Forces and plastic work in cutting

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Prolific considerations in C.I.R.P. give rise to a further step in the development of a plastic cutting model. Basically two different primary shear zone geometries are assumed: with one and with two shear planes respectively. For both geometries two energetically different approaches are developed. This leads to expressions for dimensionless friction force numbers. These are defined as the quotient of the friction force and the product of specific stress, feed and width of cut. An additional assumption that the supplied power is completely dissipated in the plastic deformation of the chip material provides expressions for dimensionless cutting force and feed force numbers. These three force numbers depend on the shear angle and the rake angle of the tool for a given value of the strain hardening exponent. All theoretical models are verified experimentally for several workpiece materials, tool rake angles and cutting conditions. The plasticity values of the material necessary to quantify the force numbers are separately determined in tensile tests. Some of the numerous cutting experiments are represented graphically and they illustrate the general tendency of a satisfying agreement of theory and experiment, especially for the model based on two shear planes.
1. INTRODUCTION

Recent discussions in C.I.R.P., partly represented in [1] and [2], made it necessary to reflect at and to expand our cutting model based on an energy criterion [3][4]. This model was based on the following assumptions:

- the cutting process is a two dimensional plastic phenomenon.
  (plain strain condition).
- the process takes place in two regions: the primary and the secondary shear zone.
- the deformation in the primary shear zone occurs in one plane.
- the friction force in the secondary shear zone is a function of the shear angle.
- the deformation process takes the least power consuming geometry.

From this a differential equation had been derived for the normalized frictional force as a function of the shear angle. The workpiece material properties were represented by the strain hardening exponent and the specific stress according to the well known Ludwik equation. The tool geometry is defined by the rake angle of the tool. The differential equation could be solved numerically with a boundary condition coming from the upsetting test.

Comparison of the theoretical and experimental results showed that the agreement is good except for small shear angles.
Moreover, it became evident that the deviation for small shear angles is the same for all investigated workpiece materials. So the quantities used seem to be good but the relations can be improved. Open to question was the assumption that the friction force in the secondary shear zone is a function of the shear angle. For the following new approaches this assumption is superfluous. Comparison of the new results with experimental data shows that the theoretical values are too high (Model I). Next, the primary shear zone has been split up into two shear zones (Model II). This model shows a better agreement with the experiments. Furthermore, a complete different approach has been made. Firstly a one plane primary shear zone is assumed, while the cutting process is split up in two deformation steps (Model III). The first step deals with shearing without friction. The friction effect is taken into account in a separate step. Accounting for the force equilibrium and the power balance for this deformation step a relation for the normalized frictional force is derived. It is called the friction force number in the following. Comparison of the calculated and measured data show that the calculated values are slightly to low. Next the primary shear zone is split up again into two shear planes (Model IV). For this modification the agreement with the experiments appears to be excellent. In order to intensify the testing possibilities the cutting and feed force numbers are also calculated. For this purpose a relation for the total cutting power consumption is used as well as a relation between the friction, cutting and feed forces. The large number of measurements could only partly be represented. From the experiments it becomes clear that model IV provides the best approach. The four models are - in essence- presented in the following chapters.

2. MODEL I

It is assumed [5]:
- two dimensional plastic process (Fig. 1)
- primary and secondary shear zone
- one shear plane in the primary zone
- minimum power geometry

As presented earlier [3] for this case we have the following expression for the power $E_{1p}$ in the primary zone:

$$E_{1p} = \frac{C}{n+1} \left( \cotan \varphi + \tan(\varphi - \gamma_0) \right)^{n+1} \frac{b f v}{\sqrt{3}}$$

$$= \frac{C}{n+1} \left( \frac{\cos \gamma_0}{\sqrt{3} \sin \varphi \cos(\varphi - \gamma_0)} \right)^{n+1} \frac{b f v}{\sqrt{3}}$$

with $C = \text{specific stress}$
$n = \text{strain hardening exponent}$
$\gamma_0 = \text{rake angle of the tool}$
$\varphi = \text{shear angle}$
Figure 1: Schematic cutting process in two dimensions (plane strain).

This shear angle is determined geometrically by:

\[ b = \text{width of cut} \]
\[ f = \text{feed} \]
\[ v = \text{cutting speed} \]
\[ \varphi = \arctan \left( \frac{h_c}{f_c - \sin \gamma_0} \right) \]  
\[ \text{(2)} \]

where \( h_c \) = chip thickness.

For the power in the secondary shear zone it holds:

\[ F_{w} \sin \varphi \]
\[ E_{ls} = \frac{\cos(\varphi - \gamma_0)}{\cos \gamma_0} \]  
\[ \text{(3)} \]

where \( F_w \) = friction force acting on the chip.

