Nonlinear parametric identification using periodic equilibrium states : application to an aircraft landing gear damper

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Nonlinear Parametric Identification Using Periodic Equilibrium States

Application to an Aircraft Landing Gear Damper

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Abstract. The subject of this paper is the development and assessment of a new nonlinear parametric identification method for dynamic systems using periodic equilibrium states or outputs. The method consists of a modified Bayesian point estimation technique which can be used in combination with a time discretization method or a shooting method to obtain the periodic equilibrium states. It is assumed that the specific excited and measured periodic solution can be computed directly from a static initial guess. An important feature of this estimator is the possibility to estimate the best parameters based on all experiments of the complete experimental set-up. The choice of using periodic states appears to be computationally efficient compared to using transient states. The new method is applied to multiple sets of nonlinear shaker-test measurements of a uniaxially loaded F-16 nose landing gear damper, for which a standard 1 dof mechanical model and a more comprehensive 2 dof thermo-mechanical model are postulated. Finally, the predictive power of the method is assessed by comparison of predictions for a transient drop-test load case of the 'best' 2 dof model with corresponding parameters and real drop-test measurements.

Key words: Periodic states, time discretization, identification, system modelling, landing gear.

Notation

$a_{,b}$	partial derivative of a with respect to vector b
$\dot{a}$	total derivative of a with respect to time t
a'	total derivative of a with respect to the dimensionless time τ
$\hat{a}$	predicted value of a
$\bar{a}$	mean value of a
$\tilde{a}$	approximate equations a
e	residuals
f	output equations
f_e	external excitation frequency
F	force
g	ordinary differential equations
l	number of parameters
n	number of samples in each experiment
N	number of experiments
$\mathcal{N}_a(b, C)$	a -dimensional normal distribution (mean b and covariance C)
p	pressure
q	degrees of freedom
s	number of outputs
t	time
T	absolute temperature

Fig. 1. Objective of research drop-test versus shaker-test.

T_e	period of the excitation and periodic solution
v	measurement error variance vector
w	parameter variance vector
y	output variable vector
z	discrete periodic solution

α	parameter label
β	output label
ϑ	model parameter vector
κ	experiment label
μ	sample label
σ	standard deviation
τ	dimensionless time
Φ	object function.

1. Introduction

For landing gear manufacturing, airworthiness authorities have defined a drop-test procedure for the certification of a landing gear. These tests will produce transient signals, which resemble the operational conditions of the system during a landing. The signals could also be used for identification of prototypes. However, because of expected low applicability for future design optimization – impact is only one of the design criteria – research was initialised on an alternative shaker-test procedure.

In linear dynamics it is common practice to identify the system under consideration by measurement of *harmonic* shaker inputs and corresponding *harmonic* outputs. Analogue to this procedure, it is believed that a new alternative shaker-test can be realized by application and measurement of *periodic* shaker inputs and measurement of the corresponding *periodic* equilibrium outputs in order to identify nonlinear dynamic systems. Both methods are illustrated in Figure 1.

In literature only one monograph by Batill [2] was found that stressed the need of parameter estimators for the identification of landing gears. Batill used the well-known nonlinear unweighted least squares estimator and time integration for transient response computations. The lower half of Figure 2 shows the conventional drop-test identification procedure.

Fig. 2. Research strategy, drop-test and alternative shaker-test computations and experiments.

In the upper branch of Figure 2 the alternative shaker-test procedure is illustrated. Starting with the periodic excitation signals, the shaker-test can be performed and the state vector, some of its components, or output signals can be measured. The periodic inputs together with approximate parameters can also be input to a postulated mathematical model and by calculation of periodic solutions the measured response can be predicted. An advantage of this approach is that it will drop the need to estimate unknown initial conditions. Based on the discrepancy between the measured and predicted output, parameter estimation procedures can be used to improve the approximate parameter vector.

The object of research has been constrained to modelling and identification of aircraft landing gear dampers, because of the uncertainties in their physical nonlinearities. In contrast, the geometrical nonlinearities of the complete gear and the linear fuselage dynamics can easily be modelled by multi-body dynamics codes. However, the accuracy of these simulations depends strongly on the correctness of the modelled physical nonlinearities of the landing gear dampers. So, the 'best' damper model and identified parameters should be able to predict the outcome of an additional transient load case, in this project a drop-test. If this ultimate test is passed, the 'best' model can be used with confidence in multi-body aircraft landing or ground operation simulations.

