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Analysis of the interaction between a dipole antenna and a large object using the Method of Moments and the Finite Difference Time Domain technique

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Abstract

In small communication products like pagers and cellular phones the antenna size becomes small and the sizes of the adjacent objects are much larger than that of the antenna element. The power from the antenna is reflected, absorbed and scattered by these objects. This effect is called the body effect in the case of a human body or the proximity effect when another object is near the antenna.

This project aims to analyse the interaction between the antenna and a large object. The antenna is modeled by a half-wave dipole antenna, because many calculations on this type of antenna can be done analytically. Also this type of antenna can easily be made for verification by experiments. The characteristics of the dipole antenna are comparable to those of antennas that are used on portable communication equipment. The objects are large compared to the considered wavelength and are modeled by large, plane plates. The considered wavelength is 12 cm (2.5 GHz).

The analysis is done with three calculation methods: analytical and two different types of computational methods. One computational method is the Method of Moments and the other is the Finite Difference Time Domain technique. The applicability of these methods is tested in this project for several configurations.

The main conclusion of this study may be formulated as follows. The matching of the antenna to the radio circuit is satisfactory when the distance between the antenna and a perfectly conducting ground is at least \(0.27\) times the wavelength. When a wall of brick is used, the matching becomes better. The effects of human tissues with respect to matching resemble those of a perfectly conducting ground. The case of a perfectly conducting ground has been found to be a worst case situation.

The issues that are discussed in this study refer mainly to the properties that can be measured at the feed-points of the antenna. Radiation patterns are analysed just briefly. Also the calculation of absorption by the objects is still case of study.

Keywords

antenna, Method of Moments, Finite Difference Time Domain, proximity effect, body effect, return loss, transmitter, dipole, receiver, matching, radiation resistance, antenna impedance.
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Chapter 1

Introduction

In small communication products like pagers and cellular phones the antenna size becomes small and the size of the adjacent conducting materials (batteries, interconnection) is comparable to that of the antenna element. In this situation the antenna element can not be separated electromagnetically, so that the radiation current flows on the adjacent conducting materials, and this effects the antenna performance significantly. This effect is called the proximity effect.

For portable communication products the antenna is located very near to a human body, the energy from the antenna is reflected, absorbed and scattered by the human body. This effect is called the body effect.

In this project the proximity and body effects will be analysed to get an insight into the interaction of electrically small antennas with their environment. From this insight general applicable design rules and design techniques can be extracted.

The used antenna is a half wavelength dipole antenna. One reason for this choice is that this type of antenna can be modeled easily and reliably. Many calculations on this type of antenna can be done analytically with this model. Furthermore, this antenna can be made in practice easily, so that the calculations can be verified by experiments.

The objects will be modeled by infinitely large plates. For example, a wall of an office or a bureau has a plane and large surface, compared to the considered wavelength (12 cm at 2.5 GHz). The human body is modeled by the same plates, with other values of the permittivity and conductivity.

In this study, only the effects of nearby objects on the behaviour at the antenna feed-points are investigated. An antenna is perfectly matched to a transmitter when the impedance of the antenna is the complex conjugate of the output impedance of the transmitter. In that case the maximum available power from the transmitter is delivered to the antenna and no power is reflected, i.e. the antenna is perfectly matched to the transmitter. An object in the proximity of the antenna will cause a change in the impedance of the antenna, which results in a reduction of the transmitted power and therefore in a reduction of the efficiency of the communication system.

Chapter 2 reveals briefly what is written in the literature about the coupling between the antenna and the body. Next an overview of some numerical simulation methods will be given. From these methods the Finite Difference Time Domain method is studied more in depth.

Before investigating the influences of dielectric grounds, Chapter 3 outlines some theory about the half-wave dipole antenna in a free space environment. Analytical calculations are compared to the results of the computer program 'Numerical Electromagnetics Code', based on the Method of Moments (MoM). Next the effects of an infinitely large and perfectly conducting ground plane on the performance of a half-wave dipole are investigated analytically and with the aid of NEC.

Besides analytical calculations and the use of NEC, the Finite Difference Time Domain method is used.
An introduction to this method is given in Chapter 4. FDTD is a pure numerically oriented method, which transforms the continuous Maxwell's equations into a set of equations in the time-and space-domain. FDTD is a very flexible simulation technique, because nearly any geometrical structure can be analysed. It can handle complex material distributions and results are obtained, for a wide frequency band, in only one computational run using a pulsed excitation. The limitation of this method is the very large resource requirements, and therefore the need of powerful computers, when the total problem space becomes large in terms of the smallest wavelength in this entire space.

In Chapter 5 the package XFDTD is used to analyse the half-wave dipole in free space and parallel to a perfectly conducting ground as well as parallel to a dielectric ground. It is expected that the case of a perfectly conducting ground is worst case. So the performance of the antenna is influenced more badly by a perfect conductors than by other types of materials. This theorem has been verified with the use of XFDTD.
Chapter 2

Literature study

2.1 Introduction

This is a literature study on the interaction between a portable antenna and a nearby object. The near-field of an antenna is scattered, diffracted and absorbed by the object which influences the performance of the complete communication system. The object can be a human body, a battery in a telephone or the case of the apparatus. Much is written about this subject but most papers focus on the health hazards of radiation, while in this project the antenna performance is of more interest. On the other hand, when the absorption of the electromagnetic field by a body is known, the radiated power in the far field is also known.

The goal of this part of the graduation project is to make an overview of existing analysis methods and their characteristics and to choose a numerical method to calculate the electromagnetic field around the antenna. In a later part these calculations will be used for the analysis of the proximity effects and body effects and to develop design rules and design techniques.

2.2 Antenna-body coupling model

The following papers about the coupling between a small antenna and the human body are found [3, 5, 15, 19, 20, 22, 24, 26].

In Nyquist et al. [3] and Karimullah et al. [5] the coupling is described analytically. In the former paper the current distribution on the antenna is assumed to be known. This is a severe restriction because the electromagnetic field that is scattered by the object induces a new current on the antenna. A small distance between the antenna and the body, in terms of wavelengths, may make this model more inaccurate. When investigating the performance of the antenna it is this mutual coupling that becomes very important. In the latter paper [5] the model is enhanced and it gives a good analytical point of view of the mutual coupling problem.

The specific absorption rate (SAR) is a measure of the amount of radiation that is absorbed by the biological body. This quantity is of interest for an investigation to potential biological hazards. This project does not focus on biological effects but on the effects of an object on the antenna performance. On the other hand, when the power that is absorbed by a human is reduced, more power will be radiated in the far field of the antenna.

Most of the first eight papers use the SAR [3, 5, 15, 22, 24, 26], while Kuster [20] gives an overview of various calculation techniques and Toftgård et al. [19] are interested in the antenna quantities.

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Numerical simulations are obtained by the method of moments (MoM) [3, 5, 22, 24], by the multiple multipole (MMP) method [15, 20], based on the generalized multipole technique (GMT) or by the finite difference time domain (FDTD) method [19, 26].

Next in INSPEC has been searched to everything about 'ANTENNA' and 'BODY' (including plurals) in 1990 until 1994. This gives 288 papers, most of them focusing on a body in another meaning as in the antenna-body coupling model (e.g. 'It is a body of evidence ...'). Some articles are written in Japanese. A lot of items are dealing with cancer therapy or with health hazards due to electromagnetic radiation in the near field of an antenna. After a selection only five publications seemed to be left to be useful [12, 18, 21, 25].


2.3 Numerical simulation methods

All simulation methods transform the Maxwell's equations into a set of algebraic equations suitable for solution on computers. The techniques available can be classified into differential and integral methods [28].

2.3.1 Differential and integral equation methods

Differential equation methods deal directly with Maxwell's equations, and evaluate electric and magnetic fields throughout the domain of interest. In principle this domain can be infinite. Therefore an artificial outer boundary has to be introduced, so the problem can be solved.

The finite difference method fills the problem-domain with a grid of points and obtains solutions for the fields at these points only. Approximations to the derivatives of the fields at one point are obtained in terms of the fields at neighbouring points.

The finite element method subdivides the problem domain in a set of elements, and defines a set of basis functions to model the fields in each element. A solution is found by obtaining the coefficients of all the basis functions.

Integral equation methods use the ability to rewrite Maxwell's equations in integral form, using Green's theorem. The equations generated by this process give the fields as the convolution of source current density with a Green's function, which gives the field at an arbitrary location due to a unit point current source at another location. The current density is defined in terms of a set of basis functions with unknown coefficients. A set of equations to obtain these coefficients is produced.

With this method the unknown quantity is generally confined to a relatively compact region, namely the objects of interest, while for differential methods the unknown quantities, electric and magnetic field components, are required over a much larger region.

Compensating for this is that the set of equations generated using an integral method is "full" because a localized current source generates fields over the whole space, while that generated using a differential method is "sparse" because the differentials of the field at one point only depend on its value at neighbouring points. This difference means that differential methods can handle much larger number of unknowns before the problem becomes too large to implement on a computer.

2.3.2 Method of Moments

The method of moments is a widely used integral method to calculate the field of an antenna [3, 5, 21, 22, 24, 25, 28]. In early years the MoM was based on line currents. With this method it is more difficult
to model dielectrics with arbitrary shapes. That is why Singer et al. [22] model an object by surface currents. The current in a body is described by a current distribution on the surface. This makes it possible to analyse the coupling of the field into a dielectric body.

When the body is not homogeneous, it is difficult to model the object. The MoM is questionable in fields of high gradients found close to EM-sources and a straightforward implementation of the method is not very convergent for an application on lossy bodies.

Scott [28] uses the MoM because he analyses the effect of a relatively small object on the fields over a large region of space.

The literature studied so far refers to several software packages for calculating electromagnetic field. Two of them, using the MoM, are CONCEPT and NEC II (Numerical Electromagnetic Code). The latter one is available at Philips Research Laboratories Eindhoven and will be used after this literature study.

2.3.3 Multiple Multipole method

The multiple multipole method (MMP) is another integral method, which is based on the generalized multipole technique (GMT). With this method only the boundaries need to be discretised [15, 20]. Strong disturbances such as local sources with high field gradients or local disturbances of the surface are handled without any difficulty. Simply shaped bodies are very efficiently simulated. But for each different object the user has to define the multipoles again. So a lot of interaction is needed between the user and a program.

The most drastic restriction of MMP is that the greatest possible number of unknowns is limited to a few thousand, not only because of the required operations but also because of roundoff errors. Bodies with angular surfaces as edges and corners and areas where several domains meet are less manageable. Usually the largest errors are found on the boundaries.

MMP is a very suitable method when the object is easy to model. This is not the case with a human body, which has a large relative permittivity.

2.3.4 Finite Difference Time Domain method

The FDTD method [18, 19, 26] is a direct implementation of the time-dependent Maxwell equations written in finite-difference form. The finite difference procedure gives difference expressions for both the space and time derivatives of the electric and magnetic field-components. The resulting algebraic equations are used to track the time evolution of the fields within a given spatial region.

This method has not been very popular for electromagnetic applications for a long time because of the need of relative fast computers with huge memory capacity. It is easy to implement bodies with inhomogeneous and large permittivity in the model.

It is pointed out that FDTD is well suited for calculating the radiation patterns of an antenna (See section 2.4). For antenna radiation problems, FDTD can produce far-zone fields during one computation.

Another advantage of FDTD is that wide-band results can be obtained from one FDTD computation using pulsed excitation and a Fourier transformation.

In the conventional FDTD algorithm the dispersion of the tissues’ dielectric properties is ignored and frequency independent properties are assumed. Therefore the materials are described by a Debye equation, which gives the frequency dependent permittivity of the body in a time dependent equation. This method, used by Furse et al. [26] is called Frequency Dependent Finite Difference Time Domain (FD)TD. This approach is overdone when investigating the performance of an antenna which operates in a relatively small bandwidth, because the permittivity is constant within this bandwidth.
2.3.5 Gradually increasing parameters

Popovic and Notaros [12] propose quite another numerical method for the analysis of electromagnetic field in the presence of dielectric bodies. The method is called PPP, an abbreviation of Serbo-Croatian words meaning 'gradual increase of parameters'.

It starts with zero susceptibility and known excitation electric field in a vacuum and then increases the electric susceptibility of the medium in small increments. So the system is solved approximately to the actual permittivity and the total electric field. The method is conceptually very simple and the number of iterations is always much smaller than the number of unknowns. Because the method is iterative, no big matrices are required and it can easily handle inhomogeneous dielectric media.

The iteration schemes that were used by the authors were found out to be divergent for larger values of relative permittivity or strong coupling between the antenna and the dielectric bodies. So this method is less convenient for this project.

2.4 FDTD used for the computation of antenna-radiation

The antenna body coupling model as found till now is approached in two different ways. One is the scattering problem, i.e. an incident plane wave is scattered by an object. A second is the effect of an object in the near field of the antenna that influences the antenna radiation pattern and efficiency as well as the impedance at the antenna feed-point.

In this section a selection is made of the papers that have been found till now. A search is made to everything that deals with antenna radiation and uses the FDTD method.

At first The IEEE Transactions on Antennas and Propagation of august 1994 gives another article about the use of FDTD in the performance analysis of antennas [29].

From the results it becomes clear that the method is mainly used by A. Taflove and K.A. Umashankar and on the other hand K.S. Kunz and R.J. Luebbers. All of them are refer to Yee [1], the first one who mentions the basics of the method in 1966.

Taflove and Umashankar merely worked on the scattering problem and have written a chapter FDTD in a book about electromagnetic scattering [11]. Other persons, among them D.M. Sullivan, D.T. Borup, O.P. Gandhi and J.Y. Chen, have in common that they have analysed the SAR quantity [7, 8].

The other group has written papers in which the antenna performance is discussed [10, 14, 16, 17].

When using the FDTD method an outer boundary has always to be defined. Therefore G. Mur [6] developed an artificial, non existing, boundary that perfectly absorbs the electromagnetic field. He specially defined the 'Absorbing Boundary Conditions' for FDTD analysis.

Because antennas are normally designed for telecommunications, the far-zone field of the antenna is of interest. Therefore the far field components have to be extracted from the field in the near zone via a 'Near Zone to Far Zone Transformation' in the time domain [13].

Kunz and Luebbers have written a very good introductory book about the FDTD method for electromagnetics [23].
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Chapter 3

Analysing the half-wave dipole using analytical calculations and NEC

3.1 Introduction

In this chapter the interaction between an antenna and a large, perfectly conducting, object is analysed. The used half-wave dipole antenna is resonant at the frequency of 2.5 GHz in free space conditions. The object is modeled by an infinitely large and perfectly conducting groundplane. The calculations are done analytically and with the Method of Moments (MoM), implemented in the software package Numerical Electromagnetics Code (NEC).

Section 3.2 introduces NEC and gives an overview of the capabilities and limitations of the program.

The half-wave dipole antenna in free space is analysed in section 3.3. NEC is compared to analytical calculations.

The antenna, parallel to a perfectly conducting ground, is modeled in section 3.4.

In section 3.5 the current distribution on the wires is calculated by NEC, while this distribution is used as a known function in the analytical calculations. The antenna impedance, the radiation patterns and the return loss, all based on the current distribution, are calculated analytically and with NEC. The return loss is defined as a measure of reflection at the antenna feed-points.

3.2 Introduction to the software package NEC

The Numerical Electromagnetics Code (NEC-2) is a computer program for the analysis of the electromagnetic response of antennas and other metal structures. It is based on the numerical solution of integral equations for the currents induced on the structure by impressed sources or incident fields. For this report, the version NEC-2 is used, although just NEC is mentioned.

The code combines an integral equation for smooth surfaces with one specialized to wires to provide for convenient and accurate modeling of a wide range of structures. A model may include non-radiating networks and transmission lines, connecting parts of the structure, perfect conductors, and lumped-element loading. A structure may also be modeled over a ground plane that may be either a perfect or imperfect conductor.
There are some limitations in the modeling capabilities of NEC. One of them is that the structures must be in free space and not in a dielectric medium. However it is possible to define a ground which has a certain relative dielectric constant and a finite conductivity. This can be done by the Sommerfeld/Norton method. This method is not used in this report. In the next chapters the FDTD method will be used to investigate objects with several material parameters, including dielectrics.

Another feature of NEC is the Numerical Green's Function option. With this option a fixed structure and its environment may be modeled and the factorized interaction matrix can be saved as a file. New parts may then be added to the model in subsequent computer runs and the complete solution can be obtained without repeating calculations for the data on the file.

A structure is modeled by wires, consisting of segments, and surfaces, consisting of patches. Proper choice of the segments and patches for a model is the most critical step to obtaining accurate results. The number of segments and patches should be the minimum required for accuracy, however the dimensions of them with respect to each other and to the wavelength are limited. In the next sections a model strategy and its limitations are investigated for a simple half-wave dipole antenna.

The output may include induced currents and charges, electric or magnetic fields in the far-zone of the antenna, radiation patterns and impedances at the sources.