Assuming a constant external friction force the power is minimum for:

\[ d(E_{ls} + E_{lp}) \frac{d\varphi}{d\varphi} = 0 \]  
\[ \text{(4)} \]

Hence, with Eqs (1), (3) and (4):

\[ F_{1w} = \frac{\cos(2\varphi - \gamma_0)}{\sqrt{3} \sin^2 \varphi} \left( \frac{\cos \gamma_0}{\sqrt{3} \sin \varphi \cos(\varphi - \gamma_0)} \right)^n \]  
\[ \text{(5)} \]

where \( F_{1w} \) = friction force number (dimensionless).

The total power consumption is represented by:

\[ F_{1v} = E_{lp} + E_{ls} \]  
\[ \text{(6)} \]

where \( F_{1v} \) = cutting force.

Hence, with Eqs (6), (1) and (3):

\[ \frac{F_{1v}}{C_{bf}} = \frac{1}{n+1} \left( \frac{\cos \gamma_0}{\sqrt{3} \sin \varphi \cos(\varphi - \gamma_0)} \right)^{n+1} \frac{F_{1w} \cdot \sin \varphi}{C_{bf} \cos(\varphi - \gamma_0)} \]  
\[ \text{(7)} \]

where \( F_{1v} \) = cutting force number.

The condition of equilibrium of the tool forces leads to:

\[ F_{1w} = F_{1v} \sin \gamma_0 + F_{1f} \cos \gamma_0 \]  
\[ \text{(8)} \]

where \( F_{1f} \) = feed force.

Hence,

\[ F_{1f} = \frac{F_{1v} \tan \gamma_0}{C_{bf}} + \frac{F_{1w}}{C_{bf}} \cdot \frac{1}{\cos \gamma_0} \]  
\[ \text{(9)} \]

where \( F_{1f} \) = feed force number.

Eqs (5), (7) and (9) provide the forces on the tool for a work-
piece material represented by \( C \) and \( n \) and a geometry represented by \( \gamma, b \) and \( f \).

As shown in Figs. 5 and 13 the force number curves calculated in accordance with this model (1) deviate considerably from the experimental cutting data.

3. MODEL II

This model \([5]\) is based on the same assumptions as model I. In this model two shear planes are assumed to intersect each other on the chip surface (Fig. 2) and shape a cavity at the tool tip. The angles between the shear planes and the connection line from the tool tip to the point of intersection of the two shear planes are taken equal \((= \alpha)\). The power in the primary zone is:

\[
E_{2p} = \frac{C}{n+1} \left( \varepsilon_1 + \varepsilon_2 \right)^{n+1} b f v
\]

where generally:

\[
\varepsilon_i = \frac{\cos \lambda_i}{\sqrt{3} \sin \psi_i \cos (\psi_i - \lambda_i)}
\]

For shear plane A:

\[
\lambda_1 = \zeta
\]
\[
\psi_1 = \varphi + \alpha
\]

![Figure 2: Geometrical representation for two shear planes.](image)

For shear plane B:

\[
\lambda_2 = \frac{\pi}{2} - \zeta + \gamma_o
\]
\[
\psi_2 = \frac{\pi}{2} + \varphi - \alpha - \zeta
\]

where \( \zeta = \arctan \left( \frac{\cos(\varphi - \alpha) \cos \gamma_o + 2 \sin \varphi \cos \alpha \sin \gamma_o}{\sin(\varphi + \alpha) \cos \gamma_o} \right) \)
The power is minimum if:
\[
\frac{\partial (E_{2p} + E_{2s})}{\partial \varphi} = 0
\]
\[
\frac{\partial (E_{2p} + E_{2s})}{\partial \alpha} = 0
\]
In which \(E_{2s} = E_{1s}\).

Numerical solution of Eqs (15) and (16) with respect to Eqs (10) \(\ddagger (14)\) gives the value of \(\frac{F_{2w}}{Cbf}\) as a function of the shear angle \(\varphi\).

The shear angle \(\varphi\) in fact only a number representing the chip reduction, is defined by Eq. (2). Conform model I, it holds for the cutting force number:

\[
\frac{F_{2v}}{Cbf} = \frac{1}{n+1} \left( \frac{\varepsilon_1 + \varepsilon_2}{n+1} + \frac{\sin \varphi}{\cos (\varphi - \gamma_o)} \right) \cdot \frac{F_{2w}}{Cbf}
\]

Similarly it holds for the feed force number:

\[
\frac{F_{2f}}{Cbf} = -\frac{F_{2v}}{Cbf} \tan \gamma_o + \frac{F_{2w}}{Cbf} \cdot \frac{1}{\cos \gamma_o}
\]

The theoretical curves (11) from Eqs. (10) \(\ddagger (14)\), (17) and (18) are compared with experimental results in Figs. (5) \(\ddagger (13)\).