2. Parametric Estimation of Dynamic Systems

The differences between identification methods used in literature mainly depend on the amount of prior knowledge available and can be classified in three groups, see Verbeek [11].

- Detection of nonlinearities and extension of linear modal analysis to weakly nonlinear dynamic systems, see, e.g. Popp [7], Blohm [3], or Ewins [4].
- Nonparametric methods, which rely on series expansions in both the time domain as well as the frequency domain, see, e.g. Tomlinson *et al.* [9, 14], or Thouverez and Jezequel [10].
- Parametric identification. For off-line applications, a parametric identification of a landing gear damper is reported by Batill [2]. On-line applications are more often reported for control problems. For structural dynamics, Seibold and Fritzen [8] applied the extended Kalman filter.

For this project, an off-line parametric method based on assumed physical models has been selected. This choice had to be made, because the results of the identification procedure should be applicable as input to a subsequent design optimization process. It is assumed that the deterministic mathematical models for structural systems can be derived by using physical laws.

2.1. PERIODIC EQUILIBRIUM STATES

A periodic solution technique for the uncertain ordinary differential equations (1) of structural systems g in the degrees of freedom q , and corresponding output equations (2) is used

$$g(q, \dot{q}, \ddot{q}, F(f_e), t, \vartheta) = 0, \quad (1)$$

$$\hat{y} = f(q, \dot{q}, \ddot{q}, F(f_e), t, \vartheta). \quad (2)$$

The l -dimensional vector ϑ in both sets of equations stands for the uncertain model parameters. The external forcing frequency is f_e (period $T_e = 1/f_e$) and F stands for the measured periodic excitation force. In this paper the basic principle will be outlined briefly. It is assumed that the periodic solutions can be computed from initial approximations without the use of path-following techniques, bifurcation or stability analyses, as reported by Fey [5]. It is assumed that a periodic solution exists and will be harmonic. If a cyclic time variable τ is used

$$\tau \equiv f_e t, \quad \tau \in [0, 1], \quad (3)$$

the sets of equations (1) and (2) can be rewritten to

$$g(q, q', q'', F, \tau, \vartheta) = 0, \quad (4)$$

$$\hat{y} = f(q, q', q'', F, \tau, \vartheta). \quad (5)$$

The prime (') stands for differentiation with respect to the cyclic dimensionless time τ .

In order to compute the predicted output values, an approximate periodic response can be obtained by application of a discretization procedure. In this paper only equidistant schemes are used. Discretization of the time variable τ on n equidistant time steps gives

$$\tau_\mu = (\mu - 1) \Delta\tau, \quad \mu \in \{1, 2, \dots, n\}, \quad (6)$$

$$\Delta\tau = 1/n.$$

According to this time grid, all time dependent variables are discretized as well and will be subscripted by μ on time step τ_μ . The discretized degrees of freedom q_μ can be stored in a column z

$$z \equiv (q_1^T, \dots, q_n^T)^T. \quad (7)$$

Subsequently, the derivatives of the discretized degrees of freedom q_μ can be approximated by any equidistant difference scheme. The frequently used second or fourth order central difference schemes have been applied successfully for many problems. However, they occasionally appeared to suffer from numerical instability for the applications in this paper, i.e. the landing gear models proposed in the sequel. As numerical stability analyses hardly can be performed for nonlinear difference equations, and moreover the equations themselves change during the estimation process, the strategy has not been to prove numerical stability. For the applications

reported in this paper, the problem is sufficiently solved by trial and error techniques, but for general use it stays a topic for further research.

