A theoretical model of mixing between hydraulically interconnected channels
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Published: 01/01/1967

Document Version
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

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• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):
A Theoretical Model of Mixing between Hydraulically Interconnected Channels.


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Summary.

In the present article a theoretical model is presented for the calculations of flow, temperature and void distribution in a multi-rod fuel assembly of a nuclear reactor. Starting from the model for intra-cell mixing, a general model is obtained by adding mixing terms describing the hydraulic interactions between an arbitrary number of open, parallel channels with arbitrary specific power distribution.

The theory is compared both with the one under development at the UKAEA Establishment at Winfrith, and with experiments carried out in the Eindhoven Laboratory. A short description of the experimental equipment and a summary of some of the results is presented.
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1. Introduction.

In nuclear reactors for power production the fuel elements most often consist of clusters of fuel rods cooled by a flowing medium. In many cases the coolant can be present in two phases, sometimes under normal operating conditions but also in accident conditions. The specific power of a nuclear reactor, or the power per fuel rod, or per flow channel, is often limited by the maximum permissible heatflux on the heat transmitting fuel rod surfaces. Usually the value of that heatflux is, somewhat arbitrarily, put at 60 to 70% of the so-called burn-out heatflux, i.e. the heatflux at which under the specified operating conditions of coolant flow, pressure, temperature and quality, the fuel rod can fail. In designing a reactor it is required to predict with fair accuracy the value of the burn-out heatflux and to argue the value of the burn-out ratio, that is the ratio between the burn-out heatflux and the maximum heatflux present in the reactor. To this purpose theoretical as well as experimental means can be made use of. For a theoretical prediction one could start from a burn-out correlation for a single channel and try to establish the distribution of liquid and vapour flow and temperature over the set of hydraulically interconnected flow channels. This approach permits a certain power distribution over the coolant channels to be taken into account as should in fact be done in the case of nuclear reactors.

In the experimental approach one could perform measurements on scaled down versions of the fuel assembly under consideration. Although such experiments must be undertaken anyway in order to
support the theoretical approach, they may be unsatisfactory to
the reactor designer both cost-wise and from the point of view of
uncertainty in the scaling-up process to actual dimensions, espe-
cially so as regards the accounting for radial power distributions.

The purpose of the study undertaken in the Laboratory for Heat
Transfer and Reactor Engineering of the Technological University
of Eindhoven, The Netherlands, is to establish a theoretical means
of predicting the flow- and property distributions in a fuel rod
bundle with an arbitrary power distribution and to verify the theory
by experiments. To this end, the theory presented in this article
is compared with experiments carried out in air-water and in steam-
water systems. Initial experiments deal with a 2-channel geometry,
both at low (1-10 atm.) pressure and at high pressure (140 atm.).
In an extension of the programme, multi-channel geometries at high
pressure (140-300 atm.) may be tested in our experimental 1 MW/150
atm. and 3MW/300 atm. loops.

2. Description of the experimental equipment.

2.1. The atmospheric air-water loop.

The arrangement of the loop is indicated in fig. 1. Water, taken
from a constant level supply tank, is pumped through a venturi
to the 2 parallel channel test section. At the end of the test
section, an air-water separator is connected to each channel,
and the air and the water mass flows from each separator can be
measured directly. The pressure in the two separators is always
equalized. Air is taken from a $20.10^5$ Nm$^{-2}$ air supply, via a reduction valve, an air filter and an inlet venturi. The injection of air into the test section could either be symmetrically at the bottom of the 2 channel test section, or asymmetrically into one of the two channels at 4 different levels along the channel. The symmetrical mode of injection was used to check the geometrical symmetry of the two parallel channels. The channels could be shown to be hydraulically identical within 1%.

