The quantitative effect of tool geometry and strain-hardening on the critical punch force in cup drawing

Citation for published version (APA):

Kals, J. A. G. (1973). The quantitative effect of tool geometry and strain-hardening on the critical punch force in cup drawing. In F. Koeningsberger, & S. A. Tobias (Eds.), *MTDR, machine tool design and research : 12th international conference : proceedings* (pp. 367-378). Macmillan.

Document status and date:

Published: 01/01/1973

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

THE QUANTITATIVE EFFECT OF TOOL GEOMETRY AND STRAIN-HARDENING ON THE CRITICAL PUNCH FORCE IN CUP DRAWING

by

J. A. G. KALS*

SUMMARY

A relation between tensile curves and critical punch force in the deep drawing of cylindrical products is developed. Both the work hardening effect and the geometry of the drawing punch are taken into account. A reasonable correspondence between the analytical results and the experimental data can be established. Finally, the practical significance of the mathematical model is shown by giving a criterion for the minimum corner radius of the punch. Moreover, the usefulness of the model is confirmed on the basis of some observations on deep drawability and geometric similarity in formability tests.

INTRODUCTION

Deep drawability can be influenced radically by many factors which may constitute the difference between the successful production of a stamping and breakage during pressworking operations. Many individual drawing steps may be required to produce a stamping. In order to reduce the number of drawing operations, the drawing ratio, defined between the blank diameter and the average cup diameter, has to be chosen as high as possible. The limit of deformation is reached when the load, required to deform the flange, becomes greater than the load-carrying capacity of the cup wall.

The required punch load depends on a large number of drawing conditions, such as forming properties of the sheet material, sheet thickness, drawing ratio, blank diameter, die-profile radius, hold-down pressure and friction conditions. On the other hand, the critical punch load is influenced by the punch profile radius, the punch diameter and by lubrication, sheet thickness and material properties as well. Changes of lubricant and material characteristics caused by speed fluctuations are other factors that may influence formability. The actual value of the limiting drawing ratio is fixed by all these coinciding forming conditions.

In this paper a theory is described which enables a calculation of the critical punch load and of a favourable dimension of the corner radius of the punch. In order to limit the complexity of the mathematical problem to a minimum, a number of validity restrictions have to be made with respect to the following theory:

- (i) it is assumed that deformation speed effects can be neglected;
- (ii) the working sheet materials are homogeneous, plastic-rigid and isotropic;
- (iii) friction effects can be neglected;
- (iv) comparatively thin sheet material only is considered, so that bending effects do not have to be taken into account;
- (v) a relatively small punch-edge radius in relation to the punch diameter.

The direct practical significance of this theory may be based on the fact that special literature of objective information concerning the selection of a useful punch-profile radius in relation to formability limits is lacking.

ANALYSIS

The current stress and strain state in the critical cross-section

In radial drawing of the flange region the material is being upset in a tangential direction. This results in an increasing sheet thickness and a hardening of the material. These effects are stronger as a volume element is moved further into the direction of the die cavity. So the increase in sheet thickness is restricted to the outer flange areas. Contrary to this, and especially under critical drawing conditions, the inner flange area is stretched very considerably during the initial increase of the punch force. This holds particularly for the material originally over the die wall. Therefore, the failure will be located exclusively in the stretched area near the

bottom of the cup wall. The exact location of the failure, caused by exceeding the stability limit in stretching, depends on the material and the forming conditions, particularly on friction.

As a preliminary to the analysis of the stretching limit, the failure location is assumed to be exactly on the borderline between the cup wall and the rounded edge of the punch. With favourable friction conditions, and a relatively large edge radius being excepted, the fore-going will be a fair approximation of reality (see Fig. 1). A laborious procedure can be avoided by representing the rounded cup area as a part of a torus.

Let σ_ϕ and σ_t be the average axial and circumferential stress components in the critical cross-section and p the local normal pressure between the punch and the cup wall. The equation of equilibrium is

$$\frac{p}{s} = \frac{\sigma_\phi}{\rho_{st}} \left(1 + \frac{s}{2r_{st}}\right) + \frac{\sigma_t}{r_{st}} \left(1 + \frac{s}{2\rho_{st}}\right) \quad (1)$$

where

s = the momentary cup wall thickness
 r_{st} = the punch radius
 ρ_{st} = the punch profile radius

Fig. 1 Failure usually occurs in the rounded edge, close to the cylindrical wall area.

According to the simplifying assumptions, failure takes place in a symmetry plane of the torus as shown in Fig. 2.

Fig. 2 Schematic stress state in the critical cross-section of the cup wall.