For both derivatives all combinations of second order central and first or second order backward schemes have been tested for the application of this paper. It could be concluded that the second order backward schemes for both the first and second derivatives appeared to perform best (i.e. showed no occurrence of numerical instabilities), of course for sufficiently fine time grids ($n = 800$ is applied for all forthcoming computations). The second order backward schemes used in this paper read

$$q'_\mu = \frac{3q_\mu - 4q_{\mu-1} + q_{\mu-2}}{2\Delta\tau} + O(\Delta\tau^2), \quad (8)$$

$$q''_\mu = \frac{2q_\mu - 5q_{\mu-1} + 4q_{\mu-2} - q_{\mu-3}}{\Delta\tau^2} + O(\Delta\tau^2), \quad (9)$$

with boundary conditions due to periodicity

$$q_i = q_{i\pm n}, \quad i \notin \{1, 2, \dots, n\} \quad \wedge \quad i \pm n \in \{1, 2, \dots, n\}. \quad (10)$$

After substitution of the difference schemes (8) and (9) in equations (4) and (5), followed by the elimination of the boundary conditions (10) and making use of the column z (7), on each time step τ_μ a set of approximate difference and discrete output equations can be derived

$$\tilde{g}_\mu(z, x_\mu, \tau_\mu, \vartheta) = 0, \quad (11)$$

$$\hat{y}_\mu = \tilde{f}_\mu(z, x_\mu, \tau_\mu, \vartheta). \quad (12)$$

For all time steps a set of nonlinear algebraic equations can be defined by

$$\tilde{g}(z, x, \tau, \vartheta) \equiv [\tilde{g}_1^T, \tilde{g}_2^T, \dots, \tilde{g}_n^T]^T = 0, \quad (13)$$

which is solved for the discrete periodic solution z from an initial static equilibrium guess by a modified Newton procedure, making use of the decomposition technique and robust step halving technique as proposed by Fey [5]. Substitution in the discretized output equations (12) will give the requested output values $\hat{y}_{\mu\kappa}$.

For the Bayesian estimator used in this paper, the required first derivatives of the outputs to the model parameters can be derived

$$\frac{d\hat{y}_{\mu\kappa}}{d\vartheta} = \left(\tilde{f}_{\mu\kappa, \vartheta} + \frac{d\tilde{f}_{\mu\kappa}}{dz} z_{, \vartheta} \right). \quad (14)$$

The derivatives $\tilde{f}_{\mu\kappa, \vartheta}$ and $d\tilde{f}_{\mu\kappa}/dz$, of the output equations to the model parameters and periodic solution respectively, can be derived by hand or by symbolic computations. For the computation of the remaining partial derivatives $z_{, \vartheta}$ of the periodic solution to the model parameters the sensitivity equations can be used. Linearization of the set nonlinear algebraic equations (13) about a periodic trajectory belonging to an approximate parameter vector gives

$$\frac{d\tilde{g}}{d\vartheta} = \tilde{g}_{, \vartheta} + \frac{d\tilde{g}}{dz} z_{, \vartheta} = 0. \quad (15)$$

The partial derivatives $\tilde{g}_{, \vartheta}$ should be derived by hand or by symbolic computations from the differential equations. The derivatives $d\tilde{g}/dz$ are already computed and can efficiently be extracted from the last Newton iteration of the periodic solver for equation (13).

2.2. BAYESIAN ESTIMATION

The parametric estimation problem is solved by a Bayesian estimator, which is capable of estimating the model parameters as well as the still unknown distribution parameters of the residuals, see Bard [1]. This estimator consists of a combination of the residuals

$$e_{\mu\kappa}(\vartheta) \equiv y_{\mu\kappa} - f_{\mu\kappa}(q, \dot{q}, \ddot{q}, F(f_e), t, \vartheta) = y_{\mu\kappa} - \hat{y}_{\mu\kappa}, \quad (16)$$

which are assumed to be normally distributed $\mathcal{N}_s(0, \text{diag}(v))$ and are defined as the difference between the measured outputs y and the predicted outputs $\hat{y}$, and normally distributed $\mathcal{N}_l(\bar{\vartheta}, \text{diag}(\sigma^2))$ a priori knowledge of the model parameters.

