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The effect of curvature on the wall shear stress distribution in the left main coronary bifurcation

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September 12, 1996

Abstract

Wall shear stress is an important atherogenic factor. In this study the effect of curvature on the wall shear stress distribution in the left main coronary bifurcation is investigated. Therefore three different cases are considered, each with a different curvature. Computational fluid dynamics is used to solve the three-dimensional Navier-Stokes equations. Wall shear stress is visualized by means of contour plots and unfolding the geometry into a flat plane. A more quantitative way of displaying the wall shear stress distribution is carried out with a cumulative distribution function. The results in this study may suggest a decrease in low shear area for increasing curvature. The complexity of the bifurcation does not allow to draw that conclusion without further investigation of flow phenomena in the bifurcation region. Improvement of the computational accuracy and scaling of the distribution function can contribute to more valuable results.

1 Introduction

Atherosclerosis is a major cause of death in the Western world. It is the principal cause of heart disease and stroke. Population studies show that some regions of the vasculature are more predisposed to atherosclerosis than others. Many of these regions, such as the inner curvature of bends, the outer wall of the carotid sinus of the neighborhood of major branch points in the abdominal and coronary vessels, are sites believed or shown to be exposed to fluid dynamic environments that are substantially different from those seen by relatively spared segments. This suggests that hemodynamic forces play a role in the localization of the disease (Caro et al., 1971; Nerem, 1992).

Among preferred sites for atherosclerosis are the proximal portions of the left anterior descending (LAD) and left circumflex (LCX) coronary arteries (see figure 1). Ding et al. (1996); Brinkman et al. (1994); Friedman et al. (1993) show that the predisposition to sudanophilia of the proximal portion of the LAD is correlated with the spatial geometry of the left main (LM) bifurcation. Also was found that geometric features such as the LM length, LAD – LCX branch angle, and the turning angles between the LM and each of its daughters, might be associated with the susceptibility of the vessels. In Caro et al. (1996) the curvature of the branch is examined. It is found that the non-planarity of the branch affects the velocity distribution significantly.

Hemodynamic forces and the location of atherosclerotic lesions are coupled by wall shear stress. A commonly accepted hypothesis is that regions with low or oscillating shear stress
are more likely to develop atherosclerotic lesions. Because wall shear stress can be seen as a physiological important parameter, investigation of geometry effect is carried out by looking at the wall shear stress. Looking at low shear stress of steady flow is useful because areas with oscillating shear stress correlate with regions with low average shear stress. Computational fluid dynamics is a powerful tool for investigating hemodynamics in complex models like arterial branches. Computational solutions give the velocity at discrete points in the model from which shear rates can be calculated. The steady flow simulations in this study are carried out with the finite element package FIDAP. This software is used to solve the full three-dimensional, steady Navier-Stokes equations for the blood flow through the bifurcation. In this study the effect of curvature of the LM on the locations of low shear stress is examined. Three cases can be distinguished, each with a different curvature of the LM. For each case a cumulative wall shear stress function is derived to show differences in shear stress distribution between the three cases.

2 Methods

2.1 Governing equations

The blood flow in the coronary bifurcation is considered laminar and the arterial wall is modelled rigid. In study the simulations are limited to steady flow. Blood is modelled as an incompressible Newtonian fluid with a dynamic viscosity \( \mu = 4 \cdot 10^{-3} \text{ Pa}\cdot\text{s} \) and a density \( \rho = 1 \cdot 10^3 \text{ kg/m}^3 \). The Navier-Stokes equations for an incompressible fluid are:

\[
\rho (\mathbf{u} \cdot \nabla) \mathbf{u} - \mu \nabla^2 \mathbf{u} = -\nabla p + \rho \mathbf{f}
\]

\( \nabla \cdot \mathbf{u} = 0 \) \hspace{2cm} (1)

with \( \mathbf{u} \) the velocity, \( p \) the pressure, \( \mathbf{f} \) the volume force and \( \mu \) the dynamic viscosity. The Reynolds number is defined as:

\[
Re = \frac{\rho U D}{\mu}
\]

(3)

with \( \rho \) the density, \( U \) the inlet velocity, \( D \) the LM diameter. The Reynolds number in the simulations was taken 240 which is in the physiological range.