An immediate simplification of equation (1) can be achieved by using the restriction $s \ll \rho_{st}$. In this case, equation (1) reduces to

$$\frac{p}{s} \approx \frac{\sigma_\phi}{\rho_{st}} + \frac{\sigma_t}{r_{st}} \quad (2)$$

In the first instance the normal stress component σ_n depends on the inner wall pressure p . Thus far ($0 \leq i \leq 1$)

$$\begin{aligned} \sigma_n &= -ip \\ &\approx -is \left(\frac{\sigma_\phi}{\rho_{st}} + \frac{\sigma_t}{r_{st}} \right) \end{aligned} \quad (3)$$

The axial stretching of the cup wall during the initial increase of the punch load is compensated exclusively by a reduction in wall thickness, as the punch effectively precludes straining in the circumferential direction. The decrease of the average cup radius r_{ss} by the reduction in thickness may be neglected when $s \ll r_{st}$. As a consequence

$$d\delta_t = 0 \quad (4)$$

Let $d\delta_t$, $d\delta_\phi$ and $d\delta_n$ be the principal components of an increment of strain. Since there is no change in volume the following relation exists

$$d\delta_t + d\delta_\phi + d\delta_n = 0 \quad (5)$$

Hence

$$d\delta_\phi = -d\delta_n \quad (6)$$

The Lévy–von Mises equations may be expressed for the normal and the axial direction, respectively:

$$d\delta_n = d\lambda \left(\sigma_n - \frac{\sigma_t + \sigma_\phi}{2} \right) \quad (7)$$

$$d\delta_\phi = d\lambda \left(\sigma_\phi - \frac{\sigma_n + \sigma_t}{2} \right)$$

where $d\lambda$ is a scalar factor of proportionality. If this is combined with the straight strain-path as expressed in equation (6), we obtain the following necessary condition for the stress state:

$$\sigma_n = 2\sigma_t - \sigma_\phi \quad (8)$$

Now, the average normal stress σ_n can be eliminated from equation (3). Thus

$$\sigma_t \approx j\sigma_\phi \quad (9)$$

where

$$j = \frac{r_{st}}{\rho_{st}} \cdot \frac{\rho_{st} - is}{2r_{st} + is} \quad (10)$$

Finally, equations (8) and (9) may be combined to give

$$\sigma_n \approx (2j - 1) \sigma_\phi \quad (11)$$

It seems fair to regard the equations (9) and (10) as a reasonably good first-approximation of the complete current stress state in the critical cross-section.

For applications requiring a high accuracy, it will eventually be necessary to exclude the simplifications from the theoretical framework. At present, however, a practical approximation is wanted. So, for the time being additional mathematical complexity does not seem to be worth while.

The current load of the cup wall

Von Mises suggested that yielding occurs when the second stress-tensor invariant reaches a critical value $\bar{\sigma}$. In connection with our problem this criterion may be written in terms of the principal components of the stress state. Thus

$$2\bar{\sigma}^2 = (\sigma_t - \sigma_\phi)^2 + (\sigma_\phi - \sigma_n)^2 + (\sigma_n - \sigma_t)^2 \quad (12)$$

where $\bar{\sigma}$, the effective stress, is a parameter depending on the amount of strain. The concept of a yield criterion is not restricted merely to loading directly from the annealed state, as is sometimes thought. In combination with equation (12), we have from (9) and (11)

$$\bar{\sigma} \approx \sqrt{3} (1 - j) \sigma_\phi \quad (0 < j < 1) \quad (13)$$

In order to include the strain hardening effect in the theoretical model, $\bar{\sigma}$ has to be related to a certain measure of the total plastic deformation. A quantity

$d\bar{\delta}$, known as the generalized or effective plastic strain increment, is defined in terms of the principal strain increments by the equation

$$d\bar{\delta} = \left[\frac{2}{3} (d\delta_1^2 + d\delta_2^2 + d\delta_3^2) \right]^{1/2} \quad (14)$$

Apart from the numerical factor, $d\bar{\delta}$ is the same invariant function of the plastic strain increment tensor, as $\bar{\sigma}$ is of the components of the deviatoric stress tensor. The use of the previous equations (4) and (6), and integration of (14), result in

$$\bar{\delta} = \int d\bar{\delta} = \frac{2\delta_\phi}{\sqrt{3}} \quad (15)$$

This integration is the simplest and most natural way to satisfy the obvious requirement that the measure of total distortion must involve the summation of some continually positive quantity over the whole strain path. In this case integration is very simple because the components of any strain increment bear constant ratio to one another. Besides, it is worth noting that this strain model has the additional advantage that the general requirement of minimum dissipation of specific strain energy is satisfied automatically.

Turning now to the strain hardening relation between $\bar{\sigma}$ and $\bar{\delta}$; it is assumed that the following generalized form of an early empirical power law, due to Nadai, fits well to many sheet materials

$$\bar{\sigma} = C (\bar{\delta} + \bar{\delta}_0)^n \quad (16)$$

where C (characteristic stress) and n (strain hardening exponent) are material constants. The quantity $\bar{\delta}_0$ may be considered to include the strain history. Extending Nadai's equation in $\bar{\delta}_0$, results in C and n are essentially independent of strain history. According to the results taken from many tensile tests on different sheet materials, the introduction of $\bar{\delta}_0$ has the additional advantage of considerably higher accuracy in approximating real stress–strain curves of materials with an unknown strain history. Typical examples are given in Figs 3 and 4.