The measurement errors and modelling errors are not distinguished and the total errors are assumed to be statistically independent over the n samples μ , the N experiments κ , and the s measurement channels β . Now the Bayesian estimation problem can be defined as a maximization problem

$$\begin{aligned} \Phi_B(\varphi) &= \Phi_B(\vartheta, v_1, \dots, v_s) = \\ &= -\frac{nN}{2} \ln \prod_{\beta=1}^s v_{\beta} - \frac{1}{2} \sum_{\beta=1}^s \frac{M_{\beta\beta}(\vartheta)}{v_{\beta}} - \frac{1}{2} \sum_{\alpha=1}^l \frac{(\vartheta_{\alpha} - \bar{\vartheta}_{\alpha})^2}{\sigma_{\alpha}^2} \end{aligned} \quad (17)$$

with

$$M_{\beta\beta}(\vartheta) = \sum_{\kappa=1}^N \sum_{\mu=1}^n e_{\mu\kappa\beta}^2(\vartheta),$$

in which $M_{\beta\beta}(\vartheta)$ stands for the diagonal terms of the moment matrix of the residuals.

The estimation problem is solved iteratively by Newton-Gauss approximation of the Taylor series of the object function (17) and by staged optimization. Firstly, from (17) the unknown measurement channel variances can be solved with bias correction for each iteration

$$v_{\beta} = \frac{s}{nNs - l} \sum_{\kappa=1}^N \sum_{\mu=1}^n e_{\mu\kappa\beta}^2. \quad (18)$$

Secondly, a modified Bayesian estimator for the uncertain model parameters ϑ can be derived by the assumption that the prior knowledge of the parameters is reset during all iterations i . The model parameter updating formula reads

$$\begin{aligned} \Delta\vartheta_i &= (-H_{\vartheta\vartheta}^{-1} q_{\vartheta})_i = -\left[\sum_{\kappa=1}^N \sum_{\mu=1}^n e_{\mu\kappa,\vartheta}^T \text{diag}(v)^{-1} e_{\mu\kappa,\vartheta} + \text{diag}(w)^{-1} \right]_i^{-1} \times \\ &\quad \times \left(\sum_{\kappa=1}^N \sum_{\mu=1}^n e_{\mu\kappa,\vartheta}^T \text{diag}(v)^{-1} e_{\mu\kappa} \right)_i \end{aligned} \quad (19)$$

with

$$w = (\sigma_1^2, \dots, \sigma_l^2)^T, \quad (20)$$

in which $H_{\vartheta\vartheta}$ stands for the ϑ -partition of the Hessian of the object function (17) and q_{ϑ} is the corresponding partition of the gradient. The first order derivatives $e_{\mu\kappa,\vartheta} = -\dot{\hat{y}}_{\mu\kappa,\vartheta}$ can be computed efficiently from a large set of periodic algebraic sensitivity equations (14) and (15),

Fig. 3. Schematic view of the shaker-test set-up, internal test-frame dimensions: 2.78 m $\times$ 0.78 m.

at least for low degree of freedom systems by symbolic algebra and sparse matrix algebra wherever possible.

Despite the fact that prior knowledge of the model parameters is taken into account, which should constrain the parameter update vector (19), sometimes an additional linear search is necessary in order to obtain convergence. For more information on the estimator, the solution procedure or the computation of derivative information the reader is referred to Verbeek [11].

The modified Bayesian estimator in combination with the time discretization method has been implemented in FORTRAN statements with application of NAG-routines making use of the band matrices and sparse matrices where possible. The derivatives of the model equations, with respect to the model parameters and the degrees of freedom and their time derivatives, are obtained in optimized FORTRAN code from the symbolic manipulation program MAPLE.

A drawback of equidistant time grids is the expected undesirable behaviour for discontinuous systems. Aircraft landing gears for instance, can display extreme friction, sudden impacts, or very stiff end-stops, which can lead to system models which will possess trajectories with very large gradients. A solution for this problem is application of shooting techniques on the cost of high CPU-times, see Verbeek [11]. All computations for the application in this paper could be performed satisfactorily with the time discretization technique.

3. Experimental Set-Up

This technique will be illustrated for measurements on a simplified F-16 nose landing gear damper under periodic excitation. A schematic view of the experimental set-up for these tests is illustrated in Figure 3. With suitable hydraulics, periodic axial forces or displacements can be applied to the landing gear. With this set-up measurements are made on: the axial stroke, the force applied to the wheel axle, and the internal gas pressure of the damper. All signals are filtered and sampled by a PC-based four channel 16 bit data acquisition system. The actual identification computations are performed by a FORTRAN code on a SGI workstation.

