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A user equilibrium model for combined activity–travel choice under prospect theoretical mechanisms of decision-making under uncertainty

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ABSTRACT

Rather than considering single trips as the unit of analysis, the activity-based modeling paradigm of travel demand analysis has led to reconceptualizations and innovations in traffic flow models by focusing on complete daily activity–travel patterns. The vast majority of these travel demand and traffic flow models have either implicitly or explicitly assumed that travelers choose between alternatives by maximizing their utility under a deterministic representation of the choice alternatives. While this behavioral assumption leads to tractable, easy-to-apply models, the validity of the assumption largely went untested. This paper investigates the user equilibrium of activity–travel patterns under uncertainty from the perspective of prospect theory. A formulation of the static activity-based user equilibrium model is proposed. In particular, we adopt the concept of a multi-state supernetwork to represent the choice space of activity–travel patterns. A numerical example using hypothetical scenarios is presented to illustrate the proposed model and solution algorithm.

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1. Introduction

Traffic assignment models have a prominent place in travel demand analysis. Since Wardrop's (1952) and Beckmann, McGuire, and Winsten's (1956) seminal work, many studies in urban transportation planning and management have derived equilibrium models to assess the effects of large-scale infrastructure improvement or transportation policies on traffic flows and system performance. However, virtually all these applications have been based on traditional trip-based approaches, which have received increasing criticisms. Ultimately, this led to the development of activity-based models of travel demand analysis (e.g. Pinjari and Bhat 2011; Rasouli and Timmermans 2014a). Rather than focusing on single trips, the activity-based approach takes interdependencies between trips and activities within a certain time frame into account. In turn, the activity-based approach also led to

reconceptualizations and innovations in traffic flow models (e.g. Lam and Yin 2001; Lam and Huang 2002, 2003; Li et al. 2010; Ouyang, Lam, and Huang 2011; Fu and Lam 2014).

A number of activity-based dynamic user equilibrium models have recently been developed. Most of these consider the utility maximization rule as the decision-making mechanism. Lam and Yin (2001) proposed a time-dependent dynamic user equilibrium model to study individuals' activity\destination\route choices. Lam and Huang (2002) developed a multi-class model for activity/route choices during the morning rush hours in a congested network with queues. Considering the imperfect perception of travelers of the transport network, Huang and Lam (2005) extended the above work in a stochastic user equilibrium model. However, the choice dimension in the above studies is limited due to the lack of representation of activity-travel patterns. Li et al. (2010) investigated the timetabling problem of transit services with an activity-based transit assignment model in multi-modal transit networks. Ouyang, Lam, and Huang (2011) formulated an activity-based user equilibrium model in congested networks.

Compared to this wealth of studies, little effort has been made on decision-making mechanisms in these user equilibrium studies in the context of daily activity-travel-scheduling problems. The vast majority of these travel demand and traffic flow models, however, have implicitly or explicitly assumed that travelers choose between alternatives by maximizing their utility under a deterministic representation of the alternatives. While this behavioral assumption leads to tractable and easy-to-apply models, the validity of the assumption of utility-maximizing behavior largely went untested. Moreover, the deterministic representation of the choice problem is unrealistic. In case of decision-making under uncertainty, Fu and Lam (2014), considering the stochastic utility of activities in multi-modal transit networks, assumed that travelers make activity-travel choices based on the budget utility maximization rule, which is defined as the summation of the expected utility and a safety margin. However, the principle of expected utility maximization has found little empirical support in consumer behavior research (e.g. Rasouli and Timmermans 2014b).

Several alternative theories of decision-making under conditions of uncertainty have been formulated, such as rank-dependent utility theory (Quiggin 1982), disappointment theory (Bell 1985; Loomes and Sugden 1986), regret theory (Loomes and Sugden 1987) and (cumulative) prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Rasouli and Timmermans (2014b) provided a recent overview of these decision models under uncertainty. A series of experiments find that people's decision-making process under uncertainty is reference dependent and different risk attitudes will occur when people face a gain or loss situation. Prospect theory assumes the existence of a reference point, which differentiates between gains and losses. A non-linear value function is estimated to represent risk-seeking and risk-avoiding behavior. For example, in reality, individual's activity choices and travel mode choices are interdependent. If the prior activity's duration is longer than planned, the individual would take a faster mode with a higher fare to commence the next compulsory activity on time, while the individual may choose a mode with a lower fare to save money for other trips to the non-compulsory activities of that day. In this case, there is a reference for both money and time. From this perspective, we can infer that individuals' activity-travel-scheduling behavior is reference dependent. There have been some efforts to apply prospect theory to route choice behavior (e.g. Connors and Sumalee 2009; Xu et al. 2011), but little effort has been made to apply prospect theory to address the user equilibrium of combined activity-travel choices.

The purpose of this study, therefore, is to investigate activity–travel behavior from the perspective of prospect theory, including the choice of mode, activity location and activity duration. In this paper, we will present a static activity-based user equilibrium model, in which prospect theory is used to represent decision-making under uncertainty. A prospect-based user equilibrium model for combined activity–travel choices with discrete endogenous reference points will be investigated. In particular, we adopt a simplified network representation (extraction) of the multi-state supernetwork (Liao, Arentze, and Timmermans 2010, 2011, 2013) to represent the choice space of conducting activity programs under uncertainty.

The remainder of this paper is organized as follows. Section 2 describes the representation of activity–travel patterns, the assumptions of this paper, and the discussions of cumulative prospect theory and reference points. Section 3 discusses the prospect-based user equilibrium conditions for combined activity–travel choices, and formulates it as a variational inequality problem. Thereafter, a heuristic solution algorithm is presented. Section 4 applies the proposed model and solution algorithm to a numerical example. The final section provides a summary and identifies directions for future research.

2. Basic considerations

This section firstly describes the network representation of activity–travel patterns and the basic assumptions of this paper; secondly, we discuss the link costs of travel and activity participation based on the representation of activity–travel patterns; thirdly, an extended weighting function is proposed according to (cumulative) prospect theory; lastly, a discrete scheme method is adopted to determine travelers' reference points.

2.1. Network representation and assumptions

To study the decision-making mechanism for combined activity–travel choices, an extracted network representation from the multi-state supernetwork (Arentze and Timmermans 2004; Liao, Arentze, and Timmermans 2010, 2013) is introduced in the following.

