Investigation of the phenomenon “chatter” in the milling process

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Abstract

The process of milling is used widely in many sectors of industry. The milling of large structures is done in e.g. the airplane building industry, where large amounts of material are removed. To make the process the most efficient, the speed of the process should be as high as possible while maintaining a high quality level.

During the milling process chatter can arise at certain combinations of spindle speed and depth-of-cut. This behavior is usually undesired, because in such a case a non-smooth surface of the workpiece is caused by heavy vibrations of the cutter. In addition the machine and cutting tool wear out rapidly. Several studies have been done to understand and model the phenomenon chatter. Both linear and nonlinear models have been developed, where nonlinearities are modeled in several different ways. Early studies have shown that the border between stable and unstable cuts in terms of the depth-of-cut can be visualized as a function of spindle speed. This results in a Stability Lobe Diagram (SLD). With the help of these diagrams it is possible to find the point with a combination of spindle speed and depth-of-cut which has the largest metal removal rate while avoiding chatter.

In this study, a literature study on both linear and nonlinear models is discussed. To answer whether linear models suffice to accurately predict the occurrence of chatter, experiments are performed. These experimental results are then compared to results from an analysis with a linear model. The program CutPro 3.0 [7] is used to analytically predict stability lobe diagrams, using a linear model developed by Altintas [1].

Impact tests have been done to calculate the modal parameters of the machine-tool system. The resulting absolute value of the frequency response function (frf) between the force of the impact and the displacement of the cutter appeared to be dependent on the magnitude of the impact of the hammer on the cutter. If the impact is higher, the stiffness of the system seems to increase, which results in higher stability lobes, up to 50%. For generating these stability lobes also tests have been done to identify the cutting parameters of the material. These parameters are only valid for the combination of material and cutter used in these experiments (aluminium 51ST and a 10 mm JH420 cutter). It seems that a nonlinearity in the mill and/or spindle exists.

Measurements at several spindle speeds using a High Speed Milling machine have been performed to verify the analytical lobes. To detect the chatter the program Harmonizer 2.1 is used [9]. For spindle speeds in the region 22.000-31.000 rpm, the experiments are in good agreement with the lobes generated using the modal parameters of the "hard" impacts. For higher spindle speeds the differences of the analytically developed stability limits and the stability limits found by Harmonizer run up to 100%.

In that region, Harmonizer marks a cut as stable, while it is actually unstable, because the mill makes a loud noise and the surface of the milled workpiece is non-smooth. The real stability limit is about 1-2 mm lower. If the real limit would be used, the difference between the analytically predicted boundary and the real boundary would be about 50%. More experiments should be done to detect the boundary more accurate.

It can be concluded that stability limits as predicted by the linear model differ significantly from the real limits. It would be more realistic to use a nonlinear model, if one desires a more accurate prediction of the stability lobes, although the computing time using a nonlinear model should not increase to much in order to remain attractive for practical, industrial use.
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Chapter 1

Introduction

The process of milling is used widely in many sectors of industry. The milling of large structures is done in e.g. the airplane building industry, where large amounts of material are removed. To make the process the most efficient, the speed of the process should be as high as possible while maintaining a high quality level. This means that a large material removal rate has to be reached, while the surface of the milled workpiece has to be smooth.

During the milling process chatter can arise at certain combinations of spindle speed and depth-of-cut. This behavior is usually undesired, because in such a case a non-smooth surface of the workpiece is caused by heavy vibrations of the cutter. In addition the machine and cutting tool wear out rapidly. Several studies have been done to understand and model the phenomenon chatter. Both linear and nonlinear models have been developed, where nonlinearities are modeled in several different ways. Early studies have shown that the border between stable (i.e. without chatter) and unstable (i.e. with chatter) cuts in terms of the depth-of-cut can be visualized as a function of spindle speed. This results in a Stability Lobe Diagram (SLD). With the help of these diagrams it is possible to find the point with the combination of spindle speed and depth-of-cut which has the largest metal removal rate while avoiding chatter.

A linear model developed by Altintas [1] is implemented in the computer program CutPro. With the help of this program it is possible to quickly find the SLD for a specific combination of machine, tool, workpiece and cutting parameters.

The goal of this study is to gain insight in the different kind of models that have been developed to study chatter and answer the question whether linear models are sufficient enough to accurately predict the occurrence of chatter.

First, in chapter 2 a literature study is presented where different models are evaluated. Second, experiments are done. These experiments are done at TNO Industry on a Mikron HSM 700 milling machine using a 10 mm JH420 cutter by Jabro Tools. The stability lobes are generated using the program CutPro [7] and verified doing actual cuts in Aluminium 51ST. For detecting the occurrence of chatter during these real cuts the program Harmonizer [9] is used. This is described in chapter 3. Finally conclusions are drawn and recommendations are made in chapter 4.
Chapter 2

Literature study on the modeling of the milling process

2.1 Introduction

Milling is a process where material is removed by a spinning tool, which has several cutting teeth. The main difference between modeling the milling and the turning process is that the chip thickness in milling is not constant, but periodic. A schematic representation of the milling process is shown in figure 2.1. Some process parameters are shown:

- Feed per tooth \( f_z \).
- Axial depth-of-cut \( a_p \).
- Spindle speed \( \omega \)

![Figure 2.1: A schematic representation of the milling process.](image)

Several types of milling exist, which are shown in figure 2.2:

- Up-milling, where the entry angle is zero and the exit angle is non-zero,
Figure 2.2: Different milling types. Upper left: up-milling; upper right: down-milling; lower left: face milling; lower right: slotting. Pictures taken from CutPro 3.0.

- Down-milling, where the entry angle is nonzero and the exit angle is zero,
- Face milling, where the entry angle $\phi_{en}$ and exit angle $\phi_{ex}$ of the milling cutter relative to the workpiece are nonzero,
- Slotting, where the entry angle is zero and the exit angle is 180°.

The first studies regarding chatter were done in the 1950's and 1960's by Tlusty [23] and Tobias [28]. The phenomenon chatter can be divided into two parts.

- Primary chatter is the consequence of
  - friction effects between the tool and the chip [31] or
  - mode coupling [23] or
  - thermodynamics of the cutting process [32].

- Secondary chatter is caused by regeneration of waviness of the surface of the workpiece [23].

Regeneration of waviness means that if there is a relative vibration between the tool and the workpiece, a wavy surface is left behind. The next tooth encounters this wavy surface and removes a chip with periodically variable thickness (figure 2.3). In case of secondary chatter, this vibrational effect increases.

Chatter can be prevented by changing the spindle speed or decreasing the depth-of-cut. To ensure a stable process, i.e. without chatter, often a rather conservative choice for the depth-of-cut is made. This causes a decrease in productivity. It might be possible that at a different spindle speed (higher or lower) a larger depth-of-cut is possible, without obtaining chatter, which results in a larger cutting volume per minute. To find the maximum depth-of-cut at a certain spindle speed so-called stability-lobe diagrams (SLD's) can be used [27]. These diagrams show the border in terms of the depth-of-cut, between a stable and an unstable cut as a function of the spindle speed. Several methods are used to produce these diagrams. By choosing a
Figure 2.3: Regeneration of waviness for a) shaping, b) turning, c) milling. The chip thickness variation $h$ depends on the phase $\varepsilon$ between vibrations of subsequent cuts. [26].

certain combination of depth-of-cut and the spindle speed, a characteristic equation can be formed. Using the Nyquist stability criterion, it can be found whether the combination is stable or not (e.g. Sridar et al. [21] and Minis et al. [15]) in terms of secondary chatter. Another way to study the process is by using time-domain simulations (e.g. Tlusty and Ismail [24] and Weck et al. [29]). Herein, the process is divided in small time steps. At every step the cutting tool is turned and the forces are calculated. Then the accelerations are calculated and this results in the new position of the tool after integrating two times. It is possible to include tool jump-out in this type of models, which is an important non-linearity of the milling process. If the calculated new position is outside the workpiece, the forces are zero. A disadvantage of these models is the large computation time needed to run the simulations which are necessary to obtain a prediction of the SLD.

2.2 Linear models

A commonly used model in practice for determining forces during the milling process has been developed by Kienzle [11]. This model is developed using stress theory and empirical work. The force is a linear function of the axial depth-of-cut and number of teeth in the cut. The general equation is formulated as

$$F_t = K_{c1.1} \overline{h}^{1-m} a_p z_{sn},$$

(2.1)

where $F_t$ is the mean tangential force, $\overline{h}$ is the mean chip thickness, $a_p$ the axial depth-of-cut and $z_{sn}$ the mean number of active teeth in the cut. The parameters $K_{c1.1}$ and the exponent $m$ are dependent of the material of the workpiece and can be found using tables. These parameters have been found empirically by Kienzle in the 1950's and hence they do not fully satisfy for modern tools.

Altintas and Budak [1] have derived an analytical model to predict chatter, which is extended in [3]. This model is implemented in the software of CutPro, which will be used for the experiments in chapter 3. It is represented by the closed loop system of figure 2.4. The metal cutting closed-loop system consists of two main steps:
The displacement of the cutter causes forces to act on the cutter, since the cutter displacement results in material deformation.

These forces, in turn, cause the cutter to displace.

