MASTER

Modelling of the dynamic behaviour of the DCPBH-DFB semiconductor laser

Ariaans, J.L.M.H.

Award date:
1990

Link to publication

Disclaimer
This document contains a student thesis (bachelor's or master's), as authored by a student at Eindhoven University of Technology. Student theses are made available in the TU/e repository upon obtaining the required degree. The grade received is not published on the document as presented in the repository. The required complexity or quality of research of student theses may vary by program, and the required minimum study period may vary in duration.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
MODELLING OF THE DYNAMIC BEHAVIOUR OF THE DCPBH-DFB SEMICONDUCTOR LASER

By J.L.M.H. Ariaans

Verslag van het afstudeerwerk
Afstudeerhoogleraar: Prof. ir. G.D. Khoe.

De faculteit Elektrotechniek van de Technische Universiteit Eindhoven aanvaardt geen verantwoordelijkheid voor de inhoud van stage- en afstudeerverslagen.
Acknowledgements

First I wish to express my thanks to prof. G.D. Khoe for asking me to perform my graduation work at Philips Research Laboratories in Eindhoven and to dr. T.J.B. Swanenburg for giving me the opportunity to work in his 'Wideband Communication Systems' group.

Moreover, I would like to thank all the members of the group for making my stay at the Philips Research Laboratories enjoyable. Special mentions go to P. Mols for his theoretical contributions, P. Drahi for his mathematical advice and J. v. Steenwijk for untying the knots that the FORTRAN-compiler presented me with. Despite this, a lot of credit goes to the members of Numerical Algorithms Group for writing some very useful routines.

Credit goes to my brother, L. Ariaans, for checking the integrations carried out.

Finally, I have to thank K. Wright for digging through the bad English of the uncorrected version of this report.
Abstract

To further the work on laser diode modelling a Distributed FeedBack type laser model was wanted, which transverse laser structure is of the Double Channel Planar Buried Heterostructure type.

In this report a model is derived for the above mentioned laser type. Basically, the model is written to predict small signal behaviour, thereby not including the parasitics introduced by the surroundings of the active layer. Despite the linearisations around threshold, the results for larger signals are promising. This is caused by the fact that the carrier density for larger values of pump current doesn't change significantly.

Apart from the AC/DC-behaviour the model predicts the optical power inside the laser as a function of all carthesian coordinates.

The calculated values for the $\kappa L$-product are relatively low compared to real conditions, which is a result from the applied theory that describes the electric field shape in the transverse plane.

Although promising, the AC/DC results can be improved by updating the values of the constants used in the rate equations as soon as they are available in literature and by including the electrical losses in the modelling.

After having carried the proposed adjustments it will be useful to compare the simulated results with experimental results.
CONTENTS

Abstract

List of Symbols .......................................................... 3

1.0 Introduction .......................................................... 8

2.0 Determination of the Field Shape in the Transverse Plane ....................... 11
  2.1 General ............................................................ 11
  2.2 Determination of the Wave Numbers ................................ 11
  2.3 Determination of the Confinement Factor ............................ 15

3.0 Calculation of the Longitudinal Modes in a DFB Laser .......................... 18
  3.1 General ............................................................ 18
  3.2 Analysis of the Electric Field inside a DFB Laser ..................... 18
    3.2.1 Theory ......................................................... 18
    3.2.2 Adapting the Mirror Phases ................................ 23
  3.3 Calculation of the Light Intensity in a DFB Laser ...................... 24

4.0 Determination of the Optical Response to an Electrical Excitation ............ 27
  4.1 General ............................................................ 27
  4.2 Carrier Rate Equation ............................................. 28
    4.2.1 Radiative and Non-radiative Recombination .................... 28
    4.2.2 Stimulated Recombination ................................... 30
    4.2.3 Diffusion Mechanism ........................................ 34
  4.3 Field Rate Equation ............................................... 35
    4.3.1 Stimulated Radiative Recombination and Internal Losses ....... 36
    4.3.2 Spontaneous Recombination .................................. 36
    4.3.3 Refractive Index Dependence ................................ 37
  4.4 Adapting the Rate Equations for Numerical Treatment ....................... 38
  4.5 Applied Constants ................................................ 40
  4.6 Modelling the Surroundings of the Active Layer .......................... 41

5.0 Simulations and Results ............................................ 42
  5.1 General ............................................................ 42
  5.2 z-Dependence of the Power and Carrier Density ............................ 43
5.3 DC-Behaviour .................................................. 47
5.4 AC-Behaviour .................................................. 50
5.5 Transient Behaviour .......................................... 54

Conclusions .......................................................... 60

Recommendations ................................................... 61

References ............................................................ 62

APPENDIX A: Applying the Boundary Conditions in the Transverse Plane ............... 66
  Determination of the Wave Numbers .................................. 66
  Determination of the Confinement Factor .......................... 69

APPENDIX B: Derivation of the Constant of the Wave Equation ............................. 75

APPENDIX C: Derivation of the Optical Intensity .................................................... 78

APPENDIX D: Developing the Rate Equations for Numerical Treatment .................... 81
  z-Dependence of the Field and the Intensity ........................ 81
  Rewriting the Rate Equations ....................................... 82
InGaAsP stands for $In_{0.4}Ga_{0.6}As_{0.85}P_{0.15}$ with a bandgap of 0.80 eV and a wavelength of 1.55 μm for the emitted light.

Wherever they appear, the subscripts f,d,a,t,s,l,r are used to denote the part of the laser structure and their meaning as follows:

- **f**: film or active layer
- **d**: dielectric or waveguiding layer
- **a**: anti meltback layer
- **t**: top layer
- **s**: substrate
- **l**: left confining layer of the active stripe
- **r**: right confining layer of the active stripe

In case the symbols 'V' and 'W' are mentioned together with the symbol 'cm' in the following list of symbols, they both have to be expressed in kg-cm-s-1 instead of in kg-m-s-1 except for μ, the electron mobility. This means: $[V] = [kg \text{ cm}^2 \text{ A}^{-1} \text{ s}^{-1}] = 10^{-8}[kg \text{ m}^2 \text{ A}^{-1} \text{ s}^{-1}]$ and $[W] = [kg \text{ cm}^2 \text{ s}^{-1}] = 10^{-4}[kg \text{ m}^2 \text{ s}^{-1}]$.

- **a**: $\tau_a(\tau_a + \tau_{rad})$ [-]
- **a**, **a**: help variables in $f_s$ [-]
- **A**: help variable in the determination of $\phi_{h}$ [-]
- **A**: $\beta_d/2\alpha_0$ $[A^2 s^-1 kg^{-1} cm^{-2}]$
- **A**: imaginary surface $[m^2]$
- **b**, **b**: help variables in $f_s$ [-]
- **B**: help variable in the determination of $\phi_{h}$ [-]
- **B**: radiative coefficient $[cm^2 s^{-1}]$
- **B**: radiative coefficient for excess carrier density $[cm^2 s^{-1}]$
- **c**: free space light velocity $[cm s^{-1}]$
- **c**: $r_3/r_1$ [-]
- **C**: Auger coefficient $[cm^6 s^{-1}]$
- capacitor, representing the parasitics in the laser structure [F]

$C_i$  
Auger coefficient for excess carrier density $[cm^3s^{-1}]$

d  
thickness of the waveguiding layer $[cm]$

d_{max}  
maximum thickness of the waveguiding layer $[cm]$

d_{min}  
minimum thickness of the waveguiding layer $[cm]$

d_1  
s_1/r_1\ [\ - \ ]$

d_2  
s_2/r_1\ [\ - \ ]$

D  
- electron diffusion constant in InGaAsP $[cm^2s^{-1}]$

$D(z)$  
imaginary part of the z-dependence of the electric field $[-]$

$D_r(z)$  
real part of the z-dependence of the electric field $[-]$

$(e_i)$  
unity vector in the x-direction

$(e_y)$  
unity vector in the y-direction

$(e_z)$  
unity vector in the z-direction

E  
space dependent part of the electric field $[V/cm]\$

$E_r$  
space dependent part of the electric field $[V/cm]\$

$E_{x,y}$  
amplitude of the electric field for x-dependence, y-dependence respectively $[V/cm^{-1}]$

$E_{x1}, E_{x2}$  
field amplitudes in the active layer with positive and negative x-dependence respectively $[V/cm^{-1}]$

$F_{gap}$  
bandgap energy of InGaAsP, 0.80 $[eV]$  

$F_{q1}, F_{q2}$  
equations determining the right hand side of the two carrier rate equations $[cm^3s^{-1}]$

$E_l$  
left travelling part of the electric field $[V/cm]\$

$(F_k)_{x}$  
the electric field in the film, dependent only on the x-coordinate and polarized in the y-direction $[V/cm]\$

$f_k(z)$  
z-dependence of the electric field $[-]$

$f_k(z)$  
imaginary part of the z-dependence of the electric field $[-]$

$f_k(z)$  
real part of the z-dependence of the electric field $[-]$

$f_k, f_k(z)$  
function containing the z-dependence of the intensity in the laser $[-]$

$f_k, f_k(z)$  
second derivative of the z-dependence of the intensity with respect to z $[-]$

G  
gain function $[cm^{-1}]$

$G_{h1...h12}$  
equations determining the gain contributions $[cm^3s^{-1}]$ and $[cm^{-1}]$ for $h7...h12$

$G_{s}$  
slope of the gain function as a function of $r_{et}[cm^2s]$  

$G_{th}$  
- threshold material gain necessary for lasing, supposed to be a constant $[cm^{-1}]$

- threshold material gain necessary for lasing, as a function of $r_{et}[cm^3s^{-1}]$

$\hbar$  
Planck's constant, 6.63.10^{-34} $[kg cm^2s^{-1}]$

$H$  
space dependent part of the magnetic field $[A/cm^{-1}]$

$\hat{H}$  
space dependent part of the magnetic field $[A/cm^{-1}]$

$H_l$  
left travelling part of the magnetic field $[A/cm^{-1}]$

$(H_k)_{x,z}$  
magnetic field dependent on the x-coordinate and polarized in the z-direction $[A/cm^{-1}]$

I  
pumping current $[A]$  

$I_{1,2}$  
equations determining the right hand side of the two carrier rate equations $[cm^3s^{-1}]$

$I_{2,3,22}$  
integration results $[-]$
\[ I_{int} \quad \text{integration results} \quad [\text{--}] \]

\[ J \quad q_{\text{eff}} \quad [A \ cm^{-3}] \]

\[ k \quad \text{Boltzmann's constant, } 1.38 \times 10^{-23} \ [J \ K^{-1}] \]

\[ k_x, k_y \quad \text{wavenumber in the x- and y-direction respectively} \quad [cm^{-1}] \]

\[ k(z) \quad \text{'constant' of the wave equation} \quad [cm^{-1}] \]

\[ k_0 \quad \text{free space wave number} \quad [cm^{-1}] \]

\[ l_l \quad \text{length associated with the left mirror round trip phase} \quad [cm^{-1}] \]

\[ l_r \quad \text{length associated with the right mirror round trip phase} \quad [cm^{-1}] \]

\[ L \quad \text{total laser length} \quad [cm^{-1}] \]

\[ L_{in} \quad \text{internal length, } 2m \lambda, m \text{ integer} \quad [cm^{-1}] \]

\[ n \quad \text{- constant part of the refractive index} \quad [\text{--}] \]

\[ n_{eff} \quad \text{effective refractive index} \quad [\text{--}] \]

\[ n_{eff,s} \quad \text{effective refractive index, resulting from applying the boundary conditions on the} \]

\[ \text{x-dependence of the fields only} \quad [\text{--}] \]

\[ n_f \quad \text{refractive index of the film} \quad [\text{--}] \]

\[ n_{ref} \quad \text{refractive index} \quad [\text{--}] \]

\[ n_h \quad \text{carrier density at threshold} \quad [cm^{-3}] \]

\[ n_s \quad \text{space independent part of the carrier density} \quad [cm^{-3}] \]

\[ n_t \quad \text{amplitude of space dependent part of the carrier density} \quad [cm^{-3}] \]

\[ N \quad N_0 + n \quad [cm^{-3}] \]

\[ N_a \quad \text{acceptor density in quaternary material} \]

\[ N_0 \quad \text{equilibrium electron density in quaternary material} \quad [cm^{-3}] \]

\[ p \quad \text{excess hole density in quaternary material} \quad [cm^{-3}] \]

\[ p_d \quad \text{decay constant in the dielectric layer} \quad [cm^{-1}] \]

\[ P \quad P_0 + p \quad [cm^{-3}] \]

\[ \dot{P} \quad \text{total power density} \quad [W cm^{-2}] \]

\[ \overline{P} \quad \text{average power density} \quad [W cm^{-2}] \]

\[ \overline{P}_{av} \quad \text{average power density} \quad [W cm^{-2}] \]

\[ P_l \quad \text{power density at left mirror} \quad [W cm^{-2}] \]

\[ \text{Pow} \quad \text{power through imaginary transverse plane in active layer} \quad [W] \]

\[ P_r \quad \text{- right travelling power density} \quad [W cm^{-2}] \]

\[ P_{rl} \quad \text{- power density at right mirror} \quad [W cm^{-2}] \]

\[ P_s \quad \text{left travelling power density} \quad [W cm^{-2}] \]

\[ P_{ls} \quad \text{left travelling power density} \quad [W cm^{-2}] \]

\[ P_h \quad \text{equilibrium hole density in quaternary material} \quad [cm^{-1}] \]

\[ q \quad \text{electron's charge, } 1.6 \times 10^{-19} \ [C] \]

\[ r_i \quad \text{imaginary part of the field amplitude} \quad [V cm^{-1}] \]

\[ r_r \quad \text{real part of the field amplitude} \quad [V cm^{-1}] \]

\[ r_{\text{rad}} \quad \text{excess radiative recombination rate} \quad [cm^{-3}s^{-1}] \]

\[ r_{\text{Auger}} \quad \text{excess rate of Auger recombination} \quad [cm^{-3}s^{-1}] \]

\[ r_1, r_2 \quad \text{complex amplitudes of } R(z) \quad [V cm^{-1}] \]

\[ R \quad \text{resistance of the laser structure} \quad [\Omega] \]
resistor, necessary to match the input resistance of the laser device to 50 Ω [Ω]

$R_{\text{loss}}$ loss term in carrier rate equation [cm⁻³s⁻¹]

$R_{\text{pump}}$ pump term in carrier rate equation [cm⁻³s⁻¹]

$R_{\text{rad}}$ radiative recombination rate [cm⁻³s⁻¹]

$R_{\text{stim}}$ stimulated recombination rate [cm⁻¹]

$R_{\text{stim,5}}$ stimulated recombination rate related to the photon density [cm⁻¹]

$R(z)$ right travelling wave in a DFB [V cm⁻¹]

$R_{\text{Auger}}$ rate of Auger recombination [cm⁻³s⁻¹]

$R_{\text{Diff}}$ diffusion rate in active layer [cm⁻³s⁻¹]

$s_1, s_2$ complex amplitudes of $S(z)$ [V cm⁻¹]

$S$ photon density [cm⁻³]

$S(z)$ left travelling wave in a DFB [V cm⁻¹]

t thickness of the quaternary region [cm]

T temperature [K]

$v_g$ group velocity [cm s⁻¹]

$V_e$ volume of the active layer [cm³]

$V_{\text{ext}}$ voltage across the quaternary junction [V]

$w_1$ thickness of the quaternary region [cm]

$w_2$ $w_2 - w_1$; thickness of the anti meltback layer [cm]

$X$ number of times that $kT/q$ is to be added to $E_{\text{gap}}$ to get a convenient measure for inversion [– –]

$X(z)$ twice the real part of the r.h.s. of the field rate equation [s⁻¹]

$Y(z)$ twice the imaginary part of the r.h.s. of the field rate equation [s⁻¹]

$\alpha$ constant part of the laser gain [cm⁻¹]

$\alpha_{th}$ threshold gain for lasing mode [cm⁻¹]

$\alpha_r$ real part of $\gamma$ [cm⁻¹]

$\beta$ actual spatial frequency [cm⁻¹]

$\beta_s$ fraction of spontaneous emitted light, coupled into the lasing mode [– –]

$\beta_r$ imaginary part of $\gamma$ [cm⁻¹]

$\beta_0$ spatial Bragg frequency, $\pi/\Lambda$ [cm⁻¹]

$\beta_{1-6}$ help phases in the determination of $I_{n,11}$ ... $I_{n,23}$ [cm⁻¹]

$\gamma$ propagation constant [cm⁻¹]

$\Gamma$ confinement factor [– –]

$\delta$ $\beta_0 - \beta$, deviation from the Bragg frequency [cm⁻¹]

$\Delta n$ amplitude of the varying part of the refractive index [– –]

$\Delta \alpha$ amplitude of the varying part of the gain [cm⁻¹]

$\epsilon_r$ relative permittivity of the active layer [– –]

$\epsilon_0$ permittivity of free space $8.85 \times 10^{-12}$ [A²kg⁻¹m⁻³]

$\eta$ rate of variation of the refractive index with carrier density [cm²]

$\zeta$ factor to match the spontaneous radiation to real conditions [s]

$\kappa$ coupling constant [cm⁻¹]

$\lambda$ wavelength of the emitted light [cm]
\( \Lambda \)  
pitch, spatial period of the corrugation \( [cm] \)

\( \mu \)  
electron mobility \( [cm^2 V^{-1} s^{-1}] \)

\( \mu_0 \)  
magnetic permeability in free space, \( 4\pi \times 10^{-7} [kg \, cm \, A^{-2} \, s^{-2}] \)

\( \nu \)  
frequency of the emitted light \( [s^{-1}] \)

\( \rho \)  
\( \rho = \rho_L \rho_R \) [ - ]

\( \rho_L, \rho_R \)  
actual left and right reflectivity [ - ]

\( \hat{\rho}_L, \hat{\rho}_R \)  
left and right mirror reflectivity [ - ]

\( \sigma \)  
electric conductivity \( [\Omega^{-1} m^{-1}] \)

\( \tau_n \)  
non-radiative carrier lifetime \( [s] \)

\( \tau_{\text{rad}} \)  
\((\tau_{\text{rad}}^L + \tau_{\text{rad}}^R)^{-1} [s] \)

\( \varepsilon \)  
radiative carrier lifetime \( [s] \)

\( \phi \)  
argument of periodically varying effective index \( [\text{rad}] \)

\( \phi_{\text{f-L}} \)  
half phase shift at the film-left confining layer boundary \( [\text{radians}] \)

\( \phi_{\text{f-D}} \)  
half phase shift at the film-dielectric layer boundary \( [\text{radians}] \)

\( \phi_{\text{f-A}} \)  
half phase shift at the film-anti meltback layer boundary \( [\text{radians}] \)

\( \phi_{\text{L}, \text{L}} \)  
left mirror round trip phase \( [\text{radians}] \)

\( \phi_{\text{R}, \text{R}} \)  
right mirror round trip phase \( [\text{radians}] \)

\( \phi_{\text{h}, \text{h}} \)  
'help' phases \( [\text{radians}] \)

\( x, \xi \)  
\( \pm y + x - i \delta [cm^{-1}] \)

\( \psi \)  
\( p(x - w_y) [ - ] \)

\( \psi_1 \)  
\( 2x, z [ - ] \)

\( \psi_2 \)  
\( 2\beta_z [ - ] \)

\( \Psi^+, \Xi \)  
integral operators \( [-] \)

\( \omega \)  
angular frequency of the electric and magnetic waves \( [\text{radians} \, s^{-1}] \)

\( \Omega \)  
phase of the refractive index and gain \( [\text{radians}] \)
1.0 Introduction

As interest in communication by means of single mode fibers has grown in the last few years, the demand for single mode light sources has also grown. For this reason Philips is interested in lasers of the Distributed Feedback type. Their monomode characteristics are far better than those of Fabry-Perot type lasers. Since optical fibre losses are less at $1.55 \mu m$ than at $1.3 \mu m$, a shift has taken place to the longer wavelength.

The DFB lasers that are grown at the Philips Research Laboratories are based on the same structure as the Fabry-Perot type lasers. The transverse structure is of the DC-PBH type, which stands for Double Channel Planar Buried Heterostructure. There are two main differences compared with the Fabry-Perot type laser operating at a wavelength of $1.3 \mu m$, namely the periodically varying structure in the longitudinal direction and the presence of an anti meltback layer. The former is necessary to realize the feedback of energy of the forward and backward travelling waves into each other in order to select only one lasing mode and the latter is necessary to prevent the active layer consisting of $1.55 \mu m$ InGaAsP from being damaged by the growth of the $0.92 \mu m$ InP top layer. One cannot grow $0.92 \mu m$ material directly on to $1.55 \mu m$ material. For this reason a protective layer of $1.3 \mu m$ InGaAsP is first grown on the active layer. The laser is schematically presented in figure 1.1. The spatial periodicity of the waveguide caused by the so-called corrugation, which in the case of the Philips laser has a triangular shape, is 240 nm.

The small diagram in figure 1.1 depicts the whole laser device whereas the large diagram details the part that actually causes the lasing action. This part is situated in the middle of the small diagram. The black area in the latter indicates the 'proton bombarded region'. After the laser device has been grown, the black part in the small diagram of figure 1.1 is proton bombarded. This is done to diminish the parasitics of this part of the laser and therefore to improve the IIF-behaviour, since the parasitics would impose a bandwidth.

