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Quantum magnetoconductance of the two-dimensional electron gas on a liquid helium surface

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QUANTUM MAGNETOCONDUCTANCE OF
THE TWO-DIMENSIONAL ELECTRON GAS
ON A LIQUID HELIUM SURFACE

Richard van de Sanden, juni 1987
Abstract

In this report the longitudinal conductivity \( \sigma_{rr} \) of the two-dimensional electron gas (2DEG) on liquid helium is examined, as function of a magnetic field perpendicular to the surface. The attention was mainly concentrated on the high magnetic field behaviour.

The conductivity is measured, by using an a.c. measuring method, employing a Corbino-type geometry. The interpretation of the experimental data is extensively studied, by comparing the usual simplified model and a new derived model of the experimental cell. It is shown, that the simplified model is equal to the new model, if certain conditions are fullfilled.

By analyzing only the measurements for which the simplified model was valid, \( \sigma_{rr} \) is measured in the range where scattering of electrons by vapour atoms is dominant. For low magnetic fields, the classical relation \( \sigma_{rr} \sim 1/B^2 \) is confirmed. By entering the quantum limit, i.e. the Landau quantum separation \( \hbar \omega_c \) becomes larger than the thermal energy \( k_b T \) (\( \hbar \omega_c / k_b T \leq 4 \) under the present conditions), a strong deviation from the classical behaviour is observed. The deviation is compared with a quantum transport theory, which however predicts a larger deviation. To account for the discrepancy between our measurements and the theory a preliminary physical explanation is suggested.
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Chapter 1 Introduction

The two-dimensional electron gas (2DEG) on the surface of liquid helium is more and more recognized as an unique system for the research in the various fields of physics. It is for only two decades that the physicist became interested in this system, and that the first theoretical predictions of surface states on liquid helium were made by Cole and Cohen [1] and independently by Shikin [2]. A number of experiments have established that electrons are bound perpendicularly to the surface and form a non-degenerate (classical) electron gas parallel to the surface [3]. From this time on great progress has been made and various interesting physical phenomena have been reported.

One of these interesting phenomena is the recently discovered [4] Wigner crystallisation of the two-dimensional electron gas on the surface of liquid helium, i.e. for low enough temperatures, when the Coulomb interaction dominates the kinetic energy of the electrons, the 2DEG on a liquid helium surface exhibits a two-dimensional solidification. This 2D phase transition is of great interest, both theoretically and experimentally, because 2D phase transitions are unlike any seen in the three dimensional systems. Studying this 2D phase transition could tell us more about, for example the surface of three dimensional solids and of anisotropic solids in which the interactions in a plane of symmetry are much stronger than the interplane coupling. Since the discovery of the Wigner crystallisation much research has been done in this field of critical phenomena [5].

Another important aspect of the 2DEG on a liquid helium surface is the fact that the system is very clean, i.e. the motion of the electrons parallel to the surface is only determined by three well known scattering mechanisms. First of all the already mentioned Coulomb interaction. However, if the density is low enough this interaction can be neglected and only two mechanisms play a role of importance, i.e. the electron vapour atom interaction and the electron-ripplon interaction. A ripplon is the quantized version of the gravity capillary waves (the same as on the ocean) on the surface of liquid helium. Above 1.1 K the vapour atom interaction is dominant, below 0.8 K the electron-ripplon interaction is dominant. Because the 2DEG on the surface of liquid helium is one of the cleanest examples of the existing two-dimensional
electron systems and essential parameters, such as the number of scatterers, the electron-ripplon coupling and the electron density, can be varied over a considerable range, the 2DEG on the surface of liquid helium is very suitable to test electron transport theories. An important parameter in these theories is the conductivity $\sigma$ of the electron gas.

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Table 1: Some typical values of parameters in 2DEG on a liquid helium surface and in semiconductor structures: $E_0$ binding energy, $n_0$ electron density, $E_F$ Fermi energy and $\mu$ mobility.

In this report the conductivity $\sigma$ of the electron gas in high magnetic fields perpendicular to the surface is measured as function of magnetic field $B$. One of the interesting aspects of a high magnetic field from a fundamental point of view, is the fact that the motion parallel to the surface is confined to Landau orbits, and as a consequence the energy spectrum is completely discrete. The system is now fully quantized. Electron transport theories in 2DEG in high magnetic fields play an important role in the understanding of quantum phenomena such as the quantum Hall effect (QHE) [33] or the fractional quantum Hall effect (FQHE) [42] in semiconductor devices. The study of the conductivity of the 2DEG on a liquid helium surface is therefore not only interesting from a fundamental point of view, but could give some more insight in the phenomena appearing in semiconductors. Some typical differences between 2DEG on a liquid helium surface and 2DEG in semiconductor structures are shown in table 1 [6]. The occupation factor $v = 2\pi n_0 e^2$, which is the number of occupied Landau levels, is for typical densities (see table 1) and a magnetic field $B = 4$ T approximately 1% in the case of the 2DEG on a liquid helium surface.
For a comparable occupation number in 2DEG in semiconductor structures this would mean a magnetic field of $B = 500 \, \text{T}$.

Other interesting non-degenerate systems which are similar to the 2DEG on a liquid helium surface are the 2DEG on a liquid helium film and the 2DEG on solid neon and hydrogen. The first, as stated by Peeters [6], could bridge the gap between the degenerate 2DEG in semiconductors and the classical 2DEG on a liquid helium surface because important parameters, for example the electron density and electron-ripplon coupling, are in a different range than on the liquid helium surface. Recently, in the 2DEG on a helium film polaron effects are observed in the region where the ripplon scattering is dominant [7]. A polaron is, in this case, an electron surrounded by a cloud of ripplons. According to Peeters [6] this effect shows that the electron system on a liquid helium film is a nice testing ground for field-theoretical calculations in the interaction of the electron with a scalar field, i.e. the surface modes of the liquid helium.

More recently, it has been shown that the 2DEG on solid hydrogen [8] is a very suitable system for testing localisation theories, because again the parameters can be varied over a considerable range. The random potential which is needed, is supplied by the frozen surface of the solid hydrogen. By introducing a small amount of helium vapour above the hydrogen surface the localisation effects can be destroyed.

From the above mentioned properties it is clear that the two dimensional electron gas on a dielectric surface, in particular on a liquid helium surface, although "classical", has been more and more recognized as an unique model system of a two-dimensional electron gas.

In chapter 2 the electron transport theory, both in zero magnetic field as in high magnetic fields, will be discussed. Chapter 3 gives a description of the experimental setup. In chapter 4 the experimental methods will be analyzed and in chapter 5 the results will be discussed. In chapter 6, we will give the conclusions resulting from the preceeding chapters and some suggestions for future research.
Chapter 2 Theory

2.1 Electrons on the surface of liquid helium

2.1.1 The motion perpendicular to the surface

An electron, a distance $z$ above the helium surface, polarizes the helium atoms of the liquid which results in an attractive force towards the surface. The penetration of the electron into the liquid is prevented by a potential barrier of height $V_b \approx 1$ eV which is a consequence of the Pauli exclusion principle: the energy of the electron inside liquid helium ("a noble gas") would be larger than outside. Since the repulsive barrier $V_b$ is large in comparison with the weak attraction of the liquid, we can approximate $V_b$ by $V_b = \infty$. Because, as will be shown, the mean distance of the electron in the ground state to the surface, is large in comparison with the interatomic distance in liquid helium, it is a good approximation to neglect microscopic polarization effects and to calculate the attractive potential classically. This calculation [9] yields

$$V(z) = \frac{e^2(\epsilon - 1)}{4(\epsilon + 1)} \frac{1}{4\pi \varepsilon_0 z} \quad z > 0$$ (2.1)

here $e$ is the electronic charge, $\varepsilon_0$ the permittivity of the vacuum and $\epsilon$ the dielectric constant of liquid helium. Together with $V(z) = \infty$ for $z \leq 0$ eq. (2.1) forms a one dimensional well. Since the motion parallel to the surface is free electron like (we ignore for the present interactions with helium vapour atoms or ripples), the Schrödinger equation splits into two parts. One part has the solution

$$\Phi(r) = A \cdot e^{ik \cdot r} = A \cdot e^{i\mathbf{v} \cdot \mathbf{r}/\hbar m}$$ (2.2)

where $p$, $v$ and $r$ are respectively the momentum, the velocity and the position vector parallel to the surface, and $m$ is the free electron mass. This is discussed in the next paragraph.

The time independent Schrödinger equation for the motion normal to the surface reads
Theory

Figure 2.1. The image potential $V(z)$ (lower curve) and the ground state wave function (upper curve) for an electron above the liquid helium surface.

$$\psi(z) = E\psi(z)$$  \hfill (2.3a)

with boundary conditions

$$\psi(0) = 0$$  \hfill (2.3b)

$$\psi(z) = 0 \quad \text{for } z \to \infty$$

Thus, in the $z$-direction, the electron behaves like a one-dimensional hydrogen atom with a reduced nuclear charge $Q_e = (\varepsilon - 1)e/4(\varepsilon + 1)$. The energy levels are easily found to be \cite{14}

$$E_n = \frac{Q^2}{2n^2} \frac{m}{4\pi\varepsilon_0 \hbar} \left(\frac{e^2}{4\pi\varepsilon_0 \hbar}\right)^2$$  \hfill (2.4)

with $n = 1, 2, \ldots$. The energy of the ground state is equal to $E_0 \simeq 0.66 \text{ meV} \simeq 7 \text{ K}$. The normalized wave function for the ground state is given by

$$\psi(z) = \frac{2}{a\sqrt{\pi}a} z e^{-z/a}$$  \hfill (2.5)
were \( a = a_0/Q \) the effective Bohr radius and \( a_0 \) the Bohr radius. With \( \varepsilon = 1.0572 \) we find \( a = 7.6 \) nm. The mean distance from the surface in the ground state is given by \( 6a/4 = 11.4 \) nm, which is much larger than the interatomic distance in liquid helium (\( \sim 0.36 \) nm). The above mentioned results are graphically shown in figure 2.1.

Since the energy difference between the ground state and the first excited state is \( E_2 - E_1 \approx 5.6 \) K while the temperatures used in experiments are typically of the order \( 0.1 \) K - \( 2.0 \) K the electrons are mainly in the ground state.

The existence of these surface states on a liquid helium surface were predicted theoretically by Cole and Cohen [1] and independently by Shikin [2]. Experimentally they were observed by Crimes and Brown [10] in a spectroscopic experiment. They found for the transition frequencies \( \nu(1 \rightarrow 2) = 125.9 \pm 0.2 \) GHz and \( \nu(1 \rightarrow 3) = 148.6 \pm 0.3 \) GHz. This compares with the theoretical predictions \( \nu^T(1 \rightarrow 2) = 119.3 \) GHz and \( \nu^T(1 \rightarrow 3) = 141.3 \) GHz. The small deviations (5.2% and 4.9% respectively) are a consequence of two approximations made in the one dimensional hydrogen atom model. First the barrier potential \( V_b \approx 1 \) eV instead of \( V_b = \infty \) and secondly the Coulomb potential \( V(z) \) will be disturbed in the neighbourhood of the helium surface due to quantum effects. In order to correct for these effects, several different potentials have been proposed. For the interested reader, we refer to Cole [11], Hipólito et al. [12] and Stern [13].

Of practical and special interest for experiments is the influence of an external electrical field \( F_n \) normal to the surface because not only the binding energy increases, but it is also possible to "press" the electron into the ground state level. The Schrödinger equation in the presence of a holding field \( F_n \) reads,

\[
\left[ \frac{\hbar^2}{2m} \frac{d^2}{dz^2} - \frac{e^2(z - 1)}{4(\varepsilon + 1)} + eF_n z \right] \psi(z) = E \psi(z) \tag{2.6}
\]

An exact analytic solution of (2.6) has not yet been obtained. By treating \( eF_n z \) as a perturbation on the original problem eq. (2.3), (which is valid for \( eF_n a \ll E_0 \) it can be shown that, the ground state energy shifts linearly with \( F_n \) [14] and that for fields of the order \( 10 - 20 \) V/cm, the electron is mainly in the ground state [15].

For a more elaborate analysis of the surface states on a liquid helium surface, including the electrical holding field \( F_n \), we refer to
the extensive literature on this subject [16,17,18,19].

We assume that the above calculations of the one electron model still holds in the case of the two dimensional electron gas on a liquid helium surface. If the electron density is low and thus electron-electron interaction can be neglected, the N-particle Schrödinger equation of the 2DEG on a helium surface reduces to N one-particle Schrödinger equations of the type similar to eq. (2.6). We only need to replace F_n in eq. (2.6) by an effective mean field F_{eff} = F_n + F_m, where F_m is the mean field in the z-direction due to the other electrons (the mean field of the electrons parallel to the surface is assumed zero because the density is low and edge effect of the electron gas are neglected).

The total energy of the electron in the ground state is given by

\[ E_{tot} = E_0 + \frac{\hbar^2 k^2}{2m} \]  

(2.7)

where k is the two-dimensional wave number. In a two-dimensional electron gas the density of states is constant and given by (including spin)

\[ g(\epsilon) = \frac{m}{\pi \hbar^2} \quad \epsilon = \hbar^2 k^2 / 2m \]  

(2.8)

2.1.2 Electrons on a liquid helium surface: a classical electron gas

In this paragraph we shall discuss the motion parallel to the surface and we shall show that the 2DEG on a liquid helium surface is a classical 2DEG.

As mentioned before, the motion is free electron like. The effective mass \( m_{eff} \) of the electron on a liquid helium surface is equal to the free electron mass. This has been experimentally confirmed by Edel'man [20] and Brown & Grimes [21]. The ratio of the effective mass \( m_{eff} \) and the free electron mass m was measured and was equal to

\[ \frac{m_{eff}}{m} = 1.00005 \pm 0.0004 \]  

(2.9)
for a holding field $F_n = 14 \, \text{V/cm}$.

If we assume, as before, that the electron density $n_0$ is low, it is possible to treat the two-dimensional electron gas on liquid helium as a non-interacting fermion gas. The calculation of the Fermi energy $E_F$, at $T = 0$, yields [9,22].

$$E_F = \frac{\pi h^2 n_0}{m}$$

(2.10)

The maximum density the surface can sustain before a hydrodynamical instability occurs [17] is of the order $10^{13} \, \text{m}^{-2}$ and thus $E_F \approx 10^{-3} \, \text{meV}$ or a Fermi temperature $T_F \approx 10 \, \text{mK}$. This is much lower than the usual experimental temperatures and also several orders of magnitude lower than for the two-dimensional electron systems in semiconductor structures ($10^2$-$10^3 \, \text{K}$). The 2DEG on a liquid helium surface thus is a classical electron gas at all practical temperatures. Quantum statistical effects can be neglected, but become important when the density is increased. Higher densities can be obtained when a helium film is used instead of bulk helium [6].

As a consequence, the electron distribution function $h(\varepsilon)$ is a Boltzmann distribution and is given by

$$h(\varepsilon) = \left(\frac{\pi h^2 n_0}{mk_b T}\right) \cdot \exp(-\varepsilon/k_b T)$$

(2.11)

here $\varepsilon$ is the kinetic energy of the electrons parallel to the surface, and $k_b$ Boltzmann's constant. The averaged kinetic energy is equal to $k_b T$. The normalization is such that

$$n_0 = \int_0^\infty \mathcal{D}(\varepsilon) \cdot h(\varepsilon) \cdot d\varepsilon$$

(2.12)

More interesting aspects of the motion parallel to surface are:
- which mechanisms determine the mobility of the electrons if an electrical field is applied parallel to the surface?
- what is the influence of a magnetic field normal to the surface on the conductivity of the electrons?

In the next paragraphs these aspects will be discussed.
2.2 The mobility of the 2DEG on a liquid helium surface

The most important parameter in electron transport theories, is the mobility \( \mu \). It provides information about the interaction mechanism of the electrons with the helium vapour and liquid. If the density of the electrons is low enough the electron-electron interaction can be neglected and mainly two mechanisms determine the mobility. The first mechanism is the electron vapour atom interaction which is dominant for temperatures above 1.1 K. If the temperature is lowered, the vapour density above the liquid helium decreases rapidly and another mechanism, the electron-ripplon interaction, becomes important. The electron-ripplon interaction is dominant for temperatures below 0.8 K. A ripplon is the quantized version of the gravity capillary waves. Physically speaking, this interaction is caused by the surface roughness of the liquid helium created by thermally excited ripplons. Therefore the electrons above the liquid helium surface do not "feel" a perfectly smooth surface but "feel" a rippled surface, and the motion parallel to the surface is influenced by this surface roughness.

