Quantitative Feedback Theory applied to a Solar Orbital Transfer Vehicle

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1 Introduction

In modelling a physical system, one can never perfectly reflect reality by using mathematical formulas. When simple linear time invariant models are used, the errors with respect to reality are bigger then when one uses more sophisticated mathematical models. But even then, these sophisticated models are not exact. This problem has already been recognised by the inventors of feedback theory, because when a system can be fully described by formulas, there is no need for feedback. The system can then be controlled by means of feedforward, and hence no delay has to be introduced with all the possibilities of unstable behaviour. Unfortunately this in general is not the case, and there definitely is a need for feedback.

In the early 1960's Horowitz invented QFT, Quantitative Feedback Theory. The thought behind this theory is to keep the models simple, and give a quantity to the uncertainties that will have to cover the errors one made by using the simple model to reflect reality.

In this report first an introduction to the use of the Nichols Chart will be given, just for convenience, and QFT will be explained with a very simple model as an example. Then the real problem is introduced. It contains a Solar Transfer Vehicle that has some time variant behaviour. The control of the Solar Transfer Vehicle will be achieved by using QFT. The report ends with results, conclusions and recommendations.
2 How to use the Nichols Chart

To tackle the problem discussed in this report numerically, the QFT-Toolbox, designed by Borghesani, Chait and Yaniv [R1], is used. The toolbox uses the Nichols Chart to represent data. On the horizontal axis the frequency is displayed, on the vertical axis the logarithm of the amplitude is displayed. In the chart itself there are some lines sketched, see fig 1.

\[
P = \frac{k}{s^2 + a \cdot s + b}, \quad \text{with } k=5, \ a=0.5 \ b=5.
\]

**figure 1: Example of the nichols chart**

These are the lines of constant closed loop response concerning amplitude, and phase. The ones for constant magnitude are labelled, the ones for constant phase all originate at phase -180 and magnitude 0dB, while ending vertically downwards. Constant magnitude lines are called M-lines, whereas constant phase lines are called N-lines. Designing a system with the use of PID techniques is time consuming when one designs in the bode-domain, without the use of a computer. For instance when a controller is added with gain 3 dB and phase +90 degrees, the transmission has to be recalculated for every frequencypoint. In the Nichols Chart one just has to move the transmission upwards with 3 dB and rightwards with 90 degrees. The bandwidth of the transmission can be read from the crossing point between the M-line that indicates 0.707 dB (closed loop) and the transmission itself (this M-line is not sketched in figure 1). When a resonance is present, its closed-loop peak magnitude can be read from the M-circle which has the highest amplitude and which is tangent to the transmission.

The following example plant is used:
The plant is plotted on a Nichols Chart in fig 2. Just for convenience the bode plot of the above system is also depicted in fig. 3.

As expected the Nichols plot starts at phase zero and magnitude approximately 
\[20 \log_{10} \left(\frac{k}{b}\right) = 20 \log_{10} \left(\frac{5}{5}\right) = 0 \text{dB},\]
and ends up at phase \(-180\) and magnitude minus infinity. The blue dot indicates the maximum peak (open loop). One disadvantage of the NC is that the frequency is not plotted along. It is retrievable however by tracking along the plot with the mouse (when using a computer). Doing so, the blue dot is indicated at \(w=2.21\) rad/sec, gain=13.1 dB, phase=-83.5 degrees. Looking at the bodeplot, this information is confirmed.

\[\text{figure 2: nichols representation of } P\]
figure 3: Bode plot representation of $P$
3 Principles of QFT

When a controller has been designed for a certain plant to meet stability and performance demands, it is not guaranteed that this very same controller is still stable and meets the performance demands for a slightly different plant. A very common example is found in mass production. When a product is being made, its dynamical behaviour will not be exactly the same as another product from the same assembly, and hence also its plant will be slightly different. Plant variations also occur when one does multiple measurements on the same device or machine. The measurements of the frequency response functions (frf's) will never be exactly the same. In general two uncertainties occur. The first one in modelling, the second one in measuring. For instance, when one doubts about the mass of an element in a structure, one can give that element several masses, and hence give it multiple frf's. These kinds of uncertainties are called parametric uncertainties. Non-parametric uncertainties usually occur from measurements and when not much is known from the model. The relevant difference between these two forms of uncertainty is in its shapes of 'templates', as will be discussed in the sequel.