Both the top layer and the substrate consist of $0.92 \mu m$ InP and both the anti meltback layer and the waveguiding layer consist of $1.3 \mu m$ InGaAsP. The length of the Philips DFB is approximately 250 $\mu m$. All other important dimensions are quoted in the figure.

A modelling of the Fabry-Perot type laser operating at a wavelength of $1.3 \mu m$ has already been developed by Versleijen [1] and Mols [2]. Versleijen has modelled the DC properties whereas Mols has extended Versleijen's model to include AC behaviour. The model is presented in an input file for the circuit analysis program 'PHILIPAC'.
We want to derive a model that predicts approximately both the DC- and AC-behaviour of a DFB laser. The word 'approximately' is used, since several assumptions will be made in deriving the equations that describe the lasing behaviour and since the electric losses caused by the surroundings of the active layer will not be taken into account. Unfortunately we will not be able to integrate the DFB-characteristics in the PIII.PAC-modelling of the DC-PB1 structure otherwise the electric losses would have been taken into account. This is not possible because of the fact that PIII.PAC has to be called from another program that first determines the transverse field shape and the laser threshold before PHILPAC can be entered. Since it is not possible to call PHILPAC from another program written in FORTRAN, we have chosen to solve the DFB laser problem completely with a FORTRAN program, neglecting the influence of the surroundings of the active layer, for the time being.
2.0 Determination of the Field Shape in the Transverse Plane

2.1 General

The optical field inside the laser is confined to the active area and its direct surroundings. This is due to the waveguiding structure. Unlike a Fabry Perot type laser the waveguiding structure of a DFB laser varies periodically between the two end facets. Therefore one may not simply refer to 'the' transverse plane inside a DFB laser, since the transverse field shape is a function of the position inside the laser cavity. The variation thus depends on the shape of corrugation. For simplicity the wave numbers are only calculated for the smallest and largest thicknesses of the waveguide. Taking the average of the values results in 'the' wave numbers in the x- and y-direction. For these two cases also the effective indices of refraction are determined, which are necessary for the DFB theory which is set out in the next chapter.

In practice the corrugation has a triangular shape, necessarily resulting in a triangular dependence of the effective refractive index of the plane between the two end facets. The DFB theory presented by Streifer [4] is based on a harmonic dependence of the effective refractive index in the longitudinal direction. According to Thompson [5] the behaviour of the field inside a periodic waveguide is mainly determined by the period and not by the shape of the waveguide, which enables us to apply Streifer's theory to the case of a triangular shaped corrugation.

2.2 Determination of the Wave Numbers

Figure 2.1 shows the structure for which the transverse field shape is determined. In this figure, t is the width of the active layer, \( w_1 \) is the thickness of the active layer, \( w_2 - w_1 \) is the thickness of the anti meltback layer and \( d \) is the average thickness of the waveguiding layer. The \( n \)'s are the refractive indices of the respective layers. The outer bounds are situated at infinite distance from the origin. Note that the positions of the anti meltback layer and the waveguiding layer are interchanged compared with figure 1.1. The analysis that will be presented in this chapter relates to y-component of the electric field and is valid only if the modes are not too close to cutoff. The polarization of the field is in the y-direction, because in the x-direction polarized modes require a much higher lasing
Figure 2.1. Transverse cross-section of the laser structure

Calculation of the field shape of a 2-dimensional structure reduces to calculating twice the field shape of a 1-dimensional structure using the effective index method. First the laser structure along the y-coordinate is considered as shown in figure 2.2.a. Again, in this figure and in figure 2.2.b the outer bounds lie at infinite distance from the origin. For this structure the wave number in the y-direction can be calculated and accordingly an effective index of refraction for this structure can be derived. This effective index of refraction is used in the structure of figure 2.2.b as the refractive index of the active layer. Now the wave number in the x-direction can be determined. Finally the effective refractive index of the 2-dimensional structure as a whole can be calculated.

As the structure along the x-coordinate is more complex to handle, the field shape along this coordinate will be calculated as proposed by Wang [6]. The dependence on the y-coordinate will not be taken into account in the formulation. This is stressed by the addition of 'x' in the subscript of the fields in the various media. Of course only real modes which have a cosine shape in the film or active layer and become evanescent in the other media are considered. If the structure is characterized by five indices of refraction, \( n_f \) for the film or active layer, \( n_d \) for the dielectric or waveguiding layer, \( n_a \) for the anti meltback layer, \( n_t \) for the top layer and \( n_s \) for the substrate, there exist certain modes if \( n_f \) is greater than the other indices of refraction. As the lasing behaviour of a DFB laser is mainly determined by y-component of the electric field, the expressions for the other components will not be presented. If the transverse electric polarization in the y-direction is denoted by the subscript 'y', the electric fields in the various media can be written as is done in equations.
\begin{align*}
(E_y)_{f,x} &= E_f^1 \exp (ik_xx) + E_f^2 \exp (-ik_xx) \\
(E_y)_{d,x} &= E_d^1 \exp (p_d x) + E_d^2 \exp (-p_d x) \\
(E_y)_{a,x} &= E_a^1 \exp (p_a x) + E_a^2 \exp (-p_a x) \\
(E_y)_{l,x} &= E_l \exp (-p_l (x - d)) \\
(E_y)_{s,x} &= E_s \exp (p_s (x + w_2))
\end{align*}

In these equations $k_x$ is the wave number in the $x$-direction. The various decay constants are given by

\begin{align*}
p_d &= \left[ (n_f^2 - n_d^2)k_0^2 - k_x^2 \right]^{1/2} \\
p_a &= \left[ (n_f^2 - n_a^2)k_0^2 - k_x^2 \right]^{1/2} \\
p_l &= \left[ (n_f^2 - n_l^2)k_0^2 - k_x^2 \right]^{1/2} \\
p_s &= \left[ (n_f^2 - n_s^2)k_0^2 - k_x^2 \right]^{1/2}
\end{align*}

$k_0$ is the free-space wave number. Applying the Maxwell equations to equations (2.1) to (2.5) leads to the following magnetic fields in the various media.
The boundary conditions require that $E$, and $H$, are continuous at $x=0$ and $x=d$. Applying the boundary conditions and eliminating $E_x$ and $E_{dl}$ as is done in appendix A leads to

\[
\frac{E_{\alpha}}{E_\alpha} = \frac{k_x - i p_d A}{k_x + i p_d A} = \exp (-i2\phi_{E,\alpha})
\]

with

\[
A = \frac{p_d \tanh (p_d d) + p_t}{p_d + p_t \tanh (p_d d)}
\]

and

\[
\phi_{E,\alpha} = \arctan \left( \frac{p_d A}{k_x} \right)
\]

Using (2.15) $(E_y)_\alpha$ can be written

\[
(E_y)_\alpha = 2 E_\alpha \exp (-i\phi_{E,\alpha}) \cos (k_x x + \phi_{E,\alpha})
\]

Similarly applying the boundary conditions at $x=-w_1$ and $x=-w_2$ gives (appendix A)

\[
\frac{E_{\alpha}}{E_\alpha} = \frac{k_x - i p_d B}{k_x + i p_d B} \exp (-2ik_x w_1) = \exp (-i2\phi_{E,\alpha})
\]

with

\[
B = \frac{p_a \tanh (p_a (w_1 - w_2)) - p_z}{p_a - p_t \tanh (p_a (w_1 - w_2))}
\]

and

\[
\phi_{E,\alpha} = \arctan \left( \frac{p_d B}{k_x} \right) + k_x w_1
\]
Using (2.19) \((E_y)_{ax}\) can be written
\[
(E_y)_{ax} = 2E_{ij} \exp(-i\phi_{Ed}) \cos(k_sx + \phi_{Ed})
\]
(2.22)

Equations (2.18) and (2.22) must be equal. This is only possible if \(\phi_{Es}\) is equal to \(\phi_{Ed}\) plus an integer times \(\pi\) (appendix A), mathematically
\[
\phi_{Es} = \phi_{Ed} + q\pi, \quad q \text{ integer}
\]
(2.23)

As \(k_s\) appears explicitly in \(\phi_{Es}\) and in the arctangent functions of both \(\phi_{Es}\) and \(\phi_{Ed}\), equation (2.23) is a transcendental equation that has to be solved numerically. The solutions of (2.23) are the guided modes that can propagate in the structure. The order of the modes depends on the value of \(q\). In this approach the fundamental mode is found by setting \(q\) equal to zero. If (2.23) has been solved the effective refractive index can be determined by means of equation (2.24).
\[
N_{eff,x} = \frac{\sqrt{(k_s^2n_2^2-k_s^2)}}{k_0}
\]
(2.24)

2.3 Determination of the Confinement Factor

Eliminating all amplitudes except \(E_{ij}\) in order to arrive at the equation that governs the selection of a proper mode means in fact, expressing all amplitudes in \(E_{ij}\). So the electric fields in the various media can all be expressed in terms of the field amplitude in the active layer. Let \(E_{E,s}\) be equal to \(2E_{ij} \exp(-i\phi_{Ed})\) then the following expressions can be derived for the fields in the various media. For completion the derivation is carried out in appendix A.

\[
(E_y)_{fx} = E_{E,s} \cos(k_sx + \phi_{Ed})
\]
(2.25)

\[
(E_y)_{dx} = E_{E,s} \cos(\phi_{Ed}) \frac{\cosh(p_\theta x - \phi_0)}{\cosh(\phi_0)}
\]
(2.26)

\[
(E_y)_{rx} = E_{E,s} \cos(\phi_{Ed}) \frac{\cosh(p_\theta x - \phi_0)}{\cosh(\phi_0)} \exp(-\rho_1(x - d))
\]
(2.27)

\[
(E_y)_{ax} = E_{E,s} \frac{\cos(p_\theta (x + w_1) - \phi_1)}{\cosh(\phi_1)}
\]
(2.28)

\[
(E_y)_{sz} = E_{E,s} \cos(k_s w_1 - \phi_{Ed}) \frac{\cosh(p_\theta (w_1 - w_2) - \phi_1)}{\cosh(\phi_1)} \exp(p_\theta (x + w_2))
\]
(2.29)

With \(\phi_\theta\) and \(\phi_1\) given by:
\[
\phi_0 = p_\theta d + \text{artanh} \left( \frac{p_1}{p_d} \right)
\]
(2.30)
If the arguments of the area tangent hyperbolic functions in the above equations become greater than unity, \(\phi_0\) and \(\phi_1\) become complex. In this case the 'artanh'-functions have to be replaced by 'arcoth'-functions in equations (2.30) and (2.31) and the 'cosh'-functions have to be replaced by 'sinh'-functions in equations (2.25) to (2.29). The practical meaning of this is that substitution is necessary if the refractive indices of the outer layers are lower than those of the inner layers.

The complete analysis of the \(y\)-dependence of the field will not be treated in this report. We will confine ourselves to presenting its results. Let \(\varepsilon_i\) be the electric permittivity of the left confining layer and \(n_i\) and \(n_r\) the refractive index of the left and right confining layers respectively. Then the \(y\)-dependence of the field, in and on both sides of the active layer can be written as

\[
\begin{align*}
\phi_1 &= p_a (w_1 - w_2) - \text{artanh}\left( \frac{p_a}{P_a} \right) \\
\phi_2 &= p_a (w_1 - w_2) - \text{artanh}\left( \frac{p_a}{P_a} \right)
\end{align*}
\]  

(2.31)

with the decay constants and phase factor given by

\[
\begin{align*}
p_i &= \left[ (\varepsilon_i^2 - n_i^2)k_0^2 - k_y^2 \right]^{1/2} \\
p_r &= \left[ (\varepsilon_r^2 - n_r^2)k_0^2 - k_y^2 \right]^{1/2} \\
\phi_{Bl} &= -\arctan \left( \frac{p_r (k_r/k_y)}{k_y} \right) + k_y t/2
\end{align*}
\]  

(2.35) (2.36) (2.37)

The field dependence in the active layer on the transverse coordinates is completely described by equations (2.25) and (2.32): We simply have to multiply both right hand sides by each other. The \(y\)-dependence of the fields in the top layer, the anti meltback layer, the waveguiding layer and the substrate will be assumed to be the same as that in the active layer. The same will be assumed for the \(x\)-dependence of the fields in the left and right confining layers. This implies that multiplication of the equations (2.26) ... (2.29) with equation (2.32) and of the equations (2.33) and (2.34) with (2.25) yields the field distribution in the surrounding media. At this point however, we have to recall the analysis we have applied to arrive at the transverse field distribution in the structure: In fact we only have taken care of the boundary conditions in the \(x\)-direction for \(y \in [-t/2,t/2]\) and in the \(y\)-direction for \(x \in [-w_0,0]\). With good approximation we know the field for \(x \in (-\infty, \infty)\) and \(y \in [-t/2,t/2]\) and for \(y \in (-\infty, \infty)\) and \(x \in [-w_0,0]\). There remain four rather large areas in the transverse structure for which we don't know the field distribution. In the analysis that has been carried out, it has been shown that the fields in these regions have to decay exponentially for proper modes. This means that the actual power of the total field will only be slightly larger than the power that can be calculated for the field we have found. This is important to know in the determination of the confinement factor. The confinement factor is a measure for the field intensity.
in the active layer with respect to the total field intensity. If we denote the confinement factor by \( \Gamma \), then its definition becomes mathematically

\[
\Gamma = \frac{\int_{-\infty}^{0} \int_{-l/2}^{l/2} |E_y|^2 \, dz \, dy \, dx}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |E_y|^2 \, dz \, dy \, dx}
\]

(2.38)

The dependence of the field on the \( z \)-coordinate will be discussed in the next chapter, but yet at this point we can say that the dependence will be the same for the various media, so the contributions to \( \Gamma \) in the numerator and the denominator will cancel. Thus, only the transverse field distribution determines the confinement factor. The importance of the confinement factor will be discussed in the chapter on the rate equations. The integrations that have to be undertaken to arrive at a value for \( \Gamma \) are straightforward, though involved. Therefore we will confine ourselves to presenting the mathematical results of the various integrations from which \( \Gamma \) can be calculated in appendix A.
3.0 Calculation of the Longitudinal Modes in a DFB Laser

3.1 General

The behaviour of the electric fields in a waveguide having an internal reflector has been described by a number of scientists, e.g. Kogelnik and Shank [3], Streifer [4], Thompson [5] and Wang [6, 7]. Of all these articles, the one written by Steifer and co-workers [4] is the most general. The effect of the external reflectors on the fields inside the laser is explained in a comprehensive though unpractical way. In Streifer's description there seems to be a relation between the phases of the left and right mirrors and the internal reflector. In practice however these phases are unfortunately completely random.

In this chapter we will take over Streifer's analysis and utilise his notation as much as possible, but we will extend the analysis to get to a more realistic description of the input parameters that influence the lasing behaviour.

3.2 Analysis of the Electric Field inside a DFB Laser

3.2.1 Theory

The main characteristic of a DFB laser is that the left and right travelling waves feed energy into each other. This is caused by a periodic feedback. The periodic feedback is effected by a physical corrugation in the vicinity of the gain region in a laser. This structure is equivalent to a periodic variation in refractive index and/or gain. Mathematically, this becomes

\[ n(z) = n + (\Delta n) \cos \left( \frac{2\pi z}{\Lambda + \Omega} \right) \]  
\[ \alpha(z) = \alpha + (\Delta \alpha) \cos \left( \frac{2\pi z}{\Lambda + \Omega} \right) \]

In these equations \( n(z) \) is the effective refractive index for \( z = -L/2 \) to \( z = L/2 \), with \( L \) the laser length, and \( \alpha(z) \) the gain on the same interval. \( n, \Delta n, \alpha \) and \( \Delta \alpha \) are constants, whereby \( n \) and \( \Delta n \) are calculated in a way, described in section 2.1. \( \Lambda \) is the spatial period of the internal reflector or corrugation and \( \Omega \) is the phase of the amplitude variations of \( n(z) \) and \( \alpha(z) \). If the spatial variation of the effective refractive index and the gain is not harmonic, then equations (3.1) and (3.2) describe only the constant and fundamental terms of the Fourier series in which the actual variation of the
index and gain can be expressed. The above equations are appropriate only for analyzing first-order Bragg scattering.

The transverse field shape has been expressed in the effective index of refraction. Thus our problem has been reduced to a 1-dimensional field problem in the z-direction. To solve this problem, we have to find solutions for the scalar wave equation, given by

\[ \left[ \Delta^2 + k^2(z) \right] E(z) = 0 \]  

(3.3)

The time dependence of the electric field \( E(z) \) is \( \exp(i\omega t) \), where \( \omega \) is the angular frequency of the travelling waves. The constant of the wave equation [4] is given by equation (3.4) under the assumptions: \( \Delta n < \beta \), \( \alpha < \beta \) and \( \Delta \alpha < \beta \), and will be derived in appendix B.

\[ k^2(z) \approx \beta^2 + 2i\alpha\beta + 4\kappa \beta \cos(2\beta_0 z + \Omega) \]  

(3.4)

The variables have the following meaning: \( \beta_0 \triangleq \pi/\Lambda \), \( \beta \triangleq \pi\alpha/c \) and \( \kappa = (\beta/2)(\Delta n/n) + i\Delta z/2 \) for \( \kappa < \beta \). \( c \) is the free space light velocity. Except for \( \alpha \) and \( \beta \) all the constants are known. Therefore the values of \( \alpha \) and \( \beta \) have to be determined. The net gain at threshold for laser oscillation is specified by the values of \( \alpha \) and the laser frequency or wavelength belonging to a certain value of \( \alpha \) is represented by \( \beta \).

The solutions of the wave equation are assumed to be of the form

\[ E(z) = R(z) \exp(-i\beta_0 z) + S(z) \exp(i\beta_0 z) \]  

(3.5)

This formula suggests that the total electric field consists of a right travelling wave with complex amplitude \( R(z) \) and a left travelling wave with complex amplitude \( S(z) \) (for 'S', think of the latin 'sinister', meaning 'left'). Both amplitudes are assumed to vary slowly, so that the second derivatives \( \frac{d^2 R(z)}{dz^2} \) and \( \frac{d^2 S(z)}{dz^2} \) can be neglected compared with \( \beta_0 \frac{dR(z)}{dz} \) and \( \beta_0 \frac{dS(z)}{dz} \). If we substitute the expected solution (3.5) in the wave equation (3.3), and neglect the second derivatives, we find

\[ \left[ -2i\beta_0 \frac{dR}{dz} - (\beta_0^2 - \beta^2 - 2i\alpha\beta) R \right] \exp(-i\beta_0 z) + \left[ 2i\beta_0 \frac{dS}{dz} - (\beta_0^2 - \beta^2 - 2i\alpha\beta) S \right] \exp(i\beta_0 z) \]  

(3.6)

\[ + 2\kappa \left[ \exp(\Omega) + \frac{d}{dz} \exp(\Omega) \right] [R \exp(-i\beta_0 z) + S \exp(i\beta_0 z)] = 0 \]

By separating the terms with similar z-dependence in equation (3.6) we can find solutions for \( R(z) \) and \( S(z) \). Omitting the exponential z-dependence, neglecting the terms that propagate as \( \exp(\pm 3\beta_0 z) \) because they introduce only a secondary effect in first-order Bragg scattering, and the assuming \( |\delta| < \beta \), with \( \delta \) defined by

\[ \delta \triangleq \beta - \beta_0 \]  

(3.7)

separation then leads to
Equations (3.8) and (3.9) are constant coefficient linear differential equations so that they admit exponential solutions for $R(z)$ and $S(z)$ of the form

$$R(z) = r_1 \exp(\gamma z) + r_2 \exp(-\gamma z)$$

$$S(z) = s_1 \exp(\gamma z) + s_2 \exp(-\gamma z)$$

In these equations $r_1, r_2, s_1, s_2$ are complex constants. We will assume, without loss of generality, that $\Re(\gamma)$ is positive, so that $r_1$ and $s_2$ grow as they propagate. Substituting the solutions (3.10) and (3.11) into the equations (3.8) and (3.9) and collecting terms with identical $z$-dependence leads to the following set of four equations

$$\tilde{\chi} r_1 = i\kappa \exp(-\kappa z)s_1$$

$$\tilde{\chi} r_2 = i\kappa \exp(-\kappa z)s_2$$

$$\tilde{\chi} s_1 = i\kappa \exp(\kappa z)r_2$$

$$\tilde{\chi} s_2 = i\kappa \exp(\kappa z)r_2$$

where

$$\tilde{\chi} = -\gamma + \alpha - i\delta$$

$$\chi = \gamma + \alpha - i\delta$$

By setting the product of the right hand sides of (3.16) and (3.17) equal to zero, we get the dispersion relation, expressed by equation (3.18)

$$\gamma^2 = (\alpha - i\delta)^2 + \kappa^2$$

If and only if $\gamma$ satisfies the dispersion relation, equations (3.12) to (3.15) have non-trivial solutions. For the actual determination of $\gamma$ and thus $\alpha$ and $\delta$ (or $\beta$) we have to impose the reflection conditions at $z = \pm L/2$. Let $\hat{\rho}$ and $\hat{\rho}'$ be the reflection coefficient at $z = -L/2$ and $\hat{\rho}$, the reflection coefficient at $z = L/2$, then we obtain at the left and right mirror respectively

$$[r_1 \exp(-\gamma L/2) + r_2 \exp(\gamma L/2)] \exp(i\beta_0 L/2) =$$

$$= \hat{\rho}_L [s_1 \exp(-\gamma L/2) + s_2 \exp(\gamma L/2)] \exp(-i\beta_0 L/2)$$
\[
\left[ s_1 \exp \left( \frac{yL}{2} \right) + s_2 \exp \left( -\frac{yL}{2} \right) \right] \exp \left( i\beta_0 L/2 \right) = \\
\hat{\rho}_r \left[ r_1 \exp \left( \frac{yL}{2} \right) + r_2 \exp \left( -\frac{yL}{2} \right) \right] \exp \left( -i\beta_0 L/2 \right)
\]  
(3.20)

Substituting (3.12) and (3.15) into (3.19) and (3.20) and combining the latter two, results in

\[
\left( 1 - \frac{\rho \hat{\chi}}{\kappa c} \right) \exp \left( -\frac{yL}{2} \right) = \left( \frac{\rho_r - \frac{\hat{\chi}}{\kappa c}}{\rho_l - \frac{\hat{\chi}}{\kappa c}} \right) \exp \left( yL \right)
\]  
(3.21)

where \( \rho_r \) and \( \rho_l \) are given by the following equations

\[
\rho_l = \hat{\rho}_l \exp \left( -i\beta_0 L + i\Omega \right) 
\]  
(3.22)

\[
\rho_r = \hat{\rho}_r \exp \left( -i\beta_0 L - i\Omega \right) 
\]  
(3.23)

Comparing equations (3.1) with (3.22) and (3.23) shows that the phase functions of (3.22) and (3.23) are equal to the phase of the DFB structure at \( z = -L/2 \) and \( z = L/2 \) respectively. The values of the phases at the left and right mirror are shown in figure 3.1 (next page) for a nonsinusoidal corrugation and first order Bragg scattering. In this case the phase of the left and right reflection coefficient is not taken into account, but since the laser material has a higher refractive index than the media beyond \( z = \pm L/2 \) the phase of both reflection coefficients is equal to zero. The figure therefore also applies to the total phase of \( \rho_l \) and \( \rho_r \).