There is a remarkable similarity between the 2DEG on a liquid helium surface and the 2DEG in semiconductors: the impurity and the electron-phonon interaction in semiconductors play an analogous role as the vapour atom and electron-ripplon interaction in the 2DEG on a liquid helium surface. The situation however is reversed, i.e. the impurity scattering in semiconductors dominates the conductivity at low temperatures.

Several theoretical approaches have been made to calculate the mobility of the 2DEG on a liquid helium surface by taking both mechanisms into account. Numerous experiments have been performed and were in reasonable agreement with the theoretical predictions made by Monarkha et al. [23], Shikin [24], Platzman [25] et al. and Saitoh [26].

One remark has to be made here: in the experiments that we have performed, we were only interested in the Ohmic mobility, i.e. the mobility which is independent of the applied electrical field \( E_0 \) parallel to the surface. Physically this means that the electrons, when set into motion, stay in the ground state level. The reader who is interested in the non-Ohmic mobility, the so called hot electron effects, is referred to refs. [27,28].
Saitoh [26] has calculated the Ohmic mobility for electrons on a liquid helium surface. The remainder of this paragraph is an adaption of his publication. We shall mainly quote the results and give the physical interpretation of these results. For details about the derivations we refer to the original article [26].

The electron mobility is determined by solving the Boltzmann transport equation (B.T.E) for the electron distribution function $f(k)$ in the single relaxation time approximation [29,30], i.e. the collision term in the B.T.E. is written as

$$\left[ \frac{\partial f(k)}{\partial t} \right]_{\text{coll.}} = -\left[ \frac{f(k)-h(\epsilon)}{\tau(\epsilon)} \right] - \left[ \frac{k_x \cdot g(\epsilon)}{\tau(\epsilon)} \right]$$

(2.13)

where the electrical drift field is in the $x$-direction and $\tau(\epsilon)$, the relaxation time is given through

$$\tau(\epsilon)^{-1} = \tau_v(\epsilon)^{-1} + \tau_r(\epsilon)^{-1}$$

(2.14)

The subscripts refer to the ripplon resp. vapour atom part. Equation (2.13) states that the non-equilibrium electron distribution function $k_x \cdot g(\epsilon)$ would, in the absence of fields, relax within a time $\tau(\epsilon)$ towards the equilibrium distribution function $h(\epsilon)$. In principal the mobility of the electrons can now be expressed in terms of $\tau(\epsilon)$ by the standard text book methods [29], while explicit forms of $\tau(\epsilon)$ can be obtained by evaluating the left hand term in eq. (2.13) for the specific interaction Hamiltonian.

To calculate the left term in eq. (2.13) some details about the correct form of the interaction Hamiltonians have to be known. The Hamiltonian for the electron-ripplon interaction is, in the adiabatic approximation, given by [26,31]

$$H_{e-r} = \frac{1}{\sqrt{S}} \sum_q \eta_q^* (b_q + b_{-q}^\dagger) e^{i q \cdot r}$$

(2.15)

here $S$ is the surface area of the system, $q$ the wave vector of a ripplon, $r$ the two-dimensional position vector and $b_q$ resp. $b_{-q}^\dagger$ the creation resp. annihilation operator of a ripplon with wave vector $q$. $\eta_r(q)$ is given by (in the one electron model [26,31])
\[ \eta_q = \left( \frac{\hbar q}{2\rho \omega_q} \right)^{1/2} \cdot (\gamma_q + eF_n) \]  
(2.16)

Here \( \rho \) is the density of liquid helium, and \( \gamma_q \) is given by

\[ \gamma_q = \left( \frac{\hbar^2 q^2}{2ma_o} \right) \cdot \ln \left( \frac{4}{ebq} \right) \]  
(2.17)

\( \eta_q \) is the coupling function between the electron and ripplon. \( \gamma_q \) and \( F_n \) in eq. (2.16) are, respectively, the coupling term due to the polarization of the helium surface and the coupling term due to the applied holding field, and \( e \) is the natural constant. Note that the ripplon interaction depends explicitly on the holding field \( F_n \).

The angular frequency of a ripplon \( \omega_q \) in eq. (2.16), is given by (if the helium is infinitely deep)

\[ \omega_q^2 = g \cdot q + (\sigma/\rho) \cdot q^3 \]  
(2.18)

where \( \sigma \) is the surface tension of liquid helium and \( g \) is the gravitational constant. The dispersion relation eq. (2.18) is essentially the same as for waves on the surface of water [32], however in the ripplon case, the wave number \( q \) is quantized.

For the electron-vapour atom scattering, a simplified \( \delta \)-function potential is assumed of the form

\[ H_{e-v} = U_v \cdot \sum_{i,j} \delta(r_i - R_j) \]  
(2.19)

where \( r_i \) and \( R_j \) stand for the position vectors of the \( i \)th electron and the \( j \)th He vapour atom respectively, and \( U_v \) is the strength of the interaction between the electrons and vapour atoms. Equation (2.19) is the hard core potential with zero range.

Since both Hamiltonians are known, the collision term in eq. (2.13) can be calculated. For the interested reader we refer to Saitoh [26]. The calculation yields the following results for \( \tau_v(\varepsilon) \) and \( \tau_r(\varepsilon) \)

\[ \tau_v^{-1} = \frac{3\pi\hbar A_{nc}}{8mb} \]  
(2.20)
\[ \tau^{-1}(\varepsilon) = \frac{T}{4\hbar a^2} \left[ (eF_n a)^2 + 2eF_n a \cdot \ln \left( \frac{16E_b}{e^2\varepsilon} \right) \right] + \frac{3e^2}{2} \left[ \left( \ln \left( \frac{16E_b}{e^2\varepsilon} \right) - \frac{1}{6} \right)^2 + \frac{\pi^2}{6} - \frac{115}{36} \right] \] (2.21)

Here \( A \) is the scattering cross section of an electron with a helium vapour atom, \( n_G \) the density of the helium vapour atoms, \( b \) is the decay length of the ground state wave function under both the image potential and the in paragraph 2.1.1 introduced effective field \( F_{\text{eff}} \) (replace \( a \) in eq.(2.5) by \( b \)), and \( E_b = \hbar^2/2mb^2 \).

The electron mobility is related to \( \tau(\varepsilon) \) through

\[ \mu = \frac{e}{m} \langle \tau(\varepsilon) \rangle = \int_0^\infty d\varepsilon \cdot \frac{e\tau(\varepsilon)}{(k_B T)^2} \cdot e^{-\varepsilon/k_B T} \] (2.22)

Since \( \tau(\varepsilon) \) has a complicated \( \varepsilon \) dependence, \( \mu \) is calculated numerically on a computer [22]. Figure 2.2 shows the mobility \( \mu \) as function of the helium vapour density \( n_G \) and the temperature \( T \) for different holding fields \( F_n \). \( F_m \) was taken zero, \( F_m = 0 \). As seen from fig. 2.2, the

\[ \begin{array}{c}
\text{Figure 2.2 The mobility } \mu \text{ as function of the temperature } T \text{ for different holding fields } F_n.
\end{array} \]
electron vapour interaction is dominant for temperatures above 1.1 K and is nearly independent of the holding field. (the $F_n$ dependence of $\tau_v$ is a consequence of the small $F_n$ dependence of $b$). Another main feature can be seen from fig. 2.2. Since $\tau_v$ (eq. (2.20)) is independent of the electron energy, $\mu$ has the same temperature dependence as $n_0^{-1}$, i.e the mobility $\mu$ is inversely proportional to $n_0$ (the dashed line in fig. 2.2).

For temperatures below 0.8 K the electron-ripplon interaction is dominant and varies only slightly with $T$. The dependence on $F_n$ is stronger than in the vapour case, which is a consequence of the explicit $F_n$ dependence of $\tau_r(\epsilon)$. If the holding field $F_n$ is increased, the mobility $\mu$ decreases. Physically speaking, this is a consequence of the fact that the electrons are more pressed against the helium surface, if the holding field is increased, and thus "feel" the surface roughness, i.e. the ripplons, more. This results in a decreasing mobility.

The conductivity $\sigma$ of the electron gas is given by

$$\sigma = n_0 e \mu$$

(2.23)

The conductivity $\sigma$ is a function of the temperature $T$, the electrical holding field $F_n$, and the electron density $n_0$. Note that the 2D conductivity has the dimension siemens with S.I. symbol S. However to avoid confusion with the symbol for the second s, we use as symbol for the 2D conductivity the mho $\Omega$.

In the next paragraph we shall consider the transport of the electrons in a magnetic field $B$ perpendicular to the surface.

2.3 The conductivity of the 2DEG in a magnetic field $B$

When a magnetic field is applied perpendicular to the surface the dynamical properties of the 2DEG on liquid helium surface change. The magnetic field causes the electrons to move in cyclotron orbits parallel to the surface. If the magnetic field is high enough i.e. $\omega_c \tau > 1$, where $\omega_c = eB/m$ is the cyclotron frequency, the motion is quantized into Landau levels. The energy spectrum of the electrons is then completely discrete (we neglect spin) [33,34].
\[ E_{\text{tot}} = E_0 + (N + 1/2) \cdot \hbar \omega_c \] (2.24)

Here \( N = 0, 1, \ldots \). Equation (2.24) is graphically shown in Fig. 2.3. Therefore, in high magnetic fields, quantum mechanical effects will play an important role. Before we discuss the high magnetic field conductivity, we shall first consider the case of low magnetic fields in the sense that \( \hbar \omega_c < k_b T \), which means that many Landau levels are occupied with a substantial number of electrons. In this case the energy quantization turns out to have not many consequences and thus the calculation can be done classically.

Since a magnetic field is present it can be shown that the generalized form of Ohm's law reads

\[ J = \sigma \cdot E_0 \] (2.25)

Here \( \sigma \) is the conductivity tensor. In Ref. [22] the calculation of the diagonal element \( \sigma_{rr} = \sigma_{rr}(B,T) \) is done for a Corbino geometry (for more information about the Corbino geometry see paragraph 3.3) which yields

\[ \sigma_{rr}(B,T) = \frac{n \mu e u}{1 + \mu^2 \beta^2} = \frac{\sigma_{rr}(B=0)}{(1 + \mu^2 \beta^2)} \] (2.26)

![Figure 2.3](image_url) The dispersion relation for the 2DEG on a liquid helium surface, a: without a magnetic field, b: with a magnetic field.
Here \( \sigma_{rr}(B,T) \) measures the conductivity in the direction of the applied electrical drift field. Equation (2.26) shows that the conductivity decreases if the magnetic field is increased, which is a consequence of the fact that in the presence of a magnetic field the electrons move perpendicular to the electrical field. If there is no scattering mechanisms, \( \mu = \infty \), consequently \( \sigma_{rr}(B,T) = 0 \) ! This strange behaviour of the conductivity, i.e. \( \sigma_{rr}(B,T) \) is low if the mobility is high, is purely a consequence of the used Corbino geometry.

In the derivation of \( \sigma_{rr} \) in ref. [22], it was implicitly assumed that the relaxation time \( \tau \) was not a function of the energy \( \epsilon \) of the electrons. We know however that the ripplon relaxation time does depend on \( \epsilon \). If the vapour atom scattering is dominant (during all our experiments) eq.(2.26) is correct, in the range where the ripplon interaction plays a role, eq.(2.26) has to be replaced by [30] (compare eq.(2.22))

\[
\sigma_{rr}(B,T) = \frac{n_0 e^2}{m} \left( \frac{\tau(\epsilon)}{1 + (\omega_c T(\epsilon))^{2}} \right) = \frac{n_0 e\mu}{(1 + \mu_H^2 B^2)}
\]

(2.27)

where the brackets stand for the same averaging procedure as in eq.(2.22), \( \mu_H \) is the Hall mobility and \( \mu \) is consequently the drift mobility in the absence of a magnetic field. The ratio \( R = \mu/\mu_H \) is called the Hall factor and is usually in the order of 1 - 2. The difference between eq.(2.26) and (2.27) is a well known effect in semiconductors. However in the case of vapour atom scattering \( R = 1 \). For classical high magnetic fields, i.e. \( \mu_H B > 1 \), but \( \hbar \omega_c \ll k_B T \), \( \sigma_{rr}(B,T) \) is equal to

\[
\sigma_{rr}(B,T) = \frac{n_0 e^2}{\hbar \omega_c} \frac{1}{\tau(\epsilon)}
\]

(2.28)

i.e. the conductivity is inversely proportional to \( B^2 \). It can also be shown that the Landau level width is given by (in essence the uncertainty relation)

\[
\Gamma = \hbar/\langle \tau(\epsilon) \rangle
\]

(2.29)

with \( \langle \tau(\epsilon) \rangle \) independent of the magnetic field \( B \). The classical behaviour eqs. (2.26-27), has been experimentally confirmed in refs. [35,36].

When the magnetic field is high, i.e. \( \hbar \omega_c \gg k_B T \), the situation is
more complicated. Since all electrons now occupy the lowest Landau level, the conductivity \( \sigma_{rr}(B,T) \) depends much on the particular form of the Landau levels. This can be easily shown qualitatively.

Consider first the ideal case, i.e. the energy levels are given by eq. (2.24). It can be shown that the density of states \( g_N(\varepsilon) \) of the Nth Landau level is given by

\[
g_N(\varepsilon) = 2\cdot \frac{eB}{h} \delta(\varepsilon-(N + 1/2)\cdot\hbar\omega_c) \tag{2.30}
\]

i.e. each Landau level has the form of a \( \delta \)-function with degeneracy factor \( 2eB/m \), including spin. This degeneracy factor of the Landau levels is calculated by assuming that the energy states in the neighbourhood of the Nth Landau level are projected on the Nth Landau level (fig. 2.4). In this idealized case therefore only inter-Landau level scattering can cause the conductivity (intra-Landau level scattering, i.e. scattering within a Landau level, does not contribute to the conductivity if the Landau level has a \( \delta \)-function form). However transition probability of \( N = 0 \rightarrow N = 1 \) is

\[ D(\varepsilon) \]

Figure 2.4 The density of states in a 2DEG in a magnetic field in an idealized system.

low because \( \hbar\omega_c \gg k_B T \) and because the density of states are \( \delta \)-functions (Fermi's Golden Rule). Consequently the conductivity is low.

In reality however, the Landau level broadens due to collisions of the electrons with vapour atoms and ripplons (fig 2.5). The width of the Nth Landau level is \( \Gamma_N \). Intra-Landau level scattering is now possible (which is more probable) and therefore the conductivity is larger than in the case of a \( \delta \)-function form of \( g_N(\varepsilon) \).
From these qualitative considerations, it can be concluded that the conductivity of the 2DEG on a liquid helium surface in a high magnetic field depends much on the particular form and width of the Landau levels.

Saitoh [37,38,39] calculated the broadening of the Landau level due to collisions with ripplons. Later in this paragraph we shall briefly report his results in the case of high magnetic fields.

So far, theoretical treatments of the Landau level broadening due to helium vapour atom scattering have not been found in the literature. However the relaxation time \( \tau_v \) due to vapour atom collisions is indepen-
dent of the energy $\varepsilon$ of the electrons. As mentioned before, this is analogous to the impurity scattering in 2DEG in semiconductor structures. Ando and Uemura [40] have calculated the Landau level broadening in the case of impurity scattering in a 2DEG in high magnetic fields. The broadening of the $N$th Landau level is calculated self consistently (the derivation is very cumbersome, for more details of this calculation, we refer to the original publication ref.[40]). The results for the density of states in the lowest Landau level $g_0(\varepsilon)$, in the case a $\delta$-function potential is assumed for the impurity scattering is

$$g_0(\varepsilon) = \frac{1}{2\pi:\ell^2} \frac{2}{\pi:1} \sqrt{1 - \left[\frac{\varepsilon - \hbar \omega_c/2}{\Gamma}\right]^2} \quad (2.31)$$

Here $\ell = \sqrt{\hbar/eB}$ is the Landau orbit radius, and $2\cdot\Gamma$ is the width of the lowest Landau level. Note that the density of states $g_0(\varepsilon)$ (shown in fig. 2.6) is semi-elliptic around $\hbar \omega_c/2$. The level width is given by

$$\Gamma = \frac{\sqrt{2\cdot\hbar \omega_c} \cdot \hbar}{\tau} \quad (2.32)$$

were $\tau$ is the relaxation time of the vapour atom scattering in $B = 0$. As can be seen from eq.(2.32), the level width is proportional to $\sim \sqrt{\omega_c/\tau}$ while classically the level width, as mentioned before, $\sim 1/\langle \tau(\varepsilon) \rangle$. For Landau levels to exist, it is necessary that the following inequality holds

$$\Gamma = \frac{\sqrt{2\cdot\hbar \omega_c} \cdot \hbar}{\tau} \ll \hbar \omega_c \quad (2.33)$$

which is, after some rearranging of terms is equivalent to $\omega_c \tau \gg 1$, as in the classical case.