The boundary conditions can be categorized into essential and natural boundary conditions. The essential boundary conditions prescribe the velocity and the natural boundary conditions specify stress. Essential boundary conditions are applied at the inlet and the wall and natural boundary conditions are applied at the outlet. The boundary conditions are:

1. At the wall, the no-slip condition leads to the following equation for the velocity: \( \mathbf{u} = 0 \)
2. At the inlet the velocity profile is prescribed over the whole cross-section. Blood entering the left main coronary arteries have a blunt velocity profile which is slightly skewed toward the epicardial and LCX side. The exact velocity profile is not known and therefore a flat inlet profile will be used.

\[ u = U, \ v = 0, \ w = 0 \] \hspace{2cm} (4)

with \( u, v \) and \( w \) the velocity in respectively the normal and the two tangential directions and \( U \) the inlet velocity.
3. At both the outlets the flow satisfies the stress-free outflow conditions. This boundary condition is necessary for convergence of the solution of the Navier-Stokes equation without having to specify a velocity profile at the outlet. This boundary condition allows a not fully developed flow at the outlet. The upstream influence of the boundary condition more than one diameter from the outlet is assumed to be negligible (van de Vosse, 1987).

2.2 Numerical method

The numerical method used in this study is based on the standard Galerkin finite element method (Zienkiewicz and Taylor, 1991). A set of non-linear differential equations is converted into a set of non-linear equations using weighing functions and partial integration. This gives the following equations that need to be solved.

\[ [S + N(U)]U + L^T P = F + B \]  \hspace{1cm} (5)

\[ LU = 0 \] \hspace{1cm} (6)

The first equation proceeds from the discretization of the Navier-Stokes equation and the second from the discretization of the continuity equation. The vector \( \mathbf{U} \) contains the velocity unknowns and \( \mathbf{P} \) the pressure unknowns in discrete points. The term \( SU \) represents the viscous forces, \( N(U)U \) the convective acceleration forces and \( L^T P \) the pressure gradient forces. The right-hand side of the first equation consists of the volume forces \( F \) and the boundary forces \( B \). In the solving process of the above set of equations FIDAP uses a Newton-Raphson iteration method to linearize the convective term. This term is replaced by (van de Vosse et al., 1986):

\[ N(U_i^{i+1})U_i^{i+1} = J(U_i^i)U_i^{i+1} - N(U_i^i)U_i^i \] \hspace{1cm} (7)

with \( i \) the iteration number and \( J \) the Jacobian matrix of \( N \). As a stop criterium for the iteration process, the maximal difference between two successive solutions in the same discrete point is used. Because no pressure unknowns appear in the discretized continuity equation, partial pivoting is necessary to solve the velocity and pressure unknowns from the equations. To overcome the problem of partial pivoting, which is very time consuming and has a negative effect on the band structure of the matrix, a penalty function method is used (Cuvelier et al., 1986). This enables the elimination of the pressure unknowns from the discretized Navier-Stokes equation. In this case the discretized continuity equation reads:

\[ LU_i^{i+1} = \varepsilon M_p \mathbf{p}_i^{i+1} \] \hspace{1cm} (8)

with \( M_p \) the pressure mass matrix and \( \varepsilon \) a very small parameter, so the right-hand side of the equation becomes very small. Substituting equation (8) into equation (5) yields the following equation that needs to be solved:

\[ [S + N(U)]U + \frac{1}{\varepsilon} L^T M_p^{-1}LU = F + B \] \hspace{1cm} (9)

The pressure can be approximated by substituting the velocity solution into equation (8). A measure for the wall shear stress is defined in this study as:

\[ \tau_w = \mu \sqrt{\mathbf{D} : \mathbf{D}} \] \hspace{1cm} (10)

with \( \mathbf{D} \) the deformation rate tensor.
### Table 1: Angles and dimensions for the left coronary bifurcation model. The length of the branches is given as multiple branch diameters.

