Shaped-multibeam antennas for the reception of TV-programs of broadcasting satellites.

by

ir. J.L.M. Buijnsters

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Philips GmbH Tuner Werk Krefeld
Division Development TV Tuners & RF Units

Supervisors: R. Schiltmans (PTWK)
             dr. M.E.J. Jeuken (EUT)


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Summary

At the moment, the number of satellites for TV-broadcasting is increasing very fast. Because of this, there is a great demand for single reflector antennas for receiving TV-signals from two or more satellites simultaneously. To satisfy this demand a new special shaped-reflector antenna has been developed. This antenna can receive TV-signals from two or more satellites simultaneously, which are positioned up to 60 degrees apart from each other.

The new shaped-reflector antenna is described comprehensively in this report. First, the shaped-reflector surface and the optimum feed locus are derived both for the symmetric and the offset-reflector antenna. Second, the radiation pattern of the shaped-reflector antenna is described. Then the calculations of the cross polarization, the antenna gain and the G/T are described. Finally, three specific designs of shaped-reflector antennas for simultaneous reception of two or three satellites are presented. These designs were made with a computer programme developed by the author of this report. The four designs are:

1. Astra 1 (19.2° east) and Eutelsat (13° east)
2. Astra 1 (19.2° east) and Astra 2 (28.2° east)
3. Astra 1 (19.2° east), Eutelsat (13° east) and Astra 2 (28.2° east)
4. Astra 1 (19.2° east), Eutelsat (13° east) and Telenor (1° west)

The advantages for the first design of the shaped-reflector antenna compared to the offset parabola are moderated. The main reason of this is that the two satellites are positioned too close together to get some profit of the shaped-reflector antenna. For the second, third and fourth design, in which the satellites are positioned further apart from each other, the shaped-reflector antenna has great advantages when compared to the offset parabola. One of these advantages is that the antenna gain for the shaped-reflector antenna remain across the total field of view pretty much the same, while the gain of the offset parabola has a strong decrease for greater angles of reception. Furthermore for greater scan angles of reception the offset parabola does not meet the requirements of the European Telecommunication Standard (E.T.S.). For that case the shaped reflector should be preferred. Another antenna with the possibility to receive satellites which are positioned further away from each other is the torus. The main disadvantage of the torus compared to the shaped-reflector antenna is the limited space for the positions of the feeds, since the "foci" of the torus are positioned very close to each other.

The main conclusion is that the shaped-reflector antenna is superior to any other conventional-reflector antenna for the reception of TV-satellites which are standing approximately more than 10 degrees apart from each other.
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1 Introduction

Reflector antennas can be thought of as mathematical surfaces that redirect microwave rays from a point source feed to an aperture and beyond, into the far-field radiation pattern. In order for the surface to be useful as an antenna, it must have a special shape that reflects all rays so that they leave in a collimated beam and traverse the same total distance from the feed to a perpendicular planar wavefront. Only the parabola accomplishes these two constraints simultaneously. However, the greatest disadvantage of the parabola is the limited scan angle. When the feed source of a parabola is moved away from the focus on the axis of symmetry, the rays reflecting from the surface point in the direction of the scan angle, measured from the axis or boresight direction, are no longer exactly parallel. This leads to phase errors across the wavefront. As the feed displacement and scan angle increase, the aberrations begin to lower the peak gain and raise the sidelobe level of the antenna. The factors that effect the radiation pattern of a parabolic reflector for a particular scan angle are the ratio of the focal length to the reflector diameter ($F/D$) and the electrical size of the aperture. As the $F/D$ ratio of a reflector increases, the surface curvature decreases, so the individual ray variations are not as great as for a deep, small $F/D$ reflector. Thus, large $F/D$ reflectors have large maximum scan angles. Also, as the electrical diameter $D/\lambda$ of the antenna increases, the geometrical errors are electrically magnified, resulting in greater phase errors and a smaller field of view. The conclusion of the parabola with a general $F/D$ ratio and electrical diameter is that scan angles of only a few degrees can be reached. This makes the parabola unsuitable for our purpose. The parabola with the feed in and out focus and the corresponding perpendicular wavefront is shown in fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{parabola_diagram.png}
\caption{The parabola with the feed in and out focus.}
\end{figure}
The torus reflector is an other type of reflector antenna, which generates much better scanned beams. The torus reflector is circular in the plane of scan or azimuth plane and parabolic in the orthogonal or elevation plane. Since the radius of curvature of a parabola at its vertex is double its focal length, the torus azimuth circle radius is chosen normally be twice the elevation parabola focal length. The important condition for this reflector with circular profile is that each feed illuminates only a small portion of the reflector for which the circle does not deviate from a parabola too much. As with paraboloids, the geometrical difference between the actual circle and the desired parabola becomes increasingly significant with increased frequency. The greatest problem with circular profile reflectors is that for large scan angles the illuminated aperture portion in only a small fraction of the entire reflector. In other words, the aperture efficiency of the torus is very bad. The aperture efficiency of the torus is also inversely related to its field of view. An other disadvantage of the torus is the positions of the "focal points". Since these "focal points" are closely together, there is limited place for the different feeds. The torus with the illuminated portions of the reflector surface is shown in fig. 2.

![Torus Reflectors](image)

**Figure 2: The torus**

This report present a new scanning reflector shaping method based on an idea proposed by Rappaport [1] and [2]. This principle is to shape the reflector to optimize simultaneously the aperture phase laws for the two different beams, one in the boresight direction and the other at the maximum angle of scan. The proposed method can roughly be summarized into three steps. Setting up of the initial geometry and optimization of the reflector surface and derivation of the optimum feed locus take place. In all the above optimization processes Geometrical Optics is applied.
2 Design methode of the shaped reflector

The shaped-reflector surface has to be optimized to minimize the phase errors for two beams. The first beam propagates along the axis of symmetry of the shaped reflector, known as the unscanned direction. The second beam propagates along an axis which is rotated $\alpha$ degrees with respect to the symmetry axis, known as the scanned direction. Purpose of this section is to find a description of the shaped-reflector surface in the plane $y = 0$ and the locations and orientations of the feeds such that the phase errors of the scanned and unscanned beams are minimum. For this reason, the starting point is a paraboloid with focal length $f_i$ and tilted by $\alpha$ degrees. The mathematical description of this paraboloid is given in the first paragraph. Then, in the second paragraph a two dimensional optimization at the plane $y = 0$ is described. And in the last paragraph a derivation of the locations and orientations of the feeds is given.

2.1 Initial antenna geometry

The starting point is a description of the tilted paraboloid with focal length $f_i$ and $\alpha$ being the maximum angle of scan. The tilted paraboloid has to meet the following two requirements:

1. The orgin of the coordinate system has to belong to the tilted paraboloid.
2. The tilted paraboloid has to be a zero partial derivative over $x$ at the orgin of the coordinate system; $\frac{\partial z}{\partial x} = 0$.

To find the expression of the tilted paraboloid we start with the expression of the paraboloid, which is not tilted or translated in the $x''', y''', z'''$ coordinate system. The symmetric paraboloid with the vertex at the orgin of the coordinate system and the $z'''$-axis as the axis of symmetry can be expressed as:

$$z''' = \frac{1}{4f_i}(x'''^2 + y'''^2)$$

Extension of equation (1) gives an expression of the offset-paraboloid antenna:

$$z'' = -\frac{y'''^2}{4f_i} + \frac{1}{4f_i} (x'''^2 + (y'''' - y''')^2)$$

The parameter $y'''$ represents the height between the centre of the reflector and the scanned feed which is called the offset-height. So, $y''' = 0$ corresponds to a symmetric paraboloid and $y''' > 0$ will be used for an offset paraboloid. It can be shown in fig. 3 that the vertex of the paraboloid $y'''$ is moved along the $y''''$-axis. The expression of the offset paraboloid (2) meets also to the two requirements mentioned above. An intersection of this paraboloid with the $y''z'''$-plane and the $x''z'''$-plane is shown in fig. 3 and fig. 4 respectively. The focus $F_i$ in the $(x''', y''', z''')$ coordinate system is given by:

$$\begin{align*}
x'' &= 0 \\
y'' &= y'''
\\z'' &= f_i - \frac{y'''^2}{4f_i}
\end{align*}$$
Making use of the property of a paraboloid that transmitted rays starting at a feed located at the focus will reflect and be collimated to form a planar aperture field without phase errors:

\[
\rho + \rho \cos \theta = 2f_t \\
\sqrt{x''^2 + (y'' - y_t')^2 + (z'' - z_t')^2 + (z'' - z'')} = 2f_t \\
x''^2 + (y'' - y_t')^2 + 4f_t(z_t'' - z'') - 4f_t^2 = 0
\]

The plane \( z'' = z_t'' \) is chosen to be the equi-phase plane.

By means of: \( z_t'' = f_t - \frac{y''^2}{4f_t} \)

we finally find:

\[
x''^2 + y''^2 - 2y''y_t'' - 4f_t z'' = 0
\]

(4)
Now the paraboloid will be rotated by an angle $-\alpha$ degrees. However, the same result is obtained by rotating the coordinate system by an angle $+\alpha$ degrees. This rotation is shown in fig. 5.

The matrix of the coordinate transformation is given by:

$$
\begin{pmatrix}
  x'' \\
  y'' \\
  z''
\end{pmatrix} =
\begin{pmatrix}
  \cos \alpha & 0 & \sin \alpha \\
  0 & 1 & 0 \\
  -\sin \alpha & 0 & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix}
$$

(5)

Substitution of (5) in (3) gives:

$$
x'_t \cos \alpha + z'_t \sin \alpha = 0
$$

$$
y'_t = y'_t
$$

$$
-x'_t \sin \alpha + z'_t \cos \alpha = f_t - \frac{y'^2_t}{4f_t}
$$

The final result of the focus $F_t$ in the $(x', y', z')$ coordinate system can be written as follows:

$$
\begin{align*}
x'_t &= f_t \sin \alpha \left( \frac{y'^2_t}{2f_t} \right)^2 - 1 \\
y'_t &= y'_t \\
z'_t &= f_t \cos \alpha \left( 1 - \left( \frac{y'^2_t}{2f_t} \right)^2 \right)
\end{align*}
$$

(6)

The expression of the paraboloid in the $(x', y', z')$ coordinate system can be found by substitution of (5) in (4):

$$
x'^2 \cos^2 \alpha + 2x'z' \sin \alpha \cos \alpha + z'^2 \sin^2 \alpha + y'^2 - 2y' y'_t + 4 f_t x' \sin \alpha - 4 f_t z' \cos \alpha = 0
$$

(7)

To meet the two requirements mentioned above, a translation of the paraboloid has to be executed. The same results can be obtained by a translation of the coordinate system, which is shown in fig. 6.
To find the transformation \((x', y', z') \rightarrow (x, y, z)\) we have to look for the point, at the plane \(y' = y = 0\), where the partial derivative over \(x'\) equals zero, or \(\frac{\partial z}{\partial x'} = 0\). The first step is to rewrite (7) as a second-order polynomial in \(x'\). Then we have to look for the value of \(z'\) with the discriminant equals zero. With the discriminant equals zero, there is precisely one solution of \(x'\) for this particular value of \(z'\). So, the value of \(x'\) and \(z'\) correspond with the origin of the \((x, y, z)\) coordinate system. Rewriting (7) as a second-order polynomial in \(x'\), gives:

\[
ax'^2 + bx' + c = 0
\]

with:

\[
a = \cos^2 \alpha \\
b = 2z' \sin \alpha \cos \alpha + 4f_t \sin \alpha \\
c = z'^2 \sin^2 \alpha + y'^2 - 2y' y_t - 4f_t z' \cos \alpha
\]

The discriminant of this equation is:

\[
D^2 = b^2 - 4ac \\
= 16z'^2 f_t \cos \alpha - 4y'^2 \cos^2 \alpha + 16f_t^2 \sin^2 \alpha + 8y' y'_t \cos^2 \alpha
\]

With the constrain that the discriminant equals zero at the plane \(y' = y = 0\), gives:

\[
z' = -f_t \frac{\sin^2 \alpha}{\cos \alpha} \quad \Rightarrow \quad z = z' + f_t \frac{\sin^2 \alpha}{\cos \alpha}
\]  

(8)

The solution of (7) with the discriminant equals zero can be written as:

\[
x' = x'_1 = x'_2 = -\frac{b}{2a} = -f_t \sin \alpha \left(1 + \frac{1}{\cos^2 \alpha}\right)
\]

\[
\Rightarrow x = x' + f_t \sin \alpha \left(1 + \frac{1}{\cos^2 \alpha}\right)
\]  

(9)
The transformation of the \((x', y', z')\) coordinate system to the \((x, y, z)\) coordinate system can be found with the help of \(y' = y\) and the expressions (8) and (9):

\[
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} - f_t \sin \alpha \begin{pmatrix}
  \cos \alpha + \frac{1}{\cos \alpha} \\
  0 \\
  \sin \alpha
\end{pmatrix}
\]

Substitution of (10) into (6) gives:

\[
x_t - f_t \sin \alpha - f_t \frac{\sin \alpha}{\cos^2 \alpha} = f_t \sin \alpha \left( \left( \frac{y_t}{2f_t} \right)^2 - 1 \right)
\]

\[
y_t = y_t
\]

\[
z_t - f_t \frac{\sin^2 \alpha}{\cos \alpha} = f_t \cos \alpha \left( 1 - \left( \frac{y_t}{2f_t} \right)^2 \right)
\]

Finally the coordinates of the focus \(F_t\) of the tilted and translated paraboloid can be written as:

\[
\begin{align*}
x_t &= f_t \sin \alpha \left( \frac{1}{\cos^2 \alpha} + \left( \frac{y_t}{2f_t} \right)^2 \right) \\
y_t &= y_t \\
z_t &= f_t \cos \alpha \left( \frac{1}{\cos^2 \alpha} - \left( \frac{y_t}{2f_t} \right)^2 \right)
\end{align*}
\]

To find the general solution of \(z_t(x, y)\) we have to rewrite expression (7) as a function of \(z\). First we have to apply the coordinate transformation to the equation (7). After the coordinate transformation the expression (7) can be written as:

\[
ax^2 + bz + c = 0
\]

with:

\[
a = \sin^2 \alpha \\
b = 2x \sin \alpha \cos \alpha - \frac{4f_t}{\cos \alpha} \\
c = x^2 \cos^2 \alpha + y(y - 2y_t)
\]

The two solutions of this equation are:

\[
z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
= \frac{1}{\sin \alpha} \left[ \frac{2f_t}{\sin \alpha \cos \alpha} - x \cos \alpha \pm \sqrt{\left( \frac{2f_t}{\sin \alpha \cos \alpha} \right)^2 + y(2y_t - y) - \frac{4f_t}{\sin \alpha} x} \right]
\]

With the constrain \(\frac{\partial z_{1,2}}{\partial x} = 0\) at the plane \(y = 0\), there remains only one solution:

\[
z_{1,2} = \frac{1}{\sin \alpha} \left[ \frac{2f_t}{\sin \alpha \cos \alpha} - x \cos \alpha - \sqrt{\left( \frac{2f_t}{\sin \alpha \cos \alpha} \right)^2 + y(2y_t - y) - \frac{4f_t}{\sin \alpha} x} \right]
\]
The tilted paraboloid $z_{ti}(x,y)$ is described in (12) and the coordinates of the corresponding focus $F_t$ are given in (11). The equations (11) and (12) are similar to expressions to be found in [1] and [2].

### 2.2 Two dimensional profile optimization

The next step is to approximate the tilted paraboloid $z_{ti}(x)$ by an even polynomial in $x$ called $z_{pol}(x)$ at the plane $y = 0$. Since the paraboloid is symmetric about the $YOZ$-plane, the polynomial has only terms of even order:

$$z_{pol}(x) = z_o + r_1 x^2 + r_2 x^4 \quad (13)$$

On account of the symmetry about the $YOZ$-plane and the constraint that the origin of the coordinate system has to be a point on the paraboloid, $z_0$ is equal to:

$$z_0 = 0$$

At the moment we assume that $z_{pol}(x)$ is the expression of the unknown profile of the reflector surface for $y=0$, so:

$$z_{pol}(x) = z_{sur}(x, 0) \quad (14)$$

The coefficients $r_1$ and $r_2$ are determined over a specified $x$-range interval. The range of this interval depends on the maximum acceptable phase error. The $x$-range interval is shown in fig. 7.

The coefficients $r_1$ and $r_2$ of the polynomial will be found by means of the least-squares method. The procedure of the least-squares method can be described as follows:

First the difference between the two functions $z_{pol}(x_i)$ and $z_{ti}(x_i, 0)$ is formed at $N$ regularly spaced points. Then the differences at these points are squared and summed. To find the minimum values at $r_1$ and $r_2$ we have to differentiate the remaining equations with respect to $r_1$ and $r_2$. Finally we obtain two linear equations for $r_1$ and $r_2$. 