If in our case the vapour atom scattering is dominant, the situation is analogous to the impurity scattering. Therefore we assume that the calculations of Ando and Uemura are applicable to our situation as well. The relaxation time in eq. (2.33), has to be replaced by $\tau_v$, the relaxation time due to vapour atom collisions.

If only the lowest Landau level is occupied, Ando and Uemura obtain for the conductivity (see also [41,42])
\[ \sigma_{rr}(B,T) = \int d\epsilon \left[ - \frac{\partial f(\epsilon)}{\partial \epsilon} \right] \epsilon^2 \frac{e^2}{\hbar} \left[ 1 - \left( \frac{\epsilon - \hbar \omega_c}{\Gamma} \right)^2 \right] \tag{2.34} \]

where the integral is over the zeroth Landau level (from \( \hbar \omega_c - \Gamma \) to \( \hbar \omega_c + \Gamma \)). In our case, \( f(\epsilon) \) is given by a Boltzmann distribution (not eq. \( (2.11) \)) since the density of states is different and thus the prefactor in eq. \( (2.11) \) shall be different

\[ f(\epsilon) = C(B,T) \cdot e^{-\epsilon/k_B T} \tag{2.35} \]

where \( C(B,T) \) may depend on the magnetic field \( B \) and the temperature \( T \).

**Figure 2.7 and 2.8** In fig. 2.7 the relative change of \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \) as function of the magnetic field \( B \) for different temperatures \( T \) based on the theory of Ando and Uemura is shown, fig.2.8 shows the relative change of \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \) as function of the magnetic field \( B \) for different temperatures \( T \) on basis of the theory of Saitoh.

\[ n_0 = C \cdot \int \mathcal{F}_0(\epsilon) \cdot f(\epsilon) d\epsilon \tag{2.36} \]

where the integral is only over the lowest Landau level. The integrals
(2.34) and (2.36) can be calculated analytically, using [58], and the results for \( C(B,T) \) and \( \sigma_{rr}(B,T) \) are

\[
C(B,T) = \pi \cdot n_0 e^2 \cdot e^{-\tilde{\omega}_c / k_b T} \cdot \frac{1}{S(\Gamma / k_b T)} \]  

\[
(2.37) \]

\[
\sigma_{rr}(B,T) = 4\pi \cdot n_0 e^2 \cdot \frac{e^2}{2\pi^2 \hbar} \cdot k_b T \cdot \frac{1}{T} \left[ \cosh \left( \frac{\Gamma}{k_b T} \right) - \frac{k_b T}{T} \sinh \left( \frac{\Gamma}{k_b T} \right) \right] \cdot \frac{1}{S(\Gamma / k_b T)} \]  

\[
(2.38) \]

where \( S(\Gamma / k_b T) \) is given by

\[
S(\Gamma / k_b T) = \sum_{n=0}^{\infty} \frac{(\Gamma / k_b T)^{2n}}{(2n)!!(2n+2)!!} \]  

\[
(2.39) \]

Equation (2.38) only holds in the case that the range of the scattering potential is zero i.e. a \( \delta \)-potential is assumed for the vapour atom scattering (see eq.(2.19)). Note that in the limit \( (\Gamma / k_b T) \rightarrow 0 \), \( \sigma_{rr}(B) \) has the form

\[
\sigma_{rr}(B,T) = 4\pi \cdot n_0 e^2 \cdot \frac{e^2}{2\pi^2 \hbar} \cdot \frac{\Gamma}{k_b T} \cdot \frac{2}{3} \]  

\[
(2.40) \]

i.e. \( \sigma_{rr}(B,T) \sim \sqrt{B} \). Equations (2.38) and (2.40) are only valid in the case the vapour atom interaction is dominant \( (T > 1.1 \text{ K}) \) and \( \tilde{\omega}_c \gg k_b T \).

In fig. 2.7 the ratio \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \) is shown as function of \( B \) for several temperatures \( T \), calculated on the basis of eqs. (2.38) and (2.23). The mobility \( \mu = \mu(T) \) and the relaxation time \( \tau_v = \tau_v(T) \) used to calculate \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \), are determined from the theory of Saitoh [26] discussed in paragraph 2.2. As seen from fig. 2.7 \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \) is nearly linear on \( B \) in contrast to the low magnetic field behaviour which is proportional to \( B^2 \) (see eq. (2.26)).

The physical interpretation of the above results is kept for the next paragraph. First we shall give the results of the calculation of \( \sigma_{rr}(B,T) \) in the case that the electron-ripplon interaction is dominant.

If the ripplons become dominant, the calculation of the broadening of the Landau level done by Ando and Uemura [40] fails because the relaxation time for ripplons \( \tau_r(\varepsilon) \) depends explicitly on the energy \( \varepsilon \). Since a magnetic field alters the energy spectrum (see eq. (2.24)).
\( \tau_r(\epsilon) \) depends explicitly on \( B \). For the ripplon case therefore a different calculation of the level broadening is necessary. Saitoh has in a number of articles treated this case \([37,38,39]\). Because the derivation is cumbersome (path integral methods are used) we only give the results in the case \( \hbar \omega_c > k_b T \). The Landau level broadening is again calculated self consistently and the "high temperature" approximation is used, i.e.

\[
\hbar \omega_q < k_b T \tag{2.41}
\]

where \( \hbar \omega_q \) is the ripplon energy. The conductivity is given by (compare eq. (2.28))

\[
\sigma_{rr}(B,T) = \frac{n_0 e^2}{m \omega_c^2} \frac{1}{\tau_r(B)} \tag{2.42}
\]

where \( \tau_r(B) \) is the ripplonic relaxation time in the presence of a magnetic field. For \( \tau_r(B) \) Saitoh \([39]\) obtains

\[
\frac{1}{\tau_r(B)} = \frac{\pi \omega_c \Theta}{\sqrt{48 k_b T}} \tag{2.43}
\]

where \( \Theta \) and \( \theta \) are given by

\[
\Theta = \frac{1}{4 \pi a^2} \left[ (eF_N a)^2 + \frac{(eF_N a \hbar \omega_c)}{2} \ln \left( \frac{64 E_b / \hbar \omega_c}{\exp(4-u)} \right) \right] + \frac{3\hbar^2 \omega_c^2}{16} \left[ \ln^2 \left( \frac{64 E_b / \hbar \omega_c}{\exp(14/3-u)} \right) + \frac{\pi^2}{2} \frac{40}{9} \right] \tag{2.44}
\]

\[
\theta = \frac{1}{4 \pi a^2} \left[ (eF_N a)^2 + (eF_N a \hbar \omega_c) \ln \left( \frac{16 E_b / \hbar \omega_c}{\exp(3-u)} \right) \right] + \frac{\hbar^2 \omega_c^2}{2} \left[ \ln^2 \left( \frac{16 E_b / \hbar \omega_c}{\exp(7/2-u)} \right) + \frac{\pi^2}{6} \frac{5}{4} \right] \tag{2.45}
\]

where \( u = 0.5772 \) is the Euler constant. In fig 2.8 \( \sigma_{rr}(B=0)/\sigma_{rr}(B) \) is shown as function of \( B \), in the case the ripplon interaction is dominant \((T < 0.8 \text{ K})\). Equations (2.44,45) are valid under the condition

\[
\hbar \omega_c > k_b T > \theta \tag{2.46}
\]
Physically speaking this means that the coupling energy $\delta$ between the ripplons and the electrons, is small compared to the thermal energy $k_bT$ and the characteristic magnetic energy $\hbar\omega_c$.

### 2.4 Simple physical interpretation in a diffusion picture

In this paragraph we shall give a simple physical interpretation of the preceding paragraphs. A qualitative understanding can be obtained by considering the transport in a diffusion picture [39,40].

Relating the mobility to the diffusion constant $D$ through the Einstein relation $\mu = eD/k_bT$, the conductivity in case of a non-degenerate statistics can be written generally as

$$\sigma = \frac{n_0 e^2}{k_bT} D$$

(2.47)

with

$$D = \frac{\ell^2}{\tau}$$

(2.48)

where $\ell$ and $\tau$ are the characteristic length and hopping time for the diffusion process, and $n_0$ is the total carrier density. Equation (2.47) follows immediately when the elementary expression for the current density, $j = \sum e \cdot v_1$ is converted to an integral over the energy:

$$j = \int_0^\infty \mathcal{D}(\epsilon) f(\epsilon) v(\epsilon) d\epsilon$$

(2.49)

here $\mathcal{D}(\epsilon)$ is the density of states, $f(\epsilon)$ the distribution function of the electrons and $v(\epsilon)$ the velocity. Only the deviation from thermal equilibrium gives rise to a net current (see fig. 2.9) so that, with

$$f(\epsilon) = f^0(\epsilon) - \frac{\partial f^0(\epsilon)}{\partial \epsilon} \Delta U$$

(2.50)

where $f^0(\epsilon)$ and $\Delta U = eE_0 \ell$, $E_0$ the applied drift field and $\ell$ the mean
free path. Using eq. (2.49) $\sigma = E_0/j$ can be written as

$$\sigma = -e^2 \cdot D \cdot \int_0^\infty \mathcal{D}(\epsilon) \cdot \frac{\partial f^0(\epsilon)}{\partial \epsilon} \, d\epsilon$$  \hspace{1cm} (2.51)$$

where average constant values for $l$, $v$ and $\tau$ are assumed with $v = l/\tau$. When $f^0(\epsilon)$ is a Boltzmann distribution

$$\frac{\partial f^0(\epsilon)}{\partial \epsilon} = \frac{f^0(\epsilon)}{k_B T}$$  \hspace{1cm} (2.52)$$

and eq. (2.47) follows with

$$n_0 = \int_0^\infty \mathcal{D}(\epsilon) \cdot f^0(\epsilon) \, d\epsilon$$  \hspace{1cm} (2.53)$$

For a degenerate electron gas, $f^0(\epsilon)$ is the Fermi-Dirac function and $-\partial f^0(\epsilon)/\partial \epsilon$ is a $\delta$-function $\delta(\epsilon - \epsilon_F)$ (for $T = 0$), so that eq. (2.47) becomes

$$\sigma = \mathcal{D}(\epsilon_F) \cdot e^2 \cdot D$$  \hspace{1cm} (2.54)$$

with $\mathcal{D}(\epsilon_F)$ the density of states at the Fermi energy.

**Figure 2.9** The shifted distribution function as function of the wave number $k_x$, the applied drift field is in the $x$-direction. The shaded part is the electrons which effectively contribute to the current. $k_0 = mv_0/h = m j / n_0 e h$ where $v_0$ is the drift velocity in the Drude model of conduction in metals.

For a classically high magnetic field, $\hbar \omega_c \ll k_B T$ but $\omega_c \tau > 1$, we
have to make modifications in eq. (2.48). The characteristic length $l$ in the diffusion process is now the classical cyclotron radius $l_c$. $l_c$ is given by

$$l_c = \frac{mv}{eB} \sim \frac{\sqrt{mk_BT}}{eB} \quad (2.55)$$

where $v \sim \sqrt{k_BT/m}$ is the mean thermal velocity. Substituting (2.55), (2.53) and (2.52) in eq. (2.47) with the help of eq. (2.48), we obtain

$$\sigma(B) = \frac{nBe^2}{m} \cdot \frac{1}{\omega_c^2 \tau} \quad (2.56)$$

Equation (2.56) is equal to the classical magnetoconductance given in eq. (2.38).

In the quantum limit, $\hbar \omega_c > k_BT$, all the electrons are in the lowest Landau level. If the temperature is not too low, i.e. if $k_BT > \Gamma$, the average values for $l$ and $\tau$ are the average values in the lowest Landau level. The characteristic length is now the quantum mechanical cyclotron radius of the lowest Landau level which is given by (one orbit area comprises one flux quantum)

$$l = \frac{\hbar}{\sqrt{eB}} \quad (2.57)$$

Thus classically $l_c \sim 1/B$ while in the quantum limit $l \sim 1/\sqrt{B}$. The scattering time is to be replaced by $\tau(B) = \hbar/\Gamma(B)$, which follows from the uncertainty relation as $\Gamma(B)$ is the width of the Landau level. Substitution of $l$ and $\tau(B)$ in eq. (2.47) yields

$$\sigma = \frac{2nBe}{B} \frac{\Gamma}{k_BT} \quad (2.58)$$

which is equal to the result eq. (2.40), apart from the prefactor. The field dependence of $\sigma = \sigma_{rr}$ therefore differs from the classical $1/B^2$ behaviour. Because $\Gamma \sim \sqrt{B}$, a $1/\sqrt{B}$ behaviour would be expected from eq. (2.58) in the limit $k_BT \gg \Gamma$. Exact calculations according to eq. (2.38) are shown in fig. 2.7 and 2.8, for the case more relevant to the
experiments: $\Gamma \sim k_b T$. An approximately linear dependence is found for this case. It should be noted that the different field dependence in both cases is mainly a consequence of the different equations for the cyclotron radii.

In case the electron-ripplon interaction is dominant it can be easily shown from eqs. (2.42), (2.43), (2.44) and (2.45) that the result eq. (2.58) is exactly reproduced in the limit $a \to \infty$, i.e. the image potential term $\gamma_a$ in eq. (2.16) is neglected. The image potential term $\gamma_a$ leads to the complex $\omega_c$ dependence in eqs. (2.44), (2.45).

Equation (2.58) can also be written as

$$\sigma = \frac{e^2}{h} \cdot \nu \cdot \frac{\Gamma}{k_b T}$$

(2.59)

where $\nu = n_\text{oh}/eB$. The first factor in eq. (2.59) is recognized as the inverse of the fundamental resistance unit. $\nu$ is the so called filling factor: the total electron density divided by the Landau level degeneracy (excluding spin degeneracy), i.e. the number of occupied Landau levels. For our non-degenerate case $\nu \ll 1$. Except for the factor $\Gamma/k_b T$, eq. (2.59) is the expression Ando and Uemura [40] obtained for the peak value of $\sigma_{rr}$ in a degenerate 2DEC for integral values of $\nu$.

As shown in this paragraph the difficult derivations of Saitoh [37,38,39] and Ando and Uemura [40] of $\sigma_{rr}(B,T)$ in a strong magnetic field can be understood qualitatively in a simple diffusion picture. However for comparison with the experiments, the exact calculations have to be performed.
Chapter 3 Experimental setup and methods

3.1 Outline of the experimental setups

The experimental setup used to measure the conductivity of the 2DEG on a liquid helium surface in the temperature range 1.3 - 2.0 K is shown in figure 3.1.

The experimental chamber is cooled to 1.3 - 2.0 K. This is achieved by first precooing with liquid nitrogen in the outer cryostat to 77 K and then filling the inner cryostat with liquid helium of 4.2 K. By pumping the helium cryostat, we can achieve the necessary temperatures.
in the range 1.3-2.0 K. At the bottom of the cryostat insert a cylindrical experimental chamber is connected. The experimental chamber is made of copper. Inside the chamber an experimental cell is suspended. From a tank at room temperature, helium gas can be condensed into the experimental chamber by means of two capillary tubes. A superconducting coil is mounted around the experimental chamber and can apply magnetic fields up to 4 T. We made sure that the experimental cell was in the heart of the coil and that all the materials used were non-magnetic. Half way the cryostat insert, radiation shields were soldered on the current wires to cool these wires with the helium vapour.

**Figure 3.2** The cryogenic part of the dilution refrigerator. See text for explanation.

Before we discuss the experimental cell, we shall first describe the experimental setup in the temperature range 0.1-1.5 K, since the exper-
The experimental cell used in both cases was the same.

To achieve temperatures in the range 0.1-1.5 K, we made use of a $^3$He circulating dilution refrigerator. For an extensive description of the $^3$He circulating refrigerator, we refer to ref. [43]. The cryogenic part is shown in fig. 3.2.

