Application of a numerical/experimental method for the mechanical characterization of wood

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APPLICATION OF A NUMERICAL/EXPERIMENTAL METHOD FOR THE MECHANICAL CHARACTERIZATION OF WOOD.

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SUMMARY

Applying the traditional procedure of identifying material parameters to complex systems, such as composites, raises a number of problems. In this article an alternative procedure, 'the identification method', is used. The application of this method on the orthotropic material wood is investigated. This is done by means of a bending test. The results are validated by a second bending test. The experiments are preceded by a numerical simulation.

1. INTRODUCTION

This paper is a contribution to the work that is done in the field of parameter identification of materials at the Technical University Eindhoven. The main issue in the characterization of materials is to determine parameters in a chosen material model. This is done by means of experiments. In traditional testing the experimental set-up is chosen in such a way that it provides a simple, often homogeneous, stress and strain distribution in a sample. Therefore applying this traditional method for the identification of complex materials raises a number of problems:
(1) Composites may have inhomogeneous properties which makes it impossible to obtain a homogeneous stress and strain distribution in the sample.
(2) It is not always possible to make samples of a determined sample shape for fibre-structure reinforced composites without deteriorating the internal coherence of the structure.
(3) The number of experiments needed for sufficient characterization of the material is large.

In Hendriks [1991] a new method, 'the identification method', is presented, which provides more freedom for the experiments. The principle of this method is shown in fig.1.1, in which the numerical/experimental character is of great importance.

The sample in an experiment is multi-axially loaded and the non-homogeneous strain distribution is registered with a position measurement system based on digital image processing. Then a theoretical model based on the finite element method is derived. With a given set of initial parameters (estimations for the real values) this model supplies computed strain data. By means of data correlation the computed and the measured data are compared with each other. Using this error a parameter adjustment of the initial parameters can be made. These new parameters are used in the mathematical algorithm for another iteration in parameter adjustment. In this way the parameters in the model are estimated recursively based on measured data.
The aim of the investigation, presented in this paper, is the application of the 'identification method' to an orthotropic material with a large stiffness ratio. Wood can be considered as such a material and hence wood (in this case Norwegian Spruce) is used in the experiment. The applied experiment is a 4-point bending test.

In section 2 the theory of the constitutive behavior and the estimation algorithm will be quoted shortly. A computer simulation of the experimental/numerical method is discussed in section 3. Section 4 describes the set-up and results of the real experiment, section 5 continues with an experimental validation and discussion of the results and finally conclusions and recommendations are given in section 6.

2. THEORY

Wood Elastic Behavior

Wood is a biological material characterized by: (a) a high ratio of Young's moduli (a high degree of anisotropy) (b) a certain degree of dependence on time, temperature, moisture content, etc. The influences mentioned under (b) will be neglected and therefore the material model can be reduced to a linear elastic orthotropic one if small deformations are considered. This orthotropic character will be described in a cartesian co-ordinate system (see fig. 2.1), which is a valid description when the size of the specimen is small compared to the size of the tree and when the specimen is made out of wood far enough from the centre of the tree.

Because the wooden bar is loaded in a bending test the problem can be reduced to a quasi 2-dimensional case based on a plane stress condition. Also the assumption has been made that the Young's moduli in tension, compression and bending are identical. With these simplifications the elastic behaviour can be described with Hooke's Law as follows:

\[ \varepsilon = C \sigma \]

(2.1)
With \( \mathbf{C} \) the compliance matrix:

\[
\mathbf{C} = \begin{pmatrix}
1/E_1 & -v_{12}/E_2 & 0 \\
-v_{21}/E_1 & 1/E_2 & 0 \\
0 & 0 & 1/G_{12}
\end{pmatrix}
\]

and \( v_{21} = v_{12} \frac{E_1}{E_2} \)

The axes are defined according fig.2.1:
- direction 1: the longitudinal direction, which is the axis of the tree
- direction 2: the radial direction, defined locally according the growth rings and in which the bar is loaded.

