Application of a dynamic vibration absorber to a piecewise linear beam system under pre-tension

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Application of a dynamic vibration absorber
to a piecewise linear beam system
under pre-tension

J.C.A. de Bruin

DCT 2005.07

Traineeship report

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Eindhoven, March, 2005
Abstract

The following question is experimentally investigated in this report: to which frequency should a dynamic vibration absorber, which is attached to a piecewise linear beam system at condition of pre-tension be tuned to reduce vibrations.

Two types of dynamic vibration absorbers are used: an undamped and a damped dynamic vibration absorber (DVA). The purpose of the application of the undamped DVA is to minimize the response at one specific frequency. And the purpose of the application of the damped DVA is to minimize the response for a frequency range.

For this investigation experiments are done mainly with the undamped DVA for different tuned natural frequencies. Additionally some experiments are done with the damped DVA. The experiments are performed for different levels of pretension.

After application of an undamped DVA, it can be said from the experimental results that the minimization of the response of one specific frequency is achieved. The amplitude of the first harmonic resonance peak is reduced after the application of the undamped DVA.

There is no clear guideline for tuning the natural frequency of the undamped DVA for an optimal result. But the amplitude can be largely decreased for a single excitation frequency just below the tuned natural frequency of the undamped DVA.

The experimental results of the application of the damped DVA give varying results. Improvement of the minimization of the response over a frequency range with respect to the application of the undamped DVA is not achieved for all experimental results.
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Chapter 1

Introduction

Vibrations appear in many systems. They often limit the performance of a system. Therefore reduction of vibrations of systems is important.

The reduction of the vibrations of a piecewise linear beam system is investigated in this report. An experimental set-up of the piecewise linear beam system is available in the DCT laboratory. The beam system is called piecewise linear because its stiffness is different for positive and negative deflection which results in two linear regimes. In the experiments the middle of the beam is excited by the rotating mass unbalance at a constant frequency and the resulting transversal acceleration of the middle of the beam is measured by an accelerometer, which is attached to the beam.

To reduce the vibrations for the middle of the beam, a dynamic vibration absorber (DVA) is used. Two types of dynamic vibration absorbers are used: an undamped and a damped DVA. The DVA can be tuned for a specific natural frequency. An undamped dynamic vibration absorber is available in the DCT laboratory. Dampers can optionally be connected to the undamped DVA to get a damped DVA.

Good results were already obtained for reducing the vibrations of the piecewise linear beam system at condition of flush for both the undamped DVA and the damped DVA for the middle of the beam [1]. The condition which will be investigated in this report, is the condition of pre-tension.

The main question is: to which frequency should a dynamic vibration absorber (DVA), which is attached to a piecewise linear beam system at condition of pre-tension be tuned to reduce vibrations. The purpose of the application of the undamped DVA is to minimize the response at one specific frequency. The specific frequency is in this case the peak of the first harmonic resonance. And the purpose of the application of the damped DVA is to minimize the response for a frequency range, around the peak of the first harmonic resonance.

Experiments are first done for the condition of flush to validate the experimental set-up and the post processing. The dynamical behavior for three levels of pre-tension is investigated next. Then the DVA is applied for two levels of pre-tension. The experiments are done mainly with the undamped DVA for different tuned natural frequencies. Additionally some experiments are done with the damped DVA.
Measurements are done for a large number of frequencies throughout the frequency range of interest for a level of pre-tension and a natural frequency of the applied DVA. The frequency range of interest is from 10 up to 50 Hz. The measurements are used for the computation of amplitude frequency characteristics, which are used for determining the effect of the applied DVA. The post processing is done with a computer.

An outline of the sections of this report:
In chapter 2 the used experimental set-up will be explained. Next, in chapter 3 the method for post processing of the measured data will be described. In chapter 4 the results of the different experiments will be presented. Conclusions and recommendations are given in chapter 5.
Chapter 2

Experimental set-up

2.1 Experimental set-up beam system

An experimental set-up of the piecewise linear system is available in the DCT-lab. See figure (2.1) for a schematic view of the set-up. The piecewise linear beam system consists of a main beam, which is attached to two leaf springs. A second beam, which is clamped at both ends, acts as one-sided spring which is only active for negative deflection of the middle of the main beam. The system is harmonically excited by a rotating mass unbalance driven by an electric motor. The pre-tension can be adjusted with a bolt at the one-sided spring. This bolt pushes against the middle of the main beam. That means that both the main beam and the one-sided spring are under pre-tension. The adjustment of the bolt has to be done exactly, because small differences in adjustment cause different behavior of the beam. The level of pre-tension is determined by measuring the displacement of the middle of the beam with respect to the position of the middle of the beam in flush situation.