<table>
<thead>
<tr>
<th>Angles [degrees]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LM – LAD</td>
<td>159</td>
</tr>
<tr>
<td>LM – LCX</td>
<td>117</td>
</tr>
<tr>
<td>LAD – LCX</td>
<td>84</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensions [mm]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of LM</td>
<td>4.0</td>
</tr>
<tr>
<td>Diameter of LAD</td>
<td>3.4</td>
</tr>
<tr>
<td>Diameter of LCX</td>
<td>3.0</td>
</tr>
<tr>
<td>Length of LM</td>
<td>2.75D</td>
</tr>
<tr>
<td>Length of LAD</td>
<td>5.88D</td>
</tr>
<tr>
<td>Length of LCX</td>
<td>6.67D</td>
</tr>
<tr>
<td>Radius of curvature (LAD)</td>
<td>42.8</td>
</tr>
<tr>
<td>Radius of curvature (LCX)</td>
<td>39.3</td>
</tr>
<tr>
<td>Radius of curvature (LM)</td>
<td>∞ / 25.0 / 16.7</td>
</tr>
</tbody>
</table>

#### 2.3 Mesh generation

In this study three cases are considered. The geometric parameters of each case are listed in table 1. The branch length is given in multiple branch diameters. The mesh is constructed with quadratic elements which have 27 nodes for the velocity and 1 node for the pressure and its derivatives. Figure 1 shows a top view of the mesh a side view of the mesh with the three different meshes for the LM. Regarding mesh generation the following assumptions were made:

1. The bifurcation region is located in a plane.
2. The LM, LCX and LAD are curved tubes with a circular cross-section. The radii of curvature are constant.
3. The curvature planes are perpendicular to the bifurcation plane.

#### 3 Results

The presentation of wall shear stress in this study uses contour plots in which the branches are cut open and unfolded into a flat plane. The LM is cut open both along the epicardial and myocardial sides and then unfolded. The LAD and LCX are cut along their inner wall and unfolded into a flat plane. Due to the complexity of the geometry, the bifurcation region is distorted during the mapping process. The generated image is only illustrative and does not fully resemble the original geometry. The horizontal axis in the figure indicates the distance from the inlet and the vertical axis gives the circumferential location. Figure 2 show colored contour plots (using linear interpolation) of the wall shear stress in the LCX and LAD for all three cases. The region of interest is the bifurcation area which ranges from 8 to 14 mm distance from the inlet. In the LCX the largest area with low shear stress can be found at the outer wall. There are two other smaller regions which are located on the myocardial and epicardial side. The high
Figure 1: Finite element mesh of the left main coronary bifurcation. The bottom graph shows the three different meshes for the left main (LM).
shear stress is mainly located on the inner wall which is the divider wall. There is also a narrow area visible at 11 mm distance from the inlet on the outer wall. The non-planarity has little effect on the spatial distribution in the LCX as the three contour plots show nearly no difference.

Areas with low shear stress are located on the myocardial and epicardial side in the LAD. When the curvature is increased the area on the epicardial side gets smaller while the area on the myocardial side increases. The high shear stress is found at the flow divider wall.

The effect of curvature on the magnitude of the wall shear stress can be displayed by means of a cumulative distribution function of the wall shear stress. The cumulative distribution function is calculated by dividing the number of nodes which have a shear stress value equal or below a certain level by the total number of nodes.

The cumulative distribution of the three cases is shown in figure 3. In each case the model is divided into a distal (from 8 to 13 mm distance from the inlet) region and a proximal (from 13 mm to the end of the branch) region. Figure 3 clearly shows no difference between the three cases in the distal region for both the LCX and LAD. The LCX has the largest amount of shear stress equal and below 1.0 Pa (approximately 79%) than the LAD (approximately 49%).

