Static compression of a circular cylinder

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<table>
<thead>
<tr>
<th>rapport van de sectie: Werkplaats techniek</th>
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<tbody>
<tr>
<td><strong>tijtel:</strong> Static compression of a circular cylinder</td>
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<tr>
<td><strong>auteur(s):</strong> ir. E. Mot</td>
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<tr>
<td><strong>hoogleraar:</strong> Prof. dr. P. C. Veenstra</td>
</tr>
</tbody>
</table>

**samenvatting**

By a method, very similar to the "Bridgman correction" of the stress in the neck of a tensile test specimen, a formula is derived for the load, needed to compress a circular cylinder. Experimental results are given for four materials.

**prognose**

This procedure could be reversed, viz. from an experimentally determined load-displacement characteristic, the values of $c$ and $m$ could be determined.
Static compression of a circular cylinder

ir. E. Mot

Summary.
By a method, very similar to the "Bridgmen correction" of the stress in the neck of a tensile test specimen, a formula is derived for the load, needed to compress a circular cylinder. Experimental results are given for four materials.

List of symbols

\( a \) radius of central plane during straining
\( a_0 \) initial radius of cylinder

\( b \) half height during straining
\( b_0 \) half initial height of cylinder

\( c \) value of \( \sigma \) for \( \delta = 1 \)

\( h \) height of element near central plane
\( h' \) height of element near central plane

\( L \) load for straining with fixed ends

\( m \) strain hardening exponent

\( r \) coordinate

\( R \) radius of curvature of the profile near central plane

\( z \) coordinate

\( \delta \) logarithmic strains
\[ \delta \] effective logarithmic strain
\[ \sigma \] normal stresses
\[ \bar{\sigma} \] effective stress
\[ \phi \] angle
\[ \phi' \] angle
\[ \theta \] coordinate
We consider a bar, compressed in such a way that the radius of the outside profile in the middle plane is $R$ (Fig. 1).

Fig. 1. Equilibrium near the central plane.
For an infinitesimal element: ABCD-EFGH we calculate the equilibrium in direction r. This element is taken as follows: ABCD is a flat plane perpendicular to OM. BCGF and ADHE are spheres with their centres in N' and N'', respectively. ABEF and DCGH are flat planes through OM; EFGH is a sphere with its centre in M.

The equilibrium yields

\[(\frac{\partial r}{\partial r} + \frac{\partial r}{\partial \phi}) h'(r+dr)d\theta - \frac{1}{2} r h r d\theta - (\frac{\partial z}{\partial z} + \frac{\partial z}{\partial r} h) r dr d\theta \sin \theta' + \frac{1}{2} \phi h dr \sin (d\theta) = 0 \quad (1)\]

From symmetry considerations, we find that near the central plane we have

\[\frac{\partial z}{\partial r} = 0 \quad (2)\]

Since \(\phi'\) is small, we have

\[\sin \phi' \approx \phi' = \frac{\phi}{a} \quad (3)\]
\[\cos \phi' = \sqrt{1 - \sin^2 \phi'} \approx 1 - \frac{1}{2} \phi'^2 \frac{r^2}{a^2} \]
\[\cos \phi' - \cos \phi \approx \phi^2 \frac{a^2 - r^2}{2a} \]

Since, from geometrical considerations:

\[h = R \phi - a (\cos \phi' - \cos \phi) \quad (4)\]
\[h' = R \phi - \frac{a}{\phi} (\cos \phi' d\phi - \cos \phi)\]
we derive, substituting (3) into (4), that

\[ h \cdot h' = \rho \left( R - \frac{a^2 - r^2}{2a} \right) \]
\[ h' - h = rd\phi' = \rho \cdot \frac{r}{a} dr \]  

(5)

(2), (3), and (5) in (1) yields

\[ \tau_r \left( R - \frac{a^2 - 3r^2}{2a} \right) + r \frac{d\tau_r}{dr} \left( R - \frac{a^2 - r^2}{2a} \right) - \sigma_\theta \frac{r^2}{a} \frac{\partial}{\partial \theta} \left( R - \frac{a^2 - r^2}{2a} \right) = 0 \]  

(6)

Near the central plane, we will assume uniform strain. If the initial radius was \( a_0 \), we have

\[ \delta_r = \log \frac{a}{a_0} \]
\[ \delta_\theta = \log \frac{a}{a_0} \]
\[ \delta_z = -2 \log \frac{a}{a_0} \]

(7)

Hence

\[ \bar{\delta} = \sqrt[3]{\frac{1}{2} (\delta_r^2 + \delta_\theta^2 + \delta_z^2)} = 2 \log \frac{a}{a_0} \]  

(8)

and

\[ \bar{\tau} = c \bar{\delta}^m = c \left( 2 \log \frac{a}{a_0} \right)^m \]  

(9)

while (7) yields

\[ \tau_r = \tau_\theta \]  

(10)
study (10) into (6), we obtain
\[ \frac{\partial \sigma_r}{\partial r} (R - \frac{a^2 - r^2}{2a}) - \frac{\sigma_r}{a} = 0 \]  
(11)