![Figure 7: x-range interval](image-url)
First we start with the following N expressions:

\[ r_1 x_1^2 + r_2 x_1^4 - z_i(x_1, 0) = \text{res}_1 \]
\[ r_1 x_2^2 + r_2 x_2^4 - z_i(x_2, 0) = \text{res}_2 \]
\[ \vdots \]
\[ r_1 x_i^2 + r_2 x_i^4 - z_i(x_i, 0) = \text{res}_i \]
\[ \vdots \]
\[ r_1 x_N^2 + r_2 x_N^4 - z_i(x_N, 0) = \text{res}_N \]

\( \text{res}_1, \text{res}_2, \ldots, \text{res}_i, \ldots, \text{res}_N \) are called the residues.

Next we minimize that:

\[ \sum_{i=1}^{N} \text{res}_i^2 = \varphi(r_1, r_2) \]

This means:

\[ \frac{\partial \varphi}{\partial r_1} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial r_2} = 0 \quad (15) \]

\[ \frac{\partial \varphi}{\partial r_1} = \sum_{i=1}^{N} \left( \frac{\partial \varphi}{\partial \text{res}_i} \cdot \frac{\partial \text{res}_i}{\partial r_1} \right) \quad \text{with:} \quad \begin{cases} \frac{\partial \varphi}{\partial \text{res}_i} = 2 \text{res}_i \\ \frac{\partial \text{res}_i}{\partial r_1} = x_i^2 \end{cases} \]

\[ \frac{\partial \varphi}{\partial r_2} = \sum_{i=1}^{N} \left( \frac{\partial \varphi}{\partial \text{res}_i} \cdot \frac{\partial \text{res}_i}{\partial r_2} \right) \quad \text{with:} \quad \begin{cases} \frac{\partial \varphi}{\partial \text{res}_i} = 2 \text{res}_i \\ \frac{\partial \text{res}_i}{\partial r_2} = x_i^4 \end{cases} \]

Substituted into (15) gives:

\[ \begin{cases} \sum_{i=1}^{N} \text{res}_i \cdot x_i^2 = 0 \\ \sum_{i=1}^{N} \text{res}_i \cdot x_i^4 = 0 \end{cases} \quad (16) \]

Substitution of \( \text{res}_i = r_1 x_i^2 + r_2 x_i^4 - z_i(x_i, 0) \) into (16) gives:

\[ \begin{cases} \sum_{i=1}^{N} (r_1 x_i^2 + r_2 x_i^4 - z_i(x_i, 0)) x_i^2 = 0 \\ \sum_{i=1}^{N} (r_1 x_i^2 + r_2 x_i^4 - z_i(x_i, 0)) x_i^4 = 0 \end{cases} \]

Rewriting these equations:

\[ \begin{pmatrix} \sum_{i=1}^{N} x_i^4 & \sum_{i=1}^{N} x_i^6 \\ \sum_{i=1}^{N} x_i^6 & \sum_{i=1}^{N} x_i^8 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N} (z_i(x_i, 0) \cdot x_i^2) \\ \sum_{i=1}^{N} (z_i(x_i, 0) \cdot x_i^4) \end{pmatrix} \quad (17) \]

We obtain a system of linear equations with \( r_1 \) and \( r_2 \) as the unknown. To determine the coefficients \( r_1 \) and \( r_2 \) with great accuracy, the value of \( N \) is equated to 100.
2.3 Derivation of the optimum feed locus

At this stage, only the location of the scanned focus is known. To find the location of the unscanned focus we start with the expression of the symmetric untilted paraboloid with focal length \( f_u \) and the z-axis as the axis of symmetry.

\[
z_{ut}(x, y) = \frac{1}{4f_u} (x^2 + y^2) \tag{18}
\]

Extension of (18) gives an expression of the offset paraboloid:

\[
z_{ut}(x, y) = -\frac{y_u^2}{4f_u} + \frac{1}{4f_u} (x^2 + (y - y_u)^2) \tag{19}
\]

The parameter \( y_u \) represents the height between the centre of the reflector and the unscanned feed. Equation (19) has to meet the two requirements from paragraph 2.1 also. With the help of fig. 8 we can rewrite (19) as follows:

![Figure 8: Unscanned offset paraboloid.](image)

\[
z_{ut}(x, y) = z_u - f_u + \frac{1}{4f_u} (x^2 + (y - y_u)^2) \tag{20}
\]

For small values of \( x \), \( z_{pol} \) and \( z_{sur}(x, 0) \) can be approached as:

\[
z_{sur}(x, 0)|_{x=0} = z_{pol}(x)|_{x=0} \approx r_1 x^2 \tag{21}
\]

Equation (21) describes in the plane \( y = 0 \) a parabola. Comparing the equations (21) with equation (18) gives an expression for \( r_1 \):

\[
r_1 = \frac{1}{4f_u} \tag{22}
\]
It should be possible to determine the position of the focus \( F_u(x_u, y_u, z_u) \) by equated (20) and (21) together with their partial derivatives at the point \( x = y = 0 \).

\[
z_{ut}(0,0) = z_u - f_u + \frac{y_u^2}{4f_u} = z_{sur}(0,0)_{|x=0} = 0 \quad (23)
\]

\[
\frac{\partial z_{ut}}{\partial x} \bigg|_{x=0} = 0 = \frac{\partial z_{sur}(x,0)_{|x=0}}{\partial x} \bigg|_{x=0} = 0 \quad (24)
\]

\[
\frac{\partial z_{ut}}{\partial y} \bigg|_{y=0} = -\frac{y_u}{2f_u} = \frac{\partial z_{sur}(x,0)_{|x=0}}{\partial y} \bigg|_{y=0} = 0 \quad (25)
\]

Substitution of (23) into (22) gives:

\[
z_u = \frac{1}{4f_1} - r_1y_u^2 \quad (26)
\]

From (24) follows:

\[
x_u = 0 \quad (27)
\]

The remaining task is to determine \( y_u \). This is not possible with (25), since the partial derivative of \( z_{sur} \) over \( y \) is unknown. However, this will be solved by replacing \( z_{pol} \) (13) with \( z_{ti} \) (12) in equation (21), so:

\[
z_{sur}(x,0)_{|x=0} = z_{pol}(x)_{|x=0}
\]

becomes:

\[
z_{sur}(x,y)_{|x=y=0} = z_{ti}(x, y)
\]

And:

\[
\frac{\partial z_{sur}(x,y)_{|x=y=0}}{\partial y} \bigg|_{x=y=0} = \frac{\partial z_{ti}(x,y)_{|x=y=0}}{\partial y} = -\frac{y_t \cos \alpha}{2f_t}
\]

Equation (25) becomes:

\[
\frac{\partial z_{ut}}{\partial y} \bigg|_{y=0} = -\frac{y_u}{2f_u} = \frac{\partial z_{sur}(x,y)_{|x=y=0}}{\partial y} \bigg|_{x=y=0} = -\frac{y_t \cos \alpha}{2f_t}
\]

By the use of equation (22) we finally find the expression of \( y_u \).

\[
y_u = y_t \cos \alpha \frac{1}{4f_t r_1} \quad (28)
\]

Summarized the unscanned focus \( F_u \):

\[
\begin{align*}
x_u &= 0 \\
y_u &= y_t \cos \alpha \frac{1}{4f_t r_1} \\
z_u &= \frac{1}{4r_1} - r_1y_u
\end{align*} \quad (29)
\]

At this stage the expressions of the reflector surface \( z_{sur} \) for \( y = 0 \) and the feed coordinates of the scanned and unscanned foci are known. However, the orientations of the feeds are
still unknown. In other words, it is still unknown which parts of the reflector surface are considered as the centre of the illuminated circles. These places can also be indicated as the places where: \( \theta_u = 0 \) for the unscanned situation and \( \theta_t = 0 \) for the scanned situation. Each feed is assumed to be at the origin of the corresponding \((r, \theta, \phi)\) coordinate system, with \( \theta = 0 \) as the axis of symmetry for the feeds. The middle of the \( x_t \)-range interval, indicated with \( x_{ct} \) and the origin of the \((x, y, z)\) coordinate system are taken as the centre of the illuminated circles for the scanned and unscanned feeds respectively. The reflector surface \( Z_{sur} \) for \( y = 0 \) and the locations and orientations of the feeds are shown in fig. 9.

It should be noticed that fig. 9 is valid for symmetric as well as offset reflectors. The parameters \( \alpha, f_t \) and \( y_t \) determine the shape of the reflector and the locations of the feeds. The offset height is determined by \( y_t \) with: \( y_t = 0 \) corresponds to a symmetric reflector and \( y_t > 0 \) will used for offset reflectors. In the next paragraph an extension of the reflector surface to three dimensions will be given.

Figure 9: *Locations and orientations of the feeds.*
3 Three dimensional optimization.

The extension of the plane $y = 0$ of the previous section to the 3 dimensional reflector surface $z_{\text{sur}}(x, y)$ is presented in this section. The derivation who describes this extension is valid for both the symmetric and offset reflector. The derivation starts with the groundplane of the reflector surface which will be divided into a grid of points $(x_i, y_j)$. This grid of points is defined in the first paragraph. To calculate the phase errors in the scanned and unscanned direction we have to introduce a reference plane and a reference distance for both directions. The reference planes and distances are described in the second paragraph. In the third paragraph we calculate for each grid point the most optimum $z$-coordinate when we use theoretical feeds with uniform radiation patterns. The most optimum $z$-coordinate means that rays from both feeds have minimum phase errors. Also in the fourth paragraph we calculate for each grid point the most optimum $z$-coordinate but now for practical feeds with non uniform radiation patterns. Since each grid point is illuminated by a different amount of radiation energy from both feeds, we have to introduce weighting functions. At the end of this section we have calculated the total reflector surface into a grid of points $(x_i, y_j, z_{ij})$.

3.1 Dimensions of the reflector surface.

A front view of the reflector surface is shown in fig. 10. We distinguish the different parts of the reflector surface which are mainly illuminated by the different feeds. The feed which is placed in the unscanned focal point will mainly illuminate the circle in the middle of the reflector surface and the feed which is placed in the scanned focal point will mainly illuminate the outer circle. Areas where these circles overlap each other, will therefore be illuminated by both feeds. These areas play a prominent part in the use of theoretical and practical feeds, which will be treated in paragraph 3.3 and 3.4 respectively. Notice that the positions of the circles depend of the choice of the height and the width of the reflector surface. The centre of the scanned circle, indicated by $x_{ct}$, can be calculated as follows:

$$x_{ct} = \frac{\text{width}}{2} - \text{radius of the circles}$$

Figure 10: Groundplane of the reflector surface.
The groundplane of the reflector surface is divided into a grid of points \((x_i, y_j)\). The aim is to find for each grid point the most optimum \(z\)-coordinate. The most optimum \(z\)-coordinate means that reflected rays from a grid point for both feeds have minimum phase errors. Since both the symmetric and offset reflector are symmetrical with respect to the \(y\)-axis, we only have to calculate the grid points of the plane \(x > 0\). Therefore, the \(x\)-axis for \(x > 0\) and the \(y\)-axis are divided into \(N\) points. The \(x\) values of the grid are indicated by \(x_i\), with \(i = 1, 2, ..., N\) and the \(y\) values of the grid are indicated by \(y_j\), with \(j = 1, 2, ..., N\) if \(x \leq x_{ct}\). For \(x > x_{ct}\) we have to determine for each \(x_i\) value the first value \(M_1\) and the last value \(M_2\) of the index \(j\). Since \(M_1\) and \(M_2\) are dependented on the value of \(i\), we can write for \(i\) and \(j\):

\[
\begin{align*}
  i &= 1, 2, ..., N \\
  j &= M_1(i), ..., M_2(i) \quad \left\{ \begin{array}{ll}
    M_1(i) & \geq 1 \\
    M_2(i) & \leq N
  \end{array} \right.
\end{align*}
\]

### 3.2 Calculations of the reference planes and reference distances.

The reference plane and reference distance in the unscanned situation are shown in fig. 11. The reference plane has to be perpendicular to the reflected rays from the feed which is placed in the unscanned focal point. For the unscanned reference distance \(d_{ref_u}\) is chosen the distance from the unscanned focal point \(F_u\) to the orgin of the coordinate system and from the orgin to the reference plane. The orgin of the coordinate system is chosen because this is the centre of the illuminated circle in the unscanned situation. We assume that each reflected ray from the feed which is placed in the unscanned focal point is perpendicular to the corresponding reference plane. The reference distance in the unscanned situation can be calculated as follows:

\[
d_{ref_u} = \sqrt{y_u^2 + z_u^2} + z_p
\]

The reference plane and reference distance in the scanned situation are shown in fig. 12. The reference plane in the scanned situation is perpendicular to the reflected rays from the feed which is placed in the scanned focal point. The expression for the reference plane in the scanned situation is:

\[
z = x \tan \alpha + z_t
\]
Notice that the reference plane crosses the z-axis through the point $z_t$. For the scanned reference distance $d_{ref}$ is chosen the distance from the focal point $F_t$ to the point $(x_{ct}, 0, z_{pol}(x_{ct}))$ of the reflector surface and from the point $(x_{ct}, 0, z_{pol}(x_{ct}))$ to the reference plane. The point $(x_{ct}, 0, z_{pol}(x_{ct}))$ is chosen because this is the centre of the illuminated circle in the scanned situation. We also assume that in the scanned situation each reflected ray is perpendicular on the corresponding reference plane. For the first part of the reference distance between the focal point $F_t$ and the point $(x_{ct}, 0, z_{pol}(x_{ct}))$ we can write:

$$d_1 = \sqrt{(x_{ct} - x_t)^2 + y_t^2 + (z_{pol}(x_{ct}) - z_t)^2}$$

The distance between the point $(x_{ct}, 0, z_{pol}(x_{ct}))$ and the reference plane can be calculated with the help of an extra plane. This extra plane has to pass through the point $(x_{ct}, 0, z_{pol}(x_{ct}))$ and has to be parallel to the reference plane. This extra plane crosses the z-axis through the point $C$, which is shown in fig. 12. Point C can be expressed as follows:

$$z = x \tan \alpha + C$$
$$z_{pol}(x_{ct}) = x_{ct} \tan \alpha + C$$
$$\Rightarrow C = z_{pol}(x_{ct}) - x_{ct} \tan \alpha$$

The shortest distance $d_2$ between both planes can be expressed as follows:

$$d_2 = (z_t - C) \cos \alpha$$
$$d_2 = z_t \cos \alpha - z_{pol}(x_{ct}) \cos \alpha + x_{ct} \sin \alpha$$

Finally, the total reference distance $d_{ref} = d_1 + d_2$ can be expressed as:

$$d_{ref} = \sqrt{(x_{ct} - x_t)^2 + y_t^2 + (z_{pol}(x_{ct}) - z_t)^2 + (z_t - z_{pol}(x_{ct})) \cos \alpha + x_{ct} \sin \alpha} \quad (32)$$
3.3 Calculations of the reflector surface with an uniform illumination.

In this paragraph we use feeds with an uniform radiation pattern. This means that within the illuminated circles, shown in fig. 10, the radiation energy uniform is. We assume that the radiated energy outside the illuminated circles equals zero.

Now, we determine for each grid point \((x_i, y_j)\) the optimum z-coordinate \(Z_{ij}\). Therefore we have to compare for each grid point \((x_i, y_j)\) the distance \(d_{u,ij}(z)\) with the reference distance \(d_{refu}\) in the unscanned situation and \(d_{s,ij}(z)\) with \(d_{refs}\) in the scanned situation. The unscanned and scanned situation are shown in fig. 13 and fig. 14 respectively. It can be shown

\[
d_{u,ij}(z) = \sqrt{(x_i - x_u)^2 + (y_j - y_u)^2 + (z - z_u)^2 + z_u - z}
\]  

Figure 13: Calculations of the grid points in the unscanned situation.

Figure 14: Calculations of the grid points in the scanned situation.

from fig. 13 that the unscanned distance \(d_{u,ij}\) can be written as:

When we replace the point \((x_{cl}, 0, z_{pol}(x_{cl}))\) from (32) by \((x_i, y_j, z)\), the scanned distance \(d_{s,ij}\) can be written as:
The difference in path length between $d_{u,ij}(z)$ and $d_{t,ij}(z)$ with the corresponding reference distances $d_{refu}$ and $d_{reft}$ can be expressed in the following two error functions:

$$
err_{u,ij}(z) = (d_{refu} - d_{u,ij}(z))^2
$$

$$
err_{t,ij}(z) = (d_{reft} - d_{t,ij}(z))^2
$$

Notice, that these error functions are only a function of $z$. Equations (35) and (36) are only valid for points of the reflector surface which are illuminated by the feed in the unscanned and scanned focus respectively. For points of the reflector surface which are illuminated by both both feeds, we can write the following error function.

$$
err_{T,ij}(z) = err_{u,ij}(z) + err_{t,ij}(z)
$$

The most optimum value of $z_{ij}$ can be found by minimizing the error functions (35), (36) and (37). This can be done by equating the derivatives of each error function to zero. For points of the reflector surface which are only illuminated by the feed which is placed in the unscanned focal point, we can write:

$$
\frac{d(\text{err}_{u,ij}(z))}{dz} = -2(d_{refu} - d_{u,ij}(z)) \frac{d(d_{u,ij}(z))}{dz} = 0
$$

\[
\frac{d(d_{u,ij}(z))}{dz} = \frac{(z - z_u)}{\sqrt{(x_i - x_u)^2 + (y_j - y_u)^2 + (z - z_u)^2}} - 1
\]

When we use the following definition:

$$
dis_u = \sqrt{(x_i - x_u)^2 + (y_j - y_u)^2 + (z - z_u)^2}
$$

, equation (38) can be expressed as:

$$
(d_{refu} - d_{u,ij}(z)) \cdot \left(\frac{(z_u - z)}{dis_u} + 1\right) = 0
$$

(39)

The definitions of $d_{refu}$ and $d_{u,ij}(z)$ can be found in the equations (31) and (33).

