Backlash in mechanical systems

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Backlash in mechanical systems

*Literature study and experiments on the piecewise linear beam*

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Contents

Summary

1 Introduction .............................................................................................................1
2 Results of the literature study ...............................................................................2
  2.1 Models of non-gear systems ...........................................................................2
  2.2 Models of gear systems ..................................................................................8
3 Experiments on the piecewise linear beam .........................................................16
  3.1 Experimental set-up .........................................................................................16
  3.1.1 Set-up of the beam system .........................................................................16
  3.1.2 Sensors used ................................................................................................17
  3.2 Experiments .....................................................................................................17
  3.2.1 Presentation of the results ..........................................................................17
  3.2.2 Overview of the experiments ......................................................................17
  3.2.3 Results: system without DVA ....................................................................18
  3.2.4 Results: system with DVA .........................................................................21
4 Conclusions and recommendations ......................................................................25
  4.1 Conclusions covering the literature study ......................................................25
  4.2 Conclusions covering the experiments on the piecewise linear beam ..........25
  4.3 Recommendations ..........................................................................................26
5 References ............................................................................................................28
Appendix A: A mathematical explanation for scaling $x_{\text{max}}$ with $F$ ...............29
Appendix B: The crash of the system ....................................................................30
  B.1 Checking system properties ..........................................................................30
  B.2 Measuring results just after the crash ............................................................34
Summary

This report covers a literature study in the field of dynamics of mechanical systems with backlash, with the focus on steady-state behaviour of these systems. Further this report covers the results of experiments with the aim to research the dynamical phenomena following from the research in the first part of the report by physical experiments on the piecewise linear beam in the CST-lab. The results of Bonsel (2003) are used as a starting point. Also the applicability of the Dynamical Vibration Absorber (DVA) as applied by Bonsel (2003) in the case of backlash is researched.
1 Introduction
The objective of this report is two fold. The first goal of this report is to investigate and present an summary of the results of the research that has done over the past ten years in the field of dynamics of mechanical systems with backlash, with the focus on steady-state behaviour of these systems.

The second goal of this report is to make an attempt to confirm the dynamical phenomena following from the research in the first part of the report by physical experiments. For the experiments the piecewise linear beam in the CST-lab is available. Bonsel (2003) makes use of the same experimental set-up. The results of Bonsel (2003) are used as a starting point. Checking the applicability of the Dynamical Vibration Absorber (DVA) as applied by Bonsel (2003) in the case of backlash is also in the scope of the experiments.

From the objective of this report as stated above a problem definition can be formulated. This problem definition is divided in two parts, the first part describes the literature study and the second part the experiments on the piecewise linear beam.

Literature study
In which categories can the mechanical systems with backlash be divided, what are the models that are used to analyse the systems in these categories respectively and which methods are used to analyse the steady-state behaviour of these models, concerning the results of the research that has been done over the past ten years in the field of dynamics in mechanical systems with backlash?

Experiments
What are the effects of flush, backlash and pretension on the steady-state dynamics of the piecewise linear beam with and without the Dynamical Vibration Absorber (DVA) as applied by Bonsel (2003)?

The structure of the report is as follows: Chapter 2 presents the results of the literature study. In chapter 3 the experiments on the piecewise linear beam are described and the results are analysed. Chapter 4 contains the conclusions and recommendations.
Results of the literature study

In this chapter the first part of the problem definition is going to be covered: "In which categories can the mechanical systems with backlash be divided, what are the models that are used to analyse the systems in these categories respectively and which methods are used to analyse the steady-state behaviour of these models, concerning the results of the research that has been done over the past ten years in the field of dynamics in mechanical systems with backlash?"

The search results can be divided in two categories: Models of non-gear systems and models of gear systems and will be covered by the sections 2.1 and 2.2 respectively. These sections will explain the models used to approach the system dynamics and will also describe the methods that are used to analyse the steady-state behaviour of these models.

2.1 Models of non-gear systems

Analysis of a sdof oscillator

Peterka et al. (2004) analyse a system of a sdof oscillator with backlash (Fig.2.1). The mass can have an impact on a stop. Here $x \ [m]$ is the position of the oscillator mass $m \ [kg]$. The position of the equilibrium position of the mass with respect to the stop is denoted by $r \ [m]$. This stop can be made rigid or soft by altering the force characteristic $F(x) \ [N]$. The system is excited by a harmonic excitation force $F_{\text{ex}} = F_0 \cos(\omega \cdot t + \varphi) \ [N]$ with $F_0 \ [N]$ the amplitude, $\omega \ [\text{rad/s}]$ the excitation frequency and $\varphi \ [\text{rad}]$ the phase angle. This will cause a strongly non-linear motion if the backlash is over bridged. The model which describes the non-linear motion is defined by:

$$m \frac{d^2x}{dt^2} + b_1 \frac{dx}{dt} + k_1 x + F(x) = F_0 \cos(\omega \cdot t + \varphi) \quad (2.1)$$

with $m \ [kg]$ the oscillator mass, $b_1 \ [N/m/s]$ the viscous damping constant and $F(x) \ [N]$ the non-linear force characteristic of the stop. The characteristic of the stop is given by:

$$F(x) = \begin{cases} 0 & \text{for } x \leq r \\ (x - r) \left( k_2 + k_3 \text{sign} \left( \frac{\sqrt{mk_1}}{F_0} \frac{dx}{dt} \right) \right) & \text{for } x > r \end{cases} \quad (2.2)$$

with $k_2 \ [N/m]$ the spring stiffness. The constant $k_3 \ [N/m]$ is introduced as to make (2.1) dimensionless. The dimensionless deflection is denoted by $X = x / x_w \ [-]$ with $x_w = F_0 / k_1 \ [m]$. Time $t \ [s]$ is transformed in $\tau = \Omega \cdot t \ [-]$ with $\Omega = \sqrt{k_1 / m} \ [\text{rad/s}]$ and the gap width is made dimensionless by introducing $\rho = r / x_w \ [-]$. Writing the dimensionless frequency and the dimensionless phase angle in the form $\eta = \omega / \Omega \ [-]$ and $\theta = \varphi / 2\pi \ [-]$ respectively makes it possible to write the equations of motion in the dimensionless form:

$$X'' + \beta X' + X + F(X) = \cos(\eta \tau + \theta) \quad (2.3)$$
where \( X' = \frac{dX}{dt} \), \( X'' = \frac{d^2X}{dt^2} \) and \( \beta = \frac{b_1}{\sqrt{k_1m}} \). After making (2.2) dimensionless the stop characteristic becomes:

\[
F(X) = \begin{cases} 
0 & \text{for } X \leq \rho \\
(X - \rho)(k_2 + k_3 \text{sign}X')/k_1 & \text{for } X > \rho
\end{cases} 
(2.4)
\]

Figure 2.1: SDOF system with backlash and non linear stop

Peterka (1996) also analyses the system as depicted in Fig. 2.1. A fundamental difference is that only rigid stops are analysed, i.e. \( k_2 \rightarrow \infty \). The latter paper gives information concerning transitions between different types of impact motion and from periodic to chaotic motion. The motion of the system is studied by the use of Newton’s elementary theory of direct and centric impact with the restitution coefficient \( R = -\dot{x}_+ / \dot{x}_- \) where \( -\dot{x}_+ \) is the after-impact velocity and \( \dot{x}_- \) the before-impact velocity (Fig. 2.2).