![Figure 2.1: schematic view of experimental set-up beam system](image_url)
2.2 Dynamic vibration absorber (DVA)

A dynamic vibration absorber (DVA) is a device consisting of an auxiliary mass-spring-damper system. If tuned correctly, the device tends to neutralize the vibration of a structure to which it is attached at one specific frequency or frequency range, depending on the type DVA which is applied to the piecewise linear beam system. The basic principle of operation is vibration out of phase with the vibration of such structure, thereby applying a counteracting force. The DVA will be attached to the piecewise linear beam system. A DVA is a typical example of a passive controller. The natural frequency of the DVA is numerically determined with the finite element method. One side of the model of the cantilever beam with the mass is divided in 8 elements. Two main types of DVA's can be distinguished, namely the undamped DVA and the damped DVA.

2.2.1 Undamped dynamic vibration absorber

The undamped DVA consists of two cantilever beams with additional masses at the ends. Together it can be seen as a mass-spring system. By moving a mass along its cantilever beam, the stiffness can be tuned very precisely. That means that the natural frequency can be tuned precisely too. The two cantilever beams will always be tuned to the same natural frequency. The undamped DVA is used to minimize the response of a specific frequency. For most cases the natural frequency is chosen to be equal to the frequency for which it is desired to minimize the response. It does not minimize the responses at frequencies near to the specific minimized frequency. So it can be said that the main goal of application of the undamped DVA is to minimize the response at one specific frequency.

Due to the presence of the excitation mechanism and one-sided stiffness construction it is not possible to attach the undamped DVA in the middle of the beam. It has to be attached to beam at 0.200 m from the middle of the beam. That is the closest position to the middle of the beam without touching the excitation mechanism during operation. A schematic view of the set-up with the undamped DVA is shown in figure (2.2).

![Figure 2.2: Schematic view of the experimental set-up beam system with undamped DVA](image-url)
2.2.2 **Damped dynamic vibration absorber**

The damped DVA does have dampers at the ends of the cantilever beams with respect to the undamped DVA. The system can be seen as a mass-spring-damper system. Because of the dampers the amplitude of the vibration out of phase is smaller with respect to the undamped DVA at the natural frequency. The level of minimization that with a damped DVA can be attained is less compared to the undamped DVA for the one specific minimized frequency. The application of dampers does not only reduce the response of the one specific minimized frequency, but also the responses of a frequency range. So it can be said that the main goal of application of the damped DVA is to minimize the responses in a frequency range.

The dampers take more space and so the distance from the position to which the damped DVA is attached, is larger. They have to be attached to the beam at 0.215 m from the middle of the beam. A schematic view of the set-up with damped DVA is shown in figure (2.3).

![Figure 2.3: schematic view of the experimental set-up beam system with damped DVA](image)
Chapter 3

Post processing of measured data

3.1 Post processing data for an amplitude frequency characteristic

From experiments the transversal acceleration of the middle of the beam is acquired for several constant excitation frequencies. To obtain displacement signals the measured acceleration signals are integrated twice. The following computations are executed to obtain the displacement of the beam:

\[ a_{corrected} = a_{measured} - \text{mean}(a_{measured}) \]  
\[ v = \int a_{corrected} \, dt \]  
\[ v_{corrected} = v - \text{mean}(v) \]  
\[ v = \int v_{corrected} \, dt \]  
\[ x_{corrected} = x - \text{mean}(x) \]

The Matlab command \texttt{cumtrapz} is used for integration. This command executes a cumulative trapezoidal numerical integration. The reason for adding a constant to the measured acceleration in (3.1) is to cancel the offset of the measured signal. The corrections of the velocity \( v \) and displacement \( x \) in (3.3) and (3.5) after integration, are executed to take the integration constants into account.

For an experiment several periods are continuously measured. The computations (3.1) up to (3.5) are executed per harmonic period or per subharmonic period. The reason for not executing the computations to continuous several periods at once is that the result presents too much deviations over the several periods, which are not physically realistic. This can be caused by numerical computations.
For each excitation frequency the maximum transversal displacement $x_{max}$ of a period is stored, which is defined as:

$$x_{max} = \max(x_p) - \min(x_p)$$

in which $x_p$ is the periodic displacement signal. For each excitation frequency many periods are measured. The mean of the computed maximum transversal displacement of the periods is taken to get a average value for each excitation frequency. This mean maximum displacement is displayed in the amplitude frequency characteristic.

To determine the maximum transversal displacement it is necessary to know if the response is a harmonic or a subharmonic. Poincaré maps are used for recognizing a harmonic or a subharmonic responses. In a Poincaré map a phase portrait is stroboscopically lighted with the excitation frequency. The position and velocity on a certain time in each period is plotted in the Poincaré map for several periods. There are some small errors in the Poincaré map caused by truncating the harmonic periods to be plotted. This is necessary because the period time is not a multiple of the sample frequency.

