Why joint-joining is applied on complex linkages

_Citation for published version (APA):_
WHY JOINT-JOINING IS APPLIED ON COMPLEX LINKAGES

E.A. DIJKSMAN, dr.
Eindhoven University
of Technology

1. Introduction

Planar kinematic chains with revolute joints only and having one degree of mobility, rapidly increase in number if more links are involved. If four links are entangled, only one (closed) chain results, to wit: the four-bar chain. For six links, we already have two possibilities, namely the Watt’s form and the Stephenson’s form. Whereas for eight links, sixteen different chains exist having one degree of mobility; for ten-links chains, finally, we find there are even 230 possibilities [1]. Most of these contain one or more four-bar loops, and are therefore less complicated than the residuals not containing a four-bar. Of the 16 eight-link chains, for instance, only one chain stands out that does not contain a four-bar, but comprises merely pentagonal loops with the least number in sides. Of the 230 ten-link chains there are only ten that do not contain a four-bar loop. For the kinematician such linkages are intriguing, the more so, if there are no sub-chains having the mobility 1.

One of the first steps for understanding the motion of a chain, involves the determination of the instantaneous centers of rotation (the poles) of the links, relative to one another. Such a determination is usually done by repeated intersection of two, so-called, Kennedy-lines, based on the fact that according to the Aronhold-Kennedy rule, always three poles stay on a straight-line if these are the relative poles for three links of the linkage. For a four-bar loop of the chain, it is easy to carry this out and to determine the exact location of
the six relative poles between the four sides of the loop. Then, four of them coincide with the turning-joints of the loop (the so-called trivial poles), whereas the remaining two are the intersections of each time two opposite sides.

For instance

\[ P_{24} = (21 - 14) \times (23 - 34) \]
\[ P_{13} = (14 - 43) \times (12 - 23) \]

in which the turning-joints of the loop are indicated by 12, 23, 34 and 41 respectively. In case a linkage contains a four-bar loop, therefore, it may be easier to determine the location of the remaining poles, by using the already obtained ones from the four-bar in the linkage. However, if a chain does not contain a four-bar, this cannot be done. If the chain does not contain a four-bar, it may contain a pentagonal loop. But a pentagonal loop in itself has two degrees of freedom in motion, so that for such a loop alone, the relative motion between the sides is not fixed, with the consequence that no relative poles can be found that are non-trivial. Then, to find the poles, we have to consider a more extensive part of the linkage, such that the (sub-)chain considered, has one degree of freedom in motion. Clearly, in (sub-)chains, the poles are only deterministic if such a chain has the mobility one. Otherwise, as for the pentagonal loop, only the trivial poles are to be indicated.

The one and only eight-bar linkage not containing a four-bar, does not have a sub-chain with the mobility one. In such a case, therefore, we have to apply other means to determine the relative poles. If we further exclude methods based on velocity-assumptions, we are naturally forced to extend the Aronhold-Kennedy rule for these linkages, such that, notwithstanding the above, intersection of lines similarly leads to the determination of poles. How this may be done, will be explained in the chapter under the heading of reduction of kinematic chains.

Before going into the details, however, we will first explain how it is possible to obtain the eight- and ten-bar linkages not having a four-bar loop in them. The derivation is somewhat different from those found in literature, since there are only 11 of the sort. However, a short cut would be to sort them out by considering the configurations made by Woo [3] and by Manolescu [4]. Particularly the ones provided by Manolescu come in very handy, as he clearly has them

\[ x \] Here, Tao's notation [2] is used.
all divided in groups indicating the number of, for instance, the quaternary links in the linkage, or the number of dyads that are contained in them.

