MASTER

Swarm parameters of C3F6

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Swarm parameters of C\textsubscript{3}F\textsubscript{6}

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This thesis reports on the work I've been doing in the High Voltage Group of the Eindhoven University of Technology in order to get my master's degree. The work has been done in a period from October 1985 until August 1986. I would like to thank Dr. Ir. J.M. Wetzer for the many discussions and the coaching during this period. I also would like to thank M.Sc. Wen Chuan for his interest in my work and A. Aldenhoven for the technical assistance. I'm very grateful to Helga Pouwels for typing this report and taking care of the lay-out.
## CONTENTS

### SUMMARY

### NOMENCLATURE

1. **INTRODUCTION.** 1
2. **BRIEF INTRODUCTION TO AVALANCHE THEORY AND THE EXPERIMENT.** 3
   1. **General theory on avalanches in gases.** 3
   2. **Theory of the experiment.** 5
   3. **Experimental setup.** 7
3. **DETERMINATION OF THE TIME-RESOLUTION OF THE AVALANCHE MEASURING SETUP BASED ON MEASUREMENTS IN VACUUM.** 10
   1. **Introduction** 10
   2. **Theory** 11
   3. **Numerical Fourier analysis** 14
   4. **Results** 14
   5. **Conclusions and recommendations** 20
4. **SCALING LAWS IN HEXAFLUOROPROPYLENE (C3F6)** 21
   1. **Introduction: scaling laws in insulating gases** 21
   2. **Scaling laws in hexafluoropropylene (literature survey)** 24
   3. **Analysis of C3F6/N2 measurements: theory** 28
   4. **Analysis of C3F6/N2 measurements: computer simulation** 35
   5. **Analysis of C3F6/N2 measurements: results** 39
   6. **Conclusions and recommendations** 52
5. **LITERATURE** 54
6. APPENDIX

I  Calculation of $\delta$ and $\beta$ from the electron current approximation with one exponential 56

II  Computer program used for the calculation of swarm parameters in mixtures $C_3F_6/N_2$ 58

III  Calculated swarm parameters for mixtures $C_3F_6/N_2$. 64
SUMMARY

In this thesis a practical method is discussed to determine the time response of the avalanche measuring set-up. Measurement of the current caused by a swarm of electrons crossing the gap in vacuum gives information on the impulse response of the measuring circuit. From the impulse response the risetime can be calculated. Secondly avalanche measurements in mixtures of C\textsubscript{3}F\textsubscript{6} and N\textsubscript{2} are analyzed for low partial C\textsubscript{3}F\textsubscript{6} pressures up to 12 torr. Values of the swarm parameters for attachment ($\eta$), detachment ($\delta$) and conversion ($\beta$) are calculated by using additional data for the electron drift velocity and the ionization coefficient from pure nitrogen. The pressure and field dependence of the electron drift velocity and the calculated swarm parameters $\eta$, $\delta$ and $\beta$ is discussed and compared to theoretical predictions. From this comparison it follows that the attachment process depends on the partial C\textsubscript{3}F\textsubscript{6} pressure. Detachment depends on the total gas pressure and further depends on the electric field, showing a well defined threshold. For conversion a dependency on the partial C\textsubscript{3}F\textsubscript{6} pressure and the electric field is found.
NOMENCLATURE

\( \alpha \) ionization coefficient \([1/cm]\)
\( \eta \) attachment coefficient \([1/cm]\)
\( \delta \) detachment coefficient \([1/cm]\)
\( \beta \) conversion coefficient \([1/cm]\)
\( v_e \) electron drift velocity \([cm/sec]\)
\( n \) number of electrons
\( N_{nu} \) number of unstable negative ions
\( N_p \) number of positive ions
\( N_{ns} \) number of stable negative ions
\( T_e \) electron transit time \([sec]\)
\( d \) gap distance \([cm]\)
\( p \) total pressure \([torr]\)
\( P_A \) partial pressure of compound A \([torr]\)
\( E \) electric field \([V/cm]\)
\( v_p \) positive ion drift velocity \([cm/sec]\)
\( v_n \) negative ion drift velocity \([cm/sec]\)
\( m \) electron mass \([kg]\)
\( q \) elementary charge \([c]\)
\( N_o, n_o \) number of electrons at \( t=0 \)
\( \lambda \) mean free path \([m]\)
\( n \) density \([1/m^3]\)
\( e \) mean energy \([eV]\)
\( T \) gas temperature \([K]\)
\( k \) Boltzmann's constant
\( Q_A \) cross-section for ionization of compound A \([m^2]\)
\( F_{pro} \) rate constant for particles p and process pro \([1/sec]\)
1. INTRODUCTION

In the high-voltage group of this university research is carried out on avalanches in insulating gases. With the experimental set-up experiments can be performed for several gases (for example air, SF₆, C₃F₆) at different pressures and different electric field-strengths. The most important feature of the experimental set-up is the high time-resolution achieved by:

1. careful layout of the electrode geometry
2. short pulsewidth laser
3. fast signal storage equipment

The experimental results can be analyzed with a simulation program in order to derive the so-called swarm-parameters that determine the dielectric strength of the gases investigated.

The goals of this research are the following:

- Determination of the swarm-parameters for several insulating gases as a function of pressure and electric field. Important are measurements with a high time-resolution to be able to observe fast processes that occur in the gas.
- Investigation of relationships between swarm-parameters and pressure and electric field in order to derive scaling-laws for insulating gases.
- Investigation of the transition from Townsend-breakdown to streamer-breakdown.

Determination of swarm-parameters will give insight in the important processes that govern the breakdown mechanism in a gas. The research on scaling laws can be helpful in determining the influence of pressure and electric field on the processes that will lead to breakdown. They can be used to predict the insulation behaviour of a gas at higher pressures (> 1 atm.) where it is more difficult to measure the pre-breakdown current.

Knowledge of the important processes will make it possible to find ways of improving the dielectric strength of a gas for example by mixing it with other gases.
The work I have been doing in order to get my masters degree concerns scaling laws in \( \text{C}_3\text{F}_6 \). The aim is to find the four swarm-parameters for this gas as a function of \( E/p \) and to interpret these results in order to formulate scaling laws for this gas. This work is described in the second part of this report. The first part describes an experimental method to obtain the time-resolution of the set-up. This method can be used to check the time-resolution of the set-up whenever changes have been made.
2. BRIEF INTRODUCTION TO AVALANCHES IN GASES

2.1. General theory on avalanches in gases

Free electrons in a gas of pressure $p$ which is subjected to a homogeneous electric field $E$ will drift in the opposite direction of the field. During this motion they will gain energy from the field and lose energy because of collisions with gas molecules. In this way they will travel through the gas with an average velocity $v_e$. This drift velocity is usually a function of $E/p$ and typical for a certain gas.

\[ v_e \]

Fig. 2.1. The path of free electrons through the gas

A collision of an electron with a gas molecule can lead to the following processes [1]:

- elastic collision; both the electron and the gas-molecule will remain unchanged. Only momentum, but no energy is transferred.