3.2.1 Wire modeling

The basic devices for modeling wires with the NEC code are short, straight segments. Modeling a wire structure with segments involves both geometrical and electrical factors. Geometrically, the segments should follow the paths of conductors as closely as possible, using a piece-wise linear fit on curves. On a segment, only currents in the axial direction are considered. Hence the current around the wire circumference may not vary.

The main modeling consideration is segment length $\Delta$ in meters relative to the free space wavelength $\lambda$. Generally, $\Delta$ should be less than about $0.1 \lambda$, while shorter segments, $0.05 \lambda$ or less, may be needed in modeling critical regions of an antenna. Extremely short segments, less than about $10^{-3}\lambda$, should be avoided since these leads to numerical inaccuracy. This gives the first modeling rule:

$$0.001 < \frac{\Delta}{\lambda} < 0.1$$

The wire radius, $a$, relative to $\lambda$ is limited by the approximations used in the kernel of the electric field integral equation. The NEC - User's Guide [1] advises to keep $2\pi a/\lambda$ much less than unity. A more convenient rule is [2]:

$$0 < a < 0.01 \lambda$$

At last the accuracy of the numerical solution is also dependent on $\Delta/a$. Small values of $\Delta/a$ may result in extraneous oscillations in the computed current near free wire ends, voltage sources, or lumped loads. In NEC there are two approximation options available, used in the kernel of the electric field integral equation: the thin-wire kernel and the extended thin-wire kernel. In the former, the current on the surface of a segment is reduced to a filament of current on the segment axis, while in the latter kernel, a current uniformly distributed around the segment surface is assumed. The method in the extended thin-wire kernel extends the accuracy for larger values of the radius. The NEC - User’s Guide [1] guarantees an inaccuracy of less than 1 % if

$$\frac{\Delta}{a} > 8$$, for thin-wire kernel

$$\frac{\Delta}{a} > 2$$, for extended thin-wire kernel

The use of NEC is illustrated by an example in the next section.
3.3 The half-wave dipole antenna in free space

As a test of the NEC program a simple, half-wave dipole antenna is investigated. The antenna is made of a wire with a length of 5.62 cm and a radius of 0.25 mm. In analytical considerations the length of the antenna is exact half the wavelength and the radius is infinitely thin. This theoretical antenna is resonant, i.e. the reactance at the antenna feed-point is zero. However, in practice the radius has a finite thickness. The resonant length becomes shorter than \( \frac{1}{2} \lambda \) with increasing the ratio between radius and length of the wire.

The program input of the structure and program calls for NEC are defined by several so-called punched cards. The input into NEC for the example is as follows:

```
CM EXAMPLE : CENTER FED LINEAR ANTENNA
CM BY : BART VAN LEERSUM
CE DATE : 11-10-1994
GW 0 25 0.00 0.00 -2.81E-02 0.00 0.00 2.81E-02 2.5E-04
GE 0
FR 0 1 0 0 2500.00 4.000
EX 0 0 13 01 1.0
RP 0 361 1 0010 0.0 90.0 1.0 1.0
EN
```

For a single run the data-card set consists of three types of data cards. The first type consists of comment cards which may contain a description of the run and begin with CM or CE.

These are followed by geometry cards which define the geometry of the antenna. In the example there is one wire specification beginning with GW. This wire is modeled by an odd number of segments. The middle segment is replaced by a voltage source of 1.0 Volt. The source has a harmonic time dependence with a fixed frequency. In this example, the wire is divided into 25 segments to satisfy the preconditions in equations (3.1) - (3.4). The GE-card ends the geometry input.

Finally, a section of program control cards defines electrical parameters as frequency (FR) and excitation (EX) and defines which quantities have to be calculated. In the example the RP-card causes program execution by which a radiation pattern will be calculated. The input is ended by the EN-card. For a full understanding of the various data cards with all their aspects see the NEC's User's Guide [1].

Calculations with NEC pointed out that the imaginary part of the antenna impedance of an antenna with the chosen dimensions in a free space environment equals zero at a frequency of 2.5 GHz. At this resonance frequency the impedance is 71.9 \( \Omega \) which is approximate 1.7 percent below the value of 73.13 \( \Omega \) which follows from analytical calculation. The gain is 2.13 dB, which is the same as in the analytical approach. The analytically obtained values that are mentioned here are discussed thoroughly in the literature about antenna theory (e.g. Collin [3]).

In NEC the wire with a finite thickness is excited by a voltage source in the gap in the middle of the wire. First NEC calculates the current distribution on the antenna. Next from this current, NEC calculates the radiation patterns.

3.3.1 The current distribution on the wire

The analytical calculations start with an impressed current distribution on the wire. The current is given by

\[
\tilde{I} = \delta(z')\delta(y') \left[ U(z' + \frac{1}{4} \lambda) - U(z' - \frac{1}{4} \lambda) \right] I(z')\hat{a}_z, \tag{3.5}
\]

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where $I(z')$ is a complex function that defines the current distribution along the wire. In most literature (e.g. [3]), the wire is taken infinitely thin and the length of the wire is half the wavelength. In that case the current distribution is approximated by a real function that is defined by:

$$I(z') = I_0 \cos(k_0 z').$$  \hspace{1cm} (3.6)

The calculated current distribution is plotted in Figure 3.1. To compare the results of NEC to the analytical calculations, a cosine function is also plotted through the calculated points of the real part of the current distribution. The real part of the current seems to match to equation (3.6). However due to the finite thickness of the wire, an imaginary part has to be added to the current function. Therefore a better approximation of the current is given by:

$$I(z') = I_0 \cos(k_0 z') + j I_1 \sin(2k_0 z'),$$  \hspace{1cm} (3.7)

where the quotient of the amplitudes $I_1/I_0$ is determined by NEC. For comparison the imaginary part of equation (3.7) is also plotted in Figure 3.1. Experiments show that the amplitude of the imaginary part of the current decreases to zero and the resonant length of the antenna increases to half the wavelength when decreasing the thickness of the wire. The quotient $I_1/I_0$ is only a function of the thickness of the wire.

Note that in the analytical calculations the antenna is exact half the wavelength, while in NEC the antenna is somewhat shorter, because of the finite thickness of the modeled wire. In the remainder of this chapter the lengths of both wires are kept their resonance lengths, so in fact the antennas are not entirely the same.

![Figure 3.1: Current distribution](image)

### 3.3.2 The directivity function

From the current on the wire the directivity function can be calculated analytically. The directivity in dB, for an infinitely thin wire with an arbitrary current distribution function is calculated in this section. This directivity function follows from the power flux per square unit in the far field $\mathbf{S}(r)$ (Poynting vector) at the surface of a sphere with a certain radius $r$ in the far field zone. This power flux in the direction $(\theta, \phi)$ is normalized to the time average of the total radiated power $P_r$:

$$D(\theta, \phi) = 10 \log \left( \frac{4\pi |r^2 \mathbf{S}(\vec{r})|^2}{P_r} \right) \text{ dB}.$$  \hspace{1cm} (3.8)
The Poynting vector is defined by

$$\vec{S}(r) = \frac{1}{2} \Re \left[ \vec{E} \times \vec{H}^* \right].$$  \hspace{1cm} (3.9)

The fields $\vec{E}$ and $\vec{H}$ in the far zone of the antenna are assumed to satisfy the Sommerfeld radiation conditions, i.e. these fields are perpendicular to each other and to the direction of the wave propagation:

$$\vec{H}(r) = \frac{1}{Z_0} \hat{a}_r \times \vec{E}(r).$$  \hspace{1cm} (3.10)

Therefore the Poynting vector becomes:

$$\vec{S}(r) = \frac{1}{2Z_0} |\vec{E}|^2 \hat{a}_r.$$  \hspace{1cm} (3.11)

The total radiated power $P_r$ is obtained by integrating the Poynting-vector over a closed surface that encloses all source points:

$$P_r = \oint_{S_{\text{closed}}} \left( \vec{S}(r, \theta, \phi) \cdot \hat{a}_r \right) r^2 \sin \theta \, d\theta \, d\phi.$$  \hspace{1cm} (3.12)

The electric field $\vec{E}(r)$ in the far zone is related to the current on the antenna [3]:

$$\vec{E} = \frac{k_0 Z_0 \exp(-j k_0 r)}{4\pi r} \int_V [(\hat{a}_r \cdot \vec{I}) \hat{a}_r - \vec{I}] \exp(j k_0 (\hat{a}_r \cdot \vec{r}')) dV',$$  \hspace{1cm} (3.13)

where

$$Z_0 = 120\pi$$  the free space wave impedance,

$$r, \theta, \phi$$  the spherical coordinates of a point in the far field,

$$\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$$  the unity vectors of the spherical coordinates,

$$\hat{a}_x, \hat{a}_y, \hat{a}_z$$  the unity vectors of the Cartesian coordinates and

$$k_0 = \frac{2\pi}{\lambda}$$  the wavenumber in free space.

The points in the far field are given by the vector $\vec{r}$, while the source points are given by $\vec{r}'$. In the far-zone region, $|\vec{r}| \gg |\vec{r}'|$ for all $\vec{r}'$ in $V$. In equation (3.13), the current $\vec{I}$ is integrated over the volume $V$, which contains the wire antenna. This equation is not restricted to currents on infinitely thin wires, but is generally valid for every current distribution. All source points must lie within this volume. The current from equation (3.5) is substituted in equation (3.13) with the following definitions ($x' = 0$ and $y' = 0$):

$$\vec{r}' = z' \hat{a}_z,$$

$$\hat{a}_r \cdot \vec{r}' = z' \cos \theta,$$

$$\exp(j k_0 (\hat{a}_r \cdot \vec{r}')) = \exp(j k_0 z' \cos \theta),$$

$$\hat{a}_r \cdot \hat{a}_z = \cos \theta \hat{a}_r,$$

$$\cos \theta \hat{a}_r - \hat{a}_z = \sin \theta \hat{a}_\theta,$$

$$\frac{k_0 \lambda}{4} = \frac{\pi}{2}.$$  \hspace{1cm} (3.14)

The electric field is evaluated as follows:

$$\vec{E} = \frac{j k_0 Z_0 \exp(-j k_0 r)}{4\pi r} f(\theta) \hat{a}_\theta,$$  \hspace{1cm} (3.15)

$$f(\theta) = \sin \theta \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dy' \int_{-\frac{k_0 \lambda}{4}}^{\frac{k_0 \lambda}{4}} \delta(z') \delta(y') I(z') \exp(j k_0 z' \cos \theta) dz',$$  \hspace{1cm} (3.16)
where $I(z)$ is an arbitrary complex current distribution function, for example the approximation of Figure 3.1:

\[
I(z') = I_r(z') + jI_i(z'),
\]

\[
I_r(z') = I_0 \cos(k_0z'),
\]

\[
I_i(z') = I_1 \sin(2k_0z').
\]

(3.17)

The real part $I_r(z)$ gives a real valued function $f_r(\theta)$ and the imaginary part $I_i(z)$ gives another real valued function $f_i(\theta)$:

\[
f_r(\theta) = I_0 \sin \theta \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dy' \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} \delta(z') \delta(y') \cos(k_0z') \exp(jk_0z' \cos \theta) dz'
\]

\[
= I_0 \sin \theta \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} \cos(k_0z') \exp(jk_0z' \cos \theta) dz'
\]

\[
= I_0 \frac{2 \cos \left( \frac{\pi}{2} \cos \theta \right)}{k_0 \sin \theta},
\]

(3.18)

\[
f_i(\theta) = I_1 \sin \theta \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dy' \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} \delta(z') \delta(y') \sin(2k_0z') \exp(jk_0z' \cos \theta) dz'
\]

\[
= I_1 \sin \theta \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} \sin(2k_0z') \exp(jk_0z' \cos \theta) dz'
\]

(3.19)

The current on the wire is an even function of the coordinate $z$, i.e. $I(z) = I(-z)$ for all current functions, both real or complex. Therefore equation (3.19) is simplified by taking half the domain of the integral:

\[
f_i(\theta) = I_1 \frac{2 \sin \theta}{k_0} \int_{0}^{\frac{1}{4} \lambda} \sin(2k_0z') \cos(k_0z' \cos \theta) dz'
\]

\[
= I_1 \sin \theta \int_{0}^{\frac{1}{4} \lambda} \sin \left( \frac{2\pi z'}{\lambda} (2 + \cos \theta) \right) dz' + I_1 \sin \theta \int_{0}^{\frac{1}{4} \lambda} \sin \left( \frac{2\pi z'}{\lambda} (2 - \cos \theta) \right) dz'
\]

\[
= -I_1 \sin \theta \left[ \frac{\cos \left( \frac{\pi z'}{\lambda} (2 + \cos \theta) \right)}{2 + \cos \theta} - 1 \right] \bigg|_{0}^{\frac{1}{4} \lambda} - I_1 \sin \theta \left[ \frac{\cos \left( \frac{\pi z'}{\lambda} (2 - \cos \theta) \right)}{2 - \cos \theta} - 1 \right] \bigg|_{0}^{\frac{1}{4} \lambda}
\]

\[
= -I_1 \frac{\sin \theta}{k_0} \left[ \frac{\cos \left( \frac{\pi \cos \theta}{2} \right) - 1}{2 + \cos \theta} + \frac{\cos \left( \frac{\pi \cos \theta}{2} \right) - 1}{2 - \cos \theta} \right]
\]

\[
= -I_1 \frac{\sin \theta}{k_0} \left[ \frac{-\cos \left( \frac{\pi \cos \theta}{2} \right) - 1}{2 + \cos \theta} + \frac{-\cos \left( \frac{\pi \cos \theta}{2} \right) - 1}{2 - \cos \theta} \right]
\]

\[
= I_1 \frac{\sin \theta}{k_0} \left[ \frac{4 \cos \left( \frac{\pi \cos \theta}{2} \right) + 4}{4 - \cos^2 \theta} \right].
\]

(3.20)

The total $\theta$-dependence is then given by

\[
f(\theta) = f_r(\theta) + jf_i(\theta)
\]

\[
= I_0 \frac{2 \cos \left( \frac{\pi \cos \theta}{2} \right)}{k_0 \sin \theta} + jI_1 \frac{\sin \theta}{k_0} \left[ \frac{4 \cos \left( \frac{\pi \cos \theta}{2} \right) + 4}{4 - \cos^2 \theta} \right].
\]

(3.21)

It is remarkable that both functions, $f_r(\theta)$ and $f_i(\theta)$, are almost the same as can be seen in Figure 3.2, where $I_0 = 1.0$ A and $I_1 = 1.0$ A.
The power-flux per square unit $\mathcal{S}(r)$ in the far field is

$$
\mathcal{S}(r) = \frac{1}{2\pi_0} |\mathbf{E}|^2 \hat{a}_r = \frac{k_0^2 Z_0}{32\pi^2 r^2} |f(\theta)|^2 \hat{a}_r
$$

(3.22)

The total radiated power $P_r$ is obtained by integrating the Poynting-vector (3.22) over a closed surface that encloses all source points:

$$
P_r = \int_{\theta=0}^{\pi} \int_{\phi=-\pi}^{\pi} \left( \mathcal{S}(r, \theta, \phi) \cdot \hat{a}_r \right) r^2 \sin \theta \, d\theta d\phi
$$

= \frac{k_0^2 Z_0}{32\pi^2} \int_{\theta=0}^{\pi} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} |f(\theta)|^2 \sin \theta \, d\theta d\phi.

(3.23)

Finally the directivity is calculated from the power flux density of equation (3.22), normalized to the total radiated power of equation (3.23):

$$
D(\theta, \phi) = 10 \log \left( \frac{4\pi r^2 |\mathcal{S}(r)|}{P_r} \right)
$$

= 10 \log \left( \frac{k_0^2 Z_0}{8\pi} \frac{|f(\theta)|^2}{P_r} \right)

(3.24)

If the current on the wire has a cosine form, $I(z) = I_0 \cos(k_0z)$, the total radiated power of equation (3.23) becomes:

$$
P_r = 30 I_0^2 \int_{0}^{\pi} \cos^2\left(\frac{1}{2} \pi \cos \theta \right) \frac{\cos \theta}{\sin \theta} \, d\theta
$$

$\approx 36.565 I_0^2.$

(3.25)
The free space wave impedance $Z_0$ has a value of $120 \, \pi$. The directivity becomes:

$$D(\theta, \phi) \approx 10 \log \left\{ 1.64 \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]^2 \right\} \text{ dB}. \quad (3.26)$$

If the current on the wire has a sine form, $I(z) = I_1 |\sin(2k_0 z)|$, the total radiated power of equation (3.23) becomes:

$$P_r = 120I_1^2 \int_0^\pi \sin^3 \theta \left( \cos \left( \frac{1}{2} \pi \cos \theta \right) + 1 \right)^2 \left( 4 - \cos^2 \theta \right)^2 d\theta$$

$$\approx 34.780I_1^2 \quad (3.27)$$

The directivity becomes:

$$D(\theta, \phi) \approx 10 \log \left\{ 1.725 \left[ 2 \sin \theta \frac{\cos \left( \frac{\pi}{2} \cos \theta \right) + 2}{4 - \cos^2 \theta} \right]^2 \right\} \text{ dB}. \quad (3.28)$$

### 3.3.3 NEC results

The directivity functions of equations (3.26) and (3.28) are plotted in Figure 3.3. The patterns are much alike, the difference between them is at most 0.072 dB. The amplitude of the imaginary part of the current is very small compared to that of the real part (1.03 mA compared to 13.9 mA). So the contribution of the imaginary part is $20 \log(13.9/1.03) = 22.6$ dB less. Therefore the imaginary current can be neglected.