For points of the reflector surface which are only illuminated by the feed which is placed in the scanned focal point, we can write:

$$
\frac{d(\text{err}_{t,ij}(z))}{dz} = -2(d_{reft} - d_{t,ij}(z)) \frac{d(d_{t,ij}(z))}{dz} = 0
$$

\[
\frac{d(d_{t,ij}(z))}{dz} = \frac{(z - z_t)}{\sqrt{(x_i - x_t)^2 + (y_j - y_t)^2 + (z - z_t)^2}} - \cos \alpha
\]

(40)
When we use the following definition:

\[ \text{dis}_t = \sqrt{(x_i - x_t)^2 + (y_j - y_t)^2 + (z - z_t)^2} \]

, equation (40) can be expressed as:

\[ (\text{drel}_t - \text{d}_{t,ij}(z)) \cdot \left( \frac{z_t - z}{\text{dis}_t} + \cos \alpha \right) = 0 \]  \hspace{1cm} (41)

The definitions of \( \text{drel}_t \) en \( \text{d}_{t,ij}(z) \) can be found in the expressions (32) en (34).

For points of the reflector surface which are illuminated by both feeds, we can write:

\[ \frac{d (\text{err}_{\text{T},ij})}{dz} = \frac{d (\text{err}_{\text{u},ij}(z))}{dz} + \frac{d (\text{err}_{t,ij}(z))}{dz} \]  \hspace{1cm} (42)

Now, we are enable to calculate the reflector surface as a grid of points for feeds with an uniform radiation pattern. However, feeds with an uniform radiation pattern do not exist in practice. Therefor we calculate the reflector surface with more practical feeds in the next paragraph.

### 3.4 Calculations of the reflector surface with a non uniform illumination.

Now we are calculating the reflector surface by using feeds with non uniform radiation patterns. Suppose that a feed with a non uniform radiation pattern is placed in the orgin of a spherical coordinate system \((r, \theta, \phi)\) with \(\theta = 0\) the main direction of radiation. The strength of the electric field for the feed with a non uniform radiation pattern decreases as the angle \(\theta\) increases. The power-current density \(\tilde{S}(\tilde{\tau})\) of the feed is proportional to the squared absolute value of the electric field. The power-current density \(\tilde{S}(\tilde{\tau})\), according [4], can be expressed as:

\[ \tilde{S}(\tilde{\tau}) = \frac{1}{2} Z_0^{-1} |\tilde{E}|^2 \tilde{\tau} \]

with \(Z_0\) the wave impedance for free space.

Fig. 15 shows the power-current density as a function of the angle \(\theta\). The normalized radiation pattern of the feed can be expressed as follows:

\[ f(\theta, \phi) = \frac{E(\theta, \phi)}{E(0, 0)} = \cos^n(\theta) \]

We assume that \(f(\theta, \phi)\) has rotation symmetry and is therefore independent from \(\phi\). Hence \(f(\theta, \phi) = f(\theta)\). We have to choose the value of \(n\) in such a way that we get an optimum between the spillover efficiency and the aperture efficiency. An edge illumination of about -12dB appears to be the optimum. If the angle \(\theta\) at the edge of the illuminated circles, fig. 10, is marked with \(\theta_0\), then the value of \(n\) can be calculated as follows:

\[ 10 \log (f(\theta_0))^2 = -12 \text{ dB} \]

\[ \Rightarrow n = \frac{-1.2}{2 \log(\cos \theta_0)} \]

In contradistinction to feeds with an uniform radiation pattern, each grid point of the reflector surface will be illuminated by both feeds with a non uniform radiation pattern. Since
each grid point will be illuminated by a different amount of radiation energy by both feeds, we have to introduce weighting functions [3] for the calculations of the error functions. We distinguish weighting functions for the unscanned $w_{u,ij}$ and for the scanned situation $w_{t,ij}$. The total error function can be written as:

$$\text{err}_{T,ij}(z) = w_{u,ij}(z) \cdot \text{err}_{u,ij}(z) + w_{t,ij}(z) \cdot \text{err}_{t,ij}(z)$$  \hspace{1cm} (43)

The definitions of $\text{err}_{u,ij}(z)$ and $\text{err}_{t,ij}(z)$ can be found in the equations (35) and (36) respectively. As shown in fig. 16, we have to calculate the angles $\theta_u$ and $\theta_t$ for each grid point $(x_i, y_j, z_{ij})$. These angles determine the values of the weighting functions according:

$$w_u = \cos^{nu} \theta_u$$ \hspace{1cm} (44)
$$w_t = \cos^{nt} \theta_t$$ \hspace{1cm} (45)

The angles $\theta_u$ and $\theta_t$ can be calculated with the help of the cosine-rule. For calculations of
Figure 17: Determining of the angle $\theta_u$.

Figure 18: Determining of the angle $\theta_t$.

For calculations of the angle $\theta_u$ we use fig. 17:

$$\theta_u = \arccos\frac{a^2 + b^2 - c^2}{2ab}$$

where:

$$a = \sqrt{\frac{y_u^2 + z_u^2}{2}}$$

$$b = \sqrt{\frac{x_i^2 + (y_j - y_u)^2 + (z - z_u)^2}{2}}$$

$$c = \sqrt{\frac{x_i^2 + y_j^2 + z^2}{2}}$$

For calculations of the angle $\theta_t$ we use fig. 18:

$$\theta_t = \arccos\frac{a^2 + b^2 - c^2}{2ab}$$
where:
\[
\begin{align*}
a &= \sqrt{(x_i - x_t)^2 + (y_j - y_t)^2 + (z - z_t)^2} \\
b &= \sqrt{(x_{ct} - x_t)^2 + y_t^2 + (z_{pol}(x_{ct}) - z_t)^2} \\
c &= \sqrt{(x_i - x_{ct})^2 + y_i^2 + (z - z_{pol}(x_{ct}))^2}
\end{align*}
\]

We see that both angles are only dependent on z, so also the weighting functions \(w_u\) en \(w_t\) are only dependent on z. Therefore, the total error function is only a function of z. To find the most optimum z coordinate we have to minimize the error function. This can be done by taking the derivative of the total error function to z and equating this derivative to zero:

\[
\frac{d \text{err}_{T,ij}}{dz} = 0
\]

where:
\[
\begin{align*}
\frac{d \text{err}_{T,ij}}{dz} &= w_u(\theta_u(z)) \cdot \frac{d \text{err}_{u,ij}(z)}{dz} + \frac{d w_u(\theta_u(z))}{dz} \cdot \text{err}_{u,ij}(z) + w_t(\theta_t(z)) \cdot \frac{d \text{err}_{t,ij}(z)}{dz} + \frac{d w_t(\theta_t(z))}{dz} \cdot \text{err}_{t,ij}(z)
\end{align*}
\]

In equation (46) are \(w_u(\theta_u(z))\), \(w_t(\theta_t(z))\), \(\text{err}_{u,ij}(z)\), \(\text{err}_{t,ij}(z)\), \(\frac{d \text{err}_{u,ij}(z)}{dz}\) and \(\frac{d \text{err}_{t,ij}(z)}{dz}\) are given by the equations (44), (45), (35), (36), (38) and (40) respectively. The remaining two terms which have to be calculated are \(\frac{d w_u(\theta_u(z))}{dz}\) and \(\frac{d w_t(\theta_t(z))}{dz}\). Using the rules of differentiation for these two unknown terms, we get:

\[
\begin{align*}
\frac{d w_u(\theta_u(z))}{dz} &= \frac{d w_u(\theta_u(z))}{dz} \cdot \frac{d \theta_u(z)}{dz} \\
\frac{d w_t(\theta_t(z))}{dz} &= \frac{d w_t(\theta_t(z))}{dz} \cdot \frac{d \theta_t(z)}{dz}
\end{align*}
\]

Substitution of (44) and (45) gives us:

\[
\begin{align*}
\frac{d w_u(\theta_u(z))}{dz} &= -nu \cdot \cos^{(nu - 1)} \theta_u \cdot \sin \theta_u \\
\frac{d w_t(\theta_t(z))}{dz} &= -nt \cdot \cos^{(nt - 1)} \theta_t \cdot \sin \theta_t
\end{align*}
\]

Finally we have to calculate \(\frac{d \theta_u(z)}{dz}\) and \(\frac{d \theta_t(z)}{dz}\). Both equations of \(\theta_u(z)\) and \(\theta_t(z)\) have the form:

\[
y = \arccos x \quad \text{with the derivative equal to:} \quad \frac{dy}{dx} = \frac{-1}{\sqrt{1 - x^2}}
\]

After some calculations we get:

\[
\begin{align*}
\frac{d \theta_u}{dz} &= \frac{z_u \cdot q + (z - z_u) \cdot p}{q \cdot \sqrt{(y_u^2 + z_u^2) \cdot q - p^2}} \\
\frac{d \theta_t}{dz} &= \frac{(z_t - z_{pol}(x_{ct})) \cdot s + (z - z_t) \cdot r}{s \cdot \sqrt{((x_{ct} - x_t)^2 + y_t^2 + (z_{pol}(x_{ct}) - z_t)^2) \cdot s - r^2}}
\end{align*}
\]
where:

\[ p = y_u^2 + z_u^2 - y_j y_u - z z_u \]
\[ q = x_t^2 + (y_j - y_u)^2 + (z - z_u)^2 \]
\[ r = x_t^2 + y_t^2 + z_t^2 - x_i x_t - x_c x_t + x_i x_c - y_j y_t - z_t z_{pol}(x_c t) + z(z_{pol}(x_c t) - z_t) \]
\[ s = (x_i - x_t)^2 + (y_j - y_t)^2 + (z - z_t)^2 \]

Since all terms of equation (46) are known, we can calculate for each grid point \((x_i, y_j)\) of the reflector surface the optimum z-coordinate. A computer program has been written for the calculations of all these z-coordinates. The input parameters of the computer program are:

- the maximum scan angle \(\alpha\), (deg.)
- the wavelength \(\lambda\), (cm.)
- the focal distance \(f_t\) of the scanned focal point, \(\lambda\)
- the offset distance \(y_t\), \(\lambda\)
- the height of the reflector surface, \(\lambda\)
- the width of the reflector surface, \(\lambda\)

The numerical method used for the calculations of the values \(z_{ij}\) is a combination of the Koorden-Newton method and the Bisection method. The Bisection method ensures convergence, while the Koorden-Newton is a much more faster convergent method, see also [5].

The output of the computer program is a matrix with the elements \(z_{ij}\) which describes the total reflector surface in a grid of points. Since this description of the reflector surface is not very usefull for further calculations, we are looking for a continuous function which describes the surface of the reflector. In the next two sections we determine these functions of the symmetrical and offset-reflector.
4 Symmetric-shaped reflector

The analytic expression of the symmetric-shaped reflector will be determined in this section. The method of calculations for both the symmetric-shaped reflector and the offset-shaped reflector are the same. The symmetric-shaped reflector will be treated separately from the offset-shaped reflector because the number of equations for the symmetric-shaped reflector is much smaller. Therefore it becomes easier for the reader to understand the method of calculations. If the method of calculations for the symmetric-shaped reflector is known, it is also very easy to understand the calculations of the offset-shaped reflector. The analytic expression of the reflector surface can be found by an extension of the 2-dimensional expression of \( z_{\text{sur}}(x, 0) \) (14) to a 3-dimensional polynomial. Since the shaped-symmetric reflector is symmetrical with respect to the XZ-plane and the YZ-plane the polynomial consists only of even terms of \( x \) and \( y \). We finally find for the polynomial:

\[
z_{\text{sym-sur}}(x, y) = (r_1 x^2 + r_2 x^4) + (P + Q x^2 + S x^4) y^2 + R y^4
\]  

(47)

The coefficients \( r_1 \) and \( r_2 \) are determined by means of the least squared method. They are already calculated in paragraph 2.2. The remaining coefficients \( P, Q, S \) and \( R \) are also determined by means of the least squared method. This will be treated in the first paragraph. Since the shaped-reflector surface is not a real parabola, there are always phase errors in the aperture. Calculations of these phase will be treated in the second paragraph.

4.1 Determining of the coefficients

As mentioned before, the coefficients \( P, Q, S \) and \( R \) will be determined by means of the least squared method. Therefore we make use of the calculated matrix of the previous section which consists of the grid of points \((x_i, y_j, z_{ij})\). Equation (47) will be sampled on the same positions \((x_i, y_j)\) of the grid. The difference between the sampled points and the exact values \( z_{ij} \) are called the residues \( \text{res}_{ij} \). These residues can be defined as follows:

\[
\begin{align*}
z_{\text{sym-sur}}(x_0, y_0) - z_{00} &= \text{res}_{00} \\
z_{\text{sym-sur}}(x_0, y_1) - z_{01} &= \text{res}_{01} \\
\vdots \\
z_{\text{sym-sur}}(x_2, y_3) - z_{23} &= \text{res}_{23} \\
\vdots \\
z_{\text{sym-sur}}(x_i, y_j) - z_{ij} &= \text{res}_{ij} \\
\vdots \\
z_{\text{sym-sur}}(x_N, y_{M_2(N)}) - z_N M_2(N) &= \text{res}_N M_2(N)
\end{align*}
\]

\( \text{res}_{00}, \text{res}_{01}, \ldots, \text{res}_{ij}, \ldots, \text{res}_N M_2(N) \) are called the residues

We have to minimize:

\[
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij}^2
\]

Now we write:

\[
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij}^2 = \varphi(P, Q, S, R)
\]
The definitions of the boundaries $N$, $M_1(i)$ and $M_2(i)$ can be found in paragraph 3.1. To minimize $\varphi$ means:

$$\frac{\partial \varphi}{\partial P} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial S} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial R} = 0$$ (48)

$$\frac{\partial \varphi}{\partial P} = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( \frac{\partial \varphi}{\partial \text{res}_{ij}} \cdot \frac{\partial \text{res}_{ij}}{\partial P} \right) \quad \text{with:} \quad \left\{ \frac{\partial \varphi}{\partial \text{res}_{ij}} \right\}_{\partial P} = 2 \text{res}_{ij}$$

$$\frac{\partial \varphi}{\partial Q} = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( \frac{\partial \varphi}{\partial \text{res}_{ij}} \cdot \frac{\partial \text{res}_{ij}}{\partial Q} \right) \quad \text{with:} \quad \left\{ \frac{\partial \varphi}{\partial \text{res}_{ij}} \right\}_{\partial Q} = 2 \text{res}_{ij}$$

$$\frac{\partial \varphi}{\partial S} = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( \frac{\partial \varphi}{\partial \text{res}_{ij}} \cdot \frac{\partial \text{res}_{ij}}{\partial S} \right) \quad \text{with:} \quad \left\{ \frac{\partial \varphi}{\partial \text{res}_{ij}} \right\}_{\partial S} = 2 \text{res}_{ij}$$

$$\frac{\partial \varphi}{\partial R} = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( \frac{\partial \varphi}{\partial \text{res}_{ij}} \cdot \frac{\partial \text{res}_{ij}}{\partial R} \right) \quad \text{with:} \quad \left\{ \frac{\partial \varphi}{\partial \text{res}_{ij}} \right\}_{\partial R} = 2 \text{res}_{ij}$$

Substituted in (48) gives:

$$\left\{ \begin{array}{l}
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij} y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij} x_i^2 y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij} x_i^4 y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \text{res}_{ij} y_j^4 = 0 \\
\end{array} \right. \quad (49)$$

Substitution of:

$$\text{res}_{ij} = z_{\text{sym-sus}}(x_i, y_j) - z_{ij}$$

$$= r_1 x_i^2 + r_2 x_i^4 + (P + Q x_i^2 + S x_i^4) y_j^2 + R y_j^4 - z_{ij}$$

in (49) gives:

$$\left\{ \begin{array}{l}
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( r_1 x_i^2 + r_2 x_i^4 + (P + Q x_i^2 + S x_i^4) y_j^2 + R y_j^4 - z_{ij} \right) y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( r_1 x_i^2 + r_2 x_i^4 + (P + Q x_i^2 + S x_i^4) y_j^2 + R y_j^4 - z_{ij} \right) x_i^2 y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( r_1 x_i^2 + r_2 x_i^4 + (P + Q x_i^2 + S x_i^4) y_j^2 + R y_j^4 - z_{ij} \right) x_i^4 y_j^2 = 0 \\
\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} \left( r_1 x_i^2 + r_2 x_i^4 + (P + Q x_i^2 + S x_i^4) y_j^2 + R y_j^4 - z_{ij} \right) y_j^4 = 0 \\
\end{array} \right. \quad (49)$$
After reorganization of the different terms we get the following matrix:

\[
\begin{pmatrix}
  m[1,1] & m[1,2] & m[1,3] & m[1,4] \\
  m[4,1] & m[4,2] & m[4,3] & m[4,4]
\end{pmatrix}
\begin{pmatrix}
P \\
Q \\
S \\
R
\end{pmatrix}
= 
\begin{pmatrix}
r[1] \\
r[2] \\
r[3] \\
r[4]
\end{pmatrix}
\]  

(50)

with the following definitions of the matrix elements \( m[i,j] \) and column elements \( r[i] \):

\[
m[1,1] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i^2 y_j^4
\]

\[
m[1,2] = m[2,1] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i^2 y_j^4
\]

\[
m[1,3] = m[2,2] = m[3,1] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i y_j^4
\]

\[
m[1,4] = m[4,1] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} y_j^6
\]

\[
m[2,3] = m[3,2] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} y_j^6
\]

\[
m[2,4] = m[4,2] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i^2 y_j^6
\]

\[
m[3,3] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i y_j^4
\]

\[
m[3,4] = m[4,3] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} x_i y_j^6
\]

\[
m[4,4] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} y_j^8
\]

\[
r[1] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} (z_{ij} - x_i^2 (r_1 + r_2 x_i^2)) y_j^2
\]

\[
r[2] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} (z_{ij} - x_i^2 (r_1 + r_2 x_i^2)) x_i^2 y_j^2
\]

\[
r[3] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} (z_{ij} - x_i^2 (r_1 + r_2 x_i^2)) x_i^4 y_j^2
\]

\[
r[4] = \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} (z_{ij} - x_i^2 (r_1 + r_2 x_i^2)) y_j^4
\]

We obtain a system of linear equations with the unknown coefficients \( P, Q, S \) and \( R \). Table 1 gives, by the way of example, the calculated values of the coefficients for a scan angle of 30
degrees. In addition the dimensions of the shaped-reflector antenna are also given in table 1. The definitions of the used quantities can be found in paragraph 2.1 and 3.1.