The test section itself was made from plexi-glass, so as to allow visual observation. Air injection and pressure taps were provided at 4 axial positions, so that axial as well as radial pressure profiles could be determined under various up-stream conditions. The shape of the test section was such as to hydraulically simulate a part of a multi-rod fuel bundle in a nuclear reactor. In fact, the cross section represents two adjacent hydraulic channels in such a geometry, with an open connection between them (see fig. 1). As indicated above, all relevant air and water flow as well as absolute pressures and pressure difference measurements could be taken. The accuracy of air flow and pressure measurements was 3-4%. In addition direct void fraction measurements were taken by means of a Cs-137 γ-ray attenuation unit. Radial traverses of each channel could be made at any level along the axis of the test section. By integrations of local void over the cross section of a channel, the average local void fraction could be determined with an
accuracy of 10 to 20% relative void in the range of void fractions from 0.40 to 0.80. A more complete description of the experimental set-up has been given in ref. 1.

2.2. The 150 atm. steam-water mixing set-up.

The high pressure water loop (design pressure 175 atm., operating pressure 150 atm.), built from stainless steel type 304 and 347 contains in the primary circuit a reactor vessel, a 45 m³/hr channeled rotor pump, a heat exchanger, a preheater and a pressurizer. The very high quality process water is continually cleaned up by means of ion exchangers. The loop has been designed for a maximum power input of app. 1.2 MW by means of a 10-70 V direct current power supply. The loop is equipped with the normal measuring equipment for flow, temperature, pressure, pressure differences, etc. For a more detailed description, see ref. 2.

In the frame work of the mixing programme, a 2-parallel channel test section was designed for installation in the reactor vessel. The section is shown schematically in fig. 2.

It consists in principle of two square channels separated by a stainless steel plate with a slit over a part of the length giving a hydraulic connection between the channels. The total length of the test section is 800 mm. The walls of the two channels are heated with different heat fluxes. At the ends of the heated part of the test section sensors can be positioned consisting of thermocouples, pitot tubes, static pressure tappings
and void gauges. The void fraction can be measured at two locations at the end of the two channels, using the impedance technique as developed in this laboratory (ref. 3). By means of this technique it then is possible to make heat balances even in subcooled boiling conditions and to establish the degree of mixing. More refined techniques will probably be necessary for separating the contributions of cross flow and diffusion to the mixing process. The experimental programme was started by checking and calibrating the void fraction measuring technique. For the first tests the ratio of the heatfluxes of the two channels has been chosen as 1.3. This means at a total channel power of 250 kW a heatflux of respectively 225 and 306 W/cm². The width of the slit is 5 mm while it is open over the whole length of the heated part. The thickness of the separating stainless steel plate is 4 mm. Concerning the operating conditions, the tests have been started with a flow velocity at the inlet of both channels of around 5 m/sec, giving a total mass flow of $10^4$ kg/h. The inlet temperatures are in the range of 270°C to 300°C.

3. Experimental results of air-water loop.

As mentioned the test section was designed such that air could be injected into channel 1 at 4 different levels (see fig. 1). The amount of injected air was varied. Results of these experiments are given in figs 3 and 4. On the vertical axis in fig. 3, the difference has been plotted between the measured outlet mass flow of water in channel
2 and the outlet mass flow of water which would have been measured when no air was injected (equal to half of the total inlet mass flow of water). This difference has been plotted as a function of the total injected mass flow of air for the 4 different levels of injection. The corresponding curves for the outlet mass flows of water of channel 1 are found by reflecting the plotted curves relative the horizontal axis. Similar results for the outlet mass flows of air are given in fig. 4. In this figure the mass flow of air at the outlet of channel 1 and 2 have been plotted vertically. For making an analysis of these results the assumption was made that the mass flows of air and water, at a distance z above the level of injection are equal to the mass flows of air and water measured at the outlet of the test section when air had been injected at a level situated at a distance z below the outlet of the test section. This assumption was verified experimentally by means of void fraction and axial pressure drop measurements. By making the above assumption the outlet mass flows of air and water in figs 3 and 4, now also can be read as a function of the distance above the lowest level of injection. In fig. 5 the mass flows of air have been plotted in this way. Horizontally the distance above level I has been plotted logarithmically, while on the vertical axis the mass flows of air in channel 1 have been plotted. From fig. 5 it becomes clear that the mass flow of air as a function of the logarithmic value of the axial coordinate can be represented by a straight line for any injected mass flow of air at level I. In extrapolating the curves towards higher values of the axial coordinate a point is found where the mass flows of air are equal in both channels. Connecting
these points, which indicate the theoretically required test section length in order to get complete mixing, a curve is found that gives the theoretically required test section length as a function of the mass flow of air injected at level I into channel 1. In fig. 5 it is shown that the required test section length for complete mixing is, for low values of the injected mass flow of air, decreasing with increasing mass flow of air. At higher mass flows of air, the inverse is true: the required test section length for complete mixing increases with increasing mass flow of air. At very high injected mass flows of air the behaviour is similar to that at the very low mass flows. It is interesting to note that in the latter region ($Q_{air}$ larger than $1.25 \times 10^{-3}$ kg/sec.) the extrapolated straight lines show one common point of intersection. This is also the case for the two other regions, e.g. $0.25 \times 10^{-3}$ kg/sec. $< Q < 1.25 \times 10^{-3}$ kg/sec. and $Q_{air} < 0.25 \times 10^{-3}$ kg/sec.

