ and $E_t(\sigma)$, which are the governing quantities for the inelastic critical stress according to Stowell (equations (6.1), (6.2) and (6.3)) are different (Figure 6.5).

Figure 6.3 – Relative resistance as a function of relative slenderness of internal plates with $n = 3.5$ or $15$ and $f_{0,2} = 20$ or $150$ N/mm$^2$

Figure 6.4 – Normalised stress strain relationships of alloys with $E = 50000$ N/mm$^2$, $n = 3.5$ and $f_{0,2} = 20$ or $150$ N/mm$^2$
6.2 Outstands

6.2.1 Plates of alloy 5083-H111 and 6060-T66

Outstands with various dimensions were analysed with the finite element method using approximated stress-strain relationships. Dimensions and results are given in Annex A. The results are graphically shown in Figure 6.6 and Figure 6.7, together with the buckling curves in EN 1999-1-1 and the buckling curve proposed for fire exposed outstands of steel by Knobloch and Fontana [11].
The figure shows that the relation between relative slenderness and relative resistance of outstands depends only in a marginal way on the shape of the stress-strain relationships considered.

Figure 6.8 gives the relation between relative slenderness and relative resistance of outstands with arbitrary stress-strain relations. For these arbitrary stress-strain relationships, it appears that the ultimate buckling resistance depends on the stress-strain relationships. As with internal plates, the relative resistance not only depends on \( n \), but also on \( f_{0.2} \).

6.3 Chapter conclusion

It is concluded that it is not possible to develop a design model which is based on calculating the relative slenderness with the elastic critical stress in combination with
buckling curves that depend only on $n$ and which can be used for any combination of material characteristics $E, f_{0.2}$ and $n$. 
7 Stowell/Mazzolani approach: iterative procedure with inelastic critical load

In this chapter, it is investigated whether the iterative approach of Stowell and Mazzolani can be used for plates with stress-strain relationships of fire exposed aluminium alloys.

7.1 Internal plates

7.1.1 Plates of alloy 5083-H111 and 6060-T66

The plates analysed in chapter 3, using the finite element method with approximated stress-strain relationships, are compared with method of Stowell/Mazzolani described in paragraph 5.1. For each plate, the ultimate buckling resistance \( F_u \) was determined by increasing step-by-step the axial strain and using equations (7.1) up to (7.5).

\[
\sigma_{cr,inel}(\varepsilon) = k_{cr} \frac{\pi^2 \cdot \eta(\varepsilon) \cdot E \cdot (t/b)^2}{12 \cdot (1-\nu^2)} \tag{7.1}
\]

\[
\eta(\varepsilon) = \frac{E_s(\varepsilon)}{E} \left(\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{4} + \frac{3}{4} \frac{E_s(\varepsilon)}{E_s(\varepsilon)}}\right) \quad \text{For internal plates} \tag{7.2}
\]

\[
\eta(\varepsilon) = \frac{E_s(\varepsilon)}{E} \quad \text{For outstands} \tag{7.2}
\]

\[
b_{eff}(\varepsilon) = \sqrt{\frac{\sigma_{cr}(\varepsilon)}{\sigma(\varepsilon)}} \left(C_1 - C_2 \sqrt{\frac{\sigma_{cr}(\varepsilon)}{\sigma(\varepsilon)}}\right) \tag{7.3}
\]

\[
F_k(\varepsilon) = b_{eff}(\varepsilon) \cdot t \cdot \sigma(\varepsilon) \tag{7.4}
\]

\[
F_u = \max(F_k(\varepsilon)) \quad \text{and} \quad \rho = \frac{F_u}{b \cdot t} \tag{7.5}
\]

For internal plates, constant \( C_1 \) in equation (7.3) is taken as 1,0. The results are evaluated for two values of \( C_2 \), being 0,11 (in accordance with Mazzolani) and 0 (in accordance to Von Karman for elastic perfectly plastic material).

Figure 7.1 gives a selection of the results. In general, the load-displacement trajectory is described adequately up to the ultimate buckling resistance. For larger strains, the Stowell-Mazzolani calculation model disagrees with the finite element models. In case of very slender internal plates (\( \lambda_{rel} = 1,75 \)), the model gives a reasonable prediction of the ultimate buckling resistance, but the strain corresponding with the ultimate buckling resistance is not predicted accurately.
5083-H111, θ = 250 ºC, λ = 0.67

5083-H111, θ = 250 ºC, λ = 1.00

5083-H111, θ = 250 ºC, λ = 1.75

5083-H111, θ = 250 ºC, λ = 2.00

Figure 7.1 – Average stress vs axial strain of internal plates of alloy 5083-H111 at 250 ºC according to the finite element method and according to the model of Stowell-Mazzolani

Figure 7.2 gives the ratio between the relative ultimate buckling resistance $\rho$ according to the calculation method and $\rho$ according to the finite element method. In general, the resulting values for $\rho$ according to the calculation model agreed well with the finite element method results. For stocky plates and plates with intermediate slenderness, the results agree well if $C_2$ is taken as 0.11. For very slender plates (for which $\rho$ is low), the calculation model gives a conservative ultimate buckling resistance even if $C_2$ is taken as 0.

Based on the results, it is expected that, with curve fitting, it will be possible to give an equation for $b_{eff}$ depending on the ratio $\sqrt{\frac{\sigma_u}{\sigma}}$ for which the ultimate buckling resistance according to the calculation model agrees with that of the finite element method.
7.1.2 **Plates with arbitrary material properties**

In order to investigate whether the calculation model is applicable for any stress-strain relationship of fire exposed aluminium, the relative buckling resistance of plates with the very different material properties \( f_{0.2} = 20 \) and \( 150 \) N/mm\(^2\), \( n = 3.5 \) and \( 15 \) and \( E = 50000 \) N/mm\(^2\) are determined with the calculation model. The same plates are investigated as mentioned in paragraph 6.1.2.

![Figure 7.2 - Ratio between relative resistance of internal plates according to the calculation model and according to finite element models](image)

**Figure 7.2** – Ratio between relative resistance of internal plates according to the calculation model and according to finite element models  

a. \( C_2 = 0 \)  
b. \( C_2 = 0.11 \)

The relation between the ratio \( \frac{\rho_{\text{calc model}}}{\rho_{\text{FEM}}} \) and \( \rho_{\text{FEM}} \) for the plates with different material properties is similar to that of the plates of alloy 5083-H111. This indicates that the Stowell Mazzolani response model is able to determine the ultimate buckling resistance of plates with arbitrary material properties.

![Figure 7.3 - Ratio between relative resistance of internal plates according to the calculation model and according to finite element models for material with](image)

**Figure 7.3** – Ratio between relative resistance of internal plates according to the calculation model and according to finite element models for material with \( n = 3.5 \) or \( 15 \) and \( f_{0.2} = 20 \) or \( 150 \) N/mm\(^2\)
7.2 Outstands

For outstands, the equations in paragraph 7.1 were used but with other values for constants $C_1$ and $C_2$. Mazzolani proposed to use $C_1 = 1.19$ and $C_2 = 0.15$. Hopperstad proposed to use $C_1 = 1$ and $C_2 = 0$.

Figure 7.4 gives a selection of the finite element simulation and calculation with the model of Stowell and Mazzolani for outstands.

![Graphs showing stress vs axial strain for outstands](image)

Figure 7.4 – Average stress vs axial strain of outstands of alloy 5083-H111 at 200 °C according to the finite element method and according to the model of Stowell-Mazzolani.

In case there are two peaks in the load-displacement trajectory according to the finite element calculation, the Stowell-Mazzolani model only describes the first peak. The ultimate buckling resistance of outstands with large slenderness is not predicted accurately. The error in ultimate buckling resistance increases as the ratio $f_{p, \theta} / f_{02, \theta}$ decreases. Figure 7.5 shows that, in general, the model by Stowell Mazolani does not accurately predict the ultimate buckling resistance of outstands with the very low ratios $f_{p, \theta} / f_{02, \theta}$ of fire exposed aluminium alloys.
Figure 7.5 – Ratio between relative resistance of internal plates according to the calculation model and according to FEM

7.3 Chapter conclusion

The Stowell Mazzolani response model is able to determine the ultimate buckling resistance of internal plates of alloys 5083-H111 and 6060-T66 and of material with arbitrary material properties. The relation for the effective width $b_{eff}$ has to be modified in order to fit the finite element analyses of plates.

The iterative nature of this design model makes that it is relatively time-consuming to determine the ultimate buckling resistance.

For outstands, a direct (i.e. non-iterative) determination of the ultimate buckling resistance is possible with the model proposed by Hopperstad. However, the results of the calculation model do not agree with finite element calculations for outstands with a very low ratio $f_{p,0} / f_{0,2,0}$. 

\[ \text{Figure 7.5 – Ratio between relative resistance of internal plates according to the calculation model and according to FEM} \]

\[ \text{a. Coefficients according to Mazzolani} \]

\[ \text{b. Coefficients according to Hopperstad} \]
8 New approach: inelastic critical stress and direct calculation of resistance

An approach similar to EC9-1-1, using alternative buckling curves depending on $n$, is a fast method to be used in the design, but it is not possible to provide buckling curves that can be used with any stress-strain relationship. The Stowell Mazzolani response model can be used for any stress-strain relationship of internal plates, but is relatively time-consuming because of its iterative nature. Moreover, it appears to be not suited for outstands with stress-strain relationships of fire exposed aluminium alloys.

A new method is developed, which combines the advantages of the two response models mentioned.

The method consists of the following steps:
1. Determine the inelastic critical stress $\sigma_{cr, inel}$.
2. Determine the inelastic relative slenderness $\lambda_{rel, inel}$.
3. Determine the relative buckling resistance $\rho$ with an appropriate buckling curve.

These steps are elaborated in the paragraphs of this chapter.

8.1 Procedure to determine the inelastic critical stress

The inelastic critical stress of a plate, according to Stowell [22], is the stress for which the following relation exists:

$$\sigma_{cr, inel} = k_{cr} \pi^2 \cdot \eta(\sigma_{cr, inel}) \cdot E \cdot \left(\frac{t}{b}\right)^2$$  \hspace{1cm} (8.1)

$$\eta(\sigma_{cr, inel}) = \frac{E_s(\sigma_{cr, inel})}{E} \left(1 + \frac{1}{2} \left(1 + \frac{3}{4} \frac{E_s(\sigma_{cr, inel})}{4 E_s(\sigma_{cr, inel})}\right)\right)$$  \hspace{1cm} (8.2) \text{Simply supported internal plates}

$$\eta(\sigma_{cr, inel}) = \frac{E_s(\sigma_{cr, inel})}{E}$$  \hspace{1cm} (8.3) \text{Simply supported outstands}

Parameters $E_s$ and $E_t$ can be described with the parameters of the Ramberg Osgood relationship $f_{0.2}$, $E$ and $n$:

$$E_s(\sigma) = \frac{\sigma}{\varepsilon} = \frac{E}{1 + \frac{E \cdot 0.002}{f_{0.2}} \left(\frac{\sigma}{f_{0.2}}\right)^{n-1}}$$  \hspace{1cm} (8.4)

$$E_t(\sigma) = \frac{d\sigma}{d\varepsilon} = \frac{E}{1 + \frac{E \cdot 0.002 \cdot n}{f_{0.2}} \left(\frac{\sigma}{f_{0.2}}\right)^{n-1}}$$  \hspace{1cm} (8.5)

Equations (8.2) and (8.3) are substituted in equation (8.1) to reduce to equation (8.4) for outstands, and equations (8.5) and (8.6) for internal plates.
Evaluating equation (8.6), it appears that $C$ approaches 1 if the ratio $\sigma_{cr,inel}/f_{0.2}$ is low (i.e. slender plates), while it becomes smaller than 1 if the ratio $\sigma_{cr,inel}/f_{0.2}$ is high. The minimum value for $C$ is 0.75. This value is obtained when $0.002E$ is much larger than $f_{0.2}$.

Figure 8.1 gives plots of $E_t/E$, $\eta_{out}$ and $\eta_{in}$ as a function of the normalised stress for stress-strain relationships with $n = 5$, $E = 50000$ N/mm$^2$ and $f_{0.2} = 150$ N/mm$^2$ (plot a) or $f_{0.2} = 20$ N/mm$^2$ (plot b).

If the relations in Figure 8.1 are compared, it appears that parameter $\eta_{in}$ is close to $\eta_{out}$. The values for $\eta$ are evaluated for a wide range of mechanical properties ($20,000 \leq E \leq 70,000$ N/mm$^2$, $3.5 \leq n \leq 35$ and $20 \leq f_{0.2} \leq 300$ N/mm$^2$), see Annex E. In all cases considered, the value of $\eta_{in}$ is close to $\eta_{out}$.

A good approximation for the inelastic critical stress of internal plates is obtained with equation (8.7), with $\alpha = 1.4$ (this equation is an alternative for the more accurate but more complex equations (8.5) and (8.6)).
\[
\alpha \cdot E \cdot 0,002 \cdot \left( \frac{\sigma_{cr,inel}}{f_{0,2}} \right)^n + \sigma_{cr,inel} = k_{cr} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1-\nu^2)} \left( \frac{t}{b} \right)^2
\]  
(8.7)

For a wide range of constitutive properties (\(20,000 \leq E \leq 70,000 \text{ N/mm}^2\), \(3,5 \leq n \leq 35\) and \(20 \leq f_{0,2} \leq 300 \text{ N/mm}^2\)), the results of equation (8.7) are compared with the results of equations (8.5) and (8.6). The maximum error in \(\sigma_{cr,inel}\) when using equation (8.7) was determined at 3.5%.

