Functional description of MINTO: a mixed integer optimizer

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Functional description of MINTO, a mixed INTeger Optimizer

M.W.P. Savelsbergh
G.C. Sigismondi
G.L. Nemhauser

(Revised October 1992)

Eindhoven, August 1991
The Netherlands
Functional description of MINTO, a Mixed INTegeR Optimizer

Version 1.3

Martin W.P. Savelsbergh ¹,³
Eindhoven University of Technology
P.O. Box 513
5600 MB Eindhoven
The Netherlands

Gabriele C. Sigismondi ²,³
George L. Nemhauser ²,³
Georgia Institute of Technology
School of Industrial and Systems Engineering
Atlanta, GA 30332-0205
USA

mwps@bs.win.tue.nl
gnemhaus@gti01.bitnet

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Martin W.P. Savelsbergh
Eindhoven University of Technology

Gabriele C. Sigismondi
George L. Nemhauser
Georgia Institute of Technology, Atlanta

Abstract
MINTO is a software system that solves mixed-integer linear programs by a branch-and-bound algorithm with linear programming relaxations. It also provides automatic constraint classification, preprocessing, primal heuristics and constraint generation. Moreover, the user can enrich the basic algorithm by providing a variety of specialized application routines that can customize MINTO to achieve maximum efficiency for a problem class. This paper documents MINTO by specifying what it is capable of doing and how to use it.

1 Introduction
MINTO (Mixed INTeger Optimizer) is a tool for solving mixed integer linear programming (MIP) problems of the form:

\[
\begin{align*}
\text{max} & \quad \sum_{j \in B} c_j x_j + \sum_{j \in I} c_j x_j + \sum_{j \in C} c_j x_j \\
\sum_{j \in B} a_{ij} x_j + \sum_{j \in I} a_{ij} x_j + \sum_{j \in C} a_{ij} x_j & \sim b_i \quad i = 1, \ldots, m \\
0 \leq x_j & \leq 1 \quad j \in B \\
\ell_{x_j} & \leq x_j \leq u_{x_j} \quad j \in I \cup C \\
x_j & \in \mathbb{Z} \quad j \in B \cup I \\
x_j & \in \mathbb{R} \quad j \in C
\end{align*}
\]

where \(B\) is the set of binary variables, \(I\) is the set of integer variables, \(C\) is the set of continuous variables, the sense \(\sim\) of a constraint can be \(\leq, \geq\), or \(=\), and the lower and upper bounds may be negative or positive infinity or any rational number.

A great variety of problems of resource allocation, location, distribution, production, scheduling, reliability and design can be represented by MIP models. One reason for this rich modeling
capability is that various nonlinear and non-convex optimization problems can be posed as MIP
problems.

Unfortunately this robust modeling capability is not supported by a comparable algorithmic
capability. Existing branch-and-bound codes for solving MIP problems are far too limited in the
size of problems that can be solved reliably relative to the size of problems that need to be solved,
especially with respect to the number of integer variables; and they perform too slowly for many
real-time applications. To remedy this situation, special purpose codes have been developed for
particular applications, and in some cases experts have been able to stretch the capabilities of
the general codes with ad hoc approaches. But neither of these remedies is satisfactory. The
first is very expensive and time-consuming and the second should be necessary only for a very
limited number of instances.

Our idea of what is needed to solve large mixed-integer programs efficiently, without having
to develop a full-blown special purpose code in each case, is an effective general purpose mixed
integer optimizer that can be customized through the incorporation of application functions.
MINTO is such a system. Its strength is that it allows users to concentrate on problem specific
aspects rather than data structures and implementation details such as linear programming and
branch-and-bound.

The heart of MINTO is a linear programming based branch-and-bound algorithm. It can
be implemented on top of any LP-solver that provides capabilities to solve and modify linear
programs and interpret their solutions. The current version can either be built on top of the
CPLEX (TM) callable library, version 1.2 and up, or on top of the Optimization Subroutine
Library (OSL), version 1.2.

To be as effective and efficient as possible when used as a general purpose mixed-integer
optimizer, MINTO attempts to:

- improve the formulation by preprocessing and probing;
- construct feasible solutions;
- generate strong valid inequalities;
- perform variable fixing based on reduced prices;
- control the size of the linear programs by managing active constraints.

To be as flexible and powerful as possible when used to build a special purpose mixed-integer
optimizer, MINTO provides various mechanisms for incorporating problem specific knowledge.
Finally, to make future algorithmic developments easy to incorporate, MINTO's design is highly
modular.

This document focuses on the mechanisms for incorporating problem structure and only
contains a minimal description of the general purpose techniques mentioned above.

The mechanisms for incorporating problem structure and customizing MINTO are discussed
in Sections 5, 6, 7, and 8 under information, application, miscellaneous, and control func-
tions. Section 2 explains how to run MINTO and Sections 3 and 4 present the overall system
design and a brief description of the system functions. Sections 9 and 10 discuss some pro-
gramming considerations and give some computational results. Finally, Section 11 contains some
remarks on availability and future releases.
MINTO 1.3 is an update of MINTO 1.1. It fixes a number of bugs, has new system functions, new inquiry functions, new control functions, new and modified application functions, new and modified command line options, and supports other LP solvers and hardware platforms.

2 Running MINTO

MINTO requires the mixed integer programming formulation to be specified in MPS format in a file < problem name > .mps in the current working directory. Since MINTO uses the LP-solver to read the initial formulation, the < problem name > .mps file must conform to the rules specified in the documentation of the LP-solver.

The run-time behavior of MINTO depends on the command line options. The following command should be used to invoke MINTO:

minto [-xo <0,1,2> m < ... > behp <0,1,2> ckfrs] < problem name >.

The meanings of the various command line options are given in Table 1. The command line options allow the user to deactivate selectively one or more system functions and to specify the amount of output desired.

<table>
<thead>
<tr>
<th>option</th>
<th>effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>assume maximization problem</td>
</tr>
<tr>
<td>o &lt;0,1,2&gt;</td>
<td>level of output</td>
</tr>
<tr>
<td>m &lt; ... &gt;</td>
<td>maximum number of nodes to be evaluated</td>
</tr>
<tr>
<td>b</td>
<td>deactivate branching</td>
</tr>
<tr>
<td>e</td>
<td>deactivate enhanced branching</td>
</tr>
<tr>
<td>h</td>
<td>deactivate primal heuristic</td>
</tr>
<tr>
<td>p &lt;0,1,2&gt;</td>
<td>level of preprocessing and probing</td>
</tr>
<tr>
<td>c</td>
<td>deactivate clique generation</td>
</tr>
<tr>
<td>i</td>
<td>deactivate implication generation</td>
</tr>
<tr>
<td>k</td>
<td>deactivate knapsack cover generation</td>
</tr>
<tr>
<td>f</td>
<td>deactivate flow cover generation</td>
</tr>
<tr>
<td>r</td>
<td>deactivate row management</td>
</tr>
<tr>
<td>s</td>
<td>deactivate all system functions</td>
</tr>
</tbody>
</table>

Table 1: Command line options

MINTO assumes that the mixed integer programming formulation specified in the file < problem name > .mps represents a minimization problem unless the x command line option is specified, in which case MINTO assumes it represents a maximization problem.

Regardless of whether MINTO has found an optimal solution or not, it will abort after evaluating 1,000,000 nodes. The m < ... > command line option can be used to change the maximum number of nodes to be evaluated.
In default mode MINTO produces very little output. The \( \theta < 0, 1, 2 > \) command line option can be used to change the amount of output generated by MINTO. There are three output levels: very little (0), normal (1), and extensive (2).

In default mode MINTO performs extensive preprocessing. The \( \rho < 0, 1, 2 > \) command line option can be used to change the amount of preprocessing done by MINTO. There are three levels: none (0), simple preprocessing (1), and simple preprocessing and probing (2). Probing, although usually very effective, can be time very consuming, especially for problems with many binary variables.

3 System design

It is well known that problem specific knowledge can be used advantageously to increase the performance of the basic linear programming branch-and-bound algorithm for mixed integer programming. MINTO attempts to use problem specific knowledge on two levels to strengthen the LP-relaxation, to obtain better feasible solutions and to improve branching.

At the first level, system functions use general structures, and at the second level application functions use problem specific structures. A call to an application function temporarily transfers control to the application program, which can either accept control or decline control. If control is accepted, the application program performs the associated task. If control is declined, MINTO performs a default action, which in many cases will be "do nothing". The user can also exercise control at the first level by selectively deactivating system functions.

Figure 1 gives a flow chart of the underlying algorithm and associated application functions. To differentiate between actions carried out by the system and those carried out by the application program, there are different "boxes". System actions are in solid line boxes and application program actions are in dashed line boxes. A solid line box with a dashed line box enclosed is used whenever an action can be performed by both the system and the application program. Finally, to indicate that an action has to be performed by either the system or the application program, but not both, a box with one half in solid lines and the other half in dashed lines is used. If an application program does not carry out an action, but one is required, the system falls back to a default action. For instance, if an application program does not provide a division scheme for the branching task, the system will apply the default branching scheme.

Formulations

The concept of a formulation is fundamental in describing and understanding MINTO. MINTO is constantly manipulating formulations: storing a formulation, retrieving a formulation, modifying a formulation, duplicating a formulation, handing a formulation to the LP-solver, providing information about the formulation to the application program, etc. We will always use the following terms to refer to elements of a formulation: objective function, constraint, coefficient, sense, right-hand side, variable, lower bound, and upper bound.