Before going into details on QFT, first the procedure that will be followed is pointed out:

Uncertainties are being translated into templates, these templates are frequency dependent and thus form frequency dependent bounds. These bounds are borders, which the nominal (for instance mean) open loop transmission may not cross. Taking into account these bounds, the loop shaping process can begin.

3.1 Determination of the templates

- Parametric uncertainty

Suppose there is a plant P:

\[ P = \frac{k}{(s + a)(s + b)}, k \in [1,10], a \in [1,5], b \in [20,30] \]

To give an idea about the size of uncertainty in this system, the bode plot is given in figure 4 for the following two sets of parameters: \([k,a,b]=[10,1,20]\) and \([1,5,30]\). All other possible plants lie between these two lines because the preceding two sets of parameters belong to the minimum and maximum plant, concerning magnitude. As can be seen from the figure, the variation of the plants is frequency dependent. In the high and low frequency range there is no difference in phase. Also in the high frequency band there is less variation in magnitude than anywhere else.

One basic step in QFT is determining the templates. In a template the variation of the plant is plotted for just one frequency. Take for example \(\omega = 1 \text{rad/sec}\). It is seen from the bode plot that the amplitude is between -9 and -44 dB, and phase is between -13 and -48 degrees. There are several ways of plotting such a template. As mentioned in section 2 the representation in this report concerns the Nichols Chart.
That makes the template a sort of a curved parallelogram instead of a cube (as in the parameter space), as can be seen in fig 5. Fig 5 indeed has its corners at (-48, -9) being upper-left and (-13, -44) being lower-right.

**figure 4: Bode plot of maximum and minimum plant**

**figure 5: Template of P at frequency 1 rad/sec.**

These kinds of templates now need to be made for an appropriate frequency band. Which frequencies this frequency band has to contain is system dependent. When for instance a resonance is present, there needs to be taken more frequency points around this resonance than at the rest of the interesting frequency range. Furthermore a good upper and lower bound for the frequency band are the frequencies where phase variation no longer exists. At that point the templates become very thin (a vertical
line). In the example under consideration, no resonances occur, so one can fulfill with an equally spaced frequency band. The templates belonging to a random frequency band is shown in fig 6.

![Plant Templates](image)

**figure 6: Templates of a random frequency band**

This figure indeed shows that there is no phase variation in high and low frequency ranges. The templates start at phase zero (low frequencies) and end at phase -180 (high frequencies) as expected. Furthermore one can derive an expression for the spread in gain of a phase-invariant template (so high and low frequencies). It is clear that the following holds:

\[
\lim_{s \to 0} P = \frac{k}{s^2}
\]

\[
\lim_{s \to \infty} P = \frac{k}{a \cdot b}
\]

So for s>0 this becomes:

\[
P_{\text{min}} = \frac{k_{\text{min}}}{a_{\text{max}} \cdot b_{\text{max}}} = \frac{1}{5 \cdot 30} = -44dB
\]

And \(P_{\text{max}} = \frac{k_{\text{max}}}{a_{\text{min}} \cdot b_{\text{min}}} = \frac{10}{1 \cdot 20} = -6dB\), and so \(\frac{P_{\text{max}}}{P_{\text{min}}} = \frac{-44}{-6} = 38dB\)

And in the high frequency range:

\[
P_{\text{min}} \cdot P_{\text{max}} = \frac{k_{\text{min}}}{k_{\text{max}}} = 20dB
\]

These results are also clearly visible in the figure.

