Stiffness characteristics of externally pressurized gas bearings

Citation for published version (APA):

Document status and date:
Published: 01/01/1990

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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STIFFNESS CHARACTERISTICS OF EXTERNALLY PRESSURIZED GAS BEARINGS

J. M. Wang and P.H.J. Schellekens
WPA-report 0960

presented in the ASPE Annual Meetings
Rochester, NY, USA, Sept. 22 - 27

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September, 1990
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ABSTRACT

In this paper the stiffness characteristics of externally pressurized gas (EPG) bearings such as the stiffness distributions, maximum stiffness locations and stiffness capacities are discussed. It is found that the different designs present different stiffness distributions and different locations of maximum stiffness. This implies that the optimum result for EPG bearings can be reached by a proper design. EPG bearings can be designed with high stiffness, high load, high damping and low gas consumption. The bearing stiffness capacity is constant for the constant load difference so that the increase of stiffness in a region of working gap heights will cause the stiffness drop in the others. The overall stiffness increases with the increase of load. The accurate stability criterion of the EPG bearings is determined by the dynamic natural frequency or dynamic natural stiffness due to the stiffness characteristics of such bearings. Furthermore, in the design of EPG bearing systems, one must take care of the connection and structure stiffness, otherwise the overall stiffness will have a great loss.

Presented in the ASPE Annual Meeting,
Rochester, USA, September 22-28, 1990
INTRODUCTION

Externally pressurized gas (EPG) bearings have their applications in the field of precision engineering, not only as motion support elements but also as sealing elements, leadscrew, coupling and actuators due to their qualified characteristics, such as low and constant friction coefficient, low rate of wear, absent mechanical noise and error-averaging etc. These characteristics meets the basic requirements in precision design. In an application, the design of the EPG bearings involves all the disciplines integrated in the precision engineering. The bearing design is not only the hydrodynamics design but also the design of the bounded mechanical bodies.

The stiffness of EPG bearing systems govern the stability limits and determine the grade of accuracy. Much research has been done on enhancing static stiffness, as summarized in 1 and the investigation of the dynamic stiffness and damping properties. The stiffness characteristics as whole has rarely concerned, at least in the publications, which is the motivation of this paper.

In this paper, stiffness, stiffness capacity, dynamic natural stiffness and stiffness loss are discussed. It can be concluded that those aspects are important in order to get better behaviours of machines. Authors will be very pleased if the information presented could help the application of EPG bearings in precision engineering.

COMPENSATION METHODS AND STIFFNESS OF BEARINGS

The compensation methods of EPG bearings can generally be divided into three ways, 1). the fixed restriction compensation (fig. 1a and
which are the basic designs in which only gap height changes with load change in a working condition. 2). the variable-restriction compensation (fig. 1b and 1d to 1h), in which there is at least one variable restriction. and 3). the external displacement compensation (Fig. 2), in which an external displacement device is employed to generate a displacement with the opposite direction to the bearing displacement. Eventually, the displacement of the machine is minimized, even zero-machine displacement can be achieved.

![Diagram of restriction elements]

- || : changeable restrictor; ⊨ : fixed restrictor,
- ⊨ : bearing gap controlled restrictor, \( P_s \) : supply pressure,
- \( P_{d1}, P_{d2} \) : downstream pressure, \( P_e \) : Environmental pressure

Fig. 1. Some possible arrangements of the restriction elements

The stiffness \((S)\) of an EPG bearing film is defined as the derivative of the load \((W)\) with respect to the bearing.
Fig. 2. A EPG bearing with an external displacement device

displacement (e), i.e. \( \frac{dW}{de} \). There is a fixed relationship between the bearing displacement and the initial working gap height \( h_0 \): \( e = h_0 - hw \) so that the stiffness is always expressed as \( S = - \frac{dW}{dhw} \). The load is the integral of pressure distribution over the bearing area (A):

\[
W = \int P \, dA,
\]

so that stiffness becomes that \( S = \int \frac{dP}{dhw} \, dA \). A one dimensional pressure distribution for circular bearing pads with a center inlet restrictor is used to illustrate the stiffness characteristics, in which pressure distribution can be found by solving Reynolds equation, i.e.

\[
P(r) = [p_d^2 - (p_d^2 - p_e^2) \frac{Ir}{IR}]^{0.6}
\]  

where

\[
Ir = \int_{r_0}^{r} \frac{dr}{rh^3}, \quad IR = \int_{r_0}^{R} \frac{dr}{rh^3}
\]  