It is beneficial to distinguish four types of formulations. The original formulation is the formulation specified in the \(< \text{problemname} >.mps\) file. The initial formulation is the formulation associated with the root node of the branch-and-bound tree. It may differ from the original formulation as MINTO automatically tries to improve the initial formulation using various pre-
Figure 1a. The underlying algorithm
Figure 1b. Application functions
processing techniques, such as detection of redundant constraints and coefficient reduction. The current formulation is an extension of the original formulation and contains all the variables and all the global and local constraints associated with the node that is currently being evaluated. The active formulation is the formulation currently loaded in the LP-solver. It may be smaller than the current formulation due to management of inactive constraints.

It is very important that an application programmer realizes that the active formulation does not necessarily coincide with his mental picture of the formulation, since MINTO may have generated additional constraints, temporarily deactivated constraints, or fixed one or more variables.

MINTO always works with a maximization problem. Therefore, if the original formulation describes a minimization problem, MINTO will change the signs of all the objective function coefficients. This is also reflected in the remainder of this functional description; everything is written with maximization in mind.

Constraints
MINTO distinguishes various constraint classes as defined in Table 2. These constraint classes are motivated by the constraint generation done by MINTO and the branching scheme adopted by MINTO. To present these constraint classes, it is convenient to distinguish the binary variables. We do this by using the symbol \( y \) to indicate integer and continuous variables. Each class is an equivalence class with respect to complementing binary variables, i.e., if a constraint with term \( a_j x_j \) is in a given class then the constraint with \( a_j(1 - x_j) \) is also in the class.

For example \( \sum_{j \in B^+} x_j - \sum_{j \in B^-} x_j \leq 1 - |B^-| \) is in the class BINSUM1UB, where we think of \( B^- \) as the set of complemented variables.

Besides constraint classes, MINTO also distinguishes two constraint types: global and local. Global constraints are valid at any node of the branch-and-bound tree, whereas local constraints are only valid in the subtree rooted at the node where the constraints are generated.

Constraints can be in one of three states: active, inactive, or deleted. Active constraints are part of the active formulation. Inactive constraints have been deactivated but may be reactivated at a later time. Deleted constraints have been removed altogether.

Variables
When solving a linear program MINTO allows for column generation. In other words, after a linear program has been optimized, MINTO asks for the pricing out of variables not in the current formulation. If any such variables exists and price out favorably they are included in the formulation and the linear program is reoptimized.

Branching
The unevaluated nodes of the branch-and-bound tree are kept in a list and MINTO always selects the node at the head of the list for processing. However, there is great flexibility here, since MINTO provides a mechanism that allows an application program to order the nodes in the list in any way. As a default MINTO always adds new nodes at the head of the list, i.e., a last-in first-out strategy which corresponds to a depth-first search of the branch-and-bound tree.
MINTO's system functions are used to perform preprocessing, probing, constraint generation and reduced price variable fixing, to enhance branching, and to produce primal feasible solutions. They are employed at every node of the branch-and-bound tree. However, their use, except for reduced price variable fixing, is optional.

In preprocessing, MINTO attempts to improve the LP-relaxation by identifying redundant constraints, detecting infeasibilities, tightening bounds on variables and fixing variables using optimality and feasibility considerations. For constraints with only 0-1 variables, it also attempts to improve the LP-relaxation by coefficient reduction. For example a constraint of the form $a_1x_1 + a_2x_2 + a_3x_3 \leq a_0$ may be replaced by $a_1x_1 + a_2x_2 + (a_3 - \delta)x_3 \leq a_0 - \delta$ for some $\delta > 0$ that preserves the set of feasible solutions.

In probing, MINTO searches for logical implications of the form $x_i = 1$ implies $y_j = v_j$ and stores these in an 'implication' table. Furthermore, MINTO uses the logical implications between binary variables to build up a 'clique' table, i.e., MINTO tries to extend relations between pairs of binary variables to larger sets of binary variables.

After a linear program is solved and a fractional solution is obtained, MINTO tries to exclude these solutions by searching the implication and clique table for violated inequalities, and by searching for violated lifted knapsack covers and violated generalized flow covers. Lifted knapsack...
covers are derived from pure 0-1 constraints and are of the form
\[
\sum_{j \in C_1} x_j + \sum_{j \in C_2} \gamma_j x_j + \sum_{j \in B \cap C} \alpha_j x_j \leq |C_1| - 1 + \sum_{j \in C_2} \gamma_j,
\]
where \( C = C_1 \cup C_2 \) with \( C_1 \neq \emptyset \) a minimal set such that \( \sum_{j \in C} a_j > a_0 \). Generalized flow covers are derived from
\[
\sum_{j \in N^+} y_j - \sum_{j \in N^-} y_j \leq a_0
\]
\[
y_j \leq a_j x_j, \quad j \in N^+ \cup N^-\]
and are of the form
\[
\sum_{j \in C^+} [y_j + (\lambda - a_j)^+(1 - x_j)] \leq a_0 + \sum_{j \in C^-} a_j + \sum_{j \in L} \lambda x_j + \sum_{j \in N^- \setminus (L \cup C^-)} y_j,
\]
with \( (C^+, C^-) \subseteq (N^+, N^-) \) a minimal set such that \( \sum_{j \in C^+} a_j - \sum_{j \in C^-} a_j - a_0 = \lambda > 0 \) and \( L \subseteq N^- \setminus C^- \).

After solving a linear program MINTO searches for non-basic 0-1 variables whose values may be fixed according to the magnitude of their reduced price. It also tries to find feasible solutions using recursive rounding of the optimal LP-solution.

MINTO uses a hybrid branching scheme. Under certain conditions it will branch on a clique constraint. If not, it chooses a variable to branch on based on a priority order it creates.

5 Information Functions

For the sequel, it is assumed that the reader has a working knowledge of the C programming language.

5.1 Current formulation

Information about the current formulation can be obtained through the inquiry functions: inq_form, inq_obj, inq_constr, and inq_var, and their associated variables info_form, info_obj, info_constr, and info_var.

Each of these inquiry functions updates its associated variable so that the information stored in that variable reflects the current formulation. The application program can then access the information by inspecting the fields of the variable.

The rationale behind this approach is that we want to keep memory management fully within MINTO. (Note that since only nonzero coefficients are stored, the memory required to hold the objective function and constraints varies.)

As it is impossible for the application program to keep track of the indices of the active constraints, due to constraint generation and constraint management done by MINTO, the only fail-safe method for accessing constraint related information is to refer to constraints through
names rather than indices. However, in some cases, for instance when an application program only wants to inspect constraints of the original formulation (which are not affected by constraint generation and constraint management), using names would be rather cumbersome.

To overcome these difficulties, the following scheme has been adopted for MINTO. All information access for variables and constraints is done through indices. For variables the valid indices are in the range 0 up to the number of variables, and for constraints the valid indices are in the range 0 up to the number of constraints. However, to provide a fail-safe access mechanism, MINTO will have in future releases, besides the default no-names operating mode, a names operating mode, in which names are associated with each variable and each constraint.

5.1.1 inq_form

This function retrieves the number of variables and the number of constraints of the current formulation.

A call to inq_form() initializes the variable info_form that has the following structure:

```c
typedef struct info_form {
    int form_vcnt; /* number of variables in the formulation */
    int form_ccnt; /* number of constraints in the formulation */
} INFO_FORM;
```

The following example shows how inq_form can be used to print the size of the current formulation.

```c
#include <stdio.h>
#include "minto.h"

/*
 * E_SIZE.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * WriteSize
 */

void
WriteSize ()
{
    inq_form ();
    printf ("Number of variables: %d\n", info_form.form_vcnt);
    printf ("Number of constraints: %d\n", info_form.form_ccnt);
}
```
5.1.2 inq_var

This function retrieves the variable class, the objective function coefficient, the number of constraints in which the variable appears with a non-zero coefficient, and for each of these constraints the index of the constraint and the non-zero coefficient, the status of the variable, the lower and upper bound associated with the variable, additional information on the bounds of the variable, and, if the variable type is continuous and the variable appears in a variable lower or upper bound constraint, the index of the associated binary variable and the associated bound.

Variable class is one of: CONTINUOUS, INTEGER, and BINARY. Variable status is one of ACTIVE, INACTIVE, or DELETED. Variable information is one of: ORIGINAL, MODIFIED_BY_BRANCHING, MODIFIED_BY_MINTO, and MODIFIED_BY_APPL.

PARAMETERS
index: An integer containing the index of the variable.

A call to inq_var() initializes the variable info_var that has the following structure:

```c
typedef struct info_var {
    int var_class;    /* class: CONTINUOUS, INTEGER, or BINARY */
    double var_obj;    /* objective function coefficient */
    int var_nz;        /* number of constraints with non-zero coefficients */
    int *var_ind;      /* indices of constraints with non-zero coefficients */
    double *var_coef;  /* actual coefficients */
    int var_status;    /* ACTIVE, INACTIVE, or DELETED */
    double var_lb;     /* lower bound */
    double var_ub;     /* upper bound */
    VLB *var_vlb;      /* associated variable lower bound */
    VUB *var_vub;      /* associated variable upper bound */
    int var_lb_info;   /* class: ORIGINAL, MODIFIED_BY_MINTO,
                        MODIFIED_BY_BRANCHING, or MODIFIED_BY_APPL */
    int var_ub_info;   /* class: ORIGINAL, MODIFIED_BY_MINTO,
                        MODIFIED_BY_BRANCHING, or MODIFIED_BY_APPL */
} info_var;
```

The following example shows how inq_var can be used to print the variables that are fixed in
the current formulation.

/ *
 * E_FIXED.C
 */

#include <stdio.h>
#include "minto.h"

/ *
 * WriteFixed
 */

void
WriteFixed ()
{
    int j;

    for (inq_form (), j = 0; j < info_form.form_vcnt; j++) {
        inq_var (j);
        if (info_var.var_lb > info_var.var_ub - EPS) {
            printf ("Variable %d is fixed at %f\n", j, info_var.var_lb);
        }
    }
}

5.1.3 inq_obj

This function retrieves the number of variables that appear in the objective function with a nonzero coefficient, and for each of these variables the index of the variable and the nonzero coefficient.