- ionisation ($\alpha$); the electron ionizes the gas molecule. The result will be two electrons and a positive ion. This will only take place if the electron has sufficient energy. $\alpha$ is defined as the mean number of ionizing collisions of one electron travelling 1 cm in the direction of the field.

- attachment ($\eta$); the electron attaches to the gas-molecule. The result will be a negative ion. $\eta$ is defined as the mean number of attachments produced by a single electron travelling 1 cm in the direction of the field.
An electron with sufficient energy can ionize a gas-molecule. If this process is repeated several times the number of free electrons will increase exponentially and an avalanche of electrons in the gas is formed. Mathematically the avalanche can be described by [1] p. 22:

\[ n_e = n_0 \cdot e^{\alpha \cdot v_e \cdot t} \] (2.1)

The avalanche will consist of a swarm of electrons travelling with velocity \( v_e \) and a region with positive and negative ions travelling at much lower speed \( (v_p, v_n \approx \frac{1}{1000} \cdot v_e) \).

\[ \text{region with pos. and neg. ions} \]

**Fig. 2.2. An avalanche in a gas with ionization and attachment**

Since the electron drift velocity is much larger than the ion drift velocities the electrons will leave the ions behind. In some gases additional processes between ions and gas-molecules occur. If the negative ion is unstable it can release its electron again. This will lead to:

- detachment (\( \delta \)): an unstable negative ion releases an electron. This can happen by collision with another molecule or spontaneously after a certain time \( \tau \). \( \delta \) is defined as the mean number of detachments per unstable negative ion in a time an electron travels 1 cm in the direction of the field.
Another possibility is that the negative ion will stabilize upon collision with another molecule. This process is called:

- conversion ($\beta$): an unstable negative ion is converted to a stable one. $\beta$ is defined as the mean number of conversions per unstable negative ion in a time an electron travels 1 cm in the direction of the field.

If we take into account the above mentioned processes to happen in an avalanche, and leave out diffusion of the electron swarm and neglect the velocity of the ions (by assuming that $v_i \ll v_e$), then the avalanche can be described by the following set of differential equations [1] pp 38-39

\[
\begin{align*}
\frac{dn_e(t)}{dt} &= (\alpha - \eta) \cdot n_e(t) \cdot v_e dt + \delta \cdot n_{nu}(t) \cdot v_e dt \\
\frac{dn_p(t)}{dt} &= \alpha \cdot n_e(t) \cdot v_e dt \\
\frac{dn_{nu}(t)}{dt} &= \eta \cdot n_e(t) \cdot v_e dt - \delta \cdot n_{nu}(t) \cdot v_e dt - \beta \cdot n_{nu}(t) \cdot v_e dt \\
\frac{dn_{ns}(t)}{dt} &= \beta \cdot n_{nu}(t) \cdot v_e dt
\end{align*}
\]

This set of differential equations has been solved by Verhaar [1] who gives an analytical solution for times smaller than the electron transit time $T_e$ ($t \leq T_e$). Takkenberg [2] has developed a simulation program to solve these equations numerically with the computer. Recently Wen [3] has derived an analytical solution that is also valid for times greater than the electron transit time ($t > T_e$).

### 2.2. Theory of the experiment

In the experiment a homogeneous E-field is applied between two electrodes. A very short light-pulse will release $n_0$ electrons almost instantaneously from the cathode. Suppose that none of the previous mentioned processes will happen then the number of electrons will
remain constant and they will travel to the anode with a drift velocity \( v_e \) (see figure 2.3).

![Diagram of electrons moving through the gap at time t](image)

**Fig. 2.3. Electrons moving through the gap at time t (from [1] p. 14)**

At \( t = T_e \) the electrons reach the anode and will disappear from the gap. When the electrons are moving through the gap they will cause a current \( i_e \) in the external circuit. This current can be calculated by considering the energy that is needed when \( n_o \) electrons travel a distance \( dx \) in the direction of the electric field in a time \( dt \). This energy equals \( n_o \cdot q \cdot E \cdot dx \) and is supplied by the external circuit:

\[
U \cdot i_e(t) \, dt = n_o \cdot q \cdot E \, dx
\]  

(2.6)

\[
i_e(t) = n_o \cdot q \cdot \frac{E}{U} \, dx = n_o \cdot q \cdot v_e \cdot \frac{E}{U}
\]  

(2.7)

If the field \( E \) is homogeneous and the voltage \( U \) is constant then \( v_e \) will also be constant and equation 2.7 can be written as

\[
i_e(t) = n_o \cdot q \cdot \frac{v_e}{e} = n_o \cdot \frac{q}{T_e}
\]  

(2.8)

In figure 2.4 one can see the current that will flow in the external circuit when the electrons are in the gap. It can be interpreted as the movement of image charges in the external circuit that will flow to the electrodes because of the swarm of electrons in the gap. The ions in the gap will also cause a current. Because of the lower drift velocity the ion-component in the current will have a lower amplitude and a longer duration than the electron component.
Fig. 2.4. The current in the external circuit

a. no $\alpha, \eta, \delta, \beta$

b. only $\alpha, \eta : \alpha \eta$

c. only $\alpha, \eta : \alpha \eta$

d. $\alpha, \eta, \delta$ and $\beta$

2.3. Experimental set-up

The experiments are carried out in a small vessel (volume ~ 15 l, Fig. 2.5), with a pressure range from $10^{-5}$ - 1140 torr. The top of the vessel is used as cathode. It is constructed with a central measuring electrode (diameter = 6 cm) surrounded by an outer ring at earth-potential. A separated cathode is important for a good high frequency response [1], p. 48. The transition from measuring electrode to the measuring cable is designed to match the impedance of the cable in order to avoid reflections [4].

Below the cathode the anode with a Bruce-profile is situated. This electrode configuration creates an almost homogeneous electric field and prevents field enhancement at the electrode boundary. The distance between anode and cathode can be varied from 0 - 3 cm but should be less than 1.5 cm to avoid the Ramo-Shockley effect [1], p. 54. This effect describes the distortion of the current when field lines from or to the swarm end on the guard ring rather than on the measuring electrode. The high voltage is applied to the anode through a 20 M$\Omega$-damping resistor. The maximum voltage that can be applied is 40 kV.

For this purpose a Wallis, type R 603/05 P high voltage source is used. The electrons are released from the cathode by means of a short light-pulse from the laser. This laser was built in the high voltage group [15] because of the demand for a very-short-pulsewidth laser. It has a pulsewidth of 0.6 nsec and a power in the order of 1 MW at an operating voltage of 30 kV [1], p. 56. The laser will also trigger the
Fig. 2.5. Vessel used for avalanche measurements
digitizer (Tektronix 7912 AD (0-1 GHz bandwidth) or the oscilloscope (Tektronix 7844 with amplifier unit 7A19 (0-400 MHz)) that will record the signal. In figure 2.6 the electrodes together with the external circuit can be seen in a diagram. R1 and R2 make up the resistive divider \((R1 = 2 \, \Omega, \, R2 = 200 \, k\Omega)\) from which the applied voltage can be read.