\( P_e \) : Environmental pressure, \( r_0 \) : inlet restrictor radius

\( R \) : bearing pad radius

As an example, we take gap shape function as \( h = C \, r^n \), then we have
\[ I_r = \begin{cases} C \ln(r - ro) & \text{when } n = 0 \\ C (r-ro)^{-3n/(-3n)} & \text{when } n \neq 0 \end{cases} \]

\[ I_R = \begin{cases} C \ln(R - ro) & \text{when } n = 0 \\ C (R - ro)^{-3n/(-3n)} & \text{when } n \neq 0 \end{cases} \]

Therefore, the ratios of \( T_r/I_R \) become

\[ \frac{I_r}{I_R} = \frac{\ln(r-ro)}{\ln(R-ro)} \quad \text{when } n = 0 \]

\[ \frac{I_r}{I_R} = \left(\frac{r-ro}{R-ro}\right)^{-3n} \quad \text{when } n \neq 0 \]

In fact, \( n = 0 \) implies that a bearing has parallel gap shape, \( n > 0 \) is for divergent gap shape and \( n < 0 \) is the convergent gap shape. It can be seen that the stiffness distribution of three types of bearings will be similar.

Most designs employ combined gap shapes: a partial convergent shape in inner part and a partial parallel part in outer part in order to get optimized properties. Instead of the simple power shaped gap, the commonly used gap is the conical shaped gap which can be simple conical, bi-conical and even multi-conical shaped gap\(^2\), generalized conical shaped gap is illustrated in Fig. 3. The gap shape function can be expressed as the follows:

\[ h_l = r_{l1} h_{l1} + \sum_{i=1}^{n} h_{l1+i} \quad \text{[6]} \]

where

\[ r_{l1} = (r - rl+1)/(rl - rl+1) \quad \text{[7]} \]

with \( h_{ln} = hw \) and \( r_n = R \)

Hence, the corresponding parameters \( I_r \) and \( I_R \) read as

\[ I_r = \ln(1+ hh rr) + 2hh rr/(1+hh) - hh^2 rr(r+ro)/2(1+hh)^2(R-ro) \]
\[ I_R = \ln(1+hh) + 2hh/(1+hh) - hh^2(R+ro)/(2(1+hh)^2(R-ro)) \]  

where 

\[ hh = \frac{hV}{\Sigma h_i} \]

\[ rro = \frac{(r-ro)/(R-ro)} \]

Fig. 3. The illustration of generalized conical shaped gap

From these expressions, one can see that the \( I_R/I_R \) depends on the working gap height so that such gap shapes will have a modification of bearing stiffness and load capacity, which can be seen in Fig. 5.

In the bearings with the variable-compensation, extra elements are introduced, the empirical expression of \( I_R \) and \( I_R \) can not be easily found, but it is sure that the term of \( I_R/I_R \) depends on both the working gap height and/or the pressure distribution. We can treat them as a combination of two bearings: one is the working gap-dependent bearing - a bearing with the fixed compensation with the stiffness \( (S_b) \), another is the equivalent gap height\( (h_e) \)-dependent bearing with the stiffness \( (S_e) \). Then, the stiffness of the bearings with variable compensation \( (S) \) is expressed as

\[ S = \frac{S_b S_e}{(S_b - S_e)} \]
In the bearings with the external displacement devices, the stiffness is also a combination of two stiffness: one is the bearing stiffness, $S_b$, and another is the equivalent stiffness of the external devices, $S_e$. Because these two stiffness connected in series have the opposite sign, the overall stiffness of the bearings with external devices has the same expression as the bearings with variable compensations.

Some possible expressions of the equivalent gaps are listed in Table 1.

<table>
<thead>
<tr>
<th>Flexible Surface</th>
<th>$h_e = h_{vl}(P)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changeable Inlet Restrictor</td>
<td>$h_e = h(P_d)$</td>
</tr>
<tr>
<td>External Displacement</td>
<td>$h_e = h(W)$</td>
</tr>
<tr>
<td>Or</td>
<td>$h_e = h(hw)$</td>
</tr>
</tbody>
</table>

**STIFFNESS CAPACITY AND STIFFNESS DISTRIBUTIONS**

In order to understand the inside of the characteristics of bearing stiffness, a term called stiffness capacity is introduced, which is defined as the integral of stiffness distribution over the range of the working gap height, i.e.

$$Sc = \int_{h_{\min}}^{h_{\max}} \frac{dW}{dh} \, dh = W_{\max} - W_{\min}$$  \[[11]\]

This formula implies that in order to increase the stiffness in all the working gap heights the load must increase. If load difference does not increase, the stiffness increase will decrease the range of
the working gap height. It reveals that the infinite stiffness bearings with the constant supply pressure could have negative stiffness counter parts. In general, it may be stated that the range of the working gap will decrease or load difference should increase if the absolute value of stiffness is increased.