One should always be careful that the templates are closed (if the uncertainty permits it), otherwise errors will occur. The QFT Toolbox makes its own templates when one is not carefully enough, see figure 34 in appendix 4. Varying the right parameters at the same time can close a template. In the example under consideration there are three
parameters. It is stressed that it is not necessary that all possible variations take place before a closed template appears. In practice that would also take too much time. Below is a figure showing the variations that were used to form closed templates. It clearly shows 4 variations. At variation one for instance k is being held at 10 and b at 20, while a is being varied from 1 to 5. Notice that the variation arrows in figure 7 span the whole parameter space. Taking more than these 4 variations would only result in points inside the contour of the template (that is already plotted) and thus does not bring along any extra information.

![Parameter space](image)

*Figure 7: Parameter space*

- **Non-parametric uncertainty**

In case of non-parametric uncertainty one usually defines a mean \( f_{\text{frf}} \), and then builds such templates so that they include the most outlying \( f_{\text{frf}} \) from the mean \( f_{\text{frf}} \). An elliptical template is usually used for this purpose. Because little is assumed to be known about the \( f_{\text{frf}} \) and its system, the trick of phase invariance at high and low frequency ranges cannot be applied anymore. So indeed the choice for an appropriate frequency band is even more difficult. The templates at other frequencies will generally have the same form, and hence so will the bounds. One can also choose to variate the elliptical shape of the template in the frequency band. Then the bounds will not have the same form.

### 3.2 Generation of the bounds

The next step in QFT is to translate the templates into bounds on the Nichols Chart. For this it is necessary to choose a nominal plant. Which of the plants is chosen to be the nominal one is irrelevant, it can even be one that is outside the parameterspace. For convenience however the maximum plant is chosen to be the nominal one in this subsection. As mentioned before, when one is dealing with a non-parametric uncertainty the nominal plant is already chosen (the mean). For now we only look at parametric uncertainty.

The nominal plant corresponds in each template to a certain point. In this case that is the upper left point. From demands on stability and for instance disturbance rejection certain circles and lines will appear (M-lines), as was seen in section 2 (see also appendix 2). One such circle is shown in figure 8. Together with this circle a template is shown.
Obviously, this template has the wrong location because some of the plants are situated inside the circle. These are marked by the black area. When the template is moved in such a way that its upper left point is tangent to the circle (simply by multiplying with a gain less than zero), one (in its mind) takes a pencil and puts its point through the upper left point of the template. Had one chosen the nominal plant to be the lower right point of the template, one had to put the point through the lower right point of the template when the upper left point was tangent to the circle. When this point is made, one moves the template vertically (simply by a gain bigger than one) in such a way that the location of the template concerning phase will be the same. Now the lower left corner will be tangent to the circle. Again put a pencil through the upper left corner (when this one was chosen to be the nominal plant). Now there are two points, belonging to one phase location of the template. The next step is to take some other phase locations and do the same trick. Be careful not to trespass the circle with any of the points on the template. When changing the phase, other points on the template will be tangent to the circle. In this way one gets several points on the Nichols Chart (two points for every phase location). When these points are connected by a curve, the bound belonging to the frequency of that template is finished. Next one has to go through the same routine for all the other templates. The result is shown in figure 9. The bounds also become isomorphic when looking at high and low frequencies. This is because in that region the templates are straight lines.

Figure 8: Generating bounds
For disturbance rejection there can also be generated bounds. However for these disturbance rejection demands normally there are no circles available but lines. Therefore the bounds will also be lines. The toolbox provides eleven types of bounds that can be generated. It is pointed out that stability commands like Gain Margin (GM) and Phase Margin (PM) are easily indicated on the Nichols Chart. The derivations are in appendix 2 and the main results are listed below.

\[
\gamma = \frac{1}{GM - 1}
\]

\[
\gamma = \frac{1}{\sqrt{2 - 2 \cdot \cos(PM)}}
\]

with \( \gamma \) representing an M-line. So by demanding a PM or GM, the M-line to stay out of can be calculated directly. Which shows again another advantage of the Nichols Chart.