To recognize a superharmonic resonance the auto power spectrum is used. From the contribution of the frequencies of the auto power spectrum it can be said if the response is a superharmonic resonance.

### 3.2 Influence of noise on measured data

Noise is dominating the measurements below 4 Hz. In figure (3.1) the result of the measurements for the case of flush is shown. Also in this figure it is displayed what the results are of computing the mean maximum displacement for the frequencies in the frequency range with only noise data as output for the acceleration. The accelerometer does receive a signal while the excitation mechanism is switched off. This signal is the above mentioned noise data.

It can be seen in figure (3.1) that from 1 Hz up to 4 Hz the original data is almost equal to the results from the noise data. These measurements are not reliable. From 4 Hz is the noise data a minor part of the measured data.

From 4 Hz up to around 10 Hz a clear harmonic or superharmonic response can often not be recognized from the obtained Poincaré maps. It can be that the solution is quasi periodic or chaotic. Another explanation can be that the system causes noise which disturbs the movement of the beam system. One can think of disturbances from the system of the motor with the flexible couplings. To find out if this is noise from system or used devices, measurements with only the linear beam can be done.

For most cases clear harmonics or subharmonics responses can be recognized from frequencies higher than 10 Hz. In figure (3.2) Poincaré maps are shown for different frequencies. The Poincaré maps for frequencies 2.0 Hz and 5.0 Hz in figure (3.2) do not show clear harmonics or subharmonic solutions.
Figure 3.1: influence of noise

Figure 3.2: Poincaré map at 2.0 Hz and 5.0 Hz
Chapter 4

Results of the experiments

4.1 Flush - compare with previous results

First measurements are done at conditions which can be compared with previous results in [1] and [2]. This condition is the flush situation, without applying a DVA. The current result is shown in figure (4.1). The first harmonic resonance peak is at 19.0 Hz. The 1/2 subharmonic resonance peak appears at 38.0 Hz. This subharmonic resonance peak is related to the harmonic resonance peak at 19.0 Hz. The line from 37.0 up to 39.0 Hz is dashed because the results are interpolated from the measurements.

At 8.0 Hz a 2\textsuperscript{nd} superharmonic resonance appears. In figure (4.2) for the power density spectrum one can see that the frequency of 16.0 Hz dominates the excitation frequency. This 2\textsuperscript{nd} superharmonic resonance is also related to the harmonic resonance peak at 19.0 Hz. From 26.0-27.0 Hz and 29.0-30.0 Hz there exists respectively 1/3 and 1/5 subharmonic responses. The 1/3 subharmonic response is related to the 2\textsuperscript{nd} superharmonic resonance at 8.0 Hz. The 1/5 subharmonic response is expected to be related to a resonance around 5.0 Hz. This resonance is not measured.

In figure (4.3) the results are compared with the results of Bonsel [1]. In this amplitude frequency characteristic the displacement is scaled by excitation force. Bonsel gives in his master’s thesis [1] a proof of the validity to use a displacement scaled by force in case of flush. Further in this report displacement scaled by force will not be used because this scaling is not valid for the case of pre-tension (and also not for the case of backlash). The solid line is the result of the numerical model of Bonsel and the markers are the results from his experiments. The dashed line is the currently obtained result. It can be seen that the 1/2 subharmonic resonance differs a bit. The 2\textsuperscript{nd} superharmonic resonances do not correspond with each other. A reason could be that the sample frequency of the excitation frequencies was too low. Because of this it is possible that the different superharmonic resonances not could be distinguished. It has to be taken into account that the set-up has slightly changed after the experiments of Bonsel. Therefore the results also are compared with the results of de Bont [2], who performed measurements after the change of the set-up. The comparison can be seen in figure (4.4). In this figure the solid line and the markers are results of the experiments of de Bont. Once again the dashed line is the currently obtained result. The heights of harmonic peak and
the 1/2 subharmonic peak differ. Also the superharmonic resonance near 12 Hz has disappeared compared to the results of de Bont. This may be due to a too low number of measured excitation frequencies in the range of 5 Hz up to 15 Hz. On the other hand, a superharmonic resonance near 12 Hz is not immediately to be expected. The last comparison shows that the results correspond well enough and it can be assumed that the set-up and post processing are correct.

Figure 4.1: amplitude frequency characteristic - flush

Figure 4.2: power density spectrum and time response of 2\textsuperscript{nd} superharmonic resonance - 8 Hz flush
Figure 4.3: current results (−−) compared with results J. Bonsel for the case of flush

Figure 4.4: current results (−−) compared with results C.G.M. de Bont for the case of flush
4.2 Pre-tension

The next step is to determine amplitude frequency characteristics for different amounts of pre-tension. This is done for amounts of pre-tension of respectively 0.35, 0.56 and 0.99 mm, see figure (4.5), (4.6) and (4.7). In the figures the amplitude frequency characteristic of flush is displayed with a grey line.