Figure 2.2: response of the system with a rigid stop(Peterka (1996))
**Impact motions**

There exist many different periodic impact motions, which can be characterized by the quantity $z = p / n$, where $p$ is the number of impacts in the motion period and $n$ is the number of excitation force periods in the motion period $T$ [s]. The quantity $z$ expresses the number of impacts related to one excitation period. Here the motion period is $T = 2\pi n / \omega$ [s] (Fig. 2.2) with $\omega$ [rad/s] the excitation frequency.

Fundamental motions are characterized by $z = p$, $n = 1$. Only the number of impacts $p$ differ. A special type of fundamental motion is the impact-less motion where $p = 0$.

Motions with $p = 1$, $z = 1/n$ are called sub-harmonic motions. There is only one impact and more than one excitation force periods in one motion period.

**Region boundaries**

There exist three types of region boundaries, grazing (g), period doubling (PD) also called flip and saddle node (SN) bifurcation boundaries.

A grazing bifurcation boundary of an impact-less vibration is defined by the motion where the mass is periodically touching the stop. In case of a rigid stop Peterka (1996) defines the grazing bifurcation boundary $\rho' [\cdot]$ of forced impact less vibrations by $\rho' = |1/(1 - \eta^2)|$ where $\eta = \omega / \Omega$ [\cdot] is the relative frequency.

Peterka et al. (2004) conclude that when the velocity is almost zero before the impact occurs, the new periodic motion in case of a grazing bifurcation boundary is characterized by $z = (p+1)/n$. When a soft stop is applied, the motion is reversible. In case of PD, an unstable periodic motion $z = p/n$ splits on $z = 2p/2n$ which is a stable periodic solution. This transition is also reversible. And in case of SN one impact disappears and the $z = p/n$ motion jumps into a $z = (p-1)/n$ motion. This kind of transition is non-reversible except when a soft stop is used.

After crossing the grazing boundary $g$ there appears a motion with a weak impact in the case of a soft stop. This motion exists in a narrow region along the grazing boundary $g$ and is limited by a weak saddle node bifurcation boundary $(SN_w)$. There arises a jump into the same type of motion but with a stronger impact. This change is non-reversible and $z$ does not change. Only the motion with a stronger impact exhibits the hysteresis and exists up to its stability saddle node bifurcation boundary $(SN_s)$.

**Stability**

The simplified stability analysis for periodic impact motions that is used (Peterka et al. (2004)) is based on the assumption that for small differences of the system state the initial motion perturbations develop according to $\Delta_k = \gamma^k \Delta_1$ where $\Delta_1$, and $\Delta_k$ are the initial perturbation and the after-$k$-impacts perturbation respectively and $\gamma$ the dominant Floquet multiplier belonging to the time-varying and non-smooth model. In the situation with the model as depicted in Fig.2.1, the number of Floquet multipliers of the system is equal to the order of the system. The order of the system is two, which results in two Floquet multipliers. The system is assumed to be stable if $|\gamma| < 1$ and the stability boundaries correspond to $|\gamma| = 1$. 


In Fig. 2.3 the system as depicted in Fig. 2.1 is analysed in case of a rigid stop and applying the restitution coefficient $R=0.6$. The damping is excluded by taking $b_1=0$. It shows the dimensionless gap width $\rho[-]$ vs. the dimensionless frequency $\eta[-]$. Various regions are shown for different values of $z$. Clearly one can see the curve of $\rho$ versus $\eta$. On the left side of this curve are no impacts. The lines $s_1$ and $s_2$ visualize the stability boundaries. Boundary $s_1$ correspond to a PD boundary, the process can stabilize to a limit cycle. Boundary $s_2$ correspond to a SN boundary, this process leads from the unstable $z=1/n$ impact motion to the impact less motion. The value of $\gamma$ is 1 for $s_2$ and $\gamma = -1$ for $s_1$. As a result of the boundaries $s_{1,2}$ and the grazing boundary denoted with $\phi$, regions are created with stable $z=1/n$ motions. These regions are hatched in Fig. 2.3. The stability boundaries $s_{1,2}$ converge at points $X_1$ to $X_5$. The transition from impact-less motion, $z=0$ to $p=1$ impact motion are continuous in these points $X$.

Figure 2.3: Rigid stop and $R = 0.6$, $b_1=0$ (Peterka (1996))

In Fig. 2.4 the system is analysed by applying a soft stop. It also shows the dimensionless gap width $\rho[-]$ vs. the dimensionless frequency $\eta[-]$. The value of the coefficient $k_2/k_1$ is 10 and $k_3/k_1$ is 2. The damping coefficient $\beta$ has value 0.01. In Fig. 2.4a one can clearly see the hysteresis regions. The area between $g_0$ and $SN_{1/16}$ is such a hysteresis region. An other hysteresis region is the region around $\eta=0.5$. 

5
In Fig. 2.4b the weak and strong stability boundaries are clearly depicted, e.g. the boundary region of shifting from $z=2/3$ to $z=1/3$ around $(\eta, \rho) = (0.83, 2.9)$. After crossing the grazing boundary $g_{1/3} = SN_{2/3w}$ there appears motion with a weak impact. This motion exists in the region along grazing boundary $g_{1/3}$. There arises a jump into the same type of motion but with stronger impact. This change is non-reversible and $z$ does not change. Only the motion with stronger impact exhibits hysteresis and exists up to its stability boundary $SN_{2/3}$.

*Figure 2.4a: Soft stop, Peterka and Tondl (2004)*
Analysis of two flexible rods

The study of Ma and Vakakis (2001) is about two flexible rods coupled by a non-linear spring with backlash included (Fig. 2.5). The variables $x_1$ [m] and $x_2$ [m] represent the global coordinates of the front edge of the two rods. The variables $w_1$ [m] and $w_2$ [m] represent the local coordinates of the two rods. The spring $k_1$ [N/m] is a spring with a clearance non-linearity. The force-displacement curve is also depicted in Fig 2.5. The parameters $a$ and $b$ [m] characterize the backlash in the spring $k_1$. Parameter $s$ [N/m] defines the stiffness for the range $\Delta w < a, \Delta w > b$. Here the relative position of the two beams is written in the local coordinates defined as $\Delta w = w_2(0) - w_1(L)$ with $w_2(0)$ the position equal to the left end of the second beam and $w_1(L)$ the position equal to the right end of the first beam. The spring $k_2$ [N/m] is a linear spring.
The order of the system is simplified by applying the Karhunen-Loève (K-L) decomposition. A K-L decomposition enables to extract spatial information from a set of time series data available on a spatio-temporal domain. This set of data consists of data measured simultaneously at different positions in the system.