---

Fig. 2.6. Diagram of the experimental set-up (from [1] p 57)
3. DETERMINATION OF THE TIME-RESOLUTION OF THE AVALANCHE MEASURING SET-UP BASED ON MEASUREMENTS IN VACUUM

3.1. Introduction

The measuring set-up as described in chapter 2.3 is designed to do very fast measurements. It is important to know the time resolution of this system in order to get an indication which processes can be measured and which processes are too fast to be measured. The experimental set-up is fast enough to measure the current caused by a swarm of electrons that crosses the gap in vacuum (total time duration of the signal typically 4 nsec at a gapwidth of 10 mm and an applied voltage of 12 kV). Since this signal is very fast it will meet the limits of the time-resolution of the set-up. This will distort the signal and from this distortion one can get information about the time-resolution. In figure 3.1 a diagram of the experimental set-up is given. For this purpose it is split-up in two parts: the laser and the measuring section (cathode, measuring cable, digitizer).

Fig. 3.1. Diagram of the experimental set-up

Three effects determine the waveshape of the measured signal: the travelling of electrons through the gap, the distortion caused by the laser because of the finite pulse width (not all electrons are
released at the same moment) and the distortion caused by the measuring section. By analyzing the measured signal using Fourier-analysis these three components can be separated. Of special interest is the component caused by the measuring section. This component represents the impulse response and integration will give the risetime of the measuring section alone. Also of interest is the combination of laser and measuring section since this is the total distortion of the whole experimental set-up. The obtained values of the risetime can be compared with other experimental values measured in nitrogen [4] p 18 or with values calculated with a computer program [5] p. 24.

3.2. Theory

An electron that crosses the gap in a homogeneous E-field in vacuum will be accelerated by the electrical force working upon it.

\[ F = q E \] (3.1)

For the distance travelled as a function of time one can write:

\[ S = \frac{1}{2} \frac{q}{m} E t^2 + v_0 t \] (3.2)

Assuming that the initial velocity \( v_0 \) can be neglected in relation to the velocity an electron acquires during its motion in the electric field, (3.2) can be simplified to:

\[ S = \frac{1}{2} \frac{q}{m} E t^2 \] (3.3)

From (3.3) the electron transit time \( T_e \) can be derived:

\[ T_e = \sqrt{\frac{2 m d}{q E}} \] (3.4)
In which \( d \) represents the gap-width. The electron velocity can be written as:

\[
v_e = \frac{ds}{dt} = \frac{qE}{m} t
\]  

(3.5)

The current \( i \) that is measured by the measuring system whenever a swarm of electrons with charge \( Q = N_0 \cdot q \) crosses the gap is given by (2.8):

\[
i = \frac{Q v_e}{d}
\]  

(3.6)

Substituting (3.5) in (3.6) gives:

\[
i = \frac{Q qE}{m d} t = \frac{N_0 q^2 E}{m d} t
\]  

(3.7)

Formula (3.7) is correct if all electrons are released at the same moment. If the number of electrons released becomes a function of time (as is the case due to the finite pulsewidth of the laser) then the measured current \( i \) will be a superposition of the currents caused by the different electrons distributed in time. In fact this signal will be proportional to the convolution of the electron velocity and the distribution of released electrons.

\[
f(t) = \frac{q}{d} \int_0^t n(\tau) v_e(t-\tau) d\tau = \frac{q}{d} (n(t) \ast v_e(t))
\]  

(3.8)

The electron velocity and the distribution of released electrons as a function of time are presented in figure 3.2. The first one is calculated using (3.5), the latter is measured earlier [6] and represents the intensity of the laser-pulse, averaged over a number of pulses.
The convolution (t) is the input signal for the measuring system. It can be seen as a Dirac-function that excites the measuring section which is characterized by its impulse-response h(t). For the measured signal i(t) one can write:

\[ i(t) = \int_0^t f(\tau)h(t-\tau)d\tau = f(t) \ast h(t) = \frac{d}{dt}(n(t) \ast v_e(t) \ast h(t)) \]

\[(3.9)\]

Fig. 3.3. The measuring system as blackbox

The impulse-response h(t) can be calculated by deconvolution of i(t) with f(t). Integrating h(t) gives the step response u(t):

\[ u(t) = \int_{-\infty}^{\infty} h(t)dt \]

\[(3.10)\]

From the step response the risetime can be determined by taking the time it takes for the step response to rise from 10% to 90% of its maximum value. For our purpose we are interested in the risetime of the whole setup with the laser included. Therefore we need the impulse response of the whole setup which can be obtained by deconvolution of the measured signal with the electron velocity only.
3.3. Numerical Fourier-analysis

The analysis of the measured signal has been done with the Burroughs computer. The Fourier-analysis can be performed using fast-fourier-transform algorithms which are available in the computer-library. A convolution in the time-domain can be replaced by a multiplication in the frequency-domain. Therefore the signals need to be analyzed with Fourier-analysis, multiplied or divided in the frequency-domain and synthesized with Fourier synthesis. Because fast-fourier-analysis has been used a Fourier sum will be calculated instead of a Fourier integral. This means that the signals are assumed to be periodical on a certain time-domain given by $[0:2\pi]$. A good convergence of the Fourier-sum can be obtained if the signals fall to zero fast enough on this domain. The time-domain which is $T$ seconds long ($T$ seconds corresponds to $2\pi$) will be split-up in frequency components with frequencies:

$$f = k \frac{1}{T} \quad k = 0,1,2,3,... \quad (3.11)$$

The lowest frequency ($k=1$) that is used to describe the signal $f(t)$ is determined by the choice of $T$. Because the frequency components that are used depend on the choice of $T$, the discrete frequency-spectrum will depend on $T$. In reality the frequency-spectrum is continuous because the signal is not periodical. The numerical treatment of the signal as being periodical gives a discrete spectrum. The envelope of this spectrum gives an indication of the continuous frequency spectrum from which the bandwidth of the system can be deduced. The risetime follows from the stepresponse after Fourier-synthesis of the analyzed signal.

3.4. Results

With the experimental set-up (Fig. 3.1.) measurements were done in vacuum. The vacuum has a pressure of $4 \cdot 10^{-5}$ torr. This is enough for our purpose since the mean free path for electrons in nitrogen is given by [7] p. 306, 307:
\[ \lambda(p, T) = 5.66 \cdot \lambda_0 \frac{P_0}{p} \frac{T}{T_0} = 5.66 \cdot 6.28 \cdot 10^{-8} \frac{760}{4 \cdot 10^{-5}} = 6.7 \text{ m} \] (3.12)

In this formula \( \lambda_0 \) stands for the mean free path for molecules in \( N_2 \) at a pressure \( P_0 \) of 760 torr and a temperature \( T_0 \) of 288 K. 5.66 is a correction factor for electrons. The mean free path is much longer than the distance between the electrodes (10 mm) and therefore the electrons will not collide during their motion through the gap.