The same information can be obtained by successive calls to inq_var, however using inq_obj is much more efficient.

A call to inq_obj() initializes the variable info_obj that has the following structure:

typedef struct {
    int obj_nz; /* number of variables with nonzero coefficients */
    int *obj_ind; /* indices of variables with nonzero coefficients */
    double *obj_coef; /* actual coefficients */
} INFO_OBJ;

The following example shows how inq_obj can be used to print the variables with a nonzero objective coefficient.

/ *
```c
#include <stdio.h>
#include "minto.h"

/*
 * WriteObj
 */

void
WriteObj ()
{
    int j;

    inq_obj ();
    for (j = 0; j < info_obj.obj_nz; j++) {
        printf ("Variable %d has objective coefficient %f\n",
                info_obj.obj_ind[j], info_obj.obj_coef[j]);
    }
}

5.1.4 inq_constr

This function retrieves the constraint class, the number of variables that appear in the constraint
with a nonzero coefficient, and for each of these variables the index of the variable and the
nonzero coefficient, the sense of the constraint, the right hand side of the constraint, the status
of the constraint, the type of the constraint, and additional information on the constraint.

Constraint class is one of: MIXUB, MIXEQ, NOBINARYUB, NOBINARYEQ, ALLBINARYUB,
ALLBINARYEQ, SUMVARUB, SUMVAREQ, VARUB, VAREQ, VARLB, BINSUMVARUB,
BINSUMVAREQ, BINSUM1VARUB, BINSUM1VAREQ, BINSUM1UB, or BINSUM1EQ.
Constraint status is one of: ACTIVE, INACTIVE, or DELETED. Constraint type is one of: LOCA­
LAL or GLOBAL. Constraint information is one of ORIGINAL, GENERATED BY BRANCHING,
GENERATED BY MINTO, and GENERATED BY APPL.

PARAMETERS
index: An integer containing the index of the constraint.

A call to inq_constr() initializes the variable info_constr that has the following structure:

typedef struct info_constr {
    int constr_class; /* classification: ... */
    int constr_nz;    /* number of variables with nonzero coefficients */
    int *constr_ind;  /* indices of variables with nonzero coefficients */
    double *constr_coef; /* actual coefficients */
```
The following example shows how inq_constr can be used to print the types of the constraints in the current formulation.

```c
#include <stdio.h>
#include "minto.h"

/*
 * E_TYPE.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * WriteType
 */

void WriteType ()
{
    int i;

    for (inq_form (), i = 0; i < info_form.form_ccnt; i++) {
        inq_constr (i);
        printf ("Constraint %d is of type %s
", i, info_constr.constr_type == GLOBAL ? "GLOBAL" : "LOCAL");
    }
}
```

A more elaborate example showing how the inquiry functions can be used to print everything there is to know about the current formulation can be found in Appendix A.

### 5.2 Active formulation

Information about the LP-solution to the active formulation and information about the best primal solution are available to the application, whenever appropriate, through the parameters passed to the application functions.

Additional information about the active formulation and the LP-solution can be obtained through the inquiry functions lp_vcnt, lp_ccnt, lp_slack, lp_pi, lp_rc, and lp_base.
5.2.1 Ip_vcnt
This function returns the number of variables in the active formulation, i.e., the number of
variables currently loaded in the LP-solver.

5.2.2 Ip_ccnt
This function returns the number of constraints in the active formulation, i.e., the number of
constraints currently loaded in the LP-solver.

5.2.3 Ip_slack
This function returns the slack or surplus of the constraint. If the index is invalid or the associ­
ated constraint is inactive, the return value will be INF.
PARAMETERS
index: An integer containing the index of the constraint.

5.2.4 Ip_pi
This function returns the dual value of the constraint. If the index is invalid or the associated
constraint is inactive, the return value will be INF.
PARAMETERS
index: An integer containing the index of the constraint.

5.2.5 Ip_rc
This function returns the reduced cost of the variable. If the index is invalid, the return value
will be INF.
PARAMETERS
index: An integer containing the index of the variable.

5.2.6 Ip_base
This function returns the status of a variable, i.e., BASIC, ATLOWER, ATUPPER, or NON­
BASIC. If the index is invalid, the return value will be UNDEFINED.
PARAMETERS
index: An integer containing the index of the variable.
6 Application Functions

A set of application functions (either the default or any other) has to be compiled and linked with the MINTO library in order to produce an executable version of MINTO. These functions give the application program the opportunity to incorporate problem specific knowledge and thereby increase the overall performance. A default set of application functions is part of the distribution of MINTO. The incorporation of these default functions turns MINTO into a general purpose mixed integer optimizer.

Since only the nonzero coefficients of a constraint are stored, a set of constraints can and will always be specified by three arrays: cfirst, cind, ccoef. Cind and ccoef contain the indices and values of nonzero coefficients respectively. Cfirst[i] indicates the position of the first nonzero coefficient of the ith constraint in the arrays cind, and ccoef; cfirst[i+1] indicates the first position after the last nonzero coefficient of the ith constraint in the arrays cind and ccoef. Note that this implies that if a set of k constraints is specified cfirst[k] has to be defined.

6.1 appl.init

This function provides the application with an entry point in the program to perform some initial actions. It has to return either STOP, in which case MINTO aborts, or CONTINUE, in which case MINTO continues.

The following example shows how appl.init can be used to open a log file.

```c
#include <stdio.h>
#include "minto.h"

FILE *fp_log;

 unsigned appl_init()
{
    if ((fp_log = fopen("EXAMPLE.LOG", "w")) == NULL) {
        fprintf(stderr, "Unable to open EXAMPLE.LOG\n");
        return (STOP);
    }

    printf(fp_log, "Solving problem %s with MINTO\n", inq_prob);

    return (STOP);
}
```
6.2 appl_initlp (NEW)

This function provides the application with an entry point in the program to initialize the LP-solver and to indicate whether column generation will be used for the solution of the linear programming relaxations. MINTO ignores the return value.

PARAMETERS

colgen: An integer to indicate whether column generation will be used, i.e., TRUE of FALSE

MINTO solves the initial linear program using a primal simplex method and all subsequent linear programs using a dual simplex method. One reason for using the dual simplex method is that the dual simplex method approaches the optimal value of the linear program from above and thus provides a valid upper bound at every iteration, not only on the linear programming solution, but also on the mixed integer programming solution. Therefore, the solution of the linear program can be terminated as soon as this upper bound drops below the current lower bound, because at that point the node can be fathomed by bounds. However, if the linear program is solved using column generation, the values no longer provide valid upper bounds and the solution of the linear program cannot be terminated earlier. It is for this reason that MINTO needs to know whether the linear programs are solved using column generation or not.

The following example shows how appl_initlp can be used to indicate that column generation will be used to solve the linear programming relaxations.

```c
#include <stdio.h>
#include "minto.h"

/*
 * E_INITLP.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * appl_initlp
 */

unsigned
appl_initlp (colgen)
int *colgen; /* indicator */
{

*colgen = TRUE;

return (CONTINUE);
```
6.3 appl_preprocessing

This function provides the application with an entry in the program to perform some preprocessing based on the original formulation. It has to return either STOP, in which case MINTO aborts, or CONTINUE, in which case MINTO continues.

In general, MINTO only stores data in the information variables associated with the inquiry functions and never looks at them again, i.e., communication between MINTO and the application program is one-way only. However, in appl_preprocessing a set of modification functions can be used by the application program to turn this one-way communication into a two-way communication. A call to a modification function signals that the associated variable has been changed by the application and that MINTO should retrieve the data and update its internal administration.

**set_var**
This function signals that the application program has changed the contents of the info.var variable and that MINTO should get the data of the variable and update its internal administration. MINTO only accepts changes of the bounds of a variable.

**PARAMETERS**

index: An integer containing the index of the variable.

**set_obj**
This function signals that the application program has changed the contents of the info.obj variable and that MINTO should get the data of the variable and update its internal administration.

**set_constr**
This function signals that the application program has changed the contents of the info.constr variable and that MINTO should get the data of the variable and update its internal administration. MINTO only accepts changes of the coefficients and the status. If the status is changed to DELETE, the constraint will be removed from the original formulation.

**PARAMETERS**

index: An integer containing the index of the constraint.

The following example shows how appl_preprocessing can be used to identify and delete redundant rows from the original formulation.

```c
/*
 * E_PREP.C
 * /

#include <stdio.h>
#include "minto.h"
```

18
unsigned appl_preprocessing ()
{
    int i, j;
    double minlhs, maxlhs, coef;

    inq_form ();
    for (i = 0; i < info_form.form_ccnt; i++) {
        minlhs = maxlhs = (double) 0;
        inq_constr (i);
        for (j = 0; j < info_constr.constr_nz; j++) {
            inq_var (info_constr.constr_ind[j]);
            if ((coef = info_constr.constr_coef[j]) > EPS) {
                minlhs += coef * info_var.var_lb;
                maxlhs += coef * info_var.var_ub;
            } else {
                minlhs += coef * info_var.var_ub;
                maxlhs += coef * info_var.var_lb;
            }
        }
        if (info_constr.constr_sense == 'G' &&
            minlhs > info_constr.constr_rhs - EPS) {
            info_constr.constr_status = DELETE;
            set_constr (i);
        }
        if (info_constr.constr_sense == 'L' &&
            maxlhs < info_constr.constr_rhs + EPS) {
            info_constr.constr_status = DELETE;
            set_constr (i);
        }
    }
}

6.4 appl_node

This function provides the application with an entry point in the program after MINTO has
selected a node from the set of unevaluated nodes of the branch-and-bound tree and before
MINTO starts processing the node. It has to return either STOP, in which case MINTO aborts,
or CONTINUE, in which case MINTO continues.