From the discussions on the stiffness concept, it is known that bearing stiffness depends on two parts: 1). the (inlet) restrictor; and 2). the bearing surface shape. These two parts present quite different contributions to the stiffness, as illustrated in Fig. 4, Fig. 4a is the stiffness distribution of the stiffness from the inlet restrictor and the stiffness from the gap shape. Fig. 4b is two possible combinations of these two, which shows that the different stiffness distribution can be designed according to the requirement.

The maximum stiffness from the inlet restrictor is located in the middle of the working gap height, normally between 10 to 20 μm and the exact position depends on the inlet restrictor dimensions. For the simple power shaped pads, stiffness distribution against the normalized working gap height (hw/hwmax) does not change. The change of stiffness capacity only shifts the absolute location of the maximum stiffness. The higher the working gap height, the poorer the bearing damping. The bearing load efficiency (the ratio of the working load over the maximum load) in this region is also smaller.

The stiffness distribution from the gap shape part increases with the decrease of the working gap height for the convergent shape. The maximum stiffness is located in the smallest working gap height. Optimum design of a convergent gap shape can have higher stiffness over whole range of working gap height, of cause, in which the stiffness capacity also increases.
Fig. 5. Some Possible Stiffness Distribution of the Two Opposite Arranged Pads

Bearing Displacement (um)

Overall Stiffness (MN/m)

tri-conical shaped pad
parallel shaped pad

tri-conical shaped pad
parallel shaped pad
The combination of more bearings will generally change the stiffness distribution. If they are arranged in the same side, the over-all stiffness is a simple summation of all pad stiffness. However, the stiffness distribution and the location of maximum stiffness may be different from case to case, due to the working gap height and bearing dimensions of those pads may be different.

If two pads are opposed, the stiffness is also the summation of each stiffness. Two typical examples of the stiffness distributions of two opposite pads are shown in Fig. 5. The initial gap height could change the distribution of the combined stiffness of two opposite pads.

DYNAMIC NATURAL STIFFNESS

The dynamic stiffness of EPG bearing films is frequency-dependent. The transfer function of the bearing film stiffness can be identified by two ways: 1). through solving the time-dependent Reynolds equation and 2). by the approach of the element model. The transfer function of the bearing films with the fixed compensations can be approximated by a function with one zero and two poles. In relative small gap heights, the transfer function reduces to the function with one zero and one pole. The transfer function for active bearings, at least, will be one zero more, i.e. two zeros and two poles.

For better understanding, a new term called dynamic natural frequency is introduced to judge the stability of the bearing system by reviewing the close-looped servo model of EPG bearings. In the closed-loop of a single pad, the transfer function of the main loop is the frequency response of a mass supported by this bearing, $M(j\omega)$.
\[ S(j\omega) = -\frac{1}{M\omega^2}\{\text{Re}[K(j\omega)] + \text{Im}[K(j\omega)]\} \tag{12} \]

encircles the point \((-1,0)\) in the complex plane, which implies that

\begin{align*}
\text{Re}[K(j\omega_1)/M\omega_1^2] &> 1 \quad \text{when } \text{Im}[K(j\omega_1)] = 0^+ \tag{13} \\
\text{Re}[K(j\omega_1)/M\omega_1^2] &< 1 \quad \text{when } \text{Im}[K(j\omega_1)] = 0^- \\
\text{or} \\
\text{Re}[K(j\omega_1)/M\omega_1^2] &\neq 1 \quad \text{when } \text{Im}[K(j\omega_1)] = 0 \\
\text{Im}[K(j\omega_1)] &> 0 \quad \text{when } \text{Re}[K(j\omega_1)/M\omega_1^2] = 1 \\
\end{align*}

i.e.

\[ \{\text{Re}[K(j\omega_1)/M]\}^{1/2}/\omega_1 \neq 1 \quad \text{when } \text{Im}[K(j\omega_1)] = 0 \]

\[ \text{Im}[K(j\omega_1)] > 0 \quad \text{when } \{\text{Re}[K(j\omega_1)/M]\}^{1/2}/\omega_1 = 1 \]

where \(\omega_1\) : a cross-over frequency,

\(\text{Re}\) : real part of bearing stiffness

\(\text{Im}\) : imaginary part, \(\text{Im}[K(j\omega)]\) related to system damping;

Therefore, the stability of a bearing system is governed by the term, \(\{\text{Re}[K(j\omega_1)]/M\}^{1/2}/\omega_1\), or say whether or not the resonant frequency determined by real part of stiffness in the cross real axis point is damped. One can see that the natural frequency is no longer determined by the static stiffness for the bearings with frequency-dependent stiffness. The term, \(\{\text{Re}[K(\omega_1)]/M\}^{1/2}\), is defined as the dynamic natural frequency and \(\text{Re}[K(\omega_1)]\) is defined as dynamic natural stiffness. Some typical examples are given in Fig. 6 in which the left figure is the stable example because the plots do not encircle the the point(-1,0). The right side figure are the unstable
Fig. 6. Illustration of Dynamic Natural Stiffness and Dynamic Natural Frequency
examples. The curve which passes through the positive real plane shows a bearing with the negative dynamic natural stiffness.