Not only due to the limited hydraulic power, but also for reasons of efficiency, a set of experiments has to be defined that should cover the total available system state and parameter

Table 1. Maximal dynamic displacement amplitudes in mm, for the static position groups high, middle, and low.

static position		frequency Hz					
mm	group	0.04	0.1	0.2	0.5	1.0	2.0
249	high						10
229	high					20	
219	high				32		
179	high			75			
133	middle	127	127	78	34	21	10
79	low			80			
39	low				35		
17	low					22	
14	low						10

Table 2. Labels of the experiments.

date: 920713		approximate frequencies Hz					
group		0.04	0.1	0.2	0.5	1.0	2.0
high				c	d	e	f
middle	a	b	g	h	i	j	
low			k	l	m	n	

space. A choice is made for three static piston positions: high, middle, and low, to which the dynamic amplitudes are summed.

Subsequently, the selection of input parameters can be made. The frequencies selected are: 0.04 Hz and 0.1 Hz, i.e. the highest frequency for which full dynamic stroke can be realized (264 mm), and 0.2, 0.5, 1.0, and 2.0 Hz with decreasing amplitudes as shown in Table I. Each of the 14 experiments is labelled by their date and a single character as defined in Table II.

The wave form could be chosen sinusoidal or triangular. Former research on an electronic Duffing system showed that the triangular signal is preferable for identification, as it contains multiple frequencies, see Verbeek [12]. The triangular signals have been measured but are discarded, because they unnecessarily excited the complete test frame. Moreover, the force discontinuities according to the velocity jumps are summed to the force discontinuities due to dry friction, which is disadvantageous for identification. Therefore, the sinusoidal wave form has been selected, because the displacement signals and their first derivatives are continuous, resulting in smooth running experiments.

The time histories of all measurements will be illustrated by two typical examples, 920713b and 920713m, shown in Figures 4 and 5 respectively. Both experiments show nearly perfect sinusoidal displacement signals, the characteristic discontinuous force signals, and smooth relative gas pressure signals. The right hand experiment shows significant force discontinuities at 0.5 and 1 s (friction). For the large amplitude experiment these frictional jumps are present as well, but can hardly be seen in the force plot at 3 and 8 s due to the different scaling.

Fig. 4. Experiment 920713b1, frequency 0.1 Hz, full amplitude. (± 120 mm)

Fig. 5. Experiment 920713m1, frequency 1 Hz, small amplitude. (± 18 mm)

4. Identification Results (1 Dof)

The very basic model for an aircraft landing gear damper has already been proposed in 1953 by Milwitzky and Cook [6]. This model is still in use as the basis for many landing gear and aircraft landing simulation codes. Batill and Bacarro [2] used the model for identification in 1988. This 1 dof mechanical model, which is sketched in Figure 6, includes inertia, velocity squared oil damping, the nitrogen gas spring pressure approximated by polytropic ideal gas behaviour, and a continuous approximation of Coulomb friction. The equation of motion for the basic damper and the output equation can be written as

Fig. 6. The basic F-16 damper.

$$g = \vartheta_1(\ddot{q} + \vartheta_2) + \vartheta_3 \dot{q} |\dot{q}| + \vartheta_4 p + (\vartheta_8 + \vartheta_9 p) \arctan(\vartheta_{10} \dot{q}) + \vartheta_{11} - F_e = 0 \quad (21)$$

with

$$p = \vartheta_5 \left(\frac{1}{1 + \vartheta_6 q} \right)^{\vartheta_7}$$

$$\hat{y} = q + \vartheta_{12}, \quad (22)$$

in which p is the *absolute* gas pressure. Unfortunately it is not possible to estimate all parameters from a single estimation. The velocities in the periodic experiments are a factor 10 too small compared to realistic sink speeds (due to hydraulic power limitations), so the damping parameter ϑ_3 cannot be estimated from these specific periodic experiments.