PARAMETERS
depth: A long containing the depth in the branch-and-bound tree of the node that has been selected for evaluation.
creation: A long containing the creation number of the node that has been selected for evaluation.
zprimal: A double containing the value of the primal solution.
xprimal: An array of doubles containing the values of the variables associated with the primal solution.
ecnt: A long containing the number of evaluated nodes.
gap: A double containing the gap between the value of the primal solution and the value of the LP-solution associated with the node that has been selected for evaluation.

The following example shows how appl_node can be used to implement a simple stopping rule.

```c
#include <stdio.h>
#include "minto.h"

#define GAPSIZE 0.5

extern FILE *fp_log;

unsigned appl_node (depth, creation, zprimal, xprimal, ecnt, gap)
int depth; /* identification: depth */
int creation; /* identification: creation */
double zprimal; /* value of primal solution */
double *xprimal; /* value of the variables */
int ecnt; /* number of evaluated nodes */
double gap; /* gap between primal and LP solution value */
{
    if (gap < GAPSIZE) {
        printf (fp_log, "Terminated since the gap is smaller than %f\n", GAPSIZE);
        return (STOP);
    }
```

} 
else {
    fprintf (fp_log, "Evaluating node (%ld,%ld)\n", depth, creation);
    return (CONTINUE);
}

6.5 appl_exit

This function provides the application with an entry point in the program to perform some final actions. MINTO ignores the return value.

PARAMETERS
zopt: A double containing the value of the final solution.
xopt: An array of doubles containing the values of the variables associated with the final solution.

If no solution vector exists the second parameter will be NULL. The following example shows how the function appl_exit can be used to write the optimal solution to a log file and afterwards close the log file.

/* *
 * E_EXIT.C
 */

#include <stdio.h>
#include "minto.h"

extern FILE *fp_log;

/* *
 * appl_exit
 */

unsigned appl_exit (zopt, xopt)
    double zopt;    /* value of the final solution */
    double *xopt;   /* values of the variables */
{
    int j;

    fprintf (fp_log, "OPTIMAL SOLUTION VALUE: %f\n", zopt);
    if (xopt) {
        fprintf (fp_log, "OPTIMAL SOLUTION:
");


for (inq_form (), j = 0; j < info_form.form_vcnt; j++) {
  fprintf (fp_log, "x[%d] = %f\n", j, xopt[j]);
}
}
fclose (fp_log);
return (CONTINUE);
}

6.6 appl.quit

This function provides the application with an entry point in the program to perform some final actions if execution is terminated by a <ctrl>-C signal. MINTO ignores the return value.

PARAMETERS
zopt: A double containing the value of the final solution.
xopt: An array of doubles containing the values of the variables associated with the final solution.

If no solution vector exists the second parameter will be NULL.

6.7 appl.primal (MODIFIED)

This function allows the application to provide MINTO with a lower bound and possibly an associated primal solution. It has to return either FAILURE, in which case MINTO assumes that no lower bound was found by the application or no attempt was made to find one and it therefore ignores the parameters zpnew, xpnew, and xpstat, or SUCCESS, in which case MINTO assumes that a lower bound has been found by the application and that it is available through the parameter zpnew and that an associated primal solution is available through the parameter xpnew if xpstat is set to TRUE.

PARAMETERS
zlp: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.
zprimal: A double containing the value of the current primal solution.
xprimal: An array of doubles containing the values of the variables associated with the current primal solution.

zpnew: A double to hold the value of the new primal solution.
xpnew: An array of doubles to hold the values of the variables associated with the new primal solution.
xpstat: An integer to indicate the existence of a solution vector, i.e., TRUE or FALSE.
The following example shows how `appl_primal` can be used to provide feasible solutions for a node packing problem.

```c
#include <stdio.h>
#include "minto.h"

#define UNDEFINED -1
#define FREE 0
#define FIXED 1

/*
 * The graph is represented as a forward star in the arrays adjnodes and edges
*/

extern int *adjnodes;
extern int *adjedges;

/*
 * appl_primal
*/

unsigned appl_primal (zlp, xlp, zprimal, xprimal, zpnev, xpnev, xpstat)
double zlp; /* value of the LP solution */
double *xlp; /* values of the variables */
double zprimal; /* value of the primal solution */
double *xprimal; /* values of the variables */
double *zpnev; /* variable for new value of lower bound */
double *xpnev; /* array for new values of the variables */
int *xpstat; /* variable for status of associated solution */
{
    register int j, k;
    int ix;
    int *mark;
    double maxxlp;

    *xpstat = TRUE;
    *zpnew = 0.0;
```
inq_form();
inq_obj();

mark = (int *) calloc(info_form.form_vcnt, sizeof (int));

for (;;) {
    ix = UNDEFINED; maxxlp = 0.0;
    for (j = 0; j < info_form.form_vcnt; j++) {
        if (mark[j] == FREE) {
            if (xlp[j] > maxxlp) {
                maxxlp = xlp[j];
                ix = j;
            }
        }
    }
    if (ix == UNDEFINED) {
        break;
    } else {
        mark[ix] = FIXED;
        xpnew[ix] = 1.0;
        *zpnew += info_obj.obj_coef[ix];
        for (k = adjnodes[ix]; k < adjnodes[ix+1]; k++) {
            mark[adjedges[k]] = FIXED;
            xpnew[adjedges[k]] = 0.0;
        }
    }
    free(mark);
    return (SUCCESS);
}

6.8 appl_fathom

This function allows the application to provide an optimality tolerance to terminate or prevent the processing of a node of the branch-and-bound tree even when the upper bound value associated with the node is greater than the value of the primal solution. It has to return either FAILURE, in which case MINTO assumes that (further) processing of the node is still required, or SUCCESS, in which case MINTO assumes that (further) processing of the node is no longer required. For an active node, processing is terminated; for an unevaluated node, MINTO deletes it from the list of nodes to be processed.
PARAMETERS
zlpr: A double containing the value of the LP solution.
sprimal: A double containing the value of the primal solution.

The following two examples show how the function appl_fathom can be used to implement optimality tolerances. The first example shows how to incorporate the fact that objective coefficients are all integer. The second example shows how to build a truncated branch-and-bound algorithm that generates a solution that is within a certain percentage of optimality.

```
#include <stdio.h>
#include "minto.h"

/*
 * E_FATHOM.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * appl_fathom
 */

unsigned appl_fathom (zlp, zprimal)
double zlp; /* value of the LP solution */
double zprimal; /* value of the primal solution */
{
    if (zlp - zprimal < 1 - EPS) {
        return (SUCCESS);
    } else {
        return (FAILURE);
    }
}

#include <stdio.h>
#include "minto.h"

#define TOLERAICE 1.05

#include <stdio.h>
#include "minto.h"

#define TOLERANCE 1.05

/*
 * appl_fathom
 */
value of the LP solution
value of the primal solution

unsigned appl_fathom (zlp, zprimal)
double zlp;  /* value of the LP solution */
double zprimal;  /* value of the primal solution */
{
    if (zlp < TOLERANCE * zprimal - EPS) {
        return (SUCCESS);
    }
    else {
        return (FAILURE);
    }
}

6.9 appl_feasible (MODIFIED)

This function allows the application to verify that a solution to the active formulation satisfying
the integrality conditions does indeed constitute a feasible solution. It has to return either YES,
in which case MINTO assumes that the solution is feasible and therefore terminates processing
this node, or NO, in which case MINTO assumes that the solution is not feasible and therefore
continues processing this node.

PARAMETERS
zip: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.

The following example shows how appl_feasible can be used to accommodate partial formula­
tions. In the linear ordering problem one usually deals with the 3-cycle inequalities $d_{ij}+d_{jk}+d_{ki} \leq 2$ implicitly, i.e, they may be generated only when they violate an LP-solution. The following
code assumes the set of variables is $d_{ij}$ for $i,j = 1,\ldots,n$, $i \neq j$ and verifies whether the given
solution is feasible or not.

/*
 * E_FEAS.C
 */

#include <stdio.h>
#include "minto.h"

#define INDEX(I,J) \
    ((I) * (n-1) + (((J) < (I)) ? (J) : (J)-1))

/*
* appl_feasible
*/

unsigned
appl_feasible (zlp, xlp)
double zlp;      /* value of the LP solution */
double *xlp;  /* values of the variables */
{
    int i, j, k, n;
    double diff;

    inq_form (); n = info_form.form_vcnt;

    for (i = 0; i < n; i++) {
        for (j = 0; j < n; j++) {
            for (k = 0; k < n; k++) {
                if (i != j && i != k && j != k) {
                    diff = xlp[INDEX(i,j)] + xlp[INDEX(j,k)] + xlp[INDEX(k,i)] - 2;
                    if (diff > EPS) {
                        return (10);
                    }
                }
            }
        }
    }
    return (YES);
}

6.10 appl_bounds

This function allows the application to modify the bounds of one or more variables. It has to return either FAILURE, in which case MINTO assumes that no bounds have to be changed and it therefore ignores the parameters vcnt, vind, vtype, and vvalue, or SUCCESS, in which case MINTO assumes that there are variables for which the bounds have to be changed and that the relevant information is available through the parameters vcnt, vind, vtype, and vvalue.

PARAMETERS
zlp: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.
z primal: A double containing the value of the primal solution.
x primal: An array of doubles containing the values of the variables associated with the primal solution.
vcnt: An integer to hold the number of variables for which bounds have to be modified.
vind: An array of integers to hold the indices of the variables for which bounds have to be modified.
vtype: An array of characters to hold the types of modification to be performed, i.e., lower bound 'L' or upper bound 'U'.

vvalue: An array of doubles to hold the new values for the bounds.

bdim: An integer containing the length of the arrays vind, vtype, and vvalue.

The following example shows how appl_bounds can be used to implement reduced cost fixing.