Table 1: Quantities of the antenna and the calculated values of the different coefficients.

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 30^0 )</td>
<td>( y_t = 0 )</td>
</tr>
<tr>
<td>( \lambda = 3 \text{ cm} )</td>
<td>width=28.8 ( \lambda )</td>
</tr>
<tr>
<td>( f_t = 23.75 \lambda )</td>
<td>height=16 ( \lambda )</td>
</tr>
</tbody>
</table>

Calculated coefficients

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>0.2372</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.8157</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>0.1149</td>
</tr>
<tr>
<td>( S )</td>
<td>-3.8142</td>
</tr>
<tr>
<td>( P )</td>
<td>0.2594</td>
</tr>
<tr>
<td>( R )</td>
<td>-0.0874</td>
</tr>
</tbody>
</table>

4.2 Phase errors

Since the reflector surface is not a real parabola, there are always phase errors in the aperture. The phase errors for the unscanned and scanned situation can be defined as follows:

\[
\text{phase}_{\text{out}} = d_{\text{ref}} - d
\]

The terms \( d_{\text{ref}} \) and \( d \) represents the reference distances for the unscanned (31) and scanned situation (32) respectively. The terms \( d_u \) and \( d_t \) represents the distances from the feed to an arbitrary point on the reflector surface and from that point to the reference plane for the unscanned (33) and scanned (34) situation respectively. For the calculations of the phase errors we use the shaped reflector from the previous paragraph. The phase errors in the unscanned and scanned situation, expressed in wavelengths, are shown in fig. 19 and fig. 20 respectively.
Figure 19: Unscanned phase errors.

Figure 20: Scanned phase errors.
5 Offset-shaped reflector

The analytic expression of the offset-shaped reflector will be determined in this section. The offset-shaped reflector is only symmetric with respect to the YZ-plane. In this case we also have to add odd terms of $y$ to the polynomial. The expression of the offset-shaped reflector surface is defined as follows:

$$z_{\text{surf}}(x, y) = (r_1 x^2 + r_2 x^4) + (N + T x^2 + U x^4)y + (P + Q x^2 + S x^4)y^2 + (V + W x^2)y^3 + R y^4 \quad (51)$$

Calculations of the coefficients happens on the same way as the symmetric-shaped reflector with the only difference that the number of coefficients of the offset-shaped reflector and with that also the number of equations is increased. Calculations of these coefficients will be treated in the first paragraph, while the phase errors will be treated in the second paragraph.

5.1 Determining of the coefficients.

The coefficients $N, T, U, P, Q, S, V, W$ and $R$ from equation (51) will be determined by means of the least squares method. Since this method has been already explained for the symmetric-shaped reflector, we will only present some interim and final results. Equation 51 will be sampled on the points $(x_i, y_j)$. For these points we can express the residues $res_{ij}$ as follows:

$$res_{ij} = z_{\text{surf}}(x_i, y_j) - z_{ij}$$

We have to minimize:

$$\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij}^2$$

Now we write:

$$\sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij}^2 = \varphi(N, T, U, P, Q, S, V, W, R)$$

To minimize $\varphi$ means:

$$\frac{\partial \varphi}{\partial N} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial T} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial U} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial P} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial Q} = 0$$

$$\text{and} \quad \frac{\partial \varphi}{\partial S} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial V} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial W} = 0 \quad \text{and} \quad \frac{\partial \varphi}{\partial R} = 0 \quad (52)$$
Equation (52) can be written as:

\[
\begin{align*}
\frac{\partial \phi}{\partial N} &= 2 \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij} y_j = 0 \\
\frac{\partial \phi}{\partial S} &= 2 \sum_{i=1}^{N} M_2(i) \sum_{j=M_1(i)}^{M_1(i)} res_{ij} x_i^4 y_j^2 = 0 \\
\frac{\partial \phi}{\partial T} &= 2 \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij} x_i^2 y_j = 0 \\
\frac{\partial \phi}{\partial U} &= 2 \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij} x_i^4 y_j^2 = 0 \\
\frac{\partial \phi}{\partial P} &= 2 \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij} y_j^2 = 0 \\
\frac{\partial \phi}{\partial Q} &= 2 \sum_{i=1}^{N} \sum_{j=M_1(i)}^{M_2(i)} res_{ij} x_i^2 y_j^2 = 0
\end{align*}
\]

Now we have nine equations with nine unknown coefficients. After substitution of \(res_{ij}\) and some reorganization we remain the following matrix:

\[
\begin{pmatrix}
N \\
T \\
U \\
P \\
Q \\
S \\
V \\
W \\
R
\end{pmatrix}
\begin{pmatrix}
M[ij]
\end{pmatrix}
= 
\begin{pmatrix}
R[i]
\end{pmatrix}
\]

(53)
with the following definitions of the matrix elements $m[i, j]$: 

\[
m[1, 1] = \sum_{i=1}^{N} \sum_{j=M_2(i)} M_2(i)
\]

\[
m[1, 2] = m[2, 1] = \sum_{i=1}^{N} x_i^2 y_j^2
\]

\[
m[1, 3] = m[2, 2] = m[3, 1] = \sum_{i=1}^{N} x_i^4 y_j^2
\]

\[
m[1, 4] = m[4, 1] = \sum_{i=1}^{N} y_j^2
\]

\[
m[1, 5] = m[2, 4] = m[4, 2] = m[5, 1] = \sum_{i=1}^{N} x_i^2 y_j^2
\]

\[
m[1, 6] = m[2, 5] = m[3, 4] = m[4, 3] = m[5, 2] = m[6, 1] = \sum_{i=1}^{N} x_i^4 y_j^2
\]

\[
m[1, 7] = m[7, 1] = m[4, 4] = \sum_{i=1}^{N} y_j^2
\]

\[
m[1, 8] = m[2, 7] = m[4, 5] = m[5, 4] = m[7, 2] = m[8, 1] = \sum_{i=1}^{N} x_i^2 y_j^2
\]

\[
m[1, 9] = m[4, 7] = m[7, 4] = m[9, 1] = \sum_{i=1}^{N} y_j^5
\]

\[
m[2, 3] = m[3, 2] = \sum_{i=1}^{N} x_i^6 y_j^3
\]

\[
m[2, 6] = m[3, 5] = m[5, 3] = m[6, 2] = \sum_{i=1}^{N} x_i^6 y_j^3
\]

\[
m[2, 8] = m[3, 7] = m[4, 6] = m[5, 5] = m[6, 4] = m[7, 3] = m[8, 2] = \sum_{i=1}^{N} x_i^4 y_j^2
\]

\[
m[2, 9] = m[4, 8] = m[5, 7] = m[7, 5] = m[8, 4] = m[9, 2] = \sum_{i=1}^{N} x_i^2 y_j^5
\]

\[
m[3, 3] = \sum_{i=1}^{N} x_i^8 y_j^2
\]

\[
m[3, 6] = m[6, 3] = \sum_{i=1}^{N} x_i^8 y_j^3
\]

\[
m[3, 8] = m[5, 6] = m[6, 5] = m[8, 3] = \sum_{i=1}^{N} x_i^6 y_j^4
\]

\[
m[3, 9] = m[5, 8] = m[6, 7] = m[7, 6] = m[8, 5] = m[9, 3] = \sum_{i=1}^{N} x_i^4 y_j^5
\]

\[
m[4, 9] = m[7, 7] = m[9, 4] = \sum_{i=1}^{N} y_j^5
\]
The definitions of the column elements \( r[i] \) are:

\[
\begin{align*}
r[1] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ y_j \\
r[2] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ y_j \\
r[3] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ x_i^4 \ y_j \\
r[4] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ x_i^2 \ y_j^2 \\
r[5] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ x_i^4 \ y_j^2 \\
r[6] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ x_i^4 \ y_j^2 \\
r[7] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ y_j^3 \\
r[8] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ x_i^2 \ y_j^3 \\
r[9] &= \sum_{i=1}^{N} \sum_{j=1}^{M_2(i)} (x_{ij} - x_i^2(r_1 + r_2 x_i^2)) \ y_j^4
\end{align*}
\]
Again we have a system of linear equations, but now with the coefficients N, T, U, P, Q, S, V, W and R. Table 2 contains the calculated values of the coefficients for a scan angle of 30 degrees. The quantities of the antenna are also given in table 2.

Table 2: Quantities of the antenna and calculated values of the coefficients.

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 30^\circ$</td>
<td>$y_t = 9.33 \lambda$</td>
</tr>
<tr>
<td>$\lambda = 3 \text{ cm}$</td>
<td>width $= 28.8 \lambda$</td>
</tr>
<tr>
<td>$f_t = 23.75 \lambda$</td>
<td>height $= 16 \lambda$</td>
</tr>
<tr>
<td>Calculated values of the coefficients</td>
<td></td>
</tr>
<tr>
<td>$r_1 = 0.2372$</td>
<td>$Q = 0.8394$</td>
</tr>
<tr>
<td>$r_2 = 0.1149$</td>
<td>$S = -3.9110$</td>
</tr>
<tr>
<td>$N = 0.1706$</td>
<td>$V = 0.0094$</td>
</tr>
<tr>
<td>$T = 0.1545$</td>
<td>$W = -0.5229$</td>
</tr>
<tr>
<td>$U = -0.2542$</td>
<td>$R = -0.0742$</td>
</tr>
<tr>
<td>$P = 0.2604$</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Phase errors

Also for the offset situation the phase errors are calculated and shown in figures (21) and (22). The phase errors for the unscanned and scanned situation can be defined as follows:

\[
\text{phase}_{\text{out}_u} = d_{\text{ref}_u} - d_u \\
\text{phase}_{\text{out}_t} = d_{\text{ref}_t} - d_t
\]

The terms $d_{\text{ref}_u}$ and $d_{\text{ref}_t}$ represent the reference distances for the unscanned (31) and scanned situation (32) respectively. The terms $d_u$ and $d_t$ represents the distances from the feed to an arbitrary point on the reflector surface and from that point to the reference plane for the unscanned (33) and scanned situation (34) respectively. For calculations of the phase errors we use the reflector from the previous paragraph. The phase errors in the unscanned and scanned situation, expressed in wavelengths, are shown in fig. 21 and fig. 22 respectively.
Figure 21: Unscanned phase errors.

Figure 22: Scanned phase errors.
6 Radiation patterns

In the previous sections we have investigated the shape of the reflector. The next task is to calculate the radiation pattern of the shaped reflector if a feedsystem is known. To that end we have to find the induced currents on the reflector. These currents act then as sources for the radiation field. The purpose of the first paragraph is to find a general expression of the radiation pattern which is valid for an arbitrary shaped-reflector surface. A proper description of the feedsystem is given in the second paragraph. Since the calculation of the radiation pattern of the reflector antenna in the unscanned situation differ some how from the scanned situation, this will be treated in the last paragraph of this section.

6.1 Calculation of the far-field radiation pattern for the shaped-reflector in the scanned situation.

The most general calculation of the far-field radiation pattern for the shaped-reflector occurs in the scanned-situation. The angle \( \alpha \) between the reflected rays and the \( z' \)-axis is called the scan-angle. The shaped-reflector with the cartesian coordinate systems is shown in fig. 23.

![Figure 23: The shaped-reflector with the cartesian coordinate systems](image)
In fig. 23 we distinguish the following cartesian coordinate systems:

\[(x_s, y_s, z_s)\] : The coordinate system of the feed. The origin of the coordinate system is in the focal point with the \(z_s\)-axis pointed in the direction of the feed main beam.

\[(x', y', z')\] : The primary coordinate system of the shaped-reflector.

\[(x'', y'', z'')\] : A new translated and rotated coordinate system of the shaped-reflector with the \(z''\)-axis pointed in the scan-direction.

\[(x, y, z)\] : The coordinate system of the far-field radiation pattern for the shaped-reflector.

To calculate the far-field pattern of the scanned beam we have to transform the far-field pattern of the feed, which is expressed in the cartesian coordinate system \((x_s, y_s, z_s)\), into the cartesian coordinate system \((x'', y'', z'')\). Hereby we make use of a translation along \(\vec{s}\) (fig. 23) and three rotations along the Eulerian angles \((\alpha, \beta, \gamma)\). The cartesian coordinate systems \((x_s, y_s, z_s)\) and \((x'', y'', z'')\) with the Eulerian angles \((\alpha, \beta, \gamma)\) are shown in fig. 24.

![Figure 24: Eulerian angles](image-url)
Definition of the Eulerian angles:

\( x^* \) : The \( x^* \)-axis is generated by the intersection of the \( x_s y_s \)-plane and the \( x'' y'' \)-plane.

\( \alpha \) : Angle \( \alpha \) describes a counterclockwise rotation about the \( z'' \)-axis which brings the \( x'' \)-axis to the \( x^* \)-axis.

\( \beta \) : Angle \( \beta \) defines a rotation about the \( x^* \)-axis in a counterclockwise sense which brings the \( z'' \)-axis to the \( z_s \)-axis.

\( \gamma \) : Angle \( \gamma \) describes a counterclockwise rotation about the \( z_s \)-axis which brings the \( x^* \)-axis to the \( x_s \)-axis.

We can derive the transformation matrix \( (\alpha A^{e''}) \) which transforms the cartesian coordinate \( \{e''\} \) into \( \{e_s\} \). The transformation matrix is defined as:

\[
(\alpha A^{e''}) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{54}
\]

And:

\[
A_{11} = \cos \gamma \cos \alpha - \sin \gamma \cos \beta \sin \alpha \\
A_{12} = \cos \gamma \sin \alpha + \sin \gamma \cos \beta \cos \alpha \\
A_{13} = \sin \gamma \sin \beta \\
A_{21} = -\sin \gamma \cos \alpha - \cos \gamma \cos \beta \sin \alpha \\
A_{22} = -\sin \gamma \sin \alpha + \cos \gamma \cos \beta \cos \alpha \\
A_{23} = \cos \gamma \sin \beta \\
A_{31} = \sin \beta \sin \alpha \\
A_{32} = -\sin \beta \cos \alpha \\
A_{33} = \cos \beta \tag{55}
\]

The far-field pattern of the feed expressed in the cartesian coordinate system \( (x_s, y_s, z_s) \) which is transformed to the cartesian coordinate system \( (x'', y'', z'') \) can be expressed as follows:

\[
\vec{H}_s(\vec{r}'') = \left\{ \begin{array}{c} H_{x''}(\vec{r}'') \\ H_{y''}(\vec{r}'') \\ H_{z''}(\vec{r}'') \end{array} \right\} = \left( \begin{array}{ccc} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{array} \right)^{-1} \left\{ \begin{array}{c} H_{x_s}(\vec{r}_s) \\ H_{y_s}(\vec{r}_s) \\ H_{z_s}(\vec{r}_s) \end{array} \right\} \tag{56}
\]
A matrix $A$ who describes one or more rotations of a cartesian coordinate system is always orthogonal. This means that:

$$
\left( \begin{array}{ccc}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{array} \right)^{-1} = \left( \begin{array}{ccc}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{array} \right)^T = \left( \begin{array}{ccc}
A_{11} & A_{21} & A_{31} \\
A_{12} & A_{22} & A_{32} \\
A_{13} & A_{23} & A_{33}
\end{array} \right) \quad (57)
$$

To express the spherical coordinate system of the feed $(r_s, \theta_s, \phi_s)$ into the spherical coordinate system of the shaped-reflector $(r'', \phi'', \theta'')$ we use the following transformation:

$$
\left\{ \begin{array}{c}
r_s \sin \theta_s \cos \phi_s \\
r_s \sin \theta_s \sin \phi_s \\
r_s \cos \theta_s
\end{array} \right\} = \left( \begin{array}{ccc}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{array} \right) \left\{ \begin{array}{c}
r'' \sin \theta'' \cos \phi'' - s_1 \\
r'' \sin \theta'' \sin \phi'' - s_2 \\
r'' \cos \theta'' - s_3
\end{array} \right\} \quad (58)
$$

For further calculations of the far-field radiation pattern we make use of fig. 25. The reflector surface $\Sigma$ can be written as follows:

$$
z'' = f(x'', y'') = \tilde{f}(\rho'', \phi'') \quad \text{with} \quad \rho'' \in \sigma \quad (59)
$$

$\tilde{f}$: means an other function as the function $f$.