Consider a scalar random field $v(x,t)$ in the spatio-temporal domain. The rod displacement at location $x_j$ at time instant $t_n$, $n=1,2,\ldots,N$ is denoted by $v(x_j,t_n)$. The aim of the K-L decomposition is to find the optimal orthogonal eigen-functions (K-L modes) that fit the measurements $v(x_j,t_n)$. To this end, a two point correlation matrix $K$ is constructed:

$$K_{i,j} = \frac{1}{N} \sum_{n=1}^{N} v(x_j,t_n)v(x_j,t_n)$$  \hspace{1cm} (2.5)

By solving the linear eigen-value problem $K\phi = \lambda\phi$, the K-L modes $\phi$ are computed. The corresponding eigenvalues $\lambda$ represent the amount of the energy of the signal captured by the corresponding modes. By construction, the resulting K-L modes form an orthogonal basis. The K-L modes are different from physical vibration modes, since they are not associated with natural frequencies; however, the K-L modes provide the primary coherent spatial structures of the system, and their energies are an indication of the dimensionality of the dynamics of the system. If most of the energy of the dynamical response is captured by $m$ K-L modes, the dimension of the system is $m$. When having a system with many elements, the way of constructing the correlation matrix is important in order to have computationally efficient order reduction. If the correlation matrix is constructed separately for each element of the system, the discretization of the governing partial differential equations is computationally inefficient. Instead the idea of coupled K-L modes is introduced, which reduces the order of the discretized system and makes the order reduction computationally accurate and efficient. Finally Poincaré maps are utilized to study the non linear behaviour caused by the clearance in the system.

2.2 Models of gear systems
Blankenship and Kahraman (1995) study the case of a preloaded mechanical oscillator having a time-varying stiffness function and a symmetric backlash between
the mass and the environment (Fig 2.6). The model and solution methodology are developed for analysing gear systems.

![Figure 2.6: The SDOF mechanical system, Blankenship and Kahraman (1995)](image)

The describing non-linear differential equation can be denoted as

$$\frac{d^2x}{d\tau^2} + 2\zeta \frac{dx}{d\tau} + \psi[\theta(\tau)]g[x(\tau)] = f(\tau)$$  \hspace{1cm} (2.6)

with \( \theta(\tau) \) [-] the system rotation angle depending on discrete time \( \tau = nh \) with \( h \) the time step and \( n \in [0, N - 1] \) with \( N \) the number of steps. The system rotation angle is given by \( \theta(\tau) = \Lambda \tau + \beta x(\tau) \) where \( \Lambda \) [1/s] is a time-invariant mean angular velocity. The term \( \beta x(\tau) \) represents a linear variation in \( \theta(\tau) \) with \( x(\tau) \), and \( \beta \) [1/m] is the so-called angle modulation depth. The parameter \( \zeta \) [N/m/s] is the viscous damping constant. The term \( \psi[\theta(\tau)] \) represents a spatially periodic stiffness function: the number of teeth making contact change periodically, the contact-positions of the teeth alter during motion. The function \( \psi[\theta(\tau)] \) is defined as

$$\psi[\theta(\tau)] = \psi[\theta(\tau) + 2\pi] = w_1 + \sum_{k=1}^{K}[w_{2k} \cos(k\theta) + w_{2k+1} \sin(k\theta)]$$  \hspace{1cm} (2.7)

The function \( g[x(\tau)] \) is a function which characterizes the symmetric, piecewise-linear backlash function: because of the rotating motion teeth make contact, but loose contact during rotating further. The function \( g[x(\tau)] \) is defined as

$$g[x(\tau)] = \begin{cases} x(\tau) - 1, & x(\tau) > 1, \\ 0, & |x(\tau)| \leq 1, \\ x(\tau) + 1, & x(\tau) < -1. \end{cases}$$  \hspace{1cm} (2.8)

The external excitation \( f(\tau) \), introduced to prescribe the force working on the teeth during contact is assumed to be periodic in \( \theta \) and therefore it can be represented as the Fourier series

$$f(\theta) = f_1 + \sum_{l=2}^{L}[f_{2l} \cos(l\theta) + f_{2l+1} \sin(l\theta)]$$  \hspace{1cm} (2.9)

For analysing the system a multiple term Harmonic Balance Method (HBM) scheme is used. Using this scheme, an analytical solution which adequately describes force response phenomena in the fundamental and sub-harmonic resonance regimes can be
obtained. The number of terms needed in the Fourier series can restrict the use of HBM. The steady state solution of the describing non-linear differential equation \( x(\tau) \) is assumed to be periodic in \( \tau \) with \( T \) [s] the period time, i.e. \( x(\tau) = x(\tau + T) \). Accordingly, \( g[x(\tau)] = g(\tau) = g(\tau + T) \) must also be periodic in \( \tau \) with the same period \( T \). Rewriting \( g[x(\tau)] \) introduces so called Fourier coefficients, \( v_i \)

\[
g(\tau) = v_i + \sum_{r=1}^{R} [v_{2r} \cos(r\theta) + v_{2r+1} \sin(r\theta)]
\]

(2.10)

The response \( x(\tau) \) is in the same form

\[
x(\tau) = u_i + \sum_{r=1}^{R} [u_{2r} \cos(r\theta) + u_{2r+1} \sin(r\theta)]
\]

(2.11)

The expressions for \( g(\tau), \psi(\tau), f(0) \) and \( x(\tau) \) are substituted into the differential equation. The harmonic balance form is enforced to form \( S=0 \) where \( S \) is a vector that consists of equations. The stability of a periodic solution \( x(\tau) \) is determined by examining the stability of the perturbed solution \( x(\tau) + \Delta x(\tau) \), using Floquet theory. The equation linearized for the perturbation \( \Delta x(\tau) \) is given by

\[
\Delta \ddot{x} + 2\zeta \Delta \dot{x} + \psi(\tau)\phi(\tau)\Delta x(\tau) = 0
\]

(2.12)

where \( \phi(\tau) \) is a discontinuous separation function. This can be rewritten in state space form as \( \dot{z} = G(\tau)z(\tau) \). Therefore one obtains \( \Phi = G(\tau)\Phi \) with \( \Phi \) the fundamental solution matrix or state transition matrix. The discretized map \( G_n \) is obtained by

\[
G_n = \frac{1}{h} \int_{(n-1)h}^{nh} G(\tau) d\tau, \quad n \in [0, N-1]
\]

(2.13)

which is used to derive an approximate value of monodromy matrix \( M = \Phi(T) \) with \( T \) the period time and \( T = N \cdot h \)

\[
M = \prod_{n=0}^{N-1} \left[ I_2 + \sum_{p=1}^{P} \left( \frac{hG_n^p}{p!} \right) \right]
\]

(2.14)

where \( I_2 = \Phi(0) \) the 2x2 identity matrix and \( P \) the number of terms in the approximation for the matrix exponential.