Fig. 3 The usual form of Nadai's equation in comparison with the generalized one and the results of tensile tests.

From equations (13) and (15), the actual form of (16) becomes

$$\sigma_\phi \approx \frac{C}{(1-j)\sqrt{3}} \left(\frac{2}{\sqrt{3}} \delta_\phi + \bar{\delta}_o \right) \quad (17)$$

Substitution of σ_ϕ in the general expression for the cup wall load (Fig. 2) gives

$$F = 2\pi s \left(r_{st} + \frac{s}{2} \right) \sigma_\phi = 2\pi s r_{ss} \sigma_\phi \quad (18)$$

which results in

$$F \approx \frac{2\pi}{\sqrt{3}} \cdot \frac{C s r_{ss}}{1-j} \left(\frac{2}{\sqrt{3}} \delta_\phi + \bar{\delta}_o \right)^n \quad (19)$$

According to the general definition of a logarithmic strain, we can write

$$s = s_o \exp(\delta_n) \quad (20)$$

where s_o is the initial sheet thickness. Combining equations (6) and (20), we find

$$s = s_o \exp(-\delta_\phi) \quad (21)$$

The required relation between the load F and the axial strain δ_ϕ is obtained by substituting this formula in equation (19)

$$F \approx \frac{2\pi}{\sqrt{3}} \cdot \frac{C r_{ss} s_o}{1-j} \left[\exp(-\delta_\phi) \right] \times \left(\frac{2}{\sqrt{3}} \delta_\phi + \bar{\delta}_o \right)^n \quad (22)$$

Finally it is to be noted that the present expression for the axial load on the critical cross-section of the cup wall is applicable for calculating also the punch force, with the limitation that friction forces can be neglected. This simplification has previously been assumed.

The critical punch load

The elongation of the partially formed cup wall is accompanied by a reduction in thickness; that is, a decrease in the cross-sectional area A , and thereby a strengthening by strain hardening. Initially the strain hardening effect dominates in view of the stretching force

$$\frac{dF}{d\delta_\phi} = \frac{d}{d\delta_\phi} (\sigma_\phi A) = \sigma_\phi \frac{dA}{d\delta_\phi} + A \frac{d\sigma_\phi}{d\delta_\phi} > 0 \quad (23)$$

Therefore, the cup wall can now support the larger deep-drawing load, so flange forming can continue. With only a few exceptions, the strain hardening effect $d\sigma_\phi/d\delta_\phi$ decreases with increasing strain level (Figs 3 and 4). In the continuation of the deep-drawing process, an ultimate strength of the cup wall will be reached when both the strain hardening and the stretching term in equation (23) cancel each other and we have

$$\frac{dF}{d\delta_\phi} = 0 \quad (24)$$

When the chosen drawing ratio implies a further increase of the drawing force to be necessary for continuous deformation of the flange region, this

load can no longer be transmitted through the lower cup wall. Finally, the load carrying capacity of this structurally weak link in the system appears to decrease with the punch going on continuously. The stamping then starts releasing elastically, with the exception of the lower region of the cup wall, and this plastic region shrinks into a circumferential constriction.

If the stability limit is once exceeded, plastic straining continues only in the necked part of the cup wall, and consequently no further straining will take place in the remaining part. Thus, equation (24) is the limiting condition of forming and, in general, it seriously reduces the achievable amount of overall deformation in those processes where stretching occurs. It is therefore the deep drawability limit.

For our purposes it may be sufficient to consider r_{ss} and j as being constant in differentiating equation (22), otherwise no explicit solution for the critical amount $\delta_{\phi k}$ of the axial component of strain can be obtained. Then, introducing the criterion of necking by differentiating (22) and setting it to zero, we may write

$$\delta_{\phi k} \approx n - \frac{\sqrt{3}}{2} \bar{\delta}_o \quad (25)$$

as a good approximation. The material with the higher n -value is characterized by a steeper stress-strain curve (Figs 3 and 4). The critical strain value at maximum punch load is larger for higher n -values. Generally the n -value primarily influences stretchability. The most important effect of a high n -value is to improve the uniformity of the strain distribution in the presence of a stress gradient, and necking happens to be a strong non-uniformity of the strain distribution. According to equation (25) and to practical experience, pre-straining diminishes formability.

Inserting this strain ceiling, in combination with equations (10) and (21), in the expression of the cup wall load, (22), we obtain

$$F_k \approx \frac{2\pi}{\sqrt{3}} C r_{ss} s_o \rho_{st} \left(\frac{2n}{e\sqrt{3}} \right)^n \times \frac{2 r_{st} e^n + i s_o \exp \left[\left(\frac{\sqrt{3}}{2} \right) \bar{\delta}_o \right]}{\rho_{st} r_{st} \exp \left[n - \left(\frac{\sqrt{3}}{2} \right) \bar{\delta}_o \right] + i s_o (r_{st} + \rho_{st})} \quad (26)$$