For the determination of the forces which act on the cutter caused by the displacement of the cutter, a material model has to be determined. The displacement of the cutter, caused by the forces acting on it, can be determined using the transfer function $H$ of the machining system (the cutter, suspension and milling machine) with respect to force and displacement.

In general orthogonal cutting the matrix equations as shown in figure 2.4 reduce to scalar equations. The chip thickness $h$ can be expressed as:

$$ h(t) = h_0 - (y(t) - y(t-T)),$$

where $h_0$ is the intended chip thickness, which is the feedrate of the machine, and $(y(t) - y(t-T))$ is the dynamic chip thickness, which is the result of the vibrations at the present time $t$ and one period $T$ before. Transferring this equation into the Laplace domain gives the delay as shown in figure 2.4:

$$ h(s) = h_0(s) - y(s) + e^{-sT}y(s) = h_0(s) + (e^{-sT} - 1)y(s).$$

(2.3)

The cutting force produced by the cut is expressed as a linear function of depth-of-cut $a_p$ and chip thickness $h$:

$$ F(s) = K_p h(s).$$

(2.4)

The cutting force excites the cutter and produces the vibration according to

$$ y(s) = F(s)H(s) = K_p h(s)H(s).$$

(2.5)

The chip thickness can thus be expressed as

$$ h(s) = h_0(s) + (e^{-sT} - 1)K_p h(s)H(s).$$

(2.6)
The resulting transfer function between the dynamic and reference chip loads becomes:

$$ h(s) = \frac{1}{b_0(s)} = \frac{1}{1 + (1 - e^{-sT}) K a_p H(s)}. $$

(2.7)

The stability of this closed-loop transfer function is determined by the roots $s$ of its characteristic equation:

$$ 1 + (1 - e^{-sT}) K a_p H(s) = 0. $$

(2.8)

The system is critically stable if the real part of the root is zero ($s = i\omega_c$), with $\omega_c$ a chatter frequency. Now it possible to calculate the critical depth-of-cut $a_c$.

Now the milling process will be discussed. The milling system is represented in figure 2.5. It consists of a cutter with $N$ teeth, rotating at an angular spindle speed $\Omega$ in a fixed $x, y$ coordinate system. Tooth $j$ makes an angle $\phi_j$ with respect to the $y$-direction, having a displacement $u_j$ in tangential direction and $v_j$ in radial direction. The angle $\phi_j$ can be expressed as $\phi_j = \phi + j\phi_p$, where $\phi$ is the angle of the tooth 1 and $\phi_p$ is the angle between two successive teeth, $\phi_p = \frac{2\pi}{N}$. The forces acting on tooth $j$ are $F_{tj}$ and $F_{rj}$ in tangential and radial direction, respectively.

The entry angle of the cutter in the material is called $\phi_{en}$ and the exit angle is called $\phi_{ex}$. The chip thickness consists of a static part caused by the feed of the cutter $f_s$ and a dynamic part, caused by the vibrations of the cutter at the present and previous tooth period. The chip thickness is expressed in radial direction as

$$ h(\phi_j) = (f_s \sin \phi_j + (v_{j,0} - v_j)) g(\phi_j), $$

(2.9)

where $g_j$ is a unit step function which determines whether a tooth is in or out of the cut, according to

$$ g(\phi_j) = \begin{cases} 1 & \text{if } \phi_{en} < \phi_j < \phi_{ex} \\ 0 & \text{if } \phi_j < \phi_{en} \text{ or } \phi_j > \phi_{ex} \end{cases}. $$

(2.10)

Since only the dynamic chip thickness is the cause of secondary chatter, the static part of equation (2.9) can be eliminated. Transferring the equation in the fixed coordinate systems yields

$$ h(\phi_j) = (\Delta x \sin \phi_j + \Delta y \cos \phi_j) g(\phi_j), $$

(2.11)
where $\Delta x = x - x_0$ and $\Delta y = y - y_0$. Here, $(x,y)$ and $(x_0,y_0)$ represent the dynamic displacements of the cutter structure at the present and previous tooth period, respectively.

The tangential and radial dynamic cutting forces acting on tooth $j$ are expressed as

$$F_{tx} = K_t a_{tx} h(\phi_j),$$
$$F_{ty} = K_r F_{tx}.$$  

(2.12)

The cutting coefficients $K_t$ and $K_r$ are constants. Resolving the cutting forces in the $x$ and $y$ directions and summing the forces contributed by all teeth, the total dynamic milling forces can be expressed as

$$F_x = \sum_{j=1}^{N} -F_{tx} \cos \phi_j - F_{ty} \sin \phi_j,$$
$$F_y = \sum_{j=1}^{N} +F_{tx} \cos \phi_j - F_{ty} \sin \phi_j.$$  

(2.14)

(2.15)

Substituting the dynamic chip thickness and tooth forces, the equations can be expressed in matrix form as

$$E(t) = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{1}{2} a_{tx} K_t h \begin{bmatrix} a_{tx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \frac{1}{2} a_{tx} K_t A(t) \Delta(t),$$

where the time-varying directional dynamic milling force coefficients are given by

$$a_{tx} = \sum_{j=0}^{N-1} g_j \sin 2\phi_j + K_r (1 - \cos 2\phi_j),$$
$$a_{xy} = \sum_{j=0}^{N-1} g_j (1 + \cos 2\phi_j) + K_r \sin 2\phi_j,$$
$$a_{yx} = \sum_{j=0}^{N-1} g_j \left(1 - \cos 2\phi_j\right) - K_r \sin 2\phi_j,$$
$$a_{yy} = \sum_{j=0}^{N-1} g_j \left[\sin 2\phi_j - K_r \left(1 - \cos 2\phi_j\right)\right].$$  

(2.17)

The matrix $A(t)$ is periodic at tooth passing frequency $\omega = N\Omega$ or tooth period $T = 2\pi/\omega$. Thus, it can be expanded into a Fourier series. For the zero-th order harmonics, this results in

$$A_0 = \frac{1}{T} \int_0^T A(t) dt.$$  

(2.18)

Since $A_0$ is only valid between the start and exit angle of the cutter, and $\phi_j = \Omega t$ and $\phi_0 = \Omega T$, it becomes equal to the average value of $A(t)$ at cutter pitch angle $\phi_p = 2\pi/N$:

$$A_0 = \frac{1}{\phi_p} \int_{\phi_p}^{\phi_p} A(\phi) d\phi = \frac{N}{2\pi} \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix},$$

(2.19)

where the integrated functions are given as

$$a_{xx} = \frac{1}{2} \left(\cos 2\phi - 2K_r \phi + K_r \sin 2\phi\right)|_{\phi_p}^{\phi_0},$$
$$a_{xy} = \frac{1}{2} \left(-\sin 2\phi - 2\phi + K_r \cos 2\phi\right)|_{\phi_p}^{\phi_0},$$
$$a_{yx} = \frac{1}{2} \left(-\sin 2\phi + 2\phi + K_r \cos 2\phi\right)|_{\phi_p}^{\phi_0},$$
$$a_{yy} = \frac{1}{2} \left(-\cos 2\phi - 2K_r \phi - K_r \sin 2\phi\right)|_{\phi_p}^{\phi_0}.$$  

(2.20)
Equation (2.16) can then be reduced to

$$ F(t) = \frac{1}{2} \sum_{j=1}^{n} 1/\beta_j K_1 A_0 \Delta(t), \quad (2.16) $$

where $A_0$ is time-invariant, but dependent on immersion $(\phi_m, \phi_c)$. Since the average cutting force per tooth period is independent of the helix angle, $A_0$ is also valid for helical end mills.

The second block in figure 2.4 is the transfer function between the force acting on the cutter and the displacement of the cutter. This transfer function is given by $H(s)$. In this model, the machine-tool combination is modeled as a linear second-order multi-degree-of-freedom system. This results in the transfer function to be

$$ H(s) = \sum_{p=1}^{n} \frac{1/m_p}{s^2 + 2\xi_p \omega_{0,p} s + \omega_{0,p}^2}, \quad (2.21) $$

where $m_p$ is the mass, $\omega_{0,p}$ the eigenfrequency and $\xi_p$ the damping factor of mode $p$.

Referring to figure 2.4, the force can be expressed in the Laplace domain as

$$ F(s) = F_0(s) + \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 \Delta(j) = F_0(s) + \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 (1 - e^{-sT}) X(s), \quad (2.23) $$

where $X(s)$ is the Laplace transformation of the displacement of the cutter. This can be expressed as

$$ X(s) = H(s) F(s), \quad (2.24) $$

which results in equation 2.23 to become

$$ F(s) = F_0(s) + \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 (1 - e^{-sT}) H(s) F(s). \quad (2.25) $$

This equation is comparable with equation 2.6. In that equation the displacement is expressed as a function of the force. Now, the milling forces are expressed as a function of displacement.