In this report the only bounds of concern are those of robust stability. Once all bounds have been plotted on the Nichols Chart, one can throw away the bounds that are not dominant (of course only at the same frequency). At this point one has reached the situation where instead of all the transmissions plotted on the Nichols Chart, one has now one nominal transmission, and some bounds to stay out of.
3.3 **Loop shaping**

After this, one is free to loop shape the transmission as close to the bounds as possible, taking attention to the complexity of the resulting controller. In the toolbox there is a file called `lpshape.m`. With this file one can design its transmission in the Nichols Chart using 8 elements. All the eight elements are listed below.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real pole</td>
<td>$\frac{1}{s/p+1}$</td>
</tr>
<tr>
<td>Real zero</td>
<td>$\frac{s}{z+1}$</td>
</tr>
<tr>
<td>Complex pole</td>
<td>$\frac{1}{s^2/w_n^2 + 2 \cdot \zeta \cdot s/w_n + 1}$</td>
</tr>
<tr>
<td>Complex zero</td>
<td>$\frac{s^2/w_n^2 + 2 \cdot \zeta \cdot s/w_n + 1}{a_1 \cdot s^2 + a_2 \cdot s + 1}$</td>
</tr>
<tr>
<td>Super 2\textsuperscript{nd}</td>
<td>$\frac{1}{s^b}$</td>
</tr>
<tr>
<td>Integrator (n&gt;0)</td>
<td>$\frac{1}{s^n}$</td>
</tr>
<tr>
<td>or differentiator (n&lt;0)</td>
<td>$\frac{s}{z+1}$ $\frac{s}{p+1}$</td>
</tr>
<tr>
<td>Lead or lag</td>
<td>$\frac{s^2/w_n^2 + 2 \cdot \zeta_1 \cdot s/w_n + 1}{s^2/w_n^2 + 2 \cdot \zeta_2 \cdot s/w_n + 1}$</td>
</tr>
</tbody>
</table>

Loop shaping in the Nichols Chart is now possible filling in the numbers for each parameter required by each element, or more advanced by selecting an element, and then moving the transmission line to the location where one wants it to be, with use of the mouse. Of course the second, at first site friendlier, method has got one disadvantage. One has to investigate in which direction the transmission can be moved, when one specific element has been selected. The loop shaping of the example is not represented in this report.
The Solar Orbital Transfer Vehicle (SOTV) is a spacecraft that has to follow an orbit around the earth. A schematic representation of such an orbit is given in figure 10. The left ball represents the earth, the right ball represents the sun. The angle $\alpha$ is called the true anomaly and is an important position indicator. It flows ranges 0 to 360 degrees.

![Figure 10: Schematic representation of the orbit](image)

A sketch of the SOTV is given in figure 11. It has three main parts:

- The central body
- The engine
- The sun-collectors

In the past there has already been a similar spacecraft, however that one was approximately 3 times bigger than this one. The restriction on the size of this edition of the SOTV comes from the limitation on mass that can be part of a spacecraft launch. During a launch of for instance Ariane 5, the maximum allowable mass the SOTV may have is 120 kg. When the SOTV is heavier it may not come along on the launch. Another disadvantage of such a situation (launch on board of Ariane 5) is that the construction for the sun-collectors has to be inflatable. When the SOTV is still attached to the Ariane 5, its collectors are fowled up, during its own journey they have to be inflated. Therefore the construction could become unstable because of resonances occurring at the sun-collectors (which are not very stiff).
The energy for maintaining its orbit around the earth has to come from sunlight energy. Therefore it is very important that the concentrators maintain pointing the sun. The energy for movements like leaving the orbit has to come from propellant (6 kg).