The last term in the numerator may be neglected according to the previous assumption for relatively thin sheet materials. Furthermore, this equation may be simplified, by the introduction of dimensionless quantities, to

$$F_k^* \approx i \frac{(4\pi/\sqrt{3}) (2n/\sqrt{3})^n}{[(1/\rho_{st}^*) + (1/r_{st}^*)] + \exp [n - (\sqrt{3}/2) \bar{\delta}_o]} \quad (27)$$

where

$$F_k^* = \frac{F_k}{C r_{ss} s_o}; \quad \rho_{st}^* = \frac{\rho_{st}}{s_o}; \quad r_{st}^* = \frac{r_{st}}{s_o} \quad (28)$$

and where r_{ss} is the average local cup radius at maximum wall load [equations (20) and (25)]

$$r_{ss} = r_{st} + \frac{s}{2} = r_{st} + \frac{s_0}{2} \exp \left(\frac{\sqrt{3}}{2} \bar{\delta}_0 - n \right) \quad (29)$$

A problem still to be solved concerns the numerical value of the stress parameter i [equation (3)]. The normal stress distribution may be approximately linear, so the value of i that we are looking for seems to be 0.5. Nevertheless it is better to choose the maximum value $i = 1$, for it is evident that instability must be initiated at the punch side of the cup wall, according to the assumption of uniformly distributed axial and tangential stresses. If a constant value $i = 1$ is combined with equation (27), the following expression is finally obtained

$$F_k^* \approx \frac{4\pi}{\sqrt{3}} \left[\frac{2n}{\sqrt{3}} \right]^n \left[\frac{1}{\rho_{st}^*} + \frac{1}{r_{st}^*} + \exp \left(n - \frac{\sqrt{3}}{2} \bar{\delta}_0 \right) \right]^{-1} \quad (30)$$

A representation of this relation is given in Fig. 5.

Theoretical results

Of course, the present solution is only a simplification of a more complex process, but this first step may shed some light on the mechanism of failure in deep drawing. Equation (30), as shown in Fig. 5, permits some interesting conclusions:

(i) Obviously, the load-carrying capacity of the cup wall vanishes very rapidly with decreasing edge radius below a definable limit of ρ_{st}^* . Practically, this effect implies the punch cutting into the cup wall. According to Oehler and Kaiser¹ the minimum value of the edge radius should preferably be chosen to equal five times the initial sheet thickness. A value $\rho_{st}^* = 15-25$ is judged as being still more recommendable. These empirical data support our foregoing theory clearly. Nevertheless, experimental investigations are necessary in order to compare the theoretical results with reality more systematically.

(ii) Strain hardening only slightly effects a change of the critical ρ_{st}^* -value.

(iii) The effects of ρ_{st}^* and r_{st}^* on the critical punch force are identical. To consider this fact may be useful in detecting failures of small stampings.

(iv) A noteworthy phenomenon being observed is that the critical punch force is smaller for larger n -values, due to larger stretchability, until instability occurs. The corresponding curves appear to pass through a minimum value at about $n = 0.8$. It can be shown (see p. 10) that the maximum punch load necessary to deform the flange region also decreases with increasing n -values. The corresponding curves $F_{\max}(n)$ appear to decline steeper than $F_k(n)$. So, ultimately, the limiting drawing ratio shows a slightly progressive increase with increasing n -values.

(v) The opposite influence of the 'strain-history' parameter $\bar{\delta}_0$ [equation (25)] is shown in Fig. 6.

Fig. 4 The usual form of Nadai's equation in comparison with the generalized one and the results of tensile tests.

Fig. 5 Theoretical curves according to equation (30) for $\bar{\delta}_0 = 0$, $r_{st} = 38.6$ mm, $s_0 = 2$ mm.

Finally a restriction has to be made with regard to the practical validity of equation (30). At very low values of the punch-edge radius in relation to sheet thickness, that is, where the edge is cutting into the wall, the validity of the presupposed deformation model may become doubtful. So Fig. 5 has to be understood merely as a representation of the mathematical relation in this region. According to the previous assumption of relatively small values of the edge radius, the validity of the theoretical equation has to be restricted in this respect too. It has been observed that the instability region is moving towards the punch centre at increasing edge radius.

Fig. 6 Theoretical relation between critical load number, strain-hardening exponent and strain-history parameter.

EXPERIMENTAL RESULTS

In order to obtain the material data, tensile tests were carried out intermittently at a mechanical tensile test machine. The local plastic strains could therefore be measured separately by measuring the cross-sectional area of the test specimen after discharging the material every now and then. The material constants have been computed according to the least-squares criterion. A number of ten sheet materials ($s_0 \approx 2\text{ mm}$) was selected on the basis of small earing in deep drawing. Nevertheless, this planar anisotropy effect increases slightly in the direction of increasing test numbers (Table 1). Tensile tests were carried out at 0° , as well as 45° , to the rolling direction. The results are given in Table 1.

Table 1. Results of tensile tests and deep-drawing tests (sheet materials as received).