By combining the equations (3.12), (3.14), (3.16) and (3.17) we come to the following relation for \( \gamma \)

\[
\gamma = -\frac{\kappa c}{2} \left( \frac{\hat{\chi}}{\kappa c} - \frac{\kappa c}{\hat{\chi}} \right) 
\]  
(3.24)

Solving (3.21) for \( \hat{\chi}/\kappa c \) finally leads to the eigenvalue equation (3.25) which has been derived by Streifer [4].

\[
\gamma L = - \frac{\kappa c \sinh \left( yL \right)}{D} \left\{ (\rho_l + \rho_r)(1 - \rho^2) \cosh \left( yL \right) \\
\pm (1 + \rho^2)[(\rho_l - \rho_r)^2 \sinh^2 \left( yL \right) + (1 - \rho^2)^2]^{1/2} \right\}
\]  
(3.25)

where

\[
D = (1 + \rho^2)^2 - 4\rho^2 \cosh^2 \left( yL \right)
\]  
(3.26)

and

\[
\rho^2 = \rho \rho_r
\]  
(3.27)
To eliminate the square root and the two possible signs in the eigenvalue equation, it may be rewritten in the form

\[(yL)^2 D + (\kappa L)^2 \sinh^2(yL)(1 - \rho_1^2)(1 - \rho_1^2)\]

\[+ 2i\kappa L (\rho_1 + \rho_2)(1 - \rho_1^2) yL \sinh(yL) \cosh(yL) = 0 \]  

(3.28)

Solving (3.28) gives values for \(\alpha\) and \(\delta\) (or \(\beta\)) if all the other variables of the final eigenvalue equation are known. As is clear, the eigenvalue is complex. By splitting the equation up into its real and imaginary parts, we find two equations which are real. Now we have two equations with two unknowns that must be solved numerically. To facilitate finding the solution of (3.28), the periodic variation in gain \(\Delta\alpha\) is assumed to be negligible resulting in a real expression for \(\kappa\) and furthermore \(\beta_0\) is substituted for \(\beta\) in the expression for \(\kappa\). In calculating the modes for a number of different laser structures, Streifer considers a frequency interval of \(|\delta L| \leq 10\). This means that the maximum deviation of \(\beta\) from the value at the Bragg frequency, \(\beta_0\), is \(\pm 10/\lambda\). If the length of the laser is not too small compared with the spatial corrugation period, or pitch, substitution of \(\beta_0\) for \(\beta\) is allowed. In the case of Philips lasers this is certainly true: the laser length is 250 \(\mu\)m and the pitch is 240 nm. These lengths result in a value for the quotient of the maximum deviation from \(\beta_0\) and \(\beta_0\) of about
0.3 percent. For a relatively short device of 100 μm the error in \( \kappa \) is still less than 1 percent. In the program that has been written the maximum deviation can be 25 percent higher than the one mentioned here, because the \( \delta L \)-interval for which solutions are sought, is 25 percent larger.

### 3.2.2 Adapting the Mirror Phases

The equations (3.22) and (3.23) suggest that a relationship exists between the left and right mirror. The word 'suggest' has been used intentionally, because, as will be shown below, Streifer's analysis is all right, only a little confusing.

In practice, the laser device first is grown and the end facets are cleaved and coated afterwards. In figure 3.2 the laser device is presented after cleaving and coating.

![Figure 3.2. Schematic representation of the cleaved and coated laser](image)

Usually the lasing behaviour as a function of both the total round trip phases of the left and right mirrors is to be determined. When we declare \( \phi_{p,l} \) and \( \phi_{p,r} \) to be the left and right round trip phases, then we have to derive a relation between these and the \( \beta_{p,l} \) and \( \Omega \) terms in the Streifer notation.

For the derivation we make use of figure 3.3.

![Figure 3.3. Schematic view of the relation between the round trip mirror phases and the 'Streifer' phases](image)

We call the length of the corrugation the internal length, denoted by \( L_{in} \). The internal length has to be equal to an integer number times the pitch. Mathematically
\[ L_{in} = 2m\Lambda, \quad m \text{ an integer} \]  

Let \( l_l \) and \( l_r \) be the lengths associated with the left and right round trip phases respectively. Then we can express \( l_l \) and \( l_r \) in terms of the phases in the following way:

\[ l_l = \frac{\phi_{p,l}}{2\beta} \]  

\[ l_r = \frac{\phi_{p,r}}{2\beta} \]  

\( \beta \) has been described after formula (3.4). Division by two in the above equations is necessary because the phases are round trip phases. To calculate the length, only the 'single phase' is needed. The total length can now be determined by adding \( l_l \) and \( l_r \) to \( L_{in} \). Thus \( \beta_0 L \) is known. It is

\[ \beta_0 L = \beta_0 (L_{in} + l_l + l_r) \]  

\( \Omega \) can be calculated from the shift between \( L_{in}/2 \) and \( l_l \) on the one hand and \( L/2 \) on the other hand. This shift has to be multiplied by \( 2\pi/\Lambda \) to express \( \Omega \) as a phase shift. \( \Omega \) becomes

\[ \Omega = \frac{2\pi}{\Lambda} \left( \frac{L}{2} - \frac{L_{in}}{2} - l_l \right) \]

\[ = \frac{\pi}{\Lambda} (l_r - l_l) \]  

In summary we can say that we have expressed the left and right mirror round trip phases in terms of the total length and the phase shift of the DFB structure. The fact that the laser length plays an important role might not have been clear from Streifer's analysis. Of course, if we say that we have extended the modelling presented by Streifer for more practical applications only by employing a substitution of variables, Streifer's model had to be complete from the outset.

### 3.3 Calculation of the Light Intensity in a DFB Laser

It has been shown in the previous paragraph that the electric field inside the laser can be considered to consist of a left and a right travelling wave. In combination with the magnetic field, these waves transport optical power. The total power through a certain transverse cross-section in the laser is built up of the left and right travelling powers. Since the transverse cross-section of a laser consists of a 2-dimensional variation of the refractive index, it is not possible to create an electric field that is exclusively polarized in the y-direction and accordingly a magnetic field directed along both the x- and z-coordinate. Calculating the average power density for time varying harmonic fields, using Poynting's theorem, equation (3.34), one find a power density vector directed in the z-direction.

\[ \hat{p}_{av} = \frac{1}{2} \text{Re}(\hat{E} \times \hat{H}^*) \]  

\[ (3.34) \]
\( \hat{H}^* \) denotes the complex conjugate of \( \hat{H} \). In practice, the electric field is only predominantly polarized in the y-direction. This implies that part of the power disappears into the surroundings of the active layer instead of leaving the device at the end facets. This loss of power will be taken into account in the rate equations, which will be discussed in the next chapter.

The electric field as a function of \( z \) derived in this chapter has to be multiplied by the \( x \)- and \( y \)-dependence, discussed in the previous chapter, to yield the total field. In the determination of the optical intensity in the laser we will omit the \( x \)- and \( y \)-dependence for reasons of convenience. Applying as much as possible the ‘Steifer’-notation and omitting the subscript ‘av’, we can write for the optical intensity

\[
\hat{P}(z) = \hat{P}_y(z) + \hat{P}_x(z)
\]

(3.35)

where

\[
\hat{P}_y(z) = A \left| s_1 \exp(zy) + s_2 \exp(-yz) \right|^2 (\varepsilon_z)
\]

(3.36)

and

\[
\hat{P}_x(z) = A \left| r_1 \exp(zy) + r_2 \exp(-yz) \right|^2 (\varepsilon_z)
\]

(3.37)

where \( A \) is a constant.

In these equations the left and right directed intensities are denoted by \( \hat{P}_y \) and \( \hat{P}_x \), respectively. For the transmitted power through the left mirror only the left travelling intensity is important. A similar statement holds of course for the right mirror power. For these cases the left and right travelling intensities have to be corrected by a factor that takes into account the total left and right mirror reflectivity, yielding the following equations for the transmitted intensities

\[
\hat{P}_y = (1 - |\rho_y|^2) \hat{P}_y(-L/2)
\]

(3.38)

\[
\hat{P}_x = (1 - |\rho_x|^2) \hat{P}_x(L/2)
\]

(3.39)

If \( y \) is thought to consist of a real and imaginary part according to equation (3.40) then expression (3.41) can be found for the constant \( A \) as will be shown in appendix C.

\[
y = \alpha_y + i\beta_y
\]

(3.40)

\[
A = \frac{\beta_0}{2\omega\mu_0}
\]

(3.41)

The above equation for \( A \) holds only if \( \beta_0 > > \alpha_y \) and \( \beta_0 > > \beta_x \). These two conditions are met as shown in the following example.
Our basis for the example will be formula (3.18) except with both sides multiplied by $L^3$. Realistic values of $\alpha_L$, $\beta_L$ and $\kappa_L$ are 2, 5 and 2 respectively, [4]. Substituting these values results in: $(y_L)^2 = -17 - i20$. Taking the square root yields: $y_L = -11 + i24$. For the Philips laser the $\beta_L$-product is of the order of magnitude of 3200. So both $\alpha_r$ and $\beta_r$ are much less than $\beta_p$.

From equations (3.35) to (3.37) it is clear that the analysis derived so far, determines the light intensity completely, except for the amplitude constants $r_1 \ldots s_2$. It seems that there are four amplitude terms to calculate to completely determine the intensity. However, there are still equations (3.12) to (3.15), (3.19) and (3.20) that can be applied. By means of these equations we are able to express all amplitude constants in, for example, $r_1$. Let the following relations hold: $r_2 = \alpha r_1$, $s_1 = d_1 r_1$ and $s_2 = d_2 r_1$, then the following equations can be derived for $c_2$, $d_1$ and $d_2$ respectively:

\[
\begin{align*}
    c_2 &= \frac{\left(1 - \frac{\hat{\beta} \hat{\kappa}}{\chi} \exp (i(\Omega - \beta_0 L))\right)}{\left(\frac{\hat{\beta} \hat{\kappa}}{\chi} \exp (i(\Omega - \beta_0 L)) - 1\right)} \exp (-yL) \\
    d_1 &= \frac{i\kappa}{\chi} \exp (i\Omega) \\
    d_2 &= \frac{i\kappa}{\chi} \exp (i\Omega)c_2
\end{align*}
\]

Now we have reduced the number of four unknowns to one unknown. $r_1$ is still to be determined. The field and thus the intensity in a laser is related to the number of carriers that fall down from higher energy levels to lower energy levels, thereby emitting radiation. The link between the carriers and the electric field is established by the rate equations, discussed in the next chapter.
4.0 Determination of the Optical Response to an Electrical Excitation

4.1 General

It is common practice to describe the coupling between the electrical input and the optical output of a laser by means of the rate equations. The rate equations are coupled differential equations. If the behaviour of the laser device only is to be described, then the unknowns are the carrier density on the one hand and the photon density on the other [2,9,10,11,12]. If the lasing behaviour of a device that is exposed to external reflections is to be described, then the second unknown usually is the electric field [13,14,15].

The literature that discusses the theoretical behaviour of lasers has thus far, mainly concentrated on Fabry-Perot type lasers. As has been mentioned before, the main difference between these type of lasers and DFB lasers is the field distribution in the device which we have considered in the previous chapter. Since the distributed feedback phenomenon is described by means of the electric field, we are forced to use the rate equations for the carrier density and the electric field in order to make use of the field description for DFB lasers.

The situation below laser threshold, where the laser acts as a LED, has not been considered. Another aspect of the laser behaviour that has been ignored is lasing in more than one mode. The reason for using DFB's is that their monomode characteristics are far better than those of Fabry-Perot type lasers. Yet, DFB lasers can exhibit multimode behaviour. In the program that has been written, a special condition is included, which tests whether a certain laser configuration will lase in one or more modes. The condition stated, is, that if the gain difference between the modes which require the lowest gain is greater than or equal to 5 cm⁻¹ the laser is monomode. If this condition is not satisfied multimode theory should be used. Experiments have shown that the P-I-curve for a DFB laser has a kink for a certain, relatively high, value of the bias current. This kink is assumed to be caused by unstable lateral mode behaviour and mode hopping. Since we only consider monomode behaviour, this crack won't appear in our results.

In the next section the rate equations will be discussed as they are used in our DFB model. The rate equations will be discussed separately. First the carrier rate equation will be considered and special attention will be given to the extension to the Fabry-Perot case that is necessary because of the distributed feedback effect. Next the field rate equation will be discussed, in particular how it is
made 'compatible' to the photon-density-equilibrium used by Mols [2] in his model of the 1.3 μm Fabry-Perot laser.

The nomenclature used in the following subparagraphs is chosen in such a way as to match Mols' nomenclature [2] in as much as possible, especially where the carrier rate equation is concerned.

### 4.2 Carrier Rate Equation

The carrier rate equation that has been used is given by equation (4.1)

\[
\frac{dn}{dt} = R_{\text{pump}} - R_{\text{loss}} - \frac{GP}{hv} + D\nabla^2 n \tag{4.1}
\]

n denotes the excess carrier density. The pump term can be denoted as \( R_{\text{pump}} = \frac{IqV_s}{e} \), where I is the external pumping-current, q the electronic charge and \( V_s \) the volume of the active layer. \( R_{\text{loss}} \) contributes to the various radiative and non-radiative processes occurring in a laser and is commonly expressed as a power-series

\[
R_{\text{loss}} = an + bn^2 + cn^3 + \cdots \tag{4.2}
\]

The physical interpretation of this series will be discussed later.

The third term in (4.1) takes into account the stimulated recombination in a laser. G is the gain function which depends on the carrier density. The exact shape of the gain function will be discussed later. \( P \) is the light intensity in the active region. \( h \) is Planck's constant and \( v \) the frequency of the emitted light. Again, it will be discussed later that the product of \( G \) and \( P \) has to be divided by the photon energy.

The last factor in the carrier rate equation charges the lateral and longitudinal diffusion.

#### 4.2.1 Radiative and Non-radiative Recombination

In this subparagraph the main processes that contribute to the rather general \( R_{\text{pump}} \) term in equation (4.1) will be elaborated upon. However, since Mols [2] has discussed these processes in detail we will confine ourselves to a more global description of the phenomena.

The radiative recombination rate \( R_{\text{rad}} \) can be expressed as follows

\[
R_{\text{rad}} = BNP = B(N_0 + nP_0 + P) \tag{4.3}
\]

where \( N \) and \( P \) are the electron and hole densities respectively, \( B \) is the radiative coefficient, \( N_0 \) and \( P_0 \) are the equilibrium densities for zero-bias and, finally, \( n \) and \( P \) are the excess densities. Since (4.1) relates the variation of the excess electron density with time to the processes that cause this variation, it is necessary to concentrate on the relation between the radiative recombination and the
carrier density on the excess carrier density. Therefore we only have to omit the product of $N_e$ and $P_0$ in formula (4.3), yielding
\[ r_{rad} = B (N_g P + n P_0 + np) \] (4.4)

According to Mols \[2\] equation (4.4) can be written as
\[ r_{rad} = \frac{n}{\tau_{rad}} + B_1 n^2 \]
\[ = \frac{a}{\tau_{nr}} n + B_1 n^2 \] (4.5)

where $\tau_{rad}$ is the carrier-independent lifetime of the radiative recombination. $B_1$ is the radiative coefficient for the excess carrier density and $a$ is the ratio of the lifetime of the non-radiative recombination, that still is to be discussed, and the sum of the lifetimes of both the radiative and the non-radiative recombination. If we call $\tau_{nr}$ the lifetime of the non-radiative stripe-edge recombination then we can write for $a$: $a = \left( \frac{1}{\tau_{nr}} \right)$. For $\tau_{nr}$ the following formulation holds:
\[ 1/\tau_{nr} = (1/\tau_{rad} + 1/\tau_{nr}) \]

The linear term is well-known from many kinds of decay processes whereas the quadratic term takes arises from the fact that there are two particles involved in causing radiative recombination, namely an electron and a hole. Values of $a$ and $\tau_{nr}$ for 1.55 $\mu m$ lasing material will be given later. Since there are no data present for $B_1$ for 1.55 $\mu m$ material a value of $10^{10}$ cm$^{-3}$ s$^{-1}$, the same as can be calculated from Mols' data for 1.3 $\mu m$ material, is assumed.

Two processes will be considered that give rise to non-radiative recombination. The first process is recombination in traps and the second is a special type of Auger process.

Recombination in traps introduces a linear contribution to the variation of the excess electron carrier density with time, which can be described by $n/\tau_{nr}$. Traps arise during the growth of the quaternary region as defects in the crystal structure. For this reason the value of $\tau_{nr}$ will be device-dependent. Since $\tau_{nr}$ is relatively small compared to $\tau_{rad}$ its contribution to the right hand side of (4.1) will be relatively large.

According to Mols \[2\] the CHSH-process is the dominant type of Auger process in InGaAsP-devices. The nature of this process is that energy and momentum are transferred to a hole in the split-off band, after recombination of an electron and a hole, and this hole is boosted up into the heavy hole band. The Auger recombination rate can thus be described by
\[ R_{Auger} = C N P^2 \] (4.6)

where $C$ is the Auger coefficient.

To arrive at a relation between the Auger recombination and the excess carrier densities it is necessary to eliminate the contribution of the zero-bias carrier densities. By expressing the excess hole density in terms of the excess electron density, as has been done by Mols, equation (4.7) results in
where $C_1$ is the Auger coefficient for the excess carrier density and $N_A$ is the acceptor density $r_{\text{Auger}}$ appears to consist of a first-, second- and third-order carrier density dependence. Again Mols [2] has shown that the first- and second-order terms may be neglected compared to the other recombination terms. Thus, we can write for the Auger recombination

$$r_{\text{Auger}} \approx C_1 n^3$$ (4.8)

The Auger recombination for 1.55 $\mu$m lasers will be larger than for 1.3 $\mu$m lasers as can be concluded from [31]. For this reason $C_1$ is set at the (arbitrary) value of $2 \times 10^{-28}$ cm$^3$s$^{-1}$ instead of $1 \times 10^{-28}$ cm$^3$s$^{-1}$ as used by Mols.

### 4.2.2 Stimulated Recombination

Apart from spontaneous recombination occurring in a laser, a process called stimulated recombin-

ation takes place. Transitions from one energy state to another can occur spontaneously in a laser giving rise to radiative or non-radiative recombination. The transitions take place between continuous energy bands and result in non-stationary solutions of the Schrödinger equation [16]. An external electromagnetic field which exercises a force on the electrons and nuclei of the atoms, is represented by the potential energy term in the Schrödinger equation. Thus we can understand why an external field is able to induce radiative emission. For a laser, this means that a field that has been induced can maintain or even enhance itself.

If we denote the stimulated emission by $R_{\text{stim}}$ then we can write for

$$R_{\text{stim}} = \frac{GP}{\hbar \nu}$$ (4.9)

where $G$ is the function describing maximum gain, $P$ the intensity of the field in the active layer, $\hbar$ Planck's constant and $\nu$ the frequency of the emitted light.

Dutta [17] has investigated the gain function for InGaAsP-lasers, emitting at a wavelength of 1.3 $\mu$m. The gain function depends on the radiative recombination rate, the doping, the wavelength and the temperature for a fixed material compound. Figure 4.1 is an example of one of the gain functions calculated by Dutta, which is the envelope of the gain versus recombination rate curve for fixed photon energy.