The transfer function between the dynamic and the reference force becomes (comparable to equation 2.7):

$$ \frac{F(s)}{F_0(s)} = \frac{1}{L - \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 (1 - e^{-sT}) H(s)}, \quad (2.26) $$

The stability of this closed-loop transfer function is determined by the roots $s$ of its characteristic equation. This equation has a nontrivial solution if its determinant is zero:

$$ \det \left[ L - \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 (1 - e^{-sT}) H(s) \right] = 0. \quad (2.27) $$

The system is critically stable if the real part of the roots are zero $(s = i\omega_c)$. The characteristic equation then becomes:

$$ \det \left[ L - \frac{1}{2} \sum_{j=1}^{n} \beta_j K_1 A_0 (1 - e^{-i\omega_c T}) H(i\omega_c) \right] = 0. \quad (2.28) $$

The notation is simplified by defining the eigenvalue of the characteristic equation as

$$ \Lambda = -\frac{N}{4\pi} \sum_{j=1}^{n} \beta_j K_1 (1 - e^{-i\omega_c T}), \quad (2.29) $$
and defining the oriented transfer function matrix, which compensates the directional differences between the forces and the displacements, as

\[
H_0(i\omega_c) = \begin{bmatrix}
\alpha_{xx}H_{xx}(i\omega_c) + \alpha_{xy}H_{yx}(i\omega_c) & \alpha_{xx}H_{xy}(i\omega_c) + \alpha_{yy}H_{yx}(i\omega_c)
\end{bmatrix}. 
\]

The characteristic equation then becomes:

\[
\det \left[ I + \Lambda H_0(i\omega_c) \right].
\]

If two orthogonal degrees of freedom in feed (x) and normal (y) direction are considered, then \(H_{xy} = H_{yx} = 0\), the characteristic equation becomes

\[
a_0\Lambda^2 + a_1\Lambda + 1 = 0,
\]

where

\[
a_0 = H_{xx}(i\omega_c)H_{yy}(i\omega_c)(\alpha_{xx}\alpha_{yy} - \alpha_{xy}\alpha_{yx}),
\]

\[
a_1 = \alpha_{xx}H_{xx}(i\omega_c) + \alpha_{yy}H_{yy}(i\omega_c).
\]

Then, the eigenvalue is obtained as

\[
\Lambda = -\frac{1}{2a_0} \left( a_1 \pm \sqrt{a_1^2 - 4a_0} \right).
\]

This eigenvalue consists of a real and an imaginary part \(\Lambda = \Lambda_R + i\Lambda_I\). By substituting this eigenvalue and the definition \(e^{-i\omega_cT} = \cos(\omega_cT) - i\sin(\omega_cT)\) into equation (2.29) gives the critical axial depth-of-cut at cutter frequency \(\omega_c\) as

\[
a_{im} = \frac{2\pi}{NK_T} \left( \frac{\Lambda_R(1 - \cos(\omega_cT)) + \Lambda_I \sin(\omega_cT)}{1 - \cos(\omega_cT)} + i \frac{\Lambda_I(1 - \cos(\omega_cT)) + \Lambda_R \sin(\omega_cT)}{1 - \cos(\omega_cT)} \right).
\]

Since \(a_{im}\) is a real value, the imaginary part is equal to zero:

\[
\Lambda_I(1 - \cos(\omega_cT)) + \Lambda_R \sin(\omega_cT) = 0.
\]

By substituting the definition

\[
\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\omega_cT)}{1 - \cos(\omega_cT)}
\]

into equation (2.35) it becomes

\[
a_{im} = -\frac{2\pi\Lambda_R}{NK_T} \left( 1 + \kappa^2 \right).
\]

The stability lobes are thus generated. These results are compared to results by numerical time-domain simulations done by Smith and Tlusty [20] for half immersion up-milling of an aluminum workpiece by a 4 inch diameter shell mill with eight teeth. (figure 2.6). The model Smith and Tlusty used is described further on in this section. The stability lobes are found using the zeroth and first harmonic of the Fourier series.

Another case, taken from [3], is shown in figure 2.7 for a bull nose cutter in an aluminum alloy. The time domain simulation used here has the same force model as described above, but nonlinear phenomena such as tool jump-out are taken into account in a way that is described in section 2.1. It is shown that the lowest points on
the stability lobes are the same for every lobe. The analytically determined stability lobes show quite good agreement with the time-domain simulations. However in some circumstances the stability limit is predicted higher or lower analytically than it is in the time-domain simulations. The main advantage of the analytical model is that it is a computationally very efficient way to generate the stability lobes.

Instead of the material model used above, Altintas has also used a different model to identify the forces as a linear function of chip thickness. For helical end mills, with helix angle $\beta$, this model is expressed as

$$
\begin{align*}
\frac{dF_{ij}}{dz} &= \left[K_{rc} h_j (\phi_j (a_p)) + K_{rc} \right] da_p \\
\frac{dF_{ij}}{dz} &= \left[K_{ac} h_j (\phi_j (a_p)) + K_{ac} \right] da_p \\
\frac{dF_{ij}}{dz} &= \left[K_{ac} h_j (\phi_j (a_p)) + K_{ac} \right] da_p 
\end{align*}
$$

where $F_{ij}$ is the axial force and the chip thickness is

$$
h_j (a_p) = f_x \sin \phi_j (a_p),
$$

and the immersion angle for flute $j$ at axial depth-of-cut $a_p$

$$
\phi_j (a_p) = \phi + j f_p - k \beta a_p.
$$

In this equations $k \beta a_p$ is called the lag angle and is calculated as $\frac{2 \tan \beta a_p}{D}$. The cutting constants $K_c$, $K_{rc}$, $K_{ac}$, $K_{te}$, $K_{rc}$ and $K_{ac}$ can be found by using an orthogonal cutting transformation of basic orthogonal cutting parameters such as the shear yield stress $\tau_s$. In orthogonal cutting tests the helix angle is zero. By using goniometric relations the cutting constants of an oblique cutting can be found using the orthogonal cutting parameters.

It is also possible to find the cutting constants experimentally. By measuring the average forces $F_x$, $F_y$ and $F_z$ in full immersion tests with different feedrates and a constant axial depth-of-cut, the cutting constants can be found using the following relations

$$
\begin{align*}
F_x &= -N a_z K_{rc} f_s - \frac{N a_z K_{rc}}{D} \\
F_y &= +N a_z K_{rc} f_s + \frac{N a_z K_{rc}}{D} \\
F_z &= +N a_z K_{ac} f_s + \frac{N a_z K_{ac}}{D}
\end{align*}
$$

Figure 2.6: Analytically determined stability limit predictions using the 0th (line) and the 1st (white circles) harmonic terms of the Fourier series compared with time-domain simulation predictions (black circles) [1].
Figure 2.7: Analytically determined stability lobes (dots) and via time domain simulations (line). For a stable (right) and unstable (left) cut time domain simulations show the forces, surface and the frequency spectrum. Experimentally measured forces are also shown. [3].
where the $x$ direction is the feed direction, the $y$ direction is the direction in the cutting plane which is perpendicular to the feed direction, and the $z$ direction is the axial direction. The average cutting forces can be expressed by a linear function of the feedrate $f$, and an offset contributed by the edge forces

$$
\bar{F}_q = \bar{F}_{qe} f_x + \bar{F}_{qe} (q = x, y, z).
$$

The forces $\bar{F}_q$ are measured, the forces $\bar{F}_{qe}$ and $\bar{F}_{qe}$ be found using a least squares method. Hence the cutting force coefficients are found as followed:

$$
K_{tc} = \frac{\bar{F}_{qc}}{N_{ap}} \quad K_{tc} = \frac{\bar{F}_{qc}}{N_{ap}} \\
K_{rc} = \frac{\bar{F}_{qe}}{N_{ap}} \quad K_{re} = \frac{\bar{F}_{qe}}{N_{ap}} \quad (2.44)
$$

This experimental method has to be repeated for every cutter geometry. The method of oblique cutting transforming using basic orthogonal cutting parameters can be used for every cutting geometry.

A similar model as described by Altintas is described by Weck, Altintas and Beer [29]. Herein, a transfer function between force acting on the cutter and displacement of the cutter is identified experimentally via modal analysis and is given as

$$
H (j\omega) = \frac{2}{F_x} \sum_{k=1}^{n} \frac{S_k}{1 - \left(\frac{j\omega}{\omega_{ik}}\right)^2 + 2jD_k \frac{\omega_{ik}}{\omega_{ik}}}.
$$

where $D_k$, $\omega_{ik}$, and $S_k$ are the modal damping factor, natural frequency and modal flexibility for the $k$-th mode, respectively. Time domain simulations, described in section 2.1, using this model are compared to experiments and shown in figure 2.8.

According to this figure, at several spindle speeds where an unstable cut is predicted by the time domain simulations, the cut is stable in practice and vice versa. So still differences exist between the model and the practice.

Also Tlusty [26] uses a force model where the total force depends linearly on the chip thickness. He also makes use of a transfer function. The simple closed loop model is shown in figure 2.9. The accompanying equations are

$$
F = K_s a_p (Y_0 - Y)
$$

$$
Y = F H_0 (\omega)
$$

$$
Y_0 = 1/ (K_s a_p) + H_0
$$

where $K_s$ is the cutting force constant, $a_p$ the chip width, $(Y_0 - Y)$ the chip thickness variation and $H_0 (\omega)$ the oriented transfer function of the system. It is the ratio of the complex amplitude of the $Y$ component of all the vibrations over the complex amplitude of a force acting in the direction $F$, as a function of frequency $\omega$; both the vibrations and the force are defined relative between the tool and the workpiece. The limit of stability is defined as the value where the vibrations do not increase or decrease i.e.:

$$
\left| \frac{Y_0}{Y} \right| = 1.
$$

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Figure 2.8: Time-domain simulations (line) verified with experiments (circles). White circles: no chatter; gray circles: light chatter; black circles: severe chatter [29].