Equation 8.7 is applicable both for internal plates and for outstands. For internal plates, \(\alpha = 1.4\) and for outstands, \(\alpha = 1.0\).

The first part of equation 8.7 describes the influence of the curvature of the stress-strain relation on the critical stress. The rest of the equation is equal to the equation for elastic critical stress. Note that for slender plates (\(\sigma_{cr,inel} < f_{0,2}\)) and elastic-plastic material (\(n \rightarrow \infty\)), equation (8.7) returns the elastic critical stress.

The root of this function still has to be determined in an iterative process.

For a plate with linear elastic material, the critical elastic stress marks the point where the stiffness deviates from the initial stiffness of the plate (Figure 4.7). Analogous to this, it is determined whether or not equation (8.4) marks the point where the stiffness of a plate with inelastic material deviates from the stiffness of the stress-strain curve. For this reason, finite element models of plates with small initial imperfections (max imperfection = \(1/2000 \times b\)) were analysed. The results of the analyses are shown with curves in Figure 8.2. The values of the inelastic critical stress of the investigated plates, determined with equation (8.4), are indicated with dots in this figure. The good agreement between the point of deviation with the stress-strain curves and the critical elastic stress according to equation (8.4) indicates that this equation gives a proper description of the (inelastic) critical stress.

![Fig. 8.2](image-url)

Figure 8.2 – Comparison of inelastic critical stress according to equation (8.4) (dots) with geometrical and physical nonlinear finite element analyses with small imperfections

a. internal plates
b. outstands
8.2 Appropriate buckling curve for internal plates

Instead of using an iterative procedure to determine the ultimate buckling resistance, as proposed by Mazzolani, it may be possible to give a direct relation between the inelastic critical stress $\sigma_{cr,inel}$ and the relative resistance $\rho$, as applied in EN 1999-1-1 (but then using the inelastic critical stress).

First, an inelastic relative slenderness is defined according to equation (8.8).

$$\lambda_{rel,inel} = \sqrt{\frac{f_{0.2}}{\sigma_{cr,inel}}}$$  (8.8)

Then, an appropriate buckling curve has to be determined. The relative resistance $\rho$ of all internal plates of alloys 5083-H111 and 6060-T66, as calculated with the finite element method in Annex A, are given in Figure 8.3 as a function of the inelastic relative slenderness. The results for all plate slenderness and all stress-strain relationships in the range of 200 °C up to 350 °C describe approximately one curve. This indicates that the proposed method can be used for fire exposed aluminium alloys.

Figure 8.4 gives the same data in a different presentation, in which the relative resistance $\rho$ is given as a function of $1 / \lambda_{rel,inel}^2$. Note that $1 / \lambda_{rel,inel}^2$ is equal to $\sigma_{cr,inel} / f_{0.2}$.

Figure 8.3 – $\rho$ as a function of $\lambda_{rel,inel}$ for internal plates with inelastic stress-strain relationships – results of simulations
The buckling curve used by Mazzolani to determine the ultimate buckling resistance of plates in an iterative process \((C_1 = 1, C_2 = 0.11)\) in combination with the new proposed method does not represent the results of the finite element calculations. Contrarily, in the iterative procedure of Stowell-Mazzolani, the effective widths calculated with \(C_1 = 1\) and \(C_2 = 0.11\) gave reasonable results for \(1 \leq \lambda_{\text{rel,inel}} \leq 1.5\). This difference is due to the fact that in the method of Stowell-Mazzolani, the resistance was calculated for any value of \(\varepsilon\), and not only for the actual value of the inelastic critical stress. Especially for slender plates, the relative resistance in the method of Stowell-Mazzolani was determined for a stress that was significantly higher than the inelastic critical stress \(\sigma_{\text{cr,inel}}\).

A buckling curve is derived which gives a good agreement with the results of the finite element calculations. The buckling curve is represented by equation (8.9) and shown in black in Figure 8.3 and Figure 8.4.

\[
\rho_{\text{internal}} = \frac{1}{\lambda_{\text{rel,inel}}^{2}} \left( 1 + \frac{0.2}{\lambda_{\text{rel,inel}}^{2}} - \frac{2.5}{\lambda_{\text{rel,inel}}^{3}} + \frac{2.3}{\lambda_{\text{rel,inel}}^{2}} \right) \tag{8.9}
\]

The resulting values of \(\rho\) as a function of \(\lambda_{\text{rel,inel}}\) are close to the curve for the critical stress \((\rho = 1 / \lambda_{\text{rel,inel}}^2)\) for values \(1 \leq \lambda_{\text{rel,inel}} \leq 1.5\). In other words: the ultimate buckling resistance of the plate is close to the inelastic, critical stress multiplied by the area of the cross-section. Only for very slender plates, this gives an underestimation of the resistance. This agrees with the experimental observations of Jombock and Clark [6] and Mennink [20] that aluminium plates buckling in the elastic range show a significant post-buckling strength, while plates buckling in the inelastic range show little post-buckling strength (cases for room temperature).

Figure 8.5 and Figure 8.6 give again the results of simulations and the proposed buckling curve. Also the results of tests are displayed in these graphs. The following tests series are displayed (background report [15]):
- The steady-state tests at elevated temperature on SHS of alloy 6060-T66;
- The steady-state tests at elevated temperature on welded SHS of alloy 5083-H111;
The transient state tests at elevated temperature on SHS of alloy 6060-T66;
- The transient state tests at elevated temperature on welded SHS of alloy 5083-H111;
- The tests on SHS carried out at room temperature by Mazzolani et al. [18], [19] and
Landolfo and Mazzolani [7].

From the graphical representation by Figure 8.6, it follows that there are differences
between the proposed buckling curve or the simulations of the parameter study, and the
tests.
The relative ultimate buckling resistance of the welded specimens in steady state tests is
lower than predicted by the proposed buckling curve. This is attributed to the extremely
large geometrical imperfections in these sections, which was not taken into account in
the parameter study. The ratio between the maximum initial imperfection and the plate
width was 1/40, while the parameter study was carried out with 1/500 (see Annex B).
The relative ultimate buckling resistance of the other specimens agrees, in general, well
with the proposed buckling curve.

The relative ultimate buckling resistance of the specimens in the transient state tests is
higher than predicted with the proposed buckling curve. In chapter 3, it was noted that
the critical temperature of simulations with the Dorn Harmathy material model is higher
than that of simulations with derived stress-strain relations. This effect is responsible for
the differences found between transient state tests and the buckling curve, which is
based on the derived stress-strain relations. The figure shows that even buckling of
welded SHS in transient state tests is safely predicted.

Figure 8.5 – $\rho$ as a function of $\lambda_{rel,inel}$ for internal plates with inelastic stress-strain
relationships – results of simulations and tests
8.3 **Appropriate buckling curve for outstands**

The inelastic plate slenderness of outstands is determined in the same way as for internal plates. Most calculations are carried out with a ratio between plate length and plate width of $l/b = 6$. A limited number was carried out with $l/b = 3$ and $l/b = 1$.

The results of all calculations are given in Annex A.

The results are given in Figure 8.7 and Figure 8.8 (only simulations of the parameter study) Figure 8.9 and Figure 8.10 (including tests results). The following tests series are displayed:

- The steady-state tests at elevated temperature on angles of alloys 5083-H111 and 6060-T66 (background report [15]);
- The transient state tests at elevated temperature on angles of alloys 5083-H111 and 6060-T66 (background report [15]);
- The tests on crucifix sections carried out at room temperature by Hopperstad [4].

The figures show that the resulting relative resistance of the simulations presented as a function of the inelastic relative slenderness (or the inelastic critical stress) have a larger scatter than in case of the internal plates.

The relative resistance of plates with $l/b = 3$ is equal to that of plates with $l/b = 6$ with the same inelastic relative slenderness. The relative resistance of plates with $l/b = 1$ is larger.

The buckling curve is formulated such, that the simulation results are on the safe side.

The buckling curve derived for outstands is represented by equation (8.10) and shown in black in Figure 8.7 up to Figure 8.10.
\[
\rho_{\text{outstand}} = \frac{1}{\lambda_{\text{rel,inel}}} \left( 1 + \frac{1,5}{\lambda_{\text{rel,inel}}} - \frac{5}{\lambda_{\text{rel,inel}}^2} + \frac{3,5}{\lambda_{\text{rel,inel}}^3} \right)
\]

The equation gives a safe approximation of the simulations, of the transient state tests and of most steady state tests carried out. The differences between tests and simulations of the parameter study are caused by the unfavourable boundary conditions applied in the parameter study and, in case of the transient state tests, the difference between the Dorn Harmathy model and the derived stress-strain relationships.

---

**Figure 8.7** – \(\rho\) as a function of \(\lambda_{\text{rel,inel}}\) for internal plates with inelastic stress-strain relationships – results of simulations and tests

**Figure 8.8** – \(\rho\) as a function of ratio \(\sigma_{\text{rel,inel}} / f_{0.2}\) for outstands with inelastic stress-strain relationships – results of simulations and tests
Figure 8.9 – $\rho$ as a function of $\lambda_{rel, inel}$ for outstands with inelastic stress-strain relationships – results of simulations and tests

Figure 8.10 – $\rho$ as a function of ratio $\sigma_{rel, inel} / f_{0,2}$ for outstands with inelastic stress-strain relationships – results of simulations and tests

Figure 8.11 and Figure 8.12 give a comparison between the results of the design model and the results of the finite element simulations for internal plates and outstands, respectively. Table 8.1 gives the average and standard deviation of the ratio between the relative resistance according to the design model and the relative resistance according to the finite element simulations. The agreement is good.
Figure 8.11 – Relative resistance of outstands according to the design model compared to relative resistance according to the finite element model

Figure 8.12 – Relative resistance of outstands according to the design model compared to relative resistance according to the finite element model

Table 8.1 – Average value and standard deviation of the ratio $\rho_{\text{design model}} / \rho_{\text{FEM}}$

<table>
<thead>
<tr>
<th></th>
<th>internal plates</th>
<th>outstands</th>
</tr>
</thead>
<tbody>
<tr>
<td>average value</td>
<td>0.98</td>
<td>0.93</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>

8.4 Summary of the proposed design model

The check on local buckling of aluminium plates exposed to fire is summarized below.

1. For the required fire resistance, determine the decisive stress-strain relationship (i.e. the relation that gives the minimum strength, this often coincides with the maximum aluminium temperature);
2. Determine the parameters $E$, $f_{0.2}$ and $n$ of the Ramberg-Osgood relationship for the decisive stress-strain relationship;
3. Determine the inelastic critical stress of the plate with equation (8.11) in an iterative procedure ($\alpha = 1.4$ for internal plates and 1.0 for outstands);
\[ \alpha \cdot E \cdot 0,002 \left( \frac{\sigma_{cr,inel}}{f_{0,2}} \right)^n + \sigma_{cr,inel} = k_{cr} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \left( \frac{t}{b} \right)^2 \] (8.11)

4. Determine the inelastic relative slenderness of the plate;
\[ \lambda_{rel,inel} = \sqrt{\frac{f_{0,2}}{\sigma_{cr,inel}}} \] (8.12)

5. Determine the relative resistance of the plate with the appropriate buckling curve;
\[ \rho_{c,internal} = \frac{1}{\lambda_{rel,inel}} \left( 1 + \frac{0,2}{\lambda_{rel,inel}} - \frac{2,5}{\lambda_{rel,inel}^2} + \frac{2,3}{\lambda_{rel,inel}^3} \right) \] (8.13)
\[ \rho_{c,outstand} = \frac{1}{\lambda_{rel,inel}} \left( 1 + \frac{1,5}{\lambda_{rel,inel}} - \frac{5}{\lambda_{rel,inel}^2} + \frac{3,5}{\lambda_{rel,inel}^3} \right) \] (8.14)

6. Determine the unity check;
\[ \frac{F_{sd}}{p_{c} \cdot b \cdot t} \leq 1,0 \] (8.15)
9 Classification of sections in compression exposed to fire

Sections in bending are to be divided into 4 cross-sectional classes. For compression members, however, only 2 cross-sectional classes remain (see background report [13]).

The classification system distinguishes sections that fail by local buckling at an average stress lower than the 0.2 % proof stress ($\rho < 1$) or sections in which case the 0.2 % proof stress is reached in the entire section ($\rho \geq 1$). This chapter gives a classification system for aluminium alloys exposed to fire.

9.1 Equations for classification border

The relative resistance for local buckling, $\rho$, is defined as follows:

$$\rho_c = \frac{F_u}{A \cdot f_{02}}$$  \hspace{1cm} (9.1)

With

- $\rho_c$ = relative resistance
- $F_u$ = ultimate load bearing resistance
- $A$ = cross-section
- $f_{02}$ = 0.2 % proof stress

If $\rho < 1$, the section fails through local buckling before the 0.2 % proof stress is reached. If $\rho = 1$, local buckling occurs if the average stress in the section is exactly equal to the 0.2 % proof stress. Due to the inelastic material characteristics, it is possible that local buckling occurs at a larger stress than the 0.2 % proof stress, i.e. for less slender sections, $\rho > 1$.