In figure 3.4b one can see clearly that for a low E-field the shape of the measured signal is determined by the linearly increasing electron velocity. At high E-fields this component cannot be distinguished anymore and the signal will be determined by the laserpulse and the impulseresponse of the system. The two signals have been digitized by hand and were fed to the computer. In Fig. 3.5, one can see the analysis for the 12 kV measurement. The risetime of the measuring section is equal to \( t = 1.2 \text{ nsec} \). This agrees well with computer calculations done by F. Schoren [5] p.24. If the laserpulse is considered as a part of the measuring system one gets a risetime of 1.6 nsec. (Fig. 6). This value is in good agreement with measurements done by M. Takkenberg [4] p. 18 who finds a value of \( 1.6 \pm 0.5 \text{ nsec} \). In Fig. 3.6 the timescale is expanded compared to Fig. 3.5. Also a frequency picture is included. From this picture the bandwidth of the system is estimated to be 220 MHz. (-3 dB).
In Fig. 3.5, the stepresponse is calculated for the measuring section with the laser excluded. In this way one only gets the risetime for the measuring section. In Fig. 3.6 the stepresponse is calculated for the whole set-up. The effects of the laserpulse on the timeresponse are now included in the stepresponse. Therefore this analysis is the most useful since it gives the total timeresponse which is important in other experiments. The analysis in Fig. 3.5 shows that the measuring system contributes the most to the total timeresponse. Analysis of the 30 Volt measurement gives a larger risetime of 2.6 nsec. (Fig. 3.7). The reason for this is not clear but since the waveshape of the two measurements differ so much from each other it is possible that at low voltages other processes become more pronounced. Such a process can be electrostatic repulsion of the electronwarm. Because the electrons remain in the gap for a relatively long time compared to the measurements at higher voltages (10-20 kV) electrostatic repulsion may cause the swarm to increase in size and influence the current measurement, especially at the tail. This effect is not taken into account in the analysis and thus it will appear in the impulseresponse as an effect of the measuring system causing the longer risetime.
Fig. 3.5. Analysis of 12 kV measurement with laser excluded. Electron velocity (a), laser pulse (b), convolution of ((a)*b)) (c), measured signal (d), impulse response of the measuring section (e) and (g), step response (f) and (h).
Fig. 3.6. Analysis of 12 kV measurement with laser included. Electron velocity (a), measured signal (b), impulse response (c) and (e), step response (d) and (f), frequency spectrum (g).
Fig. 3.7. Analysis of 30 V measurement with laser included. Electron velocity (a), measured signal (b), impulse response (c), (e) and (g), step response (d), (f) and (h).
3.5. Conclusions and recommendations

1. Measurements in vacuum are a useful tool in determining the timeresolution of the system.

2. Fourier-analysis of these measurements makes it possible to calculate the impulseresponse and the stepresponse for the total system and for the measuring section alone.

3. According to this analysis the risetime of the experimental set-up is equal to \( t = 1.6 \pm 0.1 \text{ nsec} \).

4. The risetime of the set-up with the laser excluded is equal to \( 1.2 \text{ nsec} \).

5. As soon as the digitizer will be connected to the computer it will become very easy to analyise these signals. In this way a permanent check on the timeresolution of the system can be obtained.

6. If a high electric field is applied (\( E \sim 20 \text{ kV/cm} \)) then the inputsignal will be almost a Dirac-function. In that case a simple integration of the measurement will give a good estimation of the risetime.

7. Since Fast-Fourier-software is available for the personal computer it will be handy to do the Fourier-analysis on the P.C. instead of the main frame computer.
4. SCALING LAWS IN HEXAFLUOROPROPYLENE (C₃F₆).

4.1. Introduction: scaling laws in insulating gases.

Scaling laws in insulating gases are based upon the fact that for similar discharges a number of parameters typical for the discharge is invariant. Similar discharges are discharges of which all linear dimensions differ by a constant factor (see Fig. 4.1).

\[ d_1 = ad_2 \]
\[ r_1 = ar_2 \]
\[ R_1 = aR_2 \]

Fig. 4.1. Similar discharges (from [8] p. 60)

For such discharges it is necessary that the voltage \( V \) is kept constant and the pressure \( p \) varies according to \( p_1 = \frac{p_2}{a} \).

Parameters that are invariant are for example:

\[
\begin{align*}
E/p \\
v_e \\
a/p
\end{align*}
\]

(4.1)

The invariancy of these parameters can be derived for steady state discharges leading to scaling laws as \( v_e = f(E/p) \) and \( a/p = f(E/p) \). Francis [8] p. 59-80 gives a derivation of the invariant parameters and makes a distinction between physical processes that do and physical processes that do not upset scaling laws. Processes that do not upset scaling laws are for example ionization, electron attachment, and electron detachment due to collisions of negative ions with other gas-molecules.
Another approach to check the validity of a scaling law is to look at the streamer-criterion for breakdown [7] p. 346-354. This approach is more suitable for our purposes because an avalanche is a transient phenomenon which is described by a change in the number of electrons (not the density as in the derivation for steady state discharges). The development of a streamer is also a transient phenomenon where the number of electrons in the head of the avalanche is important. If this number exceeds a critical number \( n_c \sim 10^8 \) a streamer may be initiated and breakdown will follow.

\[ n = n_0 e^{(\alpha - \eta)x} \approx 10^8 \]  \hspace{1cm} (4.2)
Fig. 4.3. Measurements in air (a), N₂/C₃F₆ (b) and N₂ (c).

a. Air 1 atm., \( p(H₂O) = 1.8 \text{ torr} \), 27 kV/cm, 20 nsec/div
b. Nitrogen 1 atm., \( p(C₃F₆) = 1.5 \text{ torr} \), 25 kV/cm, 20 nsec/div
c. Nitrogen 1 atm., 25.6 kV/cm, 20 nsec/div.

Where \( x_c \) is the critical distance. When the number of electrons in the avalanche head exceeds \( 10^8 \), the field distortion gets in the order of the applied field and may cause photoemission. This produces ionization elsewhere in the gap which will eventually lead to a conductive path and breakdown. If the electrode separation is large compared to \( x_c \) (which means that the avalanche can develop over a long path) exponential growth will lead to breakdown if

\[
\alpha - \eta > 0 \tag{4.3a}
\]

A plot of \( \frac{\alpha - \eta}{p} \) as a function of \( E/p \) will give the critical field \( E_c/p \) where \( \frac{\alpha - \eta}{p} = 0 \). This critical \( E \)-field will predict a correct value of the breakdown voltage for a homogeneous field.