Nr	sheet material	in rolling direction				45 degrees to rolling direction				F_{kw} [kN]
		s_0 [mm]	C [N/mm ²]	n	$\bar{\delta}_0$ [-]	s_0 [mm]	C [N/mm ²]	n	$\bar{\delta}_0$ [-]	
1	72/28 brass	1.97	791	0.56	0.04	1.92	786	0.57	0.04	179
2	stainless steel	2.09	1418	0.49	0.05	2.06	1391	0.53	0.06	358
3	stainless steel	2.01	1512	0.57	0.06	2.01	1460	0.61	0.08	343
4	63/37 brass	1.96	719	0.37	0.08	1.99	687	0.35	0.08	191
5	63/37 brass	1.93	697	0.46	0.12	1.93	685	0.52	0.16	181
6	alum (Si)	1.90	437	0.28	0.02	1.91	433	0.27	0.02	116
7	alum (99.5%)	1.96	140	0.33	0.01	1.95	138	0.39	0.03	34
8	nickel	2.06	1166	0.46	0.01	2.03	1104	0.44	0.01	255
9	copper	1.95	408	0.27	0.16	1.94	421	0.45	0.29	135
10	steel (Cu)	1.98	895	0.27	0.02	1.95	904	0.30	0.04	270

The best fitting stress-strain curves on the basis of the original Nadai equation (without strain history parameter) can be reconstructed with the values in Table 2.

Table 2. Experimental results according to the engineering form of the Nadai equation and measured values of the plastic anisotropy parameter

Nr	sheet material	in rolling direction		45 degrees to rolling direction		plastic anisotropy parameter	
		C [N/mm ²]	n	C [N/mm ²]	n	R_0 [-]	R_{45} [-]
1	72/28 brass	754	0.45	724	0.42	0.96	0.99
2	stainless steel	1230	0.31	1219	0.32	0.96	0.98
3	stainless steel	1346	0.37	1387	0.40	0.92	0.96
4	63/37 brass	553	0.13	583	0.16	0.77	0.88
5	63/37 brass	566	0.18	618	0.24	0.89	0.90
6	alum (Si)	410	0.22	408	0.22	0.45	0.52
7	alum (99.5%)	137	0.30	133	0.31	0.68	0.67
8	nickel	1132	0.43	1069	0.40	0.81	0.88
9	copper	335	0.06	339	0.06	0.68	0.78
10	steel (Cu)	778	0.17	714	0.14	0.79	0.88

The deep-drawing tests were carried out on a hydraulic press with low punch velocities and a rather arbitrarily chosen tool geometry with $r_{st} = 38.6$ mm and $\rho_{st} = 12.0$ mm. It is a well-known fact that the load carrying capacity of the cup wall decreases slightly as the drawing ratio further exceeds the limiting value. This is due to the introduction of local instability before the forming of the bottom rounding has been completed. In this case necking occurs nearer to the flat bottom and also the critical cross-section is not perpendicular to the moving direction of the punch. Therefore, the critical drawing load has to be measured exactly at the limiting drawing ratio. In order to obtain these values of F_k , both the maximum drawing force F_{max} and the critical punch load F_k have been measured as a function of the drawing ratio. The required value of F_k can be taken as the intersection of both of these curves. The results are given in the last column of Table 1. Fig. 7 shows a satisfying correspondence between the calculated values F_k and the experimental values F_{kw} of the critical punch force.

According to equation (28) the characteristic stress C holds a rather dominant position with relation to the absolute value of the critical punch load. By eliminating this quantity, the effect of strain hardening can be made clear. Therefore in Fig. 8 the theoretical and experimental values of the dimensionless critical load number are compared. A stronger scattering can be observed in this representation. Nevertheless, the theoretical effect of strain hardening may be considered to be verified as well. It is probable that the divergence may be partly attributable to plastic anisotropy, especially in the case of the points plotted for materials 8, 9 and 10 in Table 2. In order to compare experimental and theoretical results (Fig. 6) with respect to the hardening effect on the critical load number as well, equation (30) has been evaluated according to the standard Nadai equation ($\bar{\delta}_0 = 0$) using the values in Table 2. Fig. 9 shows the results.

Every deep-drawing experiment so far mentioned has been carried out with a constant punch geometry. In order to verify the theoretical effect of the punch-edge radius (Fig. 5) separately, an additional series of experiments had to be carried out. The experimental results and the corresponding theoretical curves according to equations (27) or (30) are shown in Fig. 10. Equations (27) and (28) have been evaluated with the following data from tensile tests

rolling direction	$C = 798 \text{ N/mm}^2$
	$n = 0.54$
	$\bar{\delta}_0 = 0.06$
45° to rolling direction	$C = 760 \text{ N/mm}^2$
	$n = 0.57$
	$\bar{\delta}_0 = 0.08$

From Fig. 10 it is again found that equation (30) is a satisfactory approximation of reality. These experiments have been repeated for the larger relative sheet thickness s_0/r_{st} as practised in the former series of experiments. The results are given in Fig. 11. From this graph, in comparison with Fig. 10, it appears that

Fig. 7 Theoretical versus experimental values of the critical punch load.

the validity restriction to comparatively thin sheet materials (see p. 2) may not be overlooked. In addition, it is worth noting that the divergence of the plotted points in both the figures equals approximately the initial sheet thickness.