Figure 2.9: The closed loop model, used by Tlusty [26].
This results in a critical chip width of:

$$a_{p,\text{lim}} = \frac{-1}{2K_s \text{Re}(G)_{\text{min}}}.$$  (2.50)

Taking into account that the oriented transfer function is the sum of different vibration modes of the machine \( G = \sum_i u_iG_i \), where \( u_i \) is a factor that depends on the direction of the mode, it can be shown that the limit of stability also depends on the direction in which the cutting takes place (x or y, up-milling or down-milling). Equation (2.50) is derived by using the Nyquist stability criterion.

Linear models are widely used to generate the SLD’s. They provide an estimate for the limit of stability of the milling process, with respect to secondary chatter. However, in practice still cuts exist where chatter occurs, while it was not predicted by the models and vice versa. It is found that the lowest points on the stability lobes have a constant value. To improve accuracy of the models nonlinear modeling is an interesting possibility.

### 2.3 Nonlinear models

Nonlinear phenomena can be modeled in several ways. For instance, the cutting force can be modeled as a nonlinear function of the cutting parameters or effects such as (partial) tool jump out can be modeled.

#### 2.3.1 Nonlinear force models

Landers and Ulsoy [12] use a face-milling model which is comparable to the model of Altintas and Budak [1]. The difference they make is reflected by the fact that they model the cutting parameters as nonlinear functions of the feed per tooth \( f_z \), the axial depth-of-cut \( a_p \), and cutting speed \( v_c \). They use a cutting and a thrust pressure, \( P_C \) and \( P_T \) respectively, where \( P_C = K_t \); and \( P_T = K_r \) \((K_t \) and \( K_r \) are the cutting parameters defined by Altintas and Budak). For the experiments done by Landers and Ulsoy with a 6061 aluminium workpiece and a carbide cutter with a tooth radius of 0.025 m, the empirical equations for the cutting and thrust pressures are

$$P_C = 0.2854 f_z^{-0.2510} a_p^{0.1331} \left( \frac{v_c}{1000} \right)^{-0.7230},$$  (2.51)

and

$$P_T = 0.1613 f_z^{-0.4042} a_p^{-0.4112} \left( \frac{v_c}{1000} \right)^{-0.5829}.  \tag{2.52}$$

For generating a single stability lobe the method of Altintas and Budak is used iteratively:

1. A nominal feed, a chatter frequency close to a dominant tool-workpiece structural frequency and a stability lobe is chosen. The stability analysis will only be valid for feeds close to the nominal feed value and for the selected stability lobe.

2. Initial guesses for the marginally stable depth-of-cut and the corresponding cutting speed are selected. The cutting and thrust pressures for these values are evaluated using equations (2.51) and (2.52). Then the marginally stable depth-of-cut and corresponding cutting speed are computed, using Altintas' method.
3. The cutting and thrust pressures are re-evaluated and the marginally stable depth-of-cut and corresponding cutting speed are computed with the new cutting and thrust pressures.

4. Step three is repeated until two successive iterations are within a specified tolerance.

Using this method, it is found that the limit of stability is lower than by using the linear method. Moreover, it is found that the lowest points on the lobes increase with an increasing spindle speed, which was not the case with the linear method. The model is validated with time domain simulations containing the nonlinear force model. In the time domain simulations the depth-of-cut is increased 0.1 mm in every simulation until marginal stability occurs. For simulations which are validated with the time domain functions the parameters for $P_T$ and $P_C$ differ from equations (2.51) and (2.52), to examine the dependence of the model of a certain parameter. In figure 2.10, the model is shown with $P_C$ equal to $P_T$. The parameters are independent of the feed and weakly dependent of the cutting speed and the depth-of-cut. The corresponding constant pressure models are constructed by fitting it into the data generated by the nonlinear models. From this figure, it is illustrated that even weak nonlinearities lead to errors up to 41% in the linear stability lobes. In figure 2.11, the limit of stability is shown as a function of the feed. Here $P_C$ equals $P_T$ and is independent of the cutting speed and strongly dependent of the feed and depth-of-cut. This figure shows that these nonlinearities cause large errors (up to 130%) in the linear stability lobe.

![Stability lobe diagram](image)

Figure 2.10: Stability lobe diagram. Thin line: linear analysis $P_C = P_T = 0.3790$. Thick line: nonlinear analysis using the iterative method. Boxes: time domain simulation using the nonlinear force model. Both nonlinear methods use the force model where $P_C = P_T = 0.5a_p^{0.1} \left( \frac{v}{1000} \right)^{-0.2}$. [12].

Also a few experiments are done to validate the model. Here, the pressure models of equations (2.51) and (2.52) are used. The results are shown in figure 2.12 and 2.13. The authors draw two conclusions: using a nonlinear model the lower limit of the stability lobe is pulled up for increasing spindle speeds, and the stability limit is dependent of the feed. This should mean that having a large feed, which increases the cutted volume, the axial depth of cut could be increased, which increases the cutted volume, and productivity, even more.
Figure 2.11: The stability limit as a function of feed. Thin line: linear analysis $P_C = P_T = 0.5337$. Thick line: nonlinear analysis. Boxes: time domain simulation using the nonlinear force model. In both nonlinear methods $P_C = P_T = 0.4f^{0.6}a^{0.4}$ [12].

Figure 2.12: Speed-depth stability lobe diagram. Thick line: nonlinear analysis using equations (2.51) and (2.52). Thin line: linear analysis. Circles: experimental results including confidence intervals [12].
Figure 2.13: Feed-depth stability lobe diagram. Thick line: nonlinear analysis using equations (2.51) and (2.52). Thin line: linear analysis. Circles: experimental results including confidence intervals [12].

Wiercigroch [31] assumes a nonlinear relation between the cutting force \( f_x \) and the relative speed between tool and workpiece in the feed direction \( v_r \), which could induce frictional chatter. The machining system is shown in figure 2.14. The forces in feed and normal direction \( (F_x, F_y) \) respectively are expressed as follows:

\[
F_x (y, \dot{x}, \dot{y}) = q_0 h \left( H (v_r) \frac{1}{1 + \mu_0} + \text{sgn} (v_r) \left( \frac{\mu_0}{1 + \mu_0} \right) \left( \rho_1 (v_r - 1)^2 + 1 \right) \right) H (h),
\]

(2.53)

\[
F_y (y, \dot{x}, \dot{y}) = \xi (v_r, v_f, h) F_x (y, \dot{x}, \dot{y}),
\]

(2.54)
where
\[\xi(v_r, v_f, h) = \left(c_2(v_f - 1)^2 + 1\right) \left(c_3(h - 1)^2 + 1\right) H(F_x) \text{sgn}(v_f),\] (2.55)
\[v_r = v_0 - \dot{\bar{x}}, \ v_f = v_0^+ - \bar{R}\dot{y}, \ h = h_0 - y,\] (2.56)
\[R = R_0 \left(c_4(v_r - 1)^2 + 1\right) = \cot \phi.\] (2.57)

Here, \(q_0, c_1, c_2, c_3\) and \(c_4\) are constant cutting parameters, \(\mu_0\) is a static friction coefficient, \(h\) the chip thickness and \(H()\) the Heaviside function. It is assumed that the force \(F_x\) is mainly due to the friction \(\xi\) acting on the rake surface. The friction velocity \(v_f\) is reduced due to shear plastic deformation \(R\) which is related to the shear angle \(\phi\). The force \(F_x\) as a function of \(v_r\) is shown in figure 2.15.

Figure 2.15: The force in x-direction as a function of the relative velocity \(v_r\) [91].

Wieczgrodz examines the dynamical behavior of the system using bifurcation diagrams. In general, these diagrams show the solutions of the equations of motion as a function of a certain parameter. It could be possible that a slight change of that parameter causes a modification of the location of a equilibrium point. However it is also possible that the number of equilibrium points or its stability behavior changes. These shifts in behavior are called bifurcations and can be visualized by means of bifurcation diagrams. For more information on bifurcations, the reader is referred to [18].

The non-dimensional equations of motions for the system of figure 2.14 are
\[\ddot{x} + 2\xi_x\dot{x} + x = F_x(y, \dot{x}, \dot{y}),\] (2.58)
\[\ddot{y} + 2\xi_y\sqrt{\omega^2} + ay = F_y(y, \dot{x}, \dot{y}),\] (2.59)
where
\[\xi_x = \frac{c_x}{2m\omega_0x}, \ \xi_y = \frac{c_y}{2m\omega_0y}, \ \alpha = \frac{k_y}{k_x}, \ \omega_0x = \frac{k_x}{m}, \ \omega_0y = \frac{k_y}{m}.\] (2.60)

Here, \(m\) is the mass, \(k_x\) and \(k_y\) are the stiffness parameters in \(x\) and \(y\) direction respectively and \(c_x\) and \(c_y\) are the damping constants in \(x\) and \(y\) direction respectively, see figure 2.14. However, for simplicity it is assumed that \(\xi_x = \xi_y = \xi\). Figures 2.16 and 2.17 show the bifurcation diagrams for the response in \(x\) and \(y\) direction respectively as a function of the cutting force magnitude \(q_0\) for different stiffness
Figure 2.16: Bifurcation diagrams \( x = f(q_o) \) for different stiffness ratios \( \alpha \); a) \( \alpha = 0.25 \), b) \( \alpha = 1 \), c) \( \alpha = 4 \), d) \( \alpha = 16 \) [31].