$\sigma$: the surface of the projected circle.
We assume that the reflector surface $\Sigma$ is a perfect conductor. Then we can write the induced current $\mathbf{J}$ on the reflector surface as follows:

$$\mathbf{J} = 2\hat{n} \times \mathbf{H}_s(\mathbf{r}^\prime)$$

The unit normal of the reflector surface is given by:

$$\hat{n} = \frac{\mathbf{N}}{N} \quad \text{with} \quad \mathbf{N} = \left[ \frac{\partial f}{\partial x''} \hat{\alpha}_{x''} - \frac{\partial f}{\partial y''} \hat{\alpha}_{y''} + \hat{\alpha}_{z''} \right]$$

and

$$N = \sqrt{\left( \frac{\partial f}{\partial x''} \right)^2 + \left( \frac{\partial f}{\partial y''} \right)^2 + 1}$$

The general equation for the scattered $\mathbf{H}$-field may be expressed as:

$$\mathbf{H}(\mathbf{r}) = \nabla \times \mathbf{A}$$

with the vectorpotential $\mathbf{A}$ expressed as:

$$\mathbf{A} = \int_{\Sigma} \frac{e^{-jk|\mathbf{r} - \mathbf{r}^\prime|}}{4\pi|\mathbf{r} - \mathbf{r}^\prime|} \mathbf{J}(\mathbf{r}^\prime) \, ds''$$

Introducing the usual far-field approximation:

- the phase term: $|\mathbf{r} - \mathbf{r}^\prime| \simeq r - \mathbf{r}^\prime \cdot \hat{r}$
- the distance: $|\mathbf{r} - \mathbf{r}^\prime| \simeq r$

We get for the magnetic far-field:

$$\mathbf{H}(\mathbf{r}) = \nabla \times \frac{e^{-jkr}}{4\pi r} \mathbf{T}(\theta, \phi) \quad \text{with} \quad \mathbf{T}(\theta, \phi) = \int_{\Sigma} \frac{\mathbf{J}(\mathbf{r}^\prime)e^{jk|\mathbf{r} - \mathbf{r}^\prime|}}{4\pi|\mathbf{r} - \mathbf{r}^\prime|} \, ds''$$

The rotation of the $\mathbf{H}$-field in spherical coordinates is defined as:

$$\mathbf{H}(\mathbf{r}) = \frac{\hat{\alpha}_r}{r \sin \theta} \left[ \frac{\partial (rA_\phi \sin \theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] + \frac{\hat{\alpha}_\theta}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (rA_\phi)}{\partial r} \right]$$

In the far field we get:

$$\mathbf{H}(\mathbf{r}) = -\frac{\hat{\alpha}_\theta}{r} \frac{\partial (rA_\phi)}{\partial r} + \frac{\hat{\alpha}_r}{r} \frac{\partial (rA_\theta)}{\partial r} + O\left( \frac{1}{r^2} \right)$$

with

$$rA_\phi = \frac{e^{-jkr}}{4\pi} T_{\phi} \rightarrow \frac{\partial (rA_\phi)}{\partial r} = -jk \frac{e^{-jkr}}{4\pi} T_{\phi}$$

$$\rightarrow -\frac{\hat{\alpha}_\theta}{r} \frac{\partial (rA_\phi)}{\partial r} = jk \frac{e^{-jkr}}{4\pi r} T_\phi \hat{\alpha}_\theta$$

and

$$rA_\theta = \frac{e^{-jkr}}{4\pi} T_{\theta} \rightarrow \frac{\partial (rA_\theta)}{\partial r} = -jk \frac{e^{-jkr}}{4\pi} T_{\theta}$$

$$\rightarrow \frac{\hat{\alpha}_r}{r} \frac{\partial (rA_\theta)}{\partial r} = -jk \frac{e^{-jkr}}{4\pi r} T_\theta \hat{\alpha}_r$$

$$\hat{\alpha}_r \frac{\partial (rA_\phi)}{\partial r} = -jk \frac{e^{-jkr}}{4\pi r} T_\phi \hat{\alpha}_\theta$$

$$\hat{\alpha}_\theta \frac{\partial (rA_\theta)}{\partial r} = -jk \frac{e^{-jkr}}{4\pi r} T_\theta \hat{\alpha}_r$$
Finally we arrive at the expression for the magnetic far-field:

$$\vec{H}(\vec{r}) = jk \frac{e^{-jkr}}{4\pi r} \left( T_\phi \hat{a}_\theta - T_\theta \hat{a}_\phi \right)$$  \hfill (69)

The electric far-field can be expressed as:

$$\vec{E}(\vec{r}) = Z_0 \vec{H}(\vec{r}) \times \hat{a}_r \quad \text{with} \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$  \hfill (70)

$$\vec{E}(\vec{r}) = - Z_0 \hat{a}_r \times \vec{H}(\vec{r})$$

$$= - jk Z_0 \frac{e^{-jkr}}{4\pi r} \hat{a}_r \times (T_\phi \hat{a}_\theta - T_\theta \hat{a}_\phi)$$

$$= - jk Z_0 \frac{e^{-jkr}}{4\pi r} \left( T_\phi \hat{a}_\phi + T_\theta \hat{a}_\theta \right)$$  \hfill (71)

The integration on the reflector surface $\Sigma$ can be transformed into an integration over the projected circular region $\sigma$. By this transformation a small surface element of the reflector $ds''$ can be expressed as:

$$ds'' = |\vec{z}_{\rho''} \times \vec{z}_{\phi''}| d\rho'' d\phi''$$  \hfill (72)

For the cartesian coordinates of the reflector surface expressed in spherical coordinates we get:

$$x'' = r'' \sin \theta'' \cos \phi'' = \rho'' \cos \phi''$$

$$y'' = r'' \sin \theta'' \sin \phi'' = \rho'' \sin \phi''$$

$$z'' = \tilde{f}(\rho'', \phi'')$$  \hfill (73)

The cross-product is given by:

$$\vec{z}_{\rho''} \times \vec{z}_{\phi''} = \begin{pmatrix} \frac{\partial x''}{\partial \rho''} \\ \frac{\partial y''}{\partial \rho''} \\ \frac{\partial z''}{\partial \rho''} \end{pmatrix} \times \begin{pmatrix} \frac{\partial x''}{\partial \phi''} \\ \frac{\partial y''}{\partial \phi''} \\ \frac{\partial z''}{\partial \phi''} \end{pmatrix}$$

$$= \begin{pmatrix} \cos \phi'' \\ \sin \phi'' \\ \frac{\partial \tilde{f}}{\partial \rho''} \end{pmatrix} \times \begin{pmatrix} - \rho'' \sin \phi'' \\ \rho'' \cos \phi'' \\ \frac{\partial \tilde{f}}{\partial \phi''} \end{pmatrix} = \begin{pmatrix} \sin \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right) - \rho'' \cos \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right) \\ - \rho'' \sin \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right) - \cos \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right) \end{pmatrix}$$

$$|\vec{z}_{\rho''} \times \vec{z}_{\phi''}|^2 = \sin^2 \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right)^2 - 2 \sin \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right) \rho'' \cos \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right)$$

$$+ \rho''^2 \cos^2 \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right)^2 + \rho''^2 \sin^2 \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right)^2$$

$$+ 2 \sin \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right) \rho'' \cos \phi'' \left( \frac{\partial \tilde{f}}{\partial \rho''} \right) + \cos^2 \phi'' \left( \frac{\partial \tilde{f}}{\partial \phi''} \right)^2 + \rho''^2$$

$$= \rho''^2 \left( \frac{\partial \tilde{f}}{\partial \rho''} \right)^2 + \left( \frac{\partial \tilde{f}}{\partial \phi''} \right)^2 + \rho''^2$$
Finally we arrive at the expression for the small surface element $ds''$:

$$ds'' = \sqrt{\left(\frac{\partial \tilde{f}}{\partial \rho''} - (\rho'')^{-2}\left(\frac{\partial \tilde{f}}{\partial \phi''}\right)^2 + 1\right)} d\rho'' d\phi''$$  \hspace{1cm} (75)

The transformation described here is called the surface Jacobian transformation with the Jacobian expressed as:

$$\sqrt{\left(\frac{\partial \tilde{f}}{\partial \rho''} - (\rho'')^{-2}\left(\frac{\partial \tilde{f}}{\partial \phi''}\right)^2 + 1\right)}$$  \hspace{1cm} (76)

In the following expressions we use $J_\Sigma$ as a notation for the Jacobian. We can rewrite (65) as follows:

$$T(\theta, \phi) = \int_{0}^{2\pi} \int_{0}^{\pi} \tilde{J}(\rho'', \phi'') e^{i k \tilde{r}''} \cdot J_\Sigma \rho'' d\rho'' d\phi''$$  \hspace{1cm} (77)

The magnitude of the normal vector to the reflector surface in spherical coordinates can be expressed as:

$$N = |\vec{e}_{\rho''} \times \vec{e}_{\phi''}| = \sqrt{\left(\frac{\partial \tilde{f}}{\partial \rho''} - (\rho'')^{-2}\left(\frac{\partial \tilde{f}}{\partial \phi''}\right)^2 + 1\right)}$$  \hspace{1cm} (79)

It can readily be shown that (61) and (79) are identical, so that:

$$J_\Sigma = N$$  \hspace{1cm} (79)

Defining an equivalent current as:

$$\tilde{J}(\rho'', \phi'') = \tilde{J}(\tilde{r}'') \cdot J_\Sigma = 2\vec{n} \times \vec{H}(\tilde{r}'') \cdot N = 2\vec{N} \times \vec{H}(\tilde{r}'')$$  \hspace{1cm} (80)

we can write (77) as:

$$T(\theta, \phi) = \int_{0}^{2\pi} \int_{0}^{\pi} \tilde{J}(\rho'', \phi'') e^{i k \tilde{r}''} \cdot \rho'' d\rho'' d\phi''$$  \hspace{1cm} (81)

with $\tilde{J}(\rho'', \phi'') = 2\vec{N} \times \vec{H}(\tilde{r}'')$
Now we are going to rewrite the exponential term of (81)

\[
\vec{r}'' = \begin{pmatrix} \rho'' \cos \phi'' \\ \rho'' \sin \phi'' \\ z'' \end{pmatrix} \quad \text{and} \quad \hat{r} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}
\]

\[
\vec{r}'' \cdot \hat{r} = \rho'' \cos \phi'' \sin \theta \cos \phi + \rho'' \sin \phi'' \sin \theta \sin \phi + z'' \cos \theta
\]

\[
= \rho'' \sin \theta(\cos \phi \cos \phi'' + \sin \phi \sin \phi'') + z'' \cos \theta
\]

\[
= \rho'' \sin \theta \cos(\phi'' - \phi) + z'' \cos \theta
\]

We can finally write (81) as:

\[
\vec{T}(\theta, \phi) = \int_0^{2\pi} \int_0^\pi \tilde{J}(\rho'', \phi'') e^{jkz'' \cos \theta} \cdot e^{jk\rho'' \sin \theta \cos(\phi'' - \phi)} \cdot \rho'' d\rho'' d\phi''
\]

with \( \tilde{J}(\rho'', \phi'') = 2\tilde{N} \times \vec{H}(\vec{r}'') \) (83)

With the following transformation we find the spherical far-field components of \( \vec{T} \):

\[
\begin{bmatrix} T_\theta \\ T_\phi \end{bmatrix} = \begin{pmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{pmatrix} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}
\]

(84)

Using (71) we finally obtain the expression of the electrical far-field in spherical components.

\[
\vec{E}(\vec{r}) = -jk Z_0 \frac{e^{-jkr}}{4\pi r} (T_\phi \hat{\phi} + T_\theta \hat{\theta})
\]

(85)

The co-polar radiation pattern with the peak gain normalized at 0dB is defined as:

\[
F(\theta, \phi) = 10 \log \frac{P(\theta, \phi)}{P(0, 0)}
\]

(86)

,with \( P(0, 0) \) the maximum radiated power by unit of solid angle in the direction of \( \theta = 0 \) and \( \phi = 0 \).

For the maximum electrical field \( \vec{E}_{\text{max}} \) we find:

\[
\vec{E}_{\text{max}} = \vec{E}(\theta = 0, \phi = 0) = -jk Z_0 \frac{e^{-jkr}}{4\pi r} (T_\phi(0, 0) \hat{\phi} + T_\theta(0, 0) \hat{\theta})
\]

(87)

The maximum power density \( \tilde{S}_{\text{max}} \) is expressed as:

\[
\tilde{S}_{\text{max}} = \tilde{S}(0, 0) = \frac{1}{2} Z_0^{-1} \left[ |\vec{E}_{\theta}(0, 0)|^2 + |\vec{E}_{\phi}(0, 0)|^2 \right]
\]

with \( \vec{E}_{\theta}(0, 0) = -jk Z_0 \frac{e^{-jkr}}{4\pi r} T_\theta(0, 0) \)

and \( \vec{E}_{\phi}(0, 0) = -jk Z_0 \frac{e^{-jkr}}{4\pi r} T_\phi(0, 0) \)

(88)
So:

\[
|\vec{E}_\theta(0,0)|^2 = k^2 Z_0^2 \frac{1}{(4\pi r)^2} |T(0,0)|^2 \\
|\vec{E}_\phi(0,0)|^2 = k^2 Z_0^2 \frac{1}{(4\pi r)^2} |T(0,0)|^2
\]  \hspace{1cm} (89)

For the maximum radiated power by unit of solid angle we find:

\[
P(0,0) = r^2 S(0,0) \\
= r^2 \frac{1}{2} Z_0^{-1} \left[ |\vec{E}_\theta(0,0)|^2 + |\vec{E}_\phi(0,0)|^2 \right]
\]  \hspace{1cm} (90)

With (90) we finally obtain an expression for the normalized radiation pattern:

\[
F(\theta, \phi) = 10 \log \frac{P(\theta, \phi)}{P(0,0)} = 10 \log \frac{\left[ |\vec{E}_\theta(\theta, \phi)|^2 + |\vec{E}_\phi(\theta, \phi)|^2 \right]}{\left[ |\vec{E}_\theta(0,0)|^2 + |\vec{E}_\phi(0,0)|^2 \right]}
\]  \hspace{1cm} (91)

The radiation pattern (91) is normalized at 0dB in the direction of \( \theta = 0 \) and \( \phi = 0 \). With this are we enable to calculate the 2-dimensional radiation pattern in an arbitrary \( \phi \)-plane and across a certain \( \theta \)-range. For the non-normalized radiation pattern we first have to calculate the antenna gain and then add this gain to the normalized-radiation pattern. The antenna gain will be calculated in section 7. In the next paragraph we derive an idealized feed pattern which gives a good approach of the Philips corrugated horn.
6.2 The optimum radiation pattern of the feed

As mentioned before the far-field pattern of the reflector cannot be determined without a proper description of its feed pattern. An idealized feed pattern which gives a good approach of the Philips corrugated horn (4 grooves) may be described as:

\[ \bar{E}_s(r_s) = [\hat{a}_{\theta_s} \sin \phi_s + \hat{a}_{\phi_s} \cos \phi_s] \cdot F(\theta_s) \cdot \frac{e^{-jkr_s}}{4\pi r_s} \]

with \( F(\theta_s) = (\cos \theta_s)^L \)

for \( 0 \leq \theta \leq \frac{\pi}{2} \) and zero otherwise \( (92) \)

The exponent \( L \) will be replaced by "nu" \( (44) \) or "nt" \( (45) \) in the unscanned and scanned situation respectively. By choosing \( L=0 \) we get an uniform illuminated reflector surface. However an uniform illuminated reflector surface will result in a great spillover-loss, because lots of energy will radiate along the reflector surface. To reduce the spillover-loss we have to taper the edge illumination of the reflector \( (L > 0) \). However reducing the edge illumination will also result in a reduction of the aperture efficiency. Therefore we have to look for a particular edge illumination where the multiplication of the aperture- and spillover-efficiency is maximum. This multiplication is called the efficiency factor \( \eta_0 \) and is for the parabola defined as:

\[ \eta_0 = \cot^2 \frac{1}{2} \Psi \left[ \int_{0}^{\Psi} \left| G_f(\psi) \right| \frac{1}{2} \tan \frac{1}{2} \psi \, d\psi \right]^2 \]

with \( G_f(\psi) : \) antenna gain function

and \( \Psi : \) antenna aperture angle

For a first estimation we can use \( (93) \) also for the shaped reflector. Finally for an accurate result we have to vary the value of \( L \) a little bit. We can write the radiation pattern of the feed as follows:

\[ G_f(\psi) = G_0 \cos^M \psi \quad 0 \leq \psi \leq \frac{\pi}{2} \] \( (94) \)

\[ G_f(\psi) = 0 \quad \psi > \frac{\pi}{2} \] \( (95) \)

Note that the radiation pattern of the feed is limited to \( \phi < \pi/2 \), and the relation between \( L \) \( (92) \) and \( M \) \( (94) \) is defined as:

\[ M = 2L \]

\( (96) \)

The gain \( G_0 \) of the antenna can be obtained by using the following expression:

\[ \int_{0}^{2\pi} \int_{0}^{\pi} G_f(\psi) \sin \psi d\psi d\xi = 4\pi \]
We have to choose the value of $M$ so that the antenna aperture angle $\Psi$ correspond with the maximum of the efficiency factor $\eta_0$ (93).