When the electrode separation is not too small the general expression for \( \alpha - \eta \) is:

\[
\alpha - \eta > \frac{1}{d} (\ln 10^8 - \ln n_0) \tag{4.3b}
\]
From this expression it follows that for breakdown starting from one electron \((n_0 = 1)\) the value of the effective ionization coefficient should be: 

\[ \alpha - \eta > \frac{18}{d} \]

which results in (4.3.a) if \(d\) is large.

If one considers a steady state discharge one cannot make a distinction between ionization and detachment. Both processes will increase the number of free electrons in the discharge and one will find a higher value of \(\alpha\) (because detachment is included in this parameter). At a different pressure the breakdown criterion \(\frac{\alpha - \eta}{p} = 0\) will be met at a different value of \(E/p\) not corresponding to the actual breakdown voltage. This phenomenon has been described by Verhaart and Van der Laan [9] and is an example of the difficulty that arises if one tries to formulate a scaling law for a gas where detachment and conversion take place. In the first instance the correct values for \(\frac{\alpha - \eta}{p}\) should be used for scaling which means that a distinction should be made between the physical processes that will allow scaling and the physical processes that will upset scaling of parameters of insulating gases. As a secondary effect detached electrons may modify the electric field near the avalanche head and thereby still have some effect on the breakdown field.

4.2. Scaling laws in hexafluoropropylene (literature survey)

Hexafluoropropylene (\(C_3F_6\)) is a colourless, non-flammable, moderately toxic gas. In the literature it can also be found under the synonyms perfluoropropylene and hexafluoropropene. Its chemical formula is drawn in Fig. 4.4.

\[
\begin{align*}
&H \\
\text{C} = &C - C - H \\
&H
\end{align*}
\]

\[\text{Fig. 4.4. Chemical formula of hexafluoropropylene}\]
It has been found that $\text{C}_3\text{F}_6$ behaves as a very good insulating gas under electric stress. Its insulation properties at 1 atmosphere are comparable with those of $\text{SF}_6$ and for higher pressures they are better [10] (see Fig. 4.5).

Fig. 4.5. Breakdown field strength to pressure ratio as a function of $pd$ at several pressures (from [10]).

$A=0,1$, $B=0,25$, $C=0,5$, $D=1$, $E=1,5$, $F=2$ bar.

$\text{C}_3\text{F}_6$ is an interesting gas since its breakdown voltage $V_B$ does not follow Paschen's law $V_B = f(p \cdot d)$. The breakdown voltage increases more rapidly with pressure $p$ at a constant gap-width than with the distance $d$ at constant pressure [11]. The scaling law $E_B/p = f(pd)$ is not valid for $\text{C}_3\text{F}_6$, different pressures give different curves for $E_B/p$ (see Fig. 4.5). This deviation of a scaling law which is valid for most insulating gases also shows up as a pressure dependence of the critical field strength $E_c/p$ where $\frac{a-n}{p} = 0$.

Aschwanden and others [12] have used swarm experiments to derive $\frac{a-n}{p} = f(E/p)$ and $v_e = f(E/p)$ values for $\text{C}_3\text{F}_6$ (see Figs. 4.6 and 4.7). Figure 4.6 shows the pressure dependence of $E_c/p$. In contrast to the scaling problem in humid air [9], where the effective ionization coefficient $\frac{a-n}{p}$ would predict a wrong breakdown voltage if detachment is neglected, these values of the critical E-field do predict the correct breakdown voltage (see Fig. 4.8).
Fig. 4.6. $\frac{\alpha_e}{p}$ v.s. $E/p$ for different pressures (from [12]).

Fig. 4.7. Electron drift velocity $v_e$ as a function of $E/p$ for different pressures (from [12]).
This indicates that although detachment takes place in $\text{C}_3\text{F}_6$ it seems not to have a significant influence on the number of electrons in the head of the avalanche. Chen, Wootton and Chantry [13] have done cross-section measurements for ionization and attachment in $\text{C}_3\text{F}_6$. From these cross-sections they have calculated rate-coefficients $k_i$ and $k_a$ for ionization and attachment in $\text{C}_3\text{F}_6$. Normally the equality $k_i = k_a$ should predict a correct value of the critical electric field. They found however that for $\text{C}_3\text{F}_6$ this is not the case and they suggest an additional attachment process as a cause for this effect. The fact that $\text{C}_3\text{F}_6$ does not conform to scaling laws indicates that processes such as detachment and conversion do influence its insulating behaviour.

A possible mechanism for detachment and conversion processes in $\text{C}_3\text{F}_6$ has been given by Hunter and others [14]. An electron attaches to a $\text{C}_3\text{F}_6$-molecule to form an unstable negative ion

$$\text{C}_3\text{F}_6 + e \rightarrow \text{C}_3\text{F}_6^-$$  \hspace{1cm} (4.4)

This unstable ion with a relatively long lifetime ($\tau \approx 10^{-6}$ sec) can form a short-living dimer-ion ($\tau \leq 10^{-12}$ sec) with another $\text{C}_3\text{F}_6$-molecule:
This dimer can detach its electron or it can be converted to a stable negative dimer upon collision with another molecule:

\[ \text{C}_3\text{F}_6^* + \text{C}_3\text{F}_6 \rightarrow (\text{C}_3\text{F}_6)_2^* \quad (4.5) \]

\[ \text{(C}_3\text{F}_6)_2^* \xrightarrow{\gamma} 2\text{C}_3\text{F}_6 + e \quad (4.6a) \]

\[ \text{(C}_3\text{F}_6)_2^* + \text{A} \xrightarrow{\beta} (\text{C}_3\text{F}_6)_2^- + \text{A} \quad (4.6b) \]

Up till now it is not clear which process is responsible for the extraordinary behaviour of \( \text{C}_3\text{F}_6 \). Hunter [14] suggests that the pressure dependence of \( E_c/p \) and \( v_e \) is caused by the existence of unstable negative ions or dimers. How the pressure dependence can be explained in terms of \( \delta \) and \( \beta \) is unclear. In the chapters that follow an attempt will be made to describe the behaviour of \( \text{C}_3\text{F}_6 \) in terms of \( \delta \) and \( \beta \) in order to see if scaling laws for \( \delta \) and \( \beta \) can be derived.