Figure 2.17: Bifurcation diagrams \( y = f(q_o) \) for different stiffness ratios \( \alpha \); a) \( \alpha = 0.25 \), b) \( \alpha = 1 \), c) \( \alpha = 4 \), d) \( \alpha = 16 \) [31].
rations α. The force \( F_x \) is a linear function of \( q_0 \) as can be seen in equation (2.53). It can be seen that the stiffness ratio has great influence on the dynamical behavior of the system. For \( \alpha = 0.25 \) and \( \alpha = 4 \) the system undergoes several changes while for \( \alpha = 1 \) and \( \alpha = 16 \) the system oscillates either periodically or almost periodically for larger values of \( q_0 \). Bifurcation diagrams are also constructed for \( x \) and \( y \) as a function of \( \xi \). These diagrams are shown in figure 2.18. For \( \xi > 0.26 \) the system vibrates periodically until a phenomenon is detected which is called \textit{deaths and births of periodic solutions}. This means that the system is asymptotically stable in the finite intervals and oscillates periodically for a finite number of discrete values of \( \xi \).

Nosyreva and Molinari [16] assume the empirical relation between friction-coefficient \( \mu \) and cutting-speed \( v_c \), of equation (2.61).

\[
\mu (v_c) = \beta v_c^q
\]  

(2.61)

The parameters \( \beta \) and \( q \) are found via experiments. Using this model it is found that under supercritical conditions (where the system is unstable according to linear stability analysis) the increase of the amplitude of the vibration as a function of time, saturates and finally decreases. Experiments done confirm this behavior.

Li and Li [13] predict cutting forces based on fundamental material properties, geometry of the tool and cutting conditions such as temperature. Results of this model are compared to experimental data and simulations done by Tlusty in 1983 [25]. Especially in the \( y \) direction of the milling machine the results coincide with the experimental results (UY and DY of figure 2.19). In the \( x \) direction the results for down-milling are slightly better than Tlusty’s results, but for up-milling they are worse.

2.3.2 Loss of contact

Balachandran and Zhao [4],[34] have made a model that accounts for both regenerative effects and loss of contact between the tool and the workpiece. Most numerical models divide the cutter in axial direction in small parts with thickness \( \Delta z \). These parts are modeled as being totally in or totally out of the cut. This is not true for helical mills. Balachandran and Zhao therefore only model the part of the cutter that is actually in contact with the workpiece (figure 2.20).

Compared to the results found by Altintas and Budak, the stability lobes are lower. This seems to correspond to the experiments done by Weck et al. [29] (the simulations done by Balachandran and Zhao have the same conditions as the lower right diagram of figure 2.8) and by Balachandran and Zhao. Though, still stable cuts occur when chatter was predicted and vice versa (figure 2.21). The model has also been tested for a 25% immersion of the cutter, i.e. the radial depth-of-cut is 25% of the cutter diameter. This time the model is compared to the analytical model by Altintas and Budak [1] and experiments, see figure 2.22. In this situation, the results of the model that accounts for loss of contact coincide better with experimental results, but still no total match is found. Especially for low immersion cuts, the model is more accurate than Altintas’ model. A main advantage of this model is that it can be used to describe post-bifurcation behavior. As the depth-of-cut is increased Neimark bifurcations or period-doubling bifurcations occur, depending on the spindle speed (figure 2.23). Another graphical way to show the system dynamics is by use of Poincaré sections. Balachandran and Zhao show both the pre- as the post-bifurcational behavior of both simulations and experiments by means of Poincaré sections. There is a close resemblance between the results of the simulation and the experiment. In figure 2.24, a Neimark bifurcation is shown at 20000 rpm.

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Figure 2.18: Bifurcation diagrams a) $x = f(\xi)$ b) $y = f(\xi)$ \cite{31}.

Figure 2.19: The critical depth-of-cut for up-milling (U) and down-milling (D) in X and Y direction \cite{19}.
Figure 2.20: Effect of system dynamics on the cutting zone. Blue solid line: path that has been followed by tooth 1. The material has been cut by this tooth. Blue dashed line: path that would be followed by tooth 2 if no movement of the cutter occurred. Red dashed line: the path that will be followed by tooth 2 as a result of movement of the cutter. The dynamical cutting zone is marked as $\phi$ [4].
Figure 2.21: Stability lobes for "slotting" (radial depth of cut = cutter diameter). Line: Numerical model by Balachandran and Zhao; dotted line: numerical model by Budak and Altintas; squares: unstable cut; circles: stable cut [4].

Figure 2.22: Stability lobes for a 25% immersion cut. Line: numerical model by Zhao and Balachandran; dotted line: analytical model by Altintas and Budak; squares: unstable cut; circles: stable cut [34].
As mentioned above, physical nonlinear phenomena are modeled in several ways. These models show that the limit of stability of the milling process is predicted more accurate than by using the linear analysis. Moreover, it is shown that nonlinear phenomena such as bifurcations can be modeled and occur in practice.

2.3.3 Other nonlinearities

The gyroscopic effect of the spindle is modeled by Tian and Hutton [22]. They use a finite element model and study the wave propagation in the spindle. The stability lobes predicted by the model are lower than the lobes predicted analytically by Altintas and Budak [1]. If the gyroscopic effect is left out of the finite element model, the results are identical to those by Altintas and Budak, see figure 2.25.

Shorr and Liang [19] have extended the closed loop model, as defined by Tlusty, see figure 2.26, to determine regenerative chatter. The resulting transfer function is mathematically elaborate and exhibits nonlinear behaviour. By linearisation of the function and using the roots of the characteristic equation, the stability of the system can be described as a function of cutting parameters, geometry of the workpiece and tool and the dynamical properties of the machine. Contact parameters for the workpiece and machine are assumed. For low spindle speeds (100-400 rpm) a stability analysis is applied, where the mean chip thickness is measured. The model is validated by experiments. Experiments and simulations done are down milling of 7075-T6 aluminium with a four flute, 12.7 mm diameter, 30° helix angle end mill set at an radial depth-of-cut of 1.5 mm and an axial depth of cut of 6.4 mm. In the experiments, three parameters are varied at a spindle speed of 135 rpm:
Figure 2.24: Pre-bifurcation and post-bifurcation movement at an axial depth-of-cut of 0.76 mm (left; stable) and 1.20 mm (right; unstable) at 20,000 rpm. Upper row: amplitude of the cutter-displacement (x) as a function of time; 2nd row: power spectral density plot; 3rd and 4th row: Poincaré section amplitude of the cutter-displacement in y-direction versus amplitude in x-direction. 3rd row: numerical results; 4th row: experimental results [34].

Figure 2.25: Stability lobes using a finite element model. Line: gyroscopic effect. Dotted line: no gyroscopic effect; circles: results by Altintas and Budak. [22].
Figure 2.26: The regenerative chatter model by Shorr and Liang. G1: angular convolution model; G2: compliance of the machine tool; G3: compliance of the mill tool; G4: compliance of the workpiece; G5: impulse generator [19].

- Radial depth of cut;
- Number of flutes. The feedrate per minute is held constant, so the feed per tooth \( f_t \) increases if the number of flutes decreases;
- Feedrate per minute.

The stability analysis is shown in figure 2.27 for the simulation. The stability analysis for the experiments is shown in figure 2.28. Note that on the vertical axis, the average chip thickness is shown. Chip thicknesses above the line result in an unstable cut, whereas chip thicknesses below the line result in a chatter-free cut.

As it is shown, the limit of stability decreases with an increasing feed. If the feedrate is 70 mm/min, the chip thickness is above the stability border. If the feedrate decreases, the chip thickness also decreases, which increases the stability. This shows the opposite effect as shown by Landers and Ulsoy in figures 2.11 and 2.13. The main difference between the two models is the way they measure the limit of stability. Landers and Ulsoy increase the axial depth of cut, until chatter occurs. Shorr and Liang define a average chip thickness above which the system is unstable.
2.4 Experiments

Experiments can be used to validate certain models. It can be verified whether a cut, which is predicted to be stable, is in fact stable or not (e.g. Weck et al. [29], Altintas et al. [1], [2] and Balachandran and Zhao [4]). Also the behavior of the cutter in stable and unstable cuts can be monitored (e.g. Zhao and Balachandran [34]) in order to qualify typical behavior.

Experimental results can also be used to study possible nonlinear behaviour without using a model.

Weinert et al. [30] have compared experimental time analysis with surrogate data of e.g. linear processes in several different ways. Assuming the milling process as a black box, they generate data using several models of well-known or well investigated processes, which have features comparable to the features of the experiments done, e.g. Fourier amplitudes or spectra. After the data has been generated, some nonlinear invariants are calculated to get test statistics for a classification of the time series. This shows whether or not the original time series provide the same features as the surrogates do. Because the experimental data differs from the surrogate data it can be concluded that the process contains nonlinear or stochastic components.