Thus the material is characterised with 4 parameters: 2 Young's moduli (\( E_1, E_2 \)), a Poisson's ratio (\( v_{12} \)) and a shear-modulus (\( G_{12} \)).

![Figure 2.1: definitions of the directions of the axis in a cartesian coordinate system](image)

**Identification Algorithm**

The estimation algorithm is a recursive minimum variance algorithm (Hendrix [1991]). The model is represented by:

\[
y_k = h_2(\bar{x}) + v_k
\]

where \( y_k \) is a column with measurement data, in this case displacements. \( \bar{x} = [x_1, x_2, x_3, x_4]^T \) is a column with the 4 material parameters and the column \( h_2(\bar{x}) \) represents the calculated displacements by the mathematical algorithm based on those material parameters. The column with observation errors is represented by \( v_k \) and \( k \) is an iteration counter. The estimation of the material parameters \( \hat{\bar{x}}_k \) is adjusted according to
The term $h_{k+1}(\hat{x})$ represents the expected observation data, based on the a priori estimate $\hat{x}_k$. Since the term $y_{k+1}$ represents the actual observed value the difference $y_{k+1} - h_{k+1}(\hat{x})$ represents the new information. This difference is multiplied by a weighting matrix $K_{k+1}$ and forms the innovation for the new estimate $\hat{x}_{k+1}$. This weighting matrix is calculated according to

$$K_{k+1} = (P_k + Q_k)H_{k+1}^T(R_{k+1} + H_{k+1}(P_k + Q_k)H_{k+1})^{-1}$$

(2.4)

where $P_k$ denotes the covariance of the estimation error in $\hat{x}_{k+1}$ and is calculated with

$$P_{k+1} = (I - K_{k+1}H_{k+1})(P_k + Q_k)$$

(2.5)

The matrix $R_{k+1}$ is the covariance matrix of the observation error $v_k$. With matrix $Q_k$ it is possible to deal with the inaccuracies in the model. In practice this matrix can be used as a convergence accelerator and is chosen on engineering experience. Matrix $H_{k+1}$ contains the partial derivatives of the components of $\hat{x}$ around $\hat{x}$ of the function $h_{k+1}$

$$H_{k+1} = \left[ \frac{\delta h_{k+1}(\hat{x})}{\delta \hat{x}} \right]_{\hat{x}=\hat{x}}$$

(2.6)

For the derivation of the algorithm see Hendriks [1991]. The equation (2.1) is used in the mathematical algorithm and the equations (2.2) - (2.6) are used in the estimation algorithm of the identification method.

3 SIMULATION

To gain an insight in the experimental behavior of the wooden bar and in the identification method, firstly a numerical simulation of the actual experiment and the subsequent identification process is performed.

A displacement distribution is given to the estimator as experimental input. This distribution is determined by means of a theoretical model. Therefore the wooden bar under loading conditions is modeled with the finite element method, a displacement distribution is calculated and distorted with a simulated measurement error. Together with initial guesses $\hat{x}_{real}$ for the original parameters $\hat{x}$ this 'experimental input' is used to estimate the parameters of the simulated experiment with the advantage that now is known to what values the estimated parameters $\hat{x}$ have to converge, namely $\hat{x}_{real}$.

A sufficiently fine element mesh for modeling the wooden beam (specimen sizes: 40*40*330 mm) is found to be 4 in height and 22 in length (see fig. 3.2), the used elements are 8-point quadratic plane stress elements. A displacement distribution of the modeled beam with loadings ($F = 1250$ N) is calculated (see fig. 3.1) as well as it’s associated strain distribution (see fig. 3.2).