The results of the simulations in chapter 8 show that the border $\rho = 1$ is obtained for an inelastic relative slenderness equal to $\lambda_{rel,inel} = 1$, both for internal plates and outstands. Also the proposed buckling curves give $\rho = 1$ for $\lambda_{rel,inel} = 1$.

An inelastic relative slenderness equal to 1 means that the inelastic critical buckling load has to be equal to the 0.2 % proof stress: $\sigma_{cr,inel} = f_{0.2}$.

Replacement of $\sigma_{cr,inel}$ by $f_{0.2}$ in equation (8.4) gives the border between the cross-sectional classes of fire exposed compression members (for internal plates, $\alpha = 1.4$; for outstands, $\alpha = 1.0$):

$$\alpha \cdot 0.002 \cdot E + f_{0.2} = k_{cr} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \left( \frac{t}{b} \right)^2$$  \hspace{1cm} (9.2)

This equation is rewritten to obtain the ratio $b / t$ at the border between the cross-sectional classes:

$$\frac{b}{t_{\text{class limit}}} = \sqrt{\frac{k_{cr} \cdot \pi^2 \cdot E}{12 \cdot (1 - \nu^2) \cdot (\alpha \cdot 0.002 \cdot E + f_{0.2})}}$$  \hspace{1cm} (9.3)
Equations (9.1 and 9.2) can be derived directly; an iteration procedure is not necessary. The resulting equations are, surprisingly, not related to parameter $n$, i.e. the border between the cross-sectional classes is independent of the fact whether the stress-strain relationship is inelastic or closer to elastic-perfectly plastic material.

The fact that the $b/t$ ratio at the classification border does not depend on parameter $n$ is attributed to the following. Consider two stress-strain relationships. Both stress-strain relationships have equal values for $f_{0.2}$ and $E$, but the first relationship has a low value for $n$ (inelastic material) while the second relationship has a high value for $n$ (more elastic-plastic material). The stress-strain relationships are illustrated in Figure 9.1.

![Figure 9.1 – Example of two stress-strain relations with equal values of $E$ and $f_{0.2}$ but different values of $n$](image)

This research has shown that slender sections (class 4 sections) collapse before the 0.2 % proof stress is reached in the gross-section (area indicated with arrow 4 in Figure 9.1). In this case, the inelastic material with low value of $n$ gives a lower ultimate buckling resistance compared to the more elastic-plastic material.

In case of very stocky cross-sections (e.g. a class 2 section), local buckling occurs only after a large axial strain (area indicated with arrow 3,2,1 in Figure 9.1), i.e. higher than the strain accompanying the 0.2 % proof stress. In this case, the inelastic material with low value of $n$ gives a higher ultimate buckling resistance compared to the more elastic-plastic material.

Sections with such a $b/t$ ratio that are on the border between classes 3 and 4 just reach, on average over the cross section, the 0.2 % proof stress at collapse (dotted line in Figure 9.1). Near the 0.2 % proof stress, there is no significant difference in strength between inelastic and more elastic-plastic material. This means that the value of $n$ does not matter.

### 9.2 Parameter study to check the equations

The validity of equations (9.2) and (9.3) is checked with finite element simulations for alloys 5083-H111 and 6060-T66 exposed to fire.

Using equation (9.2) for the plate dimensions results in values for the relative resistance of $0.94 \leq \rho \leq 1.03$. Although the values are unsafe for a number of cases ($\rho < 1$), the consequences for the fire resistance are small (assuming a constant heating rate, the case with $\rho = 0.94$ returns a fire resistance of 29.2 minutes instead of 30 minutes). This unsafe error, obtained by applying equation (9.2), is assumed to be acceptable. The obtained normalised force displacement trajectories of the plates are shown in Figure 9.2.
By applying equation (9.3), the relative resistance of the plates was $0.99 \leq \rho \leq 1.10$.

Additionally, a number of plates was analysed with various stress-strain relationships. Results of the individual plates are given in Annex A. The following results are obtained for plate dimensions determined with equation (9.2):

- For plates with the following arbitrary characteristics: $E = 50000 \text{ N/mm}^2$, $f_{0.2} = 20$ or $150 \text{ N/mm}^2$ and $n = 3.5$, $15$ or $25$, the resulting values for $\rho$ are: $0.97 \leq \rho \leq 1.12$.
- For plates with material characteristics of alloys 6060-T66 and 6082-T6 at room temperature ($E = 70000 \text{ N/mm}^2$, $f_{0.2} = 205$ or $250 \text{ N/mm}^2$ and $n = 25$ or $32$, respectively), the resulting value for $\rho$ is 0.97 (Figure 9.3 a.);
- For steel S235 and S355 ($E = 210000 \text{ N/mm}^2$, $f_y = 235$ or $355 \text{ N/mm}^2$ and $n$ is taken as 100), the resulting value for $\rho$ is 0.99 (Figure 9.3 b.).

The same types of analyses have been carried out with outstands. The b/t ratios are, again, determined with equation (9.2). The results are as follows:

- For aluminium alloys 5083-H111 and 6060-T66, the relative resistance is $1.00 \leq \rho \leq 1.18$ (Figure 9.4);
- For plates with the following arbitrary characteristics: $E = 50000 \text{ N/mm}^2$, $f_{0.2} = 20$ or $150 \text{ N/mm}^2$ and $n = 3.5$, $15$ or $25$, the resulting values for $\rho$ are: $0.97 \leq \rho \leq 1.17$;
• For plates with material characteristics of alloys 6060-T66 and 6082-T6 at room temperature \((E = 70000 \text{ N/mm}^2, f_{0.2} = 205 \text{ or } 250 \text{ N/mm}^2 \text{ and } n = 25 \text{ or } 32, \text{ respectively})\), the resulting value for \(\rho\) is 0.97 (Figure 9.5 a.);

• For steel S235 and S350 \((E = 210000 \text{ N/mm}^2, f_y = 235 \text{ or } 350 \text{ N/mm}^2 \text{ and } n \text{ is taken as } 100)\), the resulting value for \(\rho\) is 0.98. Figure 9.5 b. shows that the deformation capacity of the plates is larger than belonging to the classification border between classes 4 and 3. Equation (9.2) is thus conservative for this case.

![Figure 9.4 – Load-displacement trajectories of outstands of aluminium alloys exposed to fire with b/t ratios according to equation (9.2)](image)

- a. Alloy 5083-H111
- b. Alloy 6060-T66

![Figure 9.5 – Load-displacement trajectories of outstands at room temperature with b/t ratios according to equation (9.2)](image)

- a. Aluminium precipitation hardened alloys
- b. Steel

Analogous to Von Karman, it is investigated whether the buckling resistance of class 4 members can be approximated by taking into account a plate width \(b_{\text{lim}}\) which is on the border of classes 3 and 4 (i.e. using equation (9.3)) and ignoring the rest of the plate. This method, elaborated in Annex B, appears to give safe, but in some cases very conservative results.

### 9.3 Changing classification at increasing temperature

As the 0.2 % proof stress reduces faster than the modulus of elasticity of the investigated alloys 5083-H111 and 6060-T66, the ratio \(b/t\) at the border of the cross-sectional classes is higher than at room temperature.
This means that aluminium compression sections exposed to fire are in the same class or in a lower class than at room temperature. Contrarily, steel compression sections exposed to fire are, according to the design models in EN 1993-1-2, in the same class or in a higher class than at room temperature.

To illustrate this, Figure 9.6 gives the b/t ratios at the classification border of internal plates and outstands of alloys 5083-H111 and 6060-T66. The values at 20 °C are determined with the current design model in EN 1999-1-1, and with the values for $E$ and $f_{0,2}$ as determined in the tensile tests at room temperature (background report [13]). The values at elevated temperature are determined with the new design model.

![Figure 9.6 – b/t ratios at classification border for compression members according to EN 1999-1-1 (room temperature) and the new design model (fire)](image)

The width over thickness ratio of a fire exposed plate (between 175 and 375 °C) at the classification border, determined with equation (9.2), is 1.1 to 1.9 times higher than at room temperature. In EN 1999-1-2, the classification in fire should be taken equal to room temperature (In a note, the possibility is given to account for the actual constitutive properties in the classification in the National Annex. How these actual properties can be taken into account is not specified). Figure 9.6 shows that this rule is very conservative. It is recommended to modify this rule in future versions of the standard.

### 9.4 Chapter conclusions

An explicit function for the plate dimensions on the border between cross-sectional classes 3 and 4 of fire exposed aluminium plates is derived in this chapter (equation 9.3). The plate width to thickness ratio at this border appears to be dependent on the values for the 0.2 % proof stress and the modulus of elasticity, but not on the inelasticity of the stress-strain curve.

$$
\frac{b}{t_{\text{classlimit}}} = \sqrt{\frac{k_{cs} \cdot \pi^2 \cdot E}{12 \cdot (1 - \nu^2) \cdot (E \cdot 0.002 + f_{0,2})}}
$$

(9.3)

As the value of $f_{0,2}$ reduces faster than the value for $E$ for most fire exposed aluminium alloys, the ratio $t/b$ at the border between the cross-sectional classes is in fire higher than at room temperature. This favourable effect is not taken into account in the standard classification rule in EN 1999-1-2.
10 Design example: analysis of a simple frame

In order to show the application of the design rules given in this report on members with realistic dimensions, a design example is given in this chapter. The design example consists of the analysis of a simple aluminium frame exposed to fire.

Apart from local buckling of simply supported plates, the design example in this chapter also mentions briefly interaction between local and global buckling, and local buckling of plates with partially restrained edges.

10.1 Dimensions, loads and force distribution

Consider the frame consisting of two columns and a beam in Figure 10.1. The frame is statically determinate. Thermal expansion in length direction is thus not restrained, and the columns and beam can be checked individually.

The columns are square hollow sections, which is used to give an example of internal plates. Depending on the geometry of the column, the governing failure mechanism may be global buckling, local buckling, or interaction between global and local buckling. The sensitivity to local buckling of these columns depends on the ratio between plate width and plate thickness, while the sensitivity to global buckling depends on the ratio between the squared length and the moment of inertia.

Three sets of dimensions are considered for the column:
- A column with a small \( b/t \) ratio which is on the border between class 3 and 4 at room temperature. The moment of inertia is relatively small. This column is expected to fail through global buckling;
- A column with a very large \( b/t \) ratio, so that the relative slenderness for local buckling is large and the relative slenderness for global buckling is small.
- A column with intermediate cross-sectional slenderness.

The beam is an I-shaped beam, which is applied to study local buckling of an outstand. Therefore, the flanges have a large width over thickness ratio. The web has such dimensions that it is on the border between classes 3 and 4 at room temperature. This means that the flanges are partially restrained at the intersection with the web. The beam is laterally restrained, so that lateral-torsional buckling does not occur. The thickness of the flanges and web are equal.

Table 10.1 gives the dimensions of the frame considered. The loads to be considered in the design as well as the moment and force distribution (both at room temperature and for the fire situation) are listed in table 10.2.

It should be mentioned that the shape and dimensions of the beams and columns do not reflect an optimal design. The reason to apply these dimensions is to give some examples of the application of the design model.

In a real frame, the connections between the beam and the columns will generate eccentricity by the load application in the columns. This load eccentricity is not considered in this example. (In real designs, this eccentricity will reduce the load bearing capacity, and may not be neglected.)
The required fire resistance period is 30 minutes. The beam and columns are insulated, so that the members are heated with characteristics according to Table 10.3.

Table 10.3 – Heating rates and maximum temperature

<table>
<thead>
<tr>
<th>member</th>
<th>columns</th>
<th>beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>heating rate of the member temp after 30 minutes</td>
<td>9.3 °C / min</td>
<td>7.7 °C / min</td>
</tr>
<tr>
<td></td>
<td>300 °C</td>
<td>250 °C</td>
</tr>
</tbody>
</table>

All members are composed of alloy 6060-T66, with material properties as measured in the tensile tests carried out in this research. At room temperature, the 0.2 % proof stress is 200 N/mm² and the modulus of elasticity is 69000 N/mm². The constitutive relations in fire are according to paragraph 3.1 of this report.
10.2 Analyses at room temperature

To check the load bearing capacity of the members at room temperature, the members are analysed with finite element models and with hand calculations, using the equations and checks in EN 1999-1-1.

The finite element models consist of shell elements.

For the columns, an initial geometrical imperfection is applied, which consists of the summation of the following two imperfection patterns:

- The first Euler buckling mode of the column for local buckling, scaled in such a way that the maximum imperfection is equal to $e_{0,\text{loc}} = b / 500$.
- The first Euler buckling mode of the column for flexural buckling, scaled in such a way that the maximum imperfection is equal to $e_{0,\text{flex}} = L / 500$.

The value for $e_{0,\text{flex}}$ was determined by trial and error, in such a way that the ultimate buckling resistance at room temperature of column 1 agrees with the design model in EN 1999-1-1 for flexural buckling. This value for $e_{0,\text{flex}}$ is lower than prescribed by EN 1999-1-1 ($e_{0,\text{flex}} = L / 250$), but the code does not specify that local buckling imperfections should be applied as well.