For the results of Figure 3.4 the next quantities are used:

- length of wire: 5.62 cm
- wire radius: 0.25 mm
- frequency: 2.5 GHz
- wavelength: 11.9920 cm

The results are summarized as follows:

$$I_0 = 13.909 \, mA$$
$$I_1 = -1.0334 \, mA$$
$$R_a = 71.8942 \, \Omega$$

The radiation patterns in the YOZ-plane are plotted in Figure 3.4. The wire-antenna is placed along the $z$-axis, as defined by equation (3.5). The analytical obtained directivity function differs at most 0.066 dB from the NEC's calculation at $\theta = 16^\circ$. 
Figure 3.3: Pattern plot, analytical

Figure 3.4: Pattern plot, NEC results
3.4 The half-wave dipole antenna above a perfectly conducting ground

NEC has the possibility to define structures over a perfectly conducting ground. For this purpose the code generates an image of the structure reflected in the ground surface as shown in Figure 3.5.

![Figure 3.5: Dipole parallel to a conducting earth](image)

The field that is radiated by the antenna propagates as a spherical wave and is reflected by the surface of the ground plane. Hence currents are induced in this plane. Due to these currents the surface acts like an antenna. The electric field tangential to this plane must vanish to satisfy the general boundary conditions for the surface of a perfect conductor. To accomplish this condition, the actual surface is replaced by an image of the antenna, which is reflected in the ground plane. The antenna and its image must be excited by opposite sources.

In the next sections this model is used for analytical calculations as well as for the calculations in NEC. However there is a difference between the two calculation methods. In both methods the boundary conditions on the groundplane are satisfied by the above described image theory. NEC calculates the currents on the wires by satisfying the boundary conditions on the wires also. In the analytical calculations the boundary conditions on the wires are not perfectly satisfied, because the currents are impressed by approximated functions.

3.4.1 Limitations on the distance to the groundplane in NEC

When modeling the horizontal wire in NEC, the wire should not be too close to the ground to get optimal results. However the user's guide of NEC [1] guarantees an accuracy comparable to that for a free space model when

\[ \sqrt{h^2 + a^2} > 10^{-6} \lambda, \]

where \( a \) is the wire-radius, and \( h \) the distance to the wire radius. In this example the radius equals \( 2.5 \cdot 10^{-4} \) m, which is much larger than \( 10^{-6} \lambda = 1.2 \cdot 10^{-7} \) m. On the other hand, two wires are supposed to be connected when the separation is less than \( 10^{-3} \) times the length of the shortest segment. Therefore, in this example no calculations are made for such small distances.
3.4.2 Analytical calculations

The total field in the far field zone is obtained by a superposition of the direct field from the antenna and the scattered field from the perfectly conducting ground plane, that is replaced by the image of the antenna. So the total field is obtained by two spherical waves which have a different origin.

The model of the antenna parallel to a ground plane is the same in the analytical calculations as in the NEC calculations. In the analytical calculations the currents on the antennas are impressed. The electric and magnetic fields are calculated from these currents. The radiated power is a function of the impressed current distribution may alter by the ground plane. In the next section the exact current distribution will be calculated by NEC and will be compared to the functions that are used here.

The current through the dipole and through its image is given by

\[ I = [\delta(x - h) - \delta(x + h)] \delta(y) \left[ U(z + \frac{1}{4} \lambda) - U(z - \frac{1}{4} \lambda) \right] I(z) \delta, \]

This current is used to calculate the electric field in the far field zone. The antennas are positioned as in Figure 3.5. The electric field is evaluated from equation (3.13) in the same way as in section 3.3.2 with the following coordinate transformations \((y' = 0)\):

\[ \begin{align*}
  x' &= x \hat{a}_x + z' \hat{a}_z \\
  (\hat{a}_r \cdot \hat{r}') &= x' \cos \phi \sin \theta + z' \cos \theta \\
  \exp(jk_0 (\hat{a}_r \cdot \hat{r}')) &= \exp(jx'k_0 \cos \phi \sin \theta) \exp(jk_0 z' \cos \theta),
\end{align*} \]

where \(x'\) is the position of the antennas along the x-axis, \(x' = h\) for the actual antenna and \(x' = -h\) for its image. \(h\) is the distance between the antenna and the groundplane. Finally \(z'\) is the z-coordinate along the wire of one antenna.

The electric field is given by

\[ \vec{E} = \frac{jk_0 \mathbb{E}_0 \exp(-jk_0 r)}{4\pi r} f(\theta) g(h, \theta, \phi) \delta, \]

where

\[ f(\theta) g(h, \theta, \phi) = \frac{\sin \theta \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} \delta(x' - h) - \delta(x' + h)] \delta(y') I(z') \exp(jx'k_0 \cos \phi \sin \theta) \exp(jk_0 z' \cos \theta) dz'}{2j \sin(hk_0 \cos \phi \sin \theta) \sin \theta \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} I(z') \exp(jk_0 z' \cos \theta) dz'}. \]

These functions are separated in \(f(\theta)\) and \(g(h, \theta, \phi)\):

\[ f(\theta) = \sin \theta \int_{-\frac{1}{4} \lambda}^{\frac{1}{4} \lambda} I(z) \exp(jk_0 z' \cos \theta) dz', \]

\[ g(h, \theta, \phi) = 2j \sin(hk_0 \cos \phi \sin \theta), \]

where \(f(\theta)\) is obtained from equation (3.16). Note that all next equations of this section can also be used for the antenna in free space. For this purpose the function \(g(h, \theta, \phi)\) is replaced by the constant with the value '1'.
The power-flux per square unit $\vec{S}(\vec{r})$ in the far field is

\[
\vec{S}(\vec{r}) = \begin{gathered}
\frac{1}{2}Re \left[ \vec{E} \times \vec{H}^* \right] \\
= \frac{1}{2Z_0} \left| \vec{E} \right|^2 \hat{a}_r \\
= \frac{k_0^2 Z_0}{32\pi^2 r^2} |f(\theta)|^2 |g(h, \theta, \phi)|^2 \hat{a}_r, \quad (-\frac{\pi}{2} < \phi < \frac{\pi}{2}) \\
\vec{S}(\vec{r}) = 0, \quad (-\pi < \phi < -\frac{\pi}{2} \text{ and } \frac{\pi}{2} < \phi \leq \pi). \tag{3.36}
\end{gathered}
\]

The domain of $\phi$ in equation (3.36) is restricted to a half-space because in the actual configuration with the groundplane there is only one antenna that radiates in just this half-space. The total radiated power $P_r$ is obtained by integrating the Poynting-vector (3.36) over a closed surface that encloses all source points:

\[
P_r(h) = \int_{\theta=0}^{\pi} \int_{\phi=-\pi}^{\pi} \left( \vec{S}(r, \theta, \phi) \cdot \hat{a}_r \right) r^2 \sin \theta d\theta d\phi
\]

\[
= \frac{k_0^2 Z_0}{32\pi^2} \int_{\theta=0}^{\pi} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} |f(\theta)|^2 |g(h, \theta, \phi)|^2 \sin \theta d\theta d\phi. \tag{3.37}
\]

The radiated power in equation (3.37) is not calculated analytically but evaluated numerically by the program Mathematica [4].

Finally the directivity is calculated from the power flux density of equation (3.36), normalized to the total radiated power of equation (3.37):

\[
D(h, \theta, \phi) = 10 \log \left( \frac{4\pi |r^2 \vec{S}(\vec{r})|}{P_r} \right)
\]

\[
= 10 \log \left( \frac{4\pi |f(\theta)|^2 |g(h, \theta, \phi)|^2}{\int_{\theta=0}^{\pi} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} |f(\theta)|^2 |g(h, \theta, \phi)|^2 \sin \theta d\theta d\phi} \right), \quad (-\frac{\pi}{2} < \phi < \frac{\pi}{2})
\]

\[
D(h, \theta, \phi) = -\infty, \quad (-\pi < \phi \leq -\frac{\pi}{2} \text{ and } \frac{\pi}{2} < \phi \leq \pi). \tag{3.38}
\]
3.5 NEC calculations

In this section NEC is used to calculate the currents on the antennas, the radiation patterns in the far zone and the impedance at the feedpoint of one antenna.

The NEC calculations use the same model as described in 3.4. However, while in the analytical calculations the currents are impressed, NEC calculates the currents on the antennas, which are excited by voltage sources. The current distribution on one antenna is not only a function of its voltage source, but it is also a function of the field that it receives from the other antenna. The currents, calculated using NEC, satisfy the boundary conditions on both antennas.

3.5.1 Current distribution

In the next examples the YOZ-plane acts like a mirror at the place of the perfectly conducting ground plane and the two dipoles are placed at some distance from that plane, like in Figure 3.5. NEC computes the currents in both antennas, while the mirror antenna has the opposite excitation from the actual antenna. The important difference between the two models is the fact that in the analytical model the currents are impressed currents, while in the NEC model the voltage at the gap is impressed. The currents on both antennas are calculated as a function of both sources. The excitation in NEC is a voltage source with a harmonic time dependence,

\[ V(t) = V_0 \sin(2\pi ft), \] (3.39)

with a complex amplitude \( V_0 \) and the frequency \( f = 2.5 \text{ GHz} \).

At first the amplitude is taken real, \( V_0 = 1.0 \text{ Volt} \). The current distribution that is calculated is plotted in Figures 3.6 - 3.8. In the plots of the real parts a cosine function is fitted through the calculation points for comparison with the analytical impressed current. Note that in this way only the form of the currents may be compared and not their amplitude. The plots show that the antenna is not matched to the source, because the current at the feedpoints is not in phase with the voltage at this point. Therefore a complex power is absorbed in the antenna. So the antenna can be presented as a resistive component and a reactive component in series.

Next the voltage at the feed-point is given a time-shift in order to make a comparison with the analytical currents possible. The goal of applying a complex voltage is to get a real current on the feedpoints. The voltage and the current in the first situation are denoted by \( V_{r,1} = V_{r,1} \) and \( I_{r,1} = I_{r,1} + jI_{i,1} \). These are transformed to a second situation with voltage \( V_{r,2} = V_{r,2} + jV_{i,2} \) and current \( I_{r,2} = I_{r,2} + jI_{i,2} \). To get a real current in the second situation, the voltage must have a phase angle that is opposite the phase of the current in the first situation:

\[ \frac{V_{r,2}}{V_{r,2}} = -\frac{I_{r,1}}{I_{r,1}}. \] (3.40)

The real part of the voltage is kept \( 1.0 \text{ V} \). The result is the current distribution of Figures 3.9 - 3.11.

The imaginary part of the current distribution remains the same for all distances to the groundplane. Experiments showed that the amplitude of the sine approximation is a function of the thickness of the wire. This amplitude is larger for a larger wire radius. This effect is not further analysed in this report.

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Figure 3.6: Currents, distance = 0.1 wavelength to groundplane, $V_a = 1.0$ Volt

Figure 3.7: Currents, distance = 0.5 wavelength to groundplane, $V_a = 1.0$ Volt

Figure 3.8: Currents, distance = 0.75 wavelength to groundplane, $V_a = 1.0$ Volt
Figure 3.9: Currents, distance = 0.1 wavelength to groundplane, $V_a = 1.0 + j1.1$ Volt

Figure 3.10: Currents, distance = 0.5 wavelength to groundplane, $V_a = 1.0 - j0.25$ Volt

Figure 3.11: Currents, distance = 0.75 wavelength to groundplane, $V_a = 1.0 + j0.15$ Volt
3.5.2 Radiation patterns

For several distances between the ground and the antenna, the gain is computed in the direction of the x-axis. This is equivalent to $\theta = \pi/2$ and $\phi = 0$ in the spherical coordinate system. Figure 3.12 shows how the gain varies as function of the distance.

The solid line gives the results of the calculations of mathematica, as given by equation (3.38) for a cosine excitation. Assumed is a real impressed current that has a cosine-form. The results of NEC are represented as discrete points. For every point, the gain is calculated by one run of NEC.

In Figures 3.13 to 3.15 patterns are plotted for several distances. Again in these plots the solid lines represent calculations of equation (3.38) with only a real impressed current and the discrete points represent the NEC-calculations.

It is evident that the NEC calculations coincide with the analytical approach. Hence an important conclusion is that the radiation patterns are the same for both models, i.e. the current distribution function for free space can be used in the presence of a perfectly conducting ground as well.

Figure 3.12: Pattern versus distance to groundplane in the direction of the x-axis
Figure 3.13: Distance 0.1 wavelength to groundplane

Figure 3.14: Distance 0.5 wavelength to groundplane

Figure 3.15: Distance 0.75 wavelength to groundplane
3.5.3 Antenna impedance

The antenna impedance in a free space environment at the resonance frequency is calculated in section 3.3:

\[ R_0 = 73.13 \, \Omega. \] (3.41)

In this section the variation of this quantity is investigated for several distances to the conducting screen. The derivation of the antenna-impedance is different for the analytical model and the NEC-model. Figures 3.16 and 3.19 illustrate these derivations as well as the power that is absorbed in the real part of the impedance. This absorbed power must be equal to the radiated power, because the material of the antenna is lossless. Therefore all power that flows through the feedpoint is radiated in the far field of the antenna. The ideal sources impress a current (analytical) or a voltage (NEC) on the antenna feedpoint.

In the analytical configuration of Figure 3.16 the antenna impedance is real and follows from the total radiated power in the far field and the value of the impressed current at the feedpoint. The imaginary part of the impedance can be calculated analytically as well, but is more difficult and therefore omitted here. The NEC configuration of Figure 3.16 is used to calculate the impedance for a certain current distribution. The impressed voltage \( V_a \) has an amplitude of 1.0 V. The impedance at the feedpoint that follows from the NEC calculation is plotted in Figure 3.17 as a function of the distance to the groundplane. The dashed line is the real part of the impedance that follows from the analytically calculated radiated power.

\[
Z_a(h) = Ra(h) = \frac{2Pr(h)}{Io} \\
Z_a(h) = Ra + jXa = \frac{Va}{Ra(h)}
\]

Figure 3.16: Ideal source
Figure 3.17: Impedance at feedpoint

Figure 3.18: Return loss
3.5.4 Return loss

In physical equipment, the antenna is connected to a source via a transmission-line with an internal impedance $R_i$, which is chosen at 73.13 $\Omega$. In this case the source and the line are represented as in Figure 3.19. In free-space circumstances the antenna is perfectly matched to the source, i.e. $R_a = R_i$, $X_a = 0$ and the absorbed power is the maximum available power from the source. When the antenna impedance differs from $R_i$, the antenna and the source are mismatched and due to reflections not the maximum available power is absorbed.

From Figure 3.17 it follows that the antenna above a perfectly conducting ground is not perfectly matched to a source with a constant impedance of 73.13 $\Omega$. Due to this mismatch a part of the power is reflected at the feedpoint of the antenna. The reflection coefficient $\Gamma$ is defined as the amplitude of the reflected voltage wave normalized to the incident voltage wave. In terms of the impedances this reflection coefficient becomes [5]

$$\Gamma = \frac{Z_a - R_i}{Z_a + R_i}. \quad (3.42)$$

When the load is mismatched, not all of the available power from the generator is delivered to the load. This loss is called the return loss, $RL$ and is defined in dB as

$$RL = -20 \log |\Gamma| \text{ dB.} \quad (3.43)$$

![Figure 3.19: Source with internal resistance](image)

The return loss is calculated from the impedance values that are obtained by NEC and plotted in Figure 3.18 (solid line). The dashed line in this figure represents the return loss for a real impedance $R_a$ that is found by the analytical calculations.

The return loss, i.e. the loss of the returned wave, has to be as large as possible for an optimal power transmission. An often used criterium for a good performance is a return loss higher than 15.0 dB. For the NEC-curve in Figure 3.18 this is the case for distances from antenna to the ground of more than 0.27$\lambda$.

The curve of the analytical calculations in Figure 3.18 can not be used to test the criterium, because the return loss is strongly dependent on both the real part and the imaginary part of the impedance.
3.6 Conclusions

In this chapter a half-wave dipole antenna is analysed. The antenna is placed parallel to a perfectly conducting ground with variable distances between the antenna and the ground. The results are compared to free space conditions. The program NEC is used for the calculations and is compared to analytical calculations. In NEC the determination of the segment length, relative to the wire thickness and the wavelength, is very critical to get accurate results and has to be chosen very carefully.