By observing the Floquet multipliers \( \gamma \), obtained from calculating the eigen values of the monodromy matrix, bifurcations can be classified. For instance a period doubling bifurcation occurs when one multiplier leaves the unit circle at \(-1\) when the other remains inside. The solution is unstable if one multiplier \( \gamma \) in absolute value is larger than one. To obtain different harmonic solutions the above equation of motion is divided in three different contact regimes, no impact, single sided impact and double-sided impact. These can approximately characterize the system. For these three separate functions the HBM procedure can be applied.

In Fig. 2.7 a reconstruction with the HBM method, compared with numerical integration and experimental results is shown. The root mean square amplitude values \( u_{rms} \) versus the dimensionless time-invariant mean angular velocity \( \eta [-] \) are plotted.
It includes the fundamental and first two super-harmonic resonance's. The data exhibits only no-impact motion and single-sided-impact motion. The three peaks that are shown are pointing to the lower frequencies. The cause of this phenomenon might be a large pretension in the system. In systems with no pretension but having initial a clearance the resonance peaks can point to the right and systems with initial no pretension or clearance show no bending peaks.

Figure 2.7: A comparison of HBM solutions (-----) with numerical integration (□) and experimental results (O), Blankenship and A. Kahraman 1995

Litak and Friswell (2003) also investigate gearbox dynamics. For their analysis of gear systems a low dimensional system is used (2 or 3 DOF). In the 2 DOF model a one stage transmission gear system is investigated. Due to backlash, the system has a non-linear stiffness and can vibrate periodically or chaotically. This behaviour may change if a shaft flexibility is introduced on one side of the gearbox (3 DOF model). The system contains three wheels with mass moments of inertia $I_1$, $I_2$, $I_3$ and between the first two a spring stiffness $k$, [Nm/rad], a viscous damper $c$, [Nm/rad/s] and a clearance. The second wheel and third are connected by a spring stiffness $k$, [Nm/rad] (Fig. 2.8). The symbols ψ and M represent the angles of the wheels and moments applied to the wheels respectively.
The three describing differential equations are

\[ \begin{align*}
I_1 \ddot{\psi}_1 + [k_z (r_1 \psi_1 - r_2 \psi_2) + c_z (r_1 \dot{\psi}_1 - r_2 \dot{\psi}_2)] \tau_1 &= M_1 \\
I_2 \ddot{\psi}_2 - [k_z (r_1 \psi_1 - r_2 \psi_2) + c_z (r_1 \dot{\psi}_1 - r_2 \dot{\psi}_2)] \tau_2 - k_z (\psi_3 - \psi_2) &= -M_2 \\
I_3 \ddot{\psi}_3 + k_z (\psi_3 - \psi_2) &= -M_3
\end{align*} \] (2.15)

with \( r_1 \) through \( r_3 \) [m] the radii of the three gears respectively. In the latter equations of motion (2.15), backlash is left out. The three variables \( \psi_{1,2,3} \) which are representing the rotations of the three gears are used to define two new variables \( x_1 \) and \( x_2 \) [m]. The new variables are denoted by

\[ x_1 = r_1 \psi_1 - r_2 \psi_2 \quad \text{and} \quad x_2 = r_2 (\psi_3 - \psi_2) \] (2.16).

A backlash function \( g(x_1, \eta) \) is modelled as

\[ g(x_1, \eta) = \begin{cases} 
 x_1 & x_1 \geq 0, \\
 0 & -\eta < x_1 < 0, \\
 x_1 + \eta & x_1 \leq \eta.
\end{cases} \] (2.17)

with \( \eta \) [m] the clearance and is applied by defining

\[ \frac{k_z \left( r_1^2 + r_2^2 \right)}{\omega^2 (I_1 + I_2)} x_1 = \frac{k(\tau) g(x_1, \eta)}{\omega^2} \] (2.18).

The meshing stiffness function \( k(\tau) \) [s^{-2}] as used in (2.18) depending on the number of teeth in contact is displayed in figure 2.9 with \( \tau = \omega \cdot t \) [rad] and where \( \omega \) [rad/s] is the frequency of the moments on the gear system.
Figure 2.9: The meshing stiffness function $k(\tau)$, Litak and Friswell (2003)

New coefficients $\beta_{1,2,3}$ and $2\zeta$ are defined as

$$
\beta_1 = \frac{1}{I_2}, \\
\beta_2 = \frac{1}{I_2} + \frac{1}{I_3}, \\
\beta_3 = \left(\frac{r_2^2}{I_2}\right)\left(\frac{1}{I_1} + \frac{r_2^2}{I_2}\right) \\
0 = r_2M_2/I_2 - r_2M_3/I_3, \\
2\zeta = c_1\left[\frac{r_2^2}{I_1} + \frac{r_2^2}{I_2}\right]
$$

(2.19)

Two more functions are defined. These concern the right hand side of the equations of motion

$$
M_1\frac{r_1}{I_1\omega^2} + M_2\frac{r_2}{I_2\omega^2} = \frac{B_0 + B_1 \cos(\omega \cdot t + \Theta) + D \cos(\tau + \Theta')}{\omega^2},
$$

(2.20)

$$
0 = r_2M_2/I_2 - r_2M_3/I_3.
$$

Using (2.16) through (2.20) in the differential equations (2.15) gives

$$
\begin{align*}
\frac{d^2}{d\tau^2}x_1 + \frac{2\zeta}{\omega} \frac{d}{d\tau}x_1 + \frac{k(\tau)}{\omega^2}g(x_1, \eta) + \frac{\beta_1k}{\omega^2}x_2 &= \frac{B_0 + B_1 \cos(\omega \cdot t + \Theta) + D \cos(\tau + \Theta')}{\omega^2} \\
\frac{d^2}{d\tau^2}x_2 + \frac{2\beta_3\zeta}{\omega} \frac{d}{d\tau}x_1 + \frac{\beta_1k}{\omega^2}x_2 + \frac{\beta_2k(\tau)}{\omega^2}g(x_1, \eta) &= 0
\end{align*}
$$