The dilution refrigerator is accommodated in a setup consisting of a liquid nitrogen cryostat ($T = 77$ K) and a liquid helium cryostat ($T = 4.2$ K). The part of the dilution refrigerator below 4.2 K is sealed in a stainless steel vacuum chamber in the liquid helium cryostat. Inside this vacuum chamber, the experimental chamber (a cylindrical stainless tube of 35 mm diameter and 100 mm height) is sealed on the top of the mixing chamber. A sintered layer of 90% copper and 10% French silver on the exterior top of the mixing chamber facilitates the heat exchange between the mixing chamber and the liquid helium sample for the experiment. The experimental chamber can be filled in the same manner as described earlier. A superconducting coil can be mounted around the vacuum chamber. A severe problem in this setup is the liquid helium level inside the experimental chamber during an experiment. This shall be discussed later in this chapter.

The temperature in the experimental setups is measured with carbon resistance thermometers. For the temperature range 1.1-2.2 K, an Allen & Bradley resistance is used, for temperatures below 1.1 K a thermometer of the Speer type is used. The thermometers were placed in the neighbourhood of the experimental cell.

3.2 The experimental cell

The experimental cell, shown in fig. 3.3a, is used in both experimental setups and is similar to the cell used in the earlier experiments [44, 45, 9, 22]. The reason for a new design were the dimensions of the old cell which did not fit in the dilution refrigerator. Other considerations, such as to simplify electrostatic calculations, were also taken into account and had mainly consequences for the guard ring (see later). The design which resulted is a mixture of the old cell and a cell used in experiments in refs. [9, 44, 45, 55].

The experimental cell consists of two parts. The upper part, which is made of two brass plates, four spacers and a spring, is used to adjust
the lower part in the horizontal direction. The lower part consists of two epoxy print plates covered with copper and a concentric guard ring made of copper. These three parts are isolated from each other and are isolated from the upper part of the cell. The whole cell was kept together with eight nuts and is suspended to the flange of the cryostat insert. The structure of the two print plates are shown in fig. 3.3b. The lower print plate consists of a center part of diameter $2r_0 = 6$ mm, and a concentric ring part, width $4.82$ mm, isolated from each other by a spacing of $d = 0.5$ mm. In this spacing holes are made to make sure that liquid helium can enter the experimental cell and cover the ring and center. On the upper print plate a copper disc of radius $R = 8.32$ mm is made. A hole in the upper plate is made to put a tungsten filament through (an ordinary bulb), which is used to deposit electrons on the liquid helium surface. The tungsten filament is directly connected to the guard ring of height $L = 3.13$ mm, inner radius $R_g = 8.4$ mm and outer radius $R_{tot} = 10.0$ mm.

The guard ring is used as a spacer between upper plate and center-ring which are therefore parallel within $0.1\%$.

The upper plate, ring, center and guard ring are used to confine the electrons in horizontal and vertical direction. Ring and center are further used to employ a.c. measurements.
3.3 Electrical connections, determining the helium height

Figure 3.4 shows the electrical connections of the lower part of the experimental cell.

![Electrical connections diagram]

**Figure 3.4.** The electrical connections of the lower part of the experimental cell. For explanation see text.

During the conductivity measurements, the guard ring and upper plate are at constant potential $-V_g$ and $-V_p$ respectively. The ring electrode is grounded and is together connected to a conductance-capacitance bridge operating at bridge frequency $f$, usually in the range $1 - 10$ kHz. The electrons are supported on the helium surface by heating the tungsten filament a few times until saturation occurs, i.e. the potential of the electron layer on the helium surface is equal to $-V_p$. The positive pole of the power supply is connected to the guard ring. This makes sure, together with the negative voltage on the plate, that the electrons are blown towards the helium surface. In the dilution refrigerator this technique may cause problems, since below 1 K the necessary helium vapour above the liquid helium level is absent and the electrons, when arriving with a kinetic energy greater than 1 eV (see fig. 2.1) are shoted into the liquid helium. This problem is solved by depositing the electrons on the helium surface when the temperature is 1.2 K, and then cooling down to lower temperatures. This procedure caused an other well known problem which we shall discuss later.

The superconducting coil (not shown in fig. 3.4) is connected to a current supply which can supply currents up to 120 A. The critical current of the coil is 40 A which corresponds to 4 T.

The upper plate together with the guard confines the electrons in
the vertical and horizontal direction. When the ratio \( V_g/V_p \approx 1 \), the holding field \( E_1 \) is mainly caused by the upper plate and is equal to \( E_1 = V_p/L \), where \( L \) is the separation between plate and ring electrode [46].

During the filling of the experimental chamber with liquid helium, the electrical connections are disconnected and the capacitance between plate and center-ring is measured with the conductance-capacitance bridge. If we use the plane-plate capacitor as an approximation for the cell we can determine the height \( h \) of the liquid helium with the formulae derived in ref. [9,44]

\[
\frac{C_w}{C_1} = \frac{h}{L} \frac{(1 - \varepsilon)}{\varepsilon} + 1
\]  

(3.1)

here \( h \) is the helium height, \( \varepsilon \) is the dielectric constant of liquid helium, \( C_w \) is the capacitance between plate and ring-center without helium and \( C_1 \) the value with helium present. Later we shall discuss a possible alternative method of measuring the helium height.

If we assume that the helium height is directly proportional to the pressure drop \( \Delta P \) in the storage tank (see fig. [3.1]), it is possible by

![Figure 3.5](image.png)

**Figure 3.5** The measured inverse capacitance \( C_1^{-1} \) as function of the pressure drop \( \Delta P \) in the storage tank.

measuring \( C_1 \) as function of \( \Delta P \), with the help of eq. (3.1), to determine
the helium height. By using the value of \( \Delta P \) where \( C_1 \) suddenly changes, i.e. liquid helium enters the experimental cell as a gauging point, measurements of \( C_1 - \Delta P \) in different runs can be compared.

Figure 3.5 shows \( C_1^{-1} \) as function of \( \Delta P \) for a typical run. The dashed curve is \( C_1^{-1} \) as function of \( \Delta P \) if \( \Delta P \) is a linear function of the amount of helium condensed in the experimental chamber and if eq. (3.1) holds.

From fig. 3.5 it is clear that the assumed linear dependence of \( C_1^{-1} \) on \( \Delta P \) is not experimentally confirmed. The main reason for this discrepancy is the fact that the relation between \( \Delta P \) and the amount of helium condensed in the experimental chamber is not linear. This non linearity is caused by first of all adhesive effects just before liquid helium enters the experimental cell and just before the cell is completely filled. The second reason is the fact that the uncertainty in \( \Delta P \) is large because the amount of helium which is condensed in the experimental chamber depends much on the particular circumstances (temperature, the absolute pressure in the filling tank etc.). For this reason we assume that eq. (3.1), although not experimentally verified, holds. To "check" this assumption one can calculate \( C_1 \) theoretically when the cell is completely filled with the help of eq. (3.1). \( C_\omega \) is measured frequently and was equal to

\[
C_\omega = 0.4694 \pm 0.0005 \text{ pF} \quad (3.2)
\]

and thus \( C_1(h=d) \) theoretically \((\epsilon = 1.0572)\)

\[
C_1(h=d) = 0.4962 \pm 0.0005 \text{ pF} \quad (3.3)
\]

The calculated value eq. (3.3) is in perfect agreement with the measured value

\[
C_1(h=d) = 0.4953 \pm 0.001 \text{ pF} \quad (4.3)
\]

We therefore estimate that the calculated \( h \), starting from eq. (3.1), is accurate within 10%.

One part of fig. 3.4. has not been discussed i.e. the a.c. measurements of the conductivity of the electrons. This topic will be discussed in the next chapter.
Before we shall proceed with the next chapter we shall briefly discuss the problems we have encountered in the dilution refrigerator.

Since, as discussed earlier, the electrons must be deposited on the helium surface at $T = 1.2\,\text{K}$, the temperature gradient existing in the dilution refrigerator from 1 K bath to mixing chamber changes during cooldown. Therefore thermomechanical effects appeared, i.e. the liquid helium in hollow parts close to 1 K bath and still could flow through the filling capillaries to the warmer parts of the dilution refrigerator. As a result, the liquid helium level inside the experimental chamber was not stable. By reducing and removing the hollow parts in the filling capillaries close to 1 K bath and still, the helium level inside the experimental chamber was stabilized. In ref. [52] the same thermomechanical effects were reported.
Chapter 4 Analysis of the experimental procedure

4.1 The a.c. measurements

To measure the conductivity $\sigma_{rr}(B,T)$ or the mobility $\mu(T)$ of the 2DEG on a liquid helium surface, one main problem has to be solved, i.e. one cannot make permanent contacts to the electron gas. For this reason mainly two types of experiments have been performed based on a.c. measurement techniques (there have also been performed some time of flight methods, but this measurement of $\mu$ is discontinuous and that is probably the reason why they are not popular any more [15]).

In one type of experiments, standing wave resonance are excited by driving the electrons with an high frequency e.m. field. Whenever the frequency of the e.m. field matches the frequency of a standing wave of the electron system in the cell, a power absorption takes place. The width of the absorption peak is a measure for the damping of the system and thus for the mobility $\mu$ of the electrons.

In the other type of experiments, the low frequency mobility of the electrons parallel to the surface is measured by setting the electrons in motion with an a.c. electrical field. By measuring the induced signal in another portion of the cell, one can determine the conductivity of the electrons on the surface.

To the first type of experiments belong plasmon resonance experiments (Grimes and Adams [4, 60], Marty and Poitrenaud [48]) and cyclotron resonances experiments (Brown and Grimes [21], V.S. Edel'man [20]) while the second type of experiments were performed by several authors [7, 35, 49, 51]. Our experiment belongs to the second type.

The ring (fig 3.4) is connected to the low side of the conductance-capacitance bridge which is grounded. An a.c. voltage $V_0 e^{i\omega t}$ is applied to the high side of the bridge, which is connected to the center. The detector signal of the bridge is connected to a lock-in amplifier (not shown). By phase sensitive detection of the detector signal, the complex impedance between center and ring due to electrons on the surface is measured. The c-c-bridge is used in the conductance mode, which means that the unknown measured impedance $Z$ is monitored as a parallel circuit of a conductance $G_x$ and capacitance $C_x$ (fig. 4.1).
The measured complex impedance is monitored as a parallel circuit of a conductance $G_x$ and a capacitance $C_x$.

The measured conductivity $\sigma_{rr}$ is the d.c. conductivity if $\omega \tau \ll 1$, where $\tau$ is the collision time (= relaxation time) of the electrons parallel to the surface. If the frequency is in the range $1 - 1000$ kHz this condition is fulfilled for all the temperatures used (1.3 - 2.0 K).

The conductivity is Ohmic if it is independent of the applied electric a.c. field. This condition has been experimentally verified.

By measuring $G_x$ and $C_x$ as function of magnetic field $B$ (perpendicular to the electron layer) and temperature $T$ we can determine the conductivity $\sigma_{rr}(B,T)$, if we know what the relation is between $G_x$, $C_x$ and $\sigma_{rr}(B,T)$. In the next paragraph we shall discuss two methods of analyzing the experimental data.

4.2 Methods of analyzing the experimental data

Since the ring-center-plate-guard system is cylindrical symmetric, we can only measure radial induced currents on the ring. Our geometry has great analogy with the Corbino geometry, used to measure the diagonal element $\sigma_{rr}(B,T)$ of the conductivity tensor $\sigma$ [50]. One aspect of the Corbino geometry is that only $\sigma_{rr}$ can be measured. This can be shown quite generally.

Ohm's law in a magnetic field reads (see eq. (2.25))

\[ J = \sigma \cdot E_0 \]  

(2.25)
where \( j \) is the current density in the plane of the electrons and \( E_0 \) is the applied electric drift field, which may both depend on the coordinates \((r, \varphi)\). From the Maxwell relation \( \nabla \times E_0 = 0 \) (the magnetic field is static), it follows that if \((E_0)_r = 0\) (2DEG !)

\[
(E_0)_\varphi = 0 \tag{4.1}
\]

\[
(E_0)_\varphi \sim \frac{1}{r} \tag{4.2}
\]

In the plane of the electrons

\[
\oint (E_0)_\varphi \, r \, d\varphi = 0 \tag{4.2}
\]

must hold which leaves \((E_0)_\varphi = 0\) as the only solution. Inserting this in (2.25) gives

\[
\begin{align*}
  j_r &= \sigma_{rr}^* (E_0)_r \\
  j_\varphi &= \sigma_{\varphi r}^* (E_0)_r
\end{align*} \tag{4.3}
\]

Since as mentioned earlier, only \( j_r \) can be measured, it follows that when knowing \((E_0)_r\), only \( \sigma_{rr} (B,T) \) can be determined. One can say that in a Corbino type geometry, the Hall effect is short circuited. The above analysis is only valid when the reactive part of \( j_r \) is negligible, i.e. \( \omega \tau \ll 1 \), and if the self generated magnetic field of the electrons is small [50].

4.2.1. The old method

The first method of analyzing the measured \( C_x \) and \( C_x \) is by means of a simplified circuit shown in fig. 4.2b (compare with fig. 4.1). This method has been used by Y.Iye [35] and J.Theobald et al. [51] and in earlier experiments in our group [9,22,44,45].

In fig. 4.2b \( C_0 \) and \( C_0 \) are the measured values without electrons, i.e. the direct coupling between ring and center. If electrons are present an additional coupling appears which is represented by the upper path in fig. 4.2b. \( C_1 \) represents the capacitance between the ring and
Figure 4.2 a: the lower part of the experimental cell, b: the simplified equivalent circuit of the lower part of the experimental cell.

The electron layer above it, $C_2$ a similar quantity related with the central electrode. $G_e$ is the conductance of the electrons in the region above the spacing. A simple circuit analysis gives a relation for $C_x$ and $C_x$ as function of $\omega = 2\pi f$, $G_e$ and $C_x = C_1 C_2 / (C_1 + C_2)$

$$C_x = C_0 + \frac{G_e}{1 + (G_e/\omega C_e)^2} \quad (4.4a)$$

$$C_x = C_0 + \frac{(G_e/\omega)^2/G_e}{1 + (G_e/\omega C_e)^2} \quad (4.4b)$$

In this analysis the capacitance $C_e$ is considered to depend only on geometrical factors and natural constants. The conductance $G_e$ is a function of the electron mobility $\mu$, the electron density $n_0$ and the geometry. $G_e$ can be written as

$$G_e = \sigma_{rr}(B,T)/\gamma \quad (4.5)$$

here $\gamma$ is a geometrical factor (the ratio of the width to the circumference of the ring shaped electron layer above the spacing, between ring and center electrode).

In his experiment, Iye [35] used a magnetic field to determine the mobility of the electrons in $B = 0$. The conductivity $\sigma_{rr}(B,T)$ for small $B$ is given by eq. (2.26) (only vapour atom scattering)

$$\sigma_{rr}(B,T) = \frac{n_0 e \mu}{1 + \mu^2 B^2} \quad (4.6)$$

A seen from eq. (4.6), if the magnetic field is increased at constant $T$, 
\( \sigma_{rr}(B,T) \) decreases. Iye [35] therefore used the magnetic field for two reasons: The first reason is, that with the help of eq. (4.6) \( \mu \) can be determined from the low magnetic field measurements (Iye does not make a difference between \( \mu \) or \( \mu_H \)). The second reason is the fact that the measured \( C_x \) for \( B = 0 \) is small (this can be seen from fig. 4.2b, if \( C_e > \omega C_e \), then \( C_e \) can be considered as a short circuiting between \( C_1 \) and \( C_2 \), so the relative error in \( C_x \) is large. By applying a magnetic field, one can see from eq. (4.4a) and (4.6) that \( C_x \) becomes larger (if \( C_e > \omega C_e \)) as function of \( B \). Geometrically this is a consequence of the fact that the current density \( j \) in a magnetic field has a tangential component, resulting from the Lorentz force. Because of charge conservation and since in a Corbino geometry only radial currents can be measured, this results in an increasing resistance. Therefore \( C_x \) becomes larger and thus the relative error smaller. In the earlier experiments performed in our group the same problems were encountered [9,22].

We see from eqs.(4.4) that the limits for \( C_x \) for \( C_e \to \infty \) or \( C_e \to 0 \) are both \( C_0 \) (which can be seen directly from fig. 4.2b) while \( C_x \) is equal to \( C_0 \) for \( C_e \to 0 \) and is equal to \( C_e + C_0 \) for \( C_e \to \infty \).