For the beam, an initial geometrical imperfection is applied with a shape according to the first Euler buckling mode, scaled in such a way that the maximum imperfection is equal to $e_{0,\text{loc}} = b / 500$.

The ultimate load bearing resistance of the columns and beam are listed in Table 10.5 and Table 10.6, respectively. Figure 10.2 gives deformations of columns 1 and 2. It is shown that, at ultimate buckling resistance, the deformation of column 1 is a predominantly flexural buckling mode, while the deformation of column 2 is a combination of the flexural and local buckling modes. (All deformations are scaled with an arbitrary factor, so that the deformations are visible.)

Figure 10.2 – Deformations of columns at room temperature (deformations scaled)

a. Column 1 at ultimate buckling resistance
b. Column 1 at large deformations beyond ultimate buckling resistance
c. Column 2 at ultimate buckling resistance
In the hand calculations, the columns should be checked against local and flexural buckling. The steps according to Table 10.4 are made in the hand calculations. This is conform EN 1999-1-1. The hand calculations are elaborated in Annex D.

Column 1 has such plate dimensions, that the cross-section is on the border between classes 4 and 3, so that \( \rho_c = 1 \).

Column 2 has an equal gross area, but with a much larger \( b/t \) ratio. This column is in class 4. Due to the large plate width of this column, the column is not so sensitive to flexural buckling. This column fails when subjected to the load \( N_{sd} \).

Column 3 has an equal cross section, but plate dimensions in between columns 1 and 2. The column is in class 4. This column has the highest load-bearing capacity.

The calculation results are summarised in Table 10.5. The results of the hand calculations agree reasonable with the results of the finite element model.

### Table 10.4 – Steps in the hand calculation for the columns at room temperature

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
</table>
| 1   | Check whether the member is in class 4 or not                               | For internal plates of class A alloys without welds: class 4 if \( \beta/\varepsilon > 22 \).
|     |                                                                             | \( \varepsilon = \sqrt{250/f_{0.2}} \) (\( f_{0.2} \) in N/mm\(^2\)) and \( \beta = b/t \) (for members in compression) |
| 2   | If the member is in class 4, determine the relative resistance for local buckling \( \rho_c \). (Otherwise, \( \rho_c = 1 \)) | \( \rho_c = \frac{C_1}{\beta/\varepsilon} - \frac{C_2}{(\beta/\varepsilon)^2} \) for internal plates of class A alloy: \( C_1 = 32 \) and \( C_2 = 220 \) |
| 3   | Determine the effective area of the cross-section \( A_{eff} \)              | \( A_{eff} = \sum_n b_n \cdot t \cdot \rho_c \)                                           |
| 4   | Determine the elastic critical load for flexural buckling \( N_{cr, flex} \) | \( N_{cr, flex} = \frac{\pi^2 E \cdot I}{L_{buc}^2} \) \( I \) is the second moment of inertia of the gross cross-section \( L_{buc} \) is the buckling length (in this example frame, \( L_{buc} = L \)) |
| 5   | Determine the relative slenderness for flexural buckling \( \lambda_{rel, flex} \), using the effective area | \( \lambda_{rel, flex} = \sqrt{\frac{A_{eff} \cdot f_{0.2}}{N_{cr}}} \) |
| 6   | Determine the relative resistance \( \chi \) of the column                  | \( \chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \) but \( \chi < 1,0 \)
|     |                                                                             | Where \( \phi = 0.5(1 + \alpha(\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2) \)
|     |                                                                             | Class A: \( \alpha = 0,20, \bar{\lambda}_0 = 0,10 \)                                    |
| 7   | Determine the load bearing capacity of the column \( N_{b, Rd} \)          | \( N_{b, Rd} = \chi \cdot A_{eff} \cdot f_{0.2} \)                                           |

Figure 10.3 gives the first Euler buckling mode of the beam, and Figure 10.4 gives the deformations at the ultimate buckling resistance of the beam.
Table 10.5 – Results of the calculations of the columns at room temperature

<table>
<thead>
<tr>
<th>Member</th>
<th>hand calculation</th>
<th>FEM</th>
<th>hand / FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$</td>
<td>$\lambda_{rel,flb}$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>col 1</td>
<td>1</td>
<td>1,39</td>
<td>0,42</td>
</tr>
<tr>
<td>col 2</td>
<td>0,46</td>
<td>0,55</td>
<td>0,89</td>
</tr>
<tr>
<td>col 3</td>
<td>0,65</td>
<td>0,81</td>
<td>0,78</td>
</tr>
</tbody>
</table>

Figure 10.3 – First Euler buckling mode of the beam

Figure 10.4 – Deformation at ultimate buckling resistance of the beam

The web of the beam has such dimensions that it is on the border between classes 3 and 4. The flanges are in class 4. The upper flange is loaded in compression.

For flanges with long length over width ratios loaded in compression, the buckling factor $k_{cr}$ is: $k_{cr} = \frac{6(1-V)}{\pi^2} \approx 0.41$. If this value is used to determine the elastic critical buckling stress in the ultimate fibres, the result is: $\sigma_{cr,hand} = 76 \text{ N/mm}^2$, and the corresponding elastic critical moment of the beam is: $M_{cr,hand} = 72 \text{ kNm}$.

According to the finite element calculation however, the values for the elastic critical buckling stress and the elastic critical moment are $\sigma_{cr,FEM} = 138 \text{ N/mm}^2$ and $M_{cr,FEM} = 132 \text{ kNm}$, respectively.

The difference between finite element method and hand calculation is attributed to the fact that the web acts as a stiffener to the flanges. Consequently, the value of $k_{cr}$ in the
hand calculation is underestimated. (The fact that the moment varies along the beam length, so that the compressive force in the flange varies along the beam length, has a negligible influence on \( k_{cr} \)).

This research is not aimed at determining values for \( k_{cr} \) for such situations. The problem is solved as follows: in this example analysis, the value for \( k_{cr} \) is determined from the result of the finite element calculation. This value is used in the hand calculation as the buckling factor of the flanges.

The value of \( k_{cr} \) according to finite element simulations is 0.75 at room temperature and 0.72 at 250 °C.

The steps in the hand calculation are given in Table 10.7 and elaborated in Annex D. The results of the calculations are summarised in Table 10.6. Also for the beam, the result of the hand calculation is close to the results of the finite element model.

Table 10.6 – Results of the calculations of the beam at room temperature

<table>
<thead>
<tr>
<th>Beam</th>
<th>hand calculation</th>
<th>FEM</th>
<th>hand / FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k_{cr} )</td>
<td>( \rho_c )</td>
<td>( M_{Rd} )</td>
</tr>
<tr>
<td>values</td>
<td>0.75</td>
<td>0.65</td>
<td>149 kNm</td>
</tr>
</tbody>
</table>

Table 10.7 – Steps in the hand calculation for the beam at room temperature

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Check whether the member is in class 4 or not</td>
<td>For outstands of class A alloys without welds: class 4 if ( \beta / \varepsilon &gt; 6 ). ( \varepsilon = \sqrt{250 / f_{0.2}} ) (( f_{0.2} ) in N/mm²) ( \beta = 1/2 b/t ) (for flanges in compression)</td>
</tr>
<tr>
<td>2</td>
<td>If the member is in class 4, determine the elastic critical buckling load ( \sigma_{cr} )</td>
<td>( \sigma_{cr} = k_{cr} \frac{E \pi^2}{12(1-\nu^2)} \left( \frac{t}{1/2 b} \right)^2 )</td>
</tr>
<tr>
<td>3</td>
<td>If the member is in class 4, determine the relative buckling resistance ( \rho_c ). (Otherwise, ( \rho_c = 1 ))</td>
<td>( \rho_c = \frac{0.96}{\lambda_p} \left( 1 - \frac{0.23}{\lambda_p} \right) ) ¹</td>
</tr>
<tr>
<td>4</td>
<td>Determine the effective section modulus, with the effective flange thickness ( t_{eff,0} )</td>
<td>( t_{eff,0} = t \rho_c ) ( z_{ac,eff} = f_1(t_{eff,0}, t, h, b) ) position neutral axis ( W_{eff} = f_2(t_{eff,0}, t, h, b) )</td>
</tr>
<tr>
<td>5</td>
<td>Determine the load bearing capacity ( M_{Rd} )</td>
<td>( M_{Rd} = W_{eff} f_{0.2} )</td>
</tr>
</tbody>
</table>

¹) The constants 0.96 and 0.23 in the equation for \( \rho_c \) are obtained by rewriting the equations in EN 1999-1-1 as a function of \( \lambda_p \) instead of \( \beta / \varepsilon \)
10.3 Analyses for fire loading

The columns and beam are checked for fire conditions with the finite element models, applying the Dorn Harmathy model. Results are given in Table 10.8 and Table 10.10 for columns and beam, respectively.

The members are also checked with hand calculations, applying the derived Ramberg Osgood stress-strain relationship and the newly developed design model for local buckling.

So far, in this research, design models have not been derived which reflect interaction between local and flexural buckling of fire exposed members. In the hand calculation for fire, the procedure in EN 1999-1-1 for room temperature was applied, i.e. the relative slenderness for flexural buckling is determined using the effective area $A_{\text{eff}}$ and the elastic critical elastic buckling load of the gross cross-section $F_{cr,\theta}$.

EN 1999-1-2 gives a design model for flexural buckling of columns. In this design model, the relative slenderness and relative resistance for flexural buckling should be determined with the design model in EN 1999-1-1, with the material properties at room temperature. The obtained resistance should subsequently be divided by a creep factor equal to 1.2.

The design model is conservative, as it applies the ratio between the modulus of elasticity and the 0.2 % proof stress at room temperature, while this ratio increases at elevated temperature. Langhelle [12] carried out 14 tests on flexural buckling of columns which show that the design model is indeed conservative. Therefore, instead of using the design model for fire design in EN 1999-1-2, the design model according to EN 1999-1-1 with the material properties at the temperature considered is applied.

As the stress-strain relationship in fire is inelastic, the buckling curve for flexural buckling in EN 1999-1-1 of class B material is used. The procedure is given in Table 10.9 and elaborated in Annex D. Results are given in Table 10.8.

Table 10.8 – Results of the calculations of the columns exposed to fire

<table>
<thead>
<tr>
<th>Member</th>
<th>$\lambda_{\text{c,inel}}$</th>
<th>$\rho_{\theta,0}$</th>
<th>$\lambda_{\text{rel,flb}}$</th>
<th>$\chi_{\theta}$</th>
<th>$\theta_{cr}$</th>
<th>$\Delta \theta_{cr}$</th>
<th>$\Delta t_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>col 1</td>
<td>&gt; 1</td>
<td>1.00</td>
<td>0.92</td>
<td>0.61</td>
<td>299 °C</td>
<td>-10 °C</td>
<td>-1 min</td>
</tr>
<tr>
<td>col 2</td>
<td>1.48</td>
<td>0.48</td>
<td>0.43</td>
<td>0.86</td>
<td>242 °C</td>
<td>280 °C</td>
<td>4 min</td>
</tr>
<tr>
<td>col 3</td>
<td>1.01</td>
<td>0.77</td>
<td>0.59</td>
<td>0.80</td>
<td>300 °C</td>
<td>309 °C</td>
<td>9 °C</td>
</tr>
</tbody>
</table>

Table 10.8 shows that there is a good agreement between the critical temperatures determined with the finite element method and with the hand calculations, except for column 2. This is attributed to the following:

- The interaction of local and global buckling according to EN 1999-1-2 is conservative for the considered case. A finite element analysis of the column with lateral restraints against flexural buckling resulted in a critical temperature of 295 °C. The hand calculation without the reduction coefficient for flexural buckling gave a critical temperature of 278 °C. The difference between hand calculation and the finite element method is then reduced from 38 °C to 17 °C.

- The buckling curve derived for local buckling is conservative for the considered case. The buckling curve derived for local buckling in paragraph 8.2 is slightly (about 10%) conservative for the given alloy, temperature and inelastic relative slenderness. Applying a 10 % higher relative resistance and omitting the reduction...
The coefficient for flexural buckling results in a critical temperature of 293 °C, i.e. almost equal to the finite element analysis with restraints against local buckling. This indicates how sensitive the critical temperature is to the occurrence of local buckling for very slender cross-sections.

It should be noted that the relatively large difference in critical temperature between hand calculation and finite element analysis of 38 °C is equivalent to a difference in fire resistance of only 4 minutes. This error on the fire resistance is still regarded as acceptable. Moreover, the hand calculation results in a fire resistance which is on the safe side.

Column 1 is not in class 4, this column fails due to flexural buckling. The difference in critical temperature between the hand calculation and the finite element analysis of 10 °C is attributed to the fact that the buckling curve for flexural buckling used was not developed for fire exposed alloys. The buckling curve was developed for inelastic (class B) alloys at room temperature. It is well possible that the stress-strain relationships in fire are more inelastic as considered in the construction of the buckling curve for class B at room temperature.

For columns 2 and 3, local buckling is more dominant and flexural buckling is less dominant than in case of column 1. The critical temperatures of these columns are predicted more accurately than in case of column 1.