(2.21)

Besides the external periodic excitation, an additional external periodic excitation is working on the system. The amplitude of this additional periodic excitation has a relatively small amplitude $D$. Furthermore the phase angle $\Theta'$ differs from $\Theta$. This kind of excitation is applied for chaos control. The subject chaos control is out of the scope of this report and is therefore not explained here. The type of motion is analysed using Lyapunov exponents. To estimate the convergence and divergence of two trajectories over infinite time intervals, one defines the Lyapunov exponent associated with the divergence of two trajectories. Sastry (1999) defines the Lyapunov exponent $\lambda$ with $x(t)$ and $y(t)$ the two trajectories as

$$
\lambda = \limsup_{t \to \infty} \left[ \frac{1}{t} \log \left( \frac{|x(t) - y(t)|}{|x_0 - y_0|} \right) \right]
$$

(2.22)
The results are presented in plots with the calculated vibrational amplitude of the relative motions of the gear wheels \( A_i = \frac{x_{i_{\text{max}}} - x_{i_{\text{min}}}}{2} \) (with \( i=1,2,3 \) the number of the gear) versus the excitation frequency \( \omega \) and the maximal Lyapunov exponent \( \lambda_i \) versus excitation frequency \( \omega \) (Fig. 2.10). Here \( \lambda_i > 0 \) represents a chaotic and \( \lambda_i < 0 \) a regular system.

The results for the amplitude \( A \) with various stiffness \( k_s \) are summarized in Fig. 2.10a. Curves 1, 2 and 3 correspond to \( k_s = 0, 0.1, 1.0 \), respectively. One can see that altering \( k_s \) leads to changes of the amplitude. Comparing the corresponding values of the maximal Lyapunov exponents one can investigate the possible chaotic behavior of the vibrations (Fig. 2.10b). Namely, the case \( k_s = 0.1 \) (curve #2) does not reduce the original chaoticity in region around \( \omega \approx 1.0 \) and \( \omega \in [1.24, 1.62] \) (curve #1) while \( k_s = 1.0 \) (curve #3) is large enough to tame the chaos in these regions as well as to make the vibration amplitude smaller. For \( k_s \neq 0 \) there are four nonzero exponents to be examined (Figs. 2.10c,d). For a small coupling stiffness, \( k_s = 0.1 \), the most of the chaotic regions are characterized by two positive exponents (Fig. 2.10c), which is a signal that the system is hyperchaotic. The results of calculations with \( k_s = 1.0 \) are different (Fig. 2.10d), where regions of chaotic motion with only one positive Lyapunov exponent are found. This points at typical chaotic behavior.
Figure 2.10: The amplitude of vibration for $k_s = 0(\#1), 0.1(\#2), 1(\#3)$ (a) and the corresponding maximal Lyapunov exponent $\lambda_1$ (b) versus $\omega$ and Lyapunov exponents $\lambda_i (i = 1, 2, 3, 4)$ versus frequency $\omega$ are shown for $k_s = 0.1$ (c) and $k_s = 1.0$ (d). Litak and Friswell (2003)
3 Experiments on the piecewise linear beam

In this chapter the second part of the problem definition is going to be covered: “What are the effects of flush, backlash and pretension on the steady-state dynamics of the piecewise linear beam with and without the Dynamical Vibration Absorber (DVA) as applied by Bonsel(2003)?”

Section 3.1 describes the experimental set-up, the piecewise linear beam on which the experiments are done. In section 3.2.1. the way of presenting the measurement data is explained and in section 3.2.2 an overview of the experiments is presented. In order to answer the second part of the problem definition as stated above, the results of experiments with flush, backlash and pretension are presented in sections 3.2.3 (without DVA) and 3.2.4 (with DVA). Appendix B discusses the effects of the crash of the experimental set-up.

3.1 Experimental set-up

3.1.1 Set-up of the beam system

The experimental set-up as used in this research and as used by Bonsel(2003) (Fig.3.1) consists of an elastic beam mounted on two leaf springs. The middle of the beam is harmonically excited by a rotating mass unbalance consisting of three mass unbalances positioned on an axle, called the shaker. The shaker is connected to the beam via a force transducer. This shaker is driven by a synchronic electric motor with a variable speed in the range of 5 to 60 Hz with steps of 0.5 Hz. The moment of the motor is transmitted via two flexible couplings and an axle. Parallel to the main beam a second beam is placed. This beam is clamped on both sides. In the middle of this beam a pin is mounted. In the case of a negative deflection of the main beam, these two beams are in contact. In the case of a positive deflection, there is no contact with the second beam. This introduces a one-side spring stiffness to the beam system and makes it piecewise linear. Additionally, the Dynamical Vibration Absorber (DVA) can be placed.

By reading the display of the frequency controller of the electric motor the harmonic excitation frequency is determined.

On the beam just below the position of the mass unbalance an accelerometer is placed. Between the shaker and the beam a force transducer is mounted. Additionally
two Linear Voltage Displacement Transducers can be placed on the main beam. The measurements are recorded using Siglab equipment.

3.1.2 Sensors used
It is possible to obtain the displacements and acceleration of points on the beam by using the Linear Voltage Displacement Transducers or the accelerometer. Because of the fact that Bonsel(2003) used the accelerometer to obtain the signals and the very sensitive construction of the Linear Voltage Displacement Transducer to the beam, here the accelerometer is applied. By integrating the accelerometer signal twice in Matlab the displacement of the beam is obtained:

\[
\begin{align*}
\dot{v} &= \int_0^t a \cdot dt + v_0 \rightarrow v_{\text{final}} = v - \text{mean}(v) \\
x &= \int v_{\text{final}} \cdot dt + x_0 \rightarrow x_{\text{final}} = x - \text{mean}(x)
\end{align*}
\]

(3.1)

with \(a [m/s^2]\) the accelerometer signal and \(v_0 [m/s]\) and \(x_0 [m]\) the integration constants. Subtracting the mean value of the velocity \(v\) of the velocity signal makes sense in the physical reality. If the mean value wouldn’t be zero, the beam would drift away.

3.2 Experiments

3.2.1 Presentation of the results
For analysing the system frequency amplitude plots are used. The frequency range is 5 to 60Hz with a resolution of 0.5Hz. In the cases of backlash and pretension the maximum displacement of the middle of the beam is plotted vs. the excitation frequency in Hz. The maximum displacement is defined as \(x_{\text{max}} = \max(x_p) - \min(x_p)\) with \(x_p\) the displacement signal of one response period obtained from the displacement signal \(x\). In the case of flush and in the linear case it is possible to scale \(x_{\text{max}}\) with the force amplitude without changing the characteristics of the system. A mathematical explanation can be found in Appendix A.