The approximations which are made by using this simplified circuit are 1. the coupling of the electron gas to guard ring and upper plate are neglected
2. the capacitance \( C_e \) is considered to depend only on the geometry. In experiments (see later), it is however observed that \( C_e \) also depends on \( B \) and on \( \omega \).
3. by using this analysis to measure \( \sigma_{rr}(B,T) \) in high magnetic fields the implicit assumption is made that the geometrical factor \( \gamma \) does not depend on \( B \)
4. the charge which is available is finite, therefore the idealized form of \( C_e \) will in general not be fullfilled. This corresponds with the condition \( C_e/C_e = R_e C_e < 1/\omega \), since this condition expresses that displaced charge is small (the RC-time of the system \( C_e \) and \( C_e \) is small in comparison with the inverse frequency). The condition \( R_e C_e < 1/\omega \) is used in ref. [51]

To reduce the geometrical effects, the ratios \( V_p/V_0 \) are kept constant during all the experiments.

As mentioned earlier, the experiments showed that for large \( B \) the capacitance \( C_e \) varied as function of \( B \). Since, just in this regime, the
conductivity $\sigma_r(B,T)$ deviated from the classical behaviour, a more precise analysis was needed.

4.2.2 The dynamical method

The second method of analyzing the experimental data is similar to the analysis made by E. Andrei [7] and M.L. Ott-Rowland et al. [56].

Figure 4.2a shows the experimental cell in more detail. For the present we neglect the guard ring and assume that the electron density is a disc of radius $R$ and constant density $n_0\epsilon$. The disc form is fully determined by the static potentials on the upper plate, guard ring, center-ring and the potential of the electron layer $U$ [53,54,55]. The homogeneous electron density $n_0$ can be calculated as function of $U$, $V_p$, $\epsilon$ and the height of the helium level $h$ (see Appendix A.2 ref. [9]). We assume that the electrons are in a charge neutralizing background. The magnetic field $B$ is perpendicular to the electron layer and is much larger than the magnetic field generated by the motion of the electrons. We assume that the magnetic field $B$ is uniform. The upper plate and guard have potential $-V_p$ and $-V_g$ respectively. The center and ring are equally biased at zero potential. In calculations, we assume that the spacing between ring and center can be chosen at zero potential, since the width of the spacing is small compared to the radius of the cell. To the center a harmonic a.c. voltage $V_0 e^{i\omega t}$ is applied.

It can be shown that the linearized dynamical problem, i.e. the time dependent problem, can be reduced to an electrostatic one. The generalized Ohm's law and the continuity relation yield

$$J = (\sigma + i\xi) \cdot E_0$$

$$\frac{dn}{dt} + V_r \cdot J = 0 \quad \left[ V_r \cdot J = \frac{\partial J_r}{\partial r} + \frac{J_r}{r} \right]$$

where $\sigma$ is the conductivity tensor, $i$ the imaginary constant and $\xi$ a tensor which takes the reactive properties of the electrons into account [7,56,61]. For simplicity we have assumed, because the cell is cylindrical symmetric, that the electron density $n = n(r,t)$ and the current density $J = J(r,t)$ are only a function of the distance $r$ from the cylinder axis and the time $t$. $E_0 = E_0(r,z,t)$ in eq. (4.7) is the time dependent electrical field inside the cylinder, and has to be
evaluated at the height of the electron layer $z = h$.

We assume that all the terms in eq. (4.7) and (4.8) can be written in the following linearized form [7,56,57,61]

\begin{align*}
n(r,t) &= n_0 + n_1(r) e^{i\omega t} \tag{4.9a} \\
J(r,t) &= J(r) e^{i\omega t} \tag{4.9b} \\
E_0(r,z,t) &= E_0(r,z) e^{i\omega t} \tag{4.9c}
\end{align*}

The time dependent terms are assumed small (harmonic approximation). Substitution of eqs. (4.9) in (4.7) and (4.8), neglecting second and higher harmonics, leads to a single dynamical equation ((E_0) = 0 see paragraph 4.1)

\[
\left( \frac{\epsilon_0 \xi_{rr} \omega}{(\sigma_r^2 + \xi_r^2)} + \frac{i\epsilon_0 \sigma_{rr} \omega}{(\sigma_r^2 + \xi_r^2)} \right)n_1 = -\nabla_r \cdot E_0(r,h) = \Delta_r \Phi_1(r,h) \tag{4.10}
\]

where $\Delta_r = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}$ and $\Phi_1(r,h)$ the time dependent potential inside the cell at height $h$.

Equation (4.10) and the Poisson equation for the time dependent potential $\Phi_1(r,z) e^{i\omega t}$ are simultaneous solved to yield $n_1(r)$ and $\Phi_1(r,z)$. The boundary conditions for the dynamical problem are: $\Phi_1 = 0$ on the upper plate, ring and spacing, and $\Phi_1 = V_0$ at the center. Further we assume that the radial component of the current density $J_r = 0$ at the edge of the electron disc [49,56]. The calculation is done in the Appendix.

The potential $\Phi_1(r,z)$ is then used to calculate the induced current on the receiving electrode

\[
I = i e \epsilon_0 \omega \int_{r_0}^R \frac{d\Phi_1}{dz} \bigg|_{z=0} 2\pi rdz \tag{4.11}
\]

Since the bridge is in the conductance mode (see fig. (4.1)), the induced current on the ring is equal to
\[
I = V_0 \cdot (C_x + i\omega C_x)
\]

(4.12)

The resulting relations for \(C_x\) and \(C_x\) are (see A.1)

\[
C_x = C_0 + n_0 e^2 \sum_q A_q \cdot \frac{\tanh(q(L-h))}{\epsilon \cdot \tanh(q(L-h))+\tanh(qh)} \left[ \begin{array}{c}
K_q \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} \\
\frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} + \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2}
\end{array} \right]
\]

\[
C_x = C_0 + n_0 e^2 \sum_q A_q \cdot \frac{\tanh(q(L-h))}{\epsilon \cdot \tanh(q(L-h))+\tanh(qh)} \left[ \begin{array}{c}
K_q \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} \\
\frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} + \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2}
\end{array} \right]
\]

(4.13a)

\[
\]

(4.13b)

where \(A_q\) and \(K_q\) are given by

\[
A_q = 4\pi \epsilon \cdot \frac{\rho_0}{R} f_1(q) \frac{f_0(qR)}{f_0(q)}
\]

(4.14)

\[
K_q = \frac{n_0 e^2 q}{\epsilon} \frac{\tanh(q(L-h)) \cdot \tanh(qh)}{\tanh(q(L-h)) + \tanh(qh)}
\]

(4.15)

with \(f_n\) the Besselfunctions of order \(n\), \(q\) is the wavenumber given by

\[
q = \frac{y_n}{R} \quad \text{\(y_n\) the \(n\)th zero of \(f_1\)}
\]

(4.16)

First we shall consider some physical elements resulting from eqs. (4.13), (4.14) and (4.15).

If we compare the results of our calculation, in particular \(K_q\), with the results of E. Andrei [7] for \(B = 0\), i.e. eq. (4.7) has the form

\[
J = \frac{n_0 e^2}{1 + i\omega} E_0
\]

(4.17)

then it is clear that these are equal when we set \(d = 0\) in eq. (3) in ref. [7]. This is a striking result since eq. (3) in ref. [7] was
derived for a rectangular geometry. The main differences, i.e. the cylinder symmetrical properties are found in $A_q$ and in $q$.

$K_q$ can be rewritten as [7]

$$K_q = m\omega_q^2 = \frac{n_0 e^2 q}{\varepsilon_0} \frac{\tanh(q(L-h)) \cdot \tanh(qh)}{\varepsilon \cdot \tanh[q(L-h)] + \tanh(qh)} = \frac{n_0 e^2 q}{\varepsilon_0} N_q \quad (4.18)$$

here $\omega_q$ is the screened plasmon frequency. Equation (4.18) is the dispersion relation for the plasmon waves inside the experimental cell. The function $N_q$ is of order unity and depends on the spacing between the upper plate and center-ring (the background neutralizing charge is in these electrodes), the helium height $h$ and the dielectric constant $\varepsilon$. In the fully unscreened limit i.e. $h/R \to \infty$, $N_q \to 1$ and thus $\omega_q \sim \sqrt{q}$. In three dimensions, the plasmon frequency is independent of the wave number $q$ [62], because the average restoring electrical field in three dimensions is due to large charged planes. In two dimensions however, the average restoring field is due to infinitely long lines of charges and therefore inversely proportional to the wavelength.

By adjusting the frequency $\omega$, it is possible to excite a plasmon wave with frequency $\omega_q$. The particular value of $\omega$, for which such resonance occurs, depends on the magnetic field and on $\omega_q$. The plasmon resonance experiments are essentially based on this effect [4,53,60]. In our experiments the frequencies used were much lower than $\omega_q$, in fact the inequality

$$K_q \geq \frac{\omega_0 e^2 \xi_{rr}}{(\sigma_{rr}^2 + \xi_{rr}^2)} \quad (4.19)$$

was valid throughout our experiments, for all the frequencies used.

It is further easily verified that (from the equation of motion of the electron)

$$\sigma_{rr} \geq \xi_{rr} \quad (4.20)$$

if $\omega \tau \ll 1$. The reactive term however becomes important when the mass of the electrons becomes larger, for example in the case of Wigner crystallization [4] or polaron effects [7]. In further calculations the reactive term is therefore neglected.
The approximations made in the dynamical method are:
1. we have neglected the possible r-dependence of $n_0$
2. the equations for $G_x$ and $C_x$ are only valid when $n_1 < n_0$. This condition has been experimentally verified.
3. the coupling to guard ring has been neglected but can be incorporated by introducing an effective radius $R = R_1$ as an adjustable parameter, this is similar to the effective length introduced in ref. [49]

Since $C_x$ and $C_x$ also depend on the helium heigth $h$, eqs. (4.13), (4.14) and (4.15) provide an alternative way of determining $h$.

The comparison of the old model (eqs. 4.4) and the dynamical model (eqs. 4.13) are kept for the next paragraph.

4.3 Comparison of the simplified model and the dynamical model

This paragraph is dedicated to the comparison of the old simplified model and the new dynamical model, derived in paragraph 4.2.1.

The equations which have to be compared are:

the simplified model

$$G_x = G_0 + \frac{C_e}{1 + (C_e/\omega C_e)^2} \tag{4.4a}$$

$$C_x = C_0 + \frac{(C_e/\omega)^2/C_e}{1 + (C_e/\omega C_e)^2} \tag{4.4b}$$

the dynamical model

$$C_x = C_0 + n_{pe}^2 \sum_q A_q \frac{\tanh(q(L-h))}{\epsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{K_q}{K_q + \left[ \frac{n_{pe}^2 \omega}{\sigma_{rr}(B,T)} \right]^2} \right] \tag{4.21a}$$
where we have neglected the reactive terms in eqs. (4.13). $K_q$ and $A_q$ are given in eqs. (4.14) and (4.15). To compare the two models properly and to get more insight in eqs. (4.21), we rewrite eqs. (4.21) to (dividing numerator and denominator by $(n_0 \varepsilon \omega / \sigma_{rr}(B,T))^2$ and using the explicit expression of $K_q$ eq. (4.15))

\[
C_x = C_0 + \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{n_0 \varepsilon^2 \omega^2}{\sigma_{rr}(B,T)} \left( \frac{\varepsilon}{K_q + \left( \frac{n_0 \varepsilon^2 \omega}{\sigma_{rr}(B,T)} \right)^2} \right) \right]
\]

(4.21b)

where $N_q$ is given in eq. (4.18). Note that the electron density dependence of $C_x$ and $C_y$ only appears through $\sigma_{rr}(B,T)$.

Since $q = y_n / R_1$, where $R_1$ is the effective radius of the electron disc, introduced in paragraph 4.2.1, we can define a dimensionless number $Q$, which is given by

\[
Q = \frac{\sigma_{rr}(B,T)}{\omega_0 R_1}
\]

(4.23)

Finally eqs. (4.22) can be written in the form

\[
C_x = C_0 + \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{\varepsilon_0 R_1 Q^2 B_2^2}{1 + Q^2 B_2^2} \right]
\]

(4.24a)
\[ G_x = G_0 + \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{\sigma_{rr}(B,T)}{1 + Q^2 B_q^2} \right] \]  \hspace{1cm} (4.24b)

Since \( B_q = qR_1 \cdot N_q \) is an ascending function of \( q \), it is easily verified that if

\[ Q > 1 \quad \text{i.e.} \quad K_q > \frac{\eta e^2 \omega}{\sigma_{rr}(B,T)} \]  \hspace{1cm} (4.25)

eqs. (4.24) are identical with eqs. (4.4) in the case

\[ D_e = \frac{G_e}{\omega C_e} > 1 \]  \hspace{1cm} (4.26)

Equation (4.26) is therefore the analog of eqs. (4.24) if the simplified model is used.

By comparing eqs. (4.4) and eqs. (4.24) in the limit (4.25) and (4.26), we obtain an analytical expression for the geometrical factor \( \gamma \) in eq. (4.5) and the capacitance \( C_e \), due to the presence of the electrons on the surface

\[ \gamma = \frac{\sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)}}{\left( \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \right)^2} \]  \hspace{1cm} (4.27)

\[ C_e = \varepsilon_0 R_1 \cdot \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \]  \hspace{1cm} (4.28)

The sums in eqs. (4.27) and (4.28) are calculated numerically on a computer. The values of \( L \), \( r_0 \) and \( d \) were chosen equal to the dimensions of the cell. Table 4.1 shows some results for different values of the helium height \( h \) and the effective radius \( R_1 \).

In the simplified model, \( \gamma \) is given approximately by, since the conduction of the electron gas only takes place above the spacing

\[ \gamma = \frac{d}{\pi(2r_0+d)} \]  \hspace{1cm} (4.29)
which is independent of \( R_t \) and \( h \). From table 4.1 it is seen that \( \gamma \) in

<table>
<thead>
<tr>
<th>( R_t ) (mm)</th>
<th>( h ) (mm)</th>
<th>( C_e ) (pF)</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.32</td>
<td>1.00</td>
<td>0.221</td>
<td>0.290</td>
</tr>
<tr>
<td>6.26</td>
<td>1.00</td>
<td>0.185</td>
<td>0.255</td>
</tr>
<tr>
<td>4.20</td>
<td>1.00</td>
<td>0.094</td>
<td>0.254</td>
</tr>
<tr>
<td>6.26</td>
<td>0.50</td>
<td>0.356</td>
<td>0.171</td>
</tr>
<tr>
<td>6.26</td>
<td>1.50</td>
<td>0.142</td>
<td>0.270</td>
</tr>
</tbody>
</table>

Table 4.1 Calculated values of \( C_e \) and \( \gamma \) for different helium height \( h \) and effective radius \( R_t \).

the dynamical model is about a factor of 10 larger, indicating that the conduction not only takes place above the spacing but in a larger region. If \( h \) is lowered \( \gamma \) becomes smaller, which means that the simplified model is approached. From table 4.1 it is further seen that \( C_e \) has the proper \( R_t \) and \( h \) dependence, i.e. \( C_e \) becomes larger if \( R_t \) is increased or \( h \) lowered.

As mentioned before, if \( Q > 1 \) the two models are equal and it is possible to analyse the measurements with the simplified model. This has much advantages, for example the simplified method is simple, while it is difficult to invert eqs. (4.24). Thus analyzing the measured data starting from eq. (4.24), one should probably need iterative methods and one is forced to use a computer. In principle there are no objections to use a computer, however if it can be shown that during the experiment \( Q \) is always in the range that the dynamical and the simplified model are equal, the simplified method is preferred. Important questions therefore are

- for what range of \( Q \) are the dynamical model and the simplified model the same, besides \( Q > 1 \) or \( D_e > 1 \).
- during the experiments \( C_x \) and \( C_x \) are measured as function of the magnetic field \( B \) and the temperature \( T \). The question rises, whether it is possible to determine qualitatively, from the behavior of \( C_x \) and \( C_x \) as function of \( B \), in which range of \( Q \) or \( D_e \) the measurements are performed. Note that both \( Q \) and \( D_e \) are functions of the magnetic field \( B \) through \( \sigma_r(B,T) \) and that thus \( Q \) and \( D_e \) become smaller in a magnetic
field (see for example eqs. (2.26)). Consequently, the low magnetic field measurements could be in the range where it is permitted that the simplified model is used, while the high magnetic field measurements should be analyzed with the dynamical model.