The perfectly conducting ground and the antenna are used in a model, where the ground is replaced by an image of the antenna with an opposite excitation. The perfectly conducting ground becomes a mirror plane.

In the analytical calculations the fields are obtained by superposition of two antennas. A physical source is not defined, but the current distribution on the wires of both dipoles is defined by impressed functions. The currents on both wires are equal, but pointed in opposite directions to satisfy the boundary conditions of the perfectly conducting ground. If the distribution function is chosen properly, the boundary conditions on the wire may be satisfied as well.

In the NEC calculations the dipoles are excited by a harmonic voltage source in the gap in the middle of the antennas. The current distribution on the antennas is calculated by NEC as a function of the voltage sources as well as the current distribution on the other antenna. By this method the boundary conditions on the wire as well as on the groundplane are satisfied.

A typical value of the radiated power of one transmitter that is designed by Philips Research at the frequency of 2.5 GHz is 200 mW, divided over two antennas in a diversity system. To produce this amount of power a minimum total power of 400 mW is required from the battery if a typical amplifier with an efficiency of 50 % is used and free space conditions are assumed. In stand-by condition, the equipment consumes only less than about 400 pW. When the apparatus is receiving, the consumption is only a few mW. So it is clear that the type and the size of the battery are chosen for the transmitting state.

Two quality determining features of mobile communication handsets are

1. The size of the equipment (as small as possible),
2. The lifetime of the battery (as long as possible).

To achieve a long lifetime with a certain battery, the maximum available power from the transmitter must be used for radiation, i.e. transmission of information.

From the results of the calculations the following conclusions are made:

1. One conclusion from this chapter is that the radiation patterns are the same for both the analytical and the NEC-model. The real part of the antenna impedance is almost the same in both calculation methods. So the chosen impressed current distribution function for the analytical calculations is well suited in the used model for the antenna, despite of the presence of the perfectly conducting ground.

2. In free space conditions, when the antenna and the source are perfectly matched, the maximum available power is radiated. This amount of power is just half the power that is generated by the source in the Thévenin equivalent of the transmitter. If the antenna is not matched, power is reflected at the feed-points of the antenna. Because this reflected power is generated by the source but not radiated by the antenna a mismatch reduces the efficiency. The analytical calculation of this effect of reflection gives no proper design rule for a communication system, because in this report the imaginary part of the impedance is not calculated. However with NEC both parts of the complex antenna impedance are calculated, which results in the next conclusion.
3. A significant deviation of the impedance of a half-wave dipole parallel to a perfectly conducting ground is detected when the distance between the antenna and the ground is smaller than about 0.27 \( \lambda \). For a smaller distance the return loss becomes less than 15.0 dB, i.e. more than 3.2 % of the available power is reflected. Hence if an antenna is placed at a distance larger than 0.27 \( \lambda \) from a perfectly conducting ground, most of the available power is radiated and the battery is used most effectively.

In this chapter the transmitting behaviour of the antenna parallel to a conducting ground is analysed. Not analysed is the behaviour of the field in the propagation medium between the two communicating parties. Electromagnetic waves may be reflected, absorbed or scattered by various objects. Reflections cause multipath, i.e. the information reaches the receiver more than once with an arbitrary phase shift. Power that is absorbed by objects can not be used for communication purposes. Also scattering disturbs the propagation of the waves. Hence the signal at the receiving antenna is influenced by the presence of objects and may reduce the performance of the portable antenna system. The effects of multipath are not the issue of this study. In the next part of this report, absorption and scattering by walls and human tissues in the neighbourhood of the antenna will be discussed.
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Chapter 4

An introduction to the Finite Difference Time Domain method

4.1 Introduction

This chapter reviews the formulation of the Finite Difference Time Domain method, that is first proposed in 1966 by Yee [1] and further developed in 1975 by Taflove and Brodwin [2]. The FDTD technique is a computational method that calculates the temporal evolution of the electromagnetic field within a region of space by stepping through time. At each time step, finite difference approximations are used to calculate the evolution of field components on a lattice of points in terms of the field components at previous time steps at nearby points.

The behaviour of the electromagnetic field is described by the differential Maxwell equations. Because the field components are evaluated by a computer, they are sampled in the space- and time-domain. The essence of FDTD is that these components are all evaluated at different points in the lattice, that is defined by the so-called 'Yee-cell'. The electric and magnetic fields are evaluated at interlacing time-steps. The needed differentiations for the Maxwell equations are approximated by the technique of central finite differencing as will be explained in section 4.2, which section gives the complete FDTD scheme as defined by Yee.

Section 4.3 shows that the size of the Yee cells has to be determined carefully to get a certain accuracy. This cell size depends on the minimum wavelength throughout the problem space and on the material parameters of the scatterers. With a given cell size and wavelength, a stability analysis limits the time step between two samples at which the fields are evaluated to get a stable algorithm.

With computational methods the data storage and therefore the problem space is limited by the available computer resources. Section 4.4 gives the absorbing boundary conditions that are introduced by Mur [6]. These define an artificial outer radiation boundary that prevents waves to reflect at this boundary. The absorbing boundary conditions are given by the paper of Mur but are fully derived in this report using the same central finite difference scheme as before.

At last section 4.5 gives some basic features of the FDTD fundamentals, like the way of building an object and specification of the incident field. Also an assessment of the requirements of hardware resources like memory storage and computation time is made as a function of the number of cells.
4.2 Finite Difference Time Domain equations

In this section the differential Maxwell equations are approximated by finite difference equations. The derivation of the FDTD equations is started with examining the homogeneous Maxwell equations in the time domain for a linear and source-free medium. These equations contain all information needed to completely specify the field behavior, when the initial field values are known and the boundary conditions are applied:

\[ \nabla \times \vec{E} = -\partial_t \vec{B} \] (4.1)
\[ \nabla \times \vec{H} = \partial_t \vec{E} + \vec{J} \] (4.2)

where

\[ \vec{B} = \epsilon \vec{E} \] (4.3)
\[ \vec{H} = \mu \vec{E} \] (4.4)
\[ \vec{J} = \sigma \vec{E} \] (4.5)

In this report the time derivative \( \partial / \partial t \) is notated as \( \partial_t \). In the so called constitutional equations (4.3) - (4.5), the linear isotropic medium is defined by the material parameters \( \epsilon, \mu \) and \( \sigma \). Substituting these relations in the Maxwell equations (4.1) - (4.2) gives:

\[ \nabla \times \vec{E} = -\mu \partial_t \vec{H} \] (4.6)
\[ \nabla \times \vec{H} = \epsilon \partial_t \vec{E} + \sigma \vec{E} \] (4.7)

The Maxwell equations are rearranged into the form that is used for FDTD:

\[ \partial_t \vec{H} = -\frac{1}{\mu} \nabla \times \vec{E} \] (4.8)
\[ \partial_t \vec{E} = \frac{1}{\epsilon} \nabla \times \vec{H} - \frac{\sigma}{\epsilon} \vec{E} \] (4.9)

In a Cartesian coordinate system the Maxwell equations (4.8) and (4.9) become for each component:

\[ \partial_t E_x = \frac{1}{\epsilon} (\partial_y \mathcal{H}_z - \partial_z \mathcal{H}_y) - \frac{\sigma}{\epsilon} E_x \] (4.10)
\[ \partial_t E_y = \frac{1}{\epsilon} (\partial_z \mathcal{H}_x - \partial_x \mathcal{H}_z) - \frac{\sigma}{\epsilon} E_y \] (4.11)
\[ \partial_t E_z = \frac{1}{\epsilon} (\partial_x \mathcal{H}_y - \partial_y \mathcal{H}_x) - \frac{\sigma}{\epsilon} E_z \] (4.12)
\[ \partial_t \mathcal{H}_x = -\frac{1}{\mu} (\partial_y E_z - \partial_z E_y) \] (4.13)
\[ \partial_t \mathcal{H}_y = -\frac{1}{\mu} (\partial_z E_x - \partial_x E_z) \] (4.14)
\[ \partial_t \mathcal{H}_z = -\frac{1}{\mu} (\partial_x E_y - \partial_y E_x) \] (4.15)

The equations (4.10)-(4.15) will be solved numerically. Therefore the six field components, \( E_x, E_y, E_z, \mathcal{H}_x, \mathcal{H}_y \) and \( \mathcal{H}_z \), are sampled at equidistant points in time and space. The derivatives of these components are evaluated from these sampled data. The way to approximate the derivatives with polynomial interpolation will be explained next.
4.2.1 Type of differencing, polynomial interpolation

In this paragraph a function $f(x)$ is sampled and approximated by polynomial interpolation. The derivative $f'(x)$ of this function will be expressed in terms of the interpolation coefficients. Next a type of differencing will be chosen that is appropriate for numerical differentiation. This type of differencing will be used to evaluate the differences in equations (4.10)-(4.15). Three types of differences will be analysed, i.e. forward, central and backward differences [5].

The function $f(x)$ is sampled at $x_n = n \Delta x$, $n = 0, \ldots, N$.

$$f_n = f(x_n). \quad (4.16)$$

This function is approximated by a series that is called the Lagrange polynomial interpolation formula plus an error term [3]:

$$f(x) = \sum_{n=0}^{N} f_n l_n(x) + \frac{f^{(N+1)}(\xi)}{(N+1)!} \tau(x), \quad (4.17)$$

where

$$\tau(x) = \sum_{n=0}^{N} (x - x_n)$$

$$l_n(x) = \prod_{i=0 \atop i \neq n}^{N} \frac{x - x_i}{(x_n - x_i)}, \quad (4.18)$$

The value of $\xi$ must be taken at a certain point, somewhere in the interval between $x_0$ and $x_N$. Since the exact location of $\xi$ is not known, the maximum value of $|f^{(N+1)}(\xi)|$ is taken in the error estimate. The derivative of $f(x)$ to $x$ is obtained by taking the derivatives of each term in equation (4.17):

$$f'(x) = \sum_{n=0}^{N} f_n l'_n(x) + \frac{f^{(N+1)}(\xi)}{(N+1)!} \tau'(x) + \frac{f^{(N+2)}(\xi)}{(N+1)!} \tau(x). \quad (4.19)$$

The last term in equation (4.19) becomes zero when $x$ is taken at a sampling point. The error estimate will be smaller for larger values of the order $N$ of the polynomials in $l_n(x)$. Let $N = 2$ and take the samples at equidistantial points, i.e.

\begin{align*}
  x_1 &= x_0 + h \\
  x_2 &= x_0 + 2h,
\end{align*}

where $h$ is the distance between two samples. The function $l_n(x)$ then becomes

\begin{align*}
  l_0(x) &= \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x - x_0 - h)(x - x_0 - 2h)}{2h^2} \\
  l_1(x) &= \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x - x_0)(x - x_0 - 2h)}{-h^2} \\
  l_2(x) &= \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{(x - x_0)(x - x_0 - h)}{2h^2},
\end{align*}

with their derivatives given by

\begin{align*}
  l'_0(x) &= \frac{(x - x_0 - h) + (x - x_0 - 2h)}{2h^2}
\end{align*}
\[ I'(x) = \frac{(x-x_0) + (x-x_0-2h)}{-h^2} \]
\[ I''(x) = \frac{(x-x_0) + (x-x_0-h)}{2h^2}. \]  

(4.22)

These derivatives are calculated at the three sampling points:

\[ I_0'(x_0) = -\frac{3}{2h}, \quad I_0'(x_1) = -\frac{1}{2h}, \quad I_0'(x_2) = \frac{1}{2h} \]
\[ I_1'(x_0) = \frac{2}{h}, \quad I_1'(x_1) = 0, \quad I_1'(x_2) = -\frac{2}{h} \]
\[ I_2'(x_0) = -\frac{1}{2h}, \quad I_2'(x_1) = \frac{1}{2h}, \quad I_2'(x_2) = \frac{3}{2h} \]  

(4.23)

The same as for the function \( l_n(x) \) is done for \( \pi(x) \):

\[ \pi(x) = (x-x_0)(x-x_1)(x-x_2) \]
\[ \pi'(x) = (x-x_1)(x-x_2) + (x-x_0)(x-x_2) + (x-x_0)(x-x_1) \]
\[ \pi'(x_0) = (x_0-x_1)(x_0-x_2) = 2h^2 \]
\[ \pi'(x_1) = (x_1-x_0)(x_1-x_2) = -h^2 \]
\[ \pi'(x_2) = (x_2-x_0)(x_2-x_1) = 2h^2 \]  

(4.24)

Then the derivative of \( f(x) \) becomes:

\[ f'(x) = I_0'(x)f_0 + I_1'(x)f_1 + I_2'(x)f_2 + \frac{f^{(3)}(\xi)}{6} \pi'(x) + \frac{f''(\xi)}{6} \pi(x) \]  

(4.25)

The last term in equation (4.25) vanishes when \( f(x) \) is taken at one of the sampling points. The values of the coefficients are substituted and equation (4.25) is evaluated at each point:

\[ f'(x_0) = \frac{1}{h} \left[ -\frac{3}{2} f_0 + 2f_1 - \frac{1}{2} f_2 \right] + \frac{h^2}{3} f^{(3)}(\xi) \]  

(4.26)

\[ f'(x_1) = \frac{1}{2h} [f_2 - f_0] - \frac{h^2}{6} f^{(3)}(\xi) \]  

(4.27)

\[ f'(x_2) = \frac{1}{h} \left[ \frac{3}{2} f_2 - 2f_1 - \frac{1}{2} f_0 \right] + \frac{h^2}{3} f^{(3)}(\xi) \]  

(4.28)

Choosing \( h \) as small as possible will quadratically reduce the error in numerical differentiation for all three types of differencing. From equations (4.26)-(4.28) it becomes clear why the central differencing is most suitable for numerical differentiation:

1. Differentiating in the middle of the interval (central differences) is twice as accurate as differentiating at the ends (forward and backward differences). The error term for central differencing \( h^2/6 f^{(3)}(\xi) \) is half the error term of the other two types of differencing.

2. The function value at the middle of the interval is not needed to evaluate the derivative at this point, because the value \( f_1 \) is not needed for central differencing.
4.2.2 FDTD-scheme (1-D)

The technique of the FDTD algorithm will be explained here for a simple, one dimensional configuration. In the three dimensional space the principles of the method are the same.

A transverse electromagnetic wave propagates in the direction $z$ through a medium that is discretised in this direction only (Figure 4.1). In this example, no electric conductivity is assumed, i.e. $\sigma = 0$. The components of the wave are, for example, $E_x$ and $H_y$. So the differential equations (4.10) - (4.15) that define the problem become

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon} \frac{\partial H_y}{\partial z}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu} \frac{\partial E_x}{\partial z}. \tag{4.29} \tag{4.30}$$

These equations are discretised in a way that they are easy implemented on a computer. Because the time derivative of the electric field is a function of the space derivative of the magnetic field, the components $E_x$ and $H_y$ are interleaved spatially and temporally as illustrated by Figure 4.2. The discretisation is given by

$$E^n[m] = E_x(m\Delta z, n\Delta t) \tag{4.31}$$

$$H^{n+\frac{1}{2}}[m + \frac{1}{2}] = H_y((m + \frac{1}{2})\Delta z, (n + \frac{1}{2})\Delta t), \tag{4.32}$$

where the subscripts $x$ and $y$ are omitted for short notation purposes. This is allowed because the $E$ and $H$ fields have only one component each. The differentials in equation (4.29) are replaced by the central differences

$$\frac{\partial E_x}{\partial t} \big|_{t=(n-\frac{1}{2})\Delta t} = \frac{E^n[m] - E^{n-1}[m]}{\Delta t} \tag{4.33}$$

$$\frac{\partial H_y}{\partial t} \big|_{z=m\Delta z} = \frac{H^{n-\frac{1}{2}}[m + \frac{1}{2}] - H^{n-\frac{1}{2}}[m - \frac{1}{2}]}{\Delta z}. \tag{4.34}$$

Combining these equations (4.33) and (4.34) with (4.29) gives the FDTD-equation for the component $E_x$

$$E^n[m] = E^{n-1}[m] - \frac{\Delta t}{\epsilon\Delta z} \left( H^{n-\frac{1}{2}}[m + \frac{1}{2}] - H^{n-\frac{1}{2}}[m - \frac{1}{2}] \right). \tag{4.35}$$
The FDTD-equation for the component $H_y$ is obtained in the same way

$$
H^{n+\frac{1}{2}}[m + \frac{1}{2}] = H^{n-\frac{1}{2}}[m + \frac{1}{2}] - \frac{\Delta t}{\mu \Delta z} (E^n[m + 1] - E^n[m]).
$$

(4.36)

Equation (4.35) shows that the component $E^n[m]$ is calculated by three other components. These are the two neighbouring components of the $H$-field of half a timestep earlier and the same $E$-field component of one timestep earlier. When all values of the $E$-field are evaluated at the time $n \Delta t$, the same is done for the $H$-field at the time $(n + \frac{1}{2}) \Delta t$ and so on. This so called leap-frog scheme is illustrated in Figure 4.2.