The relative resistance for local buckling in fire of columns 2 and 3, $\rho_c$, is considerably higher than at room temperature.

Table 10.9 – Steps in the hand calculation for the columns exposed to fire

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Check whether the member is in class 4 or not</td>
<td>Equation (9.2) of this report</td>
</tr>
<tr>
<td>2</td>
<td>If the member is in class 4, determine the relative resistance for local buckling $\rho_{c, \theta}$ (Otherwise, $\rho_{c, \theta} = 1$)</td>
<td>Equations (8.8) to (8.11) of this report</td>
</tr>
<tr>
<td>3</td>
<td>Determine the effective area of the cross-section $A_{\text{eff}}$</td>
<td>$A_{\text{eff}, \theta} = \sum b \cdot t \cdot \rho_{c, \theta}$</td>
</tr>
<tr>
<td>4</td>
<td>Determine the elastic critical load for flexural buckling</td>
<td>$N_{\text{cr}, \text{flex, } \theta} = \frac{\pi^2 E_{\theta} \cdot I}{L_{\text{dיע}}}$</td>
</tr>
<tr>
<td>5</td>
<td>Determine the relative slenderness for flexural buckling $\lambda_{\text{rel, flex, } \theta}$ using the effective area</td>
<td>$\lambda_{\text{rel, flex, } \theta} = \sqrt{\frac{A_{\text{eff}, \theta} \cdot f_{0.2, \theta}}{N_{\text{cr, } \theta}}}$</td>
</tr>
<tr>
<td>6</td>
<td>Determine the relative resistance $\chi$ of the column Use parameters $\alpha$ and $\overline{\lambda}_0$ of class B alloys</td>
<td>$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \frac{2}{\overline{\lambda}}}}$ but $\chi &lt; 1.0$</td>
</tr>
<tr>
<td></td>
<td>Where $\phi = 0.5(1 + \alpha(\overline{\lambda} - \overline{\lambda}_0) + \overline{\lambda}^2)$</td>
<td>Where $\phi = 0.5(1 + \alpha(\overline{\lambda} - \overline{\lambda}_0) + \overline{\lambda}^2)$</td>
</tr>
<tr>
<td></td>
<td>Class B: $\alpha = 0.32$, $\overline{\lambda}_0 = 0$</td>
<td>Class B: $\alpha = 0.32$, $\overline{\lambda}_0 = 0$</td>
</tr>
<tr>
<td>7</td>
<td>Determine the load bearing capacity of the column $N_{b, Rd, \theta}$</td>
<td>$N_{b, Rd, \theta} = \chi_{\theta} \cdot A_{\text{eff, } \theta} \cdot f_{0.2, \theta}$</td>
</tr>
</tbody>
</table>

The procedure for the hand calculation of the fire exposed beam is similar to that at room temperature, but using the newly developed design model for local buckling. The
steps are summarised in Table 10.11 and elaborated in Annex D. Results are given in Table 10.10. The results of the hand calculation agree well with the finite element analysis.

Table 10.10 – Results of the calculations of the beam exposed to fire

<table>
<thead>
<tr>
<th>Beam</th>
<th>hand calculation</th>
<th>FEM</th>
<th>hand - FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_{cr,\theta}$</td>
<td>$\lambda_{cr,\theta}$</td>
<td>$\rho_{c,\theta}$</td>
</tr>
<tr>
<td>values</td>
<td>0.72</td>
<td>1.03</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 10.11 – Steps in the hand calculation for the beam exposed to fire

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Check whether the member is in class 4 or not</td>
<td>Equation (9.2) of this report</td>
</tr>
<tr>
<td>2</td>
<td>If the member is in class 4, determine the relative buckling resistance $\rho_c$ (Otherwise, $\rho_c = 1$)</td>
<td>Equations (8.8) to (8.11) of this report</td>
</tr>
<tr>
<td>3</td>
<td>Determine the effective section modulus, with the effective flange thickness $t_{eff,fl,\theta}$</td>
<td>$t_{eff,\theta} = t \cdot \rho_{c,\theta}$, $z_{nc,eff,\theta} = f_1(t_{eff,\theta}, t, h, b)$, $W_{eff,\theta} = f_2(t_{eff,\theta}, t, h, b)$</td>
</tr>
<tr>
<td>4</td>
<td>Determine the load bearing capacity $M_{Rd,\theta}$</td>
<td>$M_{Rd,\theta} = W_{eff,\theta} \cdot f_{02,\theta}$</td>
</tr>
</tbody>
</table>

10.4 Chapter conclusions

This chapter shows that the critical temperature of members with realistic dimensions can be determined accurately with the new design model for local buckling (provided restrained thermal expansion in axial direction may be neglected in the design or does not occur).

Slightly conservative values for the fire resistance were obtained for the three columns analysed in this design example, when calculated using the interaction equation between local and global buckling in EN 1999-1-1. The fire resistance of the beam, with partially restrained, slender flanges, was accurately determined with the design model, with the precondition that the value for $k_{cr}$ has to be known. It should be mentioned, however, that these observations are based on one design example only. A larger parameter study is required to determine whether this holds in general.

The design model can be applied as an alternative for finite element calculations. This is advantageous, as members need to be modelled accurately, using many elements, in order to incorporate the local buckling failure mechanism in the finite element model. For the simple frame analysed in the current chapter, it is time consuming, but possible to model the frame so accurately with the finite element method. Many structures in practice, however, will contain many more members.
11 Conclusions and recommendations

The reasons why the relative ultimate buckling resistance for local buckling of fire exposed aluminium sections is different from that at room temperature are:
- The ratio between 0.2% proof stress and modulus of elasticity depends on the temperature;
- The stress-strain relationships are more inelastic as at room temperature;
- The stress distribution in the plate changes throughout the fire exposure, resulting in different stress-strain relations for the supported plate ends compared with the plate middle or unsupported plate ends (for internal plates and outstands, respectively);
- Restrained thermal expansion may influence the ultimate buckling resistance.

This report gives an investigation into the first three causes. The first and the third reasons cause that the relative ultimate buckling resistance is larger in fire than at room temperature. Their combined effect is partially counterbalanced by the second reason, which is unfavourable for local buckling in fire.

A design model is developed for local buckling of fire exposed internal plates and outstands of aluminium. The method incorporates the inelastic material properties. It gives a safe approximation of the ultimate buckling resistance of fire exposed aluminium alloys.

The method consists of a number of steps:
- The inelastic critical stress has to be determined in an iterative way;
- The inelastic relative slenderness has to be calculated;
- Buckling curves are determined which give the relation between the ultimate buckling resistance and the inelastic relative slenderness.

The method yields a simple classification border for sections in compression that fail before or after obtaining the plastic capacity (0.2% proof stress times gross area). Simulations show that the ratio b/t at the classification border of fire exposed plates increases with approximately 30 to 90%, compared to room temperature. The rule in EN 1999-1-2, that the classification in fire should be taken equal to room temperature, is thus very conservative.

An example of the analysis of a simple frame shows the application of the design model for members with realistic dimensions. The results of the hand calculations with the design model are reasonably close to the results of finite element calculations in terms of critical temperature and fire resistance (max. difference of 4 min on a fire resistance of 30 minutes). Applying the design model may be economical in comparison with finite element models in which the members have to be modelled with many (shell) elements in order to take into account local buckling.

Restrained thermal expansion is not taken into account in the current design model. This means that the model can be applied in cases where thermal expansion in axial direction does not occur (i.e. statically determinate structures) as well as in cases where restrained thermal expansion may be neglected, i.e. in a member analysis according to EN 1999-1-1. In practice, a member analysis is applied in combination with a nominal temperature-time curve such as the standard fire.

It is recommended to carry out research to the influence of restrained thermal expansion on the local buckling behaviour of members (not to be done in this PhD research).
The design model gives in principle results for any combination of 0.2 % proof stress, modulus of elasticity and hardening parameter in the Ramberg Osgood relationship. Results are checked extensively for alloys 5083-H111 and 6060-T66 in fire, and marginally for other combinations of material properties. It is recommended to investigate whether the design model is suited for local buckling of fire exposed steel structures and for aluminium structures at room temperature, and to compare the results with existing design models.

It is recommended to study whether the design model is also applicable to slender members in bending.
References


# Results of individual calculations

## A1 Internal plates, modelled with Ramberg-Osgood stress-strain curves

### Table A 1 – Internal plates, alloy 5083-H111, Ramberg-Osgood stress-strain curves

<table>
<thead>
<tr>
<th>θ [°C]</th>
<th>E [N/mm²]</th>
<th>ν</th>
<th>f₀.₂ [N/mm²]</th>
<th>n</th>
<th>b/t</th>
<th>ρ/FEM</th>
<th>σ_cr,inel</th>
<th>λ_rel,inel</th>
<th>ρ_hand</th>
<th>ρ_hand / ρ/FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>61600</td>
<td>0.35</td>
<td>96</td>
<td>5.6</td>
<td>33.39</td>
<td>0.95</td>
<td>89.5</td>
<td>1.04</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>200</td>
<td>61600</td>
<td>0.35</td>
<td>96</td>
<td>5.6</td>
<td>49.83</td>
<td>0.66</td>
<td>67.6</td>
<td>1.19</td>
<td>0.64</td>
<td>0.97</td>
</tr>
<tr>
<td>200</td>
<td>61600</td>
<td>0.35</td>
<td>96</td>
<td>5.6</td>
<td>87.20</td>
<td>0.43</td>
<td>29.7</td>
<td>1.80</td>
<td>0.41</td>
<td>0.94</td>
</tr>
<tr>
<td>200</td>
<td>61600</td>
<td>0.35</td>
<td>96</td>
<td>5.6</td>
<td>112.81</td>
<td>0.36</td>
<td>17.8</td>
<td>2.32</td>
<td>0.35</td>
<td>0.95</td>
</tr>
<tr>
<td>250</td>
<td>56375</td>
<td>0.37</td>
<td>67</td>
<td>4.8</td>
<td>38.77</td>
<td>0.90</td>
<td>58.6</td>
<td>1.07</td>
<td>0.83</td>
<td>0.92</td>
</tr>
<tr>
<td>250</td>
<td>56375</td>
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### Table A 2 – Internal plates, alloy 6060-T66, Ramberg-Osgood stress-strain curves

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Table A 3 – Internal plates, arbitrary material, Ramberg-Osgood stress-strain curves

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|-------|-------|--------------|-------|--------|------------|----------------|----------------|---------------|---------------|-------------------|
| 50000 | 0.35  | 20           | 15    | 67.75  | 0.80       | 17.7          | 1.06          | 0.84          | 1.06            |
| 50000 | 0.35  | 150          | 15    | 24.75  | 0.97       | 151.0         | 1.00          | 1.00          | 1.03            |
| 50000 | 0.35  | 20           | 3.5   | 77.46  | 0.57       | 11.4          | 1.32          | 0.54          | 0.95             |
| 50000 | 0.35  | 150          | 3.5   | 28.27  | 0.96       | 135.8         | 1.05          | 0.86          | 0.90             |

Table A 4 – Internal plates with b/t at the classification border, Ramberg-Osgood stress-strain curves

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## A.2 Internal plates, modelled with the Dorn Harmathy model

### Table A 5 – Internal plates, alloy 5083-H111, Dorn-Harmathy model

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### Table A 6 – Internal plates, alloy 6060-T66, Dorn-Harmathy model

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### A.3 Outstands, modelled with Ramberg-Osgood stress-strain curves

#### Table A 7 – Outstands, alloy 5083-H111, Ramberg-Osgood stress-strain curves

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#### Table A 8 – Outstands, alloy 6060-T66, Ramberg-Osgood stress-strain curves

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Table A 9 – Outstands, arbitrary material, Ramberg-Osgood stress-strain curves

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Table A 10 – Outstands with b/t at the classification border, Ramberg-Osgood stress-strain curves

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A.4 Outstands, modelled with Dorn-Harmathy model

Table A 11 – Outstands, alloy 5083-H111, Dorn-Harmathy model

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Table A 12 – Outstands, alloy 6060-T66, Dorn-Harmathy model

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B Geometrical imperfections

The initial imperfections were modelled by scaling the first Euler buckling mode. The size to be applied for the maximum imperfections for the parameter study was taken as 1/500 times the plate width. It is known that the ultimate buckling resistance of plates decreases as the initial imperfection increases. This Annex gives the results of a study to the sensitivity of the ultimate buckling resistance on the size of the maximum initial imperfection.

11.1.1 Size of the amplitude of the initial imperfections

The amplitude applied in the models is based on the following considerations.

EN 1999-1-1 does not give a standard value of the geometrical initial imperfection applied for the buckling curves for local buckling. The standard specifies in paragraph 5.3.1 that geometrical imperfections equal or less than the fundamental geometrical tolerances in prEN 1090-3 have to be considered in the buckling curves in EN 1999. prEN 1090-3 gives tolerances on e.g. the straightness of columns, but not on the flatness of plates. Only the out-of-straightness of webs of I-shaped girders is specified, but the given value of 1/40 x b is considered very conservative.

The allowed fabrication tolerances on the straightness of sections in various other codes depend on the plate width. Figure 11 gives an overview. Except for very small plates and for alloys other than 6xxx with normal fabrication tolerances, the allowed tolerance is approximately equal to or smaller than 1/200 times the plate width.

