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Generalised Method of Determining Blank Size in Deep Drawing

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Summary.

It is shown that when deep drawing rectangular products having corner radii and a bottom radius the common graphical procedures used to determine the appropriate blank size can be replaced by analytical relations.

Next it is explained that when describing the blank in terms of a super ellipse the constants defining this curve can be connected with experimental data obtained.

The procedure shows a way towards systematized computer aided calculation of determining blank size and N.C. manufacturing of both drawing tools and blanking tools.

Experiments verify the fair reliability of the method.

1. Introduction.

In the common practice of deep drawing of rectangular products the size of the blank is determined by applying graphical methods based on empirical evidence [1,2,3]. Application of these methods demands considerable skill and some sort of feeling for the geometry of the flow of the material during the drawing process. These procedures have a limited range of validity related to the dimensional ratios of the product and anyhow the accuracy is relatively poor. It may be presumed that the current methods of determining blank size do not guarantee optimal economy in the use of labor and material mainly due to the lack of analytical systematizing. From the latter also arises the impossibility of achieving numerical data by computer routines.

In the present paper it will be shown that when describing both the drawing edge and the edge of the blank in terms of super ellipses the process of deep drawing can be considered being a transformation from the one super ellipse into another. Thus once the rules of the transformation known the blank size can be determined from the geometry of the drawing edge.

The method developed proves to offer apart from its general applicability different advantages

1. the possibility of generalisation of shapes in a quantitative system
2. an outlook for the industrial designer towards systematical research into aesthetic of shapes
3. the possibility of efficient communication between the designer and the manufacturer as the boundaries for economical production in relation to the design can be marked
4. the including of the corner radius of the product in its analytical description, which is of importance for numerically controlled manufacturing both of the drawing tools and the blanking tools
5. the probability of description of the deep drawing process in a generalised plasticity mechanical model.

2. The Super Ellipse and some of its properties.

The function

\[ \left| \frac{x}{a} \right|^\alpha + \left| \frac{y}{b} \right|^\alpha = 1 \]

\[ 0 < \alpha < \infty \]

is called a super ellipse or a hyper ellipse \(|4|\).

Evidently for \( \alpha = 2 \) a circle or an ellipse is represented. When plotting the function in dependence on the value of the exponent \( \alpha \) as shown in fig. 1, it can be distinguished between

1. \( 0 < \alpha < 1 \), describing concave surfaces
2. \( 1 < \alpha \leq 2 \), describing convex surfaces between a rhomb and an ellipse
3. \( 2 < \alpha < \infty \), describing convex surfaces between an ellipse and a rectangle.

For this reason the exponent \( \alpha \) is referred to as the shape factor of the curve.

Restricting ourselves to the case \( \alpha > 1 \) it is obvious that when the shape factor increases more and more a rectangle is approximated where the corner radius is an intrinsic part of the curve. Thus any super ellipse is fully defined by its circumscribed rectangle \( \{a,b\} \) and its shape factor \( \alpha \). These three data can be looked at being the designer's data defining the cross-section of the product wanted.

After transforming eq. 1 to polar-coordinates \( \{r,\phi\} \) and with an eye to

Fig. 1. A family of super ellipses and its circumscribed rectangle.

mathematical simplicity introducing of the new variable

\[ \frac{b}{a} \cot \phi = t \]

the area factor \( q \) being the ratio of the surface areas of a super ellipse
and the circumscribed rectangle can be formulated as

\[ q = \int_0^1 \frac{\alpha}{(\sqrt{\alpha^2 + 1})^2} \, dt \]  

(2)

Fig. 2 shows the results of numerical computation.

By now a comparison can be made between a given super ellipse on the one hand and its corresponding rectangle provided with corner radii \( r_c \) on the other on the condition of equality of surface areas. In fig. 3, the results of numerical analysis have been plotted from which the conclusion is drawn that to any rectangle \( \{a, b, r_c\} \) an appropriate super ellipse \( \{a, b, \alpha\} \) corresponds.