![Figure 4.2: Leap-frog scheme for a one dimensional problem space](image)

Because the FDTD algorithm has to be implemented on a computer, which has a finite amount of memory, the problem space has to be truncated at some place by an absorbing boundary. For example, the space is truncated after $z = (m + \frac{1}{2}) \Delta z$ in Figure 4.2. The variable $E^n[m + 1]$ is needed for the calculation of $H^{n+\frac{1}{2}}[m + \frac{1}{2}]$ and is evaluated in such way that an incident plane wave that propagates in the positive $z$-direction does not reflect at the artificial boundary at $z = (m + 1) \Delta z$. These so called absorbing boundary conditions are derived in section 4.4.
4.2.3 FDTD-scheme (3-D)

The three dimensional FDTD-equations are derived in the same way as in the one dimensional case. The six field components are sampled at certain places in a so called Yee-cell (Figure 4.3), that are given by equations (4.37)-(4.42).

\[ E_{x}^{n}[k + \frac{1}{2}, l, m] = E_{x}((k + \frac{1}{2})\Delta x, l\Delta y, m\Delta z, n\Delta t) \]  
(4.37)

\[ E_{y}^{n}[k, l + \frac{1}{2}, m] = E_{y}(k\Delta x, (l + \frac{1}{2})\Delta y, m\Delta z, n\Delta t) \]  
(4.38)

\[ E_{z}^{n}[k, l, m + \frac{1}{2}] = E_{z}(k\Delta x, l\Delta y, (m + \frac{1}{2})\Delta z, n\Delta t) \]  
(4.39)

\[ H_{x}^{n+\frac{1}{2}}[k + \frac{1}{2}, l, m + \frac{1}{2}] = H_{x}(k\Delta x, (l + \frac{1}{2})\Delta y, (m + \frac{1}{2})\Delta z, (n + \frac{1}{2})\Delta t) \]  
(4.40)

\[ H_{y}^{n+\frac{1}{2}}[k + \frac{1}{2}, l, m + \frac{1}{2}] = H_{y}(k + \frac{1}{2})\Delta x, l\Delta y, (m + \frac{1}{2})\Delta z, (n + \frac{1}{2})\Delta t) \]  
(4.41)

\[ H_{z}^{n+\frac{1}{2}}[k + \frac{1}{2}, l, m] = H_{z}(k + \frac{1}{2})\Delta x, (l + \frac{1}{2})\Delta y, m\Delta z, (n + \frac{1}{2})\Delta t) \]  
(4.42)
The derivatives with respect to time and space in equation (4.10) are replaced with central differences:

\[
\frac{\partial E_x}{\partial t} \bigg|_{t=(n-\frac{1}{2})\Delta t} = \frac{1}{\epsilon} \left( \frac{\partial_y H_z |_{y=\Delta y} - \partial_z H_y |_{z=\Delta z}}{\Delta y} \right) - \frac{c}{\epsilon} E_x \tag{4.43}
\]

becomes

\[
\frac{E^n_x[k + \frac{1}{2}, l, m] - E^{n-1}_x[k + \frac{1}{2}, l, m]}{\Delta t} = \frac{1}{\epsilon} \left( \frac{H^{n-\frac{1}{2}}_z[k + \frac{1}{2}, l + \frac{1}{2}, m] - H^{n-\frac{1}{2}}_z[k + \frac{1}{2}, l - \frac{1}{2}, m]}{\Delta y} \right) - \frac{c}{\epsilon} E^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m] \tag{4.44}
\]

A problem with this interleaving principle is that not all of the \(E_x\) terms in equation (4.44) are sampled at the appropriate time. On the left hand of equation (4.44) the \(E_x\) field is calculated at the time \(t = n\Delta t\) or \(t = (n - 1)\Delta t\), but the \(E_x\) field on the right hand is sampled at the time \(t = (n - \frac{1}{2})\Delta t\), which data is not available from the algorithm. In the originating paper of Yee [1] this problem is not mentioned. He has chosen the medium either nonconducting or perfect conducting, so the current term is not relevant. In the paper of Taflowe and Brodwin [2] the problem is solved by letting

\[
E^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m] = E^n_x[k + \frac{1}{2}, l, m] \tag{4.45}
\]

using the most recent value of the electric field. A better solution to the mentioned problem is obtained by linear interpolation in time to approximate the intermediate value [5]

\[
E^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m] = \frac{E^{n-1}_x[k + \frac{1}{2}, l, m] + E^n_x[k + \frac{1}{2}, l, m]}{2} \tag{4.46}
\]

With the approximation of equation (4.45) the FDTD equation (4.44) becomes

\[
E^n_x[k + \frac{1}{2}, l, m] = \frac{\epsilon}{\epsilon + \sigma \Delta t} E^{n-1}_x[k + \frac{1}{2}, l, m] + \frac{\Delta t}{\epsilon + \sigma \Delta t} \left( \frac{H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l + \frac{1}{2}, m] - H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l - \frac{1}{2}, m]}{\Delta y} \right) - \frac{c}{\epsilon} E^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m] \tag{4.47}
\]

With the approximation of equation (4.46) the same FDTD equation becomes

\[
E^n_x[k + \frac{1}{2}, l, m] = \frac{2\epsilon - \sigma \Delta t}{2\epsilon + \sigma \Delta t} E^{n-1}_x[k + \frac{1}{2}, l, m] + \frac{2\Delta t}{2\epsilon + \sigma \Delta t} \left( \frac{H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l + \frac{1}{2}, m] - H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l - \frac{1}{2}, m]}{\Delta y} \right) - \frac{c}{\epsilon} E^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m] \tag{4.48}
\]
In the remainder of this section the first order approximation for $E^{n-\frac{1}{2}}$ of equation (4.45) is used, although the second order method is more accurate and consistent with the FDTD scheme that uses also a second order interpolation. The reason for this choice is that the used software package uses the first order method. The complete FDTD scheme then becomes

$$E^n_x[k + \frac{1}{2}, l, m] = \frac{\varepsilon}{\varepsilon + \sigma \Delta t} E^{n-1}_x[k + \frac{1}{2}, l, m]$$

$$+ \frac{\Delta t}{\varepsilon + \sigma \Delta t} \left( \frac{H^{n-\frac{1}{2}}_y[k + \frac{1}{2}, l, m] - H^{n-\frac{1}{2}}_y[k + \frac{1}{2}, l - \frac{1}{2}, m]}{\Delta y} \right)$$

$$- \frac{H^{n-\frac{1}{2}}_y[k + \frac{1}{2}, l, m + \frac{1}{2}] - H^{n-\frac{1}{2}}_y[k + \frac{1}{2}, l, m - \frac{1}{2}]}{\Delta z} \right)$$

(4.49)

$$E^n_y[k, l + \frac{1}{2}, m] = \frac{\varepsilon}{\varepsilon + \sigma \Delta t} E^{n-1}_y[k, l + \frac{1}{2}, m]$$

$$+ \frac{\Delta t}{\varepsilon + \sigma \Delta t} \left( \frac{H^{n-\frac{1}{2}}_x[k, l + \frac{1}{2}, m + \frac{1}{2}] - H^{n-\frac{1}{2}}_x[k, l + \frac{1}{2}, m - \frac{1}{2}]}{\Delta z} \right)$$

$$- \frac{H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m + \frac{1}{2}] - H^{n-\frac{1}{2}}_x[k + \frac{1}{2}, l, m - \frac{1}{2}]}{\Delta y} \right)$$

(4.50)

$$E^n_z[k, l, m + \frac{1}{2}] = \frac{\varepsilon}{\varepsilon + \sigma \Delta t} E^{n-1}_z[k, l, m + \frac{1}{2}]$$

$$+ \frac{\Delta t}{\varepsilon + \sigma \Delta t} \left( \frac{H^{n-\frac{1}{2}}_z[k, l, m + \frac{1}{2}] - H^{n-\frac{1}{2}}_z[k - \frac{1}{2}, l, m + \frac{1}{2}]}{\Delta z} \right)$$

$$- \frac{H^{n-\frac{1}{2}}_z[k, l + \frac{1}{2}, m + \frac{1}{2}] - H^{n-\frac{1}{2}}_z[k, l + \frac{1}{2}, m - \frac{1}{2}]}{\Delta y} \right)$$

(4.51)

$$H^{n+\frac{1}{2}}_x[k, l + \frac{1}{2}, m + \frac{1}{2}] = H^{n-\frac{1}{2}}_x[k, l + \frac{1}{2}, m + \frac{1}{2}]$$

$$+ \frac{\Delta t}{\mu} \left( \frac{E^n_y[k, l + \frac{1}{2}, m + 1] - E^n_y[k, l + \frac{1}{2}, m]}{\Delta z} \right)$$

$$- \frac{E^n_y[k, l + 1, m + \frac{1}{2}] - E^n_y[k, l, m + \frac{1}{2}]}{\Delta y} \right)$$

(4.52)

$$H^{n+\frac{1}{2}}_y[k + \frac{1}{2}, l, m + \frac{1}{2}] = H^{n-\frac{1}{2}}_y[k + \frac{1}{2}, l, m + \frac{1}{2}]$$

$$+ \frac{\Delta t}{\mu} \left( \frac{E^n_z[k + 1, l, m + \frac{1}{2}] - E^n_z[k, l, m + \frac{1}{2}]}{\Delta z} \right)$$

$$- \frac{E^n_z[k + 1, l + \frac{1}{2}, m + \frac{1}{2}] - E^n_z[k + 1, l, m + \frac{1}{2}]}{\Delta y} \right)$$

(4.53)

$$H^{n+\frac{1}{2}}_z[k + \frac{1}{2}, l + \frac{1}{2}, m] = H^{n-\frac{1}{2}}_z[k + \frac{1}{2}, l + \frac{1}{2}, m]$$

$$+ \frac{\Delta t}{\mu} \left( \frac{E^n_y[k + \frac{1}{2}, l, m + 1] - E^n_y[k + \frac{1}{2}, l, m]}{\Delta y} \right)$$

$$- \frac{E^n_y[k + 1, l + \frac{1}{2}, m] - E^n_y[k + 1, l + \frac{1}{2}, m]}{\Delta x} \right)$$

(4.54)
4.3 Determining the cell size and the time step

In the previous chapter the segment length of a wire that is modeled in NEC is discussed. In general there should be at least seven to ten segments per wavelength. In FDTD this is also an often used criterium. To ensure the accuracy of the computed results, the cell size must be taken as a small fraction (e.g. 0.1\lambda) of either the minimum wavelength expected or the minimum scatterer dimension. Note that the wavelength must be determined in the medium with the lowest speed of the wave propagation, i.e. in the medium with the largest dielectric constant.

When the cell dimensions are chosen then the stability of the time stepping algorithm in the FDTD scheme is ensured by the Courant criterium:

\[ c\Delta t \leq \frac{1}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}}, \]  

(4.55)

where \( c = c_0\sqrt{\varepsilon_r} \) is the maximum wave phase velocity expected within the problem space. This velocity is located where \( \varepsilon_r \) has its highest value. This criterium is derived from the Von Neumann stability analysis and will be outlined in the next paragraph. This analysis assumes a locally homogeneous material.

Suppose that there are \( n \) cells per wavelength, i.e. \( \Delta x = \Delta y = \Delta z = \lambda/n \). Then equation (4.55) becomes

\[ \Delta t \leq \frac{\lambda}{n\sqrt{3c_0\varepsilon_r}} = \frac{1}{nf\sqrt{3}}, \]  

(4.56)

where \( f \) is the maximum frequency.

4.3.1 Stability analysis

At every time step, when the fields are evaluated from the previous values, the error in the approximations will accumulate, because these previous values already contain some error. However, the presence of exponentially increasing instabilities must be avoided. Such instable solutions are indicated by the eigenmodes of the system. A method to investigate the stability of the finite difference schemes of section 4.2 is the so called Von Neumann analysis [5]. This method is based on a medium that is considered locally homogeneous. Therefore the assumption is made that the medium properties vary sufficiently gradual, i.e. the space discretisation is sufficiently fine and has to be defined before the stability is analysed. When the problem space is infinite and homogeneous, the eigenmodes of the system are of the form

\[ H_n^{n+\frac{1}{2}}[k, l, \frac{1}{2}, m + \frac{1}{2}] = H_0 v^n + \frac{1}{2} \exp(-j\beta_x k \Delta x - j\beta_y (l + \frac{1}{2}) \Delta y - j\beta_z (m + \frac{1}{2}) \Delta z) \]  

(4.57)

\[ H_n^{n+\frac{1}{2}}[k + \frac{1}{2}, l, m + \frac{1}{2}] = H_0 v^n + \frac{1}{2} \exp(-j\beta_x (k + \frac{1}{2}) \Delta x - j\beta_y l \Delta y - j\beta_z (m + \frac{1}{2}) \Delta z) \]  

(4.58)

\[ H_n^{n+\frac{1}{2}}[k + \frac{1}{2}, l + \frac{1}{2}, m] = H_0 v^n + \frac{1}{2} \exp(-j\beta_x (k + \frac{1}{2}) \Delta x - j\beta_y (l + \frac{1}{2}) \Delta y - j\beta_z m \Delta z) \]  

(4.59)

\[ E_n^{n}[k + \frac{1}{2}, l, m] = E_0 v^n \exp(-j\beta_x (k + \frac{1}{2}) \Delta x - j\beta_y l \Delta y - j\beta_z m \Delta z) \]  

(4.60)

\[ E_n^{n}[k, l + \frac{1}{2}, m] = E_0 v^n \exp(-j\beta_x k \Delta x - j\beta_y (l + \frac{1}{2}) \Delta y - j\beta_z m \Delta z) \]  

(4.61)

\[ E_n^{n}[k, l, m + \frac{1}{2}] = E_0 v^n \exp(-j\beta_x k \Delta x - j\beta_y l \Delta y - j\beta_z (m + \frac{1}{2}) \Delta z) \]  

(4.62)
for a wave that is defined by the propagation wave-number $\vec{\beta}$. In these equations the position of the wave is not defined at the origin of each Yee cell but at the same location as the field-components are sampled within the Yee cell.

The eigenmodes, which are described by (4.57) - (4.62), are stable if

$$|\xi| \leq 1.$$  \hfill (4.63)

The field equations of the eigenmodes (4.57) - (4.62) are substituted in the discrete Maxwell equations (4.10) - (4.15). The result is:

$$\left( \xi - \frac{\epsilon}{\epsilon + \sigma \Delta t} \right) \vec{E}_0 = -j \Delta t \left( \frac{\epsilon}{\epsilon + \sigma \Delta t} \right) \sqrt{\xi} \left( \vec{\beta}^- \times \vec{H}_0 \right)$$  \hfill (4.64)

$$\left( \xi - 1 \right) \vec{H}_0 = \frac{\Delta t}{\mu} \sqrt{\xi} \left( \vec{\beta}^- \times \vec{E}_0 \right),$$  \hfill (4.65)

with the appropriate propagation vector $\vec{\beta}^- = \beta^-_x \hat{a}_x + \beta^-_y \hat{a}_y + \beta^-_z \hat{a}_z$, given by

$$\beta^-_x = \frac{2}{\Delta x} \sin \left( \frac{1}{2} \beta_+ \Delta x \right)$$  \hfill (4.66)

and similar expressions for $\beta^-_y$ and $\beta^-_z$.