By looking at the figs 3 and 4 the case of air injection at level I the same three regions can be distinguished. At very low injected mass flows of air all the injected air stays in channel 1, which corresponds, according to fig. 5 with a nearly infinite mixing length. At somewhat higher mass flows of air, part of the injected air crosses over to channel 2; this means that the required test section length for obtaining complete mixing decreased. Then at injected mass flows of air from $0.20 \times 10^{-3}$ to $1.50 \times 10^{-3}$ kg/sec. The curves for channel 1 and 2 diverge again, which means an increasing mixing length corresponding with the results of fig. 5.

Finally at injected mass flows of air higher than $1.50 \times 10^{-3}$ kg/sec. the curves converge again.
By plotting the experimental results in the "weighted mean velocity-average volumetric flux density plane" proposed by Zuber and Findlay (ref. 4), the same three regions could be distinguished. According to Zuber and Findlay the relationship between the two quantities for a fully established flow profile and for a two phase flow system in which a change of phase does not occur, is a straight line. Through the plotted data, fig. 6, three lines could be drawn with intersections at a injected mass flow of air of about $0.05 \times 10^{-3}$ and $1 \times 10^{-3}$ kg/sec. corresponding with the values mentioned before. This means that the different behaviour of the mixing process in the three regions corresponds with a change in flow profile probably from bubbly to slug and from slug to a semi-annular flow. The slug flow regime should correspond to the region where the required test section length for complete mixing increased with increasing injected mass flow of air.

Besides mass flow measurements, also void fraction data along and across the channel have been taken. The purpose of these measurements was to determine the volumetric air and water distribution in channel 1 and 2 at different levels. The levels were chosen as follows:

1. $260.0 \times 10^{-3}$ m. above level I ;
2. $35.0 \times 10^{-3}$ m. above level II ;
3. $35.0 \times 10^{-3}$ m. above level III ;
4. $180.0 \times 10^{-3}$ m. above level IV .

As has been mentioned earlier, the experimentally obtained void fraction data were recorded on a continuously writing recorder. Hence, by assuming the void fraction distribution to be symmetric about Y-axis, it has become possible to plot the void fraction distribu-
tion over the cross section at a certain level and at a certain amount of injected air, an example is given in fig. 8. From similar figures it can be clearly seen in what way the void distribution changes with the amount of injected air and along the test section. From the recorder graphs the mean void fractions for channel 1 and channel 2 were obtained by an integration procedure. For two positions the void fractions have been plotted in fig. 8 as a function of the total mass flow of air injected at level I. In section 4 these data are compared with those predicted by the HAMBO programme (ref. 5). Finally, also pressure data have been taken. In fig. 7 the measured pressure difference between channel 1 and channel 2 is plotted at the levels II, III and V with air injected at level I. The radial pressure difference has been taken positive when the static pressure in channel 1 was higher than the static pressure in channel 2 at the same level. As can be concluded from fig. 7, the pressure in the bottom part of channel 1 is higher than that in channel 2, while in the upper part of the channel the opposite is true. Also the results of these experiments are compared with the results of HAMBO-programme in section 4.