2.1.1. An extracted network

The representation of multi-state supernetwork integrates individuals' activity programs, multi-modal transport networks and locations of facilities. The supernetwork consists of private vehicle networks, which can only be accessed by private vehicles, and public transport networks, where the individual can walk and take public transport. To capture the choice space of conducting activity programs, these networks are assigned to and connected by different states of the implementation of the activity programs. A nice feature is that any path through the supernetwork expresses a particular way of conducting the activity program. To simplify the representation and without loss of generality, we extract a small network (Figure 1) to study the daily activity–travel scheduling problem for our purpose.

Define $G(N, A)$ as a connected network, where N is the set of nodes representing various location zones, and A is the set of arcs connecting these locations, divided into two types, namely, travel links A_v and activity links A_a , where $A_v \cup A_a = A$. For instance, as Figure 1 shows, a commuter may depart a residential zone (home) to conduct an activity at zone A1 in the morning by car, and directly return home by car without visiting any other zones, or

Figure 1. Representation of activity-travel patterns.

continue to conduct another activity at zone A2 by car first and then transfer to bus and then come back home by combined bus and car due to the expensive parking at A2, and so on, which form different daily activity-travel patterns. Thus, an activity-travel sequence for an individual can be represented as a path that includes both travel and activity arcs. Both origins (O) and destinations (D) of travelers' daily activity-travel paths are home zones. For each link $a \in A$, let f_a denote the flow on link a . Let R be the set of activity-travel paths between an O-D pair. For each path $r \in R$, f_r denotes the corresponding path flow. The path flows are related to the link flows by $f_a = \sum_{r \in R} \delta_a^r f_r$, where δ_a^r is a link-path incidence whose value is 1 if link a is on path r , and 0 otherwise.

2.1.2. Model assumption

In this paper, the following assumptions are adopted.

- A1. The set of feasible activity-travel patterns is assumed to be pre-specified. This assumption has been commonly adopted by related studies (e.g. Li et al. 2010; Fu and Lam 2014).
- A2. Only mode choice, activity durations and activity location choice are considered in our model.
- A3. The concept of activity utility and travel disutility is applied. The travel disutility between activity zones is determined by travel time (with crowding discomfort for transit modes and the flows on the travel route of other modes) and fare. Activity utility is determined by a modified form of the bell-shaped utility function suggested by Ettema and Timmermans (2003). As this paper only concerns the static scenario, the utility gained from activity participation mainly depends on the duration of the activity. Moreover, individuals have identical value of time;
- A4. The daily activity-travel schedules of travelers are the paths including travel links and activity links. The utility of a path is the summation of the disutility of travel links and the utility of activity links on the path. Travelers base their choice of schedules on a reference which is a tradeoff between the utility derived from activity participation at different locations and the disutility incurred by travel between activity locations. In this paper, individuals are assumed to make their decisions to maximize their prospect value, which will be discussed below.
- A5. Passenger flows on travel links and activity durations are assumed to follow normal distributions. The uncertainty of passenger flows on the travel links makes the travel time and travel disutility uncertain, and the uncertainty of activity durations makes the activity utility uncertain, together resulting in the uncertainty of path utility. We define

an expected gain probability to describe travelers risk attitude. The heterogeneity of travelers' risk attitude is not considered in this study.

2.2. Travel disutility and activity utility

Let A_{v1} and A_{v2} be the set of transit and non-transit travel links, respectively ($A_{v1} \cup A_{v2} = A_v$). Following assumption A5, we can define the travel time of each travel link as Equations (1) and (2). For the transit travel link, travel time experienced by travelers is influenced by the crowding discomfort (Lo, Yip, and Wan 2003), which is expressed as

$$T_a = t_a^0 \left(1 + \beta_1 \left(\frac{F_a}{h_a \cdot g_a} \right)^{\gamma_1} \right), \quad a \in A_{v1}, \quad (1)$$

where t_a^0 is the free flow travel time of transit travel link a ; F_a is the stochastic passenger flow on the transit link, which is assumed to follow normal distribution. We adopt the definition of $cv_{a \in A_a}$ in Fu and Lam (2014), which is a model parameter relevant to the activity type (compulsory or non-compulsory). Let f_a and $cv_{a \in A_a} \cdot f_a$ denote the mean and the standard deviation of F_a respectively, where $a \in A_a$ is the activity at the end of the travel links. h_a is the vehicle capacity of transit travel link a ; g_a is the frequency. Besides, for the non-transit travel link, the travel time is influenced by the flow of transport modes in traffic network. Assume that other modes' flow is fixed as Δ , then we can define the travel time with BPR (Bureau of Public Road) function:

$$T_a = t_a^0 \left(1 + \beta_2 \left(\frac{F_a + \Delta}{C_a} \right)^{\gamma_2} \right), \quad a \in A_{v2}, \quad (2)$$

where the notation definitions are similar to Equation (1), and C_a is the capacity of non-transit travel link.

The disutility of a travel link means the loss of time and/or money, which can be expressed as

$$DISU_a = -\theta T_a - c_a = \begin{cases} -\theta T_a - \tau_a, & a \in A_{v1} \\ -(\theta + \vartheta_a) T_a, & a \in A_{v2} \end{cases}, \quad a \in A_v, \quad (3)$$

where θ is the value of time, which transfers time to money; c_a is the fixed fare of transit; and ϑ_a is the coefficient of money per travel time on non-transit travel links. Since $F_{a \in A_v}$ follows normal distribution, the mean and the standard deviation of $DISU_a$ can be obtained as $DISU_a \sim N(\text{dis}\mu_a, (\sigma_a)^2)$ according to Shao, Lam, and Tam (2006). The detailed computation of $\text{dis}\mu_a$ and σ_a can be found in Appendix 1.

For an activity link $a \in A_a$, we suppose activity duration follows normal distribution, in which μ_a is the mean and $cv_a \cdot \mu_a$ is the standard deviation. Let U_a be the total utility of the activity a with activity duration $T_a \sim N(\mu_a, (cv_a \cdot \mu_a)^2)$. The activity utility of performing activity link a from start to end can be expressed as

$$U_a = \int_0^{T_a} \bar{u}_a(\omega) d\omega, \quad a \in A_a, \quad (4)$$

where $\bar{u}_a(\omega)$ denotes the marginal utility of performing activity in link $a \in A_a$ with duration ω , which can be measured by the modified bell-shaped marginal function proposed by

Joh, Arentze, and Timmermans (2002). In this study, we assume that activity utility is totally dependent on activity duration, independent of the time of day. We adopt the function proposed by Ettema and Timmermans (2003) to present the marginal utility:

$$\bar{u}_a(t) = \frac{\gamma_a \beta_a u_a^{\max}}{\exp[\beta_a(t - \alpha_a)](1 + \exp[-\beta_a(t - \alpha_a)])^{\gamma_a + 1}}, \quad t \geq 0, \quad a \in A_a, \quad (5)$$

where t is the duration of the activity, expressed in minutes on activity links; $u_a^{\max}$ is the maximum accumulated utility of the activity; α_a , β_a and γ_a are activity-specific parameters.

