Letter to the editor

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Letter to the Editor

To the Editor:

A recent issue of *Precision Engineering* contains the interesting article, "A statistical analysis of positional errors of a multiaxis machine tool," by Y.C. Shin and Y. Wei (*Precision Engineering* 1992, **14**, 139–146). In the article, I noted several possible errors that might cause confusion to your readers.

First order approximation in the derivation of Equation 4

In Equation 4 of the article, a coordinate transformation is presented that describes the effects of three angular and three translational errors on the transformation matrix between two succeeding coordinate frames. As noted by the authors, the matrix is only valid for small (angular) errors. This is due to a first-order approximation of the effect of angular errors [i.e., $\cos(\alpha) \approx 1$ and $\sin(\alpha) \approx \alpha$]. This first-order approximation is not consistent with the claim made in the last paragraph of the first column of page 142, stating that error terms up to the second order are considered. For example, if second-order terms such as $\alpha_1\Delta y_2$ are included in the error model (see Equation 11), one should also include the second-order approximation of the cosine effect of an angular error [i.e., $\cos(\alpha) \approx 1 - \frac{1}{2}\alpha^2$]. The neglected term $-\frac{1}{2}\alpha^2$ might have a larger effect on the position error of the tool than the term $\alpha_1\Delta y_2$, because it is magnified by the relative tool vector. Note that the introduction of second-order terms destroys the commutative and additive properties of the errors.

Elaboration of Equation 10 resulting in Equations 11–13

Equation 10 of the article describes how errors in the relative position and orientation of succeeding coordinate frames affect the errors in the relative position and orientation of the tool. In Appendix A the various terms of this equation are calculated (first and second order). The position components of the derived transformations are added to obtain the total position error of the tool. The x , y , and z components of this error are presented in Equations 11–13. In addition to a confusing use of the Δ symbol, serious errors have been made in Equations A.4–A.6. One of the consequences of these errors is that the translation error Δx_1 introduced in axis 1 results in a translation error $3\Delta x_1$ in the x position of the tool. This is a peculiar

result. The various errors in Equations 11–13 obviously affect Equations 14 and 15 and the content of Appendix B.

In my opinion the correct equations should have been:

$$A\delta ABC = \begin{bmatrix} 0 & -\alpha_1 & \beta_1 & \beta_1 z - \alpha_1 y + \Delta x_1 \\ \alpha_1 & 0 & -\gamma_1 & -\gamma_1 z + \Delta y_1 \\ -\beta_1 & \gamma_1 & 0 & \gamma_1 y + \Delta z_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.1})$$

$$AB\delta BC = \begin{bmatrix} 0 & -\alpha_2 & \beta_2 & \beta_2 z + \Delta x_2 \\ \alpha_2 & 0 & -\gamma_2 & -\gamma_2 z + \Delta y_2 \\ -\beta_2 & \gamma_2 & 0 & \Delta z_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.2})$$

$$ABC\delta C = \begin{bmatrix} 0 & -\alpha_3 & \beta_3 & \Delta x_3 \\ \alpha_3 & 0 & -\gamma_3 & \Delta y_3 \\ -\beta_3 & \gamma_3 & 0 & \Delta z_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.3})$$

$$A\delta AB\delta BC = \begin{bmatrix} -\alpha_1\alpha_2 - \beta_1\beta_2 & \beta_1\gamma_2 \\ \gamma_1\beta_2 & -\alpha_1\alpha_2 - \gamma_1\gamma_2 \\ \gamma_1\alpha_2 & \beta_1\alpha_2 \\ 0 & 0 \\ \alpha_1\gamma_2 \\ \alpha_1\beta_2 \\ -\beta_1\beta_2 - \gamma_1\gamma_2 \\ 0 \\ \alpha_1\gamma_2 z - \alpha_1\Delta y_2 + \beta_1\Delta z_2 \\ \alpha_1\beta_2 z + \alpha_1\Delta x_2 - \gamma_1\Delta z_2 \\ -z\beta_1\beta_2 - z\gamma_1\gamma_2 - \beta_1\Delta x_2 + \gamma_1\Delta y_2 \\ 0 \end{bmatrix} \quad (\text{A.4})$$

$$A\delta ABC\delta C = \begin{bmatrix} -\alpha_1\alpha_3 - \beta_1\beta_3 & \beta_1\gamma_3 \\ \gamma_1\beta_3 & -\alpha_1\alpha_3 - \gamma_1\gamma_3 \\ \gamma_1\alpha_3 & \beta_1\alpha_3 \\ 0 & 0 \\ \alpha_1\gamma_3 & \beta_1\Delta z_3 - \alpha_1\Delta y_3 \\ \alpha_1\beta_3 & \alpha_1\Delta x_3 - \gamma_1\Delta z_3 \\ -\beta_1\beta_3 - \gamma_1\gamma_3 & \gamma_1\Delta y_3 - \beta_1\Delta x_3 \\ 0 & 0 \end{bmatrix} \quad (\text{A.5})$$