In this figure the material gain has been plotted as a function of $J$,

$$J = q r_{\text{rad}}$$ (4.10)

$q$ is the electronic charge, equal to $1.6 \times 10^{-19}$ C and $r_{\text{rad}}$ is the radiative recombination expressed in
From figure 4.1 it is clear that the gain function is a smooth function that can be approximated by a linear region, a quadratic region and a region where the gain is zero. For ease of calculation, which will be evident later on, the gain function we have used, consists of a zero-gain region and three linear gain regions (see figure 4.2).

Since as far as we know there is no data available on the gain function of 1.55 µm material, the gain function depicted in figure 4.2 for 1.3 µm material is applied.

The mathematical formulation of the applied gain function is the following

\[
G = \begin{cases} 
7.40 \times 10^{-6} (J - 3.00 \times 10^7), & J \geq 3.45 \times 10^7 \text{Acm}^{-3} \\
3.81 \times 10^{-6} (J - 2.58 \times 10^7), & 2.94 \times 10^7 < J < 3.45 \times 10^7 \text{Acm}^{-3} \\
2.09 \times 10^{-6} (J - 2.28 \times 10^7), & 2.28 \times 10^7 < J < 2.94 \times 10^7 \text{Acm}^{-3} \\
0, & J \leq 2.28 \times 10^7 \text{Acm}^{-3} 
\end{cases}
\]

(4.11)

Now we will prove that the photon energy term in the denominator of the right hand side of equation (4.9) is necessary. Therefore, we compare the stimulated emission expressed in a light in-
tensity with the stimulated emission expressed in a photon concentration. The last formulation has

$$\text{G}[\text{cm}^2]$$

Figure 4.2. Gain function used in our DFB model

has been used in [2,9,10,11,12]. Let this formulation be \( R_{\text{stim}} \) then the expression becomes

$$R_{\text{stim},S} = G v_g S$$  \((4.12)\)

where \( v_g \) is the group velocity or the velocity with which the photons travel through the lasing material (in \( \text{m}^{-1} \)) and \( S \) is the photon concentration (in \( \text{m}^{-3} \)). For simplicity we assume \( S \) to be a constant, not a function of position. The photons are thought to move in a longitudinal direction, for example in the positive z-direction as is suggested in figure 4.3. In this figure 'A' denotes the transverse area through which the photons, indicated by straight arrows, travel. The electromagnetic waves are indicated by the wavy arrows.

Each photon has an energy equal to \( h v \). The total energy per second that 'flows' through a plane perpendicular to the direction of motion is equal to the number of photons that travels through that plane per second times the photon's energy. The number of photons travelling through the plane with area \( A \) per second is equal to \( S A v_g \). The terms used in this expression have been explained earlier. The total energy per second is usually called the power. If we denote the power by \( \text{Pow} \), then
the is mathematically given by equation (4.13)

\[ P_{\text{ow}} = SAV_{g}hv \quad (4.13) \]

The vector product of the electric and the complex conjugate of the magnetic field, yields a power density or a power per unit area. The explanation here is only included to show the need of the photon energy in equation (4.9) so we may assume for simplicity that the power density is constant in a plane perpendicular to the z-axis. Now we can simply calculate the power density from the total power, by dividing the latter by the area of the transverse plane. Thus, we may write

\[ P = \frac{SAV_{g}hv}{A} = S_{g}hv \quad (4.14) \]

Equation (4.14) can be rewritten in the following way

\[ S_{g} = \frac{P}{hv} \quad (4.15) \]

Now we have found the link between \( R_{\text{im}} \) and \( R_{\text{im,S}} \), because the power of the light in the photon representation is equal to the power in the wave representation. The total power density travelling through a transverse plane consists of the sum of the left and right travelling power densities. For this case we must use left and right travelling photons. The photons will travel with equal velocities in both the positive and the negative z-direction. For this reason no changes are introduced in equation (4.15) as the power density is considered to be the total power density.
4.2.3 Diffusion Mechanism

The power density in the laser is a function of all cartesian coordinates. Since the power density is related to the number of carriers that fall from higher energy states to lower energy states, the number of recombinations has to be higher where the power density is larger; therefore the number of carriers will be less. In fact the carrier concentration will also be a function of all three cartesian coordinates. As the thickness of the active layer is much smaller than the diffusion length, the dependence on the x-coordinate will be neglected: The carrier density is assumed to be constant along the x-coordinate. In figure 4.4 both the power density and the carrier density as a function of the y- and z-coordinate are shown. In the y-direction the power density will have a cosine-shaped profile and in the z-direction, roughly, a hyperbolic sine-shape. The actual z-dependence of the power can be calculated from the analysis presented in the previous chapter.

![Figure 4.4. Power density (a) and carrier density (b) as a function of the y- and z-coordinate](image)

The dependence of the two space coordinates mentioned above on the carrier density can be expressed as follows

\[ n(y,z) = n_0 - n_l \left[ 1 + \cos(2(k_y y + \phi_B)) \right] f_2(z) \]  

(4.16)

where \( n_0 \) is a constant and \( n_l \) is the amplitude of the space dependent part of the carrier density. The term between brackets expresses the power density dependence and thus the carrier density dependence on the y-coordinate. The \( f_2(z) \) term, expressing the z-dependence of both the power density and the carrier density is not elaborated upon here, but is described in appendix D.

The space dependence of the carrier density introduces a diffusion current. If we call the contribution of this diffusion term to the right hand side of (4.1) \( R_{\text{Diff}} \), then we can write

\[ R_{\text{Diff}} = DN^2 n(y,z) \]  

(4.17)
In this expression $D$ is the diffusion constant which can be calculated from the well-known Einstein relation.

In this very subparagraph we have encountered the only difference between the carrier rate equation used by Mols [2] and the carrier rate equation used in this report. The only difference is the $z$-dependence of the carrier density, yielding a $z$-dependent diffusion term. In a Fabry-Perot type laser there won’t be a ripple in carrier density along $z$, since the diffusion length is much larger than the standing wave pitch. For this reason Mols didn’t need to include a carrier density variation along the $z$-coordinate.

### 4.3 Field Rate Equation

Using the carrier rate equation (4.1) we try to find the field by means of the field rate equation, which is presented in equation (4.18).

$$\frac{dE}{dt} = \frac{1}{2} \left[ \frac{c}{n_{\text{eff}}} \left( \Gamma G - (\alpha_{sca} + \alpha_{m}) - i 2k_0\eta(n - n_{th}) \right) + \zeta \beta, \Gamma r_{rad} \right] E \quad (4.18)$$

where $c$ is the velocity of light in free space, $n_{\text{eff}}$ is the effective refractive index, $\alpha_{sca}$ are the scattering losses, $\alpha_{m}$ are the 'mirror losses' for a DFB at threshold, $\zeta$ and $\beta$, are factors that will be explained later, $k_0$ is equal to $2\pi/\lambda$, $\eta$ is the ratio of variation of the refractive index with carrier density and $n_{th}$ is the carrier density at threshold.

The meaning of the various terms appearing between brackets on the right hand side of (4.18) will be clarified in the next subparagraphs. At this point we will explain the factor $(c/2n_{\text{eff}})$ in front of the left bracket. This factor equals the group velocity of the travelling waves and has to be included since the collective terms between brackets have dimension $m^{-1}$.

The factor $(1/2)$ in front of the left bracket results from the fact that the factors between brackets are related to the intensity and not to the field. The intensity is proportional to the square of the field amplitude. Therefore we can derive the transition from the differential equation for the intensity to the differential equation for the field amplitude as follows

$$\frac{dP}{dt} = f(n)P \rightarrow \frac{d(\eta E^2)}{dt} = f(n)\eta E^2$$

$$\rightarrow 2\eta E \frac{dE}{dt} = m\eta f(n)E^2 \rightarrow \frac{dE}{dt} = \frac{1}{2} f(n)E$$

In this case $E$ is considered to be a real function of time only. As will be made clear in paragraph 4.4 this consideration is correct.
4.3.1 Stimulated Radiative Recombination and Internal Losses

The contribution of the gain factor and the losses in the laser are treated simultaneously. If we take a look at the relation that determines the threshold gain, equation (4.19), it will be obvious that the two factors are in fact one term:

\[ \Gamma G_{th} = \alpha_{scat} + \alpha_{th} \]  

(4.19)

where \( G_{th} \) is the laser gain at threshold. As has been mentioned before, \( \alpha_{scat} \) are the scattering and free carrier losses [17], \( \alpha_n \) are the losses of the DFB structure including the mirrors. These losses are found by solving the eigen equation derived by Streifer [4], represented by equation (3.28) and multiplying the result by 2. The multiplication is necessary since the 'Streifer'-losses denote 'field'-losses and (4.19) requires 'power'-losses.

To come to (4.19) the assumption that the absorption and scattering losses in both the active layer and cladding layers are equal is made. Because the field arises from the active layer, only the contribution of the gain in the active layer can cancel the total losses. For this reason the gain has to be multiplied by the confinement factor. Since the gain term increases with the carrier density it is clear that for higher values of pump current, which yields a higher carrier density, the difference between the gain and the laser losses increases rapidly.

4.3.2 Spontaneous Recombination

The third term on the right hand side of (4.18) needs some explanation. Again the photon density will be our basis for the following analysis. In subparagraph 4.2.2 we have derived the relation between the light intensity \( P \) and the photon density \( S \). Let \( (dS/dt)_{spon} \) be the contribution of the spontaneous recombination to the variation of the photon density. Then we can write [2]

\[ \left( \frac{dS}{dt} \right)_{spon} = \beta \Gamma \tau_{rad} \]  

(4.20)

where \( \beta \) is the contribution due to photons that actually participate in the lasing mode. This contribution is relatively low, since the radiation has to be coherent. Values of \( \beta \) are of the order of magnitude \( 10^{-4} \).

Using (4.14) we find, the variation of the light intensity due to the spontaneous recombination

\[ \left( \frac{dP}{dt} \right)_{spon,S} = v_g \hbar \beta \Gamma \tau_{rad} \]  

(4.21)

The addition of 'S' in the subscript in the above equation denotes the derivation of the equation from the photon density. Let \( P \) be equal to the square of the field amplitude times \( (\beta_0/2)\omega_0 \). Then (4.21) becomes

\[ 2E \left( \frac{dE}{dt} \right)_{spon,S} = \frac{2\omega_0}{\beta_0} v_g \hbar \beta \Gamma \tau_{rad} \]
or
\[
\left( \frac{dE}{dt} \right)_{\text{spont.,S}} = \frac{\omega \mu_0}{\beta_0} c \cdot \frac{1}{n_{\text{eff}}} \cdot \frac{\nu}{\beta_s} \cdot \frac{r_{\text{rad}} E}{E}
\]
(4.22)
where for the group velocity \( \nu_g \), the actual value \( c/n_{\text{eff}} \) has been substituted.

Let \( \zeta \) be given by equation (4.23)
\[
\zeta = \frac{2 \omega \mu_0}{\beta_0} \frac{\nu}{E^2} = \frac{\nu}{P}
\]
(4.23)
then we can write for \( (dE/dt)_{\text{spont}} \)
\[
\left( \frac{dE}{dt} \right)_{\text{spont}} = \frac{c}{2n_{\text{eff}}} \zeta \beta_s \Gamma r_{\text{rad}} E
\]
(4.24)
Equation (4.24) originates from the inclusion of a contribution of the spontaneous recombination in the field rate equation. Since the spontaneous recombination is included in the carrier rate equation, it would not have been consistent to omit it in the field rate equation.

From (4.23) it is clear that \( \zeta \) equals the quotient of the photon energy and the optical power density. For increasing power the value of \( \zeta \) decreases. So, for larger values of pump current the contribution of the spontaneous recombination to the right hand side of (4.1R) diminishes.

Another, more realistic possibility is to include a relation between \( (dE/dt) \) and \( (1/E) \) in the rate equation for the field instead of that already done between \( (dF/dt) \) and \( F \). This gives rise to rather complex integrals which must be solved in order to make the rate equations suitable for solving by means of a computer.

Thus far however, we have used the rate equation for the field as it has been presented in formula (4.18) and we have considered \( \zeta \) to be a constant. If realistic values for the quantities determining \( \zeta \) are chosen, values for \( \zeta \) of about \( 10^{-5} \) will be found above threshold, yielding a small contribution of the radiative recombination rate. We have set \( \zeta \) equal to \( 10^{-5} \) [15]. However, setting \( \zeta \) equal to zero yielded a laser power that deviated from the case in which \( \zeta \) had been set equal to \( 10^{-5} \), only in the second decimal.

4.3.3 Refractive Index Dependence

It is known that the refractive index of lasing material depends on the carrier density. Usually only the variation of the refractive index with the carrier density variation is important. The relation between them appears to be linear, according to Stubkjaer [18]. Stubkjaer and co-workers have experimentally determined the relation between the variation of the refractive index of the active layer and the carrier density for 1.6 \( \mu m \) InGaAsP/InP lasers. They have found [18]
where \( n_n \) denotes the refractive index of the active layer and \( n \) the carrier density. Although we need the dependence of the effective refractive index with carrier density, we have used (4.25). This will give a dependence of the effective refractive index on the carrier density, which is slightly too large, since the changes of the effective refractive index will only be caused by the changes of the index of the active layer and not by the indices of the surrounding layers.

In (4.18) we have denoted \( dn_{\text{eff}}/dn \) by \( \eta \).

To come to the real contribution of the carrier dependent refractive index, we have to multiply \( \eta \) by \( (c/2n_{\text{eff}})k_0 \). In section 4.3 we have already explained the necessity of the factor \( c/2n_{\text{eff}} \). The factor \( 2k_0 \) originates from the fact that the terms between braces in (4.18) represent the gain and phase of the electric field. The phase is given by \( k_0 n_{\text{eff}} \). By gain we mean in this context the net gain, so material gain minus the 'DFB'- and scattering losses.

The contribution of the effective refractive index is linearized around the threshold value of the carrier density, \( n_{th} \), which yields: \( k_0 \eta(n - n_{th}) \). In section 4.3.1 we have shown that equation (4.19) holds. For \( \Gamma G(n) \) we can write \( \Gamma G(n_{th}) \). Now, the first and second term between braces in (4.18) denote \( \Gamma G(n) - \Gamma G(n_{th}) \), which also is a linearization.

Although the field rate equation is linearized around laser threshold and therefore only is valid to determine small signal behaviour, we also use it for larger signals, since we have numerically determined that the carrier density doesn't change significantly for larger values of pump current.

### 4.4 Adapting the Rate Equations for Numerical Treatment

First we will restate the rate equations as they have been presented in the previous sections, only with \( R_{\text{pump}} \) and \( R_{\text{int}} \) in the carrier rate equations replaced by equivalent terms

\[
\frac{dn}{dt} = \frac{l}{qV_a} - \frac{n}{\tau_{nr}} - B_1 n^2 - C_1 n^3 - \frac{GP}{hv} + D \nabla^2 n
\]

(4.26)

\[
\frac{dE}{dt} = \frac{1}{2} \left[ \frac{c}{n_{\text{eff}}} (\Gamma G - (\alpha_{\text{scat}} + \alpha_{\text{th}})) - i 2k_0 \eta(n - n_{th}) \right] E
\]

(4.27)

In subparagraph 4.2.3 we have seen that the space dependent carrier density can be described by a constant term plus a term that varies along the \( y \)- and \( z \)-axis. We have called the constant part \( n_c \) and the amplitude of the varying part \( n_l \). At this point we state that the word 'constant' as used in this context is taken to mean: independent of the space coordinates. Both \( n_c \) and \( n_l \) are time dependent, so they both have to be solved.
We can split the carrier rate equation into two differential equations by applying a technique used by Mols [2] and Furuya [9]. The technique consists of multiplying both sides of the carrier rate equation with the same cosine term appearing behind \( n \) and integrating the thus found result between \(-t/2\) and \(t/2\), where \( t \) is the width of the active layer. As will be pointed out in appendix D, we still have to do some manipulations to find two differential equations of the form \( (dn_1/dt) = ... \) and \( (dn_2/dt) = ... \).

At this point we need to have a careful look to the meaning of the field amplitude that we want to calculate. We want to know this amplitude as a function of time only. If we neglect the space dependence for convenience, we can write the field as a real amplitude which is a function of time, times \( \exp(i\omega t) \). If we had assumed the amplitude to be complex, we could have represented it by a time dependent modulus times the exponent of a time dependent phase. This phase now, could be combined with the actual exponential time dependence giving \( \exp[i(\omega t + \phi(t))] \). Since we haven't assumed a time dependence like this, the time dependent field amplitude has to be real.

From the previous chapter we know that the \( z \)-dependence of the field is complex. This means that we have to solve a field rate equation of the form given in equation (4.28). If we neglect the \( x \)- and \( y \)-dependence

\[
\frac{dr_1(t)}{dt} = \frac{-1}{2D_r(z)} \left[ X(z)D_r(z) - Y(z)D_i(z) \right] r_1(t) \\
\frac{dr_2(t)}{dt} = -\frac{1}{2D_i(z)} \left[ X(z)D_i(z) + Y(z)D_r(z) \right] r_2(t)
\]  

where \( r_1 \) is the field amplitude, \( D_r(z) \) and \( D_i(z) \) are the real and imaginary parts of the \( z \)-dependence, respectively, and \( X(z) \) and \( Y(z) \) are the real and imaginary contributions to the part between brackets in equation (4.27). From (4.28) two equations can be derived by taking the respective real and imaginary parts of (4.28), yielding

\[
\frac{dr_1(t)}{dt} = \frac{-1}{2D_r(z)} \left[ X(z)D_r(z) - Y(z)D_i(z) \right] r_1(t) \\
\frac{dr_2(t)}{dt} = -\frac{1}{2D_i(z)} \left[ X(z)D_i(z) + Y(z)D_r(z) \right] r_2(t)
\]

The solution for \( r_1(t) \) must obey both equations (4.29) and (4.30). This can be accomplished by taking the time-derivative of (4.29) with the right hand side of (4.30) substituted for \( dr_1/dt \). Now we have a second order differential equation, represented by equation (4.31). The solutions of this equation may be different from the solutions of (4.30), so \( r_1 \) has to obey both (4.31) and (4.30). This set of two equations will be solved numerically, together with the two differential equations for the carrier density.

\[
\frac{d^2r_1(t)}{dt^2} = \frac{\left[ X(z)D_r(z) - Y(z)D_i(z) \right] [X(z)D_i(z) + Y(z)D_r(z)]}{4D_r(z)D_i(z)} r_1(t)
\]

Since the rate equations give only the relationship between the time dependent carrier density and the time dependent field amplitude we only want to have the time dependence from both the time and space dependence left in the equations that are to be solved. This means that we will integrate
the equations over the volume of the active layer. Mols [2] has done the same to come to his set of rate equations.

Rewriting the rate equations into a form which can be handled by a computer is a rather involved operation. For this reason and since it gives the reader no extra knowledge, the transformation won't be carried out here, but in appendix D.

4.5 Applied Constants

In this section we summarize the constants we have used to model a laser (lasing at 1.55 µm). As has been mentioned before, not all of the constants have been found in literature, based on experiments. For this reason it is recommended to adapt the constants as soon as reliable experimental values become available in literature. In table 4.1 the constant symbols and their values have been assembled.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>1.6.10^{-19}</td>
<td>C</td>
</tr>
<tr>
<td>a</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>τ&lt;sub&gt;e&lt;/sub&gt;</td>
<td>5.5.10^{-9}</td>
<td>s</td>
</tr>
<tr>
<td>B&lt;sub&gt;1&lt;/sub&gt;</td>
<td>10^{-10}</td>
<td>cm&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>C&lt;sub&gt;i&lt;/sub&gt;</td>
<td>2.10^{-28}</td>
<td>cm&lt;sup&gt;3&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>h</td>
<td>6.63.10^{-30}</td>
<td>kg cm&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>λ (→ ν, k&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>1.55.10^{-4}</td>
<td>cm</td>
</tr>
<tr>
<td>T (→ D)</td>
<td>300</td>
<td>K</td>
</tr>
<tr>
<td>μ (→ D)</td>
<td>6500</td>
<td>cm&lt;sup&gt;2&lt;/sup&gt;V&lt;sup&gt;-1&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>c</td>
<td>3.10&lt;sup&gt;10&lt;/sup&gt;</td>
<td>cm/s</td>
</tr>
<tr>
<td>α&lt;sub&gt;ext&lt;/sub&gt;</td>
<td>60</td>
<td>cm&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>ζ</td>
<td>10&lt;sup&gt;-25&lt;/sup&gt;</td>
<td>s</td>
</tr>
<tr>
<td>β&lt;sub&gt;i&lt;/sub&gt;</td>
<td>1.3.10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>-</td>
</tr>
<tr>
<td>η</td>
<td>3.6.10&lt;sup&gt;-20&lt;/sup&gt;</td>
<td>cm&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Some quantities appear in the rate equations (4.26) and (4.27) which are not constants. These are now described:

V<sub>e</sub>, the volume of the active layer, is not a constant but depends on the laser structure for which the optical behaviour is to be calculated. G, the gain function has been presented in expression (4.11). The effective refractive index n<sub>eff</sub> and the confinement factor Γ result from the transverse field shape, and α<sub>e</sub> results from the eigenequation derived by Streifer [4], that has to be solved before the rate equations can be solved.
4.6 Modelling the Surroundings of the Active Layer

To improve the modelling of the optical response to an electrical excitation it is necessary to include electrical losses in the model. The pump current appearing in the carrier rate equation is not equal to the current that actually flows into the laser device, at least not for an AC pump current.