As can be seen from figure 12 the SOTV has got three main axis, x, y and z. It has also got three thrusters in these directions. The steering however comes from the rotation around these three main axes. So when the SOTV wants to perform the orbit of figure 11, it has to rotate around its x-axis (see figure 12). But when doing this, there is a situation where the sun-collectors are pointed to the opposite of the sunlight and hence do not receive any energy. Therefore there has to be another rotation axis that can rotate the collectors in such a way that they keep pointing to the sun, even when the SOTV itself has rotated 180 degrees around its x-axis. This extra rotation possibility is called the concentrator angle, and is represented in figure 12 by the dashed line. The concentrator-axis can rotate over [0-180] degrees.

During the modelling of the SOTV each main part got its own reference frame (body, engine and collectors), and the reference frame of the collectors are called the concentrators’ frame. The concentrators’ reference frame is exactly the same frame as the body frame, except that its origin is a few centimetres higher due to a higher centre of mass of the collectors. Therefore the control strategy can be formulated with respect to two reference frames without too much effort. As will be seen later on, the concentrators’ reference frame is chosen instead of the body frame.

As stated above the position indicator $\alpha$ ranges over [0-360] degrees. During this cycle the concentrators’ axis keeps changing for optimally catching the sunlight. This relation is formulated in equations 1, and plotted in figure 13 for $\beta = 30^\circ$.

\[
\tan \theta_{\text{roll}} = \frac{\cos \alpha \cos \beta}{\sin \beta} \\
\tan \theta_{\text{conc}} = \frac{\sqrt{\sin \beta^2 + (\cos \alpha \cos \beta)^2}}{\sin \alpha \cos \beta}
\]

where $\theta_{\text{roll}}$ = Roll angle (which is the rotation around the x-axis) and $\theta_{\text{conc}}$ = Concentrators’ angle. For a graphical interpretation of $\alpha$ and $\beta$, see figure 11.

By following this orbit the inertia of the SOTV keeps changing. Also during the flight some propellant mass will be burned because of possible evoking actions. Therefore the SOTV has a time variant behaviour.
A specialised company provided information about the vibration modes of the construction, and carefully studied the construction of the inflatable collectors. It seemed that the lowest natural frequency was around 1 Hz. In figure 14 and 15 there are two situations. In figure 14 there are shown some Bode plots for varying true anomaly, when the body axes are kept as reference. In figure 15 there are some bode plots for the same varying true anomaly, when the concentrators' axes are kept as reference. As can be seen, the body reference situation will cause more trouble in the control framework. Therefore the reference frame was chosen to be the concentrators' axes.

**Figure 13:** $\theta_{roll}$ and $\theta_c$ as a function of time (and thus $\alpha$)

**Figure 14:** Bode plots with body-reference frame
Now that the reference frame is chosen, the control problem can be formulated. The general purpose is to build a controller that is dependent on the true anomaly ($\alpha$). Therefore linear time invariant controllers have to be designed for several positions $\alpha$. The concentrators' rotation will be $\pi/2$ radians. In this situation all angles are thus fixed ($\alpha$ has got two values for one $\theta_1$), so all inertias are known. The mass of the SOTV however is not exactly known because it is not certain how much propellant has been burned each time the SOTV passes the point $\alpha$ where the concentrators' rotation is $\pi/2$. So actually this is a time variant situation.

The following uncertainties are given:

- Propellant mass: $0 - 6$ kg. (time variation)
- Damping: $0.005 - 0.05$ (modelling uncertainty)
- Spacecraft centre of mass: 1 cm variation (time variation)
- Frequency of the modes: 15% variation (time variation)
- Inertia: 20% variation (time variation)
- Thrust direction: 0.5 degrees variation (modelling uncertainty)
- Misalignment: Not known yet (modelling uncertainty)

It is noted that the variation in inertia is mainly due to the variation of inertia from the collector. Variation from the body's inertia is neglected because the collectors are much bigger than the central body. Furthermore it can be noted that there was still no information available about the uncertainty of the misalignment at this time.