In reality however, geometrical imperfections larger than 1/1000 times the plate width are rarely encountered (Mazzolani [17]). Also in the test program of Mennink [20], the
measured out-of-straightness was in every occasion smaller than 1/1000 times the plate width, and on average it was 1/3600 x b.
Moreover, background report [16] showed that an imperfection pattern with a shape according to the first Euler buckling mode gives a considerably lower strength as compared to the actual (measured imperfection pattern). Also, residual stresses in extruded sections are very low (Mazzolani [17]).

In the calculations in this research, an equivalent imperfection of 1/500 x b was taken into account. This is considered as an upper limit for imperfections obtained in practice in extruded sections. Note that the equivalent imperfections for welded sections could be larger.

In this Annex, the influence of imperfections on the buckling resistance is determined for ratios between amplitude and plate width ranging from 1/10000 to 1/40.

11.1.2 Influence on the buckling resistance of square hollow sections

FEM calculations are carried out on internal plates with two types of stress-strain relationships:
- alloy with a significant inelastic behaviour;
- alloy with a more elastic-plastic behaviour.

Further three plate slendernesses are studied:
- $\lambda_{rel} = 1.75$: slender plate
- $\lambda_{rel} = 1.0$: plate of intermediate slenderness
- $\lambda_{rel} = 0.67$: stocky plate

For these six cases, analyses are carried out with various values for the imperfection. As an example, Figure 0.2 gives the buckling resistance of plates with $\lambda_{rel} = 1.0$ and an alloy with inelastic behaviour. Figure 0.3 gives the normalised buckling resistance as a function of the ratio between imperfection amplitude and plate width. The normalised buckling resistance is defined as the ratio between the buckling resistance for the amplitude considered and the buckling resistance for an amplitude $w_0/b = 1/500$.

Figure 0.4 and Figure 0.5 give the same results for the alloy with a more elastic-plastic behaviour.

It is shown that the amplitude can have a significant influence on the buckling resistance.

It is estimated that the error in the measurement of the imperfections is smaller than 0.05 mm, i.e. $w_0/b < 1/1000$. The maximum influence of this possible error on the buckling resistance is then 5%.
Figure 0.2 – Buckling resistances for plates with $\lambda_{rel} = 1.0$ for an inelastic stress-strain relationship with various values for the amplitude of the initial imperfection

Figure 0.3 – Normalised ultimate buckling resistance as a function of ratio between imperfection and plate width for an inelastic stress-strain relationship
Figure 0.4 – Buckling resistances for plates with $\lambda_{rel} = 1.0$ for a more elastic-plastic stress-strain relationship with various values for the amplitude of the initial imperfection

Figure 0.5 – Normalised ultimate buckling resistance as a function of ratio between imperfection and plate width for a more elastic-plastic stress-strain relationship
11.1.3 Influence on the buckling resistance of angles

FEM calculations are carried out on outstands with two types of stress-strain relationships:
- alloy with a significant inelastic behaviour;
- alloy with a more elastic-plastic behaviour.

Further three plate slendernesses are studied:
- \( \lambda_{rel} = 1.75 \): slender plate
- \( \lambda_{rel} = 1.0 \): plate of intermediate slenderness
- \( \lambda_{rel} = 0.67 \): stocky plate

For these six cases, analyses are carried out with various values for the imperfection. Figure 0.6 gives the buckling resistance of plates with \( \lambda_{rel} = 1.0 \) and an alloy with inelastic behaviour and Figure 0.7 gives the normalised buckling resistance as a function of the ratio between imperfection amplitude and plate width. Figure 0.8 and Figure 0.9 give the same results for the alloy with a more elastic-plastic behaviour.

In case of very inelastic behaviour, the strength at large deformations is larger than the first peak in the load-displacement diagram (Figure 0.6). This strength at large deformations is independent of the initial imperfections.

The initial imperfections of the angles of 5083-H111 were not measured. For these slender sections, the initial imperfection does not influence the ultimate buckling resistance.
Figure 0.7 – Normalised ultimate buckling resistance as a function of ratio between imperfection and plate width for an inelastic stress-strain relationship

Figure 0.8 – Buckling resistances for plates with $\lambda_{\text{rel}} = 1.0$ for a more elastic-plastic stress-strain relationship with various values for the amplitude of the initial imperfection
Figure 0.9 – Normalised ultimate buckling resistance as a function of ratio between imperfection and plate width for a more elastic-plastic stress-strain relationship
C Reduced width method to determine the resistance of class 4 sections

The derived b/t ratio at the border between cross-section classes 3 and 4 in chapter 9 allows for a simple but conservative estimation of the relative resistance of class 4 sections. A lower limit of the ultimate buckling resistance is obtained by only taking into account that part of the plate width that corresponds to the plate width at the border of classes 3 and 4. The stress in this part of the plate is taken equal to the 0.2 % proof stress at ultimate buckling resistance, while the stress in the other part of the cross-section is taken equal to zero, according to Figure 0.10. The reduced widths of outstands and internal plates are described with equations (9.6) and (9.7), respectively.

![Figure 0.10 – Effective width of method](image)

a. Plate at the border between class 3 and 4  
b. Plate in class 4

\[
\begin{align*}
\frac{b_{\text{eff}}}{b} &= \frac{t}{b} \cdot \sqrt{\frac{k_{\text{cr}} \cdot \pi^2 \cdot E}{12 \cdot (1-\nu^2) \cdot (E \cdot 0.002 + f_{0.2})}} \\
\frac{b_{\text{eff}}}{b} &= \frac{t}{b} \cdot \sqrt{\frac{k_{\text{cr}} \cdot \pi^2 \cdot E}{12 \cdot (1-\nu^2) \cdot (E \cdot 0.002 + f_{0.2})} \left(\frac{1}{2} + \frac{1}{4} \cdot \frac{f_{0.2} + E \cdot 0.002}{f_{0.2} + E \cdot 0.002 \cdot n}\right)}
\end{align*}
\]

The results of the finite element calculations are compared with the results using this method in Figure 0.11 for internal plates and in Figure 0.12 for outstands.
The figures show that this method, based on reducing the width, is more conservative than the method proposed in chapter 8 (buckling curve method, compare with Figure 8.11 and Figure 8.12). However, it is simpler to apply the reduced width method. The method based on the reduced width can be used as a first approximation of the relative resistance.

Note that the method based on reducing the width is similar to the effective width method of Von Karman [9]. In both cases, a plate width is considered with such dimensions that the ultimate critical buckling load is equal to the yield stress (or 0.2 % proof stress). In the method of Von Karman, the effective width is theoretically derived, and it represents a plate where the elastic critical stress is just equal to the yield stress. As shown by Winter [24], the Von Karman effective gives unsafe values for the buckling resistance. In the current research, the reduced width is determined with finite element models. The reduced width appears to be represented by a plate for which the inelastic critical stress is just equal to the 0.2 % proof stress. Figure 0.11 and Figure 0.12 show that the reduced width method of the current research gives (very) conservative values for the ultimate buckling resistance.
D Calculations for the simple frame

Loads and material properties

Life load

floor

roof

Self weight

floor

roof

Partial safety factors

Material properties

0.2 % proof stress

modulus of elasticity

Poisson ratio

0.2 % proof stress in fire (30 minutes)

\[
f_{02,\theta}(\theta) = \begin{cases} 
120 - 10 \frac{\theta - 175}{25} & \text{N/mm}^2 \quad \text{if} \quad 175 \leq \theta \leq 200 \\
110 - 10 \frac{\theta - 200}{25} & \text{N/mm}^2 \quad \text{if} \quad 200 < \theta \leq 225 \\
100 - 12 \frac{\theta - 225}{25} & \text{N/mm}^2 \quad \text{if} \quad 225 < \theta \leq 250 \\
88 - 13 \frac{\theta - 250}{25} & \text{N/mm}^2 \quad \text{if} \quad 250 < \theta \leq 275 \\
75 - 15 \frac{\theta - 275}{25} & \text{N/mm}^2 \quad \text{if} \quad 275 < \theta \leq 300 \\
60 - 14 \frac{\theta - 300}{25} & \text{N/mm}^2 \quad \text{if} \quad 300 < \theta \leq 325 \\
46 - 12 \frac{\theta - 325}{25} & \text{N/mm}^2 \quad \text{if} \quad 325 < \theta \leq 350 
\end{cases}
\]

modulus of elasticity in fire

\[E_{\theta}(\theta) := (69000 - 10 \theta - 0.21\theta^2) \text{N/mm}^2\]

inelasticity parameter

\[n_{\theta}(\theta) := 19 - 0.04 \theta\]

Poisson ratio

\[\nu_{\theta}(\theta) := 0.37\]
Dimensions

span \( L := 7.2 \text{m} \)
distance in between frames \( H := 3.6 \text{m} \)
height \( h := 2.7 \text{m} \)
thickness column 1 \( t_1 := 3.4 \text{mm} \)
width column 1 \( b_1 := 25t_1 \) \( b_1 = 85 \text{mm} \)

Distribution of forces and moments, room temp

Distributed load beam
\[ q_{Sd} := \left( \gamma_G \cdot q_{\text{floor.rep}} + \gamma_Q \cdot q_{\text{floor.rep}} \right) H \]
\[ q_{Sd} = 22.68 \frac{\text{kN}}{\text{m}} \]

Force column
\[ F_{Sd} := \left( \gamma_G \cdot q_{\text{roof.rep}} + \gamma_Q \cdot q_{\text{roof.rep}} \right) H \frac{1}{2} L \]
\[ F_{Sd} = 10.886 \text{kN} \]

Moment beam
\[ M_{Sd} := \frac{1}{8} q_{Sd} L^2 \]
\[ M_{Sd} = 146.966 \text{kN.m} \]

Normal force column
\[ N_{Sd} := q_{Sd} \frac{1}{2} L + F_{Sd} \]
\[ N_{Sd} = 92.534 \text{kN} \]

Distribution of forces and moments, fire

Distributed load beam, fire
\[ q_{Sd.\theta} := \left( \gamma_{\text{floor.rep}} + q_{\text{floor.rep}} \cdot \psi_{\text{floor}} \right) H \]
\[ q_{Sd.\theta} = 10.8 \frac{\text{kN}}{\text{m}} \]

Force column, fire
\[ F_{Sd.\theta} := \left( \gamma_{\text{roof.rep}} + q_{\text{roof.rep}} \cdot \psi_{\text{roof}} \right) H \frac{1}{2} L \]
\[ F_{Sd.\theta} = 2.592 \text{kN} \]

Moment beam
\[ M_{Sd.\theta} := \frac{1}{8} q_{Sd.\theta} L^2 \]
\[ M_{Sd.\theta} = 69.984 \text{kN.m} \]

Normal force column
\[ N_{Sd.\theta} := q_{Sd.\theta} \frac{1}{2} L + F_{Sd.\theta} \]
\[ N_{Sd.\theta} = 41.472 \text{kN} \]
Check column 1, room temperature

Dimensions column 1

\[ b_1 = 85 \text{ mm} \quad t_1 = 3.4 \text{ mm} \]

Buckling length column

\[ \lambda_{\text{buc}} := \frac{h}{h_{\text{buc}}} \]

Area cross-section

\[ A_1 := b_1^2 - (b_1 - 2t_1)^2 \quad A_1 = 1.11 \times 10^3 \text{ mm}^2 \]

Moment of inertia

\[ I_1 := \frac{1}{12} \left[ b_1^4 - (b_1 - 2t_1)^4 \right] \quad I_1 = 1.234 \times 10^6 \text{ mm}^4 \]

Euler buckling load for flexural buckling

\[ F_{E,1} := \frac{\pi^2 E I_1}{h_{\text{buc}}^2} \quad F_{E,1} = 115.24 \text{kN} \]

Relative slenderness for flexural buckling

\[ \lambda_{\text{rel.fb.1}} := \sqrt{\frac{A_1 f_{0.2}}{F_{E,1}}} \quad \lambda_{\text{rel.fb.1}} = 1.388 \]

Parameters flex. buckl.

\[ \alpha := 0.2 \quad \lambda_0 := 0.10 \]

\[ \phi_1 := 0.5 \left( 1 + \alpha \left( \lambda_{\text{rel.fb.1}} - \lambda_0 \right) + \lambda_{\text{rel.fb.1}}^2 \right) \quad \phi_1 = 1.592 \]

Relative resistance

\[ \chi_1 := \frac{1}{\phi_1 + \sqrt{\phi_1^2 - \lambda_{\text{rel.fb.1}}^2}} \quad \chi_1 = 0.422 \]

Load bearing resistance column

\[ F_{Rd,1} := \chi_1 A_1 f_{0.2} \quad F_{Rd,1} = 93.60 \text{kN} \]

Unity check column

\[ u_{\text{col.1}} := \frac{N_{\text{Sd}}}{F_{Rd,1}} \quad u_{\text{col.1}} = 0.989 \]
Check column 1, fire

Temperature and mat. prop.  \[ \theta := 299 \quad F_\theta(\theta) = 4.724 \times 10^4 \dfrac{N}{mm^2} \quad f_{02,\theta}(\theta) = 6.06 \times 10^7 \text{ Pa} \]