The frequency dependent stiffness for one constant spring $K$ is a constant, so

$$\text{Re}[K(j\omega)] = K, \quad \text{Im}[K(j\omega)] = 0 \quad [15]$$

Therefore, the product plot reads $- (K/m/\omega^2)$ so that the stability criteria become that

$$K/M\omega_1^2 \neq 1 \quad \text{due to } D = 0 \quad [16]$$

i.e.

$$(K/M)^{1/2}/\omega_1 \neq 1 \quad \text{due to } D = 0$$

This result show that in the system with frequency-independent springs the stability criteria are determined by the (static) natural frequency which is the basic concept of the vibration theory.

**STIFFNESS LOSS**

Stiffness of EPG bearings, or generally speaking any kind of fluid film bearings, will have great loss when a weak connection is used, especially in the cases of higher film stiffness. The bar connection is a common "element" to connect the pad to a structure, such as carriages. From the Hooke law, $\sigma = E \varepsilon$, for axial loaded bars the stress is simply expressed as $\sigma = F/A$ and the strain can be expressed as elongation over the original length, $\Delta L/L$, therefore the stiffness expression reads

$$S_b = AE/L = \pi d^2 E/L \quad [17]$$

where

$A$ : section area of bar $(m^2)$, $L$ : bar length (m), $F$ : Force (N)

$E$ : Yong's modulus of materials, for steel $E = 208 \text{ GN/m}^2$
If a bar is made of steel with 3 mm radius and 18 mm length, the stiffness of this bar is 312 MN/m. If a bearing with stiffness of 100 MN/m is connected to a structure by this bar, the overall stiffness of the system is only 75 % of the bearing stiffness.

Another common used element in the connection is the ball for self-adjustment. This arrangement creates a line contact between the spherical and the conical surfaces. The general formula on the deformation of line contact can be adapted to the situation by using an effective length and effect load. The deformation of the line contact reads

\[ \delta_k = 4.05 \times 10^{-5} \frac{Q_{\text{ef}}^{0.925}}{L_{\text{ef}}^{0.85}} \text{ [mm]} \]  

so that stiffness becomes

\[ S_e = 120 (Q \cos \phi)^{0.075} (r \sin \phi)^{0.85} \]  

where

- \( Q \): effective load, \( Q_{\text{ef}} = Q \cos \phi \) [N]
- \( L_{\text{ef}} \): effective contact length, \( L_{\text{ef}} = 2\pi r \sin \phi \) [mm]
- \( \phi \): the angle between the pad normal direction and conical surface normal direction

If a "ball" has radius 6.35 mm and the conical angle 45° with 500 N load, for example, the stiffness is about 668 N/\( \mu \)m. If a "ball" with 4 mm radius the stiffness is only 451 N/\( \mu \)m.

Other part of connections will also bring the stiffness loss, e.g. the connecting plate or beam of two or more bearing pads in a machine. The overall stiffness is not only the bearing film stiffness in most cases, since the bearing stiffness is not so small in comparison with the connections. It is easier to enhance the connection stiffness then to increase the bearing stiffness.
CONCLUSIONS

From the discussion above, we can conclude that different types of bearings show quite different stiffness characteristics. In an application, more aspects should be taken into account. The shape modified bearings have the maximum stiffness in the low working gap height which is also the region of high load capacity, high damping and low mass consumption. By this good characteristics we can make better use of the bearings, with better performance at low cost.

The dynamic natural frequency should be used to determine the bearing stability criteria in order to get a better design of bearing system. One important conclusion is that only if the dynamic natural stiffness becomes negative, the bearing system is instable.

In addition, with the increase of bearing film stiffness, it is necessary to take care of the stiffness of connection elements such as self-adjustment ball, bar connection etc. in order to ensure that bearing system stiffness increases.

ACKNOWLEDGEMENTS

The research is supported by the Boogers Technology in Precision Engineering and Metrology Systems and Mitutoyo Netherlands Precision B.V.

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