2. Formation of Kinematic chains

2.1 The one and only eight-bar linkage not containing a four-bar loop.

According to Grübler's formula

\[ 1 = f = 3(n-1) - 2d \]

relating the number of links \( n \), the number of turning-joints \( d \), and the number of degrees of freedom \( f \) for a linkage, we see that \( f = 1 \), \( n = 8 \) and \( d = 10 \) for an eight-bar linkage. If we further have \( n_b \) binary links, \( n_t \) ternary links and \( n_q \) quaternary links,\(^{4}\), we find that

\[ n_b + n_t + n_q = n = 8 \]  \hspace{1cm} (2)

As, in addition, the turning-joints are counted for each link, we naturally count them twice, therefore

\[ 2d = 2n_b + 3n_t + 4n_q \]  \hspace{1cm} (3)

Thus,

\[ n_t + 2n_q = 4 = n_b - n_q \]  \hspace{1cm} (4)

From this we derive the list

\[
\begin{array}{c|c|c|c}
\hline
n_q & n_t & n_b & \text{number of eight-bars} \\
\hline
0 & 4 & 4 & 9 \\
1 & 2 & 5 & 6 \\
2 & 0 & 6 & 1 \\
\hline
\end{array}
\]

According to Paul [5], the number of independent loops \( C \) of a linkage, is determined by the equation

\[ 2C = n - 2 \]  \hspace{1cm} (5)

Thus, there are 3 independent loops in the eight-bar linkage, we are looking for. As further no four-bar loops exist in the chain, these

\(^{4}\) pentagonal- or five-sided links are not possible with eight-bar linkages. This stems from the fact that with such a link at least \( 1+2j+(j-1) \) links are needed to compose a closed chain, where \( j \) corresponds to the number of joints for such a link. Thus,

\[ j_{\text{max}} = n/2 ............... \]  \hspace{1cm} (6)

Hence, for eight-bar linkages \( j_{\text{max}} = 4 \) and so, only quaternary links and those of lower order are allowed here.
Two pentagons connected
\( \ell_1 = 6 \)

eight-bar linkage with 3 independent pentagonal loops
\( \ell = 4 \)
loops have to be pentagons at least. Clearly, some joints in these loops have to coincide, otherwise there are at least five too many. Thus, two pentagonal loops must at least have one common joint.

Then, as a consequence, four bars meet at this joint. As generally, the eight-bar doesn't have multiple joints, the bars that meet at this joint, must form in fact, two triangular links. This is shown in figure 1, where two of the pentagonal loops are indeed combined. It so happens that all the links of the eight-bar we are looking for, are then presented in the figure. Only a singular turning-joint is missing. This then has to be adjoined such that no four-bars are to be formed. The only way to do this, will be to connect the bars numbered 2 and 6 by means of the missing turning-joint 26. This operation finally turns the configuration into the desired eight-bar not having four-bar loops. (See again figure 1). The eight-bar derived shows three independent pentagonal loops. (There is a fourth one, but that one is dependent on the first three and is to be derived from them by means of the closed loop vector equations that represent the three pentagonal ones).

We may also represent the linkage by a graph such that the links are transformed in points, and the joints are represented by connecting-lines (edges) (see again figure 1). For such a transformation n-sided loops turn into n-sided loops again. The graph that represent the linkage gives a better insight of the possibilities. For instance, if we try to locate the missing joint as before and we connect the points 1 and 3 in the graph instead of the points 2 and 6 as done, it is easier to see that then the four-sided loop 0-1-5-4 appears, which we not intended to have.

The operation where a kinematic chain is transformed into a graph is usually called a graphization and denoted by the symbol (G). The inverse operation, by which the kinematic chain is obtained from its graph, may be denoted by the symbol (G⁻¹).

If further a graph is regarded as a new linkage, the number of degrees of freedom for such a linkage, will be

$$F_{\text{graph}} = 3(d-1) - 2(n_b + 2n_t + 3n_q + 4n_p + 5n_h + ... - 1)$$

In here, we used Grünber's formula (1) and further the equations (2a) and (3a):

$$n_b + n_t + n_q + n_p + n_h + ... = n$$

(2a)
\[ 2d = 2n_b + 3n_t + 4n_q + 5n_p + 6n_n + \cdots \] (3a)

For an eight-bar, therefore, its graph regarded as a linkage, has 3 degrees of freedom in motion. (or 6, if we do not appoint a frame). Similarly, graphization of a ten-bar leads to a linkage having 4 degrees of freedom in motion.