Thus, the daily utility gain can be obtained by summing the disutilities of travel links and utilities of activity links on path r :

$$U_r = \sum_{a \in A_v} DISU_a \delta_a^r + \sum_{a \in A_a} U_a \delta_a^r. \quad (6)$$

In Equation (6), $DISU_a$ follows a normal distribution, while U_a does not. Using the method proposed by Wu and Nie (2009), we can approximate the cumulative distribution of U_r in the discrete scheme.

2.3. Cumulative prospect theory

Prospect theory (PT) was initially advanced to describe people's decision-making behavior when confronted with uncertainty. Based on a series of experiments, PT represents a pattern of risk attitudes that differs from that of expected utility-maximizing theory. PT incorporates the following features: people distinguish gains from losses before making decisions; the outcomes (x_i) are framed as gains (i.e. $\Delta x_i \geq 0$; see Equation (7)) or losses (i.e. $\Delta x_i < 0$) when compared to a reference point (x_0); the loss looms larger than the gain ($\lambda \geq 1$; see Equation (8) and Figure 2(a)); people tend to overweight extreme, but unlikely events. One typical inverse S-shaped weighting function is presented in Equation (9) (Prelec 1998) and further illustrated in Figure 2(b).

$$\Delta x_i = x_i - x_0, \quad (7)$$

$$v(x_i) = \begin{cases} \Delta x_i^\alpha, & \Delta x_i \geq 0 \\ -\lambda(-\Delta x_i)^\beta, & \Delta x_i < 0, \quad \lambda \geq 1 \end{cases} \quad (8)$$

$$w(p_i) = \exp(-(-\ln p_i)^\gamma), \quad 0 < \gamma < 1. \quad (9)$$

The parameters α , β measure the degree of diminishing sensitivity. For $0 < \alpha, \beta < 1$, the value function exhibits risk aversion over gains and risk-seeking over losses. The parameter λ is the loss aversion coefficient, which indicates that individuals are more sensitive to losses than gains. With γ , people usually overweight unlikely events and underweight likely events. Then, the prospect value of a gamble $(x_1, p_1; x_2, p_2)$ is calculated as in the following equation:

$$u(x_1, p_1; x_2, p_2) = w(p_1)v(\Delta x_1) + w(p_2)v(\Delta x_2). \quad (10)$$

Cumulative prospect theory (CPT) is an extension of PT, which employs cumulative rather than single decision weights. Suppose an alternative, denoted by the pair $(x; p)$, is composed of $m + n + 1$ possible outcomes, $x_{-m} < \dots < x_0 < \dots < x_n$, which occur with probabilities $p_{-m}, \dots, p_n$, respectively. Then, we have $x = (x_{-m}, \dots, x_n)$ and $p = (p_{-m}, \dots, p_n)$.

Figure 2. Value function and weighting function of PT. (a) Value function and (b) weighting function.

The cumulative decision weights and prospect value are defined as follows:

$$\pi^+(p_i) = w(p_i + \dots + p_n) - w(p_{i+1} + \dots + p_n), \quad 0 \leq i \leq n, \quad (11)$$

$$\pi^-(p_{-j}) = w(p_{-m} + \dots + p_{-j}) - w(p_{-m} + \dots + p_{-j-1}), \quad m \leq -j \leq 0, \quad (12)$$

$$PV(x, p) = \sum_{i=0}^n v(\Delta x_i) \pi^+(p_i) + \sum_{i=-m}^{-1} v(\Delta x_i) \pi^-(p_i). \quad (13)$$

By substituting p_i as the cumulative probability, we can replace (11)–(12) by (15)–(16). We assume that travelers behave as prospect maximizers when they make their activity–travel choice under uncertainty. More specifically, using a reference point, travelers may consider the outcome of an activity–travel path as a gain, if the utility is more than the reference point; as a loss if otherwise. We consider a daily activity–travel path r with $\tilde{u}$ as the reference point. The relative payoff for choosing a path r can be defined as $u_r - \tilde{u}$. Consequently, the prospect value of r can be calculated through (14) and (18).

$$v(u_r) = \begin{cases} (u_r - \tilde{u})^\alpha, & \tilde{u} \leq u_r \\ -\lambda(\tilde{u} - u_r)^\beta, & \tilde{u} > u_r \end{cases}, \quad (14)$$

$$\pi^+(p_i) = w(1 - P_{i-1}) - w(1 - P_i), \quad 0 \leq i \leq n, \quad (15)$$

$$\pi^-(p_{-j}) = w(P_{-j}) - w(P_{-j-1}), \quad m \leq -j \leq 0, \quad (16)$$

$$m + n = \frac{1}{L}, \quad 0 < L < 1, \quad (17)$$

$$PV_r = \sum_{i=0}^n v(u_{ri}) \pi^+(p_i) + \sum_{i=-m}^{-1} v(u_{ri}) \pi^-(p_i). \quad (18)$$

Consider a set of discrete points in the space of cumulative probability, namely $L = 0.01, 0.02, \dots, 0.09$ (Wu and Nie 2009). PV_r is the prospect value of activity–travel path r . Consistent with the CPT, we assume that the travelers base their choices of activity–travel patterns on the prospect values and always choose the path with the maximal PV_r .

2.4. Reference point

The endogenous reference utility of travelers' daily activity–travel patterns can be determined by the method proposed in Xu et al. (2011). The cumulative distribution of path utility in the discrete scheme can be expressed as a sequence of cumulative probability $P_{r1}, P_{r2}, \dots, P_{ri}, \dots, P_{r(m+n)}$ for corresponding utilities $u_{r1}, u_{r2}, \dots, u_{ri}, \dots, u_{r(m+n)}$ of path r . Suppose that every traveler expects to select a path getting as much utility as possible with a certain probability η . According to the cumulative probability P_{ri} for each path utility u_{ri} , the reference point can be selected as the maximal u_{ri} when P_{ri} is closest to $1 - \eta$, which is expressed as

$$\tilde{u} = \max\{u_{ri} | i = \min\{i | |P_{ri} - (1 - \eta)| \approx 0\}, r \in R\}. \quad (19)$$

Further,

$$\tilde{u} = \max\{\varphi_r(1 - \eta), r \in R\}, \quad (20)$$

where φ_r is the inverse cumulative distribution function of the path utility. The points will be further adjusted and the next section presents a prospect-based user equilibrium model where the resulting reference points are consistent with the equilibrium flow pattern. With α, β and λ being 1 and the reference point being 0, the above model will reduce to the rank-dependent expected utility (RDEU)-based (rank-dependent expected utility based theory, Quiggin 1982) model.