4. The HAMBO hydraulic model.

The experimentally obtained mass flow, void fraction and pressure drop data are compared with those predicted by a computer programme called "HAMBO", ref. 5. This comparison became possible thanks to the kind cooperation of Mr. R.W. Bowring, under whose direction this computer programme is being developed at the Atomic Energy Establishment in Winfrith, England.
The purpose of HAMBO is the subchannel analysis of the steady state hydraulic and burn-out characteristics of a multirod cluster assembly, cooled by boiling water in a vertical up-flow.

In the HAMBO model the following effects are included:

a) Turbulent mixing and shear between subchannels;
b) The effect of subcooled void on pressure drop;
c) Radial cross flow pressure drop with radial cross flow;
d) Pressure losses at grids;
e) A slip ratio expressed in terms of the local quality with the coefficients given as input data;
f) The subchannel friction factor is taken as proportional to \( \text{Re}^{-m} \) where \( \text{Re} \) = Reynolds Number and \( m \) is a variable defined in the input data.

In this programme the axial and interchannel variations of mass velocity, local enthalpy and pressure drop are computed from the equations of mass, energy and momentum conservation, by taking into account turbulent mixing effecting heat and momentum exchange between subchannels. The subchannels must be chosen so that their internal turbulent mixing length is short compared to the mixing length for interaction between adjacent subchannels. Average conditions are assumed to prevail in each subchannel. In the programme the channel length is divided up into a number of intervals. The calculations for obtaining mass velocity, pressure drop and void fraction data in a certain interval are carried out by means of an iteration procedure, based on guessing an interval exit mass velocity. In doing so the pressure difference between the subchannels can be calculated. Once having calculated this pressure difference a new
exit mass velocity can be estimated from the error in the pressure balance. This can be done until there is a pressure balance between the corresponding subchannel intervals.

The mass-, heat- and momentum balance equations for each subchannel and for each interval of length can be formulated as follows:

1) Mass flow leaving interval = mass flow entering + cross flow to adjacent channel;

2) Exit heat flow = inlet heat flow + heat carried by cross flow fluid + heat carried by turbulent mixing + heat addition;

3) Exit pressure = inlet pressure + frictional component + acceleration component + hydrostatic head + momentum exchange between subchannels (drag) + spacer pressure drop.

As already mentioned, the simultaneous equations for the subchannels are solved by satisfying the condition relating the cross flow to the radial pressure difference. Each calculation interval is treated in turn starting from the bottom until the end of the channel is reached. From the enthalpy and flow variations along each subchannel the local void fraction may be calculated. By using the HAMBO model for the air-water system described in this paper the system had to be transformed into an equivalent boiling water system. This has been done by simulating the asymmetrical way of air-injection by a small heated area in channel 1 at level I.

By means of the measured inlet pressure as function of the injected mass flow of air, the air-water system could be translated into a
boiling water system in which the amount of produced steam was equivalent to the injected mass flow of air. Calculations with the HAMBO computer programme were carried out for 8 injected mass flows of air (5.10^{-3} to 4.0.10^{-3} kg.s^{-1}) with air injection at level I of channel 1. In this programme the channel length had been divided up into 17 intervals, not all of them having the same height, as, especially at the inlet region of the test section a strong cross flow gradient was found to be present. One special remark should be made about the so-called "Mixing" coefficient \( F_m \). The mixing coefficient multiplies the theoretical turbulent mixing exchange between subchannels by the value of \( F_m \). We tried out several values of \( F_m \) for the above mentioned injected mass flows of air and it was found that the best results were obtained with a value of \( F_m = 3 \). Fig. 8 gives the calculated and measured void fractions as a function of the injected mass flow of air. As can be concluded from these figures the agreement between experiment and theory is good keeping in mind the extreme conditions. Also the calculated mass flow of air and water data at the exit of the test section have been plotted and compared with the results of the experiments (figs 3 and 4). This agreement is less good. The theory did not predict a point of complete mixing, even not for the case where air was injected at level I. The difference between the mass flows of air, respectively water in both channels calculated, is in the upper part of the channel larger and in the bottom part smaller than the one predicted experimentally. Even more striking was the disagreement between the estimated difference in static pressure between the two channels. According to the data output of
the computer programme there was no radial pressure difference between the subchannels. In the measured results, however, the pressure in the bottom part of channel 1 is higher than that in channel 2, whereas in the upper part of the channel the opposite is true. Assuming that the experiments are right, then the predicted cross flow by HAMBO is too large in the bottom part and too small in the upper part of the channel, which is in agreement with the difference in predicted mass flows of air and water for both channels by theory and experiments.