4.2.1 Pre-tension 0.35 mm

Figure (4.5) shows the amplitude frequency characteristic for the amount of 0.35 mm pre-tension. Some aspects to be mentioned are:

- The harmonic resonance with a peak at 19.0 Hz. Multiple solutions exist in the region of the harmonic resonance.
- A 1/2 subharmonic resonance with a peak at 38.5 Hz, figure (4.8) shows the Poincaré map and the auto power spectrum. Multiple solutions exist in the region of the 1/2 subharmonic resonance.
- A 2\textsuperscript{nd} superharmonic resonance at 13.0 Hz, figure (4.9) shows the auto power spectrum and time response. Multiple solutions exist in the region of the 2\textsuperscript{nd} superharmonic resonance.
- Small resonance at 11.0, 11.5 Hz. After applying auto power spectrum analysis, a small contribution of the frequency three times the excitation frequency exists.
- Some narrow-banded subharmonic responses: 1/3 and 1/5 subharmonic responses.

4.2.2 Pre-tension 0.56 mm

Figure (4.6) shows the amplitude frequency characteristic for the amount of 0.56 mm pre-tension. Some aspects to be mentioned are:

- The harmonic resonance with a peak at 19.0 Hz. Multiple solutions exist in the region of the harmonic resonance.
- A 1/2 subharmonic resonance with a peak at 42.0 Hz. Multiple solutions exist in the region of the 1/2 subharmonic resonance.
- A 2\textsuperscript{nd} superharmonic resonance at 16.0 Hz. Multiple solutions exist in the small region of the 2\textsuperscript{nd} superharmonic resonance.
- Small resonance at 13.0 Hz. After applying auto power spectrum analysis, a small contribution of the frequency twice the excitation frequency exists.
- 3\textsuperscript{rd} superharmonic resonance at 11.0 Hz, figure (4.10) shows the auto power spectrum and time response. The auto power spectrum shows a contribution of the frequency three times the excitation frequency.
4.2.3 Pre-tension 0.99 mm

Figure (4.7) shows the amplitude frequency characteristic for the amount of 0.99 mm pre-tension. Some aspects to be mentioned are:

- The harmonic resonance with a peak at 19.5 Hz. Multiple solutions exist in the region of the harmonic resonance.
- A 2nd superharmonic resonance at 16.5 Hz.
- A 3rd superharmonic resonance at 11.5 Hz.
- Some narrow-banded subharmonic responses: 1/3, 1/5, 1/7 and 1/10 subharmonic responses. In figure (4.11) Poincaré maps of 1/5 subharmonic response at 28.5 Hz and 1/7 subharmonic response at 34.5 Hz are displayed.

4.2.4 Comparison of results

For the region of the first harmonic resonance around 20 Hz multiple solutions exist for all amounts of pre-tension. For all amounts of pre-tension the maximum displacement at the peak and the frequency of the peak of the first harmonic resonance stay also approximately the same. The region with multiple solutions around the first harmonic resonance will increase for a higher level of pre-tension. The frequency for which the backbone curve starts will also increase due to the higher stiffness for excitations with a smaller amplitude.

For a pre-tension of 0.35 mm and 0.56 mm the amplitude frequency characteristics show also multiple responses at the 2nd superharmonic resonance around 15 Hz and around the 1/2 subharmonic resonance around 40 Hz. This is not the case for a pre-tension of 0.99 mm, the region around the harmonic resonance around 20 Hz is the only region with multiple responses. A difference between 0.35 mm and 0.56 is the difference between the maximum displacements at the 1/2 subharmonic resonance. The 1/2 subharmonic resonance peak has totally disappeared at a pre-tension of 0.99 mm. It is clear to see that the maximum displacement of the 2nd superharmonic resonance decrease for increasing level of pre-tension. So more pre-tension results in suppression of the 2nd superharmonic and the 1/2 subharmonic resonances. It is possible to suppress the resonances by applying a high enough level of pre-tension or by making the excitation amplitude small enough. The system would behave linear, the main beam and the one-sided spring stay in touch.
Figure 4.5: amplitude frequency characteristic - pre-tension 0.35 mm

Figure 4.6: amplitude frequency characteristic - pre-tension 0.56 mm
Figure 4.7: amplitude frequency characteristic - pre-tension 0.99 mm

Figure 4.8: 1/2 subharmonic resonance peak at 38.5 Hz - pre-tension 0.35 mm
Figure 4.9: $2^{nd}$ superharmonic resonance peak at 13.0 Hz - pre-tension 0.35 mm

Figure 4.10: $3^{rd}$ superharmonic resonance peak at 11.0 Hz - pre-tension 0.56 mm

Figure 4.11: $1/5$ subharmonic response at 28.5 Hz and $1/7$ subharmonic response at 34.5 Hz - pre-tension 0.99 mm
4.3 Pre-tension with application of undamped DVA

4.3.1 Pre-tension 0.35 mm and application of an undamped DVA

An undamped DVA is applied for the case of pre-tension of 0.35 mm. The DVA is tuned at different natural frequencies for the measurements. These frequencies are: 18.0, 19.0, 20.5, 22.0 and 23.0 Hz. The results are shown in the figures (4.12)-(4.16). In the figures the amplitude frequency characteristic of pre-tension 0.35 mm without DVA is displayed with a grey line.