3. User equilibrium model for activity–travel choice and solution algorithm

3.1. Prospect-based user equilibrium condition

With the extracted network representation, individuals' activity choices (i.e. activity locations, sequences and durations) and travel choices (i.e. mode choices) are explicitly represented by different links in a directed path. Each path from origin to destination represents a feasible daily activity–travel pattern. Therefore, the proposed activity-based traffic assignment model is equivalent to a static user equilibrium problem with fixed demand and uncertain travel time and activity duration. It falls into the category of static models for long-term planning at the strategic level: at the equilibrium state, the prospect values of all used paths are equal, and are the same as (or greater than) that of all unused paths. The prospect-based user equilibrium condition can be formally expressed as

$$\begin{cases} PV_r = PV_{\max}, & f_r > 0 \\ PV_r \leq PV_{\max}, & f_r = 0 \end{cases}, \quad r \in R, \quad (21)$$

where f_r is the flow on path r and $PV_{\max} = \max\{PV_r\}$.

3.2. Model formulation

As discussed earlier, the interdependent link flow and link time of each link lead to the variation of disutility/utility. The prospect value of each path is a function of the overall path flow pattern $\mathbf{f}$. The above equilibrium conditions can be represented as follows. For

completeness, some previously presented equations are repeated here.

$$f_r \cdot (PV_{\max} - PV_r) = 0, \quad r \in R, \quad (22)$$

$$PV_{\max} - PV_r \geq 0, \quad r \in R, \quad (23)$$

$$f_r \geq 0, \quad r \in R, \quad (24)$$

$$\sum_{r \in R} f_r - q = 0, \quad (25)$$

$$f_a = \sum_{r \in R} \delta_a^r f_r, \quad a \in A, \quad (26)$$

$$disu_a = -\theta t_a - c_a, \quad a \in A_v, \quad (27)$$

$$u_a = \int_0^{t_a} \bar{u}_a(\omega) d\omega, \quad a \in A_a, \quad (28)$$

$$u_r = \sum_{a \in A_v} disu_a \delta_a^r + \sum_{a \in A_a} u_a \delta_a^r, \quad r \in R, \quad (29)$$

$$\tilde{u} = \max\{\varphi_r(1 - \eta), r \in R\}, \quad 0 < \eta < 1, \quad (30)$$

$$v(u_{ri}) = \begin{cases} (u_{ri} - \tilde{u})^\alpha, & \tilde{u} \leq u_{ri} \\ -\lambda(\tilde{u} - u_{ri})^\beta, & \tilde{u} > u_{ri} \end{cases}, \quad r \in R, \quad 0 \leq i \leq 1/L, \quad (31)$$

$$\pi^+(p_i) = w(1 - P_{i-1}) - w(1 - P_i), \quad 0 \leq i \leq n, \quad (32)$$

$$\pi^-(p_{-j}) = w(P_{-j}) - w(P_{-j-1}), \quad m \leq -j \leq 0, \quad (33)$$

$$PV_r = \sum_{i=0}^n v(u_{ri}) \pi^+(p_i) + \sum_{i=-m}^{-1} v(u_{ri}) \pi^-(p_i), \quad (34)$$

where q denotes the total travel demand; $\varphi(\cdot)$ is the inverse cumulative distribution probability of path utilities. The lower case letters of disutility/utility in Equations (27)–(29) denote the deterministic variables, different from the capital expression in Equations (1)–(4) and (6), which represent random variables. Equations (22)–(26) define the user equilibrium flow patterns; Equations (27)–(29) specify the calculation of the utility of each path; Equations (30)–(34) determine the prospect value of each path in the discrete scheme based on the cumulative probability of path utility.

The equivalent variational inequality (VI) formulation of the prospect-based user equilibrium activity/travel choice condition can be further elaborated as follows.

Theorem 1: *The traffic flow pattern satisfying conditions and constraints Eqs. (22)–(34) is a prospect-based user equilibrium state for combined activity–travel choices if and only if it satisfies the following VI problem:*

$$\sum_{r \in R} (f_r - f_r^*) (-PV_r^*) \geq 0 \quad \text{for all } f \in \left\{ f \mid \sum_{r \in R} f_r = q \right\}, \quad (35)$$

where f_r^* is the equilibrium path flow. The equivalence between the VI and the user equilibrium conditions can be proved as follows.

Proof of necessity: We need to prove that the user equilibrium activity/route choices conditions in Equations (22)–(25) imply the VI (35). For any path r , a feasible path flow is $f_r \geq 0$. Multiplied by condition Equation (23), we have

$$f_r \cdot (PV_{\max} - PV_r^*) \geq 0. \quad (36)$$

As we know, the equilibrium path flow satisfies Equation (22) that

$$f_r^* \cdot (PV_{\max} - PV_r^*) = 0. \quad (37)$$

Subtracting the above equation, Equation (37) becomes

$$(f_r - f_r^*) \cdot (PV_{\max} - PV_r^*) \geq 0. \quad (38)$$

Summing Equation (38) for all paths r , it can be written as

$$\begin{aligned} \sum_{r \in R} (f_r - f_r^*) \cdot (PV_{\max} - PV_r^*) &= \sum_{r \in R} PV_{\max} \cdot (f_r - f_r^*) - \sum_{r \in R} PV_r^* \cdot (f_r - f_r^*) \\ &= PV_{\max} \cdot \sum_{r \in R} (f_r - f_r^*) - \sum_{r \in R} PV_r^* \cdot (f_r - f_r^*) \\ &= - \sum_{r \in R} PV_r^* \cdot (f_r - f_r^*) \geq 0. \end{aligned} \quad (39)$$

Therefore, we obtain the VI Equation (35). ■

Proof of sufficiency: We need to prove that any user equilibrium solution $\{f_r^*\}$ to the VI (Equation (35)) would satisfy conditions Equations (22)–(24). Following the definition, it is obvious that Equations (23) and (24) always hold; thus, it is only required to prove that condition (Equation (22)) also holds. As assuming $\{f_r^*\}$ is a solution to the VI Equation (35), which means for each path r , we can always find a maximal prospect value based on the flow pattern $\{f_r^*\}$. That means, we can always find

$$PV_{\max} = PV_r^*, \quad \forall r. \quad (40)$$

The set of feasible path flows can make the following equality always hold:

$$f_r \cdot (PV_{\max} - PV_r^*) = 0. \quad (41)$$

Combine (41) with the VI (35), we can construct that

$$\sum_{r \in R} f_r \cdot (PV_{\max} - PV_r^*) - \sum_{r \in R} (f_r - f_r^*) \cdot (PV_{\max} - PV_r^*) = \sum_{r \in R} f_r^* \cdot (PV_{\max} - PV_r^*) \leq 0. \quad (42)$$