5. The Eindhoven mixing model.

5.1. General.

In the Laboratory for Heat Transfer and Reactor Engineering of the Technological University of Eindhoven theoretical studies of mixing models have been started. The HAMBO model (see section 4), used for the air-water system gives results that correspond reasonably well with the measurements. But the HAMBO model predicts values of the radial pressure gradients which are very small and furthermore no iteration is included for the total pressure drops on all subchannels. The calculations start at the entry with a given flow division between the subchannels, based upon the same pressure loss at entry and no further check at the end of the channel for the total pressure drop. The object of the present study is to obtain a theoretical model for two phase fluid mixing in parallel open channels, which can be generally applied. The relationships obtained are expressed in
a non-dimensional form. A numerical computer programme has been written to calculate the two phase characteristics along the height of the individual channels as well as the inlet mass flow distribution across the channels for the boundary conditions imposed.

The model is an extension of an existing single channel hydraulic model described in ref. 6. The turbulent mixing terms in the HAMBO model are based on Boussineck, in the Eindhoven model on the mixing theory of Prandtl but principally there is no difference regarding the basic physics between the two approaches.

Differences are present regarding the mathematical treatment of the equations and the formulations of the boundary conditions imposed on the channels resulting in a more complex iteration procedure. The effect of intra-channel void and velocity distribution on the exchange terms is at present not taken into account in either model. Because of the complexity of the mixing terms the validity of the theoretical model will first be determined for a simple test section consisting of two parallel channels.

5.2. Basic equations for a single channel.

The basic equations are formed by application of the laws of conservation of mass, momentum and energy and the equation of state. These equations can be solved if one adopts, in addition, correlation functions for the slip, friction and the heat division parameter and the boundary. For the correlation functions there is a choice of several existing correlations possible.
The study here is restricted to the steady state condition and no radial transport of mass, momentum and energy because of convection will take place within a single channel.

5.3. Mixing terms.

The reactor core is assumed to consist of a large number of cylindrical fuel elements arranged in a regular pattern. Between these fuel elements the coolant flows upwards. This arrangement can be divided into a number of identical channels, whose boundaries are formed partly by fuel elements and partly by the fluid flow in the neighbouring channels. In case interaction effects could be neglected, each of these channels could be treated with the equations mentioned in the previous section (5.2.). However, interaction effects will in general be present. As a consequence, there will be a transverse transport of fluid and vapour across the boundaries of the coolant subchannel that mixes with the fluid and vapour in the receiving channel.

We recognize two mechanisms that cause mixing in the transverse direction. Firstly, there is microscopic transport due to turbulent mixing. This takes place as a consequence of difference in velocity, temperature and steam quality in the neighbouring channels.

Secondly, there is a macroscopic transport due to differences in pressure in neighbouring channels, called cross flow. The mechanism of turbulence is based on the mixing length theory. Suppose that on a microscopic scale particles of fluid and vapour are moving backwards and forwards across the channel boundary,
carrying some transferable property M, which may be mass, momentum or thermal energy. It will be assumed that the mass flow in the individual channels is always fully mixed over the cross section of the flow channel.