The same conclusion is drawn by Beule and Herzel [5]. They have performed milling experiments. By removing 50% of the experimental data and predicting that data using both linear and nonlinear techniques, they conclude that the milling process contains important nonlinearities.

Ismail and Soliman [10] use experiments to generate stability lobes directly. At a certain depth-of-cut they increase the spindle speed and measure the static and dynamic components of the cutting force. The quotient of these two indicates extent of stability.
2.5 Discussion

Several researchers and groups have investigated the chatter phenomenon since the 1950's. Often models are made which can be divided into linear and nonlinear models. The linear model which is developed by Altintas et al. provides an analytical way to predict stability lobes. This is a fast way to find the boundary between stable cuts (i.e. cuts where no chatter occurs) and unstable cuts (i.e. cuts where chatter occurs). Although it is a fast method and the model can be used for several types of cutters and milling properties, still chatter occurs when it is not predicted by the model and vice versa. This means that in some occasions, it might be possible to increase the depth-of-cut, without the occurrence of chatter, which increases the productivity. On the other hand, it could be possible that chatter occurs, when it is not predicted by the model. This causes a bad surface of the workpiece and faster tool wear.

Nonlinear models have been developed in several different ways. Landers and Ulsoy have modified the model by Altintas in a way that is suitable for a nonlinear force model. A remarkable conclusion is that they find that the limit of stability increases with an increasing feed rate. This should result in a larger productivity, because of both an increasing feed rate and a larger depth-of-cut. In contrast with this, Shorr and Liang show that an increasing feedrate causes a decreasing dynamic stability.

Researchers have assumed different physical phenomena to be nonlinear. Wiercigroch emphasizes the nonlinearity of the friction force between the cutter and the workpiece, while Balachandran and Zhao model the loss of contact between tool and workpiece. Other nonlinearities which are modeled are e.g. the gyroscopic effects of the spindle. The stability lobes generated using these models show a better resemblance between the predicted stability limit and the actual limit that occurs in practice than linear models provide. Several researchers find that the actual stability limit is lower than the linear models predict.

In the next chapter, the linear method provided by Altintas is followed, and the resulting SLD's are compared to an SLD following from actual experiments.
Chapter 3

Experiments

3.1 Introduction

This chapter consists of two main parts. First, in section 3.2, the method to determine the analytical stability lobes using Altintas’ method [3] is described. We used of the computer-program CutPro 3.0, in which this method is implemented to determine the SLD’s for combinations of material, tool, machine, etc. The related results are described in section 3.3. Second, in section 3.4, this model is validated by identifying actual cuts as stable or not using the computer-program Harmonizer 2.1 and comparing this results to the results of the application of the method of Altintas.

3.2 Method of the derivation of the analytical stability lobes

The linear model as described by Altintas [3] (see also section 2.2) is implemented in the computer-program CutPro 3.0. It needs the following information to determine the stability lobes analytically:

- The geometry of the mill;
- The dynamical behavior of the mill;
- Material parameters (the parameters $K_{ij}$ of equation (2.44));
- The type of cut (e.g. the radial depth-of-cut and feed).

In order to determine the dynamical behavior of the mill and the material parameters two types of experiments are performed.

3.2.1 Identification of the dynamical behavior of the mill

The input needed by CutPro consists of a set of modal parameters such as an eigenfrequency, a stiffness and a damping ratio. These parameters are necessary to build a frequency response function (frf) between the force on the cutter and the displacement of the cutter. This transfer function $H_0$ is used in equation (2.31) to find the stability limit. In order to determine these parameters hammer tests are done. A schematic representation of the setup is shown in figure 3.1 and a picture of the equipment used is shown in figure A.3 of appendix A. The tool used is a 10 mm diameter JH420 endmill manufactured by Jabro Tools (figure A.2). It is held
in a Kelch shrink-fit toolholder and used on a Mikron HSM 700 milling machine (figure A.1).

Figure 3.1: The experimental setup used for the hammer tests.

A small accelerometer, type Endevco model 25A, is mounted on the endmill using wax. The impact hammer used is a PCB miniature hammer type 086D80. The calibration data of the accelerometer and hammer are shown in figures B.1 and B.2 in appendix B. For more details on the equipment used the reader is referred to [17]. The computer-program MalTF 3.0, which is a part of CutPro, is used as data-acquisition software. When the impulshammer hits the tool, the force of the hammer and the acceleration of the mill are measured at a samplefrequency of 50 kHz, during 0.01s, see (figure 3.2). MalTF calculates the frequency-response-function (frf) between the force and acceleration. To eliminate the effect of noise, several tests have to be done and the mean value of the frf’s of these tests is used. These tests have to be done in both the x and y direction. This means that the cutter is rotated over 90° in the spindle and excited at the same tooth, while the accelerometer also remains sticked to the same tooth. This second measurement is done to account for differences of the stiffness of the suspension of the mill in x and y direction. If long slender endmills are used at a high axial depth-of-cut (e.g. 2 times the mill-diameter or higher) impact measurements have to be done at several places in axial direction z. Either the impact-location or the measurement-location can be changed. In this case it is not necessary to have tests done at several places in axial direction.

To calculate the modal parameters, CutPro also exhibits a modal analysis part. The measured data, concerning the mean frf between the force applied to and acceleration of the cutter for discrete values of the angular frequency ω, $H_{Fa}(j\omega)$, is read and the user has to select one or several peaks which will be taken into account. Subsequently, a preliminary set of modal parameters will be computed. The transfer function of the system with respect to force and displacement $H_{Fx}(j\omega)$ is given in the Laplace domain by equation (2.22). The conversion between $H_{Fa}(j\omega)$ and $H_{Fx}(j\omega)$ can be done automatically by the modal analysis software by dividing $H_{Fa}(j\omega)$ by $(j\omega)^2$.

In the Laplace domain, the transfer function can be written for a single mode as

$$H(s) = \frac{1/m}{s^2 + 2\xi\omega_0 s + \omega_0^2}, \quad (3.1)$$
where the characteristic equation is given by the denominator. This equation has two complex conjugate roots:

\[ s_1 = -\xi \omega_d + j \omega_d \quad \text{and} \quad s_1^* = -\xi \omega_d - j \omega_d, \]

\[ (3.2) \]

where \( \omega_d \) is the damped natural frequency. The transfer function (3.1) can be expressed by its partial fraction expansion:

\[ H(s) = \frac{r}{s - s_1} + \frac{r^*}{s - s_1^*}. \]

\[ (3.3) \]

For a single degree of freedom system, the residue \( r \) has the following value:

\[ r = \lim_{s \to s_1} \frac{1}{s - s_1} \frac{1/m}{(s - s_1)(s - s_1^*)} = \frac{1/m}{s_1 - s_1^*} = \frac{1/m}{2\xi \omega_d} \]

\[ r^* = \lim_{s \to s_1^*} \frac{1}{s - s_1^*} \frac{1/m}{(s - s_1)(s - s_1^*)} = \frac{1/m}{s_1^* - s_1} = -\frac{1/m}{2\xi \omega_d}. \]

\[ (3.4) \]

After selecting all the desired peaks, an optimization algorithm can be selected to optimize the modal parameters. This can be done with respect to eigenfrequency, damping or residue or a combination of these, see figure 3.3. The algorithm used is a two stage linear least squares algorithm. The modal parameters that are found can be exported to be used in CutPro.

### 3.2.2 Identification of the material parameters

CutPro includes a database of orthogonal cutting parameters of several materials, such as some types of aluminium, steel and titanium. These parameters, as described in section 2.2, on page 11, can be transformed into oblique cutting parameters using goniometric relations. Therefore, it is possible to use this database for
several sorts of cuts and tools. For materials that are not included in the database, the material parameters can be found experimentally. These parameters are only valid for the workpiece-tool combination that is used during the experiments. The experiments consist of milling a few paths in x-direction at full immersion. For these paths a single spindle speed and a single axial depth-of-cut is chosen. The only thing that is changed is the feedrate. The steps followed are:

1. The forces in x, y and z direction are measured using a dynamometer (type Kistler 9255). A schematic representation of the experimental setup is shown in figure 3.4 and a picture of the equipment used can be found in figure A.4 in appendix A.

2. The measured forces \( \mathbf{F}_q \), \((q = x, y, z)\) are imported in CutPro.

3. The program calculates the forces \( \mathbf{F}_q \) of equation (2.43).

4. A least squares algorithm is used to determine \( \mathbf{F}_{qc} \) and \( \mathbf{F}_{qc} \).

5. Using equation (2.43), the material parameters \( K_{ij} \) of equation (2.44) are found. This can be done automatically using CutPro, if \( \mathbf{F}_x \) and \( \mathbf{F}_z < 0 \) and \( \mathbf{F}_y > 0 \).

### 3.2.3 Stability lobe generation

In the CutPro main program all the cutting parameters are defined step by step. First, some general selections have to be made e.g. the type of machining process, in this case milling, and the simulation mode, in this case single analytical stability lobes, see figure 3.5.

The next step is to define the machine and tool parameters, see figure 3.6. The type of cutter and number of flutes are defined, in this case a cylindrical cutter.
Figure 3.4: *The setup used for the tests to identify the material properties.*

Figure 3.5: *Selecting the machining process and the simulation mode in CutPro 3.0. Left: selecting the milling process. Right: selecting the computation of the single analytical stability lobes.*
with two flutes, which are equally spaced. The dimensions of the cutter have to be
defined e.g. radius, length and helix angle. Moreover here the modal parameters as
determined experimentally by the hammertests, can be imported.