### 2.2 The 10 ten-bar linkages having mobility 1 and not containing a four-bar loop.

From Grübler's formula we see that a ten-bar linkage with turning-joints only will have 13 singular joints. Thus, if \( n = 10 \), then \( d = 13 \). Further, according to Paul's equation, we find that for such a linkage four independent loops or circuits are to be indicated. (Thus, \( C = 4 \)). Finally, from eq. (6) we find that \( j_{\text{max}} = 5 \). Thus, hexagonal links are not allowed for a ten-bar linkage. So,

\[ 10 = n = n_b + n_t + n_q + n_p \] (2b) and further:

\[ 26 = 2d = 2n_b + 3n_t + 4n_q + 5n_p \] (3b)

Substitution in eq. (1) gives

\[ 4 = n_b - n_q - 2n_p \] (4b) and also:

\[ 6 = n_t + 2n_q + 3n_p \] (4a)

From this we draw the list:

<table>
<thead>
<tr>
<th>( n_p )</th>
<th>( n_q )</th>
<th>( n_t )</th>
<th>( n_b )</th>
<th>number of ten-bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>57</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>95</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>6</td>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>totally</td>
<td></td>
<td></td>
<td></td>
<td>230</td>
</tr>
</tbody>
</table>

Ten-bar linkages without four-bar loops, may first be obtained if we
eight-bar linkage
($f_1=4$)

dyad
($f_2=0$)

non-symmetrical structure
admitting the dyad (8-9)

0-1-2-3-4
0-4-5-6-7
0-1-2-6-7
0-4-5-8-9

four independent
pentagons

Ten-bar chain ($f=4$)
Woo 118; Manolescu 10/123

Graph, having admitted
a graphized dyad (8-9)

DA = Dyad Amplification
G = Graphisation

Figure 2a
Eight-bar linkage
\((f_1=4)\)

dyad
\((f_2=0)\)

Symmetrical structure admitting the dyad \((8-9)\)

\[
\begin{align*}
0 &- 1 - 2 - 3 - 4 \\
0 &- 4 - 5 - 6 - 7 \\
2 &- 3 - 4 - 5 - 6 \\
3 &- 4 - 5 - 8 - 9
\end{align*}
\]

Ten-bar chain \((f=4)\)

Woo 16; Manolescu 10/28

Figure 2b
Symmetrical structure
admitting the dyad (8-9)
\{0-1-2-3-4, 0-4-5-6-7\} three pentagons
2-3-4-5-6
3-4-0-7-8-9 one hexagon

Two-bar chain (I=4)
Woo 41; Nanolecu 10/29

Graph, admitting the
graphenized dyad (8-9)
Symmetrical structure admitting the dyad (8-9)

\[
\begin{align*}
0-1-2-3-4 \\
2-3-4-5-6 \\
0-4-5-6-7 \\
0-7-6-8-9
\end{align*}
\]

\(n_b = 6\)
\(n_t = 2\)
\(n_q = 2\)

Ten-bar chain \((f=4)\)

Woo 175; Manolescu 10/197

Figure 2d
Symmetrical structure
0-1-2-3-4
0-4-5-6-7 four independent
2-3-4-5-9 pentagons
3-4-5-6-8

Ten-bar chain (f=4)
Woo 48; Manolescu 10/49

Graph
remark: If 3 and 7 are connected instead of 3 and 6, a similar chain results.

Figure 3a
Symmetrical structure

\begin{align*}
0-1-2-3-4 & \\
0-4-5-6-7 & \text{four independent} \\
9-7-0-4-3 & \text{pentagons} \\
0-1-8-5-4 & 
\end{align*}

Ten-bar chain (f=4)

Woo 46; Manolencu 10/50

**Graph**

Figure 3b
Point-symmetric structure
0-1-2-3-4
0-4-5-6-7 four independent
3-4-5-6-9 pentagons
0-1-2-8-7

Ten-bar chain (f=4)
Woo 24; Manolescu 10/60

Figure 3c
Symmetrical Structure
0-1-2-3-4
0-4-5-6-7 \{ four independent
6-7-9-2-8 \} pentagons
0-1-2-9-7