Versleijen [1] has modelled the laser structure, except for the active layer, of a Fabry-Perot type laser of 1.3 μm. In its simplest form, we can think the active layer to be part of a RC-network as is schematically depicted in figure 4.5. In this figure, R is the resistance of the top structure of the laser and is approximately 5 Ω, \( R_n \) is the resistor that is connected in series with the laser to make the total resistance equal to 50 Ω. The capacitor C takes into account the capacitance of the junction and the capacitance of the lateral laser structure and is approximately 10 pF.

In fact, we can represent the active layer by a diode with a junction voltage of 0.85 V. This value needs a little explanation. The junction voltage is equal to the following formulation

\[
V_{ac} = E_{gap} + x \frac{kT}{q} \tag{4.28}
\]

where the bandgap energy \( E_{gap} \) is expressed in eV. \( x \) is taken to be a constant, indicating the number of times \( (kT/q) \) has to be added to the bandgap energy to bring the Fermi-levels out of the bandgap to induce inversion. The modelling that has been used by Mols [2] to derive the junction voltage to carrier density relation is valid for values of \( x \) less then 2. For convenience we have taken \( x \) equal to 2, so the junction voltage becomes, together with the 0.8 eV bandgap energy for 1.55 μm lasers [19], 0.85 V.

However, this network structure is still to be implemented in the program that has thus far been realized. We have not succeeded in solving both the rate equations and the differential equation that describes the electrical behaviour of the RC circuit within acceptable limits of computing time.

Figure 4.5. Electrical equivalent of the active layer and its surroundings
5.0 Simulations and Results

5.1 General

In this chapter examples of the behaviour of a few DFB lasers will be given. We will concentrate, in particular, on a DFB device, which is presented without coating schematically in figure 5.1. The boundaries in the x- and y-directions are taken to lie at infinity. For the indices of refraction of the various media we have taken the following values: $n_1 = 3.55$, $n_2 = 3.20$, $n = n = n = 3.16$. The dimensions are indicated in the figure.

![Figure 5.1. Schematic representation of the laser device used in the calculations](image-url)

The left facet consists of a simple InGaAsP-air transition, which is not shown in the figure. The calculated effective refractive index of the laser is 3.21. With this value we find the amplitude of the left mirror reflectivity to be 0.526. The phase of the left mirror reflectivity is zero. The left mirror gives rise to a power reflection of about 28 percent. We expect the right facet to be anti-reflection-coated. In practice this means that the amplitude of the reflectivity is not equal to zero, as we would wish, but that it is equal to 0.224, resulting in a power reflection of about 5 percent.
Only from the various refractive indices and the dimensions of the various layers of the laser we calculate the values for $\kappa L$, the confinement factor and the effective refractive index. A value for the latter has just been given whereas this laser structure has a value of about 1.1 for the $\kappa L$-product and about 0.16 for the confinement factor.

The value for $\kappa L$ is a rather low value, indicating only weak feedback of the travelling waves in the laser. The cause for this low value of the coupling factor-length product is found in the value of $\Delta n$ that is calculated from the waveguiding structure. In fact, the difference between the values of the effective refractive indices for the minimum and maximum thicknesses of the waveguiding layer is too small. We expect this to be a shortcoming of the method applied to calculate the transverse field structure. However, apart from the effective index method that has been described in chapter 2, we have initially tried a method of calculating the field structure proposed by Marcatilli [30]. We haven't however applied it in the end, because it only treats a structure consisting of three layers in both transverse directions, whereas we actually have to deal with five layers in the x-direction (although we omit the anti meltback layer in the laser structure considered here). This method would immediately meant making estimates of the values of the refractive indices of the active layer surroundings, since the five layered structure would have to be reduced to the necessary three layers. We have seen that it is difficult to find a mode that is not below cutoff. In order to adhere more to real conditions we avoid this and use the method applied here. Even with this method we had to adapt the values of the refractive indices of all layers except the active layer, in order to get a realistic, though low, value of $\kappa L$ and also a realistic value of the confinement factor. Therefore the method applied here may be further improved.

In the relatively low coupling region around $\kappa L$ equal to 1, it is difficult to find a combination of the left and right facet phases, $\Omega$ and $\beta_0 L$, according to Streifer, that yields a monomode device. For stronger coupling it is easier to find a 'monomode combination' of the facet phases, because the monomode yield is higher. Values of 2.09 and 0.17 for the left and right round-trip facet phases respectively, resulted in monomode laser behaviour. Thus far, we have completely described the laser device, that has been used in the simulations.

### 5.2 z-Dependence of the Power and Carrier Density

As has already been stated a few times earlier in this report, the main difference between a DFB and a Fabry-Perot laser is the way in which the optical power varies in the longitudinal direction. The corrugation in a DFB laser causes the left and right travelling waves to grow in their respective propagation directions, whereas the travelling waves in a Fabry-Perot laser remain constant in amplitude.

Since we are interested in the optical power that is emitted by the laser, we will only refer to the power and not the electric field, even when we examine the electromagnetic behaviour in the laser interior.

In figure 5.2 we present the so called left and right travelling powers. (In fact the powers don't travel, but we use this nomenclature to indicate that the powers are associated with the left and right
travelling waves.) The vector product of the electric and magnetic fields yields a power density. This power density has been integrated over the transverse cross-section of the active layer and the results for both the left and right travelling waves are given in figure 5.2.

\[ \text{Confinement Factor: } 0.157 \]
\[ \rho_l: 0.526 \exp(\text{i}0.000) \]
\[ \rho_r: 0.224 \exp(\text{i}0.000) \]
\[ \text{Corrugation-Mirror Phase: } -0.966 \text{ rad} \]
\[ \beta_{\text{oL}}: 1.138 \text{ rad} \]
\[ \text{Modulation Frequency: } 0.00 \text{ Mhz} \]
\[ \text{Time Of View At: } 0.0020 \mu s \]
\[ \text{Constant Part Of Modulation: } 50.00 \]
\[ \text{Varying Part Of Modulation: } 0.00 \text{ m} \]

DC, No Modulation

Figure 5.2. Left and right travelling wave powers as a function of position inside the laser

To show the reader what kind of additional information is given by the program realized, to clarify the curves, the figure is a cut down copy of a plot as it is usually generated. In future figures in this report the additional information will be left out.

If we want to get values for the total power inside the laser as a function of \( z \), we must divide the values in figure 5.2 by the confinement factor, which means multiplying the values with 6, approximately. It will become clear that the \( z \)-dependence of the optical intensity is more complicated than just a hyperbolic sine-dependence. This is something we might already have expected from function \( f_1 \) (in appendix D), which gives the \( z \)-dependence of the total power. Clearly it can be seen that the power associated with the right travelling waves is about 30, or better, 28 percent of the power associated with the left travelling waves, just at the left mirror, and also that the left travelling power is about 5 percent of the right travelling power at the right mirror. This has to be the case, since we imposed these reflectivities.

The total optical intensity at a certain point along the longitudinal axis in the active layer consists simply of the sum of the left and right travelling powers. A plot of the total power is shown in figure
Figure 5.3. Total power in the active layer as a function of position inside the laser.

Based on simple physical arguments, we have assumed the carrier density to be maximum where the optical intensity is minimum and vice versa. This is only an assumption. The real situation in a laser might differ from this. In figure 5.4 we present a part of the carrier density as a function of the longitudinal coordinate. This is explained as follows: The carrier density as a function of the spatial coordinates is given by equation (4.16). First we integrate this dependence over a transverse cross-section of the active layer. This causes the carrier density to be a function of the z-coordinate only, and to have a dimension of cm$^{-1}$ as is indicated in the figure. Furthermore, we neglect the contribution of the constant term $n_0$ to the total carrier density as a function of z. So we only have a contribution of $-n_0[1 + \cos (2(k_y y + \phi))]f(z)$, integrated over the x- and y-coordinates. This now, we have plotted in figure 5.4. The reason for omitting the contribution of the constant part of the carrier density is that it is about a factor of 270 larger than the varying part, causing the latter to be suppressed in the plot. The value for $n_0$, integrated over the x- and y-coordinates is about $4.83 \times 10^9$ cm$^{-1}$, so the influence of the z-dependent optical intensity on the total carrier density profile...
To conclude the section on the z-dependence of the optical intensity, we will consider the left and right travelling powers, in case stronger feedback of the travelling waves occurs. For this case we have assumed the same wave numbers in both the x- and y-direction and the same phases that are necessary to determine the laser behaviour. We have however imposed a value of 2.0 for the $\kappa L$-product, instead of calculating $\kappa L$ from the transverse field structure. The result of enlarging $\kappa L$ is presented in figure 5.5. It is clear that the powers no longer have their maxima near the mirrors but near the centre of the laser. This result is consistent with the results obtained by Kogelnik and Shank [3] and Streifer [4]. Furthermore, if we examine the values of the powers at the mirrors, it is clear that they are smaller than in the case $\kappa L$ is 1.1, for the same pump current. This indicates that the optical output will be less. It is known from practice that DFB lasers with a larger value of $\kappa L$ yield a smaller value of the optical output for a given pump current. To verify this in
greater detail, we will present the P-I curve for $\kappa L$ equal to 2 in the next section.

![Graph showing travelling powers as a function of position inside the laser]

Figure 5.5. Powers of the left and right travelling waves as a function of position inside the laser; $\kappa L$ is 2.0

## 5.3 DC-Behaviour

In this section we will present the output powers as a function of the DC-input current. The optical intensities at both mirrors will be given. In practice, only one mirror is considered, namely the coated one with the lowest reflectivity. Mostly, the laser diode is mounted in such a way, that the other mirror cannot be used for measurements. For this reason, only the right mirror, that with the lowest reflectivity, will be considered when commenting about the curves.
The P-I curves of the laser are presented in figure 5.6.

![Figure 5.6. Output powers as a function of the DC-current](image)

From this figure we can derive a threshold current of about 37 mA and a differential external efficiency of about 0.25 W/A. These are both realistic values, although they seem a little high. For most DFB lasers that are grown at the Philips Research Laboratories, values between 20 and 25 mA are measured for the threshold current and 0.10 to 0.15 for the differential efficiency. The difference between the simulated and real measured values of both threshold current and differential efficiency, are caused by uncertainty in threshold gain for the lasing mode, that is calculated from Streifer's eigenequation, and the scattering losses in a DFB laser in combination with the material gain. As far as the material gain is concerned, we can restate here that we have used a gain function applicable for lasers emitting at a wavelength of 1.3 \( \mu m \), since there were no data available for 1.5 \( \mu m \)-lasers.

The corrugation of a DFB laser causes the electric waves not only to feed energy into each other but also to be scattered into the surrounding media. Therefore, the scattering losses will be higher for a DFB laser than for a Fabry-Perot laser. For these scattering losses we have assumed a value of 60 cm\(^{-1}\), which is twice the value used by Mols [2] in the modelling of the 1.3 \( \mu m \)-Fabry-Perot laser. If we use a value of 30 cm\(^{-1}\) for the scattering losses in our model for the DFB laser, we get the results shown in figure 5.7 with a threshold current of approximately 28 mA and a differential...
efficiency of 0.34 W/A.

![Graph showing output powers as a function of DC current](image)

Figure 5.7. Output powers as a function of the DC-current; $\sigma_{\text{scat}}$ is 30 cm$^{-1}$

If we examine the rate equation for the field, (4.27), the change in both the threshold current and the differential efficiency can be easily explained: First, since $\sigma_{\text{scat}}$, the total laser losses are smaller and therefore the gain need not be so high. This results in lasing action for lower values of the pump current. Second, the difference between the material gain and the losses is larger for a given value of the carrier density, resulting in a relatively higher value for the optical output. This explains the increased differential efficiency.

In the explanation above, we have only examined the real term between braces in equation (4.27). We have found numerically that this term yields the main contribution to the right hand side of (4.27). For this reason it is allowed to consider only this term.

The effects of reducing the scattering losses by a factor of two are significant. Since the value of 60 cm$^{-1}$ is arbitrary, a somewhat lower value will suffice in order to reduce the threshold current. However, for the time being we will carry out all calculations with the above mentioned value of 60 cm$^{-1}$ for the scattering losses.

The value of the threshold gain for the lasing mode is based on the $\kappa L$ -product. The Philips lasers are designed to have a $\kappa L$-product of about 2. The product we have calculated from the transverse laser structure is about 1.1. From Streifer [4] we know that a shift in $\kappa L$ for the possible modes can result in a higher or a lower value for the threshold gain. Since the lasing mode must have the lowest threshold gain of all possible modes, its gain will be lower when $\kappa L$ is 2, then when it is equal to
The threshold gain we find for our laser is about 51.6 \text{ cm}^{-1}. The effect of a different value of the coupling coefficient-length product on the threshold current and differential efficiency is not expected to be large [4].

Applying a $\kappa L$-product of 2 yields a threshold gain of 35.8 \text{ cm}^{-1}. The P-I curve for $\kappa L$ equal to 2 is presented in figure 5.8. Indeed, the effect is not as large as the effect of decreasing $\sigma_{\text{sat}}$, but it still is significant. From figure 5.8 we can derive a threshold current of 31 mA and a differential efficiency of 0.17. By adjusting the value of the scattering losses in case $\kappa L$ is 2, it is possible to reduce both the threshold current and efficiency in order to get values that better match the experiments.

![Figure 5.8. Output powers as a function of the DC-current; $\kappa L$ is 2](image)

**5.4 AC-Behaviour**

The AC-behaviour of a laser is expressed in its spectrum. This suggests that a laser has only one spectrum. Just as has been the case in the previous section, the plots in this paragraph also consist of two curves, indicating the responses at both facets. Comment will only be made, however, on the curves that show the laser behaviour at the right mirror.

The laser spectra that are measured in general are flat for low frequencies, accordingly, show a slight decrease followed by a peak, caused by the relaxation oscillation, and finally show a roll-off with approximately 40 dB per decade. Figure 5.9 shows the simulated spectrum for our laser in case we
apply a bias current of 50 mA, corresponding to a CW output power of 3.6 mW.

Figure 5.9. Laser spectrum; CW output power is 3.6 mW

In section 4.6 we have indicated how the surroundings of the active layer can be integrated in the program. This means that their influence hasn't been taken into account yet. The lateral region of the laser can be represented by a capacitor. From Mols' modelling of the Fabry-Perot laser we know that this capacitor causes the frequency response to roll-off at lower frequencies. In practice, this causes the slight dip in the spectrum. The relaxation however is strong enough to pull the frequency response up, despite the capacitor, and accordingly the response drops down.

Taking a look at the difference between the peak height and the flat part of the frequency response, we commonly find values of about 2 to 4 dB. In our case we first don't see the decrease in the spectrum caused by the absence of the influence of the lateral region and secondly, for the same reason, the peak-flat part difference is larger. The difference we find, is about 14 dB. Furthermore, a realistic value for the relaxation oscillation is about 4 to 5 Ghz whereas we find a value of 6 Ghz for the simulated relaxation oscillation. This deviation cannot be explained by the absence of the influence of the lateral region, since this influence results only in lower values of the response for higher frequencies. We have already pointed out in this report that a lot of constants necessary in yielding a realistic model of the DFB laser behaviour are not known for devices operating at a wavelength of 1.55 \( \mu m \). Some constants used, are simply taken over from the modelling of devices operating at 1.3 \( \mu m \), and others are roughly adapted from physical reasoning. One of the constants we have adapted is \( C_1 \), the Auger coefficient for the excess carrier density. From [31] we expect the Auger recombination rate to be larger for 1.55 \( \mu m \)-devices than for 1.3 \( \mu m \)-devices. In our program
we have taken $C_i$ to be twice as large as in the case of Mols' model [2]. Now we will vary the value of $C_i$ from our value of $2.0 \times 10^{-8}$. First we will reduce $C_i$ to a value of $1.5 \times 10^{-8} \text{ cm}^2\text{s}^{-1}$, and thereafter increase it to $2.5 \times 10^{-8} \text{ cm}^2\text{s}^{-1}$. The resulting spectra are shown in figures 5.10 and 5.11.

![Figure 5.10. Laser spectrum; $C_i$ is $1.5 \times 10^{-8} \text{ cm}^2\text{s}^{-1}$ and the CW output power is $5.3 \text{ mW}$.](image)
In figure 5.10 we see a shift of the relaxation oscillation to 14 GHz and in figure 5.11 to 8 GHz. From these figures we can conclude that increasing the Auger recombination rate leads to a smaller value of the modulation bandwidth. From the CW output power we can conclude that the threshold current increases with increasing value of $C_1$, since the value of the CW output power is smaller then is the case for $C_1$ equal to $2.0 \times 10^{-28}$ cm$^2$s$^{-1}$. The value of the bias current is in both cases 50 mA. We can explain this easily by looking at the physical influence of increasing $C_1$: We are increasing the non-radiative recombination. This means that from the carriers a larger part will recombine without emitting photons, so the number of emitted photons for a given number of carriers will be less. Therefore we need a larger part of carriers to reach lasing action.

For $C_1$ equal to $2.5 \times 10^{-28}$ cm$^2$s$^{-1}$ we have seen that the output power shows a distorted sine-shape when a sinusoidal pump current is applied. This accounts for the irregularities in the spectrum, which has to be taken with suspicion.

The slope of the spectrum is 18 dB per octave for all cases, indicating that we are dealing with a third-order system. If we take a look at the equations that describe the link between the carrier density and the optical field, given by (D.15) to (D.22) and (D.62) and (D.63), we see that the system really is of order three. We can differentiate (D.63) once more and accordingly substitute (D.62) in the thus found equation, yielding a third order equation in $r_t$.

The reason for using a second order differential equation to describe the laser behaviour originates from the fact that the $z$-dependence of the field in the laser is complex, as has already been explained.

Figure 5.11. Laser spectrum; $C_1$ is $2.5 \times 10^{-28}$ cm$^2$s$^{-1}$ and the CW output power is 1.9 mW.
in section 4.4. In the case of Fabry-Perot type lasers we can describe the z-dependence of the field by a 'simple' sine function, thus a real function. For this reason, we only need a first order differential equation to describe the time-dependent photon density, resulting in an over-all second order system.

To conclude this section we will show the effect of increasing the bias current on the spectrum. With the original value of $2 \times 10^{-28}$ cm$^2$ s$^{-1}$ for $C_1$ we find the result presented in figure 5.12.

![Figure 5.12: Laser spectrum; CW output power is 6.1 mW](image)

In this figure the laser spectrum is depicted for a bias current of 60 mA, corresponding to a CW output power of 6.1 mW. Compared to figure 5.8 there is a clear shift of the relaxation to a frequency of 15 GHz.

### 5.5 Transient Behaviour

To conclude this chapter on simulations and results, we will present some curves showing the transient behaviour of the laser. We start with showing the response in case a DC-current of 50 mA is applied to the laser. This response is given in figure 5.13.
Figure 5.13. Step response of the laser

First, the step response shows a number of relatively large peaks before reaching its steady state end value. The first peak exceeds the value of 35 mW. This is not seen in practice. The difference between the simulation and real conditions is caused by the difference in the relaxation oscillation
peak height. In practice, the peak is much lower, causing the initial oscillations to be damped more.

Figure 5.14. Optical output due to a rectangular modulation current

The large value of the relaxation oscillation peak causes the optical output to be rather 'wild' when we apply rectangular modulation current with an amplitude of 3 mA, superponed on a bias current of 50 mA, presented in figure 5.14. In the situation considered, the modulation frequency is 2 GHz.

In practice, the relaxation oscillation of a laser is used to create very short light pulses. By pumping with a short pulse-shaped current, we can cause only the first peak in the initial overshoot of the optical response to show up. In figure 5.15 we illustrate this. However, in this figure the current pulse still is too broad, since not only the first, but also the second peak shows up in the optical
Finally we will show the response of the optical output when a harmonic modulation current is applied to the laser. In figure 5.16 we show the optical output in case a pump current with a relatively low modulation frequency of 500 MHz is applied and in figure 5.17 for a frequency of 4 GHz.
The figures are only included to show the AM-modulation of the optical laser output.

Figure 5.16. Optical output due to a time-harmonic pump current; the modulation frequency is 500 Mhz
Figure 5.17. Optical output due to a time-harmonic pump current; the modulation frequency is 4 GHz
Conclusions

We have succeeded in deriving a modelling for DFB type lasers to predict the response of the optical output power due to an electrical excitation.

The results of the theory that describes the electric field shape in the transverse plane differ from real conditions as can be concluded from the deviation between the actual and calculated value of the coupling factor-length product.

Although the modelling is based on small signal conditions, it gives promising results even when larger signals are applied. This is caused by the fact that the carrier density above threshold doesn't change significantly for larger values of pump current.

In order to obtain a more realistic value of both the peak height and position caused by the relaxation oscillation in the laser spectrum, the electrical losses of the surroundings of the active layer have to be taken into account and the values of the constants used in the rate equations must be adjusted as soon as reliable data become available in literature.
Recommendations

From the conclusions on the previous page a few recommendations are easily found.