3.2.2 Overview of the experiments
As mentioned in paragraph 3.1 the measurements are done with and without the DVA. Experiments were carried out for the situations with flush, backlash or pretension. The experiments that are done are schematically listed in table 3.1. The list is divided in two parts: the measurements that are done before the crash and the measurements that are done after the recovering of the experimental set-up. Appendix B discusses the effects of the crash on the experimental set-up.

At first, the experiment of Bonsel(2003) with flush and without the DVA is repeated to get clarity about the state of the whole system. After that, the experiments with backlash and pretension are done. Next the experiment of Bonsel(2003) with the DVA is repeated. After these experiments several experiments with backlash with the DVA are done.
Table 3.1: list of experiments

<table>
<thead>
<tr>
<th>DVA</th>
<th>relative position pin [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>backlash: + pretension: -</td>
</tr>
<tr>
<td></td>
<td>Before the crash</td>
</tr>
<tr>
<td>no</td>
<td>flush</td>
</tr>
<tr>
<td>no</td>
<td>+1</td>
</tr>
<tr>
<td>no</td>
<td>+0.6</td>
</tr>
<tr>
<td></td>
<td>After the recovering of the experimental set-up</td>
</tr>
<tr>
<td>no</td>
<td>+0.15</td>
</tr>
<tr>
<td>no</td>
<td>+0.05</td>
</tr>
<tr>
<td>yes</td>
<td>flush</td>
</tr>
<tr>
<td>yes</td>
<td>+0.1</td>
</tr>
<tr>
<td>yes</td>
<td>+0.3</td>
</tr>
</tbody>
</table>

3.2.3 Results: system without DVA

In this paragraph the results of the experiments on the system without the DVA are discussed.

In Fig. 3.2 the experimental and numerical results of Bonsel with flush are presented. Clearly, it can be seen that when comparing his numerical and experimental results the harmonic frequency at 19Hz and several sub-harmonic responses correspond. In the upper figure of Fig. 3.3 reconstruction measurement results are presented. Clearly the harmonic response and the 1/2 and 1/3 sub-harmonic responses can be seen. These correspond to the results as presented by Bonsel(2003). Absent are the sub-harmonic responses at 1/5 and 1/4 of the harmonic response. At 5, 8.5 and 11.5Hz one can see little peaks. These are super harmonic resonances. In the case of flush it can be proved that the frequency responses can be scaled by $F_0$. In the case of pretension or backlash this is not possible. (Appendix A). In the lower figure of Fig.3.3 the same data is presented but not scaled by $F_0$. 

18
Figure 3.2: A comparison of numerical integration (--) with experimental results by Bonsel(2003)

Figure 3.3: Experimental results: harmonic response at 19 Hz, 1/2 sub harmonic response at 28Hz and 1/3 sub harmonic response at 57Hz
Subsequently, the measurements with backlash and pretension are presented. In Fig. 3.4 responses are shown for backlash values +0.6 and +1 mm respectively. The larger the initial clearance, the larger the linear motion amplitude has to become to touch the pin mounted on the additional spring. At low frequencies, with low amplitudes the beam will not touch the one sided spring and it should behave like a linear system. The maximum amplitude caused by the resonance in the piece wise linear system with backlash is at approximately the same frequency as in the case of flush. The resonance peak begins at lower frequencies like in the linear case. When the excitation frequency passes the resonance frequency the amplitude decreases and the system behaves like a linear system again. If the amplitude is too low, the sub-harmonic motions will not occur.

![Graph showing responses for backlash and linear cases](image)

*Figure 3.4; O: +0.6, *:+1 mm backlash and V: linear case; before crash*

In Fig. 3.5 one can see the responses that are obtained after correcting the shaker when clearances of +0.15mm and +0.05mm applied. These sets of data are compared with the flush situation before the crash. In the situation of +0.05mm backlash the amplitude of the beam after the first harmonic response is large enough to cause a 1/2 sub-harmonic motion. Here, at higher frequencies the amplitude is not high enough anymore to let the system go in a 1/3 sub-harmonic motion. In the case of +0.15mm backlash the clearance is too large to cause any sub-harmonic motions.
3.2.4 Results: system with DVA

In this section the results of the experiments on the system with DVA are discussed. The DVA is tuned to suppress the resonance frequency at 19 Hz which is present in the case of flush.

Figure 3.6 shows measurement results in the case of flush and 0.6 mm backlash. One can see that the maximal displacement in case of backlash is approximately the same value and at the same frequency as it is in case of flush. The experiments with the DVA and backlash that are presented in this section are done to find out the effects of the DVA in case of backlash where the DVA is tuned at 19 Hz.

In Fig. 3.7 the experimental and numerical results of Bonsel are presented. Clearly it can be seen that when comparing his numerical and experimental results the suppression of the harmonic frequency at 19 Hz and several sub-harmonic responses correspond. In Fig. 3.8 reconstruction measurement results are presented. Again, the suppression of the harmonic response can be seen. These results correspond to the results as presented by Bonsel (2003). Less clear but also perceptible is the 1/3 sub-harmonic response around 45 Hz. At 42 Hz one can see another 1/2 sub-harmonic response, probably caused by the peak at 21 Hz. Peaks at 15 and 21 Hz are caused by applying the DVA (Bonsel (2003)). The unclear 1/5 sub-harmonic response around 38 Hz can be linked with the super harmonic response at 7.5 Hz. In the lower plot of Fig. 3.8 the same data as in the upper plot is presented, only without scaling by $F_0$. 
Figure 3.6: Flush compared with 0.6mm backlash

Figure 3.7: A comparison of numerical integration (--) with experimental results by Bonsel (2003) of the system with DVA and flush
Figure 3.8: Experimental results: damped harmonic response at 19 Hz

The plots in Fig. 3.9 and Fig. 3.10 show the system with DVA with +0.1 and +0.3 mm backlash applied respectively. In both cases the compression of the 19 Hz resonance is shown clearly. The system with +0.1 mm backlash shows at 30 Hz the 1/2 sub-harmonic motion caused by the 15 Hz resonance. In the system with +0.3 mm this sub-harmonic motion is no longer present. In both cases one can see peaks at 52 and 57 Hz. The effect between 52 and 55 Hz is caused by the DVA, because of the fact that this effect is not visible in the situations without DVA.
Figure 3.9: +0.1mm backlash with DVA

Figure 3.10: +0.3mm backlash with DVA
4 Conclusions and recommendations

The conclusions covering the problem definition are divided in two parts. First the conclusions that cover the first part of the problem definition, the literature study are presented. Next the conclusions that cover the second part, the experiments on the piecewise linear beam are presented. Finally suggestions for further research are made.