Main effect of the undamped DVA

The purpose of the application of an undamped DVA is to minimize the response of one specific frequency. For all tuned natural frequencies of the undamped DVA it can be said that the minimization of the response of one specific frequency is achieved. Without application of the DVA, the first harmonic resonance is one peak. When the DVA is applied, there are two peaks. The first peak increases and the second peak decreases when the natural frequency of DVA increases. Between the two new peaks one can see the frequency with the minimized response. The minimized frequency is not equal to the original resonance. Moreover it is lower than the tuned natural frequency of the DVA (in contrast to the case for flush, see [1]).

An overview of the frequencies of the minimized responses is shown in the next table:

<table>
<thead>
<tr>
<th>natural frequency of undamped DVA</th>
<th>frequency for which response is minimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.0 Hz</td>
<td>17.5 Hz</td>
</tr>
<tr>
<td>19.0 Hz</td>
<td>18.5 Hz</td>
</tr>
<tr>
<td>20.5 Hz</td>
<td>19.5 Hz</td>
</tr>
<tr>
<td>22.0 Hz</td>
<td>20.5 Hz</td>
</tr>
<tr>
<td>23.0 Hz</td>
<td>21.5 Hz</td>
</tr>
</tbody>
</table>

Two resonance peaks, the first harmonic and 1/2 subharmonic resonance, are reduced after the application of the undamped DVA. But the amplitude of the responses of some other frequency ranges is increased. This is caused by the second first harmonic peak and the appeared subharmonic resonances. The subharmonic resonances appear due to the two first harmonic peaks.

Other effects after application of an undamped DVA

The effects mentioned in this section are less important for considering the main goal of the applied undamped DVA. However the effects which appear after application of an undamped DVA, could be interesting. The following aspects are mentioned for all tuned natural frequencies of the applied undamped DVA:

- As already mentioned the first harmonic resonance peak is divided in two peaks after applying an undamped DVA. Between these peaks the response is minimized.
In figure (4.17) a Poincaré map is displayed for a minimized response at frequency of 18.5 Hz with natural frequency of the undamped DVA of 19.0 Hz. The response is low and noise does have a large influence on the signal.

- The region with multiple solutions around first harmonic resonance seems to have disappeared when the DVA is applied. Instead of the region with multiple responses, there is a smaller region which is the transition of responses with small amplitude to responses to large amplitude. In this region exist solutions which commute between stable solutions in the region from 19.0 up to 20.5 Hz. They are marked with a dot in the amplitude frequency characteristics. An explanation for this phenomenon would be the inability of the driving system which disturbs attaining a stable solution.

Figure (4.18) shows the time response at the frequency of 20.5 Hz that commutes between solutions after applying an undamped DVA with natural frequency of 19.0 Hz.

- The 2\textsuperscript{nd} superharmonic resonance still exists. The region around the 2\textsuperscript{nd} superharmonic resonance with multiple is smaller if the first peak of the first harmonic resonance is near (in particular for the natural frequencies 18.0 and 19.0 Hz of the undamped DVA).

- After the two peaks of the first harmonic resonance one can see a region with \(1/2\) subharmonic responses. In this region is a transition from responses with smaller amplitudes to responses with larger amplitudes. This region is related to respectively the 2\textsuperscript{nd} superharmonic resonance and the first peak of the first harmonic resonance.

- There is another \(1/2\) subharmonic resonance at higher frequencies. This \(1/2\) subharmonic resonance is related to the second peak of the first harmonic resonance.

- Between the \(1/2\) subharmonic resonances is a \(1/5\) subharmonic resonance. It can be seen that there are solutions that seem to be aperiodic in this region. They are marked with a dot. This region moves from 36.0-42.0 Hz for 18.0 Hz tuned DVA up to 41.0-49.0 Hz for 23.0 Hz tuned DVA. There is a clear transition before and after this region. The stable subharmonics which can be determined in these regions are \(1/5\) subharmonics. For the other frequencies in the region is \(1/5\) subharmonic assumed for the computation of maximum displacement. The \(1/5\) subharmonic resonance could be related to a superharmonic resonance at a frequency lower than 10 Hz, which is not measured.