The distribution of the region proximal to the bifurcation changes when the curvature is increased. In the LCX the change occurs in the mid range of shear stress (1.0 to 2.5 Pa). At the 1.0 Pa level the percentage of points decreases from 50% in the first case to 44% in the third case. The LAD shows a stronger effect and the greater part of the affected range of shear stress lies below the 1.0 Pa level. At that level the percentage drops from 72% in the first case to 51% in the third case. Also there is a small increase in points with a shear stress below 0.5 Pa.

4 Discussion

The results in this study may suggest a positive effect of curvature on wall shear stress. Although a decrease in the number of points with low shear stress can be seen, it is not possible to determine the hemodynamic factor which could be the cause. The complexity of the bifurcation region does not allow to draw any conclusions from the results without further investigation of flow phenomena in this region.

In this study the effect of geometry on wall shear stress distribution is investigated. A necessary condition is accurate modelling of the in vivo situation. In Karino et al. (1990) a glass model T-junction was made and flow patterns within the model were determined. The sharp angle in the model causes spiral secondary flows and recirculation zones at the entrance of the daughter vessel. After a reattachment point on the wall the flow continues its normal flow pattern. The in vivo geometry was shaped such that there is a sharp curvature of the wall at the flow divider and a gentle round curvature at the bend opposite to it. The study shows a less pronounced secondary flow and recirculation zone in the in vivo junction compared to the model. The wall shear stress distribution is the recirculation zone is very different from the rest of the branch and may affect the development atherosclerotic lesions.

In the left main bifurcation there is a similar situation as described above. The location where the LCX bifurcates from the LM is characterized by a sharp angle in the model, while in vivo the bifurcation is much smoother. The effects described in Karino et al. (1990) can also be found in the model used in the study. Because in this study the bifurcation geometry
Figure 2: Wall shear stress contour plot in the LCX (left) and LAD (right) for case 1 (upper), case 2 (middle) and case 3 (lower).
Figure 3: Cumulative distribution in the LCX (upper) and LAD (lower) in the regions proximal and distal to the bifurcation for case 1 (solid), case 2 (dashed) and case 3 (dotted).
is the topic of interest it is necessary to eliminate flow effects which result from the modelling process.

The geometry of the bifurcation plays an important role in the interaction between the effects of the bifurcation and curvature. The angle between the LM and the LCX is relatively small compared to the angle between the LM and the LAD. Presumably bifurcation effects play the dominant role in the LCX while curvature effects determine the flow in the LAD. Curvature effects will be found more likely in the LAD than in the LCX.

Visualizing wall shear stress by means of a cumulative distribution function gives information about spatial magnitude distribution. In this study the nodal values of wall shear stress were used. Therefore the function depends strongly on the mesh as the nodes are not uniformly spaced over the mesh. Scaling the distribution function with the element size yields a dimensionless distribution function which does not depend on nodal spacing.

In this study the magnitude of the wall shear stress is used. The secondary flow introduces a circumferential component to the axial component of shear stress. The circumferential component may be of importance to the development of atherosclerotic lesions.

5 Recommendations

Studies of flow phenomena in a bifurcation are necessary to obtain knowledge about how geometry affects wall shear stress. A necessary condition is a good defined mesh to prevent mesh artefacts which occurred in this study. This can be done through a mesh refinement which can carried out locally. A different solver can be used to keep computation times acceptable with a fine mesh.

The current definition of the cumulative distribution function needs to be expanded with a scaling factor which makes it dimensionless. A dimensionless function allows comparison of results with different spatial distribution of the nodes. A possible scaling factor can be the element area.

Performing unsteady flow simulations can give insight on how curvature affects time dependent shear stress, which is also considered to be a atherogenic factor. In this study blood is modelled as an incompressible Newtonian fluid. Using a non-Newtonian fluid model may yield results which have more physiological importance. The model used in this study is a simplification of the in vivo bifurcation. Adding flexible walls and coupled heart-vessel motion allow for a realistic simulation of the flow in the left main coronary bifurcation.

References


Z. Ding, T. Biggs, W.A. Seed, and M.H. Friedman. Influence of the geometry on the left main coronary artery bifurcation on the distribution of sudanophilia in the daughter vessels. *In revision*, 1996.