The friction force is modelled by an arctan-function with large derivatives, therefore it is difficult to estimate the parameters ϑ_8 to ϑ_{10} . For the identifications, these parameters are extracted directly from the data. For each experiment the friction arctan-function is fitted to the measured force discontinuities (see, e.g., Figure 5) manually. This leads to experiment dependent approximations for the parameters ϑ_8 to ϑ_{10} as function of the absolute pressure p which can be used as fixed values in the identifications, see Figure 7. For the final predictions average values for parameters ϑ_8 , ϑ_9 , and ϑ_{10} must be obtained. The average values can be computed from a linear fit through the friction force amplitude data, see Figure 7.

The three most uncertain parameters, ϑ_4 , ϑ_6 , and ϑ_7 have been estimated. Not all estimations converged for the case that the parameter vector was *estimated from a single measurement*. All converged estimations showed acceptable fits, despite the fact that the parameters ϑ_4 and ϑ_6 are by far not constant over the experiments. Moreover, the gas exponent ϑ_7 did not meet the assumption that it should be in the range of isothermal to adiabatic processes ($1 < \vartheta_7 < 1.4$).

The three parameters can also be *estimated from the complete standard set-up* in a single estimation. The parameters stayed close to their initial guesses and the gas exponent ϑ_7 was in the assumed admissible range. The mean values of the residuals and superposed 2σ intervals for each experiment are given in Figure 8, from which it can be concluded that the nonzero mean values are dependent on the group (high, middle, low) to which the experiments belong. In other words the assumed polytropic gas spring model is not perfect over the total stroke, and should therefore be improved or extended.

Fig. 7. Linear polynomial fit of the friction force amplitudes, for the standard experimental set-up.

5. Additional Experiments

Re-evaluation of the data showed thermodynamic behaviour even for the almost static 0.04 Hz experiment (920713a). The pressure-displacement plot clearly shows heat loss to the environment, see Figure 9. For all other frequencies of the experimental set-up, this thermodynamic behaviour is observed as well.

It has also been observed that a combination of the following potential phenomena: gas solubility or gas/oil mixing, oil compression, cylinder expansion, additional gas pressure measurement volume, and cavitation, is present. Static stepwise experiments have been performed (10 steps), for which the amount of gas n can be computed if it is assumed that the gas temperature T equals the environmental temperature and the realistic ideal gas law $pV = nRT$ is valid. From these measurements a perfect, but nonrealistic, linear gas 'solubility' relation without saturation can be found, see Figure 10. Nevertheless, this phenomenological model can be used to cover all five suggested phenomena.

In order to cover the total experimental set-up, the 1 dof model should be improved. The extensions that are believed to make the model more accurate are: addition of simple thermodynamics for the gas spring, also proposed by Wahi [13], and the observed gas 'solubility'.

6. Identification Results (2 Dof)

If a single dof model for the thermo-dynamics can be assumed to be applicable, the total 2 dof thermo-mechanical model consists of the equation of motion and the first law of thermodynamics (23) and the output equation (24) in which only ϑ_5 and ϑ_7 are redefined and T is

Fig. 8. Mean values and superposed 2σ intervals of the residuals for the 1 dof model. Experiments 920713 a to n.

Fig. 9. Relative gas pressure-displacement diagram.

Fig. 10. Gas 'solubility'.

the absolute temperature. The thermo-dynamical model includes a gas heat capacity, Fourier heat conduction through a cylindrical wall, and a source term due to gas compression and extension. The phenomenological gas 'solubility' model is implemented in the pressure stroke temperature p , q , T ideal gas constitutive equation

$$g_1 = \vartheta_1(\ddot{q} + \vartheta_2) + \vartheta_3 \dot{q} |\dot{q}| + \vartheta_4 p + (\vartheta_8 + \vartheta_9 p) \arctan(\vartheta_{10} \dot{q}) + \vartheta_{11} - F_e = 0 \quad (23)$$

$$g_2 = \vartheta_{15} \dot{T} + \vartheta_{16}(1 + \vartheta_6 q)(T + \vartheta_{17}) - (\vartheta_4 p + \vartheta_{11}) \dot{q} = 0$$

with

$$p = \frac{\vartheta_5 T}{1 + \vartheta_6 q + \vartheta_7 T}$$

$$\hat{y} = q + \vartheta_{12} . \quad (24)$$

In order to make a fair comparison between the 1 and the 2 dof model, the number of parameters to be estimated, is chosen the same. Fortunately both models share identical parameters, from which ϑ_4 and ϑ_6 will be kept in the set that will be estimated. It should be noted that these parameters are present in both the equation of motion and the energy balance equation. The third parameter that will be estimated is the most uncertain parameter in the energy balance equation, ϑ_{16} , dealing with heat conduction.