4. MODEL III

This model consists of two separate steps [6]:
- frictionless cutting using one shear plane
- addition of friction and its upsetting effect
For the rest it has the same geometry as model I. Fig. 3 gives a representation of the cutting process.

Frictionless cutting: The assumption of one shear plane in the primary zone and frictionless cutting includes a chip thickness equal to the feed.
Because any other geometry requires additional deformation energy.
This proposition leads geometrically to:

\[
\gamma_o = \frac{90^\circ + \gamma_o}{2} = 45^\circ + \frac{\gamma_o}{2}
\]

The same relation can be derived from Eqs. (1), (3) and (4) with the addition that for frictionless cutting \(F_{3w} = 0\).

Addition of friction: The friction force on the chip is indicated with \(F_{3w}\). Because of equilibrium there is also a force \(F_{3w}\) with opposite sign in the primary zone parallel to the friction
force on the tool. The power balance for this part reads:

\[ F_{3w}v = F_{3w}v_c + \Delta E \]  \hspace{1cm} (20)

where \( \Delta E = \) upsetting power from \( f \) to \( h_c \).

With Eq. (1) it holds:

\[
\Delta E = \frac{C}{n+1} \left( \frac{\cos \gamma_o}{\sqrt{3} \sin \varphi \cos (\varphi - \gamma_o)} \right)^{n+1} bfv - \frac{C}{n+1} \left( \frac{\cos \gamma_o}{\sqrt{3} \sin \varphi_o \cos (\varphi_o - \gamma_o)} \right)^{n+1} bfv
\]

\hspace{1cm} (21)

Invariancy of volume leads to a relation between \( v \) and \( v_c \). Eq. (20) gives with Eq. (21):

\[
\frac{F_{3w}}{Cbf} = \frac{1}{n+1} \left( \frac{\cos \gamma_o}{\sqrt{3} \sin \varphi \cos (\varphi - \gamma_o)} \right)^{n+1} - \frac{1}{n+1} \left( \frac{\cos \gamma_o}{\sqrt{3} \sin \varphi_o \cos (\varphi_o - \gamma_o)} \right)^{n+1}
\]

\[
\cdot \left( 1 - \frac{\sin \varphi_o}{\cos (\varphi - \gamma_o)} \right)
\]

\hspace{1cm} (22)

With Eq. (6) the cutting force number is:

\[
\frac{F_{3w}}{Cbf} = \frac{1}{n+1} \left( \frac{\cos \gamma_o}{\sqrt{3} \sin \varphi \cos (\varphi - \gamma_o)} \right)^{n+1} + \frac{\sin \varphi_o}{\cos (\varphi_o - \gamma_o)} \cdot \frac{F_{3w}}{Cbf}
\]

\hspace{1cm} (23)
And further with Eq. (9) for the feed force number:

\[
\frac{F_{3f}}{Cbf} = \frac{F_{3v} \tan \gamma_o}{Cbf} + \frac{F_{3w}}{Cbf} \cdot \frac{1}{\cos \gamma_o}
\]

(24)

The theoretical curves (III) from Eqs. (22), (23) and (24) are compared with experimental results in Figs. (5) – (13).

5. MODEL IV

The same basic assumptions are made as in model III. Two shear planes are assumed now (Fig. 4). For the power of frictionless cutting it can be written [6]:

\[
E_{401} + E_{402} = \frac{C}{n+1} \left( \varepsilon(\psi_{01}) + \varepsilon(\psi_{02}) \right)^{n+1} \text{bfv}
\]

(25)

with \( \psi_{01} = \) shear angle of the first shear plane (I) for frictionless cutting.

\( \psi_{02} = \) shear angle of the second shear plane (II) for frictionless cutting.

Figure 4: Representation of model IV for a primary zone with two shear planes.