4.1 Conclusions covering the literature study

In chapter two the first part of the problem definition is covered:

“In which categories can the mechanical systems with backlash be divided, what are the models that are used to analyse the systems in these categories respectively and which methods are used to analyse the steady-state behaviour of these models, concerning the results of the research that has been done over the past ten years in the field of dynamics in mechanical systems with backlash?”

The search results can be divided in two categories: Models of non-gear systems and models of gear systems. A typical property of the models of gear systems is that the stiffness parameter of the spring between two gear wheels is time varying because of the changing of the number of teeth that make contact as a result of the rotating of the gears.

Models that are used to analyse non-gear systems are single degree of freedom mass-spring models and flexible rod models. For analysing gear systems single degree and multi degree of freedom models are used.

For analysing the steady state behaviour of the models, several methods are applied. The type of steady-state motion can be characterized by the quantity \( z = p/n \) where \( p \) is the number of impacts in the motion period and \( n \) is the number of excitation force periods in the motion period \( T \) [s]. Three types of region boundaries can be defined: grazing, period doubling and saddle node. These impact motion regions bounded by the region boundaries are visualized in figures with on the axes the dimensionless clearance vs. the dimensionless frequency. The stability of the impact motions can be analysed by using Floquet multipliers.

The type of motion can also be analysed using Lyapunov exponents. Comparing the corresponding values of the maximal Lyapunov exponents with different stiffness constants applied, the chaotic nature of the vibrations is investigated.

Ways to visualize the dynamic behaviour are drawing bode plots or using poincaré maps. These poincaré maps are utilized to study the possible sub harmonic responses.

4.2 Conclusions covering the experiments on the piecewise linear beam

In chapter three the second part of the problem definition is covered:

“What are the effects of flush, backlash and pretension on the steady-state dynamics of the beam with and without the Dynamical Vibration Absorber (DVA) as applied by Bonsel(2003)?”
Measurements with flush and without the DVA are done with the aim of reconstructing the measurement results as presented by Bonsel(2003). Clearly the harmonic response and the $1/2$ and $1/3$ sub harmonic responses can be seen. It can be concluded that the state of the mechanical system and its measurement equipment are in the original shape. From measurements done with 1 and 0.6mm initial clearance and presented in the frequency domain it can be concluded that the effects of backlash can clearly be seen in the form of resonance peaks. Before that the beam touches the one sided spring the system behaves like the linear system without the one sided spring. The larger the initial clearance the higher the maximum displacement and in this case the higher the frequency before the frequency response will differ from the linear case. When backlash is compared with flush the resonance peak starts at lower frequencies with backlash than with flush. The maximum amplitude is approximately on the same frequency as in the case of flush. With backlash the resonance peaks bend over to the right.

After these experiments the mechanical system crashed. As a result of the crash the properties of the shaker and the force transducer changed. The accelerometer used to determine the displacement of the beam did not change.

After approximately reconstructing the shaker, two more measurements with backlash are done from which one can conclude that with sufficiently small initial clearances besides the fundamental resonance peak the $1/2$ sub harmonic resonance peak can be found. These peaks are both bending to the higher frequencies.

Finally three measurements with flush and backlash are done with the DVA applied. First the measurement done by Bonsel(2003) in the case of flush is reconstructed. The measurements done by Bonsel(2003) and the reconstruction correspond. One can conclude that DVA is still tuned like it was by Bonsel(2003) and it is still possible to suppress the 19Hz resonance frequency. Next two more experiments are carried out with 0.1 and 0.3mm backlash respectively. In these cases the DVA is still able to suppress the harmonic resonance peak. From these experiments one can conclude that there are situations in which the DVA is applicable in systems with backlash.

### 4.3 Recommendations

**Literature study**

When the research would cover more papers as a result of more advanced search methods, a better overview of the research that has done could be made. It is possible that when more time will be invested more results can be found, because of the better understanding of the field of research after writing this report.

**Experiments**

In general more detailed information could be obtained if the resolution of the clearance range would be higher and the frequency steps could be smaller.

In this research relatively too little experiments are done with pretension to give a good view of the effects of pretension. To get more insight it could be convenient to carry out more experiments with pretension.
In order to investigate the general applicability of the DVA in systems with backlash or pretension it is recommended to do more experiments with different values for the backlash and pretension values.

Also it should be investigated if the eigen frequency of the DVA (19 Hz) should be decreased or increased respectively in case of backlash and pretension to obtain better results.
5 References


F. Peterka, T. Kotera and S. Cipera 2004 *Chaos, Solitons & Fractals* 19, 1251-1259. Explanation of appearance and characteristics of intermittency chaos of the impact oscillator

F. Peterka and A. Tondl 2004 *Chaos, Solitons & Fractals* 19, 1283-1290. Phenomena of sub harmonic motions of oscillator with soft impacts

Grzegorz Litak and Michae1 I. Friswell 2003 *Nonlinear Sciences CD/0302050v1.* Nonlinear Vibration in Gear Systems

Grzegorz Litak and Michael I. Friswell 2003 *Chaos, Solitons & Fractals* 16(5), 795-800. Vibration in gear systems


Xianghong Ma and Alexander F. Vakakis 2001 *Journal of Vibration and Acoustics* 123, 36-44. Nonlinear transient localization and beat phenomena due to backlash in a coupled flexible system


J.H. Bonsel 2003, Master's thesis DCT 2003.77, Application of a dynamic vibration absorber to a piecewise linear beam system

A. Doris and C.G.M.deBont 2004, Validation of the Piecewise Linear Beam Components
Appendix A: A mathematical explanation for scaling $x_{\text{max}}$ with $F$

The equation of motion of a periodically forced piecewise linear single degree of freedom mass-spring system is as follows:

$$m\ddot{x} + k_1 \cdot x + \sigma(x) \cdot k_{\text{ad}} \cdot x = F \cdot \cos(\omega_\text{e} t) \quad (A.1)$$

with $k_1$ and $k_{\text{ad}} [\text{N/m}]$ the main stiffness and the additional spring respectively and $m [\text{kg}]$ the mass. The coefficient $\sigma(x) [-]$ represents the discontinuity having value 0 or 1 respectively and is a function of displacement $x$:

$$\sigma(x) = \begin{cases} 
0 & \text{if } x - a \geq 0 \\
1 & \text{if } x - a < 0 
\end{cases} \quad (A.2)$$

with $a [\text{m}]$ the initial clearance between the mass and the additional spring. The excitation force is characterized by the amplitude $F [\text{N}]$ and the excitation frequency $\omega_\text{e} [\text{rad/s}]$.