In the figures in appendix 3 it is shown what the influence is from each variation on the transfer function. Only rotation around the x-axis is shown, investigation on the other axes showed comparable results.
As a comparison the transfer function of full order (86) is shown in figure 16 together with the reduced transfer function (order 8). Looking at the figure and comparing it with the ones in appendix 3, one can conclude that the only relevant variations are the following:

- Damping : 0.005 0.05
- Frequency of the modes : 15% variation
- Inertia : 20% variation

This conclusion is valid because the difference between the reduced and full transfer function is bigger than the differences between the transfer functions due to the just neglected variations.

![Bode Diagram](image)

*figure 16: Comparison of Bode plots between full and reduced system (x-axis)*

Before starting with the actual QFT, it is stressed that for this moment it is assumed that the system can be handled as three independent SISO (single input, single output) systems instead of one MIMO (multiple input, multiple output) system. Simulation pointed out that cross-coupling terms were negligible because the transfer functions of the cross-coupling terms were at least 50 dB lower than the diagonal ones.
5 Application of QFT on the SOTV

The data can be generated and when the specifications are given the QFT-procedure can start in order to develop a controller. The specifications are the following:

All axes:

- 6 dB Gain margin
- 30 degrees Phase margin.
- 15 seconds settling time to come into a precision of 0.01 degrees on a response of a step-function

Indeed nothing is mentioned about sensitivity reduction. This is of later concern. The QFT-procedure will proceed just as explained in section 3.

5.1 Generation of templates

Just as in the example, there are again three parameters that span up the parameter space. This time however, the templates will not look as smooth as in the example, because the variations of the SOTV-model do not represent coefficients in the numerator or denominator of the transfer function, but are far more hidden in the dynamics. One still needs a nice bounded template, and therefore a lot of time is spent on studying what parameters to change at a time, and especially in what rate.

Figure 17 shows a template where the edges are not good enough. Performing the calculations again, but now with some more plants, gives the result of figure 18. As can be seen from this figure, there are actually calculated too many plants, that result in inner points of the template. This results in a larger calculation time, but one is now sure that the edges are smooth, and thus the right bounds are being calculated.

\[ \text{figure 17: Template with not smooth enough edges} \]
The figure above represents of course only one frequency (35 rad/sec), the used parameter changes are given in figure 19.

Figure 19 shows that absolutely all the edges from the template were calculated (according to the maximum principle, see [R1]). An arrow stands for a variation and all edges of the cube are used.

5.2 Generation of the bounds

The templates are finished, so one can start computing the bounds. Before doing that, a decision has to be made. The question is what frequencies should to take for computing bounds? Since the SOTV contains resonances, one should not randomly choose a frequency band. Looking at a figure that contains most of the plants (Bode plots) it is very hard to choose frequencies. The figure is not even printed in the
report, because there are 790 plants and nothing can be seen from it, without zooming functions. But one still has to start somewhere. With the command zpkdata.m from Matlab, the poles and zeros from the eight plants that represent the corners of the cube from figure 19, are calculated. This is done because at poles and zeros the variations seem the toughest. Later on, when the design is finished, one can always choose other frequency points and check whether the new bounds are still fulfilled with the same controller. For the SOTV that had to be done several times, but the starting point, using the locations of the poles and zeros of the most extreme plants seemed a good one. For the final frequency points used, the toolbox computed two bounds for each frequency (Gain Margin and Phase Margin) and then threw away that part of the bound (after intersection) that was redundant. In figure 20 some of the bounds are shown. Plotting all the bounds is useless because then nothing is visible anymore.

![Combined Bounds](image)

*figure 20: Some of the bounds of the SOTV*

As can be seen from the figure, the design will be very hard. For only four bounds, there already is an enormous area to stay out of.