Since $f_r^* \geq 0$ and $PV_{\max} \geq PV_r(f_r^*)$, then

$$\sum_{r \in R} f_r^* \cdot (PV_{\max} - PV_r^*) = 0. \quad (43)$$

The above equation is the same as the condition Equation (22). Thus, the conditions (Equations (22)–(24)) always hold for any solution $\{f_r^*\}$ to the VI (35). ■

Apparently, the feasible set of the above VI is convex and continuous. It can be proved that at least one solution exists for the VI problems. In general, the uniqueness of the solution cannot be guaranteed, given the complex form of path utility and reference point. If the activity utility follows normal distribution, the mean and standard deviation of path utility will be obtained (Shao, Lam, and Tam 2006; Fu and Lam 2014). Thus, the reference point and the prospect value can be calculated in the continuous scheme. Furthermore, the model can be reduced to the trip-based user equilibrium model with endogenous reference points proposed by Xu et al. (2011).

3.3. Solution algorithm

In this section, an iterative algorithm is presented to solve the equilibrium solutions of the proposed modeling framework. The iterative algorithm is analogous to the 'day-to-day' route swapping process. Individuals schedule their activities and trips to maximize their prospect value. This is equivalent to finding the path with the maximum prospect value from origins to destinations under their expected gain utility probability in the activity–travel network. Therefore, to satisfy the equilibrium conditions, individuals will adjust their travel choices and activity durations based on increasing daily activity–travel experiences (Smith and Wisten 1995; Nagurney and Zhang 1997).

Due to the uncertainty of passenger flows and activity durations, individuals cannot find the absolute optimal activity–travel path but the first-order stochastic dominant path. The definition of first-order stochastic paths is that, path r_A has first-order stochastic dominance over path r_B if for any path utility u , r_A gives at least as high a probability of obtaining at least u as r_B . The definition can be expressed as follows:

Definition 1 (First-order stochastic dominance path): *Given two paths $r_A \neq r_B$, if r_A first-order stochastic dominance over r_B , so that for any path utility u , $P(u_{r_A} \geq u) \geq P(u_{r_B} \geq u)$, and for some path utility u , $P(u_{r_A} \geq u) > P(u_{r_B} \geq u)$.*

In terms of the cumulative distribution functions of the two paths, r_A first-order stochastic dominating r_B means that $\Psi_A(u) \leq \Psi_B(u)$, for any path utility u , with strict inequality at some u . Based on this definition, the utility of the first-order stochastic dominant path in this study is considered as the reference point. Given the expected gain probability for each individual, they will schedule their activity–travel path based on the reference point so as to get their maximum prospect value.

Most conventional solution algorithms cannot be directly used to solve the proposed model, as the probability distributions of activity utility and path utility do not follow the normal distribution. The discrete method to calculate the cumulative distribution function of both utilities proposed by Wu and Nie (2009) makes the problem easier, as the cumulative probability can be applied to the weighting function of CPT directly. In addition, as a simulation-based method, the route swapping method has the forced convergence property. Thus, a solution algorithm based on the route swapping method is proposed for solving the problem. The reference points will be adjusted until the equilibrium flow pattern has been obtained.

The solution algorithm for solving the proposed prospect-based user equilibrium model is elaborated in the following steps. As the convergence threshold is 0.001, it is proved to converge after thousands of iterations.

Step 0: Initialization. Set parameters $\rho > 0$, iteration index $\kappa = 1$ and the convergence tolerance $\varepsilon > 0$. Choose scale parameters sequence $\rho_\kappa = \rho / \text{INT}((\kappa + 1)/\nu + 0.5)$ as Huang and Lam (2002) and specify an initial path flow pattern;

Step 1: Calculation of the cumulative distribution of path utilities. Set $k = 0$. For $l_1 = 1, \dots, (m + n)$; for $l_2 = 1, \dots, (m + n)$: set $k = k + 1$, $z_k = \varphi_r^j(l_1 \cdot L) + \hat{b}_{l_2}^{ij}$, $L \cdot (m + n) = 1$, ij denotes one link on activity–travel path r , and $\hat{b}_{l_2}^{ij}$ is a value on each interval $[\hat{b}_{l_2-1}^{ij}, \hat{b}_{l_2}^{ij}]$, according to Wu and Nie (2009). Sort z_k in an ascending order. Construct the inverse CDF φ_r of path utility.

Step 2: Determination of the reference point. Based on the current flow pattern $f(\kappa)$, calculate the link flow pattern, and the discrete scheme cumulative distribution of each path utility. Set expected probability of gaining utility η and find the subscript i with minimal gap from $1 - \eta$. Further determine the reference points that satisfy the equation $\tilde{u} = \max\{u|u_{ri}, i = \min\{i||P_{ri} - (1 - \eta)| \approx 0\}, r \in R\}$.

Step 3: Calculation of prospect values. Calculate the activity–travel prospect value for each path according to (30)–(34);

Step 4: Convergence check. Due to the complexity of the CPT and the discrete nature of endogenous generation of reference points, the monotonicity of the prospect value is unsure. Thus, assume the swapping to be proportional to the flow on the more prospect route multiplied by the prospect difference as $|\sum_{r \in R} f_r^\kappa \cdot (PV_{\max}^\kappa - PV_r^\kappa) / \sum_{r \in R} f_r^\kappa \cdot PV_{\max}^\kappa|$. The iteration terminates if the swapping vector is less than or equal to the tolerance ϵ , then stop. Otherwise, go to step 5.

Step 5: Update path flows. Compute

$$f_r^{\kappa+1} = \begin{cases} f_r^\kappa + \frac{\sum_{r \notin \Theta^\kappa} (f_r^\kappa - f_r^{\kappa+1})}{|PV_{\max}^\kappa - PV_r^\kappa|}, & r \in \Theta^\kappa \\ \max\{0, f_r^\kappa - \rho_\kappa \cdot f_r^\kappa \cdot (PV_{\max}^\kappa - PV_r^\kappa)\}, & r \notin \Theta^\kappa, \end{cases} \quad (44)$$

where $\Theta^\kappa = \{r: PV_{\max}^\kappa = PV_r^\kappa, \forall r\}$. Let $\kappa = \kappa + 1$, and go back to Step 1.