Two Poincaré maps of responses that seem to be aperiodic are displayed in the figures (4.19) and (4.20) for the frequencies of 40.0 and 43.0 Hz; the natural frequency of the undamped DVA is 22.0 Hz.

- The above mentioned subharmonic resonance in the amplitude frequency characteristics are successively a \(1/2\) subharmonic, a \(1/5\) subharmonic and again a \(1/2\) subharmonic resonance. The frequencies at which these subharmonic resonances occur, increase when the frequency of the DVA increases.
Figure 4.12: amplitude frequency characteristic - pre-tension 0.35 mm with undamped DVA 18.0 Hz

Figure 4.13: amplitude frequency characteristic - pre-tension 0.35 mm with undamped DVA 19.0 Hz
Figure 4.14: amplitude frequency characteristic - pre-tension 0.35 mm with undamped DVA 20.5 Hz

Figure 4.15: amplitude frequency characteristic - pre-tension 0.35 mm with undamped DVA 22.0 Hz
Figure 4.16: amplitude frequency characteristic - pre-tension 0.35 mm with undamped DVA 23.0 Hz

Figure 4.17: most minimized response at 18.5 Hz - frequency DVA 19.0 Hz
Figure 4.18: response that commutes at 20.5 Hz - frequency DVA 19.0 Hz

Figure 4.19: response with unspecified subharmonic at 40.0 Hz - frequency DVA 22.0 Hz

Figure 4.20: response with unspecified subharmonic at 43.0 Hz - frequency DVA 22.0 Hz
4.3.2 Pre-tension 0.99 mm and application of an undamped DVA

An undamped DVA is also applied for a pre-tension of 0.99 mm. For the measurements the DVA is again tuned at different frequencies. These frequencies are: 18.0, 19.0, 20.5, 22.0 and 23.0 Hz. The results are shown in the figures (4.21)-(4.25). In the figures the amplitude frequency characteristic of a pre-tension 0.99 mm without DVA is displayed with a grey line.

Main effect of the undamped DVA

For all tuned natural frequencies of the undamped DVA it can be said that the minimization of the response of one specific frequency is achieved. The frequency with the minimized response is lower than the tuned natural frequency of the undamped DVA, except for the undamped DVA with natural frequency of 18.0 Hz.

An overview of the frequencies of the minimized responses is shown in the next table:

<table>
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</tr>
<tr>
<td>22.0 Hz</td>
<td>21.0 Hz</td>
</tr>
<tr>
<td>23.0 Hz</td>
<td>22.0 Hz</td>
</tr>
</tbody>
</table>

The first harmonic resonance is reduced after the application of the undamped DVA. But the amplitude of the responses of some other frequency ranges is increased. This is caused by the second first harmonic peak and the appeared subharmonic resonances. The subharmonic resonances appear due to the two first harmonic peaks.

Other effects after application of an undamped DVA

The effects mentioned in this paragraph are less important for considering the main goal of the applied undamped DVA. However the effects which appear after application of an undamped DVA, could be interesting. The aspects are divided in three sections to keep an overview.

The following aspects hold for all tuned natural frequencies of the applied undamped DVA:

- The region with multiple solutions of 19.0 up to 20.5 Hz around the first harmonic resonance remains when the DVA is applied. A difference with a pre-tension 0.35 mm is that the responses which commutes between solutions in this region, do not exist. In this region two first harmonic resonances occur after application of an undamped DVA. Between the two first harmonic resonances one can see the frequency with the most minimized response.

- The 3\textsuperscript{rd} superharmonic resonance at 11.0 Hz still exists. For the higher tuned natural frequencies of the undamped DVA, the resonance peak at 11.0 Hz becomes more visible.
Figure (4.26) shows an auto power spectrum and a time response for the frequency 11.0 Hz after application of an undamped DVA with frequency of 23.0 Hz.

- There exists a 1/5 subharmonic resonance. It can be seen that there are aperiodic solutions in this region. They are marked with a dot in the amplitude frequency characteristics. This region moves from 38.0-46.0 Hz at 18.0 Hz tuned DVA up to 43.0-50.0 Hz at 23.0 Hz tuned DVA. There is a clear transition before and after this region. The stable subharmonics which can be determined in these regions are 1/5 subharmonics. For the other frequencies in the region 1/5 subharmonic is assumed for computation of maximum displacement. The 1/5 subharmonic resonance could be related to a superharmonic resonance at a frequency lower than 10 Hz, which is not measured. The region with 1/5 subharmonic responses moves to higher frequencies when the natural frequency of the DVA increases.

A Poincaré map of an aperiodic response at 41.0 Hz is displayed in figure (4.27); frequency of the undamped DVA is 19.0 Hz.