### 5.3 Loop shaping

Now that the bounds are computed, one can begin to loop shape the transmission. Before doing that, first a controller is developed in the Design It Easy Toolbox, in order to flatten the resonances from the nominal transmission. If that is not done upfront, the design will be very difficult, because resonances appear as big circles in the Nichols Chart. The flattening proceeds very well when one again uses the Matlab command zpkdata.m, in order to perfectly place notches and inverse notches. After doing this the transmission looks like figure 21. The dots on the transmission represent the other frequencies where a bound has been computed (but not plotted).
As can bee seen from the figure, the nominal system is not stable, and besides that it also crosses the bound with the lower frequencies.

![Nominal transmission to start with (after having it smoothened by notched)](image)

*figure 21: Nominal transmission to start with (after having it smoothened by notched)*

There are generally three actions to solve this problem:

- Low bandwidth
- High bandwidth
- Desired bandwidth

The first two options will be applied. When using the third option the bandwidth has to be around a frequency where resonances occur. Therefore the transmission should get a phase of +135 degrees from one single controller. This is not achievable. Therefore first the lower bandwidth option is tried, and afterwards the higher bandwidth option is tried out.

5.3.1 Lower bandwidth option

Due to the characteristics of the system, the templates of very low frequencies are all the same. Therefore the bounds will be the same at the low frequencies. One such bound is seen in figure 21 (the red one). All frequency points that belong to that form of bound, should stay out of that circle. Therefore the controller has to bring a phase of +90 degrees. This is achieved by using a real zero. After the transmission has passed the low-frequency bounds, it is turned back to the –180 degrees line, in order to get a proper transmission. This is done by applying a real pole. When this is done,
the gain can be adjusted so that all bounds are satisfied. The final transmission of the nominal plant can be seen in figure 22. Again, only four bounds are plotted for convenience.

**Figure 22: Final transmission in case of lower bandwidth option**

The controller that was used to create the above transmission is plotted in figure 23. It is plotted in a Bode plot because when it is plotted in the Nichols Chart it is not clear enough to distinguish the elements that are in the controller (especially the notches). The openloop bandwidth is now 0.0336 rad/sec.
As can be seen in the figure the controller has to contain a wide phase lead in order to achieve robust stability.

5.3.2 Higher bandwidth option

As will be seen in section 6 the bandwidth of the just discussed transmission is far too low. The specs 15 seconds settling time are not met. Therefore it is tried to put the resonances before the bandwidth instead of after it. Thereto the transmission of figure 24 was developed.
This time another bound was dominant, the bound of frequency 12.13 rad/sec, this one is indicated by the cyan coloured bound. The red bound represents high frequency bounds. The controller needed to get the above transmission is depicted in figure 25. As can be seen from this figure, here an even bigger control effort is needed to achieve robust stability, which is logical in case of a higher bandwidth. The bandwidth of this transmission is no less than 1800 Hz, and thus physically not achievable due to hardware limitations.
6 Results

Only the results for the lower bandwidth option will be shown, for two reasons:

- The higher bandwidth option is physically unachievable
- With the higher bandwidth option the error by reducing the order of the plant will be disastrous, because with the reduction the higher order modes were thrown away. In reality the bounds will look much more different at high frequencies, and therefore the results are not useful.

Although the bandwidth of the lower bandwidth option is far too low, the results are still plotted in this section, to see whether the QFT-application worked correct or not. With generating the bounds, there was a demand on the Gain Margin and the Phase Margin, namely 6 dB and 30 degrees. In figure 26 the nyquist plots of several (randomly chosen out of the 790 plants) transmissions are plotted. Indeed the demands are fulfilled.

Looking at figure 27 one can see the step responses of several transmissions (again randomly chosen out of the 790 plants). These are of course the closed loop responses of the step function, and the specification of 15 seconds settling time is by far not reached.

figure 26: Nyquist plots of controlled transmissions
figure 27: Step responses of controlled transmissions
7 Conclusions and recommendations

In using QFT to the SOTV the uncertainties of the transfer functions are too heavy to achieve a normal bandwidth. It is shown however that the QFT-technique works because designed transmissions all fulfil the demands of Gain margin and Phase margin. It can be concluded that another control strategy has to be applied in order to achieve the desired settling time of 15 seconds, or it should have to be decided that the demands on the settling time could be relaxed, so that the QFT-controller may be applied.