Similar iterative rout-swapping algorithms have been adapted to solve many equilibrium models, for example, in Huang and Lam (2002) for combined departure time and route choice, Szeto and Lo (2006) for tolerance-based route choice, Ramadurai and Ukkusuri (2010) for combined activity location and route choice, Liu et al. (2015) for dynamic activity–travel assignment problem and Liu and Smith (2015) extend the algorithm for the day to day traffic re-routing problem. According to Nagurney and Zhang (1998), if $PV_e(\cdot)$ is continuous with any path flow pattern $\mathbf{f}$, the algorithm will converge to a stationary point when ρ_κ is set in a particular way, that is, $\lim_{\kappa \rightarrow \infty} \rho_\kappa = 0$ and $\sum_\kappa \rho_\kappa = +\infty$. Given that the discrete scheme is used to approximate the cumulative probability of path utilities, the obtained equilibrium state is only the simulated equilibrium satisfying the convergence criterion (Step 4).

4. Numerical example

The purpose of the numerical example is to illustrate the influence of reference points on the equilibrium state by investigating the interactions of activity duration and travel mode choice. This example only considers the work activity and bus\ taxi mode alternatives. Figure 3 shows the network representation of commuters who do not own a car and thus can choose either bus or taxi to commute in the morning to go to work. A mode decision will also be made when they go back home after work. Hence, commuters have four choice patterns, namely, ‘bus-work-bus’, ‘bus-work-taxi’, ‘taxi-work-bus’ and ‘taxi-work-taxi’. The parameters are shown in Table 1. The activity-specific parameters and parameters of the value and weighting function are set as Ettema and Timmermans (2003) and Xu et al. (2011). Other parameters in the BPR function are assumed values. To each group of setting, the simulated equilibrium state can be found within 2000–3000 iterations, taking 78.23 s on averages on a personal computer.

Concerning the interactions of activity duration and travel mode choice, some interesting results are obtained. Table 2 shows the equilibrium results of each path, with $\eta = 0.4$, $\theta = 5$, $\vartheta_a = 20$ and $q = 150$. It is obvious that when the mean work duration changes, passenger flows and prospect values of each path will change as well. For example, if the mean work duration is 7 h, every path has passengers on it, but most people will choose the bus-work-bus due to the low value of time. When the mean work duration is longer, the degree of uncertainty in the activity is greater. Note that by definition the variance of work duration changes in the same direction as the mean. It turns out no passenger will choose the fourth path. The increased work utility makes the reference point increase, and individuals will decrease the disutility of travel to make sure they can get their expected utility gain to compensate for the probable loss in activity participation. The higher reference will make individuals more sensitive to the processes, which could generate more losses.

Table 3 shows the equilibrium results for each path when θ is increased to 20 and ϑ_a is decreased to 5. Since the value of time increased, less people will choose the first path.

Figure 3. An activity–travel network with H-W-H pattern.

Table 1. Parameters of the functions.

BPR function	$\beta_{1,2}$ 0.15	$\gamma_{1,2}$ 4	$h_a \cdot g_a$ 60	C_a 90	Δ 30
Marginal utility function	$u_a^{\max}$ 1000	α_a 700	β_a 0.01	γ_a 1	
Prospect function	α 0.37	β 0.59	λ 1.51	γ 0.74	

Table 2. Comparison of equilibrium state on each path ($\eta = 0.4, \theta = 5, \vartheta_a = 20, q = 150$).

Path	Mean duration of work = 7 (h)			Mean duration of work = 8 (h)		
	f_r	PV_r	$\tilde{u}$	f_r	PV_r	$\tilde{u}$
1	90.1722	-36.6836	77.1542	73.2902	-53.8430	154.7875
2	21.6086	-36.6531		38.3547	-53.8253	
3	21.6086	-36.6531		38.3547	-53.8253	
4	16.6105	-36.8169		0	-54.2270	

Table 3. Effects of increasing value of time and decreasing unit price of taxi, compared to equilibrium state of each path ($\eta = 0.4, \theta = 20, \vartheta_a = 5$ and $q = 150$).

Path	Mean duration of work = 7 (h)			Mean duration of work = 8 (h)		
	f_r	PV_r	$\tilde{u}$	f_r	PV_r	$\tilde{u}$
1	58.0477	-36.6998	68.8354	11.1347	-53.8600	146.4297
2	0	-36.6748		46.7143	-53.8303	
3	0	-36.6748		46.7143	-53.8303	
4	91.9523	-36.6741		45.4367	-53.8756

Figure 4. The sensitivity analysis of expected probability of gaining utility for the reference points in equilibrium flow on each path ((a) $\theta = 20, \vartheta_a = 5$ and $\mu_a = 8$; b. $\theta = 5, \vartheta_a = 20$ and $\mu_a = 8$).

According to Tables 2 and 3, the reference utility and the uncertainty of activity duration have substantial influences on the equilibrium state.

Figure 4 shows the variation of path flow at equilibrium states with the expected probability η of gaining utility for the reference point. In Figure 4(a), the parameters are set as $\theta = 20, \vartheta_a = 5$ and $\mu_a = 8$; in Figure 4(b), $\theta = 5, \vartheta_a = 20$ and $\mu_a = 8$, both $q = 150$. As Figure 4(a) shows, when η is less than 0.8, no big changes in passenger flows on each path are observed. But when η becomes higher than 0.8, most people will choose the fourth path, since the travel disutility of taxi is low enough. It means there is a threshold of expected gain utility probability since the value of time is very low. When the probability is lower than the threshold, travelers are not very sensitive, and it will make differences when the probability is higher than the threshold. Figure 4(b) shows that increasing the expected probability to gain more utility, most passengers will choose the first path to decrease the disutility of travel. There is no difference between path 2 and path 3, because the time window of the

Figure 5. The sensitivity analysis of expected probability of gaining utility in prospect value of each path (a. $\theta = 20$, $\vartheta_a = 5$ and $\mu_a = 8$; b. $\theta = 5$, $\vartheta_a = 20$ and $\mu_a = 8$).

Figure 6. The sensitivity analysis of expected probability of gaining utility in the reference utility.

work activity has not been considered in this paper (we leave it for future work). The results above indicate that our model can be used to illustrate the influence of value of time and the cost coefficient on travelers' mode choice.

Figure 5 gives the sensitivity analysis of expected probability of gaining utility in prospect value of each path. Combined with Figure 6, when the expected probability increases, the reference utility will decrease and the prospect value will increase. There is little difference of prospect value among the paths, which states that results obtained are approaching to equilibrium. Besides, the value of time and the cost coefficient has little influence on prospect value on each path and the reference utility at equilibrium.