Aspect for application of an undamped DVA with natural frequency 18.0 and 19.0 Hz:

- Applying of an undamped DVA results in a new first harmonic resonance peak. For the cases with application of undamped DVA with natural frequency of 18.0 and 19.0 Hz the response of the frequency peak does have a small amplitude compared to the responses of the other first harmonic resonance peak. For the undamped DVA with natural frequency 18.0 Hz the 2\textsuperscript{nd} superharmonic resonance at 17.0 Hz has disappeared. In that case the first harmonic peak coincides with the 2\textsuperscript{nd} superharmonic resonance at 17.0 Hz. The response at the resonance peak contains a small contribution of twice the excitation frequency. The 2\textsuperscript{nd} superharmonic resonance still exists when the undamped DVA with natural frequency 19.0 Hz is applied.

In figure (4.28) the auto power spectrum is displayed for a response at the first harmonic resonance peak at 16.0 Hz which coincides with the 2\textsuperscript{nd} superharmonic resonance; natural frequency of the undamped DVA is 18.0 Hz.

Aspects for application of an undamped DVA with natural frequency 20.5, 22.0 and 23.0 Hz:

- There are two first harmonic resonance peaks when an undamped DVA is applied. The first harmonic resonance peak increases and the second peak decreases when the natural frequency of DVA increases.

- After the two first harmonic resonances a 1/2 subharmonic resonance occurs. If the natural frequency of the DVA increases, the frequency range of the region with the 1/2 subharmonic responses increases. In this region is a transition from responses with smaller amplitudes to responses with larger amplitudes. This region is related to respectively the 2\textsuperscript{nd} superharmonic resonance at 16.5 Hz and the first peak of the first harmonic resonance.

- For the applied DVA’s with the natural frequencies 19.0, 20.5 and 22.0 Hz, a resonance exists at 13.5 Hz.
4.3.3 Comparison of results after application of an undamped DVA

After application of an undamped DVA for both amounts of pre-tension minimization of the response of one specific frequency is achieved. The frequency for which the response is minimized, is below the natural frequency, except for one case.

The amplitude of the first harmonic resonance peak is reduced after the application of the undamped DVA. But there are also frequency ranges for which the response is increased after the application of the undamped DVA.

There are some differences between the effects that occur after application of an undamped DVA for a pre-tension of 0.35 mm and a pre-tension of 0.99 mm. The next mentioned differences hold for all tuned natural frequencies of the undamped DVA.

After application of the undamped DVA for a pre-tension of 0.35 mm the region around the first harmonic resonance with multiple solution has disappeared. In this region exist solutions which commute between stable solutions in the region from 19.0 up to 20.5 Hz. The region with multiple solutions of 19.0 up to 20.5 Hz around the first harmonic resonance remains when the DVA is applied for a pre-tension of 0.99 mm.

Only a few harmonic responses in small frequency ranges can be found for frequencies higher than 35 Hz for a pre-tension 0.35 mm after application of an undamped DVA. For a pre-tension of 0.99 mm harmonic responses still exist for larger frequency ranges.

Figure 4.21: amplitude frequency characteristic - pre-tension 0.99 mm with undamped DVA 18.0 Hz
Figure 4.22: amplitude frequency characteristic - pre-tension 0.99 mm with undamped DVA 19.0 Hz

Figure 4.23: amplitude frequency characteristic - pre-tension 0.99 mm with undamped DVA 20.5 Hz
Figure 4.24: amplitude frequency characteristic - pre-tension 0.99 mm with undamped DVA 22.0 Hz

Figure 4.25: amplitude frequency characteristic - pre-tension 0.99 mm with undamped DVA 23.0 Hz
Figure 4.26: 3rd superharmonic response at 11.0 Hz - frequency DVA 23.0 Hz

Figure 4.27: response with unspecified subharmonic 41.0 Hz - frequency DVA 19.0 Hz

Figure 4.28: first peak of harmonic resonance which coincides with other resonance at 16.0 Hz - frequency DVA 18.0 Hz
4.4 Pre-tension with application of damped DVA

To see the difference between the application of the undamped DVA and the damped DVA, measurements are done with the damped DVA tuned at a frequency of 20.5 Hz, for both 0.35 mm and 0.99 mm pre-tension. The results are shown in the figures (4.29)-(4.32).

In the figures (4.29) and (4.31) the amplitude frequency characteristic of only pre-tension without DVA is displayed with a grey line. In the other figures (4.30) and (4.32) the amplitude frequency characteristic of pre-tension with undamped DVA with natural frequency of 20.5 Hz is displayed with a grey line.

It has to be taken into account that the position of attachment of the damped and the undamped DVA differs. It is attached to the beam at 0.215 m from the middle of the beam (the undamped DVA is attached to the beam at 0.200 m from the middle of the beam). This of course influences the results.