### 4.3. Analysis of \( \text{C}_3\text{F}_6/N_2 \) measurements: theory

Measurements in pure \( \text{C}_3\text{F}_6 \) and mixtures of \( \text{C}_3\text{F}_6 \) with \( N_2 \) were done by Verhaart in 1982 (For a description of the experimental set-up see [1] p. 47-59). The measurements in pure \( \text{C}_3\text{F}_6 \) cover a pressure range from 50 to 446 torr. The experiments in mixtures were done at 380 and 760 torr total pressure \( (pN_2 + p\text{C}_3\text{F}_6) \) with the partial \( \text{C}_3\text{F}_6 \) pressure ranging from 1.5 to 380 torr. The partial pressure of \( \text{C}_3\text{F}_6 \) was measured with a pressure gauge: MKS Baratron, type 170 m-7 A with head 145 AHS-10,000. Although this pressure gauge showed some strange behaviour when in contact with \( \text{C}_3\text{F}_6 \), checks with other pressure gauges gave reasons to estimate the accuracy of the partial \( \text{C}_3\text{F}_6 \) pressure to be around 3%. These measurements were used to describe the phenomena that occur in \( \text{C}_3\text{F}_6 \) qualitatively ([11]) but they were never analysed in terms of \( \alpha, \gamma, \delta \) and \( \beta \). The major obstacle for this analysis is the fact that the electron current does not give sufficient information to derive the four swarm parameters. Takkenberg has shown [16] that for gases with ionization, attachment, detachment and conversion only three combinations of parameters can be determined with the electron current.
From the solution for the electron current for \( t \leq T_e \):

\[
i_e(t) = \frac{i_0}{A_1 - A_2} \left\{ (A_1 + \delta + \beta) e^{(A_1 v_e t)} - (A_2 + \delta + \beta) e^{(A_2 v_e t)} \right\} \quad (4.7)
\]

\[
A_1 = \frac{1}{2}(\alpha - \eta - \delta - \beta) + \frac{1}{2} \left[ (\alpha - \eta - \delta - \beta)^2 - 4(\beta \eta - \alpha \beta - \alpha \delta) \right] \quad (4.8)
\]

\[
A_2 = \frac{1}{2}(\alpha - \eta - \delta - \beta) - \frac{1}{2} \left[ (\alpha - \eta - \delta - \beta)^2 - 4(\beta \eta - \alpha \beta - \alpha \delta) \right] \quad (4.9)
\]

it follows that only the sets

\[
\begin{align*}
\alpha - \eta \\
\eta + \delta \\
\delta + \beta
\end{align*}
\]

(4.10)

can be uniquely determined. In the solution derived by Wen [3] for the electron current for \( t > T_e \) (4.11) the same combinations of parameters appear.

\[
i_e(t) = \int_0^\infty i_0 \delta \eta x e^{(\alpha - \eta)x + (\delta + \beta)(x - v_e t)} \cdot \left\{ \sum_{k=0}^{\infty} \frac{1}{k!(k+1)!} [\delta \eta (v_e t - x)]^k \right\} dx \quad (4.11)
\]

The analytical solution for \( t > T_e \) does not give additional information on the separate parameters. In order to obtain a complete set of unique parameters additional experimental information is needed. For the measurements that are available this information can be obtained in two ways:

1. From the ion-current (see Fig. 4.9.a). Analysis of this current is difficult because several ions (positive ions, negative unstable ions, negative stable ions) with different drift velocities contribute to the current. Since little is known on the different properties of these ions several assumptions have to be made.
2. From the ionization-coefficient of nitrogen:

The assumption is that in mixtures of $C_3F_6$ with nitrogen with very low $C_3F_6$ partial pressure the ionization-coefficient $\alpha$ and the drift-velocity $v_e$ are determined by nitrogen. For ionization this is the case if $p_{N_2} > p_{C3F6}$.

$C_3F_6$ causes the other phenomena such as $\eta$, $\delta$ and $\beta$. Since $\alpha$ is known from measurements in pure nitrogen $\eta$, $\delta$ and $\beta$ can be uniquely determined by analysing the electron current of a mixture.

For the analysis the latter approach is chosen. It has the advantage that from the step in the electron current at $t = T_e$ (see Fig. 4.9.b) the effective ionization coefficient $\alpha - \eta$ can be calculated according to ([1] p. 39):

$$i_e(T_e) - i_e(T_e + dt) = i_0 e^{(\alpha - \eta)d}$$  \hspace{1cm} (4.12)

This leaves only $\delta$ and $\beta$ to be calculated with the computer simulation model.

---

Fig. 4.9. Avalanches in pure $C_3F_6$ and $C_3F_6/N_2$ mixture

a. $p_{C3F6} = 400$ torr, $E/p = 110$ V/cm-torr
   upper trace: electron component hor: 20 nsec/div
   lower trace: ion component hor: 10 $\mu$sec/div

b. $p_{C3F6} = 3$ torr, $p_{N2} = 750$ torr,
   $E/p = 29,33$ V/cm-torr
   electron component hor: 20 nsec/div
At higher partial $C_3F_6$ pressures ($pC_3F_6 > 6$ torr) the step in the electron current at $t = T_e$ disappears and $\eta$ has to be calculated from the computer simulation. The fact that this step disappears means that no electrons reach the anode directly. All electrons will have attached at least once before reaching the anode. Therefore we expect that for higher partial $C_3F_6$ pressures the apparent electron transit time $T_{app}$ to be made up of two components. One is the real travelling time, the time that an electron is moving through the gap ($T_e$), the other is the time that an electron is attached before detaching ($\tau$).

For one electron:

$$T_{app} = T_e + \tau$$

(4.13)

For a swarm of electrons this relationship is more complicated but one can expect that the maximum in the electron current will occur at a later time $T_{app}$ than $T_e$ (see Fig. 4.10).

This effect will make the analysis of the pure $C_3F_6$ experiments and the higher partial $C_3F_6$ pressure measurements more difficult because an extra unknown variable $\nu_e$ is added. In the computer simulation model the real travelling time $T_e$ has to be entered and an assumption has to be made on its value.

With the theoretical model that takes detachment and conversion into account the following scaling laws are expected for $C_3F_6$:
\[ \frac{a-n}{p} = f(E/p) \]  
\[ v_e = f(E/p) \]  
(4.14)

The pressure dependence of \( \frac{a-n}{p} = f(E/p,p) \) as observed by Aschwanden [12] (see Fig. 4.6) should be attributed to a pressure dependence of the detachment or conversion process. We expect that these processes might show a pressure dependence which follows from the rate equations. Consider equation (2.4.).

For the detachment process follows:

\[
\left[ \frac{1}{n_{nu}} \cdot \frac{d n_{nu}}{dt} \right] = -\delta \ v_e = F_{nu}^{det} \text{detachment} \tag{4.15a}
\]

and

\[
\begin{cases} 
F_{nu}^{det} = \text{const.} & \text{(autodetachment)} \\
F_{nu}^{det} \propto p & \text{(collional detachment)} 
\end{cases}
\]

Also for conversion one finds from formula (2.4):

\[
\left[ \frac{1}{n_{nu}} \cdot \frac{d n_{nu}}{dt} \right] = -\beta \ v_e = F_{nu}^{con} \text{conversion} \tag{4.15b}
\]

with

\[
F_{nu}^{con} \propto pC_3F_6 \text{ (conversion to dimer (form 4.6))}
\]

Since \( v_e = f(E/p) \propto \sqrt{E/p} \) it is expected that for the detachment and conversion coefficient a field dependency will be present according to \( \delta, \beta \propto 1/\sqrt{E/p} \).