**Example of calculation:**
Calculate the value of $L$ and the corresponding edge taper for the antenna given in fig. 26. First we have to calculate the antenna aperture angle $\Psi$ with the help of fig. 26. The antenna aperture angle $\Psi$ is equal to:

$$\Psi = \arctan \frac{0.25}{1.033} = 13.6^\circ$$

The efficiency factor $\eta_0$ as a function of the antenna aperture angle $\Psi$ is for $M=90$ plotted in fig. 27 We can see in fig. 27 that for $M=90$ the antenna aperture $\Psi$ correspond with the maximum of the efficiency factor $\eta_0$. With the help of (96) the value of $L$ is equal to:

$$L = \frac{M}{2} = 45$$

The edge-taper of the reflector for $L=45$ is equal to:

$$-20 \log \left( \cos^{45}(13.6^\circ) \right) = 11.1\text{dB}$$

The edge-taper of the reflector is not only caused by the taper of the feed but also by the path loss. The path loss is caused by the path length difference between the feed to the center and the upper tip of the reflector. In our case the path loss is equal to:

$$20 \log \frac{\sqrt{(0.25)^2 + (1.033)^2}}{1.048} = 0.122\text{dB}$$

This path loss is negligible with respect to the egde-taper of 11.1 dB caused by the feed taper.
**Figure 26:** The antenna aperture angle $\psi$

**Figure 27:** The efficiency factor $\eta_0$
6.3 Calculations of the far-field radiation pattern for the shaped-reflector in the unsanned situation

Calculations of the radiation pattern in the unscanned situation are less complex than in the scanned situation. The reason for this is that for the derivation of the unscanned-radiation pattern less coordinate systems and thus less rotations will be used. Because of this differences we will treated the derivation of the unscanned-radiation pattern separately.

Transmitted waves starting at the feed located at the unscanned focus reflect and propagate along the z-axis, known as the boresight, or unscanned direction. This makes the rotated coordinate system $x''y''z''$ from fig. 23 superfluous. The unscanned situation of the shaped reflector with the cartesian coordinate systems is shown in fig. 28.

The equation for the symmetrical-reflector surface $\Sigma$ is given by:

$$z'(x',y') = -b + a_1 x'^2 + a_2 x'^4 + P y'^2 + Q x'^2 y'^2 + R y'^4 + S x'^4 y'^2$$

(101)

The coefficients $b, a_1, a_2, P, Q, R$ and $S$ can be calculated with the techniques describes in the previous sections. To calculate the far-field pattern of the shaped reflector in the unscanned situation we have to transform the expression of the feed into the expression of the far-field pattern of the reflector. This transformation is obtained by three rotations about the Eulerian angles $(\alpha, \beta, \gamma)$. These Eulerian angles are shown in fig. 29.

Since the $x_s y_s$-plane coincide with the $x'y'$-plane, the line of intersection $x^*$ is chosen equal to the $x'$-axis and $x_s$-axis.
By this choice the Eulerian angles are equal to:

\[
\begin{align*}
\alpha &= 0^\circ \\
\beta &= 180^\circ \\
\gamma &= 0^\circ
\end{align*}
\]

For the elements of the transformation matrix \( (e^e A'') \) we find:

\[
\begin{align*}
A_{11} &= \cos \gamma \cos \alpha - \sin \gamma \cos \beta \sin \alpha = 1 \\
A_{12} &= \cos \gamma \sin \alpha + \sin \gamma \cos \beta \cos \alpha = 0 \\
A_{13} &= \sin \gamma \sin \beta = 0 \\
A_{21} &= -\sin \gamma \cos \alpha - \cos \gamma \cos \beta \sin \alpha = 0 \\
A_{22} &= -\sin \gamma \sin \alpha + \cos \gamma \cos \beta \cos \alpha = -1 \\
A_{23} &= \cos \gamma \sin \beta = 0 \\
A_{31} &= \sin \beta \sin \alpha = 0 \\
A_{32} &= -\sin \beta \cos \alpha = 0 \\
A_{33} &= \cos \beta = -1
\end{align*}
\]

And:

\[
( e^e A'' ) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\]

It was also possible to derive the transformation matrix immediately from fig. 29, since the \( x' \)-axis and \( y' \)-axis coincide with the \( x_s \)-axis and \( y_s \)-axis respectively and the \( z' \)-axis is rotated about \( 180^\circ \) from the \( z_s \)-axis.

The magnetic field of the feed expressed in the cartesian components of the reflector is given
by:

\[
\begin{align*}
H_s(\vec{r}') &= \{H_x(\vec{r}') \quad H_y(\vec{r}') \quad H_z(\vec{r}') \} \\
&= \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
\end{pmatrix}
\begin{pmatrix}
\sin\theta_s \cos\phi_s & \cos\theta_s \cos\phi_s & -\sin\phi_s \\
\sin\theta_s \sin\phi_s & \cos\theta_s \sin\phi_s & \cos\phi_s \\
\cos\theta_s & -\sin\theta_s & 0 \\
\end{pmatrix}
\cdot \left\{\begin{array}{c}
\frac{1}{Z_0} F(\theta_s) \cdot e^{-jkrs} \\
\sin\phi_s \\
\end{array}\right\}
\cdot \left\{\begin{array}{c}
-\cos\phi_s \\
\sin\phi_s \\
\end{array}\right\}
\end{align*}
\]

\[
= \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
\end{pmatrix}
\begin{pmatrix}
-\cos\theta_s \cos^2\phi_s - \sin^2\phi_s \\
\cos\theta_s \sin\phi_s \cos\phi_s - \sin\phi_s \cos\phi_s \\
\sin\theta_s \cos\phi_s \\
\end{pmatrix}
\cdot \frac{1}{Z_0} F(\theta_s) \cdot e^{-jkrs} \cdot \sin\phi_s
\]

\[
\begin{align*}
H_s(\vec{r}') &= \begin{pmatrix}
-\cos\theta_s \cos^2\phi_s - \sin^2\phi_s \\
\cos\theta_s \sin\phi_s \cos\phi_s - \sin\phi_s \cos\phi_s \\
\sin\theta_s \cos\phi_s \\
\end{pmatrix}
\cdot \frac{1}{Z_0} F(\theta_s) \cdot e^{-jkrs} \\
&= \begin{pmatrix}
-\cos\theta_s \cos^2\phi_s - \sin^2\phi_s \\
\cos\theta_s \sin\phi_s \cos\phi_s - \sin\phi_s \cos\phi_s \\
\sin\theta_s \cos\phi_s \\
\end{pmatrix}
\cdot \frac{1}{Z_0} F(\theta_s) \cdot e^{-jkrs}
\end{align*}
\]

The next step is to express the spherical coordinates \(\theta_s, \phi_s\) and \(r_s\) into \(\theta', \phi', r'\) with the help of (58).

\[
\begin{align*}
\begin{pmatrix}
r_s \sin\theta_s \cos\phi_s \\
r_s \sin\theta_s \sin\phi_s \\
r_s \cos\theta_s \\
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
\end{pmatrix}
\begin{pmatrix}
r' \sin\theta' \cos\phi' - s_1 \\
r' \sin\theta' \sin\phi' - s_2 \\
r' \cos\theta' + x'_{\text{unsc}} \\
\end{pmatrix}
\end{align*}
\]

The components of the \(\vec{S}\) vector of the shaped-symmetric reflector in the unscanned situation are given by:

\[
s_1 = 0 \quad s_2 = 0 \quad \text{and} \quad s_3 = z'_{\text{unsc}} \neq 0
\]

with: \(z'_{\text{unsc}}\) the \(z'\)-coordinate of the unscanned focal point

This results in the following three expressions:

\[
\begin{align*}
r_s \sin\theta_s \cos\phi_s &= r' \sin\theta' \cos\phi' \\
r_s \sin\theta_s \sin\phi_s &= -r' \sin\theta' \sin\phi' \\
r_s \cos\theta_s &= -r' \cos\theta' + z'_{\text{unsc}}
\end{align*}
\]

Formula (107) divided by formule (106) gives an expression for \(\phi_s\):

\[
\frac{r_s \sin\theta_s \sin\phi_s}{r_s \cos\theta_s} = -\frac{r' \sin\theta' \sin\phi'}{r' \cos\theta' \cos\phi'} \rightarrow \tan\phi_s = -\tan\phi' \rightarrow \phi_s = -\phi'
\]

Formula (107) divided by formule (108) gives an expression for \(\theta_s\):

\[
\frac{r_s \sin\theta_s \sin\phi_s}{r_s \cos\theta_s} = -\frac{r' \sin\theta' \sin\phi'}{-r' \cos\theta' + z'_{\text{unsc}}}
\]
\[
\tan \theta_s = \frac{-r' \sin \theta' \sin \phi'}{\sin(-\phi')( -r' \cos \theta' + z'_{unsc})}
\]

\[
\theta_s = \arctan \left( \frac{-r' \sin \theta' \sin \phi'}{\sin(-\phi')( -r' \cos \theta' + z'_{unsc})} \right)
\]

Substitution of formula (110) into formula (108) gives an expression for \(r_s\):

\[
r_s = \frac{-r' \cos \theta' + z'_{unsc}}{\cos \theta_s}
\]

With the aid of fig. 28 we can find an expression for \(r'\):

\[
\cos(\pi - \theta') = \frac{|z'|}{r'} \quad \Rightarrow \quad r' = \frac{|z'|}{\cos(\pi - \theta')}
\]

The different cartesian components of the magnetic field are given by:

\[
H_x' = \frac{-1}{Z_0}(\cos \theta_s \cos^2 \phi_s + \sin^2 \phi_s)F(\theta_s) \cdot \frac{e^{-jr \rho}}{4\pi r_s}
\]

\[
H_y' = \frac{1}{Z_0}(\cos \theta_s \sin \phi_s \cos \phi_s - \sin \phi_s \cos \phi_s)F(\theta_s) \cdot \frac{e^{-jr \rho}}{4\pi r_s}
\]

\[
H_z' = \frac{-1}{Z_0} \sin \theta_s \cos \phi_s F(\theta_s) \cdot \frac{e^{-jr \rho}}{4\pi r_s}
\]

For \(\phi_s, \theta_s\) and \(r_s\) we substitute (109), (110) and (111) respectively. The cartesian components of the unit normal are given by:

\[
N_{x'} = -\frac{\partial f}{\partial x'} = -(2a_1 x' + 4a_2 x'^3 + 2Q x'y'^2 + 4S x'^3 y'^2)
\]

\[
N_{y'} = -\frac{\partial f}{\partial y'} = -(2P y' + 2Q x'^2 y' + 4R y'^3 + 2S x'^2 y')
\]

\[
N_{z'} = 1
\]

The equivalent current on the reflector surface can write as:

\[
\tilde{J}(\rho', \phi') = 2\tilde{N} \times \tilde{H} = 2 \left( \begin{array}{c} N_{x'} \\ N_{y'} \\ N_{z'} \end{array} \right) \times \left( \begin{array}{c} H_{x'} \\ H_{y'} \\ H_{z'} \end{array} \right)
\]

In cartesian components:

\[
\tilde{J}_{x'} = 2(N_{y'} H_{x'} - N_{z'} H_{y'})
\]

\[
\tilde{J}_{y'} = 2(N_{x'} H_{x'} - N_{z'} H_{x'})
\]

\[
\tilde{J}_{z'} = 2(N_{x'} H_{y'} - N_{y'} H_{x'})
\]
Integration over the projected circular region \( \sigma \) with a substitution of the equivalent current can be expressed as:

\[
\vec{T}(\theta, \phi) = \int_{0}^{a} \int \vec{J}(\rho', \phi') e^{jk\vec{r}' \cdot \cos \theta} \cdot e^{jk\rho' \sin \theta \cos (\phi' - \phi)} \cdot \rho' d\rho' d\phi'
\]

with \( \vec{J}(\rho', \phi') = 2\vec{N} \times \vec{H}(\vec{r}') \) \((121)\)

To find the spherical far-field components of \( \vec{T} \), one then simply uses the following transformation:

\[
\begin{bmatrix}
T_{\theta} \\
T_{\phi}
\end{bmatrix} = \begin{bmatrix}
\cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
-\sin \phi & \cos \phi & 0
\end{bmatrix} \begin{bmatrix}
T_{x} \\
T_{y} \\
T_{z}
\end{bmatrix}
\]

\((122)\)

The spherical far-field components of \( \vec{E} \) may be expressed as:

\[
\vec{E}(\vec{r}) = -jkZ_0 \frac{e^{-jk\rho}}{4\pi r} (T_{\phi} \hat{a}_{\phi} + T_{\theta} \hat{a}_{\theta})
\]

\((123)\)

We finally obtain an expression for the normalized radiation pattern:

\[
P(\theta, \phi) = 10 \log \frac{P(\theta, \phi)}{P(0, 0)} = 10 \log \frac{|\vec{E}_{\phi}(\theta, \phi)|^{2} + |\vec{E}_{\phi}(\theta, \phi)|^{2}}{|\vec{E}_{\theta}(0, 0)|^{2} + |\vec{E}_{\phi}(0, 0)|^{2}}
\]

\((124)\)
7 Basic antenna parameters

Antenna parameters are very important to design reflector antennas. Therefore, a few basic antenna parameters will be treated in this section. A comprehensive description of the antenna gain is given in the first paragraph. Then a short description of the cross polarization is given and finally the G/T-ratio will be treated on the basis of an example in the last paragraph.