Figure 7 depicts the sensitivity analysis of uncertain degree of work duration with various η on reference utility (a) and prospect value of second path (b) at equilibrium state. Figure 7(a) shows that when the mean duration of work increases, the reference utilities

Figure 7. Sensitivity analysis of uncertain degree of work duration with various η on reference utility (a) and prospect value of second path (b) at equilibrium state.

Table 4. Equilibrium state on each path for RDEU-based model.

Path		1	2	3	4
$\eta = 0.5, \mu_a = 7$	f_r	57.8495	0	0	92.1505
	u_r	173.6971	173.2084	173.2084	173.9367
$\eta = 0.4, \mu_a = 7$	f_r	57.8495	0	0	92.1505
	u_r	173.6971	173.2084	173.2084	173.9367
$\eta = 0.4, \mu_a = 8$	f_r	54.7660	0	0	95.2340
	u_r	390.9749	390.6150	390.6150	391.7432

turn out to be bigger than when duration is smaller. This is because the work utility has a great influence on path utility. A small change in duration will have a big impact on the reference utility. When the expected gain probability increases, all reference utilities decrease. That means that if travelers expect to derive more utilities from the same situation, they would lower their reference point. However, as Figure 7(b) shows, the prospect values of the second path are reduced with longer working duration under the same expected gain utility probability, when it is less than 0.8. Otherwise, the prospect values will increase as the expected gain probability increases.

In order to further study the properties of the prospect-based model, we compare its performance with the RDEU based model. Let α, β, λ be 1 and the reference point be zero, the equilibrium state on each path with different mean work duration is shown in Table 4 ($\theta = 20, \vartheta_a = 5$). It shows that without the reference, the passenger flow mainly reflects the mean duration since the equilibrium flow is unchanged when η decreases to 0.4. Only a few people will move to the other paths when work duration is longer, which indicates that the influence of the uncertain degree of work duration is smaller than that in the prospect-based model as Table 2 shows. However, the RDEU-based model does not consider the individuals' risk-seeking preferences. Thus, the expected gain utility probability does not work. Figure 8 depicts the comparison of the equilibrium passenger flow on each path for different mean durations on RDEU-based model and prospect-based model with $\eta = 0.5$. As Figure 8(a) shows, people would always prefer path 4 to other alternatives even the uncertain degree of work duration is greater. Besides, when the mean duration is longer than 9 h, which is longer enough to compensate the disutility of travel, most travelers will

Figure 8. Sensitivity analysis of the uncertain degree of work duration in equilibrium passenger flow on each path for two UE models.

choose the second and the third path. However, as Figure 8(b) shows, the variation of equilibrium flow occurs after 7 h. It means that when the risk attitude of individuals is fixed, there is a threshold range where people are sensitive to changing activity duration.

As illustrated above, the activity-based modeling approach can be used to model travelers' decision-making process pertaining to mode choice under uncertain travel time and activity duration. Moreover, the proposed model demonstrates the interaction between activity duration and transport mode choice. However, the model demands far more detailed input data. For instance, the parameters of the marginal utility function (see Equation (4)) for each activity and the prospect functions (see Equations (8) and (9)) need to be calibrated.

5. Conclusions

This paper proposed a prospect-based user equilibrium model for combined activity–travel choices with discrete endogenous reference in an extracted network from the multi-state supernetwork representation. Each route from the origin (home) to the destination (home) denotes a feasible daily activity–travel pattern, which consists of activity links and travel links. Individual's daily scheduling is made under a reference point so as to maximize the prospect value. Based on the reference point, individuals consider the outcome of an activity–travel path a gain or a loss. It is shown that the daily scheduling problem can be transformed into the prospect-based traffic assignment problem. Assuming that individuals are prospect maximizers, user equilibrium can be achieved in a long run, where the prospect values of all used paths are equal, and no less than the prospect values of any unused paths. The user equilibrium model is formulated as a VI problem and solved by a heuristic algorithm. In conclusion, the daily activity–travel scheduling problem on a network is transformed into a static traffic assignment problem from the perspective of PT. The influence of people's activity-related decisions on transport mode choices can be addressed by our model. Moreover, the expected gain probability and the uncertain degree of activity duration have a major influence on the equilibrium state.

However, there are still several limitations in the current study. First of all, the assumption that utility of activity participation only depends on activity duration is limited. For example, Ettema et al. (2007) found that activity duration, timing and chain are the three major determinants of activity participation. The integration of the proposed PT-based equilibrium model into the full multi-state supernetwork representation potentially solves this limitation. Second, the scheduling problem under time constraints has not been considered, while activity participation is generally subject to these constraints. Third, time and money belong to different mental accounts of travelers, meaning that multi-reference points may exist. Fourth, the monotonicity of the prospect value needs to be theoretically analyzed. Furthermore, the heterogeneity of travelers has not been taken into account. These issues for elaborating the model will be addressed in our future work.

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Appendix 1

The detailed manipulations on the computation of the mean and standard deviation of $DISU_a$ are given as follows.

The mean travel time and the variance of travel link a is

$$E(T_a) = \int_{-\infty}^{+\infty} t_a(x) \phi_a(x) dx, \quad \forall a \in A_v, \quad (A1)$$

$$\text{Var}(T_a) = E(T_a^2) - (E(T_a))^2, \quad \forall a \in A_v. \quad (A2)$$

For transit links $a \in A_{v1}$, the mean travel time is

$$E(T_a) = \int_{-\infty}^{+\infty} t_a^0 \left(1 + \beta_1 \left(\frac{F_a}{h_a \cdot g_a} \right)^{\gamma_1} \right) \cdot \frac{1}{\sqrt{2\pi} c v_{a \in A_a} \cdot f_a} \cdot \exp \left(-\frac{(x - f_a)^2}{2 \cdot (c v_{a \in A_a} \cdot f_a)^2} \right) dx, \quad \forall a \in A_{v1}. \quad (A3)$$

Substituting $z = (x - f_a)/(c v_{a \in A_a} \cdot f_a)$ into the Equation (A3), then we can have

$$\begin{aligned} E(T_a) &= \frac{t_a^0}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(1 + \beta_1 \left(\frac{z \cdot c v_{a \in A_a} \cdot f_a + f_a}{h_a \cdot g_a} \right)^{\gamma_1} \right) \cdot \exp \left(-\frac{z^2}{2} \right) dz \\ &= t_a^0 + \frac{t_a^0 \beta_1}{\sqrt{2\pi} \cdot (h_a \cdot g_a)^{\gamma_1}} \int_{-\infty}^{+\infty} (\beta_1 (z \cdot c v_{a \in A_a} \cdot f_a + f_a)^{\gamma_1}) \cdot \exp \left(-\frac{z^2}{2} \right) dz, \quad \forall a \in A_{v1}. \end{aligned} \quad (A4)$$