The answer to the last question is particularly interesting because, as already mentioned before, the conductivity \( \sigma_{rr}(B,T) \) in a high magnetic field showed a deviation from the classical behaviour (see chapter 5), when the measured values of \( C_x \) and \( C_x \) are analyzed starting from eqs. (4.4). To know whether this deviation is a consequence of the simplified model used or a real physical effect, the two methods of analyzing the experimental data should be compared. This is done by systematically determining the influence of the parameters

- \( \sigma_{rr} \)
- \( R_t \)
- \( h \) and
- \( f = \omega/2\pi \)

In the dynamical model \( C_x \) and \( G_x \) are calculated numerically as function of \( \sigma_{rr} \) using eqs. (4.24). To calculate \( C_x \) and \( C_x \) in the simplified model as function of \( \sigma_{rr} \), \( C_e \) and \( \gamma \) have be to known. To compare \( C_x \) and \( C_x \) correctly, \( C_e \) and \( \gamma \) are determined from the limiting behaviour of eqs. (4.24), i.e. eqs. (4.27) and (4.28). This is done for each value of \( h, R_t \) and \( \omega \).

Figure 4.3 shows a typical result for \( C_x \) and \( C_x \) as function of \( \sigma_{rr} \) for both models (indicated by the different symbols, the symbols are due to the used plotting program of the computer). The values of \( G_0, C_0, h, \) and \( \omega \) are chosen close to the experimental values. \( R_t \) is estimated from the formulas given in ref. [53]. \( L, r_0 \) and \( d \) where chosen equal to the dimensions of the cell. The value of \( \sigma_{rr} \) for which \( Q = 1 \) respectively \( D_e = 1 \) are marked with an arrow respectively a dashed arrow. Figure 4.3 is the "standard" figure, i.e. to determine the influence of the parameters \( h, R_t \) and \( \omega \), only one is changed at a time starting from fig. 4.3.

Note that \( \sigma_{rr} \) is the experimentally important parameter: by measuring \( C_x \) and \( C_x \) as function of \( B \) and \( T \) we want to determine \( \sigma_{rr}(B,T) \) from eqs. (4.4) or (4.24).

Some important preliminary characteristics are seen from fig. 4.3. First of all, the global form of \( C_x \) and \( C_x \) are in both models the same,
i.e. $C_x$ has in both models a maximum as function of $\sigma_{rr}$, and $C_x$ varies from $C_e + C_0$ to $C_0$. Secondly, a striking result of the exact calculation

\[ C_x = 0.185 \text{ pF} \]
\[ \gamma = 0.255 \]
\[ f = 10 \text{ kHz} \]
\[ R_1 = 6.26 \text{ mm} \]
\[ h = 1.00 \text{ mm} \]

- ■ = dynamical model
- ○ = simplified model

Figure 4.3 The "standard" plot of $C_x$ and $G_x$ in both models. The values of the parameters are tabulated. For explanation see text.

of $C_x$ in the dynamical model is, that for particular values of $\sigma_{rr}$, a negative change of $C_x$ occurs, with other words, the change of the imaginary part of the measured complex impedance $Z$ (fig. 4.1) is negative. In the simplified model this feature is not predicted because only capacitive and dissipative terms are taken into account. This is principally the historical reason why the imaginary part of $Z$ was called a capacitance. In the dynamical model, it appears that for certain values of $\sigma_{rr}$ inductive terms play a role. This typical behavior of $C_x$ is also observed experimentally, as will be shown in the next paragraph. A third characteristic of fig. 4.3 is the fact that for $Q > 1$ or equivalently $D_e > 1$, the two models are identical as shown analytically.
Figure 4.4 $G_x$ and $C_x$ as function of $\sigma_r$ for both models. Except for the frequency (a: 100 kHz, b: 1 kHz), all the parameters are the same as in fig. 4.3. For explanation see text.
The maximum of $G_x$, in respectively the simplified model and the dynamical model, is found in the neighbourhood of respectively $D_e = 1$ and $Q = 1$, the value of $C_x$ in both model is then approximately equal to $(C_0 + C_e)/2$. The conditions $D_e = 1$ and $Q = 1$ differ slightly, which is a consequence of the fact that the geometrical factors in $D_e$ and $Q$ are not equal. If $\sigma_{rr}$ is to be determined from $G_x$ and $C_x$, if they are measured in the the neighbourhood of the maximum in $G_x$, the two models of analyzing the measurements differ slightly, the main differences are found for $Q < 1$ and $D_e < 1$. In this region however, the direction and the absolute value of the error made in determining the conductivity from the simplified model, can be estimated by comparing the measured values of $C_x$ and $G_x$ (the intersection of the simplified and the dynamical model on the left in fig. 4.3, is in the neighbourhood of the minimum in $C_x$).

To determine if this preliminary characteristics of $C_x$ and $C_x$ as function of $\sigma_{rr}$, are a general feature of eqs. (4.4) and (4.24) $h$, $R_1$ and $f$ where varied. The results are shown in fig. 4.4, 4.5, and 4.6.

In fig. 4.4 $C_x$ and $G_x$ are shown as function of $\sigma_{rr}$. Except for $f$, all the parameters are the same as in fig. 4.3. By comparing fig. 4.4 with fig. 4.3, it appears that again the maxima of $C_x$ are in the neighbourhood of resp. $Q = 1$ and $D_e = 1$ and at the same time $C_x \approx (C_0 + C_e)/2$. As a result fig. 4.3 is shifted to lower values of $\sigma_{rr}$ for a higher frequency and to higher values for lower frequencies. The form and absolute values of $C_x$ are independent of the frequency $f$ as seen from fig. 4.4. For $G_x$ the width of the curve becomes smaller for lower frequencies and the absolute values of $G_x$ increase approximately linear with $f$. This property of $G_x$ as function of $f$ can be used to make the absolute value of $G_x$ larger and as a consequence the relative error in $G_x$ smaller. The limit $G_x = G_0$ for $\sigma_{rr} \to 0$, in both models, is earlier approached for lower frequencies.

From fig. 4.4, it is again clear that the two models are equal for $Q > 1$ and $D_e > 1$. The differences in the neighbourhood of the maximum in $G_x$ seem to be small. For $Q < 1$ or $D_e < 1$ the two models seem to differ more for higher frequencies. The "negative" capacitance is again present and the absolute value seems to be independent of $f$.

Another interesting question is, is it possible to check the validity of the use of the simplified model by varying the frequency $f$. As seen from fig. 4.4, if $f$ is varied in the region where $Q > 1$ or $D_e > 1$, the calculated $\sigma_{rr}$ will not differ because both models are in
\[ C_s = 0.221 \text{ pF} \]
\[ \gamma = 0.290 \]
\[ f = 10 \text{ kHz} \]
\[ R_1 = 8.32 \text{ mm} \]
\[ h = 1.00 \text{ mm} \]

\[ \square = \text{dynamical model} \]
\[ \bigcirc = \text{simplified model} \]

**Figure 4.5** \( G_x \) and \( C_x \) as function of \( \sigma_r \) for both models. Except for the effective radius \( R_1 \) (a: 8.32 mm, b: 4.20 mm), all the parameters are the same as in fig. 4.3. For explanation see text.
this region the same. Consequently the calculated $C_x$ and $C_x$, starting from eqs. (4.4) will be the same for different $f$ ($C_x$ and $\gamma$ are independent of the frequency eqs. (4.27) and (4.28)). However, if $Q < 1$ or $D_e < 1$, the calculated value of $\sigma_{rr}$ in both models for different $f$ will in general be different.

Figure 4.5 shows $C_x$ and $G_x$ as function of $\sigma_{rr}$. Except for $R_t$ all the parameters have the same values as in fig. 4.3. Besides the equal behaviour of the two models for $Q > 1$ or $D_e > 1$, it is seen from fig. 4.5 that the values of $G_x$ in the neighbourhood of $Q = 1$ or $D_e = 1$ differ slightly for different $R_t$. The absolute values of $C_x$ in the dynamical model, as in the simplified model, depend much on $R_t$. For example, if the effective radius is increased, the "negative" capacitance change is increased and a maximum arises. This maximum is not observed experimentally, which is an indication that the effective radius is in the neighbourhood of $R_t$ of fig. 4.3. In both models, the limiting behaviour $C_x = C_0$ for $\sigma_{rr} \to 0$ is earlier approached for small $R_t$.

Figure 4.6 shows $C_x$ and $G_x$ as function of $\sigma_{rr}$ for different helium height $h$. The other parameters have the same value as in fig. 4.3.

Changing the helium height $h$ has no effect on the behaviour of $C_x$ and $G_x$ in both models, for $Q > 1$ and $D_e > 1$. The major differences between the two models are again in the region where $Q < 1$ or $D_e < 1$. In the neighborhood of the maximum the two models seem to differ slightly.

The two models differ more and the negative change of the imaginary part of the measured complex impedance is larger for higher helium heights (see fig. 4.3, the sharp bend in fig. 4.6a is due to the number of points, which are used to draw the figure). This is probably due to screening effects: if the helium height is low, the upper plate is screened. Consequently, the simplified model is more approached by the dynamical model (this is also the reason that $\gamma$ in table 4.1 is lower for lower helium heights).

The absolute value of $C_x$ increases as the helium height is decreased, which is a consequence of the fact that $C_x$ is approximately proportional to $1/h$. This property of $C_x$ or equivalently of $C_e$ can be used to measure the helium height $h$ in an alternative way.

From fig. 4.6 it is seen that the absolute value of $G_x$ is nearly independent of the helium height. However since the electron density $n_0$ is given by

$$n_0 = \frac{e_0 e_r V_p}{\varepsilon h} \quad (4.30)$$

the helium height is an important parameter. By keeping $V_p/h$ large, i.e. large density $n_0$, the range in which the simplified and the dynamical
Figure 4.6 $G_x$ and $C_x$ as function of $\sigma_{rr}$ for both models. Except for the helium height $h$ (a: 1.50 mm, b: 0.50 mm), all the parameters are the same as in fig. 4.3. For explanation see text.
model are equal is enlarged.

From the above systematic analyses of $G_x$ and $C_x$ in both models, it can be concluded that it is possible to determine qualitatively from the measured values of $G_x$ and $C_x$, whether the simplified model is valid and can be used to calculate the conductivity $\sigma_{rr}(B,T)$.

To be sure that the measurements are performed in the range where $Q > 1$ and $D_x > 1$ (the range where the dynamical model and the simplified model are equal), it has to be verified that the experimentally measured $G_x$ increases for decreasing $\sigma_{rr}(B,T)$, while $C_x$ remains larger than approximately $(C_1 + C_0)/2$, where $C_1$ is the measured value of $C_x$ for large $\sigma_{rr}(B,T)$. For the present experiments, this can easily be done with the only assumption that $\sigma_{rr}(B,T)$ decreases with increasing magnetic field, irrespective of the functional behavior.

The minimum in $C_x$ is only present in the dynamical model. An observation of this negative change of $C_x$ during the measurements of $\sigma_{rr}(B,T)$ is a justification of the dynamical model on qualitative ground.

An independent check of the validity of the simplified model is provided by a frequency check, for which one must demand that the calculated $G_x$ in the simplified model is independent of $f$ for the whole measured range of $B$. This demand is particularly important if measurements are performed near the region where $C_x$ has a maximum.

The above mentioned arguments, whether the simplified model is valid or not, will be illustrated in the next paragraph on the basis of some typical experimental results.

4.4 Experimental tests of the model calculations

Note: In considering experimental figures as a function of $B$, it should be kept in mind that $\sigma_{rr} = \sigma_{rr}(B)$ is a decreasing function of $B$ with an (as yet) undetermined functional relationship.

To illustrate in which range $Q$ or $D_x$ are during a typical measurement of $\sigma_{rr}(B,T)$, a quick estimate shows that $Q$ and $D_x$ are much larger than 1 for $B = 0$ T. For example, if the electron density is
saturated, $n_0$ is given by eq. (4.30) and for a typical helium height and upper plate voltage ($h \approx 1$ mm and $V_p = -10$ V during our measurements) the density is approximately equal to $n_0 = 6 \times 10^{11}$ m$^{-2}$. For $B = 0$, and upper plate voltage $V_p = -10$ V, $Q$ and $D_e$ are given by (eqs. (4.25) and (4.26))

$$D_e = \frac{C_e \eta_e}{\omega C_e} = \frac{n_0^* e \mu}{2 \pi f \gamma \epsilon_0 C_e} \quad (4.31a)$$

$$Q = \frac{n_0^* e \mu}{2 \pi f \epsilon_0 \frac{1}{R_1}} \quad (4.31b)$$

By substituting typical values of $\mu = 10$ m$^2$/V·s, $R_1 = 6.26$ mm, $C_e = 0.1$ pF (experimental), $\gamma = 0.26$ (calculated from eq. (4.27)) and $f = 10$ kHz, $Q$ and $D_e$ are equal to

$$Q \approx 275 \quad (4.32a)$$

$$D_e \approx 590 \quad (4.32b)$$

and thus for $B = 0$ the simplified model is valid. If, however, a magnetic field is applied normal to the surface and the classical behavior is assumed for $\sigma_{rr}(B,T)$, then $Q$ and $D_e$ are given by

$$Q(B) = Q(B=0)/(1 + \mu^2 B^2) \quad (4.33a)$$

$$D_e(B) = D_e(B=0)/(1 + \mu^2 B^2) \quad (4.33b)$$

and thus for $B = 4$ T

$$Q(B=4 \text{ T}) \approx 1 \quad (4.34a)$$

$$D_e(B=4 \text{ T}) \approx 2 \quad (4.34b)$$

From the values of $Q$ and $D_e$ for $B = 4$ T, it is seen that it is important to know, whether it is permitted to analyse the experimental data in high magnetic fields with the help of the simplified model.
Figure 4.7 a: The directly measured $G_x$ and $C_x$ as function of the magnetic field $B$ for $T = 1.56$ K. The values of the other parameters are tabulated, b: The calculated values of $C_x$ and $G_x(B=0)/G_x(B)$ as function of the magnetic field $B$ for $T = 1.56$ K by using the simplified model. For explanation see text.

In fig. 4.7a $G_x$ and $C_x$ are shown as function of the magnetic field $B$ for $T = 1.56$ K. The values of $C_0$, $G_0$, $f$ and $h$ are tabulated in fig. 4.7a. It is assumed that the electron density is saturated. From the form and absolute values we conclude, on basis of the arguments given at the end of paragraph 4.3, that the simplified model is valid for the entire range of $B$ in this particular measurement. In fig. 4.7b $C_x$ and $C_x(B=0)/C_x(B)$ are shown (by normalizing $C_x(B)$ on $C_x(B=0)$, the dependence of $C_x(B)$ on $n_0$ and $\gamma$ is eliminated and the measurements in different runs can be compared), calculated from the values $G_x$ and $C_x$ in fig. 4.7a. This is done by inverting eqs. (4.4), which yields

$$G_x = G_x - G_0 + \frac{\omega^2(C_x - C_0)^2}{C_x - C_0}$$

(4.35a)
\[ C_e = C_x - C_0 + \frac{(C_x - C_0)^2}{\omega^2(C_x - C_0)} \]  \hspace{1cm} (4.35b)

As seen from fig 4.7b, \( C_e \) varies only slightly as function of \( B \). This slight \( B \)-dependence of \( C_e \) is not completely understood but a possible reason might be found in a small \( B \)-dependence of \( R_1 \).

The measurements used to determine the conductivity \( \sigma_{rr}(B,T) \) in the next paragraph are similar to the measurements shown in fig. 4.8 and thus the use of the simplified model is permitted.

Figure 4.8a shows \( G_x \) and \( C_x \) as function of \( B \) for two different frequencies \( (f = 165 \text{ kHz}) \), at the same temperature \( T = 1.49 \text{ K} \). The values of the other parameters are tabulated in fig. 4.8. Figure 4.9, where \( G_e(B)/G_e(B=0) \) and \( C_e \) are shown as function of \( B \), is an
illustration of what happens if the simplified model is still used although it is not valid. This is easily seen from fig. 4.8, where \( C_x \), for both cases, exhibits a maximum as function of the magnetic field \( B \) (note that the value of \( D_e \) in the maximum is nearly equal to 1), and that \( C_x - C_0 \) varies more than 50% from the initial value in \( B = 0 \). From

\[ B(T) \]

**Figure 4.9** \( C_e(B) \) and \( G_e(B=0)/G_e(B) \) for two frequencies \((f = 165.3 \text{ and } 16.5 \text{ kHz})\) as function of the magnetic field \( B \) for \( T = 1.49 \text{ K} \). The values of the other parameters are tabulated. \( C_e(B) \) and \( G_e(B=0)/G_e(B) \) are calculated from \( G_x \) and \( C_x \), shown in fig. 4.8, on the basis of the simplified model. For explanation see text.