Even though some other variables to some extent exercise control over the deep-drawing process, equation (30) seems to give a true picture of the main conditions effecting the load-carrying capacity of the cup wall. Of course this study was only a first attempt to analyse the deep-drawing process and greater accuracy could probably be achieved with the aid of numerical calculation procedures. Many useful purposes, however, do not appear to be served by the application of rigour in an analysis for the sake of exactness.

Fig. 8 Theoretical versus experimental values of the critical-load number.

Fig. 9 Experimental results verifying the approximate validity of equation (30) with respect to the work-hardening effect (numerical data from Table 2).

Fig. 10 Experimental and theoretical relationship between the critical-load number and the punch-edge radius for a relatively thin sheet material.

Fig. 11 Experimental and theoretical relation of the critical-load number and the punch-edge radius for a larger relative sheet thickness.

APPLICATIONS

Finally, some significant engineering aspects of the foregoing theoretical failure model will be elucidated briefly. In trying out stamping tools, it is often necessary to change to a more formable material, to modify the die design and even to change the stamping design in order to form a new product successfully. This takes time and money, and illustrates the need for a better understanding of sheet-metal formability and for objective formability testing methods. Of course, formability alone is not the sole criterion which has to be taken into consideration when sheet metal, tool geometry and production conditions have to be selected; but it is an inevitable one.

Punch geometry and formability

It is convenient to introduce a parameter

η = F_k* / (F_k*)_max (31)

defining a practical, useful value of F_k in proportion to an imaginary maximum value

(F_k*)_max =

4π / √3 * (2n / √3)^n * { 1 / r_st* + exp [n - (√3/2) δ_0] }^-1 (32)

which results from equation (30) for ρ_st* → ∞. Substitution of (30) and (32) in (31), results in

ρ_st* = η / (1 - η) * { 1 / r_st* + exp [n - (√3/2) δ_0] }^-1 (33)

This expression enables the evaluation of a favourable punch-edge rounding as a function of the initial sheet thickness, the strain hardening exponent, the punch diameter and the chosen η-value. In the case represented in Fig. 10, for example, the following values are obtained from equation (33)

η	ρ_st*
0.75	5
0.86	10
0.90	15

Another more complex criterion might be defined in terms of a steepness limit as

∂F_k* / ∂ρ_st* ≤ q (34)

In general, the admissible slope tangent q has to be selected depending on the maximum drawing force in proportion to the critical punch load. Though this criterion would be a better one it is not going to be developed here. At present the experimental data appear to be too slight to make the additional mathematical complexity worth while.

As indicated in the introduction, the present study is part of a study directed to a theoretical analysis of some factors influencing deep drawability. In deep drawing, the overall deformation limit—limiting drawing ratio β_0 —can be defined as the ratio of the maximum blank diameter, that can be drawn into a cup

without failure, to the average diameter of the cup wall. This limit of deformation is reached when the load $F_{\max}$, required to deform the flange, becomes equal to the load carrying capacity F_k of the cup wall. A noteworthy aspect of taking $F_{\max}$ into account is that the die-edge radius ρ_{zt} has an effect on it that is opposite to the effect of the punch-edge radius on the critical punch load. Experimental values illustrating this are shown in Fig. 12. Several experimental curves are shown in Fig. 13 for different drawing ratios β_o . The corresponding measured F_k -values are also plotted. In the particular case of equal values of ρ_{st} and ρ_{zt} being selected—as often happens in practice—the limiting drawing ratios are fixed in dependence on the tool geometry by the intersections of the $F_{\max}$ curves and the F_k curves. Experimental and theoretical research in this field is going on in order to find a useful expression for $F_{\max}$ and, finally, for the limiting drawing ratio as a function of tool geometry and strain hardening behaviour of sheet metals. Finally, looking at Fig. 13, the observation can be made that the limiting drawing ratio has a practical maximum with respect to optimization of tool geometry.

Strain hardening and formability

It has been pointed out already in the theoretical results that the required drawing force $F_{\max}$ decreases slightly more than its critical value with increasing n -value. This results in larger values of the limiting drawing ratio as the strain hardening exponent becomes larger. This proposition still has to be made acceptable in order to give an outlook on the importance of the n -value as a basic material quantity affecting deep drawability. Let σ_ϕ and σ_t be the radial and circumferential stress components in the flange at radius r . With the restriction that friction effects and the blank holder pressure may be disregarded, the equation of equilibrium is

$$\frac{d}{dr} (\sigma_\phi s r) = \sigma_t s \quad (35)$$

where s is the local thickness of the blank. From many experiments, the strain state in the annulus appeared not to be a plane one, as is sometimes thought. The sheet thickness was found to be independent of r as a reasonably good first approximation. This leads to

$$\frac{d\sigma_\phi}{dr} = \frac{\sigma_t - \sigma_\phi}{r} \quad (36)$$

The relation between the radial stress component σ_ϕ and the circumferential one σ_t , if r_a = external blank radius is given by

$$\sigma_t = \sigma_\phi \frac{r^2 + r_a^2}{r^2 - r_a^2} \quad (37)$$

as can be shown² with the aid of the Lévy–von Mises equations. Substitution in the equation of equilibrium, followed by integration, leads to

$$\sigma_\phi = k \left(\frac{r_a^2}{r^2} - 1 \right) \quad (38)$$

where k is the integration constant.