As \( \sigma_r = \sigma_b \), we find
\[ \overline{\sigma} = \sqrt{\sigma_r^2 + \sigma_b^2 + \sigma_2^2 - \sigma_r \sigma_b - \sigma_b \sigma_2 - \sigma_2 \sigma_r} = \sigma_r - \sigma_2 \quad (\sigma_r > \sigma_2) \]  
(12)

(12) in (11) then gives
\[ \frac{\partial \sigma_r}{\partial r} (R - \frac{a^2 - r^2}{2a}) + \frac{\sigma_r}{a} = 0 \]  
(13)

Separation of variables and integration yields
\[ \sigma_r = -\overline{\sigma} \log (r^2 + 2aR - a^2) + c_1 \]  
(14)

with boundary condition \( \sigma_r(a) = 0 \). We find
\[ \sigma_r = \overline{\sigma} \log \frac{2aR}{r^2 + 2aR - a^2} \]  
\[ -\sigma_2 = \overline{\sigma} \left(1 - \log \frac{2aR}{r^2 + 2aR - a^2}\right) \]  
(15)

For the load we find:
\[ L = \int_0^a 2\pi r \sigma_r \, dr = \pi \overline{\sigma} a (2R - a) \log \frac{2R}{2R - a} \]  
(16)

or, using (9):
Next, we will construct a situation in which the initial diameter at the outer face is maintained, while the outside profile becomes a circle. If the height of the deformed cylinder is \(2b\), the circle has the following properties (Fig. 2). If \(Z\) is the centre of gravity of \(ACBD\), it holds that
\[
\overline{NZ} = \frac{2b^3}{3F}, \text{ in which}
\]
\[
F = \pi R^2 - bR \cos \alpha \quad (18)
\]
in which \(\sin \alpha = \frac{b}{R}\)  \( (19)\)

Further, we have
\[
\overline{NC} = \frac{2b^3}{3F} - R \cos \alpha \text{, hence, substituting (18)}:
\]
\[
\overline{NZ} = a_0 + \frac{2b^3}{3\pi R^2 - 3bR \cos \alpha} - R \cos \alpha \quad (20)
\]

\(\alpha\) follows from
\[
a = a_0 + R(1 - \cos \alpha) \quad (21)
\]
Fig. 3. Undeformed and deformed shape of the cylinder.

Then, application of the Guldin theorem yields

\[(\alpha R^2 - bR \cos \alpha)(a_0 - R \cos \alpha) + \frac{2}{3} b^3 - a_0^2 b = a_0^2 b_0 \]  \hspace{1cm} (22)

Together with (19), (22) permits of a solution of \( R \) by successive approximation, if \( a_0, b_0 \) and \( b \) are given. Together with the values of \( c \) and \( m \), which were independently calculated from a tensile test, the load can now be calculated from (21) and (17).

Experimental verification.

The value of the load \( L \) was measured as a function of the height of the cylinder. The agreement seemed to be very good, except for the brass. This was the only material, however, which had a coarse crystal structure, hence the discrepancy may be due to
anisotropy.

<table>
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<tr>
<th>b (mm)</th>
<th>a (mm)</th>
<th>R (mm)</th>
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<tbody>
<tr>
<td>19</td>
<td>18.2</td>
<td>57.2</td>
</tr>
<tr>
<td>20</td>
<td>17.6</td>
<td>77.7</td>
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<tr>
<td>21</td>
<td>17.0</td>
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<td>22</td>
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<tr>
<td>23</td>
<td>16.0</td>
<td>278.0</td>
</tr>
<tr>
<td>24</td>
<td>15.5</td>
<td>622.0</td>
</tr>
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</table>

Table I. Calculated values of R.

For \( b = 19.2 \) a radius \( R = 97 \) was measured, the theoretical value being in the order of 60. (Table I). Thus, as was to be expected, the surface does not exactly become the shape as indicated in Fig. 3.

Fig. 4 gives the shape of the plates which were placed on either flat side of the cylinder, forcing the radius \( a_0 \) to remain unchanged.

Fig. 5. Shape of plates at either side of cylinder.
Conclusion, prospects and limitations.

We have shown that the Bridgman correction can successfully be applied to inhomogeneous compression of a circular cylinder. This gives the possibility of a reverse procedure, viz. the determination of the material constants $c$ and $m$ by a compression test. In that case, a compression graph is taken and the value of $c$ and $m$ which makes formulae (17) and (22) fit best to the graph are determined.

For industrial purposes, a compression test machine may be developed working on this principle. A large number of theoretical graphs, for different $c$ and $m$ values, should then be given on transparent paper on the same scale as the recorder. Then by simple comparison, $c$ and $m$ can be established.

A limitation of the compression test is that the strains must be kept below $2.15\%$. However, within this range, the compression test has considerable advantages compared with the tensile test and it might be well worth considering the development of a compression test machine for industrial purposes.
Finally, it is interesting to notice that this experiment is essentially a proof that for the materials investigated for moderate strains no Bauschinger effect exists.

**Material data**

St. C 22 0.22% C; P < 0.06%; S < 0.06%, Normalised (perlite + ferrite)

Ms 58°C 58% Cu; 25% Pb; 39.5% Zn. Annealed, 3 hours at 400°C.

Cu (el) 99.92% Cu; 0.05% O₂
Annealed, 3 hours at 500°C

Al. St 51 1% Si; 0.6% Mg
Annealed, 3 hours at 350°C

**Literature**