Euler buckling load for flexural buckling  \[ F_{E,1,\theta}(\theta) := \dfrac{\pi^2 F_\theta(\theta) I_1}{L_{buc}^2} \quad F_{E,1,\theta}(\theta) = 78.89 \text{kN} \]

Relative slenderness for flexural buckling  \[ \lambda_{rel.fb.1,\theta}(\theta) := \dfrac{A_1 f_{02,\theta}(\theta)}{F_{E,1,\theta}(\theta)} \quad \lambda_{rel.fb.1,\theta}(\theta) = 0.923 \]

Buckling parameters  \[ \alpha_\theta := 0.32 \quad \theta_{\alpha,\theta} = 0.0 \]

\[ \phi_{1,\theta}(\theta) := 0.5 \left[ 1 + \alpha_\theta \left( \lambda_{rel.fb.1,\theta}(\theta) - \theta_{\alpha,\theta} \right) + \lambda_{rel.fb.1,\theta}(\theta)^2 \right] \quad \phi_{1,\theta}(\theta) = 1.074 \]

Relative resistance  \[ \chi_{1,\theta}(\theta) := \dfrac{1}{\phi_{1,\theta}(\theta) + \sqrt{\phi_{1,\theta}(\theta)^2 - \lambda_{rel.fb.1,\theta}(\theta)^2}} \quad \chi_{1,\theta}(\theta) = 0.616 \]

Load bearing resistance column  \[ F_{Rd,1,\theta}(\theta) := \chi_{1,\theta}(\theta) \cdot A_1 f_{02,\theta}(\theta) \quad F_{Rd,1,\theta}(\theta) = 41.45 \text{kN} \]

Unity check column  \[ u_{col,1} := \dfrac{N_{sd,\theta}}{F_{Rd,1,\theta}(\theta)} \quad u_{col,1} = 1.001 \]
Check column 2, room temperature

Dimensions column 2

\[ b_2 := 140 \text{mm} \quad t_2 := 2 \text{mm} \]

Gross area

\[ A_2 := b_2^2 - \left( b_2 - 2t_2 \right)^2 \]

\[ A_2 = 1.104 \times 10^3 \text{ mm}^2 \]

Ratio width/thickness

\[ \beta_2 := \frac{b_2}{t_2} \]

\[ \beta_2 = 70 \]

Design model local buckling

\[ C_1 := 32 \quad C_2 := 220 \]

Design model for flexural buckling

\[ e := \sqrt[2]{\frac{250 \text{N mm}^{-2}}{f_{02}}} \]

Relative slenderness for flexural buckling

\[ \lambda_{\text{rel.fb.2}} := \frac{A_{\text{eff.2}} f_{02}}{F_{E,2}} \]

Relative resistance for flexural buckling

\[ \rho_{c.2} := \frac{\beta_2}{\left( \frac{e}{\varepsilon} \right)^2} \]

\[ \rho_{c.2} = 0.455 \]

Effective cross-section

\[ A_{\text{eff.2}} := \rho_{c.2} A_2 \]

\[ A_{\text{eff.2}} = 502.297 \text{ mm}^2 \]

Moment of inertia gross cross-section

\[ I_2 := \frac{1}{12} b_2^4 - \left( b_2 - 2t_2 \right)^4 \]

\[ I_2 = 3.505 \times 10^6 \text{ mm}^4 \]

Euler buckling load for flexural buckling

\[ F_{E,2} := \pi^2 E I_2 \]

\[ F_{E,2} = 327.407 \text{ kN} \]

Relative slenderness for flexural buckling

\[ \lambda_{\text{rel.fb.2}} = 0.554 \]

Parameters flex. buckl.

\[ \alpha := 0.2 \quad \lambda_\circ := 0.10 \]

\[ \phi_2 := 0.4 \left[ 1 + \alpha \left( \lambda_{\text{rel.fb.2}} - \lambda_\circ \right) + \lambda_{\text{rel.fb.2}}^2 \right] \]

\[ \phi_2 = 0.699 \]

Relative resistance

\[ \chi_2 := \frac{1}{\phi_2 + \sqrt{\phi_2^2 - \lambda_{\text{rel.fb.2}}^2}} \]

\[ \chi_2 = 0.889 \]

Load bearing resistance column

\[ F_{Rd.2} := \chi_2 A_{\text{eff.2}} f_{02} \]

\[ F_{Rd.2} = 89.31 \text{ kN} \]

Unity check column

\[ \psi_{\text{col.2}} := \frac{N_{\text{sd}}}{F_{Rd.2}} \]

\[ \psi_{\text{col.2}} = 1.036 \]
Check column 2, fire

Temperature and material properties
\[ \theta := 242 \quad E_\theta(\theta) = 5.428 \times 10^4 \text{ N mm}^{-2} \quad f_{02,\theta}(\theta) = 91.84 \text{ N mm}^{-2} \]
\[ \mu_\theta(\theta) = 9.32 \quad v_\theta(\theta) = 0.37 \]

Estimation \( \sigma_{\text{cr.inel}} \)
\[ \sigma_{\text{cr.inel},2,\theta} := \frac{f_{02,\theta}(\theta)}{2} \]

Determination \( \sigma_{\text{cr.inel}} \)
Given
\[ 1.4 E_\theta(\theta) 0.002 \left( \frac{\sigma_{\text{cr.inel},2,\theta}}{f_{02,\theta}(\theta)} \right) \frac{n_\theta(\theta)}{\sigma_{\text{cr.inel},2,\theta}} + \sigma_{\text{cr.inel},2,\theta} = 4 \frac{\pi^2}{12} \frac{E_\theta(\theta)}{1 - v_\theta(\theta)^2} \left( \frac{l_2}{b_2} \right)^2 \]

Output \( \sigma_{\text{cr.inel},2,\theta} \)
\[ \sigma_{\text{cr.inel},2,\theta} = 42.119 \text{ N mm}^{-2} \]

Relative slenderness (inelastic)
\[ \lambda_{\text{inel},2,\theta} := \frac{f_{02,\theta}(\theta)}{\sigma_{\text{cr.inel},2,\theta}} \]
\[ \lambda_{\text{inel},2,\theta} = 1.477 \]

Relative resistance for local buckling
\[ \rho_{c,2,\theta} := \frac{1}{\lambda_{\text{inel},2,\theta}} \left( 1 + \frac{0.2}{\lambda_{\text{inel},2,\theta}} + \frac{2.5}{\lambda_{\text{inel},2,\theta}} + \frac{2.3}{\lambda_{\text{inel},2,\theta}} \right) \]
\[ \rho_{c,2,\theta} = 0.476 \]

Effective cross-section
\[ A_{\text{eff},2,\theta} := \rho_{c,2,\theta} A_2 \]
\[ A_{\text{eff},2,\theta} = 525.768 \text{ mm}^2 \]

Euler buckling load for flexural buckling
\[ F_{E,2,\theta}(\theta) := \frac{\pi^2}{12} \frac{E_\theta(\theta)}{1 - v_\theta(\theta)^2} \left( \frac{l_2}{b_2} \right)^2 \]
\[ F_{E,2,\theta}(\theta) = 257.568 \text{ kN} \]

Relative slenderness for flexural buckling
\[ \lambda_{\text{rel.fb.},2,\theta}(\theta) := \frac{A_{\text{eff},2,\theta} f_{02,\theta}(\theta)}{F_{E,2,\theta}(\theta)} \]
\[ \lambda_{\text{rel.fb.},2,\theta}(\theta) = 0.433 \]

Buckling parameters
\[ \phi_{2,\theta}(\theta) := \frac{1}{2} \left( 1 + \alpha_\theta \left( \lambda_{\text{rel.fb.},2,\theta}(\theta) - \lambda_{\text{rel.fb.},2,\theta}(\theta)^2 \right) + \lambda_{\text{rel.fb.},2,\theta}(\theta)^2 \right) \]
\[ \phi_{2,\theta}(\theta) = 0.663 \]

Relative resistance
\[ \chi_{2,\theta}(\theta) := \frac{1}{\phi_{2,\theta}(\theta) + \sqrt{\phi_{2,\theta}(\theta)^2 - \lambda_{\text{rel.fb.},2,\theta}(\theta)^2}} \]
\[ \chi_{2,\theta}(\theta) = 0.858 \]

Load bearing resistance column
\[ F_{Rd,2,\theta}(\theta) := \chi_{2,\theta}(\theta) A_{\text{eff},2,\theta} f_{02,\theta}(\theta) \]
\[ F_{Rd,2,\theta}(\theta) = 41.443 \text{ kN} \]

Unity check column
\[ u_{\text{col},2} := \frac{N_{\text{sd},\theta}}{F_{Rd,2,\theta}(\theta)} \]
\[ u_{\text{col},2} = 1.001 \]
Check column 3, room temperature

Dimensions column 3
\[ t_3 := 2.5\, \text{mm} \quad \text{and} \quad b_3 := 115\, \text{mm} \]

Gross area
\[ A_3 := b_3^2 - (b_3 - 2t_3)^2 \quad A_3 = 1.125 \times 10^3 \, \text{mm}^2 \]

Ratio width/thickness
\[ \beta_3 := \frac{b_3}{t_3} \quad \beta_3 = 46 \]

Relative resistance for local buckling
\[ \rho_{c,3} := \frac{C_1}{\left(\frac{\beta_3}{\epsilon}ight)} - \frac{C_2}{\left(\frac{\beta_3}{\epsilon}ight)^2} \quad \rho_{c,3} = 0.648 \]

Effective cross-section
\[ A_{\text{eff},3} := \rho_{c,3} A_3 \quad A_{\text{eff},3} = 728.776\, \text{mm}^2 \]

Moment of inertia
\[ I_3 := \frac{1}{12} b_3^4 - (b_3 - 2t_3)^4 \quad I_3 = 2.374 \times 10^6 \, \text{mm}^4 \]

Euler buckling load for flexural buckling
\[ F_{E,3} := \frac{\pi^2 E I_3}{b_{\text{buc}}^2} \quad F_{E,3} = 221.79\, \text{kN} \]

Relative slenderness for flexural buckling
\[ \lambda_{\text{rel},\text{fb},3} := \sqrt{\frac{A_{\text{eff},3} f_0^2}{F_{E,3}}} \quad \lambda_{\text{rel},\text{fb},3} = 0.811 \]

Parameters flex. buckl.
\[ \alpha := 0.2 \quad \lambda_\alpha := 0.10 \]

\[ \phi_3 := 0.9 \left[ 1 + \alpha \left( \lambda_{\text{rel},\text{fb},3} - \lambda_\alpha \right) + \lambda_{\text{rel},\text{fb},3} \right] \quad \phi_3 = 0.9 \]

Relative resistance
\[ \chi_3 := \frac{1}{\phi_3 + \sqrt{\phi_3^2 - \lambda_{\text{rel},\text{fb},3}^2}} \quad \chi_3 = 0.775 \]

Load bearing resistance column
\[ F_{Rd,3} := \chi_3 A_{\text{eff},3} f_0^2 \quad F_{Rd,3} = 113.007\, \text{kN} \]

Unity check column 3
\[ u_{c,\text{col},3} := \frac{N_{\text{sd}}}{F_{Rd,3}} \quad u_{c,\text{col},3} = 0.819 \]
Check column 3, fire

Temperature and material properties
\[ \theta := 300 \quad E_0(\theta) = 4.71 \times 10^4 \text{ N mm}^{-2} \quad f_{02}(\theta) = 60 \text{ N mm}^{-2} \] \[ a_0(\theta) = 7 \quad \nu_0(\theta) = 0.37 \]

Estimation \( \sigma_{cr,inel} \)
\[ \sigma_{cr,inel,3.\theta} := \frac{f_{02}(\theta)}{2} \]

Determination \( \sigma_{cr,inel} \)
\[ 1.4 E_0(\theta) \cdot 0.002 \left( \frac{\sigma_{cr,inel,3.\theta}}{f_{02}(\theta)} \right)^2 + \sigma_{cr,inel,3.\theta} = 4 \cdot \frac{\pi^2 E_0(\theta)}{12(1-\nu_0(\theta)^2)} \left( \frac{t_3}{b_3} \right)^2 \]
\[ A := \text{Find} \left( \sigma_{cr,inel,3.\theta} \right) \]

Output \( \sigma_{cr,inel} \)
\[ \sigma_{cr,inel,3.\theta} := A \quad \sigma_{cr,inel,3.\theta} = 49.676 \text{ N mm}^{-2} \]

Relative slenderness (inelastic)
\[ \lambda_{inel,3.\theta} := \frac{f_{02}(\theta)}{\sigma_{cr,inel,3.\theta}} \]

Relative resistance for local buckling
\[ \rho_{c,3.\theta} := \frac{1}{\lambda_{inel,3.\theta}} \left( 1 + \frac{0.2}{\lambda_{inel,3.\theta}} - \frac{2.5}{\lambda_{inel,3.\theta}} + \frac{2.3}{3 \lambda_{inel,3.\theta}} \right) \]
\[ \rho_{c,3.\theta} = 0.769 \]

Effective cross-section
\[ A_{eff,3.\theta} := \rho_{c,3.\theta} A_3 \]