Taking inner products with $\vec{E}_0$, $\vec{H}_0$ and $\vec{\beta}^-$, substituted with (4.64) and (4.65) directly gives the orthogonality relations

$$\vec{E}_0 \cdot \vec{H}_0 = 0,$$  \hfill (4.67)

$$\vec{\beta}^- \cdot \vec{E}_0 = 0,$$  \hfill (4.68)

$$\vec{\beta}^- \cdot \vec{H}_0 = 0.$$  \hfill (4.69)

The possible values of $\xi$ are found by eliminating either $\vec{E}_0$ or $\vec{H}_0$. For example:

$$(\xi - 1)(\xi - \frac{\epsilon}{\epsilon + \sigma \Delta t}) \vec{E}_0 = -j \Delta t \left( \frac{\epsilon}{\epsilon + \sigma \Delta t} \right) \sqrt{\xi} \left( \vec{\beta}^- \times (\xi - 1) \vec{H}_0 \right)$$

$$= \frac{(\Delta t)^2}{\epsilon \sigma \Delta t \mu} \xi \left[ \vec{\beta}^- \times \left( \frac{\Delta t}{\mu} \sqrt{\xi} \vec{\beta}^- \times \vec{E}_0 \right) \right]$$

$$= \frac{(\Delta t)^2}{(\epsilon + \sigma \Delta t) \mu} \xi \left[ \vec{\beta}^- \left( \vec{\beta}^- \cdot \vec{E}_0 \right) - \vec{E}_0 \left( \vec{\beta}^- \cdot \vec{\beta}^- \right) \right]$$

$$= - \frac{(\Delta t)^2}{(\epsilon + \sigma \Delta t) \mu} \xi |\vec{\beta}^-|^2 \vec{E}_0$$  \hfill (4.70)

This equation has a solution when

$$(\xi - 1)(\xi - A) + B^2 \xi = 0,$$  \hfill (4.71)

where $A$ and $B$ are given by

$$A = \frac{\epsilon}{\epsilon + \sigma \Delta t},$$  \hfill (4.72)

$$B = \frac{\Delta t}{\sqrt{\epsilon (\epsilon + \sigma \Delta t) \mu}} |\vec{\beta}^-|.$$  \hfill (4.73)

At first, equation (4.71) will be solved for nonconducting material. Next a stability criterium is derived for other materials.
Stability analysis for nonconducting material

For nonconducting material, equation (4.71) becomes

\[(\xi - 1)^2 + B^2\xi = 0,\]  

which is rearranged into a quadratic equation

\[\xi^2 + (B^2 - 2)\xi + 1 = 0,\]  

where \( B \) is given by

\[B = \frac{\Delta t}{\sqrt{\mu}}|\vec{\beta}| = c\Delta t|\vec{\beta}|.\]  

Equation (4.75) has only two solutions, \( \xi_1 \) and \( \xi_2 \). The quadratic equation becomes

\[(\xi - \xi_1)(\xi - \xi_2) = \xi^2 - (\xi_1 + \xi_2) + \xi_1\xi_2 = 0,\]  

where

\[
\begin{align*}
\xi_1\xi_2 &= 1 \\
\xi_1 + \xi_2 &= 2 - B^2
\end{align*}
\]

To satisfy the stability criterium of equation (4.63), \(|\xi_1| \leq 1\) and \(|\xi_2| \leq 1\). Therefore \( \xi_1 = \exp(j\phi_\xi) \), \( \xi_2 = \exp(-j\phi_\xi) \) and \( 2 - B^2 = 2\cos(\phi_\xi) \). This equation can only be satisfied for \( B^2 \leq 4 \), i.e. \( 0 \leq B \leq 2 \). Substitution of this condition in equation (4.76) gives

\[0 \leq c\Delta t|\vec{\beta}| \leq 2,\]  

what means that the time-step \( \Delta t \) must match the condition

\[0 \leq c\Delta t \leq \frac{1}{\sqrt{\frac{\sin^2(\frac{1}{2}\beta_x\Delta x)}{\Delta x^2} + \frac{\sin^2(\frac{1}{2}\beta_y\Delta y)}{\Delta y^2} + \frac{\sin^2(\frac{1}{2}\beta_z\Delta z)}{\Delta z^2},}}\]  

for all \( \beta_x \), \( \beta_y \) and \( \beta_z \). Therefore the stability criterium becomes the Courant criterium:

\[c\Delta t \leq \frac{1}{\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}},\]  

which is the same as equation (4.55).

Stability analysis for conducting material

When the material becomes conductive, say \( \sigma \Delta t = \alpha \varepsilon (\alpha \geq 0) \), equation (4.71) has to be solved, with

\[A = \frac{\varepsilon}{\varepsilon + \sigma \Delta t} = \frac{1}{1 + \alpha}, \quad (0 < A \leq 1),\]  

and

\[B = \frac{\Delta t}{\sqrt{(\varepsilon + \sigma \Delta t)\mu}}|\vec{\beta}| = \frac{c\Delta t}{\sqrt{1 + \alpha}}|\vec{\beta}|, \quad (B > 0).\]
Equation (4.71) is rearranged into a quadratic equation

\[ \xi^2 - (A + 1 - B^2)\xi + A = 0, \]  

(4.84)

with the two solutions \( \xi_+ \) and \( \xi_- \):

\[ \xi_{\pm} = \frac{A + 1 - B^2 \pm \sqrt{(A + 1 - B^2)^2 - 4A}}{2}, \]  

(4.85)

which have to satisfy the stability criterium of equation (4.63). The derivative of \( \xi_{\pm} \) with respect to \( B \) is given by

\[ \frac{\partial \xi_{\pm}}{\partial B} = -B \left( \frac{\sqrt{(A + 1 - B^2)^2 - 4A} \pm (A + 1 - B^2)}{\sqrt{(A + 1 - B^2)^2 - 4A}} \right), \]  

(4.86)

where

\[
\begin{align*}
\frac{\partial \xi_+}{\partial B} &\leq 0 & 0 < B &\leq \sqrt{A + 1 - 2\sqrt{A}} \\
\frac{\partial \xi_-}{\partial B} &\geq 0 & B &\geq \sqrt{A + 1 + 2\sqrt{A}}.
\end{align*}
\]  

(4.87)

(4.88)

In order to analyse the solutions \( \xi_+ \) and \( \xi_- \) in equation (4.85), the next situations are considered:

1. Both solutions are real valued, \((A + 1 - B^2)^2 - 4A \geq 0\), and

\[ 0 < B \leq \sqrt{A + 1 - 2\sqrt{A}}. \]  

(4.89)

For small values of \( B \), the solutions tend to

\[ \lim_{B \to 0} \xi_{\pm} = \frac{A + 1 \pm \sqrt{(A + 1)^2 - 4A}}{2} = \frac{A + 1 \pm (1 - A)}{2}, \]  

(4.90)

which are evaluated as two stable solutions:

\[ \lim_{B \to 0} \xi_+ = 1 \]

\[ \lim_{B \to 0} \xi_- = A. \]  

(4.91)

For higher values of \( B \), the solution \( \xi_+ \) in equation (4.85) decreases, but will never exceed the point \( \xi = -1 \). So this solution is stable.

For higher values of \( B \), the value of \( \xi_+ \) in equation (4.85) is a monotone and decreasing function of \( B \), on the interval of equation (4.89), because the derivative of \( \xi_+ \) with respect to \( B \) is negative. Therefore the minimum value of \( \xi_+ \) is reached at the maximum value of \( B \), i.e. \( \sqrt{A} < \xi_+ < 1 \) and this solution is stable at the interval of equation (4.89).

The value of \( \xi_- \) in equation (4.85) is a monotone and increasing function of \( B \), on the interval of equation (4.89), because the derivative of \( \xi_- \) with respect to \( B \) is positive. Therefore the maximum value of \( \xi_- \) is reached at the maximum value of \( B \), i.e. \( A < \xi_- < \sqrt{A} \) and this solution is also stable at the interval of equation (4.89).
2. Both solutions are real valued, \((A + 1 - B^2)^2 - 4A \geq 0\), and
\[
B \geq \sqrt{A + 1 + 2\sqrt{A}}.
\] (4.92)

For \(B = \sqrt{A + 1 + 2\sqrt{A}}\), the values of both solutions are \(\xi_\pm = -\sqrt{A}\).

The derivatives of \(\xi_+\) and \(\xi_-\) with respect to \(B\) are positive and negative respectively. Therefore, instable solutions may occur in the interval of equation (4.92) at \(\xi_+ = 1\) and \(\xi_- = -1\).

Equation (4.85) is solved for \(\xi_+ = 1\) and \(\xi_- = -1\). The former gives \(B = 0\) and the latter gives \(B = \sqrt{2A + 2}\). So stability is also obtained in the interval
\[
\sqrt{A + 1 + 2\sqrt{A}} \leq B \leq \sqrt{2A + 2}
\] (4.93)

3. Both solutions are complex conjugate, \((A + 1 - B^2)^2 - 4A < 0\), i.e.
\[
\sqrt{A + 1 - 2\sqrt{A}} < B < \sqrt{A + 1 + 2\sqrt{A}}.
\] (4.94)

The solutions of equation (4.85) become
\[
\xi_\pm = \frac{A + 1 - B^2 \pm j\sqrt{4A - (A + 1 - B^2)^2}}{2},
\] (4.95)

or
\[
\xi_\pm = \left|\xi\right| \exp(\pm j\phi_\xi).
\] (4.96)

The two solutions \(\xi_+\) and \(\xi_-\) of this quadratic equation (4.84) must satisfy
\[
\xi_+ \xi_- = A
\]
\[
\xi_+ + \xi_- = 1 + A - B^2
\] (4.97)

The solutions are given by
\[
\xi_\pm = \sqrt{A} \exp(\pm j\phi_\xi)
\] (4.98)

and
\[
\xi_+ + \xi_- = 1 + A - B^2 = 2\sqrt{A} \cos(\phi_\xi)
\] (4.99)

This equation can only be satisfied if
\[
\sqrt{1 + A - 2\sqrt{A}} \leq B \leq \sqrt{1 + A + 2\sqrt{A}},
\] (4.100)

so, if \(B\) is in the interval of equation (4.94), the solutions are stable.

Resuming, if \(B\) lies in one of the intervals of equations (4.89), (4.93) or (4.94), stability is obtained. Therefore
\[
0 < B \leq \sqrt{2A + 2}.
\] (4.101)

The expressions of \(A\) and \(B\) are substituted from equations (4.82) and (4.83):
\[
0 < c\Delta t|\vec{\beta}| \leq \sqrt{4 + 2\alpha},
\] (4.102)

which means that if the time-step \(\Delta t\) matches the condition (4.79), the FDTD algorithm is stable for conducting materials also. Therefore, the stability criterium in equation (4.55) can be used for both conducting and non-conducting materials.
4.4 Outer radiation boundary conditions

In analytical electromagnetics, the Maxwell equations are generally evaluated for an unbounded space, or with boundaries that are positioned at infinity. With computational analysis the data storage is limited by the available computer resources and the domain in which the field is computed must be limited. Unless the domain is enclosed by a perfectly conducting surface, the problem space will be truncated by an artificial boundary. All scatterers are chosen within this border, so the boundary is in vacuum. The waves are not vanished at this place, but they will propagate through it. All fields near the boundary are supposed to be outgoing waves, i.e. all sources are inside the problem space. The outgoing waves may not be reflected at the outer surface. Hence the boundary can be seen as a perfect absorber. In this section the first and second order absorbing boundary conditions that are derived by Mur [6] are introduced. These so called Mur-equations assume that all waves that propagate through the boundaries are plane, normal incident waves.

The absorbing boundary conditions become more accurate when these boundaries are placed at a larger distance from the scatterers within the problem space, because the spherical waves have less power at a larger distance from the sources and scatterers. Therefore, also the reflections have less power.

4.4.1 Absorbing boundary conditions: analytical

Because the boundary is placed in a free space environment, the boundary conditions are derived for the Maxwell equations in vacuum:
\[
\begin{align*}
\nabla \times \vec{E} &= -\mu_0 \partial_t \vec{H} \\
\nabla \times \vec{H} &= \epsilon_0 \partial_t \vec{E}.
\end{align*}
\]

(4.103) (4.104)

When \( \vec{H} \) is eliminated from equations (4.103) and (4.104) the following equation is obtained:

\[
\left[ \partial_x^2 + \partial_y^2 + \partial_z^2 - \frac{\partial^2}{c_0^2} \right] \vec{E} = 0,
\]

(4.105)

i.e. each component of the electric field independently satisfies the three dimensional wave equation.

The boundaries are chosen at the edge of the Yee cells. So all field components lie in the boundary surface. The \( \vec{H} \)-field components can be evaluated by using the finite difference equations (4.49) - (4.54). The \( \vec{E} \)-field components can not be evaluated in this way since this would require \( \vec{H} \)-field components that are outside the mesh. In this section, boundary conditions are derived, that are used to compute these \( \vec{E} \)-field. Assume that the mesh is located in the region \( x \geq 0 \). Next the boundary conditions for the plane \( x = 0 \) will be derived.

The solution of equation (4.105) can be splitted into two waves, one outgoing wave and one wave that is reflected at the boundary. The outgoing wave is a locally plane wave, propagating in the negative \( x \)-direction. Each component of the \( \vec{E} \)-field is given by

\[
W = \psi(t + s_x x + s_y y + s_z z),
\]

(4.106)
such that \( s_x^2 + s_y^2 + s_z^2 = c_0^{-2} \). However the reflected wave is defined by

\[
R = \psi(t - s_x x - s_y y - s_z z) = 0.
\]

(4.107)

A boundary condition that determines an outgoing wave that is absorbed is

\[
[\partial_x - s_x \partial_t] (W + R)|_{x=0} = 0,
\]

(4.108)
where $W + R$ is the total field, i.e. the sum of the outgoing and the reflected wave. An outgoing plane wave that is defined by equation (4.106) satisfies equation (4.108), but the reflected wave of equation (4.107) does not, unless $s_x = 0$. Each component of the outgoing wave always satisfies the three dimensional wave equation (4.105).

In equation (4.108) $s_x$ is substituted via $s_x^2 + s_y^2 + s_z^2 = c_0^{-2}$, to obtain

$$\left[ \partial_x - \frac{1}{c_0} \sqrt{1 - (c_0 s_y)^2 - (c_0 s_z)^2} \partial_t \right] W \bigg|_{x=0} = 0. \quad (4.109)$$

Now this equation is approximated. The first order approximation of a square root is simply unity:

$$s_x = \sqrt{1 - (c_0 s_y)^2 - (c_0 s_z)^2} = 1. \quad (4.110)$$

Hence the first order approximation of the boundary condition becomes

$$\left[ \partial_x - \frac{1}{c_0} \partial_t \right] W \bigg|_{x=0} = 0. \quad (4.111)$$

Use of the second approximation of the square root

$$s_x = \sqrt{1 - (c_0 s_y)^2 - (c_0 s_z)^2} = 1 - \frac{1}{2}((c_0 s_y)^2 + (c_0 s_z)^2) \quad (4.112)$$

yields the second order approximation of the boundary condition:

$$\left[ \partial_x - \frac{1}{c_0} \partial_t + \frac{1}{2c_0} ((c_0 s_y)^2 + (c_0 s_z)^2) \partial_t \right] W \bigg|_{x=0} = 0. \quad (4.113)$$

These approximations of the square roots are more accurate when the wave resembles a plane wave and the direction of the propagation is perpendicular to the absorbing surface.

Equation (4.113) is rewritten by taking its time derivative and multiplying every term by $1/c_0$:

$$\left[ \frac{1}{c_0} \partial_x \partial_t - \frac{1}{c_0^2} \partial_t^2 + \frac{1}{2} (s_y^2 + s_z^2) \partial_t^2 \right] W \bigg|_{x=0} = 0. \quad (4.114)$$

The values of $s_y$ and $s_z$ are fixed and are derived from equation (4.106):

$$s_y^2 \partial_t^2 W = \partial_y^2 W \quad (4.115)$$

$$s_z^2 \partial_t^2 W = \partial_z^2 W. \quad (4.116)$$

The second order boundary condition becomes

$$\left[ \frac{1}{c_0} \partial_x \partial_t - \frac{1}{c_0^2} \partial_t^2 + \frac{1}{2} (\partial_y^2 + \partial_z^2) \right] W \bigg|_{x=0} = 0. \quad (4.117)$$

In the next paragraph the first and second order boundary conditions (4.111) and (4.117) will be fit in the finite difference scheme of section 4.2.

In chapter 5, a software package is used, that is based on the FDTD-scheme. In this package, probably a combination of the first and second order absorbing boundary conditions is used. The second order a.b.c. require field values from adjacent Yee-cells. Therefore it can not be used for determining electric field values that are adjacent to the intersection of two terminating planes. At the edges of the problem space, first order a.b.c. must be applied. (See also page 35 of [4]).
4.4.2 Absorbing boundary conditions: finite difference approximation

The application of the finite difference time domain scheme to the absorbing boundary conditions is also given by Mur [6], but it is further explained in this paragraph. As an example the $E_z$ component is evaluated. The other components are evaluated in a similar way. The idea is to truncate the grid at six boundaries: $x = 0$, $x = K\Delta x$, $y = 0$, $y = L\Delta y$, $z = 0$ and $z = M\Delta z$. In this section, only the boundary at $x = 0$ is discussed.

For example, to calculate $H_x^{n+\frac{1}{2}}[0, l + \frac{1}{2}, m + \frac{1}{2}]$ and $H_y^{n+\frac{1}{2}}[\frac{1}{2}, l, m + \frac{1}{2}]$ component $E_z^n[0, l, m + \frac{1}{2}]$ has to be known. However the value of the latter variable is unknown because it is located outside the problem space and therefore has to be given by the boundary conditions.