```c
#include <stdio.h>
#include "minto.h"

unsigned appl_bounds (zlp, double zlp;
 double *xlp;
 double zprimal;
 double *xprimal;
 int *vcnt;
 int *vind;
 char *vtype;
 double *vvalue;
 int bdim;

{ int j;
  double lb, ub;
  *vcnt = 0;

  inq_form ();
  for (j = 0; j < info_form.form_vcnt; j++) {
    inq_var (j);
    if (lp_base (j) != BASIC && info_var.var_class != CONTINUOUS) {
      lb = info_var.var_lb;
      ub = info_var.var_ub;
    }
  }
}
```
if (lb > ub - EPS) {
    continue;
}

if (xlp[j] < lb + EPS && zlp + lp_rc (j) < zprimal + EPS) {
    vind[vcnt] = j;
    vtype[vcnt] = 'U';
    vvalue[vcnt] = lb;
}

if (*vcnt == bdim) {
    break;
}

if (xIp[j] > ub - EPS && zlp - lp_rc (j) < zprimal + EPS) {
    vind[vcnt] = j;
    vtype[vcnt] = 'U';
    vvalue[vcnt] = ub;
}

if (*vcnt == bdim) {
    break;
}
}

return (*vcnt > 0 ? SUCCESS : FAILURE);

6.11 appl_variables (MODIFIED)

This function allows the application to generate one or more additional variables. It has to return either FAILURE, in which case MINTO assumes that no additional variables were found, or no attempt was made to generate any and it therefore ignores the parameters nzcnt, vcnt, vobj, vlb, vub, vfirst, vind, and vcoef, or SUCCESS, in which case MINTO assumes that additional variables have been found by the application and that they are available through the parameters nzcnt, vcnt, vobj, vlb, vub, vfirst, vind, and vcoef.

PARAMETERS
zlp: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.
zprimal: A double containing the value of the primal solution.
xprimal: An array of doubles containing the values of the variables associated with the primal solution.
nzcnt: An integer to hold the number of nonzero coefficients to be added to the current formulation.
vcnt: An integer to hold the number of variables to be added to the current formulation.
vclass: An array to hold the classification of variables to be added to the current formulation, i.e., BINARY, INTEGER, CONTINUOUS.
vobj: An array of doubles to hold the objective function coefficients of the variables to be added.
vlb: An array of doubles to hold the lower bounds on the values of the variables to be added.
vub: An array of doubles to hold the upper bounds on the values of the variables to be added.
vfirst: An array of integers to hold the positions of the first nonzero coefficients of the variables to be added.
vind: An array of integers to hold the row indices of the nonzero coefficients of the variables to be added.
vcoef: An array of doubles to hold the values of the nonzero coefficients of the variables to be added.
sdim: An integer to hold the length of the arrays vobj, varlb, varub, and vfirst.
l.dim: An integer to hold the length of the arrays vind and vcoef.

The following example shows how appl_variables can be used to implement a column generation scheme for the solution of the linear program.

/*
 * E_VARS.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * appl_variables
 */

unsigned appl_variables (zlp, xl, zprimal, xprimal, nzcnt, vcnt, vclass, vobj, varlb, varub, vfirst, vind, vcoef, sdim, ldim)
double zlp; /* value of the LP solution */
double *xl; /* values of the variables */
double zprimal; /* value of the primal solution */
double *xprimal; /* values of the variables */
int *nzcnt; /* variable for number of nonzero coefficients */
int *vcnt;              /* variable for number of variables */
int *vclass;            /* variable for classifications of vars added */
double *vobj;           /* array for objective coefficients of vars added */
double *varlb;          /* array for lower bounds of vars added */
double *varub;          /* array for upper bounds of vars added */
int *vfirst;            /* array for positions of first nonzero coefficients */
int *vind;              /* array for indices of nonzero coefficients */
double *vcoef;          /* array for values of nonzero coefficients */
int sdim;               /* length of small arrays */
int ldim;               /* length of large arrays */
{
    int j;
    int col_nz;
    int *col_ind;
    double *col_coeff;
    int col_class;
    double col_obj;
    double col_lb;
    double col_ub;

    inq_form();

    col_ind = (int *) calloc (info_form.form_ccnt, sizeof (int));
    col_coeff = (double *) calloc (info_form.form_ccnt, sizeof (double));

    *vcnt = 0;
    *nzcnt = 0;

    while (get_column (&col_nz, &col_class, col_ind, col_coeff, &col_obj, &col_lb, &col_ub))
    {
        if (*nzcnt + col_nz > ldim) {
            continue;
        }

        vfirst[*vcnt] = *nzcnt;
        vclass[*vcnt] = col_class;
        vobj[*vcnt] = col_obj;
        varlb[*vcnt] = col_lb;
        varub[*vcnt] = col_ub;
        for (j = 0; j < col_nz; j++) {
            vind[*nzcnt] = col_ind[j];
            vcoef[(*nzcnt)++] = col_coeff[j];
        }
    }

    31
if (*vcnt == sdim) {
    break;
}

vfirst[*vcnt] = *nzcnt;

free (col_ind);
free (col_coef);

return (*vcnt > 0 ? SUCCESS : FAILURE);

unsigned
get_column (col_nz, col_class, col_ind, col_coeff, col_obj, col_lb, col_ub) {

    /*
     * This function tries to generate a column. It returns 1 if it
     * successful and 0 otherwise
     */
}

6.12 appl_delvariables (NOT IMPLEMENTED)

This function allows the application to delete one or more of the previously generated variables from the active formulation, i.e., the formulation currently loaded in the LP-solver. It has to return either FAILURE, in which case MINTO assumes that no variables have to be deleted and it therefore ignores the parameters vcnt and vind, or SUCCESS, in which case MINTO assumes that variables have to be deleted and that these variables are available through the parameters vcnt and vind.

PARAMETERS
vcnt: An integer to hold the number of variables to be deleted from the current formulation.
vind: An array of integers to hold the indices of the variables to be deleted from the current formulation.

Note that variables are deleted from the active formulation. Therefore indices are considered
to be relative to the active formulation. Note also that it is only possible to delete previously generated variables, either by MINTO or by the application. It is not possible to delete variables from the initial formulation.

The following example shows how appl_delvariables can be used to examine all active variables and delete all variables whose reduced cost is greater than a certain tolerance. MINTO will ignore all indices referring to variables from the initial formulation.

```c
#include <stdio.h>
#include "minto.h"
#define TOLERANCE 100

appl_delvariables (vcnt, vind)
int *vcnt;    /* variable for number of variables to be deleted */
int *vind;    /* array for indices of the variables to be deleted */
{
    int j;

    *vcnt = 0;
    for (j = 0; j < lp_vcnt (); j++) {
        if (lp_rc (j) > TOLERANCE) {
            vind[(*vcnt)++] = j;
        }
    }