The results from the *estimations using single experiments* can be summarized as follows. On the average, the residuals for the large amplitude excitations of this model are a factor two smaller than the residuals for the mechanical model. Secondly, the spreads in the geometrical parameters ϑ_4 and ϑ_6 are much smaller and acceptable. For this model, the heat conduction parameter ϑ_{16} is far from constant, and for some small amplitude inputs (experiments j, m, n) the parameter is even negative. It can be concluded that the heat transfer is still not modelled accurately enough by application of simple Fourier heat conduction through a cylindrical wall.

For the case that the three parameters are *estimated from the complete standard set-up* in a single estimation, it appears that the parameters differ more from the initial parameters than for the single dof model, but their mean values and superposed 2σ intervals for each experiment are on the average a factor two smaller than the corresponding results for the 1 dof model (compare Figure 11 and Figure 8).

It can be concluded that the 2 dof thermo-mechanical model is superior to the commonly used 1 dof mechanical model. So, the prediction of the drop-test measurements in the next section is performed with the 2 dof model and the estimated parameters over the complete experimental set-up.

7. Drop-Test Prediction

For the assessment of the predictive power, and thus feasibility of the shaker-test identification method a small number of drop-tests has been performed. The laboratory experimental set-up is a simplified version of a realistic drop-test facility, which is as close as possible to the performed shaker-tests, according to the uniaxial loading as well as the instrumentation. See Figure 12 for a schematic view. The measurements start when a mass is suddenly set free (by a so-called parachute release device), loading the damper.

The most important experiment (930426c) is a representative landing load case, a mass load of 350 kg has been selected. The maximal velocity for this experiment is about 0.5 m/s, which already is five times as high as for the periodic measurements.

The measurements and predicted outputs for the representative landing experiment 930426c are shown in Figure 13. The measurements are plotted with solid lines and the predictions with dashed lines.

At first sight, it can be observed that the shapes of the signals are very similar. Both the measured and predicted force signals show friction discontinuities whenever the displacement signal is reaching an optimum. Differences that can be observed in the force signals are, the bending point on the middle of the first compression slope of the measured force signal, and the small oscillations in the end-value of the predicted force signal. These small oscillations

Fig. 11. Mean values and superposed 2σ intervals of the residuals for the 2 dof model and experiments 920713 a to n.

Fig. 12. Schematic view of the laboratory drop-test set-up. Dimensions between the joints: landing gear height 1.16 m, width 0.58 m, pivoting frame length 2.02 m.

are caused by the continuous arctan friction function and can be ignored. Further it can be seen that the predicted displacement seems to end in a static offset error, the period time of all three predictions is too small, and all predicted signals are clearly under-damped.

Adjustment of the damping parameter ϑ_3 (that could not be estimated) by a factor 2.0, gives a good fit for both the force and pressure measurements, see Figure 14. Note that now, due to increased damping forces, the curvature in the first compression slope of the force output, is present in the predicted force signal. The amplitudes and the frequency of the

Fig. 13. Predictions for experiment 930426c0.

Fig. 14. Predictions for experiment 930426c0. The damping is increased by a factor 2.0.

oscillations in all three signals are predicted very well, but still the displacement signal shows a large static offset component. However, at 2.5 s the gas temperature is not yet equal to the environmental temperature. Simulation for a longer time interval (see Figure 15) shows that the static equilibrium value is predicted correctly after approximately 30 s, since then the nitrogen gas has cooled down to the environmental temperature.

Despite the good overall predictions for the force and pressure signals, it must be concluded again that the thermo-dynamic modelling probably is over-simplified.

8. Discussion and Conclusions

A new nonlinear identification method for dynamic systems using multiple sets of periodic equilibrium states or outputs has been derived and successfully applied to a landing gear damper.