Top-bar chain (ε=4)
Woo 137; Manolescu 10/45
Non-symmetrical structure

0-1-2-3-4  
0-4-5-6-7  
0-4-3-9-7  

three pentagons

8-6-5-4-3-2  hexagon

Ten-bar chain (f=4)

Woo 47; Manolescu 10/47

Figure 3e
Symmetrical structure

0-1-2-3-4
0-4-5-6-7  four independent pentagons
0-1-2-8-9
0-7-6-8-9

Tam-bar chain \( \ell = 4 \)
Woo 139; Banolese 10/144

**Figure 4**

If 0 is 5 instead of 0 to 6 then fig 2a emerges again.
adjoin a dyad to the eight-bar obtained in the last section. By looking at its graph, we find there are exactly four ways to adjoin a graphized dyad as long as no quadrilaterals are admitted. (See figure 2a,b,c,d). For three of them, four independent pentagonal loops exist. The remaining solution shows only three pentagonal loops in addition to a hexagon. Five other solutions are obtained by adjoining two binary links to the basic configuration for which two pentagonal loops are connected by a common joint. They are found if the adjoining is carried out through graph-theory. The chains obtained, however, do not have sub-chains with the mobility 1. Four of them contain four independent pentagons, whereas the remaining solution shows only three pentagons in addition to a hexagon. (See figure 3 a,b,c,d,e).

One final solution is obtained if the basic configuration is adjoined by a ternary link which is in itself connected to a binary one. The configuration obtained shows four independent pentagons. (See figure 4).

3. Reduction of Kinematic Chains (Instantaneous equivalency of linkages)

3.1 The creation of multiple joints by joint-joining

If the corner of a link incident to a multiple joint is expanded, so that the link becomes triangular and the joint is reduced to a simple joint, we have carried out an operation named by Nanolescu [6] as Joint Simplification or (JS). The inverse operation or (JS)\(^{-1}\), where two singular joints merge into a double joint, may be named Joint Joining or (JJ). Clearly, such an operation where double joints are created, will simplify any kinematic chain into one that is easier to understand. However, if there are no restrictions for the location of the double joint created that way, there is only superficial resemblance between the chains before and after this (JJ)-operation. Therefore, joint-joining will be applied only if the instantaneous motion of the linkage is not affected.

In order to attain this, we intersect two binary links (1) and (7), that are connected to a ternary one (0) at their intersection point \(I_{17}\). (See figure 5, showing a sub-chain of the linkage concerned). Then, the two joints 10 and 07 are replaced by this intersection point \(I_{17}\), simultaneously creating a double joint and also turning the ternary link into a binary one—namely, into the link \(I_{17} - 40\), as shown in the figure.
The Joint-joining-operation

Graph of (JJ)-operation

Figure 5
The proof that such a transformation does not affect the instantaneous motion of the linkage follows from the next reasoning: Suppose, the ternary link observed is connected to the links 1, 7 and 4; the links 1 and 7 being binary links which are connected to the links 2 and 6 of the linkage as shown in Figure 5. We further extend the linkage with the dyad 1'-7' that consists of the bars 1' and 7' respectively falling along the bars 1 and 7. The dyad is further connected to the linkage at the joints 12 and 67 respectively, whereas the dyad-joint coincides at the intersection point I17. It is easily seen then that the dyad-joint 17 has a zero-velocity with respect to the observed ternary link. The dyad-joint, therefore, may be rigidly connected to this link without any effect on the instantaneous motion of the linkage. Thus, we may connect the joints 40 and 17 = I17 by a bar 0. Having done this, the linkage becomes an overconstrained one, to be undone again by untying the bars 1 and 7 altogether. In fact, we then have replaced the bars 1 and 7 respectively by the bars 1' and 7'. Also, the former turning-joints 10 and 07 are now replaced by the double-joint 1'0 = 07' = 17'. The derivation shows that this specific way of joint-joining does neither effect the movability nor the instantaneous motion of the linkage. Figure 5 also demonstrates the graphization of the (JJ)-operation as it is carried out for a rigid triangle of a linkage. (Note that for the graph, the points 1' and 7' are chosen such that each time three points are aligned).