The finite element model is also used in the mathematical algorithm of the estimation method. The 'experimental input' is distorted with random noise with a normal distribution and a standard deviation of $10^3$ mm, so the covariance matrix $R$ is chosen to be a diagonal matrix with all the same diagonal terms $10^4$. The terms chosen in matrix $P$ ($V(P_k; x_{real}, \hat{x})$) are shown in table 3.1 as well as the beginning and final values $\hat{x}$ of the estimation process. After 5 iterations the parameters have almost converged to the real values (see Appendix A). The only parameter that is not yet converged is $v_{12}$.
From this simulation it can be concluded that it should be possible to determine the parameters from a similar real experiment.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Real Value(^*)</th>
<th>Initial Value</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{ij} )</td>
<td>( \xi_{\text{real}} )</td>
<td>( \xi_{\text{init}} )</td>
<td>( \sqrt{P_{\text{ini}}} )</td>
</tr>
<tr>
<td>( E_1 ) ([\text{kN/mm}^2])</td>
<td>17000</td>
<td>16000</td>
<td>1000</td>
</tr>
<tr>
<td>( E_2 ) ([\text{kN/mm}^2])</td>
<td>830</td>
<td>600</td>
<td>230</td>
</tr>
<tr>
<td>( v_{ij} ) ([-])</td>
<td>0.0181</td>
<td>0.030</td>
<td>0.012</td>
</tr>
<tr>
<td>( G_{ij} ) ([\text{kN/mm}^2])</td>
<td>640</td>
<td>900</td>
<td>260</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter estimation results after 5 iterations (Simulation).

*) source: F. Rouger [1991]

![Displacement field of the simulated experiment](image)

Figure 3.1: displacement field of the simulated experiment
- reference position
- deformed position

![Strain distribution of a beam under bending conditions](image)

Figure 3.2: strain distribution of a beam under bending conditions (simulation)

4 MEASUREMENTS

In this section the results of the actual experiment, a 4-point bending test, is presented.

The test is performed on a tensile testing machine (Zwick 1484 200 kN) and the deformation of the wooden beam is measured using a digital image technique. Retro-reflective markers are attached to the surface of the specimen. The position of these markers are measured with a video tracking system (Hentschell Gmbh, Hannover) based on Hamamatsu random access cameras. Illumination from the camera position causes reflections from the markers with higher intensity than the environment, which are used to identify the markers (Zamzow [1990]). The system also determines a measurement error
for each marker (for this experiment a range of 0.0091 - 0.021 mm). After loading the beam up to $F = 1970$ N, a 5 minute relaxation time is allowed before taking a measurement run.

In the numerical part the model of section 3 is used. The displacements of a series of markers (the upper row in fig. 4.1), just outside the plastic zone around the supports, are used as kinematic boundary conditions in the model. In this way the rigid body motion due to plastic deformation around those supports, caused by local stress concentrations, is eliminated. The initial and final values $\hat{a}_i$ of the estimation process, as well as the diagonal terms of matrix $p_i$, are shown in table 4.1. The parameter values of a function of iterations are shown in appendix B. Fig. 4.2 gives the residual field, which is the difference between the measured and calculated displacement field, after 30 iterations. The random distribution gives an indication that the residuals have decreased to their minimum values and are completely determined by the measurement error.

---

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Value</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_j$</td>
<td>unit</td>
<td>$\langle \hat{a}_j \rangle$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>[kN/mm²]</td>
<td>10000</td>
</tr>
<tr>
<td>$E_2$</td>
<td>[kN/mm²]</td>
<td>800</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>[-]</td>
<td>0.030</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>[kN/mm²]</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 4.1: Parameter estimation results after 30 iterations (4-point bending test).
To make a sensible statement about the correct behavior of the identification method in this test a second experiment is performed to compare the measurement results.

This second experiment is a 3-point bending test. The same testing circumstances and numerical model of section 4 are valid, with the exception of the single load $F = 3880$ N. The measurement errors in this test are also in the same range of values: 0.0091 - 0.020 mm.