Geometrically we have with Eq. (1):

\[
\varepsilon(\psi_{01}) = \frac{\cos \gamma_1}{\sqrt{2} \sin^2 \left(45^0 + \frac{\gamma_1}{2}\right)}
\]

(26)
\[
\varepsilon(\phi_{02}) = \frac{\cos \gamma_2}{\sqrt{3} \sin^2(45^\circ + \frac{\gamma_2}{2})}
\]

and \( \gamma_2 = 90^\circ + \gamma - \gamma_1 \) \hspace{1cm} (27)

In Eq. (26) the variable is \( \gamma_1 \) taking Eq. (27) into account. If the process takes the geometry connected with minimum power dissipation it holds:

\[
\frac{3(E_{401} + E_{402})}{\partial \gamma_1} = 0 \hspace{1cm} (28)
\]

Eqs. (25) \& (28) give:

\[ \gamma_1 = \gamma_2 = 45^\circ + \frac{\gamma_o}{2} \hspace{1cm} (29) \]

For the constructed shear angle \( \phi^* \) in the case of cutting with friction it is obtained with Eq. (1):

\[
\varepsilon(\phi^*_2) = \frac{\cos(45^\circ + \frac{\gamma_o}{2})}{\sqrt{3} \sin \phi^*_2 \cos(\phi^*_2 - 45^\circ - \frac{\gamma_o}{2})} \hspace{1cm} (30)
\]

where

\[
\phi^*_2 = \arctan\left(\frac{\sin(45^\circ - \frac{\gamma_o}{2})}{\cos \gamma_o + \sin \gamma_o - \cos(45^\circ - \frac{\gamma_o}{2})}\right) \hspace{1cm} (31)
\]

Taking friction into account and in agreement with model III (Eq. 22) the following expression is obtained:

\[
C_{bf} = \frac{1}{n+1} \left(\varepsilon(\phi^*_2) + \varepsilon(\phi_{01})\right)^{n+1} - \frac{1}{n+1} \left(\varepsilon(\phi_{02}) + \varepsilon(\phi_{01})\right)^{n+1}
\]

\[
\left(1 - \frac{\sin \phi}{\cos(\phi - \gamma_o)}\right)^3 \hspace{1cm} (32)
\]

Consequently for the cutting force number (with Eq. (7)) is:

\[
\frac{F_{4w}}{C_{bf}} = \frac{1}{n+1} \left(\varepsilon(\phi^*_2) + \varepsilon(\phi_{01})\right)^{n+1} + \frac{F_{4w}}{C_{bf}} \cdot \frac{\sin \phi}{\cos(\phi - \gamma_o)} \hspace{1cm} (33)
\]

and the feed force number with Eq. (9):

\[
\frac{F_{4f}}{C_{bf}} = - \frac{F_{4v}}{C_{bf}} \cdot \tan \gamma_o + \frac{F_{4w}}{C_{bf}} \cdot \frac{1}{\cos \gamma_o} \hspace{1cm} (34)
\]

The theoretical curves (IV) from Eqs. (32), (33) and (34) are compared with experimental results in Figs. 5 \& 13.
6. EXPERIMENTAL RESULTS

The experimental and theoretical values of the cutting, feed and friction force numbers are represented in Figs. 5 to 13 as a function of the chip reduction. The chip reduction is represented by the "shear angle". The experimental results are a representative selection from more than 2000 different measurements. In order to obtain the values the following quantities were varied:
- feed (between 0.1 - 0.5 mm/rev.)
- cutting speed (from 1 to 3 m/s)
- tool material (at least three for each workpiece material)
- workpiece material (C15, C22, C45)
- rake angle (-6°, 0°, 18°).

In these figures the variation of feed, cutting speed and tool material is implicitly included. The experimental force numbers can be derived from the measured feed, width of cut and cutting forces with Eq. (8). The specific stress C has been determined as an average from three tensile tests (Table 1). For the strain hardening exponent only one value (= 0.24) has been taken, because the difference between the various workpiece materials is very small and therefore negligible. The values of the chip-thickness have been established by using the mass method [4]. Except for the experiments represented in Figs. 5, 8 and 11, which were carried out with a micrometer.

Summarizing it can be concluded that model IV gives the best agreement between theory and experiment for any of the three different force numbers.

<table>
<thead>
<tr>
<th>DIN</th>
<th>STANDARD NO.</th>
<th>% C</th>
<th>% Si</th>
<th>% Mn</th>
<th>% P</th>
<th>% S</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15</td>
<td>1.0401</td>
<td>0.12-0.18</td>
<td>0.15-0.35</td>
<td>0.30-0.60</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
</tr>
<tr>
<td>C22</td>
<td>1.0402</td>
<td>0.18-0.25</td>
<td>0.15-0.35</td>
<td>0.30-0.60</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
</tr>
<tr>
<td>C45</td>
<td>1.0503</td>
<td>0.42-0.50</td>
<td>0.15-0.35</td>
<td>0.50-0.80</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
</tr>
</tbody>
</table>

Table 1: Plastic material properties and chemical composition of the workpiece materials.