Aschwanden [12] also observed a pressure dependence in the electron drift velocity as a function of \( E/p \) (Fig. 4.7). We expect that the drift velocity \( v_e \) calculated with the real travelling time \( T_e \) instead of the transit time \( T_{app} \) is not pressure dependent. The fact that \( T_{app} \neq T_e \) can be explained by the detachment and conversion processes.
For mixtures of $C_3F_6$ and $N_2$ it is plausible that attachment is a function of the partial $C_3F_6$ pressure.

From the rate equation for attachment:

$$\left[ \frac{1}{n_e} \cdot \frac{dn_e}{dt} \right] = -\eta \cdot v_e = f_{\text{att}}^{\alpha \frac{n_{C_3F_6}}{p}}$$

(4.16)

How detachment and conversion relate to scaling depends on the mechanisms that are involved. Autodetachment is pressure independent since no other molecules or electrons are needed in this process ($\delta = f(E/p)$). If detachment occurs upon collision with another molecule then a dependence of $\delta$ on the total gas-pressure is expected ($\delta = f(E/p)$) (see formula 4.15a).

For conversion the same arguments can be used. If the mechanism proposed by Hunter et al. is valid then a second $C_3F_6$-molecule is necessary for conversion and $\beta$ will be a function of: $\frac{\beta}{p_{C_3F_6}} = f(E/p)$ (see formula 4.15b). If any molecule can stabilize an unstable ion then the relationship will be: $\frac{\beta}{p} = f(E/p)$. The remaining question is if anything can be said about the relationship between $\eta$, $\delta$, $\beta$ and $E/p$. The assumption is that in mixtures with a low partial $C_3F_6$
pressure the energy distribution of the electrons will be determined by the nitrogen. Therefore the mean free path $\lambda$ and the electron velocity $v_e$ are supposed to be the same as in pure nitrogen. The mean energy for drifting electrons in a gas can be expressed as:

$$e = q E \lambda = \frac{1}{2} m v_e^2$$  \hspace{1cm} (4.17)

for the mean free path one can write:

$$\lambda = \frac{1}{n Q} = \frac{kT}{pQ}$$  \hspace{1cm} (4.18)

Here $Q$ is the total elastic collision cross section

Substituting (4.18) in (4.17) gives:

$$v_e = \sqrt{\frac{2q E \lambda}{m}} = \sqrt{\frac{2q E kT}{m pQ}} \propto \sqrt{\frac{E}{p}}$$  \hspace{1cm} (4.19)

From (4.19) it follows that $V_e$ is proportional to $\sqrt{E/p}$. The attachment process is expected to be energy-independent. From the rate equation for attachment (4.16) it follows that scaling of $\eta$ can be expressed as:

$$\frac{\eta}{pC_3F_6} \sim \frac{c}{\sqrt{E/p}}$$  \hspace{1cm} (4.20)

From the rate equations (4.15a and b) for $\delta$ and $\beta$ one can derive in the same way

$$\delta \sim \frac{c}{\sqrt{E/p}} \hspace{1cm} \text{(autodetachment)} \hspace{1cm} (4.21)$$

$$\frac{\beta}{pC_3F_6} \sim \frac{c}{\sqrt{E/p}} \hspace{1cm} \text{(conversion to dimer (form 4.6))} \hspace{1cm} (4.22)$$

with the assumption that both processes are energy-independent.
4.4. Analysis of \( C_3F_6/N_2 \) measurements: computer simulation

For the analysis of the electroncurrent a computer simulation program FIV/SIMU 3 is available [2]. The use of this program for analyzing electroncurrents is not very handy. The program calculates the electroncurrent for specified \( \alpha, \eta, \delta \) and \( \beta \). The result has to be compared with the real measurement and by changing the values of \( \alpha, \eta, \delta \) and \( \beta \) one may obtain a good simulation for the measurements. This trial and error method is rather clumsy and time consuming. If the swarm parameters are known the program is very handy as a check to see if these parameters represent the right electroncurrent. In case the swarm-parameters have to be determined from the measurements other methods have to be used.

A common approach to problems where a theoretical expression has to be fitted to a set of datapoints is the use of a least squares approximation. A first attempt was made by using the theoretical expression for the electroncurrent valid for \( 0 < t \leq T_e \) (4.7) and let the computer calculate the best fit to 8 equidistantial datapoints on \( 0 \leq t \leq T_e \) (see Fig. 4.12).

![Fig. 4.12. Datapoints used for least squares method](image)

For the calculation the procedure MINIQUAD was used and the parameters \( \eta, \delta \) and \( \beta \) had to be calculated (\( \alpha \) is known from nitrogen). This method didn't work very well. Very good first approximations of \( \eta, \delta \) and \( \beta \) had to be made to make sure that the process would converge.
Since the theoretical expression is a sum of two exponentials (see form 4.7) one of which becomes rapidly very small it is very hard to derive these exponentials from the measurement. The major problems are:

- The inaccuracy in the datapoints is of the same order of one of the exponentials for \( t > \frac{T_e}{10} \).

- To determine the rapidly declining exponential more datapoints on \( 0 < t < \frac{T_e}{10} \) should be used. In the measurements this region is very faint which makes it hard to derive datapoints.

In Fig. 4.13 a measurement together with its simulation on \( 0 < t < T_e \) is given. It is clear that for \( t > \frac{1}{2} T_e \) the electroncurrent is determined by one exponential.

![Graph showing measurement and simulation](image)

Fig. 4.13. a. Measurement in 1 atm \( \text{N}_2 \), \( \text{pC}_3\text{F}_6 = 6 \) torr

\[ E = 23 \text{ kV/cm}. \]

b. Simulation with the two exponentials drawn separately.
A second attempt was made with the assumption that the current can be represented by one exponential on \( \frac{1}{2} T_e < t < T_e \):
\[
i = \frac{i_0}{A_1 - A_2} \left\{ (A_1 + \delta + \beta) e^{A_1 v e t} \right\}
\]
(4.23)

With MINIQUAD this expression was fitted to four datapoints on \( \frac{1}{2} T_e < t < T_e \) but again the minimization proved difficult. A different method was employed by calculating \( A_1 \) and \( C = \frac{i_0}{A_1 - A_2} (A_1 + \delta + \beta) \) from the four datapoints \( i_1, i_2, i_3 \) and \( i_4 \).

From the increment in the current between two datapoints \( A_1 \) can be derived:
\[
\ln \frac{i_2}{i_1} = \ln (e^{A_1 v e (t_2 - t_1)}) = A_1 v e (t_2 - t_1)
\]
(4.24)

This is done for four datapoints and \( A_1 \) is assigned the average of the three values. If \( A_1 \) is known then \( C \) can be calculated according to:
\[
i_1 = C e^{A_1 v e t_1}
\]
(4.25)

Again \( C \) is taken as the average value of the \( C \)'s calculated from the four datapoints. With these values for \( C \) and \( A_1, \delta \) and \( \beta \) can be calculated because they are the remaining unknowns if \( \eta \) is calculated from the step in the current at \( t = T_e \). The calculation of \( \beta \) and \( \delta \) from \( A_1 \) and \( C \) is given in appendix I.