First order absorbing boundary conditions

To transform the continuous first order absorbing boundary conditions into the discrete time-and space-domain, both the left hand and the right hand of

$$\frac{\partial}{\partial t}E_z = \frac{1}{c_0}\frac{\partial}{\partial x}E_z \quad (4.118)$$

are evaluated at $x = \frac{1}{2}\Delta x$, $y = k\Delta y$, $z = (m + \frac{1}{2})\Delta z$ and $t = (n + \frac{1}{2})\Delta t$. The reason for the choice of this particular space-time point will become clear after the discretisation. The derivatives of equation (4.118) are evaluated as follows:

$$2\partial_x E_z = \partial_x E_z(x, l\Delta y, (m + \frac{1}{2})\Delta z, n\Delta t)|_{x=\frac{1}{2}\Delta x} + \partial_x E_z(x, l\Delta y, (m + \frac{1}{2})\Delta z, (n + 1)\Delta t)|_{x=\frac{1}{2}\Delta x} \quad (4.119)$$

$$2\partial_tE_z = \partial_tE_z(0, l\Delta y, (m + \frac{1}{2})\Delta z, t)|_{t=\frac{1}{2}\Delta t} + \partial_tE_z(1, l\Delta y, (m + \frac{1}{2})\Delta z, t)|_{t=\frac{1}{2}\Delta t} \quad (4.120)$$

which are transformed into the next central difference equations:

$$2\partial_x E_z \Delta x = E_z^n[1, l, m + \frac{1}{2}] - E_z^n[0, l, m + \frac{1}{2}] + E_x^{n+1}[1, l, m + \frac{1}{2}] - E_x^{n+1}[0, l, m + \frac{1}{2}] \quad (4.121)$$

$$2\partial_tE_z \Delta t = E_t^{n+1}[0, l, m + \frac{1}{2}] - E_t^{n+1}[0, l, m + \frac{1}{2}] + E_x^{n+1}[1, l, m + \frac{1}{2}] - E_x^{n+1}[1, l, m + \frac{1}{2}] \quad (4.122)$$

These differences are substituted in equation (4.118) and rearranged to eventually obtain the desired relation for $E_z^{n+1}[0, l, m + \frac{1}{2}]$:

$$E_z^{n+1}[0, l, m + \frac{1}{2}] = E_z^n[1, l, m + \frac{1}{2}] + \frac{c_0\Delta t - \Delta x}{c_0\Delta t + \Delta x} \left( E_x^{n+1}[1, l, m + \frac{1}{2}] - E_x^n[0, l, m + \frac{1}{2}] \right) \quad (4.123)$$

Second order absorbing boundary conditions

To obtain the finite difference approximation of the second order boundary condition (4.117) the following steps are taken.

1. The equation that has to be discretised is

$$\left[ \frac{1}{c_0^2} \partial_x^2 - \frac{1}{c_0^2} \partial_t^2 + \frac{1}{2} (\partial_x^2 + \partial_t^2) \right] E_z = 0. \quad (4.124)$$

2. The observation point is at $x = \frac{1}{2}\Delta x$, $y = \frac{1}{2}\Delta y$, $z = \frac{1}{2}\Delta z$ and $t = n\Delta t$.

3. The derivative with respect to $z$ is approximated by a central difference with step $\Delta x$. The time derivative in the same term is approximated by a central difference with step $2\Delta t$. This will become clear after the next step.

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4. The second time derivative is approximated by

\[ \frac{\partial^2 f(t)}{\partial t^2} \approx \frac{\partial f(t + \Delta t) - 2f(t) + f(t - \Delta t)}{\Delta t^2} \]

which may be regarded as using the central difference for the first derivative twice. This term is averaged over the values for \( x = 0 \) and \( x = \Delta x \).

5. The same procedure is used for the terms involving \( \partial_x^2 \) and \( \partial_z^2 \).

These steps are applied to each term of (4.124) to obtain:

\[
\begin{align*}
\frac{1}{c_0} \partial_x\partial_t E_z & \approx \frac{1}{2c_0\Delta x\Delta t}\left\{ E_z^{n+1} [1, l, m + \frac{1}{2}] - E_z^{n-1} [1, l, m + \frac{1}{2}] - E_z^{n+1} [0, l, m + \frac{1}{2}] + E_z^{n-1} [0, l, m + \frac{1}{2}] \right\} \\
\frac{1}{c_0} \partial_x^2 E_z & \approx \frac{1}{2c_0^2\Delta t^2}\left\{ E_z^{n+1} [0, l, m + \frac{1}{2}] + E_z^{n+1} [1, l, m + \frac{1}{2}] - 2(E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}]) + E_z^{n-1} [0, l, m + \frac{1}{2}] + E_z^{n-1} [1, l, m + \frac{1}{2}] \right\} \\
\partial_z^2 E_z & \approx \frac{1}{2\Delta y^2}\left\{ E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}] - 2(E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}]) + E_z^n [0, l, m - \frac{1}{2}] + E_z^n [1, l, m - \frac{1}{2}] \right\} \\
\partial_y^2 E_z & \approx \frac{1}{2\Delta z^2}\left\{ E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}] - 2(E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}]) + E_z^n [0, l, m - \frac{1}{2}] + E_z^n [1, l, m - \frac{1}{2}] \right\}
\end{align*}
\]

These terms are gathered together and rearranged in an appropriate way. The result is the second order absorbing boundary condition in terms of the FDTD scheme. The derivation is given in the appendix of this chapter.

The result for a grid with cubical Yee-cells is

\[
E_z^{n+1} [0, l, m + \frac{1}{2}] = -E_z^{n-1} [1, l, m + \frac{1}{2}] + \frac{c_0 \Delta t - \Delta x}{c_0 \Delta t + \Delta x}\left\{ E_z^{n+1} [1, l, m + \frac{1}{2}] + E_z^{n-1} [0, l, m + \frac{1}{2}] \right\} + \frac{2\Delta x}{c_0 \Delta t + \Delta x}\left\{ E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}] \right\} \\
+ \frac{c_0^2 \Delta t^2}{2\Delta x (c_0 \Delta t + \Delta x)}\left\{ E_z^n [0, l + 1, m + \frac{1}{2}] + E_z^n [1, l + 1, m + \frac{1}{2}] \right\} \\
- 4(E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m + \frac{1}{2}]) + E_z^n [0, l - 1, m + \frac{1}{2}] + E_z^n [1, l - 1, m + \frac{1}{2}] \\
+ E_z^n [0, l, m - \frac{1}{2}] + E_z^n [1, l, m - \frac{1}{2}] + E_z^n [0, l, m + \frac{1}{2}] + E_z^n [1, l, m - \frac{1}{2}].
\]
4.5 FDTD-basics

4.5.1 Building objects in Yee cells

In each time step of the computational FDTD method one set of matrices is evaluated that contains all electromagnetic field components of the whole problem space. For each cell three components of the $E$ field and three of the $H$ field have to be updated. Another matrix specifies the material parameters of each cell. To update the $H$ field in three orthogonal directions, the values of the permeability $\mu_x$, $\mu_y$, and $\mu_z$ are needed. The update of the $E$ field requires both $\varepsilon$ and $\sigma$ for the same three directions. On one hand this provides the capability to define anisotropic materials. On the other hand this feature can be used to more accurately specify an object's boundary, because in one cell, the material parameters can be set to a different value for each component, which is sampled at an other place within the cell.

A material parameter is not specified by fully quantizing the value of it but just to store an integer that specifies the type of the material in the appropriate field of the matrix. The matching value of the parameter is listed in a separate table. The advantage of this method is the requirement of little computer storage to define the values of $\varepsilon$, $\sigma$ and $\mu$, because each value is stored once, while the matrix that defines objects only contains integers that require much less storage than floating point variables.

For example a value of 0 could specify free space and a 1 could be reserved for a perfect conductor. Other numbers define dielectric and magnetic materials. The number of possible values for each parameter determines the memory that has to be reserved for these integers.

4.5.2 Specifying the excitation

One advantage of FDTD is the capability of evaluating a range of frequencies in one run. Therefore an analytical formulation of the excitation is given that has a large frequency content. Other types of sources may be specified but in general it will be a Gaussian pulse.

$$f(t) = \exp(-\alpha((t - \beta \Delta t)^2)), \quad 0 < t < 2\beta \Delta t,$$

(4.133)

which function is sampled at the time $t = n \Delta t$. The parameters $\alpha$ and $\beta$ have to be chosen properly. The domain of $t$ in equation (4.133) can not range from $-\infty$ to $\infty$, because the duration of the time stepping
must be finite for implementation in a computer program. Hence the domain of \( f(t) \) in equation (4.133) is truncated at \( t = 0 \) and \( t = 2\beta \Delta t \). Choose \( \alpha = \left(\frac{4}{\beta \Delta t}\right)^2 \). At the truncation, the power of the pulse has the value of \( \exp(-16) \), or -139.0 dB in respect with its maximum value at \( t = \beta \Delta t \). The size of the time step \( \Delta t \) follows from the Courant criterium. Next \( \beta \) is determined by the desired frequency content of the pulse. This will be shown by an example.

Assume a frequency of 2.5 GHz. Then in free space the wavelength is 12.0 cm. Define a grid of cubical Yee cells in Cartesian coordinates with a cell size of 1.0 cm. The Courant criterium gives a time step size of 19.25 ps. Choose \( \beta = 32 \), where \( \beta \) equals the number of time steps in the Gaussian pulse from the peak value to the truncation value. For these parameters, the pulse and the Fourier transform of it are plotted in Figure 4.4.

Note that the power contents is considerably high at the higher frequencies, so the stability analysis must be carried out for these higher frequencies to avoid instabilities at all frequencies. (See page 35 of [4])

### 4.5.3 Resource requirements

The maximum size of the problem space that can be defined for an FDTD application is limited by the available computer resources. Therefore the method has not been very popular in the early years after Yee proposed it in 1966. Nowadays, the computers have more memory storage and are much faster. In future years they will even be more powerful. This section estimates in general the resource requirements that are needed for an FDTD application.

The problem space has to be fit in the computer, i.e. the matrices that are mentioned in section 4.5.1 must be able to contain all electromagnetic field components and also an identification for the material parameters of all cells. So for each cell, six variables for the field components and six variables for the material parameters have to be stored. Assume that a Fortran code is used that needs four bytes of memory for a single precision field variable and one byte for an integer variable that contains the material information. Using these material parameters (IDs) of one byte, eight different material parameters can be defined in the problem space for both the \( E \) and the \( H \) field in the three orthogonal directions. With these assumptions the required storage for a total number of \( N \) Yee cells can be estimated by

\[
\text{Storage} = N \times \left( \frac{6 \text{Components}}{\text{Cell}} \times 4 \text{Bytes/Component} + \frac{6 \text{IDs}}{\text{Cell}} \times 1 \text{Byte/ID} \right).
\]  

Equation (4.134) implies a required storage of 30 bytes per Yee cell. This number can be updated when other types of variables are used to get more accuracy or when more different material parameters have to be defined. The total amount of storage has to be summed with the amount of the memory needed to store the executable instructions and auxiliary variables to run the program.

Another limitation, imposed by the available computer resources is the time that the computer needs to complete the calculation of the whole problem, because a calculation time of days or weeks may not be acceptable. Assume again a total number of \( N \) cells. Consider the total number of time steps \( T \). The size of the time step is set by the Courant stability condition. For cubical cells with a cell side dimension of \( \Delta x \) this time step is

\[
\Delta t = \frac{\Delta x}{\sqrt{3}c}.
\]  

When the structure is not too much resonant, the number of time steps is typically ten times the number of cells on one side of the problem space [4], so an estimate for \( T \) is

\[
T \approx 10 \times \sqrt{3} N^{1/3}.
\]
Assume that ten floating point operations are needed to calculate the field of one component. With the above assumptions the total number of floating point operations is estimated by

\[
\text{Operations} = 10\sqrt{3} N^{4/3} \times 6 \frac{\text{Components}}{\text{Cell}} \times 10 \frac{\text{Operations}}{\text{Component}}.
\] (4.137)

For a very big problem space, the cell dimensions must be chosen small to avoid cumulative errors. Therefore some cautiousness must be taken when buying a computer for a certain problem.

At the Philips Research Laboratories Eindhoven workstations are used of the type HP 9000/735 and HP 9000/819 that have a memory storage capability of between 400.0 MB and 1.0 GB. The floating point unit is of the type PA7100 and PA7200 which run with a maximum speed of approximate 200 MFLOPS or 200 million floating point operations per second.

For example, a room has the dimensions 3.6 m x 3.6 m x 3.6 m. This room is discretised in vacuum at 2.5 GHz. Every cell has the dimensions of 0.1λ along every side of the cube. Then the information about 27 million cells have to be kept in the computer. To do this a computer is needed with a memory storage capacity of 800 MB. This computer will need 12 hours to evaluate all field components. After these 12 hours some time is needed for the post-processing, i.e. for Fourier transformations or calculation of impedances at certain points.
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Capita Selecta Elektromagnetisme: Computational Techniques for Transient Fields (Course 5N190)

Appendix, Derivation of the second order Mur equations

\[
\left[ \frac{1}{c_0} \frac{\partial x}{\partial t} - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} + \frac{1}{2} \left( \frac{\partial_y^2 + \partial_z^2}{c_0} \right) \right] W \bigg|_{x=0} = 0.
\]

\[
\frac{1}{c_0} \frac{\partial x}{\partial t} E_z \approx \frac{1}{2c_0 \Delta x \Delta t} \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] 
- E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[
\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} E_z \approx \frac{1}{2c_0^2 \Delta t^2} \left\{ E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n+1} [1, l, m + \frac{1}{2}] 
- 2(E_x^n [0, l, m + \frac{1}{2}] + E_x^n [1, l, m + \frac{1}{2}]) 
+ E_x^{n-1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [1, l, m + \frac{1}{2}] \right\}
\]

\[\partial_y^2 E_z \approx \frac{1}{2\Delta y^2} \left\{ E_x^n [0, l + 1, m + \frac{1}{2}] + E_x^n [1, l + 1, m + \frac{1}{2}] 
- 2(E_x^n [0, l, m + \frac{1}{2}] + E_x^n [1, l, m + \frac{1}{2}]) 
+ E_x^n [0, l, m - \frac{1}{2}] + E_x^n [1, l, m - \frac{1}{2}] \right\}
\]

\[\partial_z^2 E_z \approx \frac{1}{2\Delta z^2} \left\{ E_x^n [0, l, m + \frac{3}{2}] + E_x^n [1, l, m + \frac{3}{2}] 
- 2(E_x^n [0, l, m + \frac{1}{2}] + E_x^n [1, l, m + \frac{1}{2}]) 
+ E_x^n [0, l, m - \frac{1}{2}] + E_x^n [1, l, m - \frac{1}{2}] \right\} = 0
\]

\[\Rightarrow \left\{ E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n+1} [1, l, m + \frac{1}{2}] - 2(E_x^n [0, l, m + \frac{1}{2}] + E_x^n [1, l, m + \frac{1}{2}]) 
+ E_x^{n-1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [1, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]

\[\Rightarrow \left\{ E_x^{n+1} [1, l, m + \frac{1}{2}] - E_x^{n-1} [1, l, m + \frac{1}{2}] - E_x^{n+1} [0, l, m + \frac{1}{2}] + E_x^{n-1} [0, l, m + \frac{1}{2}] \right\}
\]
\[ E_{z}^{n+1} [0, l, m + \frac{1}{2}] \left(1 + \frac{c_{0}\Delta t}{\Delta x}\right) \]
\[ = -E_{z}^{n+1} [1, l, m + \frac{1}{2}] + 2E_{z}^{n} [0, l, m + \frac{1}{2}] + 2E_{z}^{n} [1, l, m + \frac{1}{2}] \]
\[ -E_{z}^{n-1} [0, l, m + \frac{1}{2}] - E_{z}^{n-1} [1, l, m + \frac{1}{2}] \]
\[ + \frac{c_{0}\Delta t}{\Delta x} \{ E_{z}^{n+1} [1, l, m + \frac{1}{2}] - E_{z}^{n-1} [1, l, m + \frac{1}{2}] \} \]
\[ + \frac{c_{0}\Delta t^{2}}{2\Delta y^{2}} \{ E_{z}^{n} [0, l + 1, m + \frac{1}{2}] + E_{z}^{n} [1, l + 1, m + \frac{1}{2}] - 2(E_{z}^{n} [0, l, m + \frac{1}{2}] + E_{z}^{n} [1, l, m + \frac{1}{2}]) \]
\[ + E_{z}^{n} [0, l - 1, m + \frac{1}{2}] + E_{z}^{n} [1, l - 1, m + \frac{1}{2}] \]
\[ + \frac{c_{0}\Delta t^{2}}{2\Delta z^{2}} \{ E_{z}^{n} [0, l, m + \frac{3}{2}] + E_{z}^{n} [1, l, m + \frac{3}{2}] - 2(E_{z}^{n} [0, l, m + \frac{1}{2}] + E_{z}^{n} [1, l, m + \frac{1}{2}]) \]
\[ + E_{z}^{n} [0, l, m - \frac{1}{2}] + E_{z}^{n} [1, l, m - \frac{1}{2}] \]

\[ E_{z}^{n+1} [0, l, m + \frac{1}{2}] \left(1 + \frac{c_{0}\Delta t}{\Delta x}\right) \]
\[ = -E_{z}^{n-1} [1, l, m + \frac{1}{2}] \left(1 + \frac{c_{0}\Delta t}{\Delta x}\right) \]

\[ E_{z}^{n+1} [0, l, m + \frac{1}{2}] = -E_{z}^{n-1} [1, l, m + \frac{1}{2}] \]