7. DISCUSSION AND CONCLUSIONS

Numerous cutting experiments demonstrate a general tendency of satisfying agreement between theory and experiment, especially in the case of model IV. From these results it can be concluded that, contrary to previously proposed models, cutting can be explained by the plasticity theory. Apparently the necessary plastic material properties can be derived from a tensile test or any other equivalent test. So, the knowledge of the plastic
quantities $C$ and $n$ of the workpiece material and the geometry of the process ($f$, $b$, $\gamma$) enables to calculate the three forces on the tool and the chip reduction if one of these values is already known. Although the agreement between theory and experiment is rather well, especially in the case of model IV, still there is some scatter in the experimental results. This could be imputed to one or more of the following reasons:

- For the determination of the $C$ value results from tensile tests at room temperature and at low strain rates were used. The $C$ value is necessary for the computation of the total power in the primary zone. In order to get an impression of the temperature in this zone an approximate calculation can be made. If no energy loss in this zone is assumed the maximum temperature rise is:

$$
\Delta T = \frac{C \cos \gamma_0}{n+1} \left( \frac{\cos \gamma_0}{\sqrt{3} \sin \gamma \cos(\varepsilon - \gamma_0)} \right)^{n+1} \frac{1}{\rho \cdot c}
$$

where $\rho$ = specific mass
$\gamma$ = specific heat

For the average temperature rise it holds:

$$
\bar{\Delta T} = \frac{1}{2} \Delta T
$$

Table 2 indicates the values which can be expected in the cutting experiments. The values of the shear angle extrema are derived from Figs. 5-13. Besides an enhanced temperature in the primary zone, there is also a very high strain rate. This increases the yield stress value contrary to the temperature rise. In the first instance these effects will compensate each other [7]. Nevertheless there will be a deviation which increases with increasing temperature: for cutting with small values of the shear angle.

- Up to now we assumed that the predeformation of the workpiece material is zero. Taking this effect in account in model IV results in higher values of the friction force number, according to Eq. (32).

<table>
<thead>
<tr>
<th>Material</th>
<th>Shear angle variation</th>
<th>Average temperature rise $[{^\circ}C]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15</td>
<td>12,5 - 27,5</td>
<td>317 - 134</td>
</tr>
<tr>
<td>C22</td>
<td>17,5 - 27,5</td>
<td>256 - 160</td>
</tr>
<tr>
<td>C45</td>
<td>20 - 35</td>
<td>302 - 181</td>
</tr>
</tbody>
</table>

Table 2: Estimated average temperature rise of the workpiece material in the primary zone.
- For the sake of simplicity orthogonal cutting was assumed and a chip flow in the direction of the feed force. Otherwise it would be necessary to account for a contribution of the thrust force. This correction might be very important for fine cutting.

- In the models II and IV a cavity on the tool tip was assumed. It is also possible to replace this cavity by workpiece material (built up edge). In that case the position of the theoretical curves in Figs. 5 to 13 will change a little.

- Another source of deviations could be the roundness of the cutting edge. Measurement gives values for the radius between 20 - 90 µm for unground tips. The relative magnitude of this value compared with a common feed of 160 µm in our experiments suggests some effect.

A similar effect can be expected from worn tips. Therefore the cutting experiments were always executed with fresh tips (VB < 0.2 mm). Nevertheless such a tip has a friction zone on its flank and gives a contribution to the feed and the cutting force.

- A measuring error can be introduced by the measurement of the chip length value of curled chips. This length is necessary to determine the chip reduction with the mass method. For these chips the average of the inner and outer length has been taken. As there was found more accurate measurement of that length results in a decrease of scatter.

Acknowledgements: The authors are indebted to Mr. A. van Sorgen, Mr. M.Th. de Groot and J.C.M. Manders for their help in performing the experiments.

<table>
<thead>
<tr>
<th>C[N/mm$^2$]</th>
<th>C[N/mm$^2$]</th>
<th>n tensile test</th>
<th>n average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>0.22</td>
<td>0.23</td>
<td>830</td>
</tr>
<tr>
<td>0.23</td>
<td></td>
<td></td>
<td>810</td>
</tr>
<tr>
<td>0.24</td>
<td>0.26</td>
<td>0.26</td>
<td>937</td>
</tr>
<tr>
<td>0.29</td>
<td></td>
<td></td>
<td>965</td>
</tr>
<tr>
<td>0.25</td>
<td>0.29</td>
<td>0.23</td>
<td>1377</td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td></td>
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</tr>
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C.I.R.P. Annals Vol 31/1; pg. 91-96; 1982.


A velocity-modified temperature for the plastic flow of metals.
Figures 5-13: Theoretical and experimental results for different workpiece materials and rake angles

Model I ——— Model II ——— Model III ——— Model IV ———