When the settling time is relaxed it is recommended that more investigation is performed on parameter variations, this time including misalignment. After that it is necessary to develop controllers for other points of concentrators’ angles, so that the final controller (which is position dependent) can be composed.
Appendix 1

Reference List

[R1] Craig Borghesani, Yossi Chait and Oded Yaniv, “Users guide for the QFT Frequency Domain Control Toolbox”.
Appendix 2

Derivation of expressions between GM, PM and Y

Gain margin is defined as:

\[
GM = \left| \frac{1}{L(jw)} \right| \quad \angle L(jw) = -180^\circ
\]

So GM has to be bigger than one because L has to be smaller than one

Now define

\[
L(jw) = l \cdot e^{j\phi(jw)}, \text{ because } \phi = -180^\circ, \Rightarrow |L(jw)| = l
\]

M-circles are defined as \( \frac{L(jw)}{1 + L(jw)} \), which is exactly the complementary sensitivity function \( T(jw) \).

So the following equations should hold

\[
T(jw) = \left| \frac{L(jw)}{1 + L(jw)} \right| = \left| \frac{-l}{1-l} \right| = \frac{l}{1-l} \Rightarrow |T(jw)|(1-l) = l \Rightarrow |T(jw)| \cdot l = l = \frac{|T(jw)|}{1 + |T(jw)|}
\]

So the absolute value of the transmission is related to the absolute value of the complementary sensitivity function.

Then

\[
GM = \left| \frac{1}{L(jw)} \right| = \left| \frac{1}{l} \cdot \frac{T(jw)}{|T(jw)|} \right| = 1 + \frac{1}{|T(jw)|}
\]

or otherwise formulated:

\[
GM = 1 + \frac{1}{\gamma}, \text{ since } \gamma = |T(jw)|, \text{ which is an M-circle on the NC. Hence there is the relationship that was already used in section 3.}
\]

For the PM, another trick can be used:

\[
PM = 180^\circ + \angle L(jw) |L(jw)| = 1
\]

\[
L(jw) = l \cdot e^{j(\text{PM}-180)} = \cos(\text{PM}-180) + i \cdot \sin(\text{PM}-180) = -\cos(\text{PM}) - i \cdot \sin(\text{PM}),
\]

using Euler's rule.

So:
\[
|T(jw)| = \left| \frac{L(jw)}{1 + L(jw)} \right| = \left| \frac{-\cos(\overline{PM}) - i \cdot \sin(\overline{PM})}{1 - \cos(\overline{PM}) - i \cdot \sin(\overline{PM})} \right|
\]

Taking the absolute values of the numerator and denominator (considering them as complex numbers) one gets:

\[
|T(jw)| = \frac{1}{\sqrt{2 - 2 \cdot \cos(\overline{PM})}}
\]

again \( |T(jw)| = \gamma \), and thus a relationship is found between PM and \( \gamma \), which was already used in section 3.
Appendix 3

*Influence of parameters on transmission*

**figure 28:** Influence of propellant mass

**figure 29:** Influence of damping
figure 30: Influence of Centre of Gravity

figure 31: Influence of modes (frequency shift)
Figure 32: Influence of inertias

Figure 33: Influence of thrust direction
Appendix 4

*Example of a bad template*

When the edges of a template are not filled enough with data points the toolbox linearly interpolates the template, see thereto figure 34.

*figure 34: Example of a bad template*

The black line indicates the border that the toolbox makes. In figure 35 it can be seen that the actual border is a lot smoother than the linear approximation of figure 34.
figure 35: Example of a good template