$$AB\delta BC\delta C = \begin{bmatrix} -\alpha_2\alpha_3 - \beta_2\beta_3 & \beta_2\gamma_3 \\ \gamma_2\beta_3 & -\alpha_2\alpha_3 - \gamma_2\gamma_3 \\ \gamma_2\alpha_3 & \beta_2\alpha_3 \\ 0 & 0 \\ \alpha_2\gamma_3 & \beta_2\Delta z_3 - \alpha_2\Delta y_3 \\ \alpha_2\beta_3 & \alpha_2\Delta x_3 - \gamma_2\Delta z_3 \\ -\beta_2\beta_3 - \gamma_2\gamma_3 & \gamma_2\Delta y_3 - \beta_2\Delta x_3 \\ 0 & 0 \end{bmatrix} \quad (A.6)$$

$$\begin{aligned} \varepsilon_x = & \Delta x_1 + \Delta x_2 + \Delta x_3 - \alpha_1 y + \beta_1 z + \beta_2 z + \\ & \alpha_1 \gamma_2 z - \alpha_1 \Delta y_2 + \beta_1 \Delta z_2 - \alpha_1 \Delta y_3 + \\ & \beta_1 \Delta z_3 - \alpha_2 \Delta y_3 + \beta_2 \Delta z_3 \end{aligned} \quad (11)$$

$$\begin{aligned} \varepsilon_y = & \Delta y_1 + \Delta y_2 + \Delta y_3 - \gamma_1 z - \gamma_2 z + \alpha_1 \beta_2 z + \\ & \alpha_1 \Delta x_2 - \gamma_1 \Delta z_2 + \alpha_1 \Delta x_3 - \gamma_1 \Delta z_3 + \\ & \alpha_2 \Delta x_3 - \gamma_2 \Delta z_3 \end{aligned} \quad (12)$$

$$\begin{aligned} \varepsilon_z = & \Delta z_1 + \Delta z_2 + \Delta z_3 + \gamma_1 y - z\beta_1\beta_2 - z\gamma_1\gamma_2 - \\ & \beta_1 \Delta x_2 + \gamma_1 \Delta y_2 - \beta_1 \Delta x_3 + \gamma_1 \Delta y_3 - \\ & \beta_2 \Delta x_3 + \gamma_2 \Delta y_3 \end{aligned} \quad (13)$$

Zero diagonal elements in Equation 10

In Equation 10 of the article, a transformation ΔE is presented that describes the errors in the relative orientation and position of the tool with respect to its reference. It is more consistent with the general notation of Equation 10 to denote this transformation as δE . A more serious error is that the first three diagonal elements of this transformation are presented as zeros. This is true, in case a first-order approximation is used in the derivation of this matrix. In this article, however, second-order terms are included in the model. The result is that the first three diagonal components of the transformation ΔE are not equal to zero, as can be deduced from the revised Equations A.4–A.6.

Independency δ_x , δ_y , and δ_z

In Equations 16–18 the authors introduce the probability density functions $f_1(\delta_x)$, $f_2(\delta_y)$, and $f_3(\delta_z)$ of the random components δ_x , δ_y , and δ_z in the position error of the tool. In the second paragraph of the first column of page 143, the authors state that these functions are independent. This statement is used to calculate the density function of δ_e (the random component in the distance between the actual and nominal position of the tool) as the product of the density functions $f_1(\delta_x)$, $f_2(\delta_y)$, and $f_3(\delta_z)$. The subsequent part of the article is largely based on this result. (Note that the calculation of the average positional error μ_e according to Equation 19 of the article is a statistical first-order approximation only valid if the random errors are small compared to the systematic errors.)

Unfortunately, the random errors δ_x , δ_y , and δ_z are not mutually independent. This can be easily deduced from Equations 11–13. Consider for example the roll error γ_1 introduced in the first axis x_1 of the machine tool. As can be seen in *Figure 4* of the article, this error has an effective arm in both y (axis x_2) and z (axis x_3) direction. It therefore affects the position error of the tool in both z and y direction, as can be seen in Equations 12 and 13. As with many of the angular errors, the random component of this error introduces a random component in the position of the tool in more than one coordinate direction. The statement made by the authors that the random errors in the x , y , and z directions of the tool are mutually independent is therefore not true, which severely undermines the remainder of the article. This situation is aggravated if a relatively small number of angular errors dominate the ‘random’ errors of the machine tool.

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