The analytical expression for k can be obtained by using the boundary condition of a uniaxial peripheral stress state. Hence, with the tensile stress-strain relation (16), we may write

$$(\sigma_t)_{r=r_a} = -(\bar{\sigma})_{r=r_a} = -C \left(\ln \frac{r_{ao}}{r_a} + \bar{\delta}_o \right)^n \quad (39)$$

where r_{ao} is the initial radius of the blank and r_a the external radius at a certain moment. Substitution of equation (38) in (37), followed by combination with equation (39), gives

$$k = \frac{C}{2} \left(\ln \frac{r_{ao}}{r_a} + \bar{\delta}_o \right)^n \quad (40)$$

and

$$\sigma_\phi = \frac{C}{2} \left(\frac{r_a^2}{r^2} - 1 \right) \left(\ln \frac{r_{ao}}{r_a} + \bar{\delta}_o \right)^n \quad (41)$$

To investigate the influence of work-hardening on the drawing force we must find the sheet thickness. With the restriction of s being independent of r , and further of a uniaxial peripheral stress state in combination with the condition of constant volume and the Lévy–von Mises equations, the current flange thickness appears to be

$$s = s_o \sqrt{\frac{r_{ao}}{r_a}} \quad (42)$$

Since we are interested in the work-hardening effect only, within the scope of this paper, the effect of the punch edge and the—for the rest important—local friction may be represented in a greatly simplified way. Let r_s be the average radius of the drawing clearance. Then, the equation for the current drawing force is

$$F \approx 2\pi r_s e^{\mu\pi/2} s(\sigma_\phi)_{r=r_s} \quad (43)$$

where μ is the friction coefficient. Substitution of equations (41) and (42) gives

$$F \approx \pi s_o r_s C \left(\frac{r_{ao}}{r_a} \right)^{1/2} \left(\ln \frac{r_{ao}}{r_a} + \bar{\delta}_o \right)^n \left(\frac{r_a^2}{r_s^2} - 1 \right) \quad (44)$$

or

$$F^* \approx C \left(\frac{r_{ao}}{r_a} \right)^{1/2} \left(\ln \frac{r_{ao}}{r_a} + \bar{\delta}_o \right)^n \left(\frac{r_a^2}{r_s^2} - 1 \right) \quad (45)$$

where

$$F^* = F / (s_o r_s C) \quad (46)$$

The punch force reaches its maximum value for $r_a = r_{ak}$. Then, with

$$\left. \begin{aligned} \beta_o &= r_{ao}/r_s \text{ ('drawing ratio')} \\ \beta_k &= r_{ak}/r_s \end{aligned} \right\} \quad (47)$$

we obtain

$$\begin{aligned} F^*_{\max} &\approx \pi e^{\mu\pi/2} \left(\frac{\beta_o}{\beta_k} \right)^{1/2} \\ &\times \left(\ln \frac{\beta_o}{\beta_k} + \bar{\delta}_o \right)^n (\beta_k^2 - 1) \end{aligned} \quad (48)$$

where β_k can be calculated with

$$\beta_o = \beta_k \exp\left(2n \frac{\beta_k^2 - 1}{3\beta_k^2 + 1}\right) \quad (49)$$

This expression has been obtained by differentiating equation (45) with respect to r_a , followed by equating to zero.

Now, the nature of the work-hardening effect on deep drawability can be studied by evaluating the general condition $F_{\max}^* = F_k^*$ with the aid of equations (30) and (48). The theoretical values—represented by the curves in Fig. 14—are obtained with a digital computer, omitting the geometrical terms in equation (30) and for $\delta_o = 0$. Thus, both calculated force numbers $F_{\max}^*$ and F_k^* may be considered maximum values with respect to tool geometry. The substantial correctness of the theoretical tendency of the work-hardening effect may be demonstrated by the experimental work of Arbel³. His results (Table 3) are also shown in Fig. 14.