The displacement $x$ can be divided by the forcing amplitude $F$ resulting in the scaled displacement $X [\text{m/N}]$:

$$X = \frac{x}{F} \quad (A.3)$$

Substituting (A.3) in (A.1) results in the scaled equation of motion:

$$m \cdot F \cdot \ddot{X} + k_1 \cdot F \cdot X + \sigma(X,F) \cdot k_{\text{ad}} \cdot F \cdot X = F \cdot \cos(\omega_\text{e} t)$$

or

$$m \cdot \ddot{X} + k_1 \cdot X + \sigma(X,F) \cdot k_{\text{ad}} \cdot X = \cos(\omega_\text{e} t) \quad (A.4)$$

The new discontinuity coefficient $\sigma(X,F)$ becomes:

$$\sigma(X,F) = \begin{cases} 
0 & \text{if } X - \frac{a}{F} \geq 0 \\
1 & \text{if } X - \frac{a}{F} < 0 
\end{cases} \quad (A.5)$$

In the case that $a = 0$, the scaled discontinuity function (A.5) becomes $\sigma(X)$. In combination with (A.3) it can be concluded that the forcing amplitude $F$ only influences the real displacement as a linear scaling factor.

In the case of $a \neq 0$ the scaled discontinuity function (A.5) depends on $F$ explicitly and $F$ influences the dynamics of (A.4). $F$ influences the real displacement more than only as a linear scaling factor.
Appendix B: The crash of the system

B.1 Checking system properties
During measurements on the set-up at a high frequency the mounting element between the force transducer and the shaker failed. As a result of the accident the characteristics of the excitation mechanism are changed. The middle wheel has rotated with respect to the outer wheels and so the resultant unbalance of the shaker has decreased. In the process of trying to repair the system the pretension in the force transducer has also changed. The position of the middle wheel has been corrected by hand. The maximum displacement \( x_{\text{max}} \) vs. the excitation frequency [Hz] in the range of 5 to 20Hz in the linear case is depicted in Fig. B.1a,b. Three sets of data are plotted; data obtained before the crash, just after the crash and after the shaker is corrected. Fig. B.1a shows the data on linear scales. Here one can clearly see the deviation between the three sets of measuring data. Fig B.1b shows the same data with the vertical axe in logarithmic scale. Here one can clearly see the non linear behaviour of the system just after the crash. Also a nonlinearity can be seen in the plot of the data measured before the crash around 7Hz. The deviations between the data before the crash and after correcting the shaker may be caused by an inaccurate correction or other changes in the system due to the crash. The strong non-linear behaviour of the response of the system just after the crash at frequencies between 5 and approximately 10 Hz might be due to the fact that in situations with low frequencies the system is excited with a low force amplitude so that non-linearities that are not identified are dominant.
Figure B.1: linear system; O: before crash; □: after crash, ▽: after correction of the shaker
In Fig. B.2 one can see the frequency-force amplitude characteristics of the shaker obtained by force-measurements with the force transducer placed between the shaker and beam. Theoretically the relation should be \( F_0 = r e m (2\pi f)^2 \) with \( r e [m] \) the eccentricity, \( m e [kg] \) the eccentric mass of the shaker and \( f [Hz] \) the frequency of the shaker. The \( r e m [m \cdot kg] \) values for the least squares fit to the measured data in the cases ‘before crash’, after crash’ and ‘after correction of the shaker’ as depicted in Fig. 3.3 are 0.0010, 0.0002 and 0.0009 mkg respectively. The state of the force transducer should be investigated to get more information about the quality of the measured signal.

![Figure B.2: force characteristic shaker, obtained by force-measurements; O: before crash; □: after crash, ▼: after correction of the shaker](image)

The force transducer, accelerometer and Linear Voltage Displacement Transducers have been tested. The test procedures and the conclusions drawn from these tests are described in a report written by Doris and De Bont(2004).

In the report has been concluded that the force transducer performs satisfactory in a frequency range from 5 to 500Hz and for loads between 0 - 10N and 40 - 150N. According to the constructor of the Force Transducer, the line that describes its sensitivity function (relation between \( Q \) and \( F \)) is

\[
Q = \begin{cases} 
5.92 \cdot F & \text{for } 0 \leq F \leq 5000 \\
5.96 \cdot F & \text{for } -5000 \leq F \leq 0,
\end{cases}
\]

where \( F \) is the force output of the force transducer in [N] and \( Q \) is the charge of its amplifier in [\( \mu C \)]. Since in the present experimental set-ups the maximum absolute value of the force \( F \) is around 150N, a better approximation of the sensitivity
function between $Q$ and $F$ is obtained. This approximation can be expressed as

$$Q = \begin{cases} 
3.92 \cdot F & \text{for } 0 \leq F \leq 10 \\
3.52 \cdot F + 4 & \text{for } 10 \leq F \leq 40 \\
3.62 \cdot F & \text{for } 40 \leq F \leq 150 
\end{cases}$$

Figure B.3: sensitivity function of the FT

About the accelerometer has been concluded that it works good. After fixing a cable problem it has been concluded that both the Linear Voltage Displacement Transducers work properly.
**B.2 Measuring results just after the crash**

As mentioned in paragraph 3.1 measurements are done with and without the DVA. Experiments were carried out for the situations with flush, backlash or pretension. In table B.1 an overview of all the experiments that are done is given. This section covers the experiments just after the crash.

**Table B.1: experiments listed**

<table>
<thead>
<tr>
<th>DVA</th>
<th>relative position pin [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>backlash: +</td>
</tr>
<tr>
<td></td>
<td>pretension: -</td>
</tr>
<tr>
<td></td>
<td>Before the crash</td>
</tr>
<tr>
<td>no</td>
<td>flush</td>
</tr>
<tr>
<td>no</td>
<td>+1</td>
</tr>
<tr>
<td>no</td>
<td>+0.6</td>
</tr>
<tr>
<td></td>
<td>Just after the crash</td>
</tr>
<tr>
<td>no</td>
<td>-0.8</td>
</tr>
<tr>
<td>no</td>
<td>+0.15</td>
</tr>
<tr>
<td></td>
<td>After the reconstruction of the shaker</td>
</tr>
<tr>
<td>no</td>
<td>+0.15</td>
</tr>
<tr>
<td>no</td>
<td>+0.05</td>
</tr>
<tr>
<td>yes</td>
<td>flush</td>
</tr>
<tr>
<td>yes</td>
<td>+0.1</td>
</tr>
<tr>
<td>yes</td>
<td>+0.3</td>
</tr>
</tbody>
</table>

Fig. B.4 also shows the situation of a system with a pretension value of -0.8 mm. As expected, the peak points to the lower frequencies. This data has been obtained just after the crash and its shape is reasonable when comparing it with the linear case as measured just after the crash. This also holds for the +0.15mm backlash data obtained just after the crash.
Figure B.4: □; linear; O; -0.8 mm pretension; *; +0.15mm backlash; just after crash