\[ \frac{\Delta x}{c_{0}\Delta t + \Delta x} \{ E_{z}^{n+1} [1, l, m + \frac{1}{2}] - 2E_{z}^{n} [0, l, m + \frac{1}{2}] - 2E_{z}^{n} [1, l, m + \frac{1}{2}] + E_{z}^{n-1} [0, l, m + \frac{1}{2}] \} \]
\[ + \frac{\Delta x}{c_{0}\Delta t + \Delta x} \frac{c_{0}\Delta t}{\Delta x} \{ E_{z}^{n+1} [1, l, m + \frac{1}{2}] + E_{z}^{n-1} [0, l, m + \frac{1}{2}] \} \]
\[ + \frac{\Delta x}{c_{0}\Delta t + \Delta x} \frac{c_{0}\Delta t^{2}}{2\Delta y^{2}} \{ E_{z}^{n} [0, l + 1, m + \frac{1}{2}] + E_{z}^{n} [1, l + 1, m + \frac{1}{2}] - 2(E_{z}^{n} [0, l, m + \frac{1}{2}] + E_{z}^{n} [1, l, m + \frac{1}{2}]) \]
\[ + E_{z}^{n} [0, l - 1, m + \frac{1}{2}] + E_{z}^{n} [1, l - 1, m + \frac{1}{2}] \]
\[ + \frac{\Delta x}{c_{0}\Delta t + \Delta x} \frac{c_{0}\Delta t^{2}}{2\Delta z^{2}} \{ E_{z}^{n} [0, l, m + \frac{3}{2}] + E_{z}^{n} [1, l, m + \frac{3}{2}] - 2(E_{z}^{n} [0, l, m + \frac{1}{2}] + E_{z}^{n} [1, l, m + \frac{1}{2}]) \]
\[ + E_{z}^{n} [0, l, m - \frac{1}{2}] + E_{z}^{n} [1, l, m - \frac{1}{2}] \]
\[ E_z^{n+1} \{0, l, m + \frac{1}{2}\} = -E_z^{n-1} \{1, l, m + \frac{1}{2}\} \]
\begin{align*}
&+ \left( \frac{\Delta x}{c_0 \Delta t + \Delta x} - \frac{\Delta x}{c_0 \Delta t + \Delta z} \right) \left\{ E_z^{n+1} \{1, l, m + \frac{1}{2}\} + E_z^{n-1} \{0, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{2 \Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l, m + \frac{1}{2}\} + E_z^n \{1, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{\Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l + 1, m + \frac{1}{2}\} + E_z^n \{1, l + 1, m + \frac{1}{2}\} - 2(E_z^n \{0, l, m + \frac{1}{2}\}) \\
&+ E_z^n \{1, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{\Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l, m + \frac{1}{2}\} + E_z^n \{1, l, m + \frac{1}{2}\} - 2(E_z^n \{0, l, m + \frac{1}{2}\}) \right\} \\
&+ E_z^n \{0, l, m - \frac{1}{2}\} \right\} + E_z^n \{1, l, m - \frac{1}{2}\} \\
\end{align*}

\[ E_z^{n+1} \{0, l, m + \frac{1}{2}\} = \]
\begin{align*}
&-E_z^{n-1} \{1, l, m + \frac{1}{2}\} + \frac{c_0 \Delta t - \Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^{n+1} \{1, l, m + \frac{1}{2}\} + E_z^{n-1} \{0, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{2 \Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l, m + \frac{1}{2}\} + E_z^n \{1, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{\Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l + 1, m + \frac{1}{2}\} + E_z^n \{1, l + 1, m + \frac{1}{2}\} - 2(E_z^n \{0, l, m + \frac{1}{2}\}) \\
&+ E_z^n \{1, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{\Delta x}{c_0 \Delta t + \Delta x} \left\{ E_z^n \{0, l, m + \frac{1}{2}\} + E_z^n \{1, l, m + \frac{1}{2}\} - 2(E_z^n \{0, l, m + \frac{1}{2}\}) \right\} \\
&+ E_z^n \{0, l, m - \frac{1}{2}\} \right\} + E_z^n \{1, l, m - \frac{1}{2}\} \\
\end{align*}

\( \Delta x = \Delta y = \Delta z \Rightarrow \)
\begin{align*}
& E_t^{n+1} \{0, l, m + \frac{1}{2}\} = \\
&-E_t^{n-1} \{1, l, m + \frac{1}{2}\} + \frac{c_0 \Delta t - \Delta x}{c_0 \Delta t + \Delta x} \left\{ E_t^{n+1} \{1, l, m + \frac{1}{2}\} + E_t^{n-1} \{0, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{2 \Delta x}{c_0 \Delta t + \Delta x} \left\{ E_t^n \{0, l, m + \frac{1}{2}\} + E_t^n \{1, l, m + \frac{1}{2}\} \right\} \\
&+ \frac{\Delta x}{2 \Delta x (c_0 \Delta t + \Delta x)} \left\{ E_t^n \{0, l + 1, m + \frac{1}{2}\} + E_t^n \{1, l + 1, m + \frac{1}{2}\} \\
&- 4(E_t^n \{0, l, m + \frac{1}{2}\}) + E_t^n \{1, l - 1, m + \frac{1}{2}\} \right\} \\
&+ E_t^n \{0, l, m + \frac{1}{2}\} + E_t^n \{1, l, m + \frac{1}{2}\} + E_t^n \{0, l, m - \frac{1}{2}\} + E_t^n \{1, l, m - \frac{1}{2}\} \\
\end{align*}
Chapter 5

Analysing the half-wave dipole using XFDTD

5.1 Introduction

In this chapter, the finite difference time domain (FDTD) method is used to analyse the interaction between an antenna and a large object. The FDTD method is implemented in the computer program XFDTD [1]. The used antenna is a half-wave dipole antenna that is resonant at a frequency of 2.5 GHz in free space conditions. In fact, this antenna is not exact half the wavelength, but somewhat shorter to get a resonant antenna at the considered frequency.

As a start, in section 5.2 the antenna is analysed in a free space environment. The antenna impedance at the feed-points and the radiation patterns are calculated to compare them to the results of the earlier used calculation methods.

In section 5.3, the antenna is placed parallel to a perfectly conducting ground. Because of the finite properties of the memory-size on a computer board, this ground has no infinite dimensions but is rather large enough to approximate an infinitely large ground. The impedance and the return loss at the antenna feed-points are calculated, as well as the radiation patterns. The results are again compared to analytical calculations and to calculations with NEC.

At last, in section 5.4 a dielectric ground is introduced. The question is whether the effects of the perfectly conducting ground on the antenna performance are worst case, i.e. the power reflection is highest when the ground is perfectly conducting.

5.2 The half-wave dipole antenna in free space

To test the program XFDTD a simple half-wave dipole antenna is investigated. To design the antenna, several parameters have to be set. The antenna is modeled by two wires. Each of them take four cells with the parameters of perfectly conducting material. Within each cell only the z-component of the material parameters is set to 'perfect conducting' (PEC). A voltage source is put in the gap between these two wires. A transient analysis will determine the resonance of the antenna. The length of the antenna is chosen such, that the antenna is resonant at the frequency of 2.5 GHz.
5.2.1 Transient analysis

The length of the antenna in resonance is exact half the wavelength in case of an infinitely thin wire. In FDTD simulations as well as in practical situations the wire has a finite thickness. Therefore the resonance-length is somewhat shorter than half the wavelength.

An FDTD-grid is defined with a total number of \( N = 38,440 \) cells \((31 \times 31 \times 40)\). In the \( x\)- and \( y\)-direction there are 20 cells per wavelength, while in the \( z\)-direction there are 18 cells per wavelength. The wavelength is \( \lambda = c_0/f_0 = 0.12 \) m. Therefore the cell dimensions become: \( \Delta x = \Delta y = 0.006 \) m; \( \Delta z \approx 0.00666 \) m.

The time step \( \Delta t \) is evaluated by the Courant criterium, equation (4.55):

\[
\Delta t = \frac{1}{c_0 \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} = 6.97 \text{ ps},
\]

(5.1)

The number of timesteps is given by equation (4.136):

\[
T = 10\sqrt{3}N^{1/3} = 584.
\]

(5.2)

For a wide-band frequency determination the gap is excited with a Gaussian pulse, like in equation (4.133):

\[
f(\tau) = \exp(-\alpha((\tau - \beta \Delta t)^2)), \quad (0 < \tau < 2\beta \Delta t),
\]

(5.3)

where the width of the pulse is \( 2\beta \Delta t \). For \( \beta \) is taken a value of 32 timesteps.

The program XFDTD has been run with these parameters. The plot of the impedance versus frequency (not included here) shows that this antenna is resonant at approximate 2.1 GHz. Therefore the cell dimension in the \( z\)-direction is multiplied with a factor 0.8245 to make the antenna resonant at 2.5 GHz:

\[
\Delta z = \frac{c_0}{18 f_0} 0.8245 \approx 0.00550 \text{ m}.
\]

(5.4)

In this case there are 14.841 cells per wavelength in the \( z\)-direction and the time step size becomes 6.54 ps. The program XFDTD has been rerun with these parameters. Figure 5.1 shows the impedance at the antenna feedpoint as a function of the frequency. The calculation of this figure on a machine with a PA7200 processor takes 1.53 minutes. The memory usage is 1.8 MB.

5.2.2 Steady state analysis

To calculate the radiation patterns the antenna is excited with a harmonic wave at the frequency of 2.5 GHz. The obtained complex impedance is \( 71.11 + j 0.045 \) \( \Omega \). The radiation pattern in the \( YOZ\)-plane is plotted in Figure 5.2. The gain is 2.09 dBi, which is 0.04 dB less than the analytical value of equation (3.26).
Figure 5.1: Impedance

Figure 5.2: Radiation pattern
5.3 The half-wave dipole antenna parallel to a perfectly conducting ground

The half-wave dipole antenna is placed parallel to a perfectly conducting ground. Because of the physical limit of computer resources it is impossible to define an infinitely large ground. Therefore the dimensions of the ground are 161 x 170 cells. In terms of wavelengths these dimensions are approximate 7.4 x 8.5 wavelengths. The size of the total problem space is $N_x \times N_y \times N_z = 22 \times 161 \times 170$ cells.

Figure 5.3 shows the result of the transient analysis of this configuration. The data is calculated in 47:12 minutes and 18.3 MB of memory is used.

![Figure 5.3: Impedance vs freq.; distance to g.p. = 0.1 wavelength](image)

Figures 5.4 - 5.6 show the radiation patterns, compared to the analytical and the NEC results. Because the ground is large compared to the dimensions of the total discretised problem space, the calculation of the far field is not very accurate, especially for small distances between the ground and the antenna. The boundaries of the ground reach the boundary of the problem space at every side of the ground. The accuracy can be improved by enlarging the total grid or by choosing smaller objects. None of these actions are taken here, because the goal of this study is not very much the analysis of the far field but rather the analysis of the performance at the antenna feed-points.

Figure 5.7 and 5.8 show the impedance and the return loss respectively for several distances to the ground-plane. For comparison the results of NEC and the analytical calculations are also plotted. The computer needed approximate 50 hours cpu-time and at maximum 400 MB of memory for these calculations, while NEC needed only 13 seconds and 138 MB. The results of the calculations of these feedpoint quantities are more satisfying than the far field results.
Figure 5.4: Distance 0.1 wavelength to groundplane

Figure 5.5: Distance 0.5 wavelength to groundplane

Figure 5.6: Distance 0.75 wavelength to groundplane
Figure 5.7: Impedance versus distance; $f = 2.5$ GHz

Figure 5.8: Return loss
5.4 The half-wave dipole antenna parallel to a dielectric ground

In this section the material parameters of the ground are varied to test whether the effect of the ground on the performance of the antenna is worst case when the ground is perfectly conducting. Two types of materials are modeled here. To model a wall of brick, the conductivity $\sigma$ and the relative permittivity $\varepsilon_r$ are kept low, while human tissues are modeled with a higher value of $\varepsilon_r$. The values of the material parameters $\sigma$ and $\varepsilon_r$ are dependent on the humidity $\eta$ of the wall of brick [2] and the type of the human tissue, e.g. bone, brain, muscle, etc... as well as the considered frequency [3]. The magnetic parameter, the magnetic permeability, is the same as for vacuum, i.e. $\mu = \mu_0$. The next table shows the parameters of brick with several values of the humidity and the parameters of several types of human tissues.

<table>
<thead>
<tr>
<th>Material parameters (2.5 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>type of material</td>
</tr>
<tr>
<td>brick ($\eta = 0%$)</td>
</tr>
<tr>
<td>brick ($\eta \approx 5%$)</td>
</tr>
<tr>
<td>brick ($\eta \approx 10%$)</td>
</tr>
<tr>
<td>brick ($\eta \approx 15%$)</td>
</tr>
<tr>
<td>muscle</td>
</tr>
<tr>
<td>bone</td>
</tr>
<tr>
<td>lung</td>
</tr>
<tr>
<td>brain, nerve</td>
</tr>
<tr>
<td>skin</td>
</tr>
<tr>
<td>eye</td>
</tr>
</tbody>
</table>

Figure 5.9 shows the radiation patterns for some material parameters. Note that again the accuracy of these calculations is questionable because of the dimensions of the object, compared to the total discretised problem space.

![Figure 5.9: Distance 0.75 wavelength to groundplane](image)

Figure 5.9: Distance 0.75 wavelength to groundplane

Figure 5.10 shows the antenna impedance for different material parameters. These calculations are compared to the results with a perfectly conducting ground, also calculated by FDTD. For every type of ground the calculations are done for just a few distances between the antenna and the ground, because of the resource requirements of the calculations. Each calculation, with one material type and one value of the distance, takes approximate 110 minutes.

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Figure 5.10: Impedance versus distance
Figure 5.11 shows the return losses that follow from the impedances by applying equations (3.42) and (3.43):

\[ RL = -20 \log \left| \frac{Z_a - R_i}{Z_a + R_i} \right| \text{ dB.} \]  \hspace{1cm} (5.5)

An important conclusion from these results is that the case of the perfect conductor is the worst case, as expected. So a return loss of at least 15 dB will be measured when the distance from a dipole antenna and a large plain object is at least 0.27 times the wavelength.

Note that the discretisation of the problem space is 15 cells per wavelength in the z-direction and 20 cells per wavelength in the x- and y-direction in vacuum. For higher values of the permittivity than \( \varepsilon_0 \), the wavelength in the object becomes shorter. The discretisation is not corrected for this change in the permittivity. However, when the permittivity of the objects becomes higher (e.g. \( \varepsilon_r = 46 \)), the matching characteristics are more like them with perfect conductors.

The results are summarized in the next conclusions:

1. The effect of a wall of brick to the matching of a dipole antenna is much less than a perfectly conducting ground.

2. The effect of certain human tissues is much like a perfectly conducting ground.
References

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A deterministic approach to the modelling of electromagnetic wave propagation in urban environments.

The Use of the Frequency-Dependent Finite-Difference Time-Domain Method for Induced Current and SAR Calculations for a Heterogeneous Model of the Human Body
Chapter 6

Conclusions

The interaction between a dipole antenna and a large object is analysed with three calculation methods: analytical, with the Method of Moments (implemented in NEC) and with the Finite Difference Time Domain Method (implemented in XFDTD).

Many calculations can be carried out analytically. When the object is large and smooth and made of perfectly conducting material, the radiation patterns and the real part of the impedance at the feed-points of the antenna can be calculated analytically. However, to calculate the imaginary part of the impedance analytically has been omitted in this project, because this calculation is very difficult. The function of the current distribution on the half-wave dipole antenna for free space is approximated by a cosine form. It seems that the same current distribution function is also a good approximation if there is a large and perfect conducting object in the neighbourhood of the dipole antenna. The results of the analytical calculations are verified by NEC.

NEC is able to calculate the current distribution on the antenna for a given voltage excitation. The real part as well as the imaginary part of the impedance at the feed-points of the antenna are calculated.

Finally the Finite Difference Time Domain Method is used, which can handle objects that are made of arbitrary materials. FDTD is a very powerful numerical tool, but it requires very powerful computers when the total problem space becomes large compared to the wavelength.

The antenna is designed in such a way that it is matched to the transmitter when it is placed in a free space environment. The calculated complex impedance is used to calculate the mismatch due to the presence of the object at a varying distance from the antenna. It seems that the return loss is higher than 15 dB when the value of the distance between antenna and object is higher than 0.27 times the wavelength, i.e. less than 3 % of the power from the transmitter is reflected at the antenna feed-points. The remainder of the power is transmitted into the far field, because the antenna and the object are made of lossless material.

The half-wave dipole antenna is also analysed with the software package XFDTD. First, the radiation patterns and the antenna impedance are calculated for the antenna in free space and with a large, perfectly conducting plate. The results of the calculations are compared to them of the analytical calculations and to the calculations with NEC. Second, the calculations have been carried out with a dielectric plate. The matching of the antenna is less influenced by a dielectric object than by a perfectly conducting one, i.e. the situation of a perfect conductor is worst case.

The return loss at the antenna feed-point is very high (good matching) with a wall of brick, while the influence of human tissues is more like a perfect conductor.

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