In order to eliminate friction effects, these tests were carried out without a blank-holder. It was therefore essential to use a sheet thick enough to prevent folding. Contrary to the original values, the limiting drawing ratios have been recalculated according to the following relation [see equation (29)]

$$\beta_{o \max} = \frac{r_{ao}}{r_{ss}} = r_{ao} / \left(r_{st} + \frac{s_o}{2e^n}\right) \quad (50)$$

Table 3. Experimental data of Arbel³ showing the limiting drawing ratio $\beta_{o \max}$ as a function of the work-hardening exponent n ($r_{st} = 1.1$ in; $s_o = 0.125$ in).

material	$(2r_{ao})_{\max}$ [in.]	n	$2r_{ss}$ [in.]	$\beta_{o \max}$
65/35 brass	2.625	0.54	1.173	2.24
18/8 stainless steel	2.625	0.52	1.174	2.24
copper	2.553	0.34	1.189	2.15
alum	2.450	0.28	1.195	2.05
alum	2.420	0.25	1.197	2.02
hard brass	1.850	0.07	1.217	1.52

The last metal in Table 3 had a very marked directionality and was tested to assess the results obtained with a metal of low formability. From the form of the dotted line (Fig. 14) Arbel³ concluded that little progress, from the deep-drawing point of view, can be expected from new alloys of a high work-hardening exponent. Though an approximation, our foregoing theory brings to light the fact that too much importance has presumably been attached to the last metal. In that case Arbel's conclusion should have to be reversed to the opposite sense. Recent studies in superplasticity⁴ support our conclusion. Research

activities are going on in order to analyse the additional effects of friction, anisotropy and the drawing edge on formability.

Simulative testing methods

There exist three main methods for determining the forming characteristics of sheet metal.

(i) Testing the fundamental plastic properties of the sheet metal—the use of the determined quantities has been demonstrated in this study.

(ii) Comparative testing on the basis of arbitrarily chosen formability parameters—the use of the resulting values should be restricted to make sure that properties do not vary from coil to coil, etc.

(iii) Testing by simulating forming operations—even in the case of carefully controlled geometric similarity there is the problem of the scale factors. Whether or not a small diameter punch—the Swift flat-bottom cup test for example—can truly represent a punch used to draw a geometrically similar cup 10 or 20 times larger in diameter is questionable.

Complete similarity exists when the limiting drawing ratio obtained from a scale test equals the value observed in production conditions. A free choice of the material characteristics and the initial sheet thickness can be overlooked for practical reasons; also a controlled change in friction conditions. Thus, the rules of similarity can be obeyed only by adjusting the testing tool geometry. Hence, if equation (30) holds—and under the simplifying restriction that the load numbers F_k^* and $F_{\max}^*$ under testing conditions must be equal to the values under production conditions—one of the rules of geometrical similarity can be formulated from (30) as

$$\frac{1}{\rho_{st}^*} + \frac{1}{r_{st}^*} = \frac{1}{c} = \text{constant} \quad (51)$$

Solutions are shown in Fig. 15 for different c -values. Owing to the diminishing steepness of the part of the curves of practical interest, it is clear that it will be impossible to realize the right geometrical scale conditions in most of the cases. It must be noted that common testing conditions are expressed at the bottom left-hand side of the graph.

It appears that no matter how much any simulative test is perfected, no single deep-drawing test is presumably sufficient to evaluate formability in an accurate way. Similar findings have been expressed by Shawki⁵ on the basis of many attempts to correlate results from different tests. Nevertheless, it is evident that there is a real need to be able to predict or evaluate the formability of sheet metal in combination with tool geometry and working conditions. For the time being a careful theoretical analysis of deep drawing on the basis of fundamental plastic properties seems to be the only way.

Fig. 12 Experimental values of the necessary drawing force as a function of the relative die-edge radius.

Fig. 13 Experimental curves representing the required drawing force $F_{\max}$ as a function of the relative die-edge radius ρ_{zr}^* for different values of the drawing ratio β_0 and the critical-punch load F_k as a function of the relative punch-edge radius ρ_{st}^* .

Fig. 14 Theoretical work-hardening effect on the limiting drawing ratio compared with experimental data of Arbel³.

Fig. 15 Curves representing the theoretical condition for geometrical similarity in scale testing.

REFERENCES

1. OEHLER, G. W., and KAISER, K., 'Schnitt-, Stanz- und Ziehwerkzeuge', Springer-Verlag (1957), p. 292.
2. KALS, J. A. G., 'Dieptrekken', Eindhoven University Press (1969), p. 4.15.
3. ARBEL, C., 'The Relation between Tensile Tests and the Deep Drawing Properties of Metals', *Sheet Metal Industries*, 27 (1950), pp. 921-926.
4. SCHRODER, G., and WINTER, K., 'Superplastische Werkstoffe—ein Ueberblick', *Industrie-Anzeiger*, 92, nr. 20 (1970), pp. 425-430.
5. SHAWKI, G. S. A., 'Assessing Deep Drawing Qualities of Sheet'; Part 1: 'Stretch-Forming and Wedge-Drawing Tests', *Sheet Metal Ind.*, 42 (1965) pp. 363-368; Part 2: 'Deep Drawing Tests', *Sheet Metal Ind.*, 42 (1965), pp. 41-424; Part 3: 'Combined Simulative Tests', *Sheet Metal Ind.*, 42 (1965) pp. 525-532.

ADDITIONAL REFERENCES

1. SIEBEL, E., *Steel*, 94 (1934), p. 37.
2. SACHS, G., *Proc. Inst. Automobile Eng.*, 29 (1934), p. 588.
3. CHUNG, S. Y., and SWIFT, H. W., *Proc. Inst. Mech. Engrs.*, 165 (1951), p. 211.