    return (SUCCESS);
}
```

6.13 appl_terminatelp (NEW)

This function allows the application to terminate the solution of the current linear program without having reached an optimal solution, i.e., before all variables have been priced out. It has to return either NO, in which case MINTO assumes that the application wants to continue the solution of the current linear program and it therefore ignores the parameter zub, or YES, in which case MINTO assumes that the application wants to terminate the solution of the current
linear program and that an alternative upper bound is provided through the parameter zub.

PARAMETERS
zlp: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.
zub: A double to hold the alternative upper bound.

6.14 appl_constraints
This function allows the application to generate one or more violated constraints. It has to return either FAILURE, in which case MINTO assumes that no violated constraints were found, or no attempt was made to generate any and it therefore ignores the parameters nzcnt, ccnt, cfirst, cind, ccoef, and ctype, or SUCCESS, in which case MINTO assumes that additional constraints have been found by the application and that they are available through the parameters nzcnt, ccnt, cfirst, cind, ccoef, and ctype.

PARAMETERS
zlp: A double containing the value of the LP solution.
xlp: An array of doubles containing the values of the variables.
zprimal: A double containing the value of the primal solution.
xprimal: An array of doubles containing the values of the variables associated with the primal solution.
nzcnt: An integer to hold the number of nonzero coefficients to be added to the current formulation.
ccnt: An integer to hold the number of constraints to be added to the current formulation.
cfirst: An array of integers to hold the positions of the first nonzero coefficients of the constraints to be added.
cind: An array of integers to hold the indices of the nonzero coefficients of the constraints to be added.
ccoef: An array of doubles to hold the values of the nonzero coefficients of the constraints to be added.
csense: An array of characters to hold the senses of the constraints to be added.
crhs: An array of doubles to hold the right hand sides of the constraints to be added.
cctype: An array of integers to hold the types of the constraints to be added, i.e., GLOBAL or LOCAL.
sdim: An integer containing the length of the arrays cfirst, csense, crhs, and cctype.
ldim: An integer containing the length of the arrays cind and ccoef.

The following example shows how appl_constraints can be used to develop a cutting plane algorithm based on minimal covers for knapsack constraints.

/*
 * ECONS.C
 */
#include <stdio.h>
#include "minto.h"

#define INDEX(I,J) \  
   (((J) < (I))? (J):(J)-1))

unsigned appl_constraints
double zlp; /* value of the LP solution */
double *xlp; /* values of the variables */
double zprimal; /* value of the primal solution */
double *xprimal; /* values of the variables */
int *nzcnt; /* variable for number of nonzero coefficients */
int *ccnt; /* variable for number of constraints */
int *cfirst; /* array for positions of first nonzero coefficients */
int *cind; /* array for indices of nonzero coefficients */
double *ccoef; /* array for values of nonzero coefficients */
char *csense; /* array for senses */
double *crhs; /* array for right hand sides */
int *ctype; /* array for the constraint types: LOCAL or GLOBAL */
int sdim; /* length of small arrays */
int ldim; /* length of large arrays */
{  
  int i, j, k, n;
  double diff;

  *ccnt = 0;
  *nzcnt = 0;

  inq_form(); n = info_form.form_vcnt;

  for (i = 0; i < n; i++) {
    for (j = 0; j < n; j++) {
      for (k = 0; k < n; k++) {
        if (i != j && i != k && j != k) {
          diff = xlp[INDEX(i,j)] + xlp[INDEX(j,k)] + xlp[INDEX(k,i)] - 2;
          if (diff > EPS) {
            cfirst[*ccnt] = *nzcnt;
          }
        }
      }
    }
  }
cind[*nzcnt] = INDEX(i,j);
ccof[*nzcnt++] = 1.0;
cind[*nzcnt] = INDEX(j,k);
ccof[*nzcnt++] = 1.0;
cind[*nzcnt] = INDEX(k,i);
ccof[*nzcnt++] = 1.0;
csense[*ccnt] = 'L';
crhs[*ccnt] = 2.0;
ctype[*ccnt++] = GLOBAL;
if (*ccnt == sdim || *nzcnt > ldim - 3) {
   goto EXIT;
}
}
}
}
}
EXIT:
cfirst[*ccnt] = *nzcnt;
return (*ccnt > 0 ? SUCCESS : FAILURE);
}

6.15 *appl_delconstraints* (NEW)

This function allows the application to delete one or more of the previously generated constraints from the active formulation, i.e., the formulation currently loaded in the LP-solver. It has to return either FAILURE, in which case MINTO assumes that no constraints have to be deleted and it therefore ignores the parameters ccnt and cind, or SUCCESS, in which case MINTO assumes the constraints have to be deleted and that these constraints are available through the parameters ccnt and cind.

**PARAMETERS**

ccnt: An integer to hold the number of constraints to be deleted from the active formulation.

cind: An array of integers to hold the indices of the constraints to be deleted from the active formulation.

Note that constraints are deleted from the active formulation. Therefore indices are considered to be relative to the active formulation. Note also that it is only possible to delete previously generated constraints, either by MINTO or by the application. It is not possible to delete constraints from the initial formulation.

The following example shows how *appl_delconstraints* can be used to examine all active con-
straints every tenth iteration and delete all the constraints whose slack is greater than a certain TOLERANCE. MINTO will ignore all indices referring to constraints from the initial formulation.

/ *
 * E_DELCONS.C
 */

#include <stdio.h>
#include "minto.h"

#define TOLERANCE 0.1

static int lpcounter = 0;

/ *
 * appl_delconstraints
 */
aappl_delconstraints (ccnt, cind)
int *ccnt; /* variable for number of constraints to be deleted */
int *cind; /* array for indices of the constraints to be deleted */
{
    int i;

    if (++lpcounter % 10 != 0) {
        return (FAILURE);
    } else {
        *ccnt = 0;
        for (i = 0; i < lp_cnt(); i++) {
            if (lp_slack (i) < -TOLERANCE || lp_slack (i) > TOLERANCE) {
                cind[(*ccnt)++] = i;
            }
        }
        return (*ccnt > 0 ? SUCCESS : FAILURE);
    }
}

6.16 appl_terminatenode (NEW)

This function allows the application to take over control of tailing-off detection and set the threshold value used by MINTO to detect tailing-off. It has to return either NO, in which case MINTO
assumes that the application does not want to replace the default value of the threshold by its own and it therefore ignores the parameter threshold, or YES, in which case MINTO assumes that the application wants to replace the default value of the threshold by its own and that this value is available through the parameter threshold.

PARAMETERS
zIp: A double containing the value of the LP solution.
change: A double containing the total change in the value of the LP solution over the last three iterations.
threshold: A double to hold the threshold to be used to detect tailing-off

When MINTO processes a node, it monitors the changes in the value of the LP solutions from iteration to iteration. If it detects that the total change in the value of the LP solution in the last three iterations is less than 0.5 percent, i.e., 0.005 times the value of the current LP solution, it forces MINTO to branch.

The following example shows how appl_terminatenode can be used to override MINTO's default scheme and continue generating constraints as long as violated constraints are identified

```c
#include <stdio.h>
#include "minto.h"

/*
 * E_TERMND.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * appl_terminatenode
 */

unsigned
appl_terminatenode (zIp, change, threshold)
  double zIp;
  double change;
  double *threshold;
{
    *threshold = 0.0;

    return (YES);
}

6.17 appl_divide

This function allows the application to provide a partition of the set of solutions by either specifying bounds for one or more variables, or generating one or more constraints, or both.
has to return either FAILURE, in which case MINTO assumes that the application wants to use the default division scheme and it therefore ignores the parameters, or SUCCESS, in which case MINTO assumes that the application constructed a partition which is available through the parameters, or INSUFFICIENT, signaling that more memory, i.e., larger arrays, is required to store the partition, in which case MINTO increases the available memory and calls the function again.

PARAMETERS

**depth**: A long containing the depth in the tree of the node that has been selected for evaluation.

**creation**: A long containing the creation number of the node that has been selected for evaluation.

**zlp**: A double containing the value of the LP solution.

**xlp**: An array of doubles containing the values of the variables.

**zprimal**: A double containing the value of the primal solution.

**xprimal**: An array of doubles containing the values of the variables associated with the primal solution.

**ncnt**: An integer to hold the number of nodes in the division.

**vcnt**: An array of integers to hold the number of variables for which a bound is specified for each node.

**vind**: An array of integers to hold the indices of the variables for which a bound is specified.

**vtype**: An array of characters to hold the types of bounds, i.e., lower bound 'L' or upper bound 'U'.

**vvalue**: An array of doubles to hold the values of the bounds.

**nzcnt**: An integer to hold the total number of nonzero coefficients in the constraints generated for each node.

**ccnt**: An array of integers to hold the number of constraints generated for each node.

**cfirst**: An array of integers to hold the positions of the first nonzero coefficients of the constraints generated.

**cind**: An array of integers to hold the indices of the nonzero coefficients of the constraints generated.

**ceoef**: An array of doubles to hold the values of the nonzero coefficients of the constraints generated.

**csense**: An array of characters to hold the senses of the constraints generated.

**crhs**: An array of doubles to hold the right hand sides of the constraints generated.

**bdim**: An integer containing the length of the arrays vind, vtype, and vvalue.

**sdim**: An integer containing the length of the arrays ccnt, cfirst, csense, and crhs.

**ldim**: An integer containing the length of the arrays cind and ceoef.

The default division scheme partitions the set of solutions into two sets by specifying bounds for the integer variable with fractional part closest to 0.5. In the first set of the partition, the selected variable is bounded from above by the round down of its value in the current LP-solution. In
the second set of the partition the selected variable is bounded from below by the round up of
its value in the current LP solution. Note that if the integer variable is binary, this corresponds
to fixing the variable to zero and one respectively.

Each node of the branch-and-bound tree also receives a (unique) identification. This identifi-
cation consists of two numbers: depth and creation. Depth refers to the level of the node in the
branch-and-bound tree. Creation refers to the total number of nodes that have been created in
the branch-and-bound process. The root node receives identification (0,1).

The two following examples show how appl_divide can be used to implement the default branch-
ing scheme. In the first example, the variable is fixed by specifying new bounds. In the second
example, the variable is fixed by specifying new constraints.

```c
#include <stdio.h>
#include <math.h>
#include "minto.h"

/*
 * appl_divide.C
 */

#include <stdio.h>
#include <math.h>
#include "minto.h"

/*
 * appl_divide
 */

unsigned appl_divide (depth, creation, zIp, xlp, zprimal, xprimal,
    ncnt, vcnt, vind, vtype, vvalue,
    nzcnt, ccnt, cfirst, cind, ccoef, csense, crhs, bdim, sdim, ldim)
long depth;  /* identification: depth */
long creation;  /* identification: creation */
double zlp;  /* value of the LP solution */
double *xlp;  /* values of the variables */
double zprimal;  /* value of the primal solution */
double *xprimal;  /* values of the variables */
int *ncnt;  /* variable for number of nodes */
int *vcnt;  /* array for number of variables */
int *vind;  /* array for indices of variables */
char *vtype;  /* array for type of bounds */
double *vvalue;  /* array for value of bounds */
int *nzcnt;  /* variable for number of nonzero coefficients */
int *ccnt;  /* array for number of constraints */
int *cfirst;  /* array for positions of first nonzero coefficients */
int *cind;  /* array for indices of nonzero coefficients */
double *ccoef;  /* array for values of nonzero coefficients */
char *csense;  /* array for senses */
```

40
double *crhs;  /* array for right hand sides */
int bdim;  /* size of bounds arrays */
int sdim;  /* size of small arrays */
int ldim;  /* size of large arrays */
{
    register int i;
    register double frac, diff;
    int index = -1;
    double mindiff = (double) 1;

    for (inq_form (), i = 0; i < info_form.form_vc; i++) {
        if (inq_var (i), info_var.var_class != CONTINUOUS) {
            frac = xlp[i] - floor (xlp[i]);
            if (frac > EPS && frac < 1 - EPS) {
                diff = fabs (frac - 0.5);
                if (diff < mindiff) {
                    mindiff = diff;
                    index = i;
                }
            }
        }
    }