The first is trying to improve the theory that leads to the value of \( \kappa L \) in order to give results closer to experimentally determined values.

The second recommendation is to include the electrical losses of the active layer surroundings in the modelling and the third is to adjust the constants used in the rate constants used in the rate equations as soon as they are available for 1.55 \( \mu m \)-material.

After having carried out the proposed adjustments, a realistic comparison between the results of the modelling and experimental results can be made.

Since we have concentrated on the amplitude of the optical power as a function of the electrical excitation, it might be interesting to adjust the equation that describes the time dependence of the electric field and to add a rate equation describing the phase variation of the field amplitude with time, in order to be able to predict the laser behaviour when FM-modulation is applied. This might be useful as the interest in FM-modulating of laser devices has grown in the last few years.
References

[1] M.P.J.G. Versleijen,
"Modelling van het dynamische gedrag van de Double Channel Planar Buried Heterostructure Laser: DC spanningsverdeling en lekgedrag",
October 1985.

[2] P.P.G. Mols,
"Modelling of the dynamic behaviour of the Double Channel Planar Buried Heterostructure Laser",
December 1986.

[3] H. Kogelnik and C.V. Shank,
"Coupled-Wave Theory of Distributed Feedback Lasers",

"Effect of External Reflectors on Longitudinal Modes of Distributed Feedback Lasers",

[5] G.H.B. Thompson,
"Physics of Semiconductor Laser Devices",

[6] S. Wang,
"Proposal of periodic layered waveguide structures for distributed lasers",

[7] S. Wang,
"Principles of Distributed Feedback and Distributed Bragg-Reflector Lasers",

[8] C.T.A. Johnk,
"Engineering Electromagnetic Fields and Waves",
[9] K. Furuya, Y. Suematsu and T. Hong,
"Reduction of resonancelike peak in direct modulation due to carrier diffusion in injection laser",

[10] D. Marcuse,
"Classical Derivation of the Laser Rate Equation",

"Ultra-High Speed Semiconductor Lasers",

[12] R.S. Tucker and D.J. Pope,
"Circuit Modelling of the Effect of Diffusion on Damping in a Narrow-Stripe Semiconductor Laser",

"Ruis als gevolg van optische terugkoppeling bij halfgeleiderlasers",

"The Influence of Feedback Intensity on Longitudinal Mode Properties and Optical Noise in Index-Guided Semiconductor Lasers",

[15] R. Lang,
"Injection Locking Properties of a Semiconductor Laser",

[16] Th. van de Roer,
"Electronische Golfvoortplanting",
Eindhoven

[17] N.K. Dutta,
"Gain-current relation for In_{0.75}Ga_{0.25}As_{0.65}P_{0.35} lasers",

"Measurements of refractive-index variation with free carrier density and temperature for 1.6 \mu m \text{GaInAsP/InP lasers}",
[19] T.P. Pearsall (Editor),
"GaAsInP Alloy Semiconductors",

"Spontaneous Recombination, Gain and Refractive Index Variation for 1.6 \mu m Wavelength InGaAsP/InP Lasers",

[21] D. Botez,
"InGaAsP/InP Double-Heterostructure Lasers: Simple Expressions for Wave Confinement, Beamwidth, and Threshold Current over Wide Ranges in Wavelength (1.1 – 1.65 \mu m)",

[22] M. Yano, H. Nishi and M. Takusagawa,
"Theoretical and Experimental Study of Threshold Characteristics in InGaAsP/InP DH Lasers",

[23] J. Manning, R. Olshansky and C.B. Su,
"The Carrier-Induced Index Change in AlGaAs and 1.3 \mu m InGaAsP Diode Lasers",

"Band gap versus composition and demonstration of Vegard’s law for In_{1-x}Ga_xAs:P_{1-y}, lattice matched to InP",

[25] M. Osinski and J. Buus,
"Linewidth Broadening Factor in Semiconductor Lasers-An Overview",

"Theory of the Linewidth of Semiconductor Lasers",

[27] K. Wünstel, A. Moser, M. Schilling, G. Luz, K. Löscher, H. Schweizer, G. Schemmel and O. Hildebrand,
"High-speed 1.3 \mu m DFB laser with modified DCPBH structure",
[28] H. Soda and H. Imai,
"Analysis of the Spectrum Behaviour Below the Threshold in DFB Lasers",

[29] H. Nagai, T. Matsuoka, Y Noguchi, Y. Suzuki and Y. Yoshikuni,
"InGaAsP/InP Distributed Feedback Buried Heterostructure Lasers with Both Facets Cleaved Structure",

[30] E. A. J. Marcatili,
"Dielectric Rectangular Waveguide and Dielectric Coupler for Integrated Optics",

[31] A. Sugimura,
"Band-to-Band Auger Effect on the Output Power Saturation in InGaAsP LED's",

[32] J.H. Angenent,
"Onderzoek aan planaire optische golfgeleiders",

[33] D. Marcuse,
"Theory of dielectric optical waveguides",

[33] A. Balfour and D.H. Marwick,
"Programming in Standard FORTRAN 77",

[34] B.G. Streetman,
"Solid state electronic devices",
APPENDIX A: Applying the Boundary Conditions in the Transverse Plane

**Determination of the Wave Numbers**

If only the \(x\)-dependence is considered, the boundary conditions at \(x = 0\) and \(x = d\) lead to the following set equations

\[
E_f + E_i = E_{d1} + E_{d2} \quad (A.1)
\]

\[
-ik_x (E_f - E_i) = p_d (E_{d1} - E_{d2}) \quad (A.2)
\]

\[
E_{d1} \exp (p_d d) + E_{d2} \exp (-p_d d) = E_i \quad (A.3)
\]

\[
p_d (E_{d1} \exp (p_d d) - E_{d2} \exp (-p_d d)) = -p_i E_i \quad (A.4)
\]

Multiplying both the left and right hand side of (A.3) with \(p_i\) and adding this new equation to (A.4) gives

\[
E_{d1} = c_1 E_{d2} \quad (A.5)
\]

with

\[
c_1 = \frac{p_d - p_i}{p_d + p_i} \exp (-2p_d d) \quad (A.6)
\]

Substituting (A.5) into (A.1) and (A.2) results in

\[
\frac{E_{f2}}{E_{f1}} = \frac{1 + c_1 + \frac{ip_d}{k_x} (c_1 - 1)}{1 + c_1 - \frac{ip_d}{k_x} (c_1 - 1)} \quad (A.7)
\]

Substitution of (A.6) in (A.7) leads to

\[
\frac{E_{f2}}{E_{f1}} = \frac{k_x (p_d \cosh (p_d d) + p_i \sinh (p_d d)) - ip_d (p_d \sinh (p_d d) + p_i \cosh (p_d d))}{k_x (p_d \cosh (p_d d) + p_i \sinh (p_d d)) + ip_d (p_d \sinh (p_d d) + p_i \cosh (p_d d))} \quad (A.8)
\]
Dividing both the numerator and the denominator by \( p_d \cosh (p_d) + p_f \sinh (p_d) \) gives

\[
\frac{E_{f2}}{E_{f1}} = \frac{k_x - ip_dA}{k_x + ip_dA}
\]

(A.9)

where \( A \) is

\[
A = \frac{p_d \sinh (p_d) + p_f \cosh (p_d)}{p_d \cosh (p_d) + p_f \sinh (p_d)}
\]

(A.10)

The right hand side of equation (A.9) can be put in an exponent form. It is obvious that the modulus of the exponent equals one. Now we can poner

\[
\frac{k_x - ip_dA}{k_x + ip_dA} = \exp (-2i \phi_{Ed})
\]

(A.11)

When written in exponent notation the argument of the complex left hand fraction equals \(-2 \arctan (p_dA/k_x)\). So it follows that \( \phi_{Es} \) has to be equal to \( \arctan (p_dA/k_x) \).

In a similar way the boundary conditions can be applied for \( x = -w_1 \) and \( x = -w_2 \) leading to the following equations

\[
E_{f1} \exp (-ik_xw_1) + E_{f2} \exp (ik_xw_1) = E_{a1} + E_{a2}
\]

(A.12)

\[
-ik_x(E_{f1} \exp (-ik_xw_1) - E_{f2} \exp (ik_xw_1)) = p_a (E_{a1} - E_{a2})
\]

(A.13)

\[
E_{a1} \exp (p_a (w_1 - w_2)) + E_{a2} \exp (-p_a (w_1 - w_2)) = E_3
\]

(A.14)

\[
p_a (E_{a1} \exp (p_a (w_1 - w_2)) - E_{a2} \exp (-p_a (w_1 - w_2))) = p_3 E_3
\]

(A.15)

Multiplying both the left and right hand side of (A.14) with \(-p_3\) and adding this new equation to (A.15) gives

\[
E_{a1} = c_2 E_{a2}
\]

(A.16)

with

\[
c_2 = \frac{p_a + p_3}{p_a - p_3} \exp (-2p_3 (w_1 - w_2))
\]

(A.17)

Substituting (A.16) into (A.12) and (A.13) results in
\[
\frac{E_{y2}}{E_{y1}} = \frac{1 + c_2 + \frac{i\eta}{k_x} (c_2 - 1)}{1 + c_2 - \frac{i\eta}{k_x} (c_2 - 1)} \exp (-2ik_x w_1) \tag{A.18}
\]

Let \( \psi \) be \( p_3(w_1 - w_2) \), then substitution of (A.17) in (A.18) leads to

\[
\frac{E_{y2}}{E_{y1}} = \frac{k_x (p_3 \cosh (\psi) - p_3 \sinh (\psi)) - i\eta (p_3 \cosh (\psi) - p_3 \cosh (\psi))}{k_x (p_3 \cosh (\psi) - p_3 \sinh (\psi)) + i\eta (p_3 \cosh (\psi) - p_3 \cosh (\psi))} \exp (-2ik_x w_1) \tag{A.19}
\]

Dividing both the numerator and the denominator of (A.19) by \( p_3 \cosh (\psi) - p_3 \sinh (\psi) \) gives

\[
\frac{E_{y2}}{E_{y1}} = \frac{k_x - i\eta B}{k_x + i\eta B} \exp (-2ik_x w_1) \tag{A.20}
\]

where \( B \) is

\[
B = \frac{p_3 \sinh (\psi) - p_3 \cosh (\psi)}{p_3 \cosh (\psi) - p_3 \sinh (\psi)} = \frac{p_3 \tanh (\psi) - p_3}{p_3 - p_3 \tanh (\psi)} \tag{A.21}
\]

Writing the right hand side of equation (A.20) in exponent form gives

\[
\frac{k_x - i\eta B}{k_x + i\eta B} \exp (-2ik_x w_1) = \exp (-2i\phi_{Ed}) \tag{A.22}
\]

The argument of the left hand fraction of equation (A.22) is equal to \(-2\arctan (p_3 B/k_x) - 2k_x w_1\). So it is necessary that \( \phi_{Ed} \) is equal to \( \arctan (p_3 B/k_x) + k_x w_1 \).

All the above manipulations necessarily lead to the following equation

\[
\frac{E_{y2}}{E_{y1}} = \exp (-2i\phi_{Ed}) = \exp (-2i\phi_{Ed}) \tag{A.23}
\]

With the above equation the field in the active layer (as far as dependent on the x-coordinate) can be rewritten in two ways. The first way is

\[
(E_y)_{x} = 2E_{y1} \exp (-i\phi_{Ed}) \cos (k_x x + \phi_{Ed})
\]

\[
= \frac{1}{2} \left\{ \cos (k_x x + 2\phi_{Ed}) + \cos (k_x x) - i(\sin (k_x x + 2\phi_{Ed}) - \sin (k_x x)) \right\} \tag{A.24}
\]

The second way is
\[ (E_y)_{f,x} = 2E_f \exp \left( -i \Phi_{Es} \right) \cos (k_x x + \Phi_{Es}) \]
\[ = \frac{1}{2} \left( \cos (k_x x + 2\Phi_{Es}) + \cos (k_x x) - i (\sin (k_x x + 2\Phi_{Es}) - \sin (k_x x)) \right) \]

Equations (A.24) and (A.25) are identical only if the following is true
\[ 2\Phi_{Es} = 2\Phi_{Ed} + 2\pi q, \quad q \text{ integer} \]  \hspace{1em} (A.26)

If we call the arctangent term appearing in \( \Phi_{Es} \) \( \Phi_{Eo} \) then the \( k_x \) of a mode has to fulfill the following equation to be a proper mode
\[ k_x \omega_1 + \Phi_{Ed} - \Phi_{Ed} = q\pi, \quad q \text{ integer} \]  \hspace{1em} (A.27)

**Determination of the Confinement Factor**

To come to equation (A.27) all the amplitudes of the various fields had to be expressed in \( E_n \). In other words: the fields in the surrounding media can be expressed in the amplitude of the field in the active layer. If the field amplitude in the active layer is written as \( 2E_f \exp \left( -i \Phi_{Ed} \right) \) by using equation (A.23), the following expressions can be derived for the fields in the other media.
\[ (E_y)_{f,x} = 2E_f \exp \left( -i \Phi_{Ed} \right) \cos (k_x x + \Phi_{Ed}) \]
\[ = E_{f,x} \cos (k_x x + \Phi_{Ed}) \]  \hspace{1em} (A.28)

At the boundary \( x=0 \) \( (E_y)_{f,x} \) is equal to \( E_{f,x} \cos (\Phi_{Ed}) \). Setting the field in the dielectric medium at \( x=0 \), using (A.7) and (A.8), equal to this, the following equations are found for \( E_{d1}, E_{d2} \) and the total field in the dielectric layer respectively
\[ E_{d2} = \frac{p_d + p_l}{(p_d - p_l) \exp (-2\pi d) + (p_d + p_l)} E_{E,x} \cos (\Phi_{Ed}) \]  \hspace{1em} (A.29)
\[ E_{d1} = \frac{p_d - p_l}{(p_d - p_l) \exp (-2\pi d) + (p_d + p_l)} E_{E,x} \cos (\Phi_{Ed}) \exp (-2\pi d) \]  \hspace{1em} (A.30)
\[ (E_y)_{d,x} = \frac{(p_d - p_l) \exp (-2\pi d) \exp (p_d x) + (p_d + p_l) \exp (-p_d x)}{(p_d - p_l) \exp (-2\pi d) + (p_d + p_l)} E_{E,x} \cos (\Phi_{Ed}) \]
\[ = \frac{p_d \cosh (p_d (d - x)) + p_l \sinh (p_d (d - x))}{p_d \cosh (p_d d) + p_l \sinh (p_d d)} E_{E,x} \cos (\Phi_{Ed}) \]  \hspace{1em} (A.31)

Now having found the expression for \( (E_y)_{d,x} \), it is easy to find the description of the field in the top layer. For \( x=d \) the field is continuous. Reminding equation (2.4) makes clear that for \( x=d \) the amplitude of the field in the top layer is equal to \( (E_y)_{d,x} \big|_{x=d} \). Thus
\[ E_t = \frac{P_d}{P_d \cosh (p_d x) + p_x \sinh (p_d x)} E_{E,x} \cos (\phi_{E,d}) \quad (A.32) \]

and so for \((E_y)_{t,x}\) we find

\[ (E_y)_{t,x} = \frac{P_d}{P_d \cosh (p_d x) + p_x \sinh (p_d x)} E_{E,x} \cos (\phi_{E,d}) \exp (-p_x (x - d)) \quad (A.33) \]

At the boundary \(x = -w_1\) \((E_y)_{t,x}\) is equal to \(E_{E,x} \cos (\phi_{E,d} - k_x w_1)\) Setting the field in the anti meltback layer at \(x = -w_1\) equal to this, using (A.18) and (A.19), the following equations are found for \(E_{a2}, E_{a1}\) and \((E_y)_{a,x}\) respectively

\[ E_{a2} = \frac{P_a - p_x}{(P_a + p_x) \exp (-2\psi) + (P_a - p_x)} E_{E,x} \cos (\phi_{E,d} - k_x w_1) \quad (A.34) \]

\[ E_{a1} = \frac{P_a + p_x}{(P_a + p_x) \exp (-2\psi) + (P_a - p_x)} E_{E,x} \cos (\phi_{E,d} - k_x w_1) \exp (-2\psi) \quad (A.35) \]

\[ (E_y)_{a,x} = \frac{P_a \cosh (P_a (x + w_2)) + p_x \sinh (P_a (x + w_2))}{P_a \cosh (\psi) - p_x \sinh (\psi)} E_{E,x} \cos (\phi_{E,d} - k_x w_1) \quad (A.36) \]

From equation (2.5) it is obvious that the amplitude of the field in the substrate at the boundary \(x = -w_2\) is equal to \((E_y)_{a,x} \mid x = -w_2\). Accordingly multiplying with the exponential decay factor of the substrate leads to the following field description

\[ (E_y)_{t,x} = \frac{P_a}{P_a \cosh (\psi) - p_x \sinh (\psi)} E_{E,x} \cos (k_x w_1 - \phi_{E,d}) \exp (p_x (x + w_2)) \quad (A.37) \]

For ease of writing all amplitudes can be put in a form used by Wang [6]. The transition from the field descriptions presented here to those presented by Wang is made clear on the basis of \((E_y)_{t,x}\). For rewriting, only the fraction of equation (A.31) is important. First we divide both the numerator and the denominator of the fraction by \(P_a\). Then we will treat the numerator and the denominator separately, starting with the latter. Our aim is to determine \(b, \alpha, \beta\) in the following equation

\[ \cosh (p_d x) + p_x \sinh (p_d x) = b \cosh (\alpha x + \beta) \quad (A.38) \]

with \(x = P_d x\). Now we rewrite the right hand side of equation (A.38).

\[ \cosh (\alpha x + \beta) = \cosh (\alpha x) \cosh (\beta) + \sinh (\alpha x) \sinh (\beta) \quad (A.39) \]

Comparing (A.38) and (A.39) immediately yields: \(\alpha = 1, b\) and \(\beta\) remain to be determined. With \(\alpha\) set to unity, equation (A.40) must hold

\[ \cosh (x) + \frac{p_x}{P_d} \sinh (x) = b \cosh (x) \cosh (\beta) + b \sinh (x) \sinh (\beta) \quad (A.40) \]
(A.40) is true only if \( b \cosh (\beta) = 1 \) and \( b \sinh (\beta) = p_t/p_d \). Combining of these two necessary identities yields \( \tanh (\beta) = p_t/p_d \), thus \( \beta = \tanh^{-1}(p_t/p_d) \). Using \( f_3 = (\cosh(\beta) - 1)^{1/2} \) we find for \( b, b = (1 - (p_t/p_d)^2)^{1/2} \). So finally the left hand side of (A.38) can be rewritten as follows

\[
\cosh (p_d d) + \frac{p_t}{p_d} \sinh (p_d d) = \sqrt{1 - \frac{p_t^2}{p_d^2}} \cosh \left( p_d d + \tanh \left( \frac{p_t}{p_d} \right) \right)
\]

(A.41)

The argument of the area tangent hyperbolic function has to be an element of the interval \((-1, 1)\). As negative values of \( p_t \) and \( p_d \) cannot exist, this means that \( p_t \) has to be larger than \( p_d \), or that the index of refraction of the top layer has to be larger than that of the dielectric layer, otherwise expressing the denominator in one cosine hyperbolic form is not possible. In this case it is obvious to rewrite the denominator in one sine hyperbolic form, yielding equation (A.42)

\[
\cosh (p_d d) + \frac{p_t}{p_d} \sinh (p_d d) = b \sinh (ax + \beta)
\]

(A.42)

Applying the analysis presented above eventually results in

\[
\cosh (p_d d) + \frac{p_t}{p_d} \sinh (p_d d) = \sqrt{1 - \frac{p_t^2}{p_d^2}} \sinh \left( p_d d + \text{arcoth} \left( \frac{p_t}{p_d} \right) \right)
\]

(A.43)
In the next section we present all integrals necessary to calculate \( r \). \( r \) is defined by equation (2.38). Ignoring the \( z \)-dependence because of the fact that the contributions of the integrals in both the numerator and the denominator are the same and thus cancel, we can separate the double infinite integral appearing in the denominator of \( r \) in 7 part-integrals. The results of the integrations are given below. At this point it should be mentioned that the amplitude \( E_F \) is the product of \( E_{E_x} \) and \( E_{E_y} \), the amplitudes assigned to the \( x \)- and \( y \)-dependent part of the field respectively.