It can be concluded that identification by using periodic outputs is computationally more efficient than identification with application of transient signals. For transient states the com-

Fig. 15. Long time interval prediction of the displacement of experiment 930426c0.

plete new solution has to be integrated from the initial conditions for all iterations again. Not only the initial conditions can be dropped in the periodic approach, but the complete approximate periodic solutions computed during the estimation iterations can be re-used as an initial guess for the solution of the next iteration, which speeds-up convergence.

From the identifications performed for the 1 and 2 dof models on single and multiple periodic experiments, it can be concluded that the 2 dof thermo-mechanical model is the best and an acceptable model. It can also be concluded that the thermo-dynamical part probably is not modelled accurately enough. So, model improvements can be found in further investigations in the thermo-dynamic behaviour of the damper, which consists of additional temperature measurements and addition of heat capacities for the oil and the aluminium wall (at the cost of additional dofs and model parameters).

For R&D the shaker-test appears to be superior to the drop-test as numerous excitations can be forced to the dynamic system. For example, detailed studies could be performed to develop the physical model on isolated phenomena (e.g. gas 'solubility', and thermo-dynamic behaviour for the damper model).

Finally, the idea that the shaker-test (in combination with the new identification procedure) can be used in the future as a quality control instrument, was consolidated during the research.

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References

1. Bard, Y., *Nonlinear Parameter Estimation*, Academic Press, 1974.
2. Batill, S. M. and Bacarro, J. M., 'Modeling and identification of nonlinear dynamic systems with application to aircraft landing gear', in *29th AIAA, Structures, Structural Dynamics and Materials Conference*, Williamsburg, VA, pp. 871–884, Apr. 18–20, 1988.
3. Blohm, P. W., 'Anwendung der Hilbert-Transformation zur Erkennung von Nichtlinearitäten', *Z. angew. Math. Mech.* **69** (5), 1989, T 345–T 347.
4. Ewins, D. J., 'Influence of nonlinearities on modal tests', in *International Symposium on Identification of Nonlinear Mechanical Systems from Dynamic Tests – Euromech 280*, Ecully, pp. 3–11, Oct. 29–31, 1991.
5. Fey, R. H. B., *Steady-State Behaviour of Reduced Dynamic Systems with Local Nonlinearities*, Ph.D. thesis, Eindhoven University of Technology, The Netherlands, 1992.

6. Milwitzky, B. and Cook, F. E., 'Analysis of landing gear behaviour', Technical Report 1154, NACA Langley Aeronautical Laboratory, 1953.
7. Popp, K., 'On the detection of nonlinearities', in *Third German-Japanese Seminar on Nonlinear Problems in Dynamical Systems*, Ebni, pp. 3.0–3.15, Aug. 3–4, 1987.
8. Seibold, S. and Fritzen, C. P., 'Recursive identification of nonlinear systems by means of the extended Kalman filter', in *9th International Modal Analysis Conference, Florence*, pp. 802–809, IMAC, 1991.
9. Storer, D. M. and Tomlinson, G. R., 'Higher order frequency response functions and their relation to practical structures', in *9th International Modal Analysis Conference, Florence*, pp. 1206–1214, IMAC, 1991.
10. Thouverez, F. and Jezequel, L., 'Identification of non-linearity joints from dynamical tests', in *9th International Modal Analysis Conference, Florence*, pp. 1583–1588, IMAC, 1991.
11. Verbeek, G., *Nonlinear Parametric Identification Using Periodic Equilibrium States, with Application to an Aircraft Landing Gear Damper*. Ph.D. Thesis, Eindhoven University of Technology, The Netherlands, 1993.
12. Verbeek, G., de Kraker, A., and van Campen, D. H., 'Identification of mechanical systems by means of periodic excitation', in *Symposium on Identification of Nonlinear Mechanical Systems from Dynamic Tests – Euromech 280, Ecully*, pp. 75–82. Balkema, Rotterdam, Oct. 29–31, 1991.
13. Wahi, M. K., 'Oil compressibility and polytropic air compression analysis for oleopneumatic shock struts', *Journal of Aircraft* **13**(7), 1976, 527–530.
14. Worden, K., and Tomlinson, G. R., 'An experimental study of nonlinear SDOF systems using the restoring force method', in *9th International Modal Analysis Conference, Florence*, pp. 757–764, IMAC, 1991.