3.2 Joint-joining applied on a Stephenson-linkage

Since a Stephenson linkage contains two ternary links, each of them connected to three binary links, there are six possible ways of applying the (JJ)-operation. One of them shows Figure 6. In here, intersection-point I36 has been used. The resulting linkage is one that contains two four-bar loops instead of only one as before the JJ-operation.

Clearly, the positions of all poles not related to the replaced bars 3 or 6, are not effected by this operation: the trivial poles 12, 15, and 45, for instance, are still in the same place, and so are the remaining ones, such as:

\[
P_{25} = (21-15) \times (26-65) = (21-15) \times (26'-6'5)
\]

\[
P_{24} = (23-34) \times (25-54) = (23'-3'4) \times (25-54)
\]
Initial Stephenson-linkage

Reduced Linkage with instantaneous equivalency of motion

quadrilateral (3'-4-5-6')
(JJ)-operation graphized

Joint-joining applied on a Stephenson-Linkage

Figure 6
Joint-joining applied on a Stephenson-linkage

Figure 7
and

\[ P_{14} = (15-54) \times (12-24) \]

This, indeed, acknowledges the fact that instantaneous motion between the links that are not replaced, remains the same before and after joint-joining.

The combination chosen, has reduced the pentagonal loop into a four-bar one. The alternative linkage that has been created through joint-joining, therefore, is easier to handle and also easier to understand than the original configuration.

In figure 7 another way of intersection has been applied for joint-joining. This way reduces the four-bar loop of the configuration into a rigid triangular one. Then, the instantaneous motion between the links 2, 3, 4 and 5 remain the same.

3.3 Joint-joining applied on the eight-bar not having four-bar loops

Since joint-joining reduces the number of the bars in a loop, the operation will be particularly handy for linkages not containing four-bar loops.

As shown before, the most simple linkage not containing four-bar loops, is the eight-bar shown in the figures 1 and 8. On this linkage, JJ-operation may be applied four times. This then reduces the eight-bar to a basic four-bar with two dyads, each of them connecting opposite joints of the four-bar. The eight-bar so reduced, still contains four binary links, respectively having the same instantaneous motion of the former ternary links of the original eight-bar.

Successively application of Aronhold-Kennedy's Theorem then gives all locations of the poles we are looking for.

Explanation:
The first JJ-operation, herefore mentioned, introduces the intersection-point \( I_{17} \) and simultaneously turns the ternary link 0 into the binary link \( I_{17} - 40 \), to be indicated by the same digit 0. This operation also turns the pentagonal loop 0-1-2-6-7 into a four-bar one, viz.: 1'-2'-6-7'. What this particular joint-joining does to the graph that represents the eight-bar, is shown in figure 8. As further 1'7' = 7'0 = 01', the poles or joints represented by these points, are coinciding, and so do the three connecting lines in the graph representing the poles. Quite similarly the lines 12 = 21' = 1'1 coincide in the graph and so do the lines 67 = 77' = 7'6. As a consequence, the graph, like the
Reduced 8-bar

Initial 8-bar with determination of $p_{60}$

$\text{(JJ)}$-operations graphized

Joint-joining applied on a 8-bar

Figure 8
linkage, will show the quadrilateral $l'-2-6-7'$, enabling the designer to determine the poles $P_{27}$ and $P_{61}$.

A second (JJ)-operation, carried out for the links 3 and 5, introduces the double-joint $I_{35} = 3'5' = 5'4 = 43'$ and also the binary link 4, connecting $I_{35}$ with the joint 40. Here, the pentagonal loop 2-3-4-5-6 turns into the four-bar 2-3'-5'-6.

The third joint-joining, replaces the links 1' and 3' by the links 1" and 3", and additionally introduces the double joint $I_{13} = 3''1'' = 1''2 = 23''$. This operation turns the ternary link 2 into a binary one, namely the bar 2, connecting $I_{13}$ and the joint 26. (Here, the pentagonal loop 0-1'-2-3'-4 turns into the four-bar 0-1"-3"-4).