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_F^2 \cos^2 (k_x x + \phi_{Ed}) \cos^2 (k_y y + \phi_{Bi}) dy dx = \]

\[
= \frac{w_1 t}{4} \left( 1 + \frac{1}{k_x w_1} \sin (k_x w_1) \cos (2\phi_{Ed} - k_x w_1) \right) \left( 1 + \frac{1}{k_y t} \sin (k_y t) \cos (2\phi_{Bi}) \right) E_F^2
\]

\[
\int_{-t/2}^{t/2} \int_{-t/2}^{t/2} E_F^2 \cos^2 (\phi_{Ed}) \frac{\cosh^2 (p_2 x - \phi_0)}{\cosh^2 (\phi_0)} \cos^2 (k_y y + \phi_{Bi}) dy dx = \]

\[
= \frac{t}{2} \left( 1 + \frac{1}{k_y t} \sin (k_y t) \cos (2\phi_{Bi}) \right) I_1 E_F^2
\]

\[
\int_{-t/2}^{t/2} \int_{-t/2}^{t/2} E_F^2 \cos^2 (\phi_{Ed}) \frac{\cosh^2 (p_2 x - \phi_0)}{\cosh^2 (\phi_0)} \cos^2 (k_y y + \phi_{Bi}) \exp (-2\eta_1 (x-d)) dy dx = \]

\[
= \frac{t}{2} \left( 1 + \frac{1}{k_y t} \sin (k_y t) \cos (2\phi_{Bi}) \right) I_2 E_F^2
\]

\[
\int_{-w_1}^{-t/2} \int_{-w_1}^{-t/2} E_F^2 \cos^2 (k_x w_1 - \phi_{Ed}) \frac{\cosh^2 (p_2 (x + w_1) - \phi_1)}{\cosh^2 (\phi_1)} \cos^2 (k_y y + \phi_{Bi}) dy dx = \]

\[
= \frac{t}{2} \left( 1 + \frac{1}{k_y t} \sin (k_y t) \cos (2\phi_{Bi}) \right) I_3 E_F^2
\]

\[
\int_{-w_2}^{-t/2} \int_{-t/2}^{t/2} E_F^2 \cos^2 (k_x w_1 - \phi_{Ed}) \frac{\cosh^2 (p_2 (w_1 - w_2) - \phi_1)}{\cosh^2 (\phi_1)} \cos^2 (k_y y + \phi_{Bi}) dy dx = \]

\[
= \frac{t}{2} \left( 1 + \frac{1}{k_y t} \sin (k_y t) \cos (2\phi_{Bi}) \right) I_4 E_F^2
\]

\[
\int_{-t/2}^{t/2} \int_{-t/2}^{t/2} E_F^2 \cos^2 (k_x x + \phi_{Ed}) \cos^2 (\phi_{Bi} - k_y t/2) \exp (-2\eta_1 (y-t/2)) dy dx = \]

\[
= \frac{w_1}{2} \left( 1 + \frac{1}{k_x w_1} \sin (k_x w_1) \cos (2\phi_{Ed} - k_x w_1) \right) I_5 E_F^2
\]
\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_k^2 \cos^2(k_x x + \phi_E d) \cos^2(\phi_{Bd} + k_y y/2) \exp \left(2p_r(y + t/2)\right) dy \, dx = \]
\[ = \frac{w_1}{2} \left(1 + \frac{1}{k_x w_1} \sin (k_x w_1) \cos (2\phi_{E,d} - k_x w_1)\right) I_0 f_E^2 \]  
(A.50)

The integration results \( I_1 \ldots I_6 \) are given by the following expressions

\[ I_1 = \frac{\cos^2(\phi_{E,d})}{4 \cosh^2(\phi_0)} \left[ \frac{1}{p_d} \left( - \sinh (2\phi_0) - \sinh (2(p_{a,d} - \phi_0)) + 2d \right) \right], \quad p_t < p_d \]
\[ = \frac{\cos^2(\phi_{E,d})}{4 \sinh^2(\phi_0)} \left[ \frac{1}{p_d} \left( - \sinh (2\phi_0) - \sinh (2(p_{a,d} - \phi_0)) + 2d \right) \right], \quad p_t > p_d \]  
(A.51)

\[ I_2 = \frac{\cos^2(\phi_{E,d})}{4 \cosh^2(\phi_0)} \left[ \sinh^2(p_{a,d} - \phi_0) \right], \quad p_t < p_d \]
\[ = \frac{\cos^2(\phi_{E,d})}{4 \sinh^2(\phi_0)} \left[ \sinh^2(p_{a,d} - \phi_0) \right], \quad p_t > p_d \]  
(A.52)

\[ I_3 = \frac{\cos^2(k_x w_1 + \phi_{E,d})}{4 \cosh^2(\phi_1)} \left[ \frac{1}{p_a} \left( - \sinh (2(p_{a,w_1} + \phi_1)) + \sinh (2(p_{a,w_2} + \phi_1)) + 2(w_2 - w_1) \right) \right], \quad p_3 < p_a \]
\[ = \frac{\cos^2(k_x w_1 - \phi_{E,d})}{4 \sinh^2(\phi_1)} \left[ \frac{1}{p_a} \left( - \sinh (2(p_{a,w_1} + \phi_1)) \right) + \sinh (2(p_{a,w_2} + \phi_1)) - 2(w_2 - w_1) \right], \quad p_3 > p_a \]  
(A.53)

\[ I_4 = \frac{\cosh^2(p_a (w_1 - w_2) - \phi_1)}{4 \cosh^2(\phi_1)} \left[ 1 - e^{2p_a (w_1 - w_2)} \right], \quad p_3 < p_a \]
\[ = \frac{\sinh^2(p_a (w_1 - w_2) - \phi_1)}{4 \cosh^2(\phi_1)} \left[ 1 - e^{2p_a (w_1 - w_2)} \right], \quad p_3 > p_a \]  
(A.54)
The meaning of $\phi_0$ has already been described. $\phi'_0$ can be found from $\phi_0$ by replacing the 'artanh'-function by a 'arcoth'-function. $\phi_1$ is given by: $\phi_1 = p_x (w_1 - w_2) - \text{artanh} (p_x/p_y)$. $\phi'_1$ can be found from $\phi_1$ in a similar way $\phi'_0$ is found from $\phi_0$.

\[ I_5 = \frac{\cos^2 \left( \frac{\phi_{Bi} - k_y}{2} \right)}{2p_x} \quad (A.55) \]

\[ I_6 = \frac{\cos^2 \left( \frac{\phi_{Bi} + k_y}{2} \right)}{2p_y} \quad (A.56) \]
APPENDIX B: Derivation of the Constant of the Wave Equation

The wave equation results from the Maxwell equations. Therefore we start with presenting the latter in a form suited for our laser.

\[ \nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \quad (B.1) \]

\[ \nabla \times H = \frac{\partial D}{\partial t} + \sigma E \quad (B.2) \]

\[ \nabla \cdot E = 0 \quad (B.3) \]

\[ \nabla \cdot H = 0 \quad (B.4) \]

With \( D = \varepsilon E = \varepsilon_0 \varepsilon^2 E \), (B.2) can be rewritten as follows

\[ \nabla \times H = \varepsilon_0 \varepsilon^2 \frac{\partial E}{\partial t} + \sigma E \quad (B.5) \]

Taking the rotation of equation (B.1) and substituting (B.5) leads to

\[ \nabla \times \nabla \times E = -\mu_0 \frac{\partial}{\partial t} \left\{ \varepsilon_0 \varepsilon^2 \frac{\partial E}{\partial t} + \sigma E \right\} \quad (B.6) \]

which can be rewritten, using \( \nabla \times \nabla \times E = \nabla (\nabla \cdot E) - \nabla^2 E \) and relation (B.3)

\[ \nabla^2 E = \varepsilon_0 \varepsilon^2 \mu_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \sigma \frac{\partial E}{\partial t} \quad (B.7) \]

If we assume a harmonic time-dependence of the electric field like \( E = \hat{E} \exp(i\omega t) \) with \( \hat{E} \) a function of the coordinates \( x, y \) and \( z \), we find the wave equation (B.8) from (B.7). Since we expect the electric field to be polarized in the \( y \)-direction only, we omit the underscore in (B.8) and for convenience we omit the hat.

\[ \nabla^2 E + \left( \frac{\omega^2 \varepsilon^2}{c^2} - i\mu_0 \sigma \omega \right) E = 0 \quad (B.8) \]

\[ \nabla^2 E + k^2 E = 0 \]
The following relations have been used to obtain (B.8): \( c^2 = 1/\varepsilon_0\mu_0 \), with \( c \) the free space light velocity and \( k \) given by (B.9)

\[
k^2 = \frac{\omega^2 \eta^2}{c^2} - i\mu_0\sigma\omega
\]

(B.9)

For lossy dielectrics the relation \((\sigma/\omega) < < 1\) holds for the loss tangent. Johnk [8] has derived the following relation between the conductivity \( \sigma \) and the attenuation factor \( \alpha \)

\[
\alpha = \frac{\sigma}{2\omega\varepsilon_0} \sqrt{\mu_\varepsilon}
\]

(B.10)

Here we bring back in mind that \( \beta = \omega n/c \), thus \( \alpha \) can be rewritten as follows

\[
\alpha = \frac{\beta}{2} \frac{\sigma}{\omega\varepsilon_0}
\]

(B.11)

From (B.11) we can derive

\[
\frac{\beta}{\alpha} = \frac{2\omega\varepsilon_0}{\sigma}
\]

(B.12)

The derivation of (B.10) according to Johnk [8] holds for \((\sigma/\omega) < < 1\). This implies of course: \((\omega e/\sigma) > > 1\). With this knowledge, we find from (B.12): \( \beta > > \alpha \).

The lasing material we are considering introduces gain instead of loss in the wave propagation. Therefore, if \( \alpha \) denotes the net laser gain, the sign of the right hand side of (B.10) has to be changed. Changing sign, relating to our situation and rewriting with respect to \( \sigma \) of (B.10) yields

\[
\sigma = -2\alpha \sqrt{\frac{\varepsilon_0 n^2}{\mu_0}}
\]

(B.13)

(B.13) enables us to write for \( k^2 \)

\[
k^2 = \frac{\omega^2 \eta^2}{c^2} + 2\omega\varepsilon_0\mu_0\alpha \sqrt{\frac{\varepsilon_0 n^2}{\mu_0}}
\]

(B.14)

We remind the reader of the fact that in a DFB laser the index of refraction and the gain are functions of the position in the laser, in our case of the \( z \)-coordinate. So thus far has been meant by \( n \), \( n(z) \) and by \( \alpha, \alpha(z) \). This remark is made here to prevent the reader from getting confused when the equations (3.1) and (3.2) for the refractive index and the gain are substituted in (B.14), for in equations (3.1) and (3.2) \( n \) and \( \alpha \) denote the constant part of the index and the gain respectively.

Let the argument of the varying part in the expressions for the refractive index and the gain be denoted by \( \phi \), with \( \phi = 2\pi z/\Lambda + \Omega \), then we can write for the index and the gain

\[
n(z) = n + (\Delta n) \cos (\phi)
\]

(B.15)
\( \alpha(x) = \alpha + (\Delta \alpha) \cos(\phi) \)  \hspace{1cm} (B.16)

As has been shown previously, \( \beta \) has to be much larger than \( \alpha(x) \). This condition implies that both \( \alpha \) and \( (\Delta \alpha) \) have to be much smaller than \( \beta \). So, as Streifer has stated, these two conditions have to be met for the derivation of \( k \) to hold.

Substituting (B.15) and (B.16) in (B.14) and using \( (\Lambda n) < < n \), the third condition to be fulfilled, gives

\[
k^2 = \frac{\omega^2}{c^2} \left( n^2 + 2n (\Lambda n) \cos(\phi) \right) + 2i \omega \mu_0 (\alpha + (\Delta \alpha) \cos(\phi)) \sqrt{\varepsilon_0 (n^2 + 2n (\Lambda n) \cos(\phi))} \]

\[
= \frac{\omega^2 n^2}{c^2} + 2\omega^2 (\Lambda n) \cos(\phi) + 2i \omega (\alpha + (\Lambda \alpha)) \sqrt{n^2 + 2n (\Lambda n) \cos(\phi)}
\]

Entering \( \beta \) results in

\[
k^2 = \beta^2 + 2\beta \frac{\omega}{c} (\Lambda n) \cos(\phi) + 2i \beta (\alpha + (\Lambda \alpha) \cos(\phi)) \sqrt{1 + 2 \frac{(\Lambda n)}{n} \cos(\phi)}
\]

Now we extend the square root in a series. For small \( x \) we can write: \( (1 + x)^{1/2} = 1 + x/2 + \ldots \), resulting for \( k^2 \) in

\[
k^2 = \beta^2 + 2\beta \frac{\omega}{c} (\Lambda n) \cos(\phi) + 2i \beta (\alpha + (\Lambda \alpha) \cos(\phi)) \left( 1 + \frac{(\Lambda n)}{n} \cos(\phi) \right)
\]

\[
= \beta^2 + 2\beta \frac{\omega}{c} (\Lambda n) \cos(\phi) + 2i \beta (\alpha + (\Lambda \alpha) \cos(\phi))
\]

In the last step again \( (\Lambda n) < < n \) has been used. The definition of \( \beta \) has been repeated on the previous page. The same will be done here for \( \kappa \): \( \kappa = (\beta/2)(\Lambda n/n) + i\Lambda /2 \). Now we adapt \( k^2 \), using \( \beta \) and \( \kappa \), in the following way

\[
k^2 = \beta^2 + 4i \beta \kappa \cos(\phi) + 2i \alpha \beta \left[ 1 + \frac{(\Lambda n)}{n} \cos(\phi) \left( 1 + \frac{(\Lambda \alpha)}{\alpha} \cos(\phi) \right) \right] \]

\[
= \beta^2 + 2i \alpha \beta + 4\kappa \beta \cos(\phi)
\]

(B.17)

Finally, substitution of \( \phi \) and of \( \beta_0 \equiv \pi/\Lambda \) brings \( k^2 \) in the form presented by Streifer:

\[
k^2 = \beta^2 + 2i \alpha \beta + 4\kappa \beta \cos(2\beta_0 x + \Omega)
\]

(B.18)
APPENDIX C: Derivation of the Optical Intensity

In the analysis that will be undertaken here, we only pay attention to the \( z \)-dependence of the electric and magnetic fields in the laser. For this case, as has been stated in section 3.3, the amplitude term \('A'\) that shows up in equations (3.36) and (3.37) will be derived.

More than once has been stated that the electric field is polarized in the \( y \)-direction. Such an electric field is accompanied, according to Maxwell, with a magnetic field with components directed in both the \( x \)- and the \( z \)-direction if both the \( z \)- and \( x \)-dependence of the electric field are taken into account. A magnetic field only directed along the \( x \)-coordinate results if only a \( z \)-dependence is assumed. Moreover it can be proven that, by taking the real part of the vector product of \( \hat{E} \) and \( \hat{H}^* \), the contribution of the \( z \)-directed field vanishes, because its contribution to the vector product is purely imaginary.

Since the electric field consists of two counter-running waves, a magnetic field that also is built up by two counter-running waves will be created. In turn, this leads to the creation of an intensity that travels in both directions along the \( z \)-coordinate. As the calculation of both counter-running contributions of the total power is identical, only the left running part of the power, or better the intensity, will be calculated.

From section 3.1, especially from formulas (3.5) and (3.10) we recover the left travelling part of the electric field

\[
E_x = \{s_1 \exp(yz) + s_2 \exp(-yz)\} \exp(\imath \beta_0 z) e_y \tag{C.1}
\]

As has always been the case so far, we have omitted the time dependence. Now we apply Faraday’s law for time-harmonic fields:

\[
\nabla \times E = - \imath \omega \mu_0 H \tag{C.2}
\]

In (C.2) we have omitted the hat that denotes the position dependent amplitude for convenience. Substitution of (C.1) in (C.2) yields for the magnetic field

\[
H_x = \frac{1}{\imath \omega \mu_0} \{s_1 (y + \imath \beta_0) \exp(yz) + s_2 (-y + \imath \beta_0) \exp(-yz)\} \exp(\imath \beta_0 z) e_x \tag{C.3}
\]
Next we calculate the left travelling light intensity by means of Poynting's theorem, formulated in (C.4)

$$P_s = \frac{1}{2} \ Re(E_2 \times H_3^*)$$  \hspace{1cm} (C.4)

In the calculation equation (3.40) will be used for $y$.

$$P_s = \frac{1}{2 \omega \mu_0} \ Re\left[ -i\left( |s_1|^2 (\gamma^* - i \beta_0) \exp (2 \alpha y z) + |s_2|^2 (-\gamma^* - i \beta_0) \exp (-2 \alpha y z) + s_2 s_1^* (\gamma^* - i \beta_0) \exp (-2i \beta y z) + s_1 s_2^* (\gamma^* - i \beta_0) \exp (2i \beta y z) \right) \right] (\epsilon_z)$$  \hspace{1cm} (C.5)

In (C.5) has been used that the sum of a complex number and its conjugate equals twice its real part and that the difference of a complex number and its conjugate equals two times its imaginary part.

Let's continue the calculation

$$P_s = \frac{1}{2 \omega \mu_0} \ Re\left[ - |s_1|^2 (\alpha y - \beta y - \beta_0) \exp (2 \alpha y z) + |s_2|^2 (\alpha y + \beta y - \beta_0) \exp (-2 \alpha y z) + s_2 s_1^* (\alpha y - \beta y - \beta_0) \exp (-2i \beta y z) + s_1 s_2^* (\alpha y + \beta y - \beta_0) \exp (2i \beta y z) \right] (\epsilon_z)$$

$$= \frac{1}{2 \omega \mu_0} \ Re\left[ - |s_1|^2 (\beta y + \beta_0) \exp (2 \alpha y z) + |s_2|^2 (\beta y - \beta_0) \exp (-2 \alpha y z) - Re(s_2 s_1^* (\alpha y + \beta y + \beta_0) \exp (-2i \beta y z) + s_1 s_2^* (\alpha y - \beta y + \beta_0) \exp (2i \beta y z)) \right] (\epsilon_z)$$

$$= \frac{1}{2 \omega \mu_0} \ Re\left[ |s_1|^2 (\beta y + \beta_0) \exp (2 \alpha y z) + |s_2|^2 (- \beta y + \beta_0) \exp (-2 \alpha y z) + 2 Re(s_2 s_1^* (\beta_0 + i \alpha y) \cos (2 \beta y z) - i \sin (2 \beta y z)) + Re(\beta y s_2 s_1^* \exp (-2i \beta y z) - \beta y s_1 s_2^* \exp (2i \beta y z)) \right] (\epsilon_z)$$

By applying some basic mathematics on complex numbers it can be shown easily that the last factor in the above derivation equals zero, because the part between parentheses of which the real part is taken, is purely imaginary, so this term cancels from the derived expression.

In section 3.3 has been shown that for practical cases $\beta_0$ is much larger than both $\alpha$ and $\beta$. This knowledge enables us to write for $P_s$

$$P_s = \frac{\beta_0}{2 \omega \mu_0} \ Re\left[ |s_1|^2 \exp (2 \alpha y z) + |s_2|^2 \exp (-2 \alpha y z) + 2 Re(s_2 s_1^* \exp (-2i \beta y z)) \right] (\epsilon_z)$$

$$= \frac{\beta_0}{2 \omega \mu_0} |s_1 \exp (yz) + s_2 \exp (-yz)|^2 (\epsilon_z)$$  \hspace{1cm} (C.6)

$$= \frac{\beta_0}{2 \omega \mu_0} |E_s|^2 (\epsilon_z)$$
The first step in (C.6) can simply be verified by the reader. The second step is true, if we call:
\[ E_r = E_r(\varepsilon_r) \]

If we compare (C.6) with (3.36) then we can see easily that the amplitude term in (3.36) has to be equal to \( \beta_0/2\mu_0 \).

The derivation of the light intensity travelling in the positive z-direction is similar to the one treated in this appendix.
APPENDIX D: Developing the Rate Equations for Numerical Treatment

The title of this appendix is a little misleading, since it suggests that we will show the complete transition from the rate equations (4.26) and (4.27) to the rate equations suitable for being solved numerically. Although the analysis is not difficult, it is too involved to give in detail. For this reason we will omit parts of the analysis.

z-Dependence of the Field and the Intensity

The first function that will be presented is the z-dependence of the electric field, which follows from equation (3.5), with equations (3.10), (3.11) and (3.42) ... (3.44) substituted. The function will be called $f_z(z)$ and will be split up in its respective real and imaginary part, $f_z^r(z)$ and $f_z^i(z)$.

$$f_z(z) = f_z^r(z) + i f_z^i(z) \quad (D.1)$$

with

$$f_z^r(z) = e^{\gamma z}[\cos (\beta_1 z) + Re(d_1) \cos (\beta_2 z) - Im(d_1) \sin (\beta_2 z)]$$

$$+ e^{-\gamma z}[Re(c_2) \cos (\beta_2 z) + Im(c_2) \sin (\beta_2 z) + Re(d_2) \cos (\beta_1 z) + Im(d_2) \sin (\beta_1 z)] \quad (D.2)$$

and

$$f_z^i(z) = e^{\gamma z}[\sin (\beta_1 z) + Re(d_1) \sin (\beta_2 z) + Im(d_1) \cos (\beta_2 z)]$$

$$+ e^{-\gamma z}[Im(c_2) \cos (\beta_2 z) - Re(c_2) \sin (\beta_2 z) + Im(d_2) \cos (\beta_1 z) - Re(d_2) \sin (\beta_1 z)] \quad (D.3)$$

where $\beta_1 = \beta_r - \beta_o$ and $\beta_2 = \beta_r + \beta_o$.