According to Taylor's expansion, Equation (A4) becomes

$$\begin{aligned} E(T_a) &= t_a^0 + \frac{t_a^0 \cdot \beta_1}{\sqrt{2\pi} \cdot (h_a \cdot g_a)^{\gamma_1}} \int_{-\infty}^{+\infty} \sum_{i=0}^{\gamma_1} \binom{\gamma_1}{i} (z \cdot c v_{a \in A_a} \cdot f_a)^i (f_a)^{\gamma_1-i} \cdot \exp \left(-\frac{z^2}{2} \right) dz \\ &= t_a^0 + \frac{t_a^0 \cdot \beta_1}{\sqrt{2\pi} \cdot (h_a \cdot g_a)^{\gamma_1}} \sum_{i=0}^{\gamma_1} \binom{\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_1} \cdot \int_{-\infty}^{+\infty} z^i \exp \left(-\frac{z^2}{2} \right) dz \\ &= t_a^0 + \frac{t_a^0 \cdot \beta_1}{\sqrt{2\pi} \cdot (h_a \cdot g_a)^{\gamma_1}} \sum_{i=0, i=\text{even}}^{\gamma_1} \binom{\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_1} \cdot \sqrt{2\pi} (i-1)!! \\ &= t_a^0 + \frac{t_a^0 \cdot \beta_1}{(h_a \cdot g_a)^{\gamma_1}} \sum_{i=0, i=\text{even}}^{\gamma_1} \binom{\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_1} \cdot (i-1)!!, \quad \forall a \in A_{v1}. \end{aligned} \quad (A5)$$

And the variance is

$$\text{Var}(T_a) = E(T_a^2) - (E(T_a))^2 = \int_{-\infty}^{+\infty} (t_a(x))^2 \phi_a(x) dx - \left(\int_{-\infty}^{+\infty} t_a(x) \phi_a(x) dx \right)^2, \quad \forall a \in A_{v_1}. \quad (\text{A6})$$

In which,

$$\begin{aligned} & \int_{-\infty}^{+\infty} (t_a(x))^2 \phi_a(x) dx \\ &= \int_{-\infty}^{+\infty} \left(t_a^0 \left(1 + \beta_1 \left(\frac{F_a}{h_a \cdot g_a} \right)^{\gamma_1} \right) \right)^2 \cdot \frac{1}{\sqrt{2\pi} c v_{a \in A_a} \cdot f_a} \cdot \exp \left(-\frac{(x - f_a)^2}{2 \cdot (c v_{a \in A_a} \cdot f_a)^2} \right) dx \\ &= (t_a^0)^2 \cdot \int_{-\infty}^{+\infty} \left(1 + 2 \cdot \frac{\beta_1}{(h_a \cdot g_a)^{\gamma_1}} \cdot (z \cdot c v_{a \in A_a} \cdot f_a + f_a)^{\gamma_1} \right. \\ & \quad \left. + \frac{\beta_1^2}{(h_a \cdot g_a)^{2\gamma_1}} \cdot (z \cdot c v_{a \in A_a} \cdot f_a + f_a)^{2\gamma_1} \right) \cdot \exp \left(-\frac{z^2}{2} \right) dz \\ &= (t_a^0)^2 + 2 \frac{(t_a^0)^2 \cdot \beta_1}{(h_a \cdot g_a)^{\gamma_1}} \sum_{i=0, i=\text{even}}^{\gamma_1} \binom{\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_1} \cdot (i-1)!! \\ & \quad + \frac{(t_a^0 \cdot \beta_1)^2}{(h_a \cdot g_a)^{2\gamma_1}} \sum_{i=0, i=\text{even}}^{2\gamma_1} \binom{2\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{2\gamma_1} \cdot (i-1)!!, \quad \forall a \in A_{v_1}. \end{aligned} \quad (\text{A7})$$

Combine with Equation (A5), Equation (A6) can be get as

$$\begin{aligned} \text{Var}(T_a) &= \frac{(t_a^0 \cdot \beta_1)^2}{(h_a \cdot g_a)^{2\gamma_1}} \sum_{i=0, i=\text{even}}^{2\gamma_1} \binom{2\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{2\gamma_1} \cdot (i-1)!! \\ & \quad - \left(\frac{t_a^0 \cdot \beta_1}{(h_a \cdot g_a)^{\gamma_1}} \sum_{i=0, i=\text{even}}^{\gamma_1} \binom{\gamma_1}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_1} \cdot (i-1)!! \right)^2, \quad \forall a \in A_{v_1}. \end{aligned} \quad (\text{A8})$$

With the similar manipulations, the mean and the variance travel time on non-transit travel links can be obtained.

$$E(T_a) = t_a^0 + \frac{t_a^0 \cdot \beta_2}{(C_a)^{\gamma_2}} \sum_{i=0, i=\text{even}}^{\gamma_2} \binom{\gamma_2}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_2} \cdot (i-1)!!, \quad \forall a \in A_{v_2}, \quad (\text{A9})$$

$$\begin{aligned} \text{Var}(T_a) &= \frac{(t_a^0 \cdot \beta_2)^2}{(C_a)^{2\gamma_2}} \sum_{i=0, i=\text{even}}^{2\gamma_2} \binom{2\gamma_2}{i} (c v_{a \in A_a})^i \cdot (f_a)^{2\gamma_2} \cdot (i-1)!! \\ & \quad - \left(\frac{t_a^0 \cdot \beta_2}{(C_a)^{\gamma_2}} \sum_{i=0, i=\text{even}}^{\gamma_2} \binom{\gamma_2}{i} (c v_{a \in A_a})^i \cdot (f_a)^{\gamma_2} \cdot (i-1)!! \right)^2, \quad \forall a \in A_{v_2}. \end{aligned} \quad (\text{A10})$$

As Equation (3), the mean and the variance of $DISU_a$ can be gotten directly,

$$\text{dis } \mu_a = \begin{cases} -\theta E(T_a) - \tau_a, & \forall a \in A_{v_1} \\ -(\theta + \vartheta_a) E(T_a), & \forall a \in A_{v_2} \end{cases}. \quad (\text{A11})$$

$$(\sigma_a)^2 = \begin{cases} \theta^2 \text{Var}(T_a), & \forall a \in A_{v_1} \\ (\theta + \vartheta_a)^2 \text{Var}(T_a), & \forall a \in A_{v_2} \end{cases}. \quad (\text{A12})$$