7.1 Calculation of the gain

An useful measure describing the performance of an antenna is the gain. The gain of an antenna in given direction is defined as "4 \pi times the ratio of the radiation intensity in that direction \( P(\theta, \phi) \) to the net power accepted by the antenna from a connected transmitter \( P_t \)." When the direction is not stated, the gain is usually taken in the direction of maximum radiation. Thus, in general:

\[
G(\theta, \phi) = \frac{P(\theta, \phi)}{P_t} = 4\pi \frac{P(\theta, \phi)}{P_t} \quad (125)
\]

For the calculation of the net power accepted by the antenna from a connected transmitter we take the radiated power of the horn feed. Furthermore, we assume that the horn feed only radiates in the forward direction. The radiation pattern of the feed, described in section (?), is defined as:

\[
\tilde{E}_s(\vec{r}_s) = [\hat{a}_{\theta_s} \sin \phi_s + \hat{a}_{\phi_s} \cos \phi_s] \cdot (\cos \theta_s)^L \cdot e^{-jkr_s} \quad (126)
\]

The squared absolute value of (126) can be written as:

\[
|\tilde{E}_s(\vec{r}_s)|^2 = \frac{1}{(4\pi r_s)^2} [\sin \phi_s^2 + \cos \phi_s^2] (\cos \theta_s)^{2L} \quad (127)
\]

\[
|\tilde{E}_s(\vec{r}_s)|^2 = \frac{1}{(4\pi r_s)^2} (\cos \theta_s)^{2L} \quad (128)
\]

The power density \( \tilde{S} \) can be expressed as:

\[
\tilde{S} = \frac{1}{2} Z_0^{-1} |\tilde{E}_s(\vec{r}_s)|^2 \cdot \hat{r}_s \quad (129)
\]

\[
= \frac{1}{2} \frac{1}{Z_0^{-1}} \frac{1}{(4\pi r_s)^2} (\cos \theta_s)^{2L} \cdot \hat{r}_s \quad (130)
\]

Using (130) we finally obtain the expression of the radiated power of the feed:

\[
P_t = \int_{\theta_s=0}^{\pi/2} \int_{\phi_s=0}^{2\pi} \tilde{S}(\vec{r}_s) \cdot \hat{r}_s \cdot r_s^2 d\Omega
\]

\[
= \int_{\theta_s=0}^{\pi/2} \int_{\phi_s=0}^{2\pi} \frac{1}{2} Z_0^{-1} \frac{1}{(4\pi)^2} (\cos \theta_s)^{2L} \sin \theta_s d\theta_s d\phi_s
\]
The radiated intensity in a given direction can be expressed as:

\[
P(\theta, \phi) = \frac{1}{2} Z_0^{-1} r^2 \left[ |E_\theta(\theta, \phi)|^2 + |E_\phi(\theta, \phi)|^2 \right] \tag{132}
\]

with

\[
|E_\theta(\theta, \phi)|^2 = k^2 Z_0^2 \frac{1}{(4\pi)^2} |T_\theta(\theta, \phi)|^2
\]
\[
|E_\phi(\theta, \phi)|^2 = k^2 Z_0^2 \frac{1}{(4\pi)^2} |T_\phi(\theta, \phi)|^2
\]

So, for the power density \( \vec{S} \) we find:

\[
\vec{S}(\theta, \phi) = \frac{1}{2} Z_0^{-1} k^2 Z_0^2 \frac{1}{(4\pi)^2} \left[ |T_\theta(\theta, \phi)|^2 + |T_\phi(\theta, \phi)|^2 \right] \tag{133}
\]

Now, the radiation intensity in the direction \((\theta, \phi)\) can be expressed as:

\[
P(\theta, \phi) = r^2 \vec{S}(\theta, \phi) = \frac{1}{2} Z_0^{-1} k^2 Z_0^2 \frac{1}{(4\pi)^2} \left[ |T_\theta(\theta, \phi)|^2 + |T_\phi(\theta, \phi)|^2 \right] \tag{134}
\]

The radiation intensity in the main direction \((\theta = 0, \phi = 0)\) can be written as:

\[
P(0, 0) = r^2 \vec{S}(0, 0) = \frac{1}{2} Z_0^{-1} k^2 Z_0^2 \frac{1}{(4\pi)^2} \left[ |T_\theta(0, 0)|^2 + |T_\phi(0, 0)|^2 \right] \tag{135}
\]

with

\[
T_\theta(0, 0) = T_x(0, 0) \\
T_\phi(0, 0) = T_y(0, 0)
\]

The physical optics radiation integral in the x-direction can be expressed as:

\[
T_x(0, 0) = \int_0^a \int_0^{2\pi} \tilde{J}_{x''}(\rho'', \phi'') e^{jkx''} \cdot \rho'' \, d\rho'' \, d\phi''
\]

with

\[
\tilde{J}_{x''}(\rho'', \phi'') = 2 \left( N_{y''} H_{x''} - N_{x''} H_{y''} \right)
\]
The physical optics radiation integral in the x-direction can be expressed as:

\[ T_y(0,0) = \int_0^a \int_0^{2\pi} \tilde{J}_y''(\rho'', \phi'')e^{jkl} \cdot \rho'' d\rho'' d\phi'' \] (137)

with \( \tilde{J}_y''(\rho'', \phi'') = 2(N_{x'y'}H_{x''y''} - N_{x''y'}H_{x'y''}) \)

The antenna gain in the main direction \((\theta = 0, \phi = 0)\) can be written as:

\[
G(0,0) = \frac{4\pi P(0,0)}{P_t} = \frac{4\pi^2 Z_0^{-1} k^2 Z_0^2}{(4\pi)^2} \frac{1}{16\pi Z_0^{-1} 2L+1} \left[ |T_\theta(0,0)|^2 + |T_\phi(0,0)|^2 \right]
\] (138)

So:

\[
G(0,0) = 2Z_0^2 k^2 (2L + 1) \left[ |T_\theta(0,0)|^2 + |T_\phi(0,0)|^2 \right]
\]

\[
G(0,0) = 10\log \left\{ 2Z_0^2 k^2 (2L + 1) \left[ |T_\theta(0,0)|^2 + |T_\phi(0,0)|^2 \right] \right\}
\] (139)

### 7.2 Calculation of the cross polarization

The rectangular and spherical components are related by:

\[
\hat{a}_x = A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi
\] (140)

\[
\hat{a}_y = A_r \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi
\] (141)

Since the cross-polar radiation pattern is maximum for small angles of \(\theta\), we can reduce (140) and (141) to:

\[
\hat{a}_x = \cos \phi \hat{i}_\theta - \sin \phi \hat{i}_\phi
\]

\[
\hat{a}_y = \sin \phi \hat{i}_\theta - \cos \phi \hat{i}_\phi
\]

For a polarization in the y-direction, we can express the co-polar \(R(\theta, \phi)\) and the cross-polar \(C(\theta, \phi)\) as:

\[
R(\theta, \phi) = \vec{E} \cdot \hat{a}_y = \sin \phi \hat{i}_\theta + \cos \phi \hat{i}_\phi
\]

\[
C(\theta, \phi) = \vec{E} \cdot \hat{a}_x = \cos \phi \hat{i}_\theta - \sin \phi \hat{i}_\phi
\]

For a polarization in the x-direction, we can express the co-polar \(R(\theta, \phi)\) and the cross-polar \(C(\theta, \phi)\) as:

\[
R(\theta, \phi) = \vec{E} \cdot \hat{a}_x = \cos \phi \hat{i}_\theta - \sin \phi \hat{i}_\phi
\]

\[
C(\theta, \phi) = \vec{E} \cdot \hat{a}_y = \sin \phi \hat{i}_\theta + \cos \phi \hat{i}_\phi
\]
7.3 Calculation of the G/T

The derivation of the G/T-ratio will be given on the basis of an example. The requirement of the G/T-ratio according the contract is:

\[
\frac{G}{T} > 13.0 \text{ dB/K} \quad @ \quad 11 \text{ GHz} \quad @ \quad 30^\circ \text{ elevation angle}
\]

For the calculation of the G/T we assume that the antenna is mounted on a wall of infinite height. In this worst case situation we assume that the temperature of the wall and the earth are both 300 K.

For the edge illumination of -12 dB we have a spillover efficiency of about 93%. This means that 93% of the radiated energy will fall on the reflector surface and 7% will fall on the "hot" wall.

The system-noise temperature \(T_s\) exists of the noise temperature of the antenna \(T_a\) and the noise temperature of the LNB \(T_c\).

Furthermore we assume that the noise figure of the LNB equals 1.1 dB \(\equiv 1.29^*\).

Now the noise temperature of the LNB will be:

\[
T_c = (F - 1) \cdot T_0 = (1.29 - 1) \cdot 300 = 86.5K
\]

with \(T_0\) : the temperature of the surroundings

The noise temperature of the antenna exists for a part of the noise temperature of the sky and for the other part of the noise temperature of the earth/wall. According measurements of the noise temperature of the sky at a frequency of 11 GHz and at an elevation angle of 30 degrees in Eindhoven it appears that the noise temperature of the sky can be taken at 50 K \([?]\).

The noise temperature of the wall/earth will be:

\[
T_{a1} = 300 \cdot 0.07 = 21 \text{ K}
\]

The total noise temperature of the antenna will be:

\[
T_a = T_{a1} + T_{a2} = 50 + 21 = 71 \text{ K}
\]

The system-noise temperature will be:

\[
T_s = T_a + T_c = 71 + 86.5 = 157.5 \text{ K}
\]

If we assume that the antenna gain is equal to 39 dB \(\equiv 7943^*\), then the G/T-ratio will be:

\[
\frac{G}{T} = \frac{G}{T_s} = \frac{7943}{157.5} = 50.43 \equiv 17.03 \text{ dB/K}
\]

It appears that even in the worst case situation that the G/T-ratio meets the requirements.
8 Accuracy of the antenna design.

8.1 Accuracy of the feed position.

For the investigation of the feed-position accuracy, three different positions of the feed are chosen in relation to the original position. These three different positions are given in fig. 30. The radiation patterns which correspond to the different positions of the feeds are given in fig. 31 up to fig. 33. The peak gain, main-direction level and side-lobe level for the shaped reflector are given in table 3.

<table>
<thead>
<tr>
<th>position of the feed</th>
<th>peak gain (dB)</th>
<th>main direction level (dB)</th>
<th>side-lobe level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>position 1</td>
<td>38.93 (-0.5 deg)</td>
<td>38.21</td>
<td>-22.97 (2.8 deg)</td>
</tr>
<tr>
<td>position 2</td>
<td>38.85 (-1.2 deg)</td>
<td>34.97</td>
<td>-22.57 (2.1 deg)</td>
</tr>
<tr>
<td>position 3</td>
<td>38.76 (-1.8 deg)</td>
<td>28.10</td>
<td>-22.33 (1.4 deg)</td>
</tr>
</tbody>
</table>

Table 3: Antenna parameters as a function of the feed position.
Figure 31: Feed in position 1.

\[ \alpha = 3.385^\circ \]
\[ \lambda = 2.654cm \]
\[ f_t = 25.0\lambda \]
\[ width = 35.78\lambda \]
\[ height = 32.0\lambda \]
\[ off.height = 16.0\lambda \]
$\alpha = 3.385^\circ$
$\lambda = 2.654\text{cm}$
$f_t = 25.0\lambda$

width $= 35.78\lambda$
height $= 32.0\lambda$
off.height $= 16.0\lambda$
$\alpha = 3.385^\circ$
$\lambda = 2.654\,cm$
$f_t = 25.0\lambda$
$width = 35.78\lambda$
$height = 32.0\lambda$
$off.height = 16.0\lambda$

Figure 33: Feed in position 3.
8.2 Accuracy of the antenna design for different locations inside Europe.

For calculation of the difference between two satellite positions, seen from an arbitrary position on earth, expressed in angle $\Phi$, we use fig. 34.

\[ \lambda_i = 5.47^\circ \text{ E.L.} \]
\[ \varphi_i = 51.45^\circ \text{ N.W.} \]

**Figure 34:** The earth with two geostationary satellites.

Explanation of the used symbols:

- $r$: radius of the earth ($r = 6378.16$ km).
- $h$: height of the geostationary orbit with respect to the equator.
- $s_1, s_2$: satellites 1 and 2.
- $d_1, d_2$: distance of the earth position with respect to satellite 1 and satellite 2 respectively.
- $d_3$: shortest distance between satellite 1 and satellite 2.
- $\lambda_1, \lambda_2$: position of satellite 1 and satellite 2 respectively with respect to the meridian.
Figure 35: A segment of the earth.

\( \lambda_i, \varphi_i \) : arbitrary position on earth.

In the triangle with the sides \( d_1, d_2 \) and \( d_3 \) we can calculate angle \( \Phi \) with the help of the cosine rule. To find expressions for \( d_1, d_2 \) and \( d_3 \) we use fig. 35. For calculating \( d_1 \) we first have to find an expression for angle \( \varphi_1 \).

With the help of fig. 35, it can be shown that:

\[
\cos(\lambda_1 - \lambda_i) = \frac{r \cos \varphi_i}{b} \quad \Rightarrow \quad b = \frac{r \cos \varphi_i}{\cos(\lambda_1 - \lambda_i)} \quad (142)
\]

\[
\sin(\lambda_1 - \lambda_i) = \frac{a}{b} \quad \Rightarrow \quad a = \sin(\lambda_1 - \lambda_i) \cdot \frac{r \cos \varphi_i}{\cos(\lambda_1 - \lambda_i)} = r \cos \varphi \cdot \tan(\lambda_1 - \lambda_i) \quad (143)
\]

\[
c = \sqrt{(r \sin \varphi_i)^2 + (r \cos \varphi_i \cdot \tan(\lambda_1 - \lambda_i))^2}
\]

\[
= r \cdot \sqrt{(\sin \varphi_i)^2 + (\cos \varphi_i \cdot \tan(\lambda_1 - \lambda_i))^2} \quad (144)
\]

With the help of the cosine rule we find for angle \( \varphi_1 \):

\[
\cos \varphi_1 = \frac{r^2 + b^2 - c^2}{2rb} \quad (145)
\]
Substitution of (142), (143) and (144) in (145) gives us:

\[
\cos \varphi_1 = \frac{r^2 + \left( \frac{r \cos \varphi_i}{\cos(\lambda_1 - \lambda_i)} \right)^2 - r^2 \left( (\sin \varphi_i)^2 + (\cos \varphi_i \cdot \tan(\lambda_1 - \lambda_i))^2 \right)}{2r \cdot \frac{r \cos \varphi_i}{\cos(\lambda_1 - \lambda_i)}}
\]

\[
= 1 + \left( \frac{\cos \varphi_i}{\cos(\lambda_1 - \lambda_i)} \right)^2 - \left( (\sin \varphi_i)^2 + (\cos \varphi_i \cdot \tan(\lambda_1 - \lambda_i))^2 \right)
\]

\[
= \frac{1}{2} \cdot \frac{\cos(\lambda_1 - \lambda_i)}{\cos \varphi_i} + \frac{\cos \varphi_i}{2 \cos(\lambda_1 - \lambda_i)} - \frac{(\sin \varphi_i)^2}{2 \cos \varphi_i} - \frac{\cos \varphi_i \sin(\lambda_1 - \lambda_i)^2}{2(\cos(\lambda_1 - \lambda_i))^2}
\]

\[
= \frac{1}{2} \cos \varphi_i \cos(\lambda_1 - \lambda_i) + \frac{1}{2} \cos \varphi_i \cos(\lambda_1 - \lambda_i)
\]

\[
\cos \varphi_1 = \cos \varphi_i \cos(\lambda_1 - \lambda_i)
\] (146)

We can find an expression for \(d_1\) by drawing a perpendicular from the position on earth to the line between the centre of the earth and the position of satellite \(s_1\).

\[
d_1^2 = (r \sin \varphi_1)^2 + (r + h - r \cos \varphi_1)^2
\]

\[
= r^2 (\sin \varphi_1)^2 + (r + h)^2 - 2r(r + h) \cos \varphi_1 + r^2 (\cos \varphi_1)^2
\]

\[
= (r + h)^2 + r^2 - 2r(r + h) \cos \varphi_1
\]

\[
= (r + h)^2 + r^2 - 2r(r + h) \cos \varphi_i \cos(\lambda_1 - \lambda_i)
\] (147)

The distance \(d_2\) can be find on the analogy of \(d_1\).

\[
d_2 = (r + h)^2 + r^2 - 2r(r + h) \cos \varphi_i \cos(\lambda_2 - \lambda_i)
\] (148)

For deriving the the distance \(d_3\) we use fig. 36.

\[
\sin((\lambda_2 - \lambda_1)/2) = \frac{d_3/2}{r + h} = \frac{d_3}{2(r + h)}
\]

\[
d_3 = 2(r + h) \sin \left( \frac{\lambda_2 - \lambda_1}{2} \right)
\] (149)

With the help of the cosine rule we finally find the expression of the angle \(\Phi\).

\[
\cos \Phi = \frac{d_1^2 + d_2^2 - d_3^2}{2d_1 d_2}
\] (150)
Figure 36: derivation of the distance $d_3$

Example of calculation

Given: Position of the satellites $s_1$ and $s_2$.

Asked: Difference between the two satellites $s_1$ and $s_2$ seen from two different places on earth, expressed in angle $\Phi$.


Solution a. Coordinates of Eindhoven: $\lambda_i = 5.47^\circ$E.L., $\varphi_i = 51.45^\circ$N.W.

\begin{align*}
    d_1^2 &= 1.486 \cdot 10^9 \quad \rightarrow \quad d_1 = 38551.1 \text{km} \\
    d_2^2 &= 1.493 \cdot 10^9 \quad \rightarrow \quad d_2 = 38637.9 \text{km} \\
    d_3 &= 4560.4 \text{km} \\
    \cos \Phi &= 0.993 \quad \rightarrow \quad \Phi = 6.77^\circ
\end{align*}

Solution b. Coordinates of Dublin: $\lambda_i = 6.29^\circ$E.L., $\varphi_i = 53.34^\circ$N.W.

\begin{align*}
    d_1^2 &= 1.5155 \cdot 10^9 \quad \rightarrow \quad d_1 = 38929.5 \text{km} \\
    d_2^2 &= 1.5286 \cdot 10^9 \quad \rightarrow \quad d_2 = 39097.6 \text{km} \\
    d_3 &= 4560.4 \text{km} \\
    \cos \Phi &= 0.993 \quad \rightarrow \quad \Phi = 6.70^\circ
\end{align*}
The angle between two different satellite positions, seen from different places in Europe, is shown in table 4.