The second z-dependence we present here is that of the optical intensity. This can be derived from equation (3.35) with equations (3.36), (3.37) and (3.42) ... (3.44) substituted. Let $\psi_1$ be equal to $2x_1z$ and $\psi_2$ to $2\beta_1z$, then we find for $f_\psi(z)$ being the z-dependence of the light intensity in the active layer
\[ f_2(z) = \exp(\psi_1) + |c_2|^2 \exp(-\psi_1) + 2[\text{Re}(c_2)\cos(\psi_2) + \text{Im}(c_2)\sin(\psi_2)] \\
+ |d_1|^2 \exp(\psi_1) + |d_2|^2 \exp(-\psi_1) + 2[\text{Re}(d_1)\text{Re}(d_2) + \text{Im}(d_1)\text{Im}(d_2)]\cos(\psi_2) \]
\[ - 2[\text{Im}(d_1)\text{Re}(d_2) - \text{Re}(d_1)\text{Im}(d_2)]\sin(\psi_2) \]
\[ = a_+ \exp(\psi_1) + a_- \exp(-\psi_1) + b_\times \cos(\psi_2) + b_\circ \sin(\psi_2) \]

where
\[ a_+ = 1 + |d_1|^2 \]
\[ a_- = |c_2|^2 + |d_2|^2 \]
\[ b_\times = 2[\text{Re}(c_2) + \text{Re}(d_1)\text{Re}(d_2) + \text{Im}(d_1)\text{Im}(d_2)] \]
\[ b_\circ = 2[\text{Im}(c_2) - \text{Im}(d_1)\text{Re}(d_2) + \text{Re}(d_1)\text{Im}(d_2)] \]

If we take the second derivative of \( f_2(z) \) with respect to \( z \) and call it \( f_\ddot{2}(z) \) then \( f_\ddot{2}(z) \) becomes
\[ f_\ddot{2}(z) = (2d_1)^2 \left[ 1 + |d_1|^2 \right] \exp(\psi_1) + \left[ |c_2|^2 + |d_2|^2 \right] \exp(-\psi_1) \]
\[ - (2d_1)^2 \left[ \{2[\text{Re}(d_1)\text{Re}(d_2) + \text{Re}(c_2) + \text{Im}(d_1)\text{Im}(d_2)]\} \cos(\psi_2) \right. \]
\[ - \left. \{2[\text{Im}(d_1)\text{Re}(d_2) - \text{Re}(d_1)\text{Im}(d_2)]\} \sin(\psi_2) \right] \]

\( f_\ddot{2}(z) \) is necessary in the determination of the diffusion term.

**Rewriting the Rate Equations**

First, we will repeat the carrier rate equation but with equation (4.46) substituted for \( n \).

\[ \frac{d}{dt} \left[ n_0 - n_l \left[ 1 + \cos(2(k_yv + \phi_{Bl})) \right] f_2(z) \right] = \frac{I}{qV_a} + \frac{G_P}{hv} - \frac{\tau_m}{\tau_m} \left( n_0 - n_l \left[ 1 + \cos(2(k_yv + \phi_{Bl})) \right] f_2(z) \right) \]
\[ - B_1(n_0 - n_l \left[ 1 + \cos(2(k_yv + \phi_{Bl})) \right] f_2(z))^2 \]
\[ - C_l(n_0 - n_l \left[ 1 + \cos(2(k_yv + \phi_{Bl})) \right] f_2(z))^3 \]
\[ + D\nabla^2(n_0 - n_l \left[ 1 + \cos(2(k_yv + \phi_{Bl})) \right] f_2(z)) \]

Both \( n_0 \) and \( n_l \) are functions of time.

For reasons of complexity we have not split the gain contribution in its respective terms. Now we will apply two operators on (D.3), given by (D.11) and (D.12)

\[ \Psi_\ast = \int_{-L/2}^{+L/2} \int_{-L/2}^{+L/2} \int_{L/2}^{+L/2} (\ldots) dxdydz \]

(D.11)
\[ \Xi = \int_{-L/2}^{L/2} \int_{-w_1}^{0} \int_{-w_1}^{0} \left( \ldots \right) \cos(2(k_1y + \phi_B)) \, dx \, dy \, dz \]  

\hspace{1cm} (D.12)

Applying the \( \Psi \)-operator on the left hand side of (D.10) results in

\[ \Psi \cdot \frac{d}{dt} \left[ n_0 - n_1 \left[ 1 + \cos(2(k_1x + \phi_B)) \right] f(x) \right] = w_1 tL \frac{dn_1}{dt} \]

\hspace{1cm} (D.13)

where \( I_{re,13} \) is given by equation (D.35).

Applying the \( \Xi \)-operator on the left hand side of (D.10) results in

\[ \Xi \cdot \frac{d}{dt} \left[ n_0 - n_1 \left[ 1 + \cos(2(k_1x + \phi_B)) \right] f(x) \right] = w_1 tL \frac{dn_0}{dt} \]

\hspace{1cm} (D.14)

Now we have found the results of applying the two operators on the left hand side of the carrier rate equation. Applying these operators on the right hand side is more complicated since quadratic and third order terms appear. The results of the manipulations on the right hand side will be given later on in this appendix. Let \( F_{q1} \) be the result of the \( \Psi \)-operator applied on the right hand side of (D.10) and \( F_{q2} \) of the \( \Xi \)-operator, then we find \((dn_0/dt)\) and \((dn_1/dt)\) from \( F_{q1} \) and \( F_{q2} \) in the following way:

Let's call

\[ I_{dn,1} = w_1 tL \]  

\hspace{1cm} (D.15)

\[ I_{dn,2} = -\frac{w_1}{k_y} I_{re,13} \cos(2\phi_B) \sin(k_1t) \]  

\hspace{1cm} (D.16)

\[ I_{dn,3} = -\frac{L I_{dn,2}}{I_{re,13}} \]  

\hspace{1cm} (D.17)

\[ I_{dn,4} = -\frac{w_1}{2} I_{re,13} \left( 1 + \frac{\cos(4\phi_B) \sin(2k_1t)}{2k_1t} \right) \]  

\hspace{1cm} (D.18)

then \((dn_0/dt)\) and \((dn_1/dt)\) are given by

\[ \frac{dn_0}{dt} = \frac{I_{dn,2} F_{q1} - I_{dn,3} F_{q2}}{I_{dn,1} I_{dn,2} - I_{dn,2} I_{dn,3}} \]  

\hspace{1cm} (D.19)
Now we will present the above mentioned \( E_{q1} \) and \( E_{q2} \). In the expressions for \( E_{q1} \) and \( E_{q2} \), a number of 'constants' will be used, that not have been used before. These 'constants' will be described afterwards.

\[
E_{q1} = -\frac{w_1}{q} (t \nu_n - l_{re,13} [t + l_{re,4}] n_1) + \frac{I}{q}
- B_1 w_1 (L \nu_n^2 - 2l_{re,13} [t + l_{re,4}] n_1 n_1 + l_{re,14} n_1^2 [t + 2l_{re,5} + l_{re,5}])
- C_1 w_1 (t (L \nu_n^3 - 3n_1^2 n_1 l_{re,13} + 3n_1 n_1^2 l_{re,14} - n_1^3 l_{re,15})
+ l_{re,5} (-3n_1^2 n_1 l_{re,13} + 6n_1 n_1^2 l_{re,14} - 3n_1^3 l_{re,15})
+ l_{re,6} (3n_1 n_1^2 l_{re,14} - n_1^3 l_{re,15} - l_{re,7} l_{re,15} n_1^2)
+ D w_1 [l_{re,6} (2k_5) l_{re,13} - l_{re,16} (t + l_{re,4})]
- l_{re,1} [l_{re,3} G_{h1} + l_{re,10} G_{h2} + l_{re,11} G_{h3}]
\]

\[
E_{q2} = -\frac{w_1}{q} (L l_{re,4} \nu_0 - l_{re,13} [l_{re,4} + l_{re,5}] n_1) + \frac{I}{q}
- B_1 w_1 (L l_{re,4} \nu_0^2 - 2l_{re,13} [l_{re,4} + l_{re,5}] n_0 n_1 + n [l_{re,4} + 2l_{re,5} + l_{re,5}] l_{re,14})
- C_1 w_1 (l_{re,5} (L \nu_n^3 - 3n_1^2 n_1 l_{re,13} + 3n_1 n_1^2 l_{re,14} - n_1^3 l_{re,15})
+ l_{re,5} (-3n_1^2 n_1 l_{re,13} + 6n_1 n_1^2 l_{re,14} - 3n_1^3 l_{re,15})
+ l_{re,6} (3n_1 n_1^2 l_{re,14} - n_1^3 l_{re,15} - l_{re,7} l_{re,15} n_1^2)
+ D w_1 [l_{re,5} (2k_5) l_{re,13} - l_{re,16} (l_{re,4} + l_{re,5})]
- l_{re,1} [l_{re,3} G_{h5} + l_{re,10} G_{h4} + l_{re,11} G_{h3} + l_{re,12} G_{h6}]
\]

The constants \( l_{re,1} \) to \( l_{re,16} \) are the outcomes of integrals that will be given below. The 'constants' \( l_{n,3} \), \( l_{n,5} \), \( l_{n,9} \) and \( l_{n,17} \) to \( l_{n,22} \) have not been used in equation (D.21) either in equation (D.22). They will be used in the differential equations for the field amplitude.

\[
l_{re,1} = \int_{-w_1}^{0} \cos^2 (k_x x + \phi_{Ed}) dx
= \frac{w_1}{2} \left\{ 1 + \frac{1}{k_x w_1} \cos (2\phi_{Ed} - k_x w_1) \sin (k_x w_1) \right\}
\]
\[ I_{re,2} = \int_{-t/2}^{t/2} \cos(k_y y + \phi_B) dy \]
\[ = \frac{2}{k_y} \cos(\phi_B) \sin \left( k_y \frac{t}{2} \right) \]  
(D.24)

\[ I_{re,3} = \int_{-t/2}^{t/2} \cos^2(k_y y + \phi_B) dy \]
\[ = \frac{t}{2} \left\{ 1 + \frac{1}{k_y t} \cos(2\phi_B) \sin(k_y t) \right\} \]  
(D.25)

\[ I_{re,4} = \int_{-t/2}^{t/2} \cos(2(k_y y + \phi_B)) dy \]
\[ = \frac{1}{k_y} \cos(2\phi_B) \sin(k_y t) \]  
(D.26)

\[ I_{re,5} = \int_{-t/2}^{t/2} \cos^2(2(k_y y + \phi_B)) dy \]
\[ = \frac{t}{2} \left\{ 1 + \frac{1}{2k_y t} \cos(4\phi_B) \sin(2k_y t) \right\} \]  
(D.27)

\[ I_{re,6} = \int_{-t/2}^{t/2} \cos^3(2(k_y y + \phi_B)) dy \]
\[ = \frac{1}{12k_y} \{ 9 \cos(2\phi_B) \sin(k_y t) + \cos(6\phi_B) \sin(3k_y t) \} \]  
(D.28)

\[ I_{re,7} = \int_{-t/2}^{t/2} \cos^4(2(k_y y + \phi_B)) dy \]
\[ = \frac{1}{32k_y} \{ 12k_y t + 8 \cos(4\phi_B) \sin(2k_y t) + \cos(8\phi_B) \sin(4k_y t) \} \]  
(D.29)

\[ I_{re,8} = \int_{-t/2}^{t/2} \cos(k_y y + \phi_B) \cos(2(k_y y + \phi_B)) dy \]
\[ = \frac{1}{3k_y} \left\{ \cos(3\phi_B) \sin \left( \frac{3}{2} k_y t \right) + 3 \cos(\phi_B) \sin \left( \frac{1}{2} k_y t \right) \right\} \]  
(D.30)

\[ I_{re,9} = \int_{-t/2}^{t/2} \cos(k_y y + \phi_B) \cos(4(k_y y + \phi_B)) dy \]
\[ = \frac{1}{15k_y} \left\{ 3 \cos(5\phi_B) \sin \left( \frac{5}{2} k_y t \right) + 5 \cos(3\phi_B) \sin \left( \frac{3}{2} k_y t \right) \right\} \]  
(D.31)
\[ I_{re,10} = \int_{-\theta/2}^{\theta/2} \cos^2(k_y \psi + \phi_R) \cos(2k_y \psi + \phi_R) dy \]
\[ = \frac{1}{8k_y} (4 \cos (2 \phi_R) \sin (k_y \psi) + \cos (4 \phi_R) \sin (2k_y \psi) + 2k_y) \]  
(D.32)

\[ I_{re,11} = \int_{-\theta/2}^{\theta/2} \cos^2(k_y \psi + \phi_R) \cos(6(k_y \psi + \phi_R)) dy \]
\[ = \frac{1}{12k_y} \left(3 \cos (4 \phi_R) \sin (2k_y \psi) + \cos (6 \phi_R) \sin (3k_y \psi) + 3 \cos (2 \phi_R) \sin (k_y \psi) \right) \]  
(D.33)

\[ I_{re,12} = \int_{-\theta/2}^{\theta/2} \cos^2(k_y \psi + \phi_R) \cos(6(k_y \psi + \phi_R)) dy \]
\[ = \frac{1}{48k_y} \left(8 \cos (6 \phi_R) \sin (3k_y \psi) + 3 \cos (4 \phi_R) \sin (4k_y \psi) + 6 \cos (2 \phi_R) \sin (2k_y \psi) \right) \]  
(D.34)

\[ I_{re,13} = \int_{-\theta/2}^{\theta/2} f_2(z) dz = \left\{a_+ + a_-\right\} \frac{\sinh(z \lambda)}{\alpha} + \frac{b_2 \sin(\beta \lambda)}{\beta} \]  
(D.35)

The following integrals will not be determined analytically, but numerically. So we will confine ourselves to explaining what the left hand side stands for in the following 9 equations.

\[ I_{re,14} = \int_{-L/2}^{L/2} f_2^2(z) dz \]  
(D.36)

\[ I_{re,15} = \int_{-L/2}^{L/2} f_3^2(z) dz \]  
(D.37)

\[ I_{re,16} = \int_{-L/2}^{L/2} f_4(z) dz \]  
(D.38)

\[ I_{re,17} = \int_{-L/2}^{L/2} f_5(z) dz \]  
(D.39)

\[ I_{re,18} = \int_{-L/2}^{L/2} f_6(z) dz \]  
(D.40)

\[ I_{re,19} = \int_{-L/2}^{L/2} f_7(z) f_8(z) dz \]  
(D.41)

\[ I_{re,20} = \int_{-L/2}^{L/2} f_9(z) f_2(z) dz \]  
(D.42)
$$I_{re,21} = \int_{-L/2}^{L/2} f_1(x) g_1^2(x) \, dz$$  
(D.43)

$$I_{re,22} = \int_{-L/2}^{L/2} f_1(x) g_2^2(x) \, dz$$  
(D.44)

Just as has been done with the presentation of the integrals $I_{re,\ldots}$, we will present here all the variables $G_{\alpha_1 \ldots \alpha_{12}}$, although $G_{\alpha_7 \ldots \alpha_{12}}$ are not part of the carrier rate equations but of the field rate equations.

$$G_{h1} = \frac{\pi \beta_0}{\mu_0 \omega^2} G_{st}(r_{rad}) |r_1| \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,13} - n_{1/2} l_{re,14}) \right. \right.$$  

$$+ B_1 (\kappa_1^2 l_{re,13} - 2 \kappa_2 n_1 l_{re,14} + 1.5 \kappa_3^2 l_{re,15}) - G_{th}(r_{rad}) l_{re,13}]$$  
(D.45)

$$G_{h2} = \frac{\pi \beta_0}{\mu_0 \omega^2} G_{st}(r_{rad}) |r_1| \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,13} - n_{1/2} l_{re,14}) \right.$$  

$$+ 2 B_1 (\kappa_1^2 l_{re,13} - 2 \kappa_2 n_1 l_{re,14} + 1.5 \kappa_3^2 l_{re,15}) - G_{th}(r_{rad}) l_{re,13}]$$  
(D.46)

$$G_{h3} = \frac{\pi \beta_0 l_{re,15}}{2 \mu_0 \omega^2} G_{st}(r_{rad}) |r_1| 2 B_1 n_1^2$$  
(D.47)

$$G_{h4} = \frac{\pi \beta_0}{\mu_0 \omega^2} G_{st}(r_{rad}) |r_1| \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,13} - n_{1/2} l_{re,14}) \right.$$  

$$+ B_1 (\kappa_1^2 l_{re,13} - 2 \kappa_2 n_1 l_{re,14} + 1.75 \kappa_3^2 l_{re,15}) - G_{th}(r_{rad}) l_{re,13}]$$  
(D.48)

$$G_{h5} = \frac{G_{h2}}{2}$$  
(D.49)

$$G_{h6} = 1.5 G_{h3}$$  
(D.50)

$$G_{h7} = \frac{c}{\Delta_{ef}} G_{st}(r_{rad}) \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,17} - n_{1/2} l_{re,19}) \right.$$  

$$+ B_1 (\kappa_1^2 l_{re,17} - 2 \kappa_2 n_1 l_{re,19} + 1.5 \kappa_3^2 l_{re,21}) - G_{th}(r_{rad}) l_{re,17}]$$  
(D.51)

$$G_{h8} = \frac{c}{\Delta_{ef}} G_{st}(r_{rad}) \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,19} - 2 B_1 \kappa_1 n_1 l_{re,19} + 2 B_1 n_1^2 l_{re,19}) \right.$$  

$$+ B_1 (\kappa_1^2 l_{re,13} - 2 \kappa_2 n_1 l_{re,14} + 1.5 \kappa_3^2 l_{re,22}) - G_{th}(r_{rad}) l_{re,18}]$$  
(D.52)

$$G_{h9} = \frac{c l_{re,21}}{\Delta_{ef}} G_{st}(r_{rad}) B_1 n_1^2$$  
(D.53)

$$G_{h10} = \frac{c}{\Delta_{ef}} G_{st}(r_{rad}) \left[ \frac{a}{\tau_{nr}} (n_1 l_{re,18} - n_{1/2} l_{re,20}) \right.$$  

$$+ B_1 (\kappa_1^2 l_{re,18} - 2 \kappa_2 n_1 l_{re,20} + 1.5 \kappa_3^2 l_{re,22}) - G_{th}(r_{rad}) l_{re,18}]$$  
(D.54)
where

\[ r_{rad} = \frac{a}{\tau_{nr}} (n_0 - n_1 l_{re,13}) + B_1 (n_0 - 2 n_0 n_1 l_{re,13} + n_1^2 l_{re,1a}) \]

\[ + \frac{l_{re,2}}{l_e} \left( \frac{a}{\tau_{nr}} l_{re,13} + 2B_1 (n_0 n_1 l_{re,13} + n_1^2 l_{re,1a}) \right) + B_1 \frac{l_{re,2} l_{re,1a}}{l_e} n_1^2 \]

\[ G_{sl} \text{ and } G_{sh} \text{ are the slope and the threshold of the gain function respectively. As is clear from equations (4.10) and (4.11) both the slope and the threshold are functions of the radiative recombination.} \]

After all these expressions we come to the differential equations for the field. In section 4.4 we have explained how to deal with the complex \( z \)-dependence of the field. On the resulting first and second order differential equations, we'll apply the \( \Psi \)-operator, yielding the following equations

\[
\frac{d r_1}{d t} = \frac{1}{2l_{re,2} l_{re,18}} \left\{ - \frac{c}{n_{\text{eff}}} (\alpha_{\text{scat}} + \alpha_{\text{th}}) l_{re,2} l_{re,18} \right. \\
+ \zeta B_2 \left[ l_{re,2} \left( \frac{a}{\tau_{nr}} + B_1 n_0 \right) l_{re,18} - B_1 (l_{re,2} + l_{re,8}) l_{re,20} n_1 \right] \\
+ \Gamma (l_{re,2} G_{sh} + l_{re,8} G_{sh} + l_{re,8} G_{sh}) \\
- 2 \frac{c}{n_{\text{eff}}} k_0 (l_{re,2} l_{re,18} [n_0 - n_{th}]) \right\} r_1
\]

(Equation (D.59) is presented on the next page)
\[
\frac{d^2 r_i}{dt^2} = \frac{1}{2 l_{re,2} l_{re,18}} \left\{ - \frac{c}{n_{\text{eff}}} (\alpha_{\text{scat}} + \alpha_{\text{th}}) l_{re,2} l_{re,18} \\
+ \zeta \beta \Gamma \left[ l_{re,2} \left( \frac{a}{\tau_{\nu}} + B_1 n_0 \right) l_{re,18} - B_1 (l_{re,2} + l_{re,8}) l_{re,20} n_1 \right] \\
+ \Gamma (l_{re,2} G_{h7} + l_{re,8} G_{h8} + l_{re,9} G_{h9}) \\
- 2 \frac{c}{n_{\text{eff}}} k_0 \eta (l_{re,2} l_{re,17} [n_0 - n_{\text{th}}]) \right\} \\
\times \frac{1}{2 l_{re,3} l_{re,17}} \left\{ - \frac{c}{n_{\text{eff}}} (\alpha_{\text{scat}} + \alpha_{\text{th}}) l_{re,2} l_{re,17} \\
+ \zeta \beta \Gamma \left[ l_{re,2} \left( \frac{a}{\tau_{\nu}} + B_1 n_0 \right) l_{re,17} - B_1 (l_{re,2} + l_{re,8}) l_{re,19} n_1 \right] \\
+ \Gamma (l_{re,2} G_{h10} + l_{re,8} G_{h11} + l_{re,9} G_{h12}) \\
+ 2 \frac{c}{n_{\text{eff}}} k_0 \eta (l_{re,2} l_{re,17} [n_0 - n_{\text{th}}]) \right\} r_i
\] (D.59)