The fourth and last joint-joining replaces the links 5' and 7' by the bars 5" and 7" and introduces the double-joint $I_{57} = 5''7'' = 7''6 = 65''$. This last operation turns the ternary link 6 into a binary one, namely into the bar 6, connecting $I_{57}$ and the joint 26. (Here, the pentagonal loop 0-4-5'-6-7' turns into the four-bar loop 0-4-5"-7"). The resulting linkage is a basic four-bar 1"-3"-5"-7" in which the opposite vertices are connected through the dyads 2-6 and 0-4 respectively.

From the graph representing these operations, it is easily seen how the poles may be determined: For instance, using only the first two joint-joining operations, we obtain the set of poles:

\[ P_{61} = (62-21')x(67'-7'1'), \] where $21'=21;67'=67$ and $7'1'=I_{17}$

\[ P_{63} = (62-23')x(65'-5'3'), \] where $23'=23;65'=65$ and $5'3'=I_{35}$

\[ P_{3'1'} = (3'2-21')x(3'6-61'), \] where $3'2=32$ and $21'=21$.

\[ P_{3'0} = (3'1''-1'0)x(3'4-40), \] where $1'0=I_{17}$ and $3'4=I_{35}$

\[ P_{20} = (21-10)x(23'-3'0), \] where $23'=23$ and $3'0=P_{3'0}$

\[ P_{60} = (67-70)x(62-20) \]

\[ P_{27} = (20-07)x(26-67) \]

\[ P_{30} = (32-20)x(34-40) \]

\[ P_{37} = (32-27)x(30-07) \]

Or, alternatively, by using the points $I_{17}, I_{35}$ and $I_{13}$, we obtain the sequence:

\[ P_{63} = (65'-5'3')x(62-23''), \] where $23''=I_{13};65'=65$ and $5'3''=I_{35}$

\[ P_{3'0} = (3''1''-1'0)x(3'4-40), \] or $P_{3'0} = (I_{13}-I_{17})x(I_{35}-P_{40})$
\[
\begin{align*}
P_{60} &= (67-70)x(63''-3''0), \text{ where } 63'' = P_{63''} + 3''0 = P_{3''0} \\
P_{20} &= (21-10)x(23''-3''0)
\end{align*}
\]

etc.

Clearly, the last way of determination is the most simple one. Caused by the fact that no velocity-assumptions are needed, the method demonstrated, wins it in simplicity from the ones in use thus far. See for comparison, for instance, figure 321 demonstrated by Rosenauer and Willis in their book "Kinematics of Mechanisms" [7].

3.4 Joint-joining applied on a chain-belt mechanism

Figure 9 shows a chain-belt mechanism, containing three chain-wheels, of which two are rotating about a fixed center and the third centers about a point that in itself oscillates along a circle. The oscillation of the third center (57) is a consequence of the excentrical bearing of the wheel 2. If the input-link 1 rotates regularly, the output-wheel 2 will rotate irregularly, whereas the compensating motion of link 7 identifies the above mentioned oscillation. One may ask for the angular velocity \( \omega_{70} \) of this motion in relation to the angular velocity \( \omega_{10} \) of the input-wheel. Since \( \frac{\omega_{70}}{\omega_{10}} = \frac{P_{71}P_{10}}{P_{71}P_{70}} \) it then becomes necessary to find the location of the relative pole \( P_{71} \).

To find this point, we transform the chain mechanism into a linkage mechanism which turns out to be an eight-bar linkage as demonstrated in figure 9. The corresponding links of the chain mechanism and of the eight-bar linkage move in the same way, instantaneously.

The eight-bar obtained, may then be reduced by successive joint-joining.

In this case it is done three times. First, the dyad 4'-I_{46}-6' is adjoined and, simultaneously, the binary bar 1_{46}-57 introduced, that originates from the ternary link 5. Having also disregarded the links 4 and 6, the operation completed then represents the first joint-joining.