    *ncnt = 2;

    vcnt[0] = 1;
    vcnt[1] = 1;

    vind[0] = index;
    vtype[0] = 'U';
    vvalue[0] = (double) 0;

    vind[1] = index;
    vtype[1] = 'L';
    vvalue[1] = (double) 1;

    ccnt[0] = 0;
    ccnt[1] = 0;

    return (SUCCESS);
}

/
* E_DIVIDE.C
*/

#include <stdio.h>
#include <math.h>
#include "minto.h"

/*
 * appl_divide
 */

unsigned
appl_divide (depth, creation, zlp, xlp, zprimal, xprimal,
 ncnt, vcnt, vind, vtype, vvalue,
 nzcnt, ccnt, cfirst, cind, ccoef, csense, crhs, bdim, sdim, ldim)
long depth;  /* identification: depth */
long creation; /* identification: creation */
double zlp;  /* value of the LP solution */
double *xlp; /* values of the variables */
double zprimal; /* value of the primal solution */
double *xprimal; /* values of the variables */
int *ncnt;  /* variable for number of nodes */
int *vcnt;  /* array for number of variables */
int *vind;  /* array for indices of variables */
char *vtype; /* array for type of bounds */
double *vvalue; /* array for value of bounds */
int *nzcnt; /* variable for number of nonzero coefficients */
int *ccnt; /* array for number of constraints */
int *cfirst; /* array for positions of first nonzero coefficients */
int *cind; /* array for indices of nonzero coefficients */
double *ccoef; /* array for values of nonzero coefficients */
char *csense; /* array for senses */
double *crhs; /* array for right hand sides */
int bdim;  /* size of bounds arrays */
int sdim; /* size of small arrays */
int ldim; /* size of large arrays */
{
    register int i;
    register double frac, diff;
    int index = -1;
    double mindiff = (double) 1;

    for (inq_form (), i = 0; i < info_form.form_vcnt; i++) {
        if (inq_var (i), info_var.var_class != CONTINUOUS) {

        }}
frac = xlp[i] - floor (xlp[i]);
if (frac > EPS && frac < 1 - EPS) {
    diff = fabs (frac - 0.5);
    if (diff < mindiff) {
        mindiff = diff;
        index = i;
    }
}

*ncnt = 2;
vcnt[0] = 0;
vcnt[1] = 0;
*nzcnt = 2;
cnt[0] = 1;
cnt[1] = 1;
cfirst[0] = 0;
cind[0] = index;
ccoef[0] = (double) 1;
csense[0] = 'L';
crhs[0] = (double) 0;
cfirst[1] = 1;
cind[1] = index;
ccoef[1] = (double) 1;
csense[1] = 'G';
crhs[1] = (double) 1;
cfirst[2] = 2;
return (SUCCESS);
}

6.18 appl_rank
This function allows the application to specify the order in which the nodes of the branch-and-bound tree are evaluated. It has to return either FAILURE, in which case MINTO assumes
that the application wants to use the default rank function and it therefore ignores the parameter `rank`, or SUCCESS, in which case MINTO assumes that the rank for the current node is available through the parameter `rank`, or REORDER, in which case MINTO assumes that the application has switched to a different rank function. In this case, MINTO reorders the list of unevaluated nodes. Before reordering, each node receives a new rank by successive calls to `appl_rank`.

**PARAMETERS**

- `depth`: A long containing the depth in the branch-and-bound tree of the node that has been selected for evaluation.
- `creation`: A long containing the creation number of the node that has been selected for evaluation.
- `zIp`: A double containing the value of the LP solution.
- `zprimal`: A double containing the value of the primal solution.
- `rank`: A double to hold the rank to be associated with the current node.

The unevaluated nodes of the branch-and-bound tree are kept in a list. The nodes in the list are in order of increasing rank values. When new nodes are generated either by the default division scheme or the division scheme specified by the `appl_divide` function, each of them receives a rank value provided either by the default rank function or by the function provided by the `appl_rank` function. The rank value of the node is used to insert it at the proper place in the list of unevaluated nodes. When a new node has to be selected, MINTO will always take the node at the head of the list.

The default rank function takes the node creation number as rank, which results in a depth-first search of the branch-and-bound tree.