For a large number of such movements the average net rate of transfer of the general property M is given by:

\[- l' V \frac{dM}{dy} = - \rho \frac{dV}{dy} \frac{dM}{dy},\]

where \( l' \) is called the mixing length, \( l \) represents the Prandtl mixing length and \( \frac{dM}{dy} \) is the gradient of property M in the transverse direction \( y \).

The cross flow is the consequence of pressure differences between neighbouring channels and can be expressed as:

\[ p_1 - p_2 = k_s \frac{1}{2} \rho \frac{V_{rad}^2}{\frac{dV}{dy}}, \]

In this expression \( k_s \) represents the radial friction factor for the gap between the channels, \( \rho \) the average density of the donating channel and \( V_{rad} \) the average radial cross flow velocity. Both turbulent mixing and cross flow will be accounted for by adding, to the previously mentioned equations for conservation of mass, momentum and energy, mixing terms which describe the radial flow of mass, momentum and energy. In the present two channel study the mixing terms are equally large but opposite in direction which is expressed by an opposite sign.
5.4. Boundary conditions.

The boundary conditions for the channel and thus for the describing equations are formed by the characteristics of the other parts of the loop, e.g. headers, pumps, etc. For the two channels considered two boundary conditions have to be satisfied. First, the pressure drop across each channel including the inlet and outlet losses should be equally the same. Secondly, the total pressure drop over the channel and any further restriction in the loop should equal the pressure gain of the driving pump.

The calculation proceeds as follows:

The pressure between pump and inlet channel \( P_u \) is given.

The single phase inlet pressure drop for each channel can be calculated from

\[
P_{\text{inlet}} = k_{\text{in}} \frac{1}{2} \rho_{\text{in}} V_{\text{in}}^2
\]

Where \( V_{\text{in}} \) for each channel as a first approximation, has to be estimated to start the iteration procedure. Then the channel equations are solved for a large number of nodes in succession with a standard Runge Kutta procedure. The pressure in the top header \( P_h \) can be calculated for each channel from the Ramie equation for 2 phase expansion pressure changes. At this stage the total pressure drop of each channel is compared with the pressure in top of the return pipe \( P_v \) expressed as

\[
P_v = P_u - \Delta P_p - \rho g L + K_{\text{ret}} \frac{1}{2} \rho_{\text{ret}} V_{\text{ret}}^2
\]

The last term represents the frictional pressure drop in the return pipe which incorporates a heat exchanger and preheater.
If the first boundary condition \( P_{h1} - P_v = P_{h2} - P_v \) has not been satisfied new inlet mass velocities for the channels will be simultaneously altered such that \( P_{h} - P_v \) becomes smaller which iteration continues until the first condition and the second condition \( P_{h} - P_v = 0 \) have both been satisfied.

The programme has to be tested for a simple case of two parallel channels to check out the iteration procedure. Later on the programme will be tested with the conditions of an experimental programme.

6. **Concluding remarks.**

In principle, the two models mentioned in this paper and describing the mixing between hydraulically interconnected channels are similar, the difference being in the detailed description of the mixing terms. Both models simplify the actually existing situation in that they characterize each channel by its mean flow characteristics, which may not do justice to the real nature of turbulent mixing. It is felt that apart from high-pressure verification of the theory developed, a more fundamental programme on the turbulent mixing exchange between parallel open channels is required.
7. References.


2. Spigt, C.L. and Boot, P.G.M., Annual report of the research programme on the heat transfer and fluid flow characteristics of a pressurized water reactor, period January 1st. 1964 - January 1st. 1965.


5. R.W. Bowring, HAMBO, a computer programme for the subchannel analysis of the hydraulic and burn-out characteristics of rod clusters.
Fig. 1 Flowsheet of air-water mixing loop
fig. 2 MIXING TEST-SECTION FOR TESTS AT 140 atm. WITH HEAT ADDITION.
Fig. 6 Results plotted according to Zuber and Findlay (Ref. 4)
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Void fraction data compared with "HAMBO" Fig. 8