Secondly, we adjoin the dyad 6''-I_{36}-3' and the binary link 1, connecting I_{36} and 10. If we then obliterate the links 6' and 3, the second joint-joining has been carried out.

Thirdly, and finally, we adjoin the dyad 4''-I_{34}-3'' and also the binary link 2, connecting I_{34} and 20. Simultaneous disconnection of the bars 3' and 4' then specifies the third joint-joining.
Initial chain-belt mechanism with determination of $P_{71}$

\[ \omega_{70} = \frac{P_{71}P_{10}}{P_{71}P_{70}} \]

Instantaneous equivalent 8-bar linkage

Reduced 8-bar

Graph, demonstrating the triple joint-joining operation

- $6''_1 = 36$
- $6''_5 = 46$
- $6''_2 = 34$

$P_{6''0} = (I_{36} - 10) \times (I_{34} + 20)$

$P_{76}'' = (75 - 46) \times (70 - P_{6''0})$

$P_{71}'' = (76'' - 36) \times (70 - 21')$
The result is a reduced 8-bar as shown in figure 9. It still has one degree of freedom in motion. (Note that the bars 3", 4" and 6" form, in fact, a triangular link).

It is not difficult then to locate the poles for the reduced linkage. For instance,

\[ P_{60} = (I_{36} - 10) \times (I_{34} - 20) \] with \( I_{36} = 6"1 \) and \( I_{34} = 6"2 \)
\[ P_{70} = (75 - I_{46}) \times (70 - P_{60}) \] with \( I_{46} = 6"5 \).
\[ P_{71} = (76" - I_{36}) \times (70 - 01) \]

The graph of the linkage, also showing the joint-joining operations, will help the reader to find the right sequence of the poles that have to be located on our way to the pole we aim to find.

For each pole to determine, we seek a diagonal of a quadrilateral in the graph. (See again figure 9).

The final result, brought over into the initial mechanism, turns out to be a very simple and straight-forward construction. It is for this reason, the method is recommended for many practical cases, especially for those, where it is difficult to find the poles in another way.

References

WHY JOINT-JOINING IS APPLIED ON COMPLEX LINKAGES

Summary

For some complex linkages with a constrained motion, it is difficult to locate the instantaneous centers of rotation or the poles of the links. This is particularly true for linkages not having sub-chains with the mobility one. For this reason attention has been given first to the structure of eight- and ten-bar linkages of that type and second to a specific way of reducing them in order to determine the poles.

The method called joint-joining, transforms ternary links into binary ones, simultaneously turning n-sided polygons or loops into \((n-1)\)-sided ones. It has the advantage, however, of not changing the instantaneous and relative motion between the remaining links of the linkage. The determination of the poles for a reduced linkage is simply done by repeated application of the Aronhold-Kennedy rule. For the initial linkage this turns out to be a kind of generalisation of the rule, as now also binary links are intersected where they shouldn't by direct application of the rule.

Rezumat

În cazul anumitor lanșuri cinematiche complexe cu mișcarea supusă la constrângerii, determinarea centrelor instantanee de rotație sau a polurilor elementelor oferă oarecare dificultăți. Aceasta se aplică în special lanșurilor care nu au sub-lanșuri de gradul unu de mobilitate.

Din această cauză, se tratează mai întâi sisteme de bare de acest fel cu lanșuri de opt sau zece bare iar apoi o anumită metodă de reducere pentru determinarea polurilor. Aceasta metodă, denumită "suprapunerea articulațiilor", transformă elementele ternare în elemente binare, reducând toată dată numărul de laturi al poligoanelor sau circuitelor dela \(n\) la \(n-1\). Metoda are avantajul de a nu altera mișcarea instantanee și cea relativă dintre celelalte elemente ale lanșului.

Determinarea polurilor pentru un lanș redus se face apoi în mod simplu prin aplicarea repetată a regulii lui Aronhold-Kennedy. Pentru lanșul inițial, metoda poate fi privită ca o generalizare a regulii lui Aronhold-Kennedy, cu deosebirea că elementele binare sint intersectate, ceea ce nu ar fi fost cazul cu aplicarea directă a regulii.