The initial and final values of $\tilde{X}_i$ and the diagonal terms of matrix $P$ are shown in table 5.1. The parameter course for 30 iterations is shown in appendix B.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Value $\langle \tilde{X}_i \rangle_1$</th>
<th>$\sqrt{\langle P_{ij} \rangle_1}$</th>
<th>Estimates $\langle \tilde{X}<em>i \rangle</em>{30}$</th>
<th>$\sqrt{\langle P_{ij} \rangle_{30}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>$10000$</td>
<td>$10000$</td>
<td>$6658$</td>
<td>$0.12$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$800$</td>
<td>$45$</td>
<td>$583$</td>
<td>$0.43 \times 10^3$</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>$0.030$</td>
<td>$100$</td>
<td>$0.034$</td>
<td>$0.46 \times 10^3$</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>$1000$</td>
<td>$100$</td>
<td>$522$</td>
<td>$0.76 \times 10^3$</td>
</tr>
</tbody>
</table>

Table 5.1: Parameter estimation results after 30 iterations (3-point bending test).

Comparison of the parameter courses of the 4-point and 3-point bending test in appendix B indicates that:

- the parameters $E_1$ and $G_{12}$ can be estimated consonant within reasonable limits.
- the consonance in the estimation of the parameters $E_2$ and $v_{12}$ is poor. In the 4-point estimation $v_{12}$ even deviates up to 500% from the 3-point value.

The absolute values of the estimated parameters ($E_1$, $E_2$, and $G_{12}$) are lower than expected compared with literature values. But because wood is a biological material with large variations it is better to compare ratio’s, which are shown in appendix C of both the bending tests. Literature values are (see Bodig/Jayne [1982]):

$$E_2/E_1 = 0.08$$
$$G_{12}/E_1 = 0.07$$

Comparison of these values indicates good estimation results for the 3-point bending experiment and bad results for the 4-point bending experiment.

- Conclusion: estimation of $E_2$ in the 4-point bending experiment produces erroneous results, which is evident from the deviation in the ratio $E_2/E_1$ in appendix C.
- A probable cause: the large ratio of measurement error to the amount of the displacement distribution (they are of the same order of magnitude; maximum values: measurement error: 0.02 mm, strain: 0.3%, hence a displacement of 0.03 mm on a 10 mm marker distance). This is also indicated by the residuals (difference between measured and calculated displacements) in appendix D. After the first iteration they already are in the same range of magnitude as the measurement error. Hence a considerable parameter adjustment hardly effects the magnitude of the residuals. The measurement error obscures the signal that contains the parameter information.
- E2 en v12 suffer most from this effect, as the strain distribution of a beam under bending conditions provides less information on these parameters than on the parameters E1 and G12 which are prominent in the bending test.
- So the measurement error in the 4-point bending test is just too large for a good estimation of the most critical parameters in the bending test. The 3-point bending test has a larger displacement field, hence a larger strain distribution, and a measurement error in the same range as the 4-point experiment. Therefore the signal, which contains the parameter information, is not obscured to much by the measurement error. This provides a better convergence of all parameters and even an approach of the very critical parameter v12 towards it's expected value (expected values: see table 3.1).

6 CONCLUSIONS

- The application of the identification method on the orthotropic material wood has a good performance in the simulation studies.
- The succes in the experiments is limited because of the very large measurement error related to the size of the strain distribution, the measurement error obscures this strain distribution, which contains the information about the material parameters.

Recommendations

- Generally: an investigation on the minimum required strain distribution related to the measurement error, that can be obtained
- Better results can be obtained by changing the ratio measurement error/strain distribution as follows:
  * another experiment
  * same experiment, other measurement conditions, for example the investigation of a sub-area instead of the whole beam.
  * same experiment, bigger forces, which could imply linearity problems

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Identification results
Simulation

APPENDIX A
Identification results
4-point bending test  3-point bending test
Identification results

APPENDIX C

Ratio's

APPENDIX D

Identification results

Residual