<table>
<thead>
<tr>
<th>City</th>
<th>Eutelsat/Astra1</th>
<th>Astra1/Astra2</th>
<th>Eutelsat/Astra2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam</td>
<td>6.76</td>
<td>10.09</td>
<td>16.85</td>
</tr>
<tr>
<td>Dublin</td>
<td>6.70</td>
<td>9.98</td>
<td>16.67</td>
</tr>
<tr>
<td>Helsinki</td>
<td>6.63</td>
<td>9.95</td>
<td>16.58</td>
</tr>
<tr>
<td>Madrid</td>
<td>6.90</td>
<td>10.26</td>
<td>17.16</td>
</tr>
<tr>
<td>Oslo</td>
<td>6.64</td>
<td>9.94</td>
<td>16.58</td>
</tr>
<tr>
<td>Paris</td>
<td>6.80</td>
<td>10.15</td>
<td>16.96</td>
</tr>
<tr>
<td>Warsaw</td>
<td>6.77</td>
<td>10.16</td>
<td>16.93</td>
</tr>
</tbody>
</table>

Table 4: The difference in angle between two different satellites.
9 Results

This section contains the results of calculations carried out for seven shaped-reflector antennas. Each shaped-reflector antenna is compared with a corresponding offset-parabola. Corresponding means that both the shaped reflector and the offset parabola have the same size. However, the contour of both reflector surfaces are not the same, the front view of the shaped reflector is about elliptical and the front view of the parabola is circular.

The seven designs presented in this report are:

1 Reception of the satellites Eutelsat (13° E) and Astra1 (19.2° E).
   - offset-shaped reflector (width * height = 95 * 85cm)
   - offset parabola (diameter = 91.1cm)

2a Reception of the satellites Astra1 (19.2° E) and Astra2 (28.2° E).
   - offset-shaped reflector (width * height = 75 * 57cm)
   - offset parabola (diameter = 60cm)

2b Reception of the satellites Astra1 (19.2° E) and Astra2 (28.2° E).
   - offset-shaped reflector (width * height = 80 * 75.6cm)
   - offset parabola (diameter = 78.4cm)

3a Reception of the satellites Eutelsat (13° E) Astra1 (19.2° E) and Astra2 (28.2° E).
   - offset-shaped reflector (width * height = 75 * 57cm)
   - offset parabola (diameter = 60cm)

3b Reception of the satellites Eutelsat (13° E) Astra1 (19.2° E) and Astra2 (28.2° E).
   - offset-shaped reflector (width * height = 80 * 75.6cm)
   - offset parabola (diameter = 78.4cm)

3c Reception of the satellites Eutelsat (13° E) Astra1 (19.2° E) and Astra2 (28.2° E).
   - offset-shaped reflector (width * height = 110 * 85cm)
   - offset parabola (diameter = 99.7cm)

4 Reception of the satellites Eutelsat (13° E) Astra1 (19.2° E) and Telenor (1° W).
   - offset-shaped reflector (width * height = 110 * 85cm)
   - offset parabola (diameter = 99.7cm)
Design 1: Reception Eutelsat (13° E) and Astra1 (19.2° E).

dimensions shaped reflector:
- height of the aperture = 85cm
- width = 95cm
- offset height of the aperture = 42.5cm

dimensions parabola:
- diameter of the aperture = 91.1cm
- offset height of the aperture = 45.6cm

wavelength = 2.65cm (frequency = 11.3 GHz)

F/D = 0.74 (approx.)

scan-angles = 2 * 3.385° (with respect to Amsterdam)

offset-angle = 37°

elevation angle astral = 29.71° (with respect to Amsterdam)
elevation angle eutelsat = 30.68° (with respect to Amsterdam)

feed: Philips corrugated horn (4 grooves)

edge illumination = -12 dB (approx.)

---

1In accordance with the proposal the following expression for the radiation pattern of the feed is used:

\[ \tilde{E} = (\hat{a}_\theta \sin \varphi + \hat{a}_\rho \cos \varphi)(\cos \theta)^N e^{-jkr} \]

The radiation pattern of the Philips 4 grooves corrugated horn has been approximated by an appropriate choice of the value of "N". Furthermore we have chosen an edge illumination of approximately -12 dB, which gives us the value of "N".
Figure 37: Shaped reflector in the YZ-plane.

Figure 38: Frontview of the aperture plane (XY-plane).

Figure 39: Frontview of the mechanical surface.
Figure 40: Scan angles of the reflector in the XZ-plane.

Figure 41: Positions of the feeds for the shaped reflector.
Figure 42: Offset parabola in the YZ-plane.

Figure 43: Frontview of the aperture plane (XY-plane).

Figure 44: Frontview of the mechanical surface.
Figure 45: Positions of the feeds for the offset parabola.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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EUTELSAT (13°) / ASTRA1 (19.2°)

dB

co-pol.

cross-pol.

E.T.S.

SHAPED REFLECTOR

degrees
EUTELSAT (13°) / ASTRA1 (19.2°)

- dB
- co-pol.
- cross-pol.
- E.T.S.
- OFFSET PARABOLA

degrees

**Design 2a:** Reception Astra1 (19.2° E) and Astra2 (28.2° E).

- **Dimensions shaped reflector:**
  - Height of the aperture = 57cm
  - Width = 75cm
  - Offset height of the aperture = 28.5cm

- **Dimensions parabola:**
  - Diameter of the aperture = 60cm
  - Offset height of the aperture = 30cm

- **Wavelength = 2.65cm**
  - (Frequency = 11.3 GHz)

- **F/D = 0.74 (approx.)**

- **Scan-angles = 2 * 4.884°**
  - (With respect to Amsterdam)

- **Offset-angle = 37°**

- **Elevation angle astra1 = 29.71°**
  - (With respect to Amsterdam)

- **Elevation angle astra2 = 27.36°**
  - (With respect to Amsterdam)

- **Feed:** Philips corrugated horn (4 grooves)

- **Edge illumination = -12 dB (approx.)**
Figure 46: Shaped reflector in the YZ-plane.

Figure 47: Frontview of the aperture plane (XY-plane).

Figure 48: Frontview of the mechanical surface.
Figure 49: Scan angles of the reflector in the XZ-plane.

Figure 50: Positions of the feeds for the shaped reflector.
Figure 51: Offset parabola in the YZ-plane.

Figure 52: Frontview of the aperture plane (XY-plane).

Figure 53: Frontview of the mechanical surface.
Figure 54: Positions of the feeds for the offset parabola.

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**Design 2b:** Reception Astral (19.2° E) and Astra2 (28.2° E).

- Dimensions shaped reflector:
  - Height of the aperture = 75.6 cm
  - Width = 80 cm
  - Offset height of the aperture = 37.8 cm

- Dimensions parabola:
  - Diameter of the aperture = 78.4 cm
  - Offset height of the aperture = 39.2 cm

- Wavelength = 2.65 cm
  - Frequency = 11.3 GHz

- F/D = 0.74 (approx.)

- Scan-angles = 2 × 4.884° (with respect to Amsterdam)

- Offset-angle = 37°

- Elevation angle Astral = 29.71° (with respect to Amsterdam)

- Elevation angle Astra2 = 27.36° (with respect to Amsterdam)

- Feed: Philips corrugated horn (4 grooves)

- Edge illumination = -12 dB (approx.)
Figure 55: Shaped reflector in the YZ-plane.

Figure 56: Frontview of the aperture plane (XY-plane).

Figure 57: Frontview of the mechanical surface.
Figure 58: Scan angles of the reflector in the XZ-plane.

Figure 59: Positions of the feeds for the shaped reflector.
Figure 60: Offset parabola in the YZ-plane.

Figure 61: Frontview of the aperture plane (XY-plane).

Figure 62: Frontview of the mechanical surface.
Figure 63: Positions of the feeds for the offset parabola.

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ASTRA1(19.2°)/ASTRA2(28.2°)

co-pol.
cross-pol.
E.T.S.
shaped-reflector reception: ASTRA1/2
ASTRA1(19.2°)/ASTRA2(28.2°)

dB

E.T.S.

co-pol.
cross-pol.

offset-parabola reflector
reception: ASTRA1/2

degrees
Design 3a: Reception Eutelsat (13° E) Astral (19.2° E) and Astra2 (28.2° E).

dimensions shaped reflector:  
  height of the aperture = 57cm  
  width = 75cm  
  offset height of the aperture = 28.5cm

dimensions parabola:  
  diameter of the aperture = 60cm  
  offset height of the aperture = 30cm

wavelength = 2.65cm  
  (frequency = 11.3 GHz)

F/D = 0.74 (approx.)

scan-angles = 1 * 1.505° and 2 * 8.262°  
  (with respect to Amsterdam)

offset-angle = 37°

elevation angle eutelsat = 30.68°  
  (with respect to Amsterdam)

elevation angle astral = 29.71°  
  (with respect to Amsterdam)

elevation angle astra2 = 27.36°  
  (with respect to Amsterdam)

feed: Philips corrugated horn (4 grooves)

edge illumination = -12 dB (approx.)
Figure 64: Shaped reflector in the YZ-plane.

Figure 65: Frontview of the aperture plane (XY-plane).

Figure 66: Frontview of the mechanical surface.
Figure 67: Scan angles of the reflector in the XZ-plane.

Figure 68: Positions of the feeds for the shaped reflector.
Figure 69: Offset parabola in the YZ-plane.

Figure 70: Frontview of the aperture plane (XY-plane).

Figure 71: Frontview of the mechanical surface.
Figure 72: Positions of the feeds for the offset parabola.

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EUTELSAT(13°)/ASTRA1(19.2°)/ASTRA2(28.2°)

-20 -10 -8 -6 -4 -2 0 2 4 6 8 10

degrees

dB

40 30 20 10 0 -20 -30 -40

co-pol.
cross-pol.

E.T.S.

shaped-reflector(75*57cm)
reception: astra1
EUTELSAT(13°)/ASTRA1(19.2°)/ASTRA2(28.2°)

- co-pol.
- cross-pol.

shaped-reflector(75*57cm)
reception: eutelsat, astra2
EUTELSAT(13°)/ASTRA1(19.2°)/ASTRA2(28.2°)

offset-parabola (d=60cm)
reception: ASTRA 1
offset-parabola (d=60cm)
reception: EUTELSAT, ASTRA1, ASTRA2
**Design 3b:** Reception Eutelsat (13° E) Astra1 (19.2° E) and Astra2 (28.2° E).

- **dimensions shaped reflector:**
  - height of the aperture = 75.6cm
  - width = 80cm
  - offset height of the aperture = 37.8cm

- **dimensions parabola:**
  - diameter of the aperture = 78.4cm
  - offset height of the aperture = 39.2cm

- **wavelength = 2.65cm** (frequency = 11.3 GHz)

- **F/D = 0.74** (approx.)

- **scan-angles = 1 * 1.505° and 2 * 8.262°** (with respect to Amsterdam)

- **offset-angle = 37°**

- **elevation angle eutelsat = 30.68°** (with respect to Amsterdam)

- **elevation angle astral = 29.71°** (with respect to Amsterdam)

- **elevation angle astra2 = 27.36°** (with respect to Amsterdam)

- **feed:** Philips corrugated horn (4 grooves)

- **edge illumination = -12 dB** (approx.)
Figure 73: Shaped reflector in the YZ-plane.

Figure 74: Frontview of the aperture plane (XY-plane).

Figure 75: Frontview of the mechanical surface.
Figure 76: Scan angles of the reflector in the XZ-plane.

Figure 77: Positions of the feeds for the shaped reflector.
Figure 78: Offset parabola in the YZ-plane.

Figure 79: Frontview of the aperture plane (XY-plane).

Figure 80: Frontview of the mechanical surface.
Figure 81: Positions of the feeds for the offset parabola.

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**Shaped Reflector**

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**Offset Parabola**
ASTRA1 (19.2°)/ASTRA2 (28.2°)/EUTELSAT (13°)

dB

co-pol.
cross-pol.
E.T.S.
shaped-reflector reception: astra2/eutelsat

degrees
EUTELSAT(13°) / ASTRA1(19.2°) / ASTRA2(28.5°)

![Graph showing co-pol. and cross-pol. signals with offset-parabola reflector reception for ASTRA1.](image-url)
Design 3c: Reception Eutelsat (13° E) Astra1 (19.2° E) and Astra2 (28.2° E).

dimensions shaped reflector:  
  height of the aperture = 85cm  
  width = 110cm  
  offset height of the aperture = 42.5cm

dimensions parabola:  
  diameter of the aperture = 99.7cm  
  offset height of the aperture = 49.8cm

wavelength = 2.65cm  
  (frequency = 11.3 GHz)

F/D = 0.74 (approx.)

scan-angles = 1 * 1.505° and 2 * 8.262°  
  (with respect to Amsterdam)

offset-angle = 37°

elevation angle eutelsat = 30.68°  
  (with respect to Amsterdam)

elevation angle astra1 = 29.71°  
  (with respect to Amsterdam)

elevation angle astra2 = 27.36°  
  (with respect to Amsterdam)

feed: Philips corrugated horn (4 grooves)

edge illumination = -12 dB (approx.)
Figure 82: Shaped reflector in the YZ-plane.

Figure 83: Frontview of the aperture plane (XY-plane).

Figure 84: Frontview of the mechanical surface.
Figure 85: Scan angles of the reflector in the XZ-plane.

Figure 86: Positions of the feeds for the shaped reflector.
Figure 87: Offset parabola in the YZ-plane.

Figure 88: Frontview of the aperture plane (XY-plane).

Figure 89: Frontview of the mechanical surface.
Figure 90: Positions of the feeds for the offset parabola.

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EUTELSAT(13°) / ASTRA1(19.2°) / ASTRA2(28.5°)

dB

co-pol.

cross-pol.

E.T.S.

shaped-offset reflector
reception: eutelsat/astra2

degrees
EUTELSAT(13°) / ASTRA1(19.2°) / ASTRA2(28.5°)

dB

co-pol.
cross-pol.

E.T.S.

offset-parabola reflector
reception: ASTRA1
offset-parabola reflector
reception: eutelsat/astra2
Design 4: Reception Eutelsat (13° E) Astra1 (19.2° E) and Telenor (1° W).

dimensions shaped reflector:
  height of the aperture = 85cm
  width = 110cm
  offset height of the aperture = 42.5cm

dimensions parabola:
  diameter of the aperture = 99.7cm
  offset height of the aperture = 49.8cm

wavelength = 2.65cm (frequency = 11.3 GHz)

F/D = 0.74 (approx.)

scan-angles = 1 * 4.17° and 2 * 10.81° (with respect to Amsterdam)

offset-angle = 32°

elevation angle eutelsat = 30.68° (with respect to Amsterdam)

elevation angle astra1 = 29.71° (with respect to Amsterdam)

elevation angle telenor = 30.79° (with respect to Amsterdam)

feed: Philips corrugated horn (4 grooves)

edge illumination = -12 dB (approx.)
Figure 91: Shaped reflector in the YZ-plane.

Figure 92: Frontview of the aperture plane (XY-plane).

Figure 93: Frontview of the mechanical surface.
Figure 94: Scan angles of the reflector in the XZ-plane.

Figure 95: Positions of the feeds for the shaped reflector.
Figure 96: Offset parabola in the YZ-plane.

Figure 97: Frontview of the aperture plane (XY-plane).

Figure 98: Frontview of the mechanical surface.
Figure 99: Positions of the feeds for the offset parabola.

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shaped-offset reflector
4.17 degrees scanning
ASTRA 1(19.2° E)/EUTESAT(13° E)/TELENOR(1° W)

co-pol.

cross-pol.

E.T.S.

shaped-offset reflector
10.81 degrees scanning

dB

degrees
ASTRA1 (19.2° E) / EUTELSAT (13° E) / TELENOR (1° W)

-40 dB
-30 dB
-20 dB
-10 dB
 0 dB
+10 dB
+20 dB
+30 dB
+40 dB

degrees

degrees

co-pol.
cross-pol.

offset-parabola
4.17 degrees scanning

ETS.
ASTRA1 (19.2° E) / EUTELSAT (13° E) / TELENOR (1° W)

dB

co-pol.

cross-pol.

E.T.S.

offset-parabola
10.81 degrees scanning

degrees
10 Conclusions

- The method of calculation is very flexible and is usable for both the symmetrical and offset-shaped reflector. In addition the calculation can also be used for the design of parabolic reflectors (symmetrical and offset).

- The shaped reflector can be used for scan angles up to $2 \times 30^\circ$.

- For all the four designs presented in this report apply that with the shaped reflector one has less trouble with interference from nearby satellites than the offset parabola.

- The gain reduction as a function of the scan angle is smaller for the shaped reflector than for the offset parabola.

- A smaller reflector surface will result in a lower gain. However, a smaller reflector surface means also smaller phase errors in the aperture plane. So, there is less gain reduction as a result of the increased scan angle for smaller reflector surfaces.

- To compensate for the difference in elevation angle between the different satellites, we have to tilt the entire dish antenna a little bit.

- The positions of the feeds (Philips 4 grooves corrugated horns) do not need to be extra adjusted for different countries inside Europe.

- For all the four designs presented in this report apply that there is enough space to place the different feeds.

- To obtain a circular aperture, we have to choose the mechanical height of the reflector surface a little bit bigger than the diameter of the aperture.
References