Figure 3.6: Inserting the machine and tool parameters. Upper left: A cylindrical
endmill having two equally spaced flutes is chosen. Upper right: Material and
dimensions of the cutter are determined. Lower left: The modal parameters as
estimated by the experiments are imported.

Now, the workpiece parameters have to be defined. The material parameters
as found by the experiments can be retrieved here, see figure 3.7. For milling
thin walled structures, it is recommended to have also hammertests done at the
workpiece. The modal parameters can be imported in the CutPro main program.
For the determination of the analytical stability lobes, the last step involves the
definition of some cutting parameters, e.g. the radial depth-of-cut and feedrate, see
figure 3.8. Now, the analytical calculation of the stability lobes can be run.

3.3 Results of the derivation of the analytical sta-
bility lobes

In this section, the results of the experiments described in the previous section are
shown.
Figure 3.7: Selecting the workpiece material and the material model used.

Figure 3.8: Definition of the cutting conditions. In this case, slotting with a feedrate $f_s$ of 0.12 mm/tooth.
3.3.1 Results of the identification of the dynamical behavior of the mill

Hammertests are done on the Jabro Tools cutter JH420 with a diameter of 10 mm. To study the effect of the magnitude of the force-impulse on the estimated frf and the stability lobes, tests have been done where the hammer hits the tool at a peak force between 10-15 N and at a peak force between 50-70 N. At each magnitude 20 hits are applied and the mean frf of those hits is determined. Such tests have been done twice in both x and y direction.

For the x direction, the absolute value of the frequency response functions of the two datasets for the "soft" hits is shown in figure 3.9. This is the frequency response function of the system with respect to force and displacement $H_{F_x}$. The power spectral densities of the impact force of the two datasets are shown in figure 3.10. For both measurements 20 hits are made. Because both frf's are nearly the same with nearly the same inputs, it can be concluded that the measurements are reproducible.

The absolute value of the frequency response functions in y direction are shown in figure 3.11. For comparison the absolute value of the frequency response functions in x direction are also included. Because both frf's of the measurements in y direction are nearly the same with nearly the same inputs, it can be concluded that these measurements are reproducible. The absolute value of the frf in y direction is slightly higher than in x direction. This means that the stiffness of the spindle and tool in x direction might be higher than in y direction. The power spectral densities of the impact force of the hammer is shown in figure 3.12. It is shown that the inputs for the four hammertests are of the same order of magnitude.

The system is also excited at a larger peak force (50-70 N). The powerspectra for these hits is shown in figure 3.13 for the y direction. For comparison the power
Figure 3.10: Power spectral density of the impact force for the "soft" hits in x direction.

Figure 3.11: Absolute value of the frequency response functions $H_{Fx}$ of the "soft" hits in x and y direction.
Figure 3.12: power spectral density of the impact force for the "soft" hits in x and y direction.

spectral densities of the "soft" hits are also included. The absolute value of the frequency response functions of these four datasets in y direction are shown in figure 3.14. The difference between the absolute value of the frf's of the "hard" hits with respect to the absolute value of the frf's of the "soft" hits is up to 50%. The absolute value of the frequency response functions for the "hard" hits are lower than for the "soft" hits. This may very well indicate that the stiffness is increasing while the excitation force increases. This observation points at a nonlinearity which occurs in the spindle and tool, so the workpiece's material properties or friction between tool and workpiece (related to which in the literature study several nonlinearity types were discussed) has nothing to do with that.

In x direction, also two measurement-sets of 20 "hard" hits have been generated. However, the second set shows a second impact-peak at the end of the measurement which has great influence on the frf. Therefore, the modal parameters found for this measurement differ from the parameters of the first measurement. The mean force and acceleration as a function of time for both "hard" measurements in x direction are shown in figure 3.15, the power spectral densities of the forces in figure 3.16 and the absolute value of the frf in figure 3.17. Also in x-direction, the frf is dependent on the magnitude of impact.

3.3.2 Results of the identification of the material parameters

The material used for all the experiments is aluminium 51ST. This material is not included in the CutPro database, so the material parameters had to be determined experimentally. Therefore, these parameters are only valid for experiments done with the JH420-10 cutter. Cuts have been made at a feedrate of 0.08, 0.12, 0.16, 0.20 and 0.24 mm/tooth. The mean force is calculated and the cutting coefficients are found, see figure 3.18.
Figure 3.13: Power spectrum of the impact force for the "soft" and "hard" hits in y direction.

Figure 3.14: Absolute value of the frequency response function $H_{P_x}$ of "soft" and "hard" hits in y direction.
Figure 3.15: Force (upper) and acceleration (lower) as a function of time for both "hard" measurements in x direction. Left: set 1. Right: set 2.

Figure 3.16: Power spectral density of the impact force for the "soft" and "hard" hits in x direction.
Figure 3.17: Absolute value of the frequency response function of "soft" and "hard" hits in x direction.

Figure 3.18: Average cutting forces for Aluminium 51ST with a JH420-10 tool.
The cutting coefficients are found as:

\[
\begin{align*}
K_{cc} &= 575.878148 \text{ [N/mm]}, & K_{tc} &= 19.651827 \text{ [N/mm]}, \\
K_{vc} &= 83.260764 \text{ [N/mm]}, & K_{re} &= 19.273963 \text{ [N/mm]}, \\
K_{nc} &= 146.037540 \text{ [N/mm]}, & K_{te} &= 0.020051 \text{ [N/mm]}.
\end{align*}
\]

### 3.3.3 Results of the stability lobe generation

The modal parameters and the material parameters are used to generate the stability lobes for usage of the 10 mm JH420 cutter on the Mikron HSM 700 machine in aluminium 51ST. A feedrate of 0.12 mm/tooth is chosen and the radial depth-of-cut is 10 mm, i.e. the cutter is fully immersed in the material. The results of this computation are shown in figure 3.19. Lobes have been generated for combinations of "hard" and "soft" impact hits, i.e. measurement "x soft 1" is combined with "y soft 1" etc. to generate a lobe. All lobes generated by the modal parameters of the "soft" hits are in good agreement. This is not the case for the lobes generated by the modal parameters of the "hard" hits. The second measurement in x-direction causes the difference in both lobes for the "hard" hits. The difference between the two groups, "soft" and "hard", is large, up to 50%. The difference between the absolute value of the frf's of those two groups is up to 45%. Because the absolute value of the frequency response for the "hard" hits is lower, the system seems to be stiffer. This increases the stability limit for the "hard" hits. It can be concluded that a possible nonlinearity occurs in the stiffness of the machine, suspension or tool, which has influence on the limit of stability.
3.4 Determining chatter in practice

3.4.1 Setup

In order to verify the results of the analytical determination of the SLD using the method described in section 3.2 and 3.3, actual cuts are made on the Mikron HSM 700. The cuts are made using the same tool as is used with the hammertests (a 10 mm JH420 cutter) and are made in aluminium 51ST. The milling properties are the same as they are in section 3.2 and 3.3, i.e. the feedrate is 0.12 mm/tooth and the immersion ratio is 100%. Tests are done at several spindle speeds. The axial depth-of-cut is increased by steps of 0.5 mm until chatter occurs. To identify the cut as stable or not the program Harmonizer [9] is used. A schematic representation of the experimental setup is shown in figure 3.20 and a photo of the equipment is shown in figure A.5 in appendix A. The sound of the cut is recorded and analyzed by Harmonizer (see figure 3.21). The input that Harmonizer needs is the spindle speed and the number of teeth of the cutter. Harmonizer analyses the frequency spectrum of the recorded sound. It has the possibility to filter the frequency which is caused by the teeth hitting the workpiece and its related harmonics. The first step is to record the environmental sound. This sound will be filtered from the recorded sound of the cutting. The next step is to let the cutter spin in the air, without cutting the material and record the sound. Using this recording, a threshold level can be set automatically at a certain factor above the current soundlevel. If a certain frequency of the spectrum exceeds this threshold the cut is marked as unstable. It could happen that a cut is marked as stable, while the cut produces a loud noise and the surface is non-smooth. In that case, the threshold-level has to be decreased manually. The program is written to find the most efficient cut in terms of maximum cutting volume per minute. If chatter is detected the program suggests a new spindle speed where a stable cut should occur. In this study, Harmonizer is only used to identify a cut as stable or not.