In fig. 4.9 it is easily seen that the behavior of \( G_e(B)/G_e(B=0) \) differs very much as function of \( f \), but the low magnetic field measurements differ also. This is explained by the fact that, for high frequencies, hot electron effects [15, 26, 27, 35] become important. This is easily seen in fig. 4.2b. The potential difference, which the electrons feel effectively, is approximately given by (by a simple circuit analysis)
and thus for increasing $f (\omega/2\pi)$, the drift field is increased. This particular measurement shows, that in this case the experimental data should be analyzed with the dynamical model, and that it is possible to measure hot electron effects by varying the frequency.

**Figure 4.10** The directly measured $G_x$ and $C_x$ as function of the magnetic field $B$ for $T = 1.73$ K for two different frequencies, $a$: $f = 4.05$ kHz, $b$: $f = 40.5$ kHz. The values of the other parameters are tabulated. For explanation see text.

Figure 4.11 shows $C_x(B)/C_x(B=0)$ and $C_x$ as function of $B$ for two different frequencies at $T = 1.73$ K. Although $C_x$ as function of $B$ (shown in fig. 4.10) exhibits a maximum for $f = 40.5$ kHz, no differences are
found in the behavior of $G_0(B)/G_0(B=0)$ for different $f$. This is probably because the differences between the simplified and the dynamical model in the neighborhood of the maximum in $G_0$ are only small (see paragraph 4.3, note that the measurements in high $B$ in fig. 4.10 are close to the maximum). For this typical measurement, the differences between the two models, turns out to be smaller than expected from the theoretical considerations in paragraph 4.3.

Finally we shall illustrate that the negative changes shown theoretically in figs. 4.3-6 are not an artifact of eqs. (4.24) but that they were observed experimentally as well.

Figure 4.11 shows $G_0(B)$ and $G_0(B=0)/G_0(B)$ as function of the magnetic field $B$. The values of the other parameters are tabulated. $G_0(B)$ and $G_0(B=0)/G_0(B)$ are calculated from $G_0$ and $C_0$, shown in fig. 4.10, on the basis of the simplified model. For explanation see text.
the electron density is saturated respectively unsaturated. The unsaturated density is achieved by depositing the electrons at an upper plate

![Graph](image1)

**Figure 4.12** G and C as function of the magnetic field B for two different densities, a: saturated, b: unsaturated. The values of the other parameters are tabulated. For explanation see text.

... the voltage \( V_p = -1 \) V and guard voltage \( V_g = -2.5 \) V until saturation occurred, and then raise the voltage to the usual values -10 V and -25 V. The measurements shown in fig. 4.12 are performed immediately after each other, and therefore the conditions are in both cases the same, except for the densities.

If the surface is not saturated, the density is low and as a consequence \( Q \) and \( D_e \), which are proportional to \( n_0 \), are low. Therefore it is seen in fig. 12b that \( C_x \) has a maximum as function of \( B \), while in the saturated case the maximum is absent. In fig. 4.12b a negative change of \( C_x \) is seen as the maximum is passed (the dashed line is \( C_0 \)). Note that the value of \( C_x - C_0 \), for \( B = 0 \) in the saturated case, is a factor of five larger than in the unsaturated case. Figure 4.12 therefore, is a qualitative justification of the dynamical model, and
illustrates the importance of the conditions $Q > 1$ and $D_e > 1$ and the qualitative arguments given at the end of paragraph 4.3.

4.5 Influence of the excitation voltage $V_0$

Figure 4.13 shows the measured $C_x$ and $G_x$ as function of the excitation voltage $V_0$ for $T = 1.38 \, K$, $B = 1.0 \, T$ and $T = 1.79 \, K$, $B = 2.0 \, T$. As seen in fig 4.13 $C_x$ and $G_x$ are independent of $V_0$ for $V_0 < 1. \, V$. Consequently, since the simplified model is valid for this particular measurement, $C_e$ and $G_e$ are independent of $V_0$. This is shown in fig. 4.14. Physically speaking this measurement shows that the Ohmic conductivity is measured if $V_0 < 1.0 \, V$ (this does not depend on the model which is used to analyse the experimental data, since $C_x$ and $G_x$

![Figure 4.13](image)

Figure 4.13 $G_x$ and $C_x$ as function of the excitation voltage $V_0$ for $T = 1.79 \, K$, $B = 2 \, T$ and $T = 1.38 \, K$, $B = 1 \, T$. The values of the other parameters are tabulated. For explanation see text.

are independent of $V_0$ for $V_0 < 1 \, V$). Note that this measurement also proves that the harmonic approximation used in deriving the dynamical model is valid as well, because $G_x$ and $C_x$ are independent of $V_0$, and thus $n_1 \ll n_0$.

During the conductivity measurements, the excitation voltage $V_0$ was
in the range 0.1-0.5 V. Together with the fact that the frequencies used were not too high (fig. 4.8 and eq. (4.36)), this ensures that the data were taken in the Ohmic regime.

**Figure 4.14** \( G_e \) and \( C_e \), as function of the excitation voltage \( V_0 \) for \( T = 1.79 \text{ K}, B = 2 \text{ T} \) and \( T = 1.38 \text{ K}, B = 1 \text{ T} \). The values of the other parameters are tabulated. For explanation see text.
Chapter 5 Results

To determine from the measured $C_x$ and $C_x$ the conductivity $\sigma_{rr}(B,T)$ as function of $B$, only those measurements in which the simplified model is valid, are analyzed. Since in the simplified model $C_x$ and $C_x$ are determined as function of $B$ and thus $\gamma$ and $R_1$ are unknown, it is only possible to measure the relative change of $\sigma_{rr}(B=0)/\sigma_{rr}(B)=C_x(B=0)/C_x(B)$ as function of $B$, for different temperatures $T$. This is only valid if the geometrical factor is independent of $B$. From fig. 4.3-6 it is seen that $\gamma$ depends only on the helium height $h$ and the effective radius $R_1$. $h$ is constant during a experimental run and $R_1$ is assumed to be independent of $B$, and thus $\gamma$ is independent of $B$.

5.1 Determination of the mobility as a function of the temperature $T$: the classical magnetoconductance

From the region where the classical magnetoconductance is observed the mobility $\mu$ can be determined, as follows. By plotting $G_x^{-1}$ as function of $B^2$, it is seen from eq. (2.26), that the result should be (if $\hbar \omega_b < k_B T$) a straight line. In fig. 5.1, a typical measurement of $G_x^{-1}$ as function of $B^2$ is shown. The slope of the line is proportional

![Figure 5.1 The measured inverse $G_x^{-1}$ as function of $B$ squared. For explanation see text](image)
to $\mu^2$. By determining this slope and knowing the value of $\sigma_{rr}(B,T)$ in $B = 0$, the mobility $\mu$ can be calculated [22,35]. This is done by fitting the low magnetic field measurements to a polynomial. In this way the value of $\sigma_{rr}(B,T)$ in $B = 0$, for which the relative error is large, is determined with a smaller uncertainty.

For a particular value of $B$ (for this temperature $B \approx 3.0 \, T$), $G_{e^{-1}}$ deviates from the classical behavior (not shown in fig. 5.1). This will be discussed later in this chapter.

---

**Figure 5.2** The measured $\mu$ as function of the temperature $T$. The dashed line is $\mu$ as function of $T$ on the basis of the theory of Saitoh for $F_{\text{eff}} = 10000 \, \text{V/m}$

In fig. 5.2 the measured values of the mobility $\mu$ are shown as function of the temperature $T$. The dashed curve is calculated from theory of Saitoh [26] for an electrical field $F_{\text{eff}} \approx 10^4 \, \text{V/m}$, which is the sum of the applied electrical holding field $F_n = V_p/L$ and the field of the electrons $F_m = V_p/h$ if the electron density is saturated. In fig. 5.3 the measured mobility is shown in relation with other experiments of
Iye [35], Rybalko et al. [64].

Figure 5.3 The measured $\mu$ as function of the temperature $T$, compared with the results of Iye [35] and Rybalko et al. [64].

Since the lowest temperature which can be achieved in the cryostat setup (see paragraph 3.1) is $T = 1.3$ K, the measurements performed in this setup are in the regime where the vapour atom scattering is dominant. Figure 5.2 and 5.3 show that our measurements are in reasonable agreement with the theoretical calculations of Saitoh and with the mobilities obtained in the literature. The absolute values of some of our measured $\mu$ in relation to the results of Saitoh [26] and Iye [35] are tabulated in table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>$\mu$ (m²/V·s)</th>
<th>$T$ (K)</th>
<th>$\mu$ (m²/V·s)</th>
<th>$T$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present data</td>
<td>$16.1 \pm 2.0$</td>
<td>$1.41$</td>
<td>$6.83 \pm 1.0$</td>
<td>$1.56$</td>
</tr>
<tr>
<td>Iye [35]</td>
<td>$17.0$</td>
<td>$1.35$</td>
<td>$7.50$</td>
<td>$1.55$</td>
</tr>
<tr>
<td>Saitoh [26]</td>
<td>$12.3$</td>
<td>$1.41$</td>
<td>$6.44$</td>
<td>$1.55$</td>
</tr>
</tbody>
</table>

Table 5.1 The measured $\mu$, determined from the low magnetic field.
As seen from fig. 5.2-3, the measured values of $\mu$ deviate more from the literature values and theoretical calculations for higher temperatures. This can not be explained by the experimental uncertainties: the error made in determining the temperature is estimated to be $\pm 0.05$ K. A possible deviation from a saturated density does not explain the differences either, because $\mu$ in the vapour atom regime is almost independent of the electrical field $F_{eff}$ [22]. Another possible error might be caused by the non-linearity of the superconducting coil [22] due to flux trapping in the superconducting wires. However this error is first of all also present in the low temperature region and secondly it is small compared to the error in $C_x$. The non-linearity of the superconducting coil can therefore be left out of consideration. The deviation for higher temperatures is therefore not fully understood, but is probably caused by the small value of $\mu$ for high temperatures. Consequently $C_x$ does not increase much, and thus the relative error in $C_x$ stays relatively large.

5.2 The magnetoconductance in the quantum limit

In fig. 5.4 the relative change $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ is shown as function of the magnetic field $B$. We have indicated the values of $B$ where $\omega_c\tau = 1$ and $\hbar\omega_c = k_B T$ with dashed respectively with solid arrows. As seen from fig. 5.4, $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ exhibits the already discussed classical parabolic behaviour for low magnetic fields. When the magnetic field is increased a deviation from the classical behavior is observed. As discussed in chapter 2, deviation from classical transport can be expected if $\hbar\omega_c > k_B T$. Note that the condition $\omega_c\tau > 1$ must be fulfilled but that it is not a sufficient condition to get deviation from classical transport. This is seen from fig. 5.4 in the region $\omega_c\tau = 1$ to $\hbar\omega_c = k_B T$ where still a classical behavior is observed.

In fig. 5.5, $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ is shown as a function of the magnetic field $B$, calculated from the theory of Ando and Uemura [40] as discussed
Figure 5.4 The measured $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ as function of the magnetic field $B$ for different temperatures as indicated. For explanation see text.

Figure 5.5 The calculated $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ as function of the magnetic field $B$ for different temperatures resulting from the theory of Ando and Uemura. The mobilities $\mu$ are determined from the low magnetic field measurements shown in fig. 5.4. For explanation see text.
in paragraph 2.3. In the calculation of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$, it is assumed that only the lowest Landau level is occupied, i.e. $\hbar \omega_c > k_B T$. The mobility $\mu$, obtained from the low magnetic field measurements, is used. As seen from fig. 2.8 the ripplon interaction can be neglected (in a magnetic field the $\sigma_{rr}$'s have to be added since $\sigma_{rr}$ is proportional to $1/\tau$).

By comparing fig. 5.4 and 5.5, we see that there is a discrepancy by a factor of one and a half between our measurements and the theoretically calculated values. However the dependence of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ on $B$ is approximately linear in both cases.

<table>
<thead>
<tr>
<th>$B = 4 , T$</th>
<th>experiment</th>
<th>Ando et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 1.56 , K$</td>
<td>2.81</td>
<td>3.21</td>
</tr>
<tr>
<td>$T = 1.73 , K$</td>
<td>13.0</td>
<td>11.1</td>
</tr>
<tr>
<td>$T = 1.94 , K$</td>
<td>56.2</td>
<td>46.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B = 2 , T$</th>
<th>experiment</th>
<th>Ando et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 1.56 , K$</td>
<td>3.31</td>
<td>3.26</td>
</tr>
<tr>
<td>$T = 1.73 , K$</td>
<td>18.4</td>
<td>11.2</td>
</tr>
<tr>
<td>$T = 1.94 , K$</td>
<td>100.</td>
<td>49.2</td>
</tr>
</tbody>
</table>

Table 5.2 The measured ratios of $\sigma_{rr}(B=0,T=1.41)/\sigma_{rr}(B,T=1.41)$ to $\sigma_{rr}(B=0,T)/\sigma_{rr}(B,T)$, where $B$ and $T$ are indicated in the table, in relation with the calculations of Ando et al. For explanation see text.

In table 5.2 the ratios of $\sigma_{rr}(B=0,T=1.41)/\sigma_{rr}(B,T)$ for $B = 2 \, T$ and $B = 4 \, T$ are shown, determined from the measurements in fig. 5.4. As seen from table 5.2, the functional behaviour of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ on $T$ is in reasonable agreement with the theory of Ando and Uemura [40]. For higher
temperatures the deviations are larger, which is probably due to the fact that in the calculation only the lowest Landau level was taken into account. As seen from fig. 5.4 and table 5.2, the experimentally observed deviation from the classical relation is smaller than expected from the theory of Ando and Uemura.

The observed factor of one and a half, between our measurements and the theory of Ando and Uemura, is not due to experimental uncertainty in the measurement of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$, as claimed in chapter 4.

The relative error, made in determining $\mu$ from the low field measurements, is roughly 10 - 20%. This should shift the absolute values of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ in the theoretical calculations about 15 - 25% because the $\mu$ dependence in $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ is mainly caused by $\sigma_{rr}(B=0)$. Assume therefore that for example, the mobility $\mu$ and the temperature $T$ could be used as fitting parameters to fit the calculations of Ando and Uemura to our measurements. Then, to fit the $B = 4$ T measurement of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ for $T = 1.41$ K to the theory of Ando and Uemura, we would need values of $T = 1.50$ K and $\mu = 20$ m$^2$/V·s respectively. This is not only outside the range of the uncertainties in the measurements, but also inconsistent with the literature values of $\mu$ and $T$. Therefore $\mu$ and $T$ cannot be used as fitting parameters and the discrepancy between the theory of Ando and Uemura [40] and our measurements cannot be explained by the error made in determining $\mu$ or $T$.

The fact that only the lowest Landau level is taken into account cannot explain the discrepancy either, because this approximation is better for higher fields. We estimate that the relative error which is made in calculating $\sigma_{rr}(B=0)/\sigma_{rr}(B)$, with neglect of the higher Landau levels is of the order $e^{-\hbar\omega_c/k_BT}$, which is $\sim$ 1% for $T = 1.4$ and $B = 4$ T.

Another point of importance is the fact that the used form of the scattering potential, in the calculations of fig. 5.5, was a $\delta$-function. Ando and Uemura [40] also give in this case some equations to correct for a finite range of the scattering potential (in essence one must replace the Landau level width $\Gamma$ by $\Gamma/\sqrt{1+(R/\ell_0)^2}$). However the range $R$ of the vapour atom interaction potential, which is approximately equal to the square root of $A$ (eq. (2.20)), is much smaller than the characteristic length. the quantum mechanical cyclotron radius $\ell_0$ (eq. (2.57)). Corrections due to the finite scattering range of the vapour atom scattering can therefore be left out of consideration.
Since there are no adjustable parameters in the theory of Ando and Uemura, the discrepancy by a factor of one and a half is still unexplained. This is rather remarkable because the two-dimensional electron gas on a liquid helium surface approaches the ideal circumstances in which the theory of Ando and Uemura was developed. Originally, the theory of Ando and Uemura was derived for 2DEG in semiconductor structures (MOSFET's). However, in 1980 K. von Klitzing discovered the Quantum Hall Effect [33]. Before the discovery of the QHE, some measurements of $\sigma_{rr}(B,T)$ in high magnetic fields in semiconductor structures were reported [41], however degenerate phenomena dominated (the Shubnikov-de-Haas oscillations) in these cases. Very few measurements of $\sigma_{rr}(B,T)$ in semiconductor structures were reported under the condition $\nu < 1$, i.e. the lowest Landau level is only partially occupied. In 1982, Tsui et al. [42] performed, in a GaAs-AlGaAs heterojunction, measurements under this condition, and discovered the Fractional Quantum Hall Effect (FQHE). From these two examples it is clear that the theory of Ando and Uemura is much too simple for the 2DEG in semiconductor structures, since both the QHE and the FQHE can not be described on the basis of this theory.