A different way of representation in order to compare a super ellipse to a rectangle is to calculate the local radius of curvature according to standard mathematical routine and expressing this value in terms of its minimum value using the shape factor as a parameter. The result is shown in Fig. 4, which again demonstrates that when the shape factor increases the super ellipse approximates pretty soon to a rectangle provided with corner radii.

Fig. 3. Definition of the aequivalent corner radius of a super ellipse.

Fig. 4. Local curvature of a super ellipse.
Finally the length of circumference of a super ellipse can be calculated. Numerical results in terms of the dimensionless circumference factor \( p \) being the ratio of the circumference of a super ellipse and its corresponding rectangle have been plotted in Fig. 5, as a function of the shape factor.

3. Determination of the Blank Size.

When deep drawing circular products it is common practice to determine the blank size by assuming that the surface area of the blank is equal to that of the product drawn. This method cannot be flatly applied to the drawing of rectangular products as the blank in this case is no longer defined by only one dimension but rather by a shape. The deeper the product is to be drawn the more the shape of the blank approximates a circle.

It now is assumed that the initial shape of a blank in deep drawing can be generalised by super ellipses. Hence a set of three dimensionless quantities can be introduced defining the blank in relation to the dimensions of the product, thus being

- the relative blanklength \( \frac{a_b}{a} \)
- the relative blankwidth \( \frac{b}{b_p} \) \( (3) \)
- the shape factor \( \alpha_b \)

Characteristic values defining the shape of the product on the other hand are

- the slenderness ratio \( \frac{a_p}{b_p} \)
- the relative corner radius \( \frac{r_c}{b_p} \) \( (4) \)
- or the shape factor \( \alpha_p \)
- the relative bottomradius \( \frac{r_p}{b_p} \)
- the relative height \( \frac{H}{b_p} \)

The problem is to correlate each of the quantities eq. 3 to any of the values eq. 4.

Now from Romanowski's work \( |1| \) relevant information is at hand to investigate the correlations mentioned. Generally three cases are distinguished

1. small radii and low drawing ratio
2. big radii and low drawing ratio
3. high value of the drawing ratio.
Analysis shows that in the first two cases the following experimental relations hold

\[ b_b = H + b_p - \frac{4-\pi}{2} r_p - \frac{C_f R^2}{a_p - r_c} \]

\[ a_b = H + a_p - \frac{4-\pi}{2} r_p - \frac{C_f R^2}{b_p - r_c} \]

where

\[ C_f = \frac{\pi}{8} \left[ \left\{ 0.074 \left( \frac{R}{2r} \right)^2 + 0.982 \right\} - 1 \right] \]

and

\[ R^2 = r_c^2 + 2HR_c - 0.86 r_p (r_c + 0.16 r_p) \]

In the latter case where the blank is close to a circle it holds

\[ a_b = D + a_p - b_p \]

\[ b_b = \frac{D(b_p - r_c) + (b_p + H - \frac{4-\pi}{2} r_p)(a_p - b_p)}{a_p - r_c} \]

where

\[ D^2 = \frac{4}{\pi} \left\{ b_p^2 + 2HR_p - \frac{4-\pi}{2} r_c H + \frac{2(\pi-3)}{4-\pi} r_p \right\} \]

Now in order to determine the shape factor of the blank on basis of these results invariancy of surface area is assumed, resulting in

\[ q_b = \frac{b_p^2}{a_p b_b} \left[ p_p a_p (a_p) + 1 \right] \frac{H}{b_p} + q_p - C_b \]

where \( C_b \) is a geometrical factor accounting for the influence of the bottom radius

\[ C_b = \frac{4-\pi}{2} (1 + \frac{b_p}{a_p}) + \frac{\pi}{2} 3\pi \left( \frac{R}{b_p} \right)^2 \frac{b_p}{a_p} \]

In the case of rectangular products with corner radii it holds

\[ p_b = 1 - \frac{4-\pi}{2} \frac{r_c}{a_p + b_p} \]

\[ q_p = 1 - (1 - \frac{\pi}{4}) \frac{r_c^2}{a_p b_p} \]

In the case of super elliptic tools \( p_p \) and \( q_p \) are given in Figs. 5 and 2.
Finally the value of $q_b$ thus calculated from eq. 7 renders the shape factor of the blank when again using Fig. 2.