3.4.2 Results

Cuts have been made for a frequency range from 10,000 to 42,000 rpm. If a cut is stable according to Harmonizer, the axial depth-of-cut is increased by 0.5 mm until chatter occurs. At some places chatter occurs while Harmonizer marks the cut as stable. This occurs at 10,000 rpm and in the range 35,000-40,000. In both cases,
Figure 3.21: Harmonizer screenshots. Axial depth of cut 3.0 mm. Upper left: Unfiltered unstable cut at 14151 rpm. Upper right: Unfiltered stable cut at 18209 rpm. Lower left: Filtered unstable cut at 14151 rpm. Lower right: Filtered stable cut at 18209 rpm.
Harmonizer marks the cut as unstable when the axial depth-of-cut is increased by 1-2 mm above the level when chatter occurs. The results of the tests are shown in figure 3.22. The stability lobes produced by CutPro are also shown. The lobes as predicted by CutPro using the modal parameters of the "hard 1" impact measurements show good agreement for cuts between 22,000 and 31,000 rpm. Above that spindle speed the differences are large (over 100%). It should be noted that the information Harmonizer gave above 31,000 rpm is not trustworthy, having the threshold set automatically. By listening to the sound produced by the cutting and watching the milled surface, it was found that chatter occurred at cuts, which had an axial depth-of-cut that was 1-2 mm lower than the cuts Harmonizer first indicated as chatter. In that region, the threshold level should have been decreased manually by comparing it to the resulting milled surface and sound produced. Then, the difference between the boundary predicted by CutPro and the actual boundary is about 50% for cuts with a spindle speed of 35,000-40,000 rpm. To have a proper comparison between the analytically predicted stability lobes and the chatter boundary as it exists in practice more cuts should be made at different spindle speeds. Also the real chatter boundary should be used in stead of the chatter boundary produced by Harmonizer.

3.5 Discussion

The nonlinear models discussed in section 2.3 include nonlinearities related to the friction between tool and workpiece or to the material properties. Other types of nonlinearities modeled are the (partial) tool jump-out. This kind of nonlinearities are all based on the interaction between the mill and the workpiece. The impact
measurements described in section 3.2 and 3.3, show that the absolute value of the
frequency response function $H_F$, decreases when the peak magnitude of the impact
increases. This may indicate that the stiffness of the spindle or its suspension or tool
increases when the force applied on them increases. This causes the analytically
determined SLD's to increase up to 50% compared to lobes generated by "soft"
impact hits.

For some spindle speeds, a difference exists between the cuts which Harmonizer
marked as chatter, if the threshold level is set automatically, and the occurrence of
chatter by listening to the sound produced by the cutting and watching the milled
surface. This difference occurs at 10,000 rpm and in the region 35,000-40,000 rpm.
For those cuts the threshold level has to be decreased manually. In that region, the
chatter boundary is 1-2 mm lower than indicated by Harmonizer.

Even if that difference is taken into account, the analytically determined stability
lobes predict a stability border which is up to 50% lower than the border determined
experimentally for cuts at a spindle speed above 31,000 rpm. The analytically
determined SLD of the "hard 1" impact tests show good agreement for the region
22,000-31,000 rpm. The possible nonlinearity of the machine, suspension or tool
could be the cause for this difference.
Chapter 4

Conclusions and recommendations

Several researchers have studied and modeled the phenomenon chatter. Chatter is the result of several causes. Primary chatter is the consequence of friction effects between the tool and the chip, mode coupling or thermodynamics of the cutting process. Secondary chatter is caused by regeneration of waviness of the surface of the workpiece. Both linear and nonlinear models have been developed in different ways. The friction force can be modeled as a nonlinear function of the cutting parameters. Partial tool jump-out can be modeled. Also the gyroscopic effect of the spindle speed has been modeled. Experiments are performed to study chatter and to validate the models. Several researchers conclude that nonlinearities should be modeled for a more accurate prediction of chatter. They show that the milling process contains phenomena which can not be modeled using linear models.

Impact-tests have been performed on a Mikron HSM 700 milling machine using a 10 mm diameter JH420 tool. The frequency-response-function (frf) is dependent of the magnitude of the impact. A higher impact results in a lower response, up to 45%. This may indicate that the stiffness of the spindle, suspension or mill increases while the force increases. The modal parameters of the frf’s and the material parameters of the workpiece are determined and have been used to analytically construct stability lobe diagrams using the program CutPro. This program makes use of a linear model developed by Altintas et al. [1]. The lobes generated by the parameters of the "hard" hammerhits (50-70 N) are up to 50% higher than the lobes generated using the modal parameters of the "soft" hits (10-15 N).

Harmonizer has been used to identify a cut as stable or not. Using the automatic threshold of the program some cuts are marked as stable while they are actually unstable. The threshold had to be decreased manually to make harmonizer have the correct information. This occurs at a spindle speed of 10,000 rpm in the region from 35,000-40,000 rpm.

The experimental results have been compared to the analytically determined stability lobes. For the region 22,000-31,000 rpm the experiments are in good agreement with the stability lobes generated by the modal parameters of the "hard" impact tests. If the cutter is in use, the forces are also high, so the "hard" hits should be more comparable to a cutter in use. For higher spindle speeds the differences run up to 100% if the chatter boundary, according to Harmonizer, using an automatically determined threshold level, is used. This boundary is 1-2 mm too high. If the real boundary, marked by the occurrence of a bad surface and a loud sound produced by the cutting, would be used the differences would be about 50%. New experimental tests should be done where the chatter boundary should be marked.
by the occurrence of a bad surface and a loud sound produced by the cutting, instead of the boundary as given by Harmonizer, having an automatically determined threshold. It is also possible to decrease the threshold level in Harmonizer manually.

For a single combination of machine, tool and a workpiece several stability lobes are determined, dependent on the magnitude of the peak force of the hammer tests. To produce only one SLD, even if several impact tests are done, the force applied during the impact tests should always be the same. This can be achieved by having the hammer hit the tool automatically instead of manually using the hammer.

The experimental results differ from the analytically determined stability lobes using a linear model. Therefore, it is likely to use a nonlinear model. First it should be investigated which types of chatter need nonlinear modeling. Then, the milling process should be modeled. This model should include the variability of the frf as a function of the impact force. The next step is to find the stability limit using this model. The computation time of the SLD generation should not increase too much in order to remain attractive for practical use.
Bibliography


Appendix A

Photos of the experimental setups

Figure A.1: The Mikron HSM 700 at TNO Industrie.
Figure A.2: The 10 mm JH420 cutter with Endevco accelerometer in a Kelch shrink-fit holder.

Figure A.3: The equipment used for the hammertests.
Figure A.4: *The equipment used for determining the material parameters.*

Figure A.5: *The equipment used for Harmonizer.*
Calibration Data

Max Transverse Sensitivity: 2.0%

Description: IEPE Accelerometer
Manufacturer: Endevco
Model Number: Z5A
Serial Number: C176

Sensitivity: 4.54 mV/g at 100 Hz, 10.0 g pk
0.626 mV/g(±5%) at 100 Hz, 98.1 g pk

Temperature: 70°F
Humidity: 41%
Bias: 10.1 VDC
Polarity: Forward
Case: Isolated

Amplitude Response

Frequency in Hz

NIST Traceability Number: 822-282802-00

Uncertainty estimate (95% confidence, k=2)
-2.1% 100.0 Hz Sensitivity
±0.5% 20.0 Hz to ±0.0000 Hz
±0.0% 10.0000 Hz to ±0.0000 Hz
±0.0% 10.0000 Hz to ±0.0000 Hz

Date: Feb 6, 2000 11:31:26 AM

By:

Figure B.1: Calibration data of the accelerometer used for the experiments in section 4.1.1 and 4.1.2.
## CALIBRATION CERTIFICATE

**IMPULSE FORCE HAMMER**

| Model No. | 086D80 |
| Serial No. | 12399 |

- **Range**: 0 - 50 lb.
- **Linearity error**: < 2.0 %
- **Discharge Time Constant**: 100 ms
- **Output Impedance**: 100 ohms
- **Output Bias**: 10.87 volts

Traceable NIST project No. 822/259355-98 in compliance with ISO 10012-1, and former MIL-STD 45662A.

**Technician**: Travis Lodyga

**Date**: 09-02-1993

**Accelerometer Model No.**: 309A
**Serial No.**: 278
**Sensitivity**: 5.3 mV/g

**Pendulous Test Mass**: 0.06 lbs (22.7 grams) including accelerometer.

### HAMMER SENSITIVITY:

<table>
<thead>
<tr>
<th>Hammer Tip</th>
<th>STEEL</th>
<th>VINYL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration</td>
<td>Extender</td>
<td>None</td>
</tr>
<tr>
<td>Hammer Sensitivity</td>
<td>mV/lb</td>
<td>94.7</td>
</tr>
<tr>
<td></td>
<td>mV/N</td>
<td>21.3</td>
</tr>
</tbody>
</table>

**Notes:**

1. To convert a measurement (typically mV) to engineering units (lbs or g's), each channel must be divided by its sensitivity (mV/lb or mV/g).

2. Each specific hammer configuration has a different sensitivity. The difference is a constant percentage which depends on the mass of the cap and tip assembly relative to the total mass of the hammer. Calibrating the specific hammer structure being used automatically compensates for mass effects.

3. PCB hammers 086D82, B03 and B04 may be calibrated by mounting the Model 302A07 Accelerometer on the back of the hammer head, impacting a convenient surface and measuring the output of both hammer (Vh) and accelerometer (Va):

   Hammer sensitivity $S_f = S_a \left( \frac{V_h}{V_a} \right)$, where 'm' is the *Effective Mass* and 'S_a' is *Accelerometer Sensitivity*.

   Effective mass $\frac{N/A}{\text{with 302A07 attached and vinyl-capped plastic tip.}}$

**Figure B.2**: Calibration data of the impulse hammer used for the experiments in section 3.2 and 3.3.