In the case of the 2DEG on a liquid helium surface, there is to our knowledge only one measurement reported in the literature for which the condition $\hbar \omega_c > k_B T$ was fullfilled. This measurement was performed by V.S. Edel'man [52]. Edel'man did some cyclotron resonance experiments in the range where the electron ripplon scattering is dominant, and determined the relaxation time by measuring the cyclotron resonance line width. This experiment is particularly interesting because by measuring the cyclotron line width, the Landau level width is directly measured. However, Edel'man did not analyse his experiments on the basis of the theory of Ando and Uemura [40] (note that the theory of Saitoh [37,38,39] was not published yet) and the temperatures used were low ($T < 0.4$ K). Therefore a comparison with this experiment is difficult.

An interesting (preliminary) physical explanation, of the discrepancy between our measurements and the theory of Ando and Uemura, is suggested by Peeters [65]. The density of states (DOS) derived by Ando and Uemura [40] has a semi-elliptic form (see paragraph 2.3 or fig. 2.6). Peeters showed [65] that the absolute value of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ was almost independent of the particular form of the density of states, i.e. a Gaussian form and a Lorentz form lead almost to the same result.
However, if the DOS has a Gaussian form (or Lorentz, semi-elliptic) superposed on a constant background, the absolute values of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ shifted in the direction of the classical values. Interestingly, such a constant background DOS has been observed in different experiments in 2DEG’s in semiconductor structures [64]. These systems are degenerate and show the Quantum Hall effects, implying that the constant background must consist of localized (i.e. not-conducting) states. In our case however, the constant background states are assumed to be mobile and this explains why our measurements are shifted in the direction of the classical behavior. This constant background is poorly understood at present. Recent theories however [64], propose explanations in terms of some sort of sample inhomogeneities, in particular an inhomogeneous density distribution.

If this preliminary explanation of Peeters is true, this would aid in the understanding of the QHE and two-dimensional electron systems in general, because in that case our measurements show that in a system which differs so much from the electron systems in semiconductors, a similar form of the density of states is found. However the results of Peeters are preliminary and additional work, both experimental and theoretical, has to be done.
Chapter 6 Conclusions and suggestions

With the present new designed experimental setup, it is possible to measure the conductivity of the two-dimensional electron gas on a liquid helium surface.

A new method of analyzing the experimental data is derived which is more general than the method frequently used in the literature and the method used in earlier experiments in our group. By comparing the two methods, it is shown qualitatively that the two methods are identical if certain conditions are fulfilled. These conditions are:

- If \( D_e > 1 \) or \( Q > 1 \) the two methods are identical. The condition \( D_e > 1 \), in the literature frequently used, to characterise the validity of the simplified model is thus justified.
- The measured \( C_e \) must be an increasing function of \( B \). In combination with this condition a frequency check can be performed. If \( C_e \) as function of \( B \), calculated on the basis of the old model, is equal for different \( f \), for the whole range of \( B \), the simplified model is valid.
If these conditions are not fullfilled, the analysis of the experimental data has to be done on basis of the dynamical model.

The experimental observation of the "negative capacitance" is a justification of the dynamical model on a qualitative ground. This negative capacitance is not fully understood but is possibly due to inductive effects of the electrons. A quantitative justification of the dynamical model can be found in the fact that the old simplified model and the new dynamical model are identical for \( D_e > 1 \) or \( Q > 1 \).

From low magnetic field measurements the mobility of the electron gas on a liquid helium surface is measured as a function of the temperature \( T \). The measurements are mainly performed in the vapour atom scattering regime. The mobilities obtained are in reasonable agreement with the values reported in the literature and with the theoretical calculated values.
Conclusions

The relative change of the conductivity $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ as function of the magnetic field is measured for different temperatures. For low magnetic fields, i.e. $\hbar \omega_c \ll k_b T$, the classical parabolic behaviour is seen. If the quantum limit is reached, i.e. $\hbar \omega_c \gg k_b T$ a deviation from the classical behaviour is observed. A comparison with the theory of Ando and Uemura yields a discrepancy by nearly a factor of one and a half, but the predicted functional behavior of $\sigma_{rr}(B=0)/\sigma_{rr}(B)$ on $B$ and $T$, is however approximately confirmed.

The discrepancy can not be explained by the experimental uncertainties, and is still unexplained. An interesting physical explanation is suggested by Peeters.

Measurements in the dilution refrigerator were not yet performed, because problems due to thermomechanical effects were encountered, which had to be solved first.

Some suggestions to proceed this interesting research on the two-dimensional electron gas on a liquid helium surface are

- To verify the suggestion of Peeters, more experimental research has to be performed, both in the range where the vapour interaction is dominant and where the ripplon interaction is dominant.
- To extend the range of the conductivities $\sigma_{rr}$ which can be measured quantitatively, a computer program has to be developed, which can calculate from a given (measured) $C_x$ and $C_z$ the physically interesting parameters $\sigma_{rr}(B,T)$, $h$ and $R_1$. A computer program is certainly necessary if the high magnetic field conductivity is measured in the range where the ripplon interaction is dominant, because in this range the mobility of the electrons is high and consequently $Q$ decreases rapidly. Another important point of notice is the fact that in the computer program the reactive term should be taken into account because future measurements will probably be in the range where Wigner crystallisation occurs.

An interesting experiment is to determine the influence of a magnetic field on the Wigner crystallisation of the two-dimensional electron gas on a liquid helium surface. This is interesting, because in the
literature the theories on this subject are in conflict with each other: some predict a higher melting temperature, others a lower melting temperature [59].
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Appendix Derivation of $C_x$ and $C_\chi$ in the conductance mode

In this appendix, we shall derive the equations for $C_x$ and $C_\chi$ in the conductance mode. We assume that the problem is cylindrical symmetric and use cylinder coordinates. The cell is filled with helium to a height $h$. A homogeneous charged disc of density $n_0$ and radius $R$ is assumed to float above the helium surface. For the present, the guard ring is neglected. To derive the equations for $C_x$ and $C_\chi$, the harmonic approximation is used, i.e. the displaced charge $n_1$, due to the applied electrical field is much smaller than $n_0$.

First of all, the dynamical equation, derived in paragraph 4.2.2 are

\[
\left( \frac{\sigma_0 \xi_{rr} \omega}{\sigma_{rr}^2 + \xi_{rr}^2} \right) + \left( \frac{\sigma_0 \xi_{rr} \omega}{\sigma_{rr}^2 + \xi_{rr}^2} \right) \frac{n_1}{n_0} = -\nabla_e E_0 (r,h)
\]

\[
C \cdot n_1 = A_\chi \Phi_1 (r,h) \quad (A.1)
\]

with $E_0 (r,h) = \frac{\partial \Phi_1 (r,h)}{\partial r}$, and $A_\chi$ and $\nabla_e$ are defined in 4.2.2. Secondly the Poisson equation for the potential $\Phi (r,z,t) = \Phi_0 (r,z) + \Phi_1 (r,z) \cdot e^{i\omega t}$ inside the cell is:

\[
A \Phi (r,z) = A \Phi_0 (r,z) + A \Phi_1 (r,z) \cdot e^{i\omega t} = \frac{en(r,t)}{\varepsilon_0} \delta (z-h) \quad z \geq h \quad (A.2a)
\]

\[
= \frac{en(r,t)}{\varepsilon \varepsilon_0} \delta (z-h) \quad z < h \quad (A.2b)
\]

with $A$ the Laplace operator in cylinder coordinates (cylinder symmetry)

\[
A = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}
\]

Substituting the linearized form of $\Phi (r,z,t)$ and of $n(r,t)$ (eq. 4.9a) gives the dynamical and the static Poisson equation:

\[
A \Phi_0 (r,z) = \frac{en_0}{\varepsilon_0} \delta (z-h) \quad z \geq h \quad (A.3a)
\]

\[
= \frac{en_0}{\varepsilon \varepsilon_0} \delta (z-h) \quad z < h \quad (A.3b)
\]
The static problem is solved in refs. [55].

The boundary conditions (B.C.'s) for the dynamical case are:
the potential on the upper plate is zero

\[ \Phi_1(r, L) = 0 \]  \hspace{1cm} (A.5)

and a harmonic signal on the center

\[ \Phi_1(r, L) = V_0 \quad \text{for } 0 < r < r_0 \]
\[ = 0 \quad \text{for } r_0 < r < R \]  \hspace{1cm} (A.6)

here \( r_0 \) the radius of the center. We have assumed that the spacing between center and ring can be chosen at zero potential. We further assume that the radial current is zero at the edge of the electron disc [49,53,56]. Since in the electron layer, the current is proportional to the applied electrical drift field, this results in the boundary condition

\[ \frac{\partial \Phi_1}{\partial r} \bigg|_{r=R, z=h} = 0 \]  \hspace{1cm} (A.7)

We start the solving of the problem by splitting the cell into two parts, \( z > h \) and \( z < h \). In this way, two Laplace equations result, since there is only charge at \( z = h \), one for the upper part and one for the lower part. For \( z = h \), we use the usual boundary conditions [61]

\[ \varepsilon_0 \frac{\partial \Phi_1}{\partial z} \bigg|_{z=h} - \varepsilon_0 \frac{\partial \Phi_1}{\partial z} \bigg|_{z=h+} = e n_1 \]  \hspace{1cm} (A.8)

\[ \frac{\partial \Phi_1}{\partial r} \text{ and } \Phi_1 \text{ continuous in } z = h \]  \hspace{1cm} (A.9)

The general solution of the Laplace equation of the upper part, which
fits the boundary condition (A.5) is

$$\Phi_1(r,z) = \sum_q S_0 \cdot j_0(qr) \cdot \sinh(q(z-L))$$  \hspace{1cm} (A.10)

where $j_0(x)$ is the Bessel function of order zero (only $j_0(x)$ appears in (A.10) because the problem is cylindrical symmetric). $A_q$ and $q$ are to be determined from the B.C. (A.8-9) and the dynamical equation (A.1).

Using eq. (A.1) and (A.11), the following relation for $\Phi_1(r,z)$ for $z < h$ is easily found

$$\epsilon \cdot \epsilon_0 \cdot \frac{\partial \Phi_1}{\partial z} \bigg|_{z=h} = \epsilon_0 \cdot \sum_q q \cdot S_q \cdot j_0(qr) \cdot \cosh(q(z-L)) \bigg|_{z=h^+}$$

$$- \frac{\epsilon}{C} \sum_q q^2 \cdot S_q \cdot j_0(qr) \cdot \sinh(q(z-L)) \bigg|_{z=h}$$ \hspace{1cm} (A.11)

By integrating eq. (A.11) and keeping in mind that terms of the form

$$\sum_q B_q \cdot j_0(qr) \cdot \sinh(q(z-h))$$ \hspace{1cm} (A.12)

are a solution of the Laplace equation as well and can be added to eq. (A.11) without any consequence, the following form of $\Phi_1(r,z)$ for $z < h$ is obtained:

$$\Phi_1(r,z) = \sum_q S_q \cdot j_0(qr) \cdot \left[ \sinh(q(z-L)) - \frac{q}{\epsilon \cdot \epsilon_0 \cdot C} \cosh(q(z-L)) + D_q \cdot \cosh(q(z-h)) \right]$$ \hspace{1cm} (A.13)

We now fit eq.(A.13) and (A.11) to the B.C.'s. First of all, since

$$\frac{\partial j_0}{\partial x} = j_1(x)$$ \hspace{1cm} (A.14)

it follows from B.C (A.7),

$$q = \frac{V_n}{R}$$ \hspace{1cm} (A.15)

with $y_n$ the $n^{th}$ zero of $j_1(x)$. From the B.C.'s (A.9) $D_q$ can be obtained
\[ D_q = \left( \frac{-1}{\epsilon} \right) \cdot \sinh(q(h-L)) + \frac{q}{\epsilon \cdot \varepsilon_0 \cdot C} \cdot \cosh(q(h-L)) \]  

(A.16)

which immediately fits both B.C.'s.

All the unknown coefficient are now known, except for \( A_q \). \( A_q \) is determined by fitting (A.13) to the last B.C. (A.6).

\[ V_0 \cdot \Theta(r-r_0) = \sum_S q \cdot \mathcal{J}_0(qr) \cdot \left[ \sinh(q(z-L)) - \frac{q}{\epsilon \cdot \varepsilon_0 \cdot C} \cdot \cosh(q(z-L)) + D_q \cdot \cosh(q(z-h)) \right] \]  

(A.17)

where \( \Theta(r-r_0) \) is the unit step function. Equation (A.17) is a series of the type

\[ f(x) = \sum_{n} P_n \cdot \mathcal{J}_0(y_n x) \quad 0 \leq x \leq 1 \]  

(A.18)

If certain conditions are fulfilled, the coefficients \( P_n \) are easily calculated by using the standard textbook methods (see for example ref.[61])

\[ P_n = \frac{2}{\mathcal{J}_0^2(y_n)} \int_{0}^{1} x \cdot \mathcal{J}_0(y_n x) \cdot dx \]  

(A.19)

Applying eqs. (A.18-19) to our case, after some algebra, we obtain for \( S_q \)

\[ S_q = \frac{2V_0 \cdot \frac{1}{r_0} \cdot \mathcal{J}_1(qr_0)}{\mathcal{J}_0^2(y_n) \cdot qR \cdot R} \cdot \left[ -\frac{\sinh(qL)}{\epsilon} - \frac{q}{\epsilon \cdot \varepsilon_0 \cdot C} \cdot \cosh(qL) + D_q \cdot \cosh(qh) \right] \]  

(A.20)

Since the time dependent potential inside the cylinder is now fully determined through eqs. (A.10, 13, 14, 15, 16 and 20), \( C_x \) and \( C_y \) can be calculated by
where \( I \) is the complex induced current on the ring, and we have used the fact that the c-c-bridge is in the conductance mode. After some algebra, using the expression for \( C \) eq. (A.1), the following relations for \( C_x \) and \( C_y \) are determined

\[
C_x = C_x^0 + n_0 e^2 \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{K_q - \left( \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} \right)}{K_q - \left( \frac{\omega n_0 e^2 \xi_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} \right) + \frac{\omega n_0 e^2 \sigma_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2}} \right]
\]

(A.22a)

\[
C_y = n_0 e^2 \sum_q A_q \frac{\tanh(q(L-h))}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)} \left[ \frac{\omega^2 n_0 e^2 \sigma_{rr}}{\sigma_{rr}^2 + \xi_{rr}^2} \right]
\]

(A.22b)

where \( A_q \) an \( K_q \) are given by

\[
A_q = 4\pi \varepsilon \frac{R}{\varepsilon R} \frac{\frac{\partial \phi_1(qR)}{\partial q}}{\phi_0(qR)} \frac{\frac{\partial \phi_1(qR)}{\partial q}}{\phi_0(qR)}
\]

(A.23)

\[
K_q = \frac{n_0 e^2 q}{\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)}
\]

(A.24)

and \( C_x^0 \), the direct coupling between center and ring (calculated by setting \( n_0 = 0 \) in eq. (A.1)) is given by

\[
C_x^0 = 2\pi \varepsilon \varepsilon_0 \sum_q \frac{R}{\varepsilon R} \frac{\frac{\partial \phi_1(qR)}{\partial q}}{\phi_0(qR)} \frac{\frac{\partial \phi_1(qR)}{\partial q}}{\phi_0(qR)} \left[ \varepsilon \cdot N_q + 1/[\varepsilon \cdot \tanh(q(L-h)) + \tanh(qh)] \right]
\]

(A.25)

Note that eqs. (A.22-25) are symmetric in \( r_0 \) and \( r_0+d \), in other words it makes no difference, whether the harmonic signal is applied to the
center or to the ring. $C_0^h$ in eq. (A.25) is the theoretical formulae for the direct coupling. During experiments, one always deals with stray capacitances. Therefore we replace $C_0^h$, by the measured direct coupling, $C_0$ and add a measured conductance $G_0$ to $C_x$. Finally $C_x$ and $C_x$ have the form

$$C_x = C_0 + n\sigma^2 \sum_q A_q \frac{\tanh(q(L-h))}{\epsilon \tanh(q(L-h)) + \tanh(qh)}$$

$$G_x = G_0 + n\sigma^2 \sum_q A_q \frac{\tanh(q(L-h))}{\epsilon \tanh(q(L-h)) + \tanh(qh)}$$

Note that the expression for $C_0$ also can be used to determine the helium height.
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