The following example shows how `appl_rank` can be used to implement the strategy that starts with depth-first and switches to best-bound as soon as a primal feasible solution has been found.

```c
#include <stdio.h>
#include "minto.h"

static unsigned switched = FALSE;

/*
 * E_RANK.C
 */

unsigned
appl_rank (depth, creation, zIp, zprimal, rank)
long depth;     /* identification: depth */
long creation;  /* identification: creation */
double zlp;  /* value of the LP solution */
double zprimal;  /* value of the primal solution */
double *rank;  /* rank value */
{
    if (switched == TRUE) {
        *rank = -zlp;
        return (SUCCESS);
    }
    else {
        if (zprimal < -INF + EPS) {
            *rank = (double) creation;
            return (SUCCESS);
        }
        else {
            *rank = -zlp;
            switched = TRUE;
            return (REORDER);
        }
    }
}

7 Miscellaneous Functions

7.1 inq_prob

This function retrieves the name of the problem that is being solved, i.e., the name found in the NAME section of the < problem name > .mps file that was read when MINTO was invoked. The following example shows how inq_prob can be used to print the name of the problem being solved.

/*
 * E_NAME.C
 */

#include <stdio.h>
#include "minto.h"

/*
 * WriteName
 */

void
WriteName ()

{  
    printf ("Problem name: \%s\n", inq_prob ());
}

7.2 wrLprob

This function writes the active formulation, i.e., the formulation currently loaded in the LP-solver to a specified file in MPS-format.

PARAMETERS
fname: A character string specifying the name of the file to which the active formulation should be written.

The following example shows how wrLprob can be used to write the active formulation to a file.

/*
  * E_WRITE.C
  */

#include <stdio.h>
#include "minto.h"

/*
  * WriteActive
  */

void WriteActive ()
{
    wrLprob ("active.mps");
}

8 Control Functions

MINTO provides more detailed control over the run-time behavior of MINTO through a set of control functions. Each of these control functions can be called any time during the solution process and activates or deactivates one of the system functions.

8.1 ctrl_clique

This function activates or deactivates generation of clique constraints.

PARAMETERS
indicator: An unsigned integer to hold a status indicator that controls the generation of clique constraints, i.e., ON or OFF.
8.2 ctrl_implication
This function activates or deactivates generation of implication constraints.

PARAMETERS
indicator: An unsigned integer to hold a status indicator that controls the generation of implication constraints, i.e., ON or OFF.

8.3 ctrl_knaps cov
This function activates or deactivates generation of lifted knapsack covers.

PARAMETERS
indicator: An unsigned integer to hold a status indicator that controls the generation of lifted knapsack covers, i.e., ON or OFF.

8.4 ctrl_flowcov
This function activates or deactivates generation of simple and extended generalized flow covers.

PARAMETERS
indicator: An unsigned integer to hold a status indicator that controls the generation of simple and extended generalized flow covers, i.e., ON or OFF.

8.5 ctrl_output
This function sets the output level.

PARAMETERS
indicator: An unsigned integer to hold the level of output to be set, i.e., 0, 1, or 2.

9 Programming considerations
The include file minto.h is, and should always be, included in all sources of application functions, since it contains constant definitions, type definitions, external variable declarations, and function prototypes.

The variables and arrays containing information about the LP-solution associated with the active formulation and information about the best primal solution, which are passed as parameters to the application functions, are the ones maintained by MINTO for its own use. They should never be modified; they should only be examined.

MINTO allocates memory dynamically for the arrays that are passed as parameters to an application function. However, from an application program point of view they are fixed length arrays. When appropriate, the current lengths of the arrays are also passed as parameters. It
is the responsibility of the application program to ensure that memory is not overrun. MINTO will abort immediately if it detects a memory violation.

10 Test problems

The distribution of MINTO contains a set of 10 test problems. The main purpose of the test problems is to verify whether the installation of MINTO has been successful. However, MINTO’s performance on this set of test problems also demonstrates its power as a general purpose mixed integer optimizer. Table 3 shows the problem characteristics. Table 4 shows the LP value, the IP value, and the number of evaluated nodes and total cpu time when MINTO is run as a plain branch-and-bound code with all system functions deactivated, and when MINTO is run in its default setting. These runs have been made on an IBM RS/6000 using OSL as LP-solver. We have observed substantial variation in performance when running the system under different architectures because different branch-and-bound trees are generated.

11 Availability and Future Releases

MINTO 1.3a is available on SUN SPARC stations with CPLEX 1.2 installed, IBM RS/6000 workstations with either CPLEX 1.2 or 2.0 or OSL 1.2 installed, and on HP Apollo workstations with CPLEX 1.2 or 2.0 installed.

MINTO is an evolutionary system and therefore version 1.3a is not a final product. We see the development of MINTO as an evolutionary process, that should lead to a robust and flexible mixed integer programming solver. It’s modular structure makes it easy to modify and expand, especially with regard to the addition of new information and application functions. Therefore we encourage the users of this release to provide us with comments and suggestions for future releases.

We envision that future releases will have stronger support for applications using column generation and a names operating mode to provide a fail-safe mechanism for keeping track of variables and constraints that are added during the solution process.

Other developments in future releases may include parallel implementations, more efficient cut generation routines, additional classes of cuts, explicit column generation routines, better primal heuristics and different strategies for getting upper bounds, such as Lagrangian relaxation.

We welcome suggestions for improving MINTO as well as other comments.
Table 3: Characteristics of the test problems

<table>
<thead>
<tr>
<th>NAME</th>
<th>#cons</th>
<th>#vars</th>
<th>#nonzeros</th>
<th>#cont</th>
<th>#bin</th>
<th>#int</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGOUT</td>
<td>98</td>
<td>141</td>
<td>282</td>
<td>86</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>VPM1</td>
<td>234</td>
<td>378</td>
<td>749</td>
<td>210</td>
<td>168</td>
<td>0</td>
</tr>
<tr>
<td>FIXNET3</td>
<td>478</td>
<td>878</td>
<td>1756</td>
<td>500</td>
<td>378</td>
<td>0</td>
</tr>
<tr>
<td>KHB05250</td>
<td>101</td>
<td>1350</td>
<td>2700</td>
<td>1326</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>SET1AL</td>
<td>492</td>
<td>712</td>
<td>1412</td>
<td>472</td>
<td>240</td>
<td>0</td>
</tr>
<tr>
<td>LSEU</td>
<td>28</td>
<td>89</td>
<td>309</td>
<td>0</td>
<td>89</td>
<td>0</td>
</tr>
<tr>
<td>BM23</td>
<td>20</td>
<td>27</td>
<td>478</td>
<td>0</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>P0033</td>
<td>15</td>
<td>33</td>
<td>98</td>
<td>0</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td>P0201</td>
<td>133</td>
<td>201</td>
<td>1923</td>
<td>0</td>
<td>201</td>
<td>0</td>
</tr>
<tr>
<td>P0291</td>
<td>252</td>
<td>291</td>
<td>2031</td>
<td>0</td>
<td>291</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Results for the test problems

<table>
<thead>
<tr>
<th>NAME</th>
<th>minto -s -m100000</th>
<th>minto</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LP value</td>
<td>IP value</td>
</tr>
<tr>
<td>EGOUT</td>
<td>149.588</td>
<td>568.100</td>
</tr>
<tr>
<td>VPM1</td>
<td>15.4167</td>
<td>21.000</td>
</tr>
<tr>
<td>FIXNET3</td>
<td>40717.1</td>
<td>55845.0</td>
</tr>
<tr>
<td>KHB05250</td>
<td>95919464.</td>
<td>106940226.</td>
</tr>
<tr>
<td>SET1AL</td>
<td>11145.6</td>
<td>15918.7</td>
</tr>
<tr>
<td>LSEU</td>
<td>834.68</td>
<td>1120.0</td>
</tr>
<tr>
<td>BM23</td>
<td>20.5709</td>
<td>34.0</td>
</tr>
<tr>
<td>P0033</td>
<td>2520.6</td>
<td>3089.0</td>
</tr>
<tr>
<td>P0201</td>
<td>6875.0</td>
<td>7615.0</td>
</tr>
<tr>
<td>P0291</td>
<td>1705.12</td>
<td>14672.7</td>
</tr>
</tbody>
</table>
Appendix A. Inquiry functions

/*
 * E_UTIL.C
 */

#include "minto.h"

#ifdef PROTOTYPING
void WriteFormulation (void);
char * ConvertCClass (int);
char * ConvertCType (int);
char * ConvertCInfo (int);
char * ConvertVClass (int);
char * ConvertVInfo (int);
char * ConvertStatus (int);
#else
void WriteFormulation ();
char * ConvertCClass ();
char * ConvertCType ();
char * ConvertCInfo ();
char * ConvertVClass ();
char * ConvertVInfo ();
char * ConvertStatus ();
#endif

/*
 * WriteFormulation
 *
 * WriteFormulation is an example of the use of the inquiry functions
 * provided by MINTO to access the formulation in the current node
 * of the branch-and-bound tree.
 */

void WriteFormulation ()
{
    int i, j;

    printf ("\n\nCURRENT FORMULATION:\n\n");
    printf ("OBJECTIVE\n");
    for (inq_obj (), j = 0; j < info_obj.obj_nz; j++) {
        printf (" %f %d\n", info_obj.obj_coef[j], info_obj.obj_ind[j]);
printf ("CONSTRAINTS
n");  
for (inq_form (), i = 0; i < info_form.form_cnt; i++) {  
    printf ("%d\n", i);  
    for (inq_constr (i), j = 0; j < info_constr.constr Nz; j++) {  
        printf ("%f %d\n", info_constr.constr_coef[j], info_constr.constr_ind[j]);  
    }  
    printf ("%s\n", info_constr.constr_sense);  
    printf ("%s\n", ConvertCClass (info_constr.constr_class));  
    printf ("%s\n", Convert CType (info_constr.constr_type));  
    printf ("%s\n", ConvertStatus (info_constr.constr_status));  
    printf ("%s\n", ConvertClnfo (info_constr.constr_info));  
}  
printf ("VARIABLES
n");  
for (i = 0; i < info_form.form_vcnt; i++) {  
    printf ("%d\n", i);  
    for (inq_var (i), j = 0; j < info_var.var_nz; j++) {  
        printf ("%f %d\n", info_var.var_coef[j], info_var.var_ind[j]);  
    }  
    printf ("%s\n", ConvertVClass (info_var.var_class));  
    printf ("%s\n", ConvertStatus (info_var.var_status));  
    printf "%s\n", info_var.var_lb);  
    printf ("%s\n", info_var.var_ub);  
    printf ("%s\n", ConvertVVar (info_var.var_lb_info));  
    printf ("%s\n", ConvertVVar (info_var.var_ub_info));  
    if (info_var.var_vlb) {  
        printf ("%d\n", info_var.var_vlb->vlb_var);  
    }  
    else {  
        printf ("NO VLB\n");  
    }  
    if (info_var.var_vub) {  
        printf ("%d\n", info_var.var_vub->vub_var);  
    }  
    else {  
        printf ("NO VUB\n");  
    }  
}
printf ("\n");
}

static char *bsiu = "BINSUM1UB";
static char *bsie = "BINSUM1EQ";
static char *bs1vu = "BINSUM1VARUB";
static char *bs1ve = "BINSUM1VAREQ";
static char *bsvu = "BINSUMVARUB";
static char *bsve = "BINSUMVAREQ";
static char *svu = "SUMVARUB";
static char *sve = "SUMVAREQ";
static char *vu = "VARUB";
static char *ve = "VAREQ";
static char *vl = "VARLB";
static char *mixu = "MIXUB";
static char *mixe = "MIXEQ";
static char *nlu = "MOBINUB";
static char *nle = "MOBINEQ";
static char *abu = "ALLBINUB";
static char *abe = "ALLBINEQ";

/*
 * ConvertCClass --
 *
 * Convert the constraint class into a printable string.
 */

char *
ConvertCClass (class)
int class;
{
    switch (class) {
    case BINSUM1UB:
        return (bsiu);
    case BINSUM1EQ:
        return (bsie);
    case BINSUM1VARUB:
        return (bs1vu);
    case BINSUM1VAREQ:
        return (bs1ve);
    case BINSUMVARUB:
        return (bsvu);
    case BINSUMVAREQ:
        return (bsve);
    case BINSUMVARUB:
        return (bsvu);
    case BINSUMVAREQ:
        return (bsve);
    case BINSUMVARUB:
        return (bsvu);
    case BINSUMVAREQ:
        return (bsve);
    case BINSUMVARUB:
        return (bsvu);
    case BINSUMVAREQ:
        return (bsve);
    case BINSUMVARUB:
        return (bsvu);
    case BINSUMVAREQ:
return (bsve);
case SUMVARUB:
    return (svu);
case SUMVAREQ:
    return (sve);
case VARUB:
    return (vu);
case VAREQ:
    return (ve);
case VARLB:
    return (vl);
case MIXUB:
    return (mixu);
case MIXEQ:
    return (mixe);
case NOBINUB:
    return (nbu);
case NOBINEQ:
    return (nbe);
case ALLBINUB:
    return (abu);
case ALLBINEQ:
    return (abe);
}
}

static char *local = "LOCAL"
static char *global = "GLOBAL"

/*
 * ConvertCType --
 *
 *   Convert the constraint type into a printable string.
 */

char *
ConvertCType (type)
int type;
{
    switch (type) {
    case LOCAL:
        return (local);
    case GLOBAL:
        return (global);
static char *original = "ORIGINAL";
static char *genminto = "GENERATED_BY_MINTO";
static char *genbranch = "GENERATED_BY_BRANCHING";
static char *genappl = "GENERATED_BY_APPL";

/*
 * ConvertCInfo --
 *    Convert the constraint status into a printable string.
 */
char * ConvertCInfo (info)
int info;
{
    switch (info) {
    case ORIGINAL:
        return (original);
    case GENERATED_BY_MINTO:
        return (genminto);
    case GENERATED_BY_BRANCHING:
        return (genbranch);
    case GENERATED_BY_APPL:
        return (genappl);
    }
}

static char *cont = "CONTINUOUS";
static char *bin = "BINARY";
static char *integ = "INTEGER";

/*
 * ConvertVClass --
 *    Convert the variable class into a printable string.
 */
char * ConvertVClass (class)
int class;
{

switch (class) {
    case CONTINUOUS:
        return (cont);
    case BINARY:
        return (bin);
    case INTEGER:
        return (integ);
}

static char *modminto = "MODIFIED_BY_MIITO";
static char *modbranch = "MODIFIED_BY_BRANCHING";
static char *modappl = "MODIFIED_BY_APPL";

/*
 * ConvertVInfo --
 *     Convert the constraint status into a printable string.
 */

char *
ConvertVInfo (info)
int info;
{
    switch (info) {
    case ORIGINAL:
        return (original);
    case MODIFIED_BY_MIITO:
        return (modminto);
    case MODIFIED_BY_BRANCHING:
        return (modbranch);
    case MODIFIED_BY_APPL:
        return (modappl);
    }
}

static char *act = "ACTIVE";
static char *inact = "INACTIVE";
static char *del = "DELETED";

/*
 * ConvertStatus --
 *     Convert the constraint status into a printable string.
 */
char * ConvertStatus (status)
int status;
{
    switch (status) {
        case ACTIVE:
            return (act);
        case INACTIVE:
            return (inact);
        case DELETED:
            return (del);
    }
}