Euler buckling load for flexural buckling
\[ F_{E,3.\theta}(\theta) := \frac{\pi^2 E_0(\theta)}{2 h_{buc}^2} I_3 \quad F_{E,3.\theta}(\theta) = 151.396 \text{kN} \]

Relative slenderness for flexural buckling
\[ \lambda_{rel.fb,3.\theta}(\theta) := \frac{A_{eff,3.\theta} f_{02}(\theta)}{\sqrt{F_{E,3.\theta}(\theta)}} \quad \lambda_{rel.fb,3.\theta}(\theta) = 0.585 \]

Buckling parameters
\[ \phi_{3.\theta}(\theta) := 0.5 \left[ 1 + \alpha_0 \left( \lambda_{rel.fb,3.\theta}(\theta) - \lambda_{rel.fb,3.\theta}(0) \right) + \lambda_{rel.fb,3.\theta}(\theta) \right]^2 \]
\[ \phi_{3.\theta}(\theta) = 0.765 \]

Relative resistance
\[ \chi_{3.\theta}(\theta) := \frac{1}{\phi_{3.\theta}(\theta) + \sqrt{(\phi_{3.\theta}(\theta))^2 - \lambda_{rel.fb,3.\theta}(\theta)^2}} \]
\[ \chi_{3.\theta}(\theta) = 0.795 \]

Load bearing resistance column
\[ F_{Rd.3.\theta}(\theta) := \chi_{3.\theta}(\theta) A_{eff,3.\theta} f_{02}(\theta) \quad F_{Rd.3.\theta}(\theta) = 41.263 \text{kN} \]

Unity check column
\[ u_{col,3.\theta} := \frac{N_{sd,\theta}}{F_{Rd.3.\theta}(\theta)} \quad u_{col,3.\theta} = 1.005 \]
Check beam, room temperature

Dimensions beam

- thickness web beam: \( t_w := 7\text{mm} \)
- height beam: \( h_b := 430\text{mm} \)
- thickness flange beam: \( t_f := 7.0\text{mm} \)
- width beam: \( b_b := 260\text{mm} \)

Determination of buckling factor

- second moment of inertia of the gross cross-section: \( I_{b,\text{gross}} := \frac{1}{12} b_b h_b^3 - \frac{2}{12} \left( \frac{b_b - t_w}{2} \right) \left( b_b - 2 t_f \right)^3 \)
  \[ I_{b,\text{gross}} = 2.048 \times 10^8 \text{mm}^4 \]
- section modulus of the gross cross-section: \( W_{b,\text{gross}} := \frac{I_{b,\text{gross}}}{0.5 h_b} \)
  \[ W_{b,\text{gross}} = 9.527 \times 10^5 \text{mm}^3 \]
- Buckling factor hand calculation: \( k_{\text{cr.hand}} := 0.41 \)
- Elastic critical stress hand calculation: \( \sigma_{\text{cr.hand}} := \frac{k_{\text{cr.hand}} \pi^2 E}{12 \left( 1 - \nu^2 \right)} \left( \frac{t_f}{0.5 b_b} \right)^2 \)
  \[ \sigma_{\text{cr.hand}} = 75.707 \text{N/mm}^2 \]
- Elastic critical buckling moment hand calculation: \( M_{\text{cr.hand}} := \sigma_{\text{cr.hand}} W_{b,\text{gross}} \)
  \[ M_{\text{cr.hand}} = 72.127\text{kN m} \]
- Result FEM Euler analysis
  - Elastic critical stress FEM: \( \sigma_{\text{cr.FEM}} := \frac{M_{\text{cr.FEM}}}{W_{b,\text{gross}}} \)
  \[ \sigma_{\text{cr.FEM}} = 138.31 \text{N/mm}^2 \]
  - Elastic critical moment FEM: \( M_{\text{cr.FEM}} := \frac{1}{8} q_{\text{cr.FEM}} L^2 \)
  \[ M_{\text{cr.FEM}} = 131.77 \text{kN m} \]
  - Elastic critical stress FEM: \( \sigma_{\text{cr.FEM}} := \frac{M_{\text{cr.FEM}}}{W_{b,\text{gross}}} \)
  \[ \sigma_{\text{cr.FEM}} = 138.31 \text{N/mm}^2 \]
- Buckling factor FEM: \( k_{\text{cr.FEM}} := \frac{12 \left( 1 - \nu^2 \right)}{\pi^2} \left( \frac{0.5 b_b}{t_f} \right)^2 \)
  \[ k_{\text{cr.FEM}} = 0.749 \]
- Relative slenderness: \( \lambda_b := \sqrt{\frac{q_{\text{cr}}}{\sigma_{\text{cr.FEM}}}} \)
  \[ \lambda_b = 1.203 \]
Check on local buckling

Relative slenderness

\[ \lambda_b := \frac{f_{02}}{\sigma_{cr,FEM}} \]

\[ \lambda_b = 1.203 \]

Buckling factors outstand

\[ c_1 := 0.96 \]

\[ c_2 := 0.23 \]

Relative resistance flange

\[ \rho_{c.b} := \frac{c_1}{\lambda_b} \left( 1 - \frac{c_2}{\lambda_b} \right) \]

\[ \rho_{c.b} = 0.646 \]

effective flange thickness

\[ t_{fl,eff} := \frac{t_{fl}}{\rho_{c.b}} \]

\[ t_{fl,eff} = 4.519 \text{mm} \]

effective height

\[ h_{b,eff} := h_b - \frac{1}{2} \left( t_{fl} - t_{fl,eff} \right) \]

\[ h_{b,eff} = 428.76 \text{mm} \]

neutral axis

\[ \frac{b_b t_f l 0.5 t_f}{b_b t_f l + \left( h_{b,eff} - t_{fl} - t_{fl,eff} \right) t_w \left( 0.5 h_{b,eff} + t_{fl} \right) + b_b t_{fl,eff} \left( h_{b,eff} - \frac{1}{2} t_{fl,eff} \right)} \]

\[ \gamma_{nc} = 195.092 \text{mm} \]

second moment of inertia

\[ I_{b,eff} := \frac{1}{12} b_b t_f l^3 + \frac{1}{12} b_b t_{fl,eff}^3 + \frac{1}{12} \left( h_{b,eff} - t_{fl} - t_{fl,eff} \right)^3 t_w \]

\[ + b_b t_f l \left( \gamma_{nc} - 0.5 t_f \right)^2 + b_b t_{fl,eff} \left( h_{b,eff} - \gamma_{nc} - 0.5 t_{fl,eff} \right)^2 \]

\[ + \left( h_b - 2 t_f \right) t_w \left( 0.5 h_{b,eff} + t_f - \gamma_{nc} \right)^2 \]

\[ I_{b,eff} = 1.741 \times 10^8 \text{mm}^4 \]

section modulus

\[ W_{b,eff} := \min \left( \frac{I_{b,eff}}{h_{b,eff} - \gamma_{nc}}, \frac{I_{b,eff}}{\gamma_{nc}} \right) \]

\[ W_{b,eff} = 7.452 \times 10^5 \text{mm}^3 \]

load bearing resistance beam

\[ M_{Rd} := W_{b,eff} f_{02} \]

\[ q_{Rd} := \frac{M_{Rd}}{L^2} \]

\[ M_{Rd} = 149.03 \text{kN} \cdot \text{m} \]

\[ q_{Rd} = 22.999 \text{kN/m} \]

unity check beam

\[ u_{c,beam} := \frac{M_{sd}}{M_{Rd}} \]

\[ u_{c,beam} = 0.986 \]
Check beam in fire

Temperature and material properties

\[ \theta := 257 \quad E_0(\theta) = 5.256 \times 10^4 \text{N}\cdot\text{mm}^{-2} \quad f_{0.2}(\theta) = 8.72 \quad v_0(\theta) = 0.37 \]

Result FEM Euler analysis

\[ q_{\text{cr.FEM}} := \frac{1}{8} q_{\text{cr.FEM}} L^2 \]

Elastic critical buckling moment

\[ M_{\text{cr.FEM}} := \frac{q_{\text{cr.FEM}} L^2}{8} \]

Elastic critical stress

\[ \sigma_{\text{cr.FEM}} := \frac{M_{\text{cr.FEM}}}{W_{b,\text{gross}}} \]

Buckling factor

\[ k_{\text{cr.FEM}} := \sigma_{\text{cr.FEM}} \frac{12 (1 - \nu_0(\theta)^2)}{2 \pi E_0(\theta)} \left( \frac{0.5 b_b}{0.5 t_f} \right)^2 \]

Relative slenderness (inelastic)

\[ \lambda_{\text{inel}} := \frac{f_{0.2}(\theta)}{\sigma_{\text{cr.inel}}} \]

Relative resistance for local buckling

\[ \rho_{c,\theta} := \frac{1}{\lambda_{\text{inel}}} \left( 1 + \frac{3.5}{\lambda_{\text{inel}}^2} + \frac{2.5}{\lambda_{\text{inel}}^3} \right) \]

Effective flange thickness

\[ t_{f\text{f}} := t_f \rho_{c,\theta} \]

Effective height

\[ h_{b,\text{eff}} := h_b - \frac{1}{2} (t_f - t_{f\text{f}}) \]

Output \( \sigma_{\text{cr.inel}} \)

\[ \sigma_{\text{cr.inel}} := \frac{f_{0.2}(\theta)}{2} \]

Determination \( \sigma_{\text{cr.inel}} \)

Given

\[ E_0(\theta) = 0.002 \left( \frac{\sigma_{\text{cr.inel}}}{f_{0.2}(\theta)} \right) + \sigma_{\text{cr.inel}} = k_{\text{cr.FEM}} \frac{2 \pi E_0(\theta)}{12 (1 - \nu_0(\theta)^2)} \left( \frac{t_f}{0.5 b_b} \right)^2 \]

\[ A := \text{Find} \left( \sigma_{\text{cr.inel}} \right) \]

Relative slenderness (inelastic)

\[ \lambda_{\text{inel}} := \frac{f_{0.2}(\theta)}{\sigma_{\text{cr.inel}}} \]

Relative resistance for local buckling

\[ \rho_{c,\theta} \]

Effective flange thickness

\[ t_{f\text{f}} := t_f \rho_{c,\theta} \]

Effective height

\[ h_{b,\text{eff}} := h_b - \frac{1}{2} (t_f - t_{f\text{f}}) \]

\[ q_{\text{cr.FEM}} := 15.17 \text{kN}\cdot\text{m} \]

\[ M_{\text{cr.FEM}} = 98.308 \text{kN}\cdot\text{m} \]

\[ \sigma_{\text{cr.FEM}} = 103.187 \text{N/mm}^2 \]

\[ k_{\text{cr.FEM}} = 0.711 \]

\[ \sigma_{\text{cr.inel}} := 73.089 \text{N/mm}^2 \]

\[ \rho_{c,\theta} = 0.851 \]

\[ t_{f\text{f}} = 5.959 \text{mm} \]

\[ h_{b,\text{eff}} = 429.479 \text{mm} \]
neutral axis

\[
\begin{align*}
\zeta_{nc} = & \frac{b_b t_{fl} 0.5 t_l + (b_{b,\text{eff,}0} - t_l - t_{fl,\text{eff,}0}) t_w (0.5 h_{b,\text{eff,}0} + t_l) + b_{b,\text{eff,}0} (b_{b,\text{eff,}0} - \frac{1}{2} t_{fl,\text{eff,}0})}{b_b t_{fl} + (b_{b,\text{eff,}0} - t_l - t_{fl,\text{eff,}0}) t_w + b_{b,\text{eff,}0}} \\
& + (b_b - 2 t_l) t_w (0.5 h_{b,\text{eff,}0} + t_l - \zeta_{nc})^2
\end{align*}
\]

\[
\zeta_{nc} = 195.092 \text{mm}
\]

second moment of inertia

\[
I_{b,\text{eff,}0} := \frac{1}{12} b_b t_{fl}^3 + \frac{1}{12} b_{b,\text{eff,}0} t_{fl,\text{eff,}0}^3 + \frac{1}{12} (b_{b,\text{eff,}0} - t_l - t_{fl,\text{eff,}0}) t_w \cdot \ldots
\]

\[
I_{b,\text{eff,}0} = 1.94 \times 10^8 \text{mm}^4
\]

section modulus

\[
W_{b,\text{eff,}0} := \min \left( \frac{I_{b,\text{eff,}0}}{b_{b,\text{eff,}0} - \zeta_{nc}}, \frac{I_{b,\text{eff,}0}}{\zeta_{nc}} \right)
\]

\[
W_{b,\text{eff,}0} = 8.277 \times 10^5 \text{mm}^3
\]

load bearing resistance beam

\[
M_{Rd,0} := W_{b,\text{eff,}0} f_{02,\theta}(\theta)
\]

\[
q_{Rd,0} := \frac{M_{Rd,0}}{L^2}
\]

\[
M_{Rd,0} = 69.825 \text{kN} \cdot \text{m}
\]

\[
q_{Rd,0} = 10.775 \frac{\text{kN}}{\text{m}}
\]

unity check beam 2

\[
u_{c,\text{beam,}0} := \frac{M_{Sd,0}}{M_{Rd,0}}
\]

\[
u_{c,\text{beam,}0} = 1.002
\]