4.4.1 Pre-tension 0.35 mm

From figure (4.29) can be seen that the first harmonic resonance and the 1/2 sub-harmonic resonance are reduced. The response of the frequency of 19.5 Hz is minimized. The amplitude of the most minimized response is larger compared to the situation when an undamped DVA is applied. This is expected due to the dampers. The application of the damped DVA has almost the same effect as the application of the undamped DVA, this can be seen in figure (4.30).

The purpose of applying of an damped DVA is to minimize the response of a frequency range. The first harmonic and 1/2 subharmonic resonances are reduced, but this result is already achieved with a undamped DVA. Globally there is no decrease in amplitude compared to the application of the undamped DVA.

It can be said from the results that there is no minimization of the response of the frequency range around the first harmonic resonance because of the second first harmonic peak, which appear after application of the damped DVA.

4.4.2 Pre-tension 0.99 mm

The response of the small frequency range from 19.5 Hz up to 23.5 Hz is reduced, which can be seen in figure (4.31). The second solution with the largest amplitude for this frequency range has disappeared. There is no second first harmonic peak after the application of the damped DVA. Subharmonic resonances do not appear for higher frequencies. For higher frequencies, from 40 Hz up to 50 Hz, there is also small reduction of the amplitude of the response.

In figure (4.32) is the application of the damped DVA compared to the application of the undamped DVA. For 0.99 mm pre-tension gives the damped DVA a better result. The damped DVA suppresses the second first harmonic peak and the subharmonic resonances at higher frequencies. Of course the level of minimization of the response at the frequency of 19.5 Hz is less compared to the application of the undamped DVA because of the damping.

The damped DVA minimizes the response of a small frequency range around the first harmonic resonance and also minimizes the response for higher frequencies. The application of the damped DVA gives good results on suppressing the second first harmonic peak and the subharmonics at higher frequencies for the pre-tension of 0.99 mm.
4.4.3 Comparison of the results after application of a damped DVA

The application of the damped DVA reduces the response of the first harmonic resonance for both amounts of pre-tension.

For the level of pre-tension of 0.35 mm, it can be said that the effect of the applied damped DVA is almost the same compared to the situation when the undamped DVA is applied. Also a second first harmonic peak and subharmonic resonances at higher frequencies appear after the application of the damped DVA. This effect was not expected and is not fully understood yet.

For a pre-tension of 0.99 mm gives the damped DVA good results on suppressing the second first harmonic resonance and the subharmonic resonances at higher frequencies (compared to the situation when an undamped DVA is applied).

With the results of only two measurements it is difficult to understand the effect of the application of the damped DVA.
Figure 4.29: amplitude frequency characteristic - pre-tension 0.35 mm with damped DVA 20.5 Hz - compared to the situation when no DVA is applied

Figure 4.30: amplitude frequency characteristic - pre-tension 0.35 mm with damped DVA 20.5 Hz - compared to the situation when undamped DVA is applied
Figure 4.31: amplitude frequency characteristic - pre-tension 0.99 mm with damped DVA 20.5 Hz - compared to the situation when no DVA is applied.

Figure 4.32: amplitude frequency characteristic - pre-tension 0.99 mm with damped DVA 20.5 Hz - compared to the situation when undamped DVA is applied.
Chapter 5

Conclusions and recommendations

5.1 Conclusions

The purpose of the application of the undamped DVA is to minimize the response of one specific frequency. From the experimental results it can be said that the minimization of the response of one specific frequency is achieved.

The amplitude of the first harmonic resonance peak is reduced after the application of the undamped DVA. But there are also frequency ranges for which the response is increased after the application of the undamped DVA. This is caused by the second harmonic peak and the subharmonic resonances which appear after application of the undamped DVA.

There is no clear guideline for tuning the natural frequency of the undamped DVA for an optimal result. But the amplitude can be largely decreased for a single excitation frequency just below the tuned natural frequency of the undamped DVA. Probably it depends on the amount of pre-tension.

The purpose of the application of the damped DVA is to minimize the response of a frequency range. The experimental results of the application of the damped DVA give varying results.

For the level of pre-tension of 0.35 mm, the effect of the applied damped DVA is almost the same compared to the situation when the undamped DVA is applied.

For a pre-tension of 0.99 mm, the damped DVA minimizes the response of a small frequency range around the first harmonic resonance. Also the second harmonic resonance and the subharmonic resonances at higher frequencies are suppressed compared to the situation when an undamped DVA is applied.

With the results of only two measurements it is difficult to understand the effect of the application of the damped DVA.
5.2 Recommendations

It is useful to compare the experimental results with results generated by a simulation model. This for a better understanding of the effect of the undamped DVA.

A second recommendation is to investigate the effects of application of the damped DVA for different natural frequencies and different amounts of pre-tension. The current two measurements gives two different results, so further research on this would be interesting.
Bibliography