By means of numerical computation a number of graphs has been produced representing the procedure exposed.

Fig. 6 shows the relative blankwidth as a function of the relative product height, the relative corner radius acting as a parameter. Thus from the data $a_p/b_p$, $H/b_p$ and $r_c/b_p$ known from the product specification the value $b_b/b_p$ is found.

In Fig. 7, the blank slenderness ratio has been plotted as a function of relative product height, both the product slenderness ratio and the relative corner radius being parameters. From this the value $a_p/b_p$ is found from product specifications.

It is remarked that when the drawing height increases the blank approximates a circle $a_p/b_p = 1$.

Finally in Fig. 8, the shape factor of the blank is correlated to the shape factor of the product for different values of the relative blank width.

It is observed that the description of the blank in terms of super ellipses generalises the three different cases distinguished by Romanowski to one single system of calculation, which easily can be represented in a complete set of graphs or tables covering the entire field of technical application.

4. The Maximum Drawing Ratio.

In the case of the drawing of circular products the drawing ratio is defined
Panknin and Dutschke [5] have shown that in the case of rectangular and elliptical products the quantity

\[ \beta = \frac{R_{\text{blank}}}{R_{\text{punch}}} \]

represents a reliable criterion for drawability.

When using super elliptic blanks and tools and introducing the area factor \( q \) according to eq. 2 the drawing ratio becomes simply

\[ \beta_1 = \sqrt{\frac{a_b b_b q_b}{a_p b_p q_p}} \]  \hspace{1cm} (9)

This offers a possibility to relate the drawing ratio of rectangular products to that of circular products where \( a_b = b_b = r_b \) and \( a_p = b_p = r_p \), whilst \( q_b = q_r = \pi/4 \). All factors governing the process must then be related to an equivalent diameter of the rectangular product, being

\[ D_e = \sqrt{4 \frac{a_b b_b q_b}{a_p b_p q_r}} \]  \hspace{1cm} (10)

Experimental evidence shows that the maximum drawing ratios determined this way are comparable with the values observed when deep drawing circular products.

5. Experimental Verification.

Though in fact the eqs. 5 and 6 represent experimental data in a condensed form and have been used to show that the blank can be defined in terms of super ellipses, additive experiments have been performed. Romanowski states that his graphical procedures are valid up to the condition \( a_p/b_p < 2 \).

For this reason super elliptic tools have been made defined by \( a_p = 50 \text{ mm} \), \( b_p = 25 \text{ mm} \), \( r_p = 12.5 \text{ mm} \) for the drawing edge and by \( a'_p = 48.5 \text{ mm} \), \( b'_p = 23.5 \text{ mm} \), \( r_p = 10 \text{ mm} \) for the drawing punch.

The shape factor of the super ellipse has been chosen to amount to \( \alpha_p = 5 \). Experiments have been carried out using brass sheet Ms 72 with a thickness of 1.5 mm. For different values of the drawing height \( H \) the blank size has been calculated in terms of \( a_b, b_b \) and \( a_b \) or \( q_b \).
The experimental results are shown in the next figures 9, 10, 11 and 12 from which the fair reliability of the method is evident. The slight differences can easily be explained by dimensional inaccuracies in the experimental super elliptic tools which have been made by means of template copying milling and hand finishing.

Finally the drawing ratio has been calculated according to the definition eq. 9 as a function of the height of the product as shown in fig. 13. The maximum drawing ratio based on the criterion of necking proved to be in between the value 1.96 and 1.98.
| References. |
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