The above method gives a good fit for the electron current on \( 0 < t < T_e \). For \( t > T_e \) the deviation of the calculated current from the measured current can be considerable. Also the value of \( I_0 \) has an influence on the calculated \( \delta \) and \( \beta \). Because it is difficult to determine \( I_0 \) accurately (very fast processes occur at \( t = 0 \) which are difficult to measure) the uncertainty in \( I_0 \) results in an uncertainty in \( \delta \) and \( \beta \). Summarizing the problems that are left over:
- fitting of the electron current for \( t > T_e \) is still not accurate
- this method only works if \( \eta \) can be determined from the step in the current at \( t = T_e \)
- the uncertainty in \( I_0 \) results in an uncertainty in \( \delta \) and \( \beta \).

To solve these problems four extra datapoints for \( t > T_e \) have been added (see Fig. 4.13). With MINIQUAD the curve is fitted to these datapoints by using the formula derived by Wen [3]:

\[
i(t) = \int_0^d \frac{d}{\delta} \eta x e^{(\delta+\beta)(x-v_e t)} \left\{ \sum_{k=0}^\infty \frac{1}{k!(k+1)!} [\delta \eta(v_e t-x)]^k \right\} dx \quad (4.26)
\]

The variables in this calculation are \( \eta \) and \( I_0 \). For every \( \eta \) and \( I_0 \) values for \( \delta \) and \( \beta \) are calculated using the four datapoints on \( t \leq T_e \). With these parameters and formula (4.26) the current is calculated for the four datapoints on \( t > T_e \). The difference between the calculated values and the measured values is minimized. In this way a good fit to the measured electron current can be found. Whether a good fit is found depends on the choice of the starting values for \( \eta \) and \( I_0 \) which have to be chosen by trial and error. In appendix II the computer program together with an example of input and output data is presented.
4.5. Analysis of $C_3F_6/N_2$ measurements: results

The measurements in pure $C_3F_6$ have first been partly analyzed with the simple model of only ionization and attachment as was done by Aschwanden [12]. From the rise in the current between $t = 0$ and $t = T_{app}$, the effective ionization coefficient $\alpha - \eta$ can be calculated according to

$$\alpha - \eta = \frac{1}{\nu T_{app}} \ln \left[ \frac{i(T_{app})}{i_0} \right]$$

(4.27)

This is done for pressures of 50.75 and 100 torr. The results are plotted in Fig. 4.15. Also the values obtained by Aschwanden (see Fig. 4.6) are drawn in this figure. Very good agreement between the two sets of data can be seen.

Fig. 4.15. $\frac{\alpha - \eta}{p}$ as a function of $E/p$ for different pressures
From this the following can be concluded:

- Aschwanden takes only ionization and attachment into account in his analysis; detachment is neglected.
- He calculates an apparent drift velocity from $T_{\text{app}}$ which turns out to be pressure dependent.
- Our measurements compare well with other measurements if analyzed with a model that only takes into account $\alpha$ and $\eta$.

With the simulation program FIV/SIMU 3 an electron current is simulated with respectively large $\eta$ and $\delta$ (strong attachment and strong detachment) and small $\eta$ and $\delta = 0$ (little attachment and no detachment). In both simulations the drift velocity is the same. In Fig. 4.16 the influence of strong attachment and detachment can be seen.

![Graph](image)

**Fig. 4.16.** a. Simulation with large $\eta$ and $\delta$

b. Simulation with small $\eta$ and $\delta = 0$.

Comparison of the two simulations shows that
- $T_{\text{app}} > T_c$ for large $\eta$ and $\delta$. This might give an explanation for the pressure dependence of the drift velocity $v_e$.
- Fast attachment results in a rapid decrease in the current for $t = 0$. 
In the measurements this sharp drop in the current for \( t = 0 \) can be seen very faintly. Because this is such a fast process it will meet the limits of the experimental set-up with regard to the time-resolution. It is therefore questionable whether the value for \( I_0 \) that can be taken from the measurement is the real value or a value that is lower because of the limited time-response of the experimental set-up.

To check this phenomenon some of the measurements have been repeated with the new experimental set-up (see §2.3). Comparison of two electron-currents measured with different set-ups but under the same experimental conditions (see Fig. 4.17) shows that the set-up has an influence on the registration of the fast processes at \( t = 0 \).

![Fig. 4.17. a. Measurement with the old set-up (Verhaart) b. Measurement with the new set-up](image)

In Fig. 4.17b no fast attachment at \( t = 0 \) can be seen possibly due to a lower time-resolution of the new set-up. In order to get more information on the accuracy of \( I_0 \) improvement of the time resolution of the set-up remains necessary.

Analysis of measurements in a mixture of \( \text{C}_3\text{F}_6 \) and \( \text{N}_2 \) has been done for partial \( \text{C}_3\text{F}_6 \) pressures of 1.5, 3, 6 and 12 torr. The pressure of the buffer gas \( \text{N}_2 \) is either 375 or 750 torr. Values for \( \eta, \delta, \beta \) and \( I_0 \) were calculated using the method described in the previous chapter. The summation in the same formula was done for eight components. The calculated values for the electron-current with this formula were compared with the values obtained by numerical simulation. The two methods give values that compare well indicating that for these cases (\( \eta \) and \( \delta \) in the order of 10) seven components in the summation give a
good approximation of the electron current. For the minimization a maximum number of 8 iterations was used. In most cases this proved to be enough and a minimum was found. In case no minimum could be found or the calculated parameters did not give the right electron current the process was repeated with different starting values for \( \eta \) and \( I_0 \). The fact that different starting values lead to different minima can be due to the properties of the function (4.26) that has to be matched to the datapoints. In all these calculations the values for the ionization coefficient \( \alpha \) and for the electron travelling time \( T_e = \frac{d}{v_e} \) were taken from Verhaart's thesis ([1] p. 68 and 69). In Fig. 4.18 the driftvelocity of the \( \text{C}_3\text{F}_6/\text{N}_2 \) measurements is compared with the driftvelocity of nitrogen. For low partial \( \text{C}_3\text{F}_6 \) pressures (1.5-6 torr) there is very good agreement between the two. For \( p_{\text{C}_3\text{F}_6} = 12 \) torr the driftvelocity of the mixture is clearly lower than that of pure nitrogen. For this partial \( \text{C}_3\text{F}_6 \) pressure no step in the currents at \( t = T_e \) can be seen. The detachment process causes the maximum in the current to appear at a later time than the real electron travelling time \( T_{\text{app}} > T_e \) which results in a lower driftvelocity.

![Graph](image)

Fig. 4.18. Electron drift velocity in \( \text{C}_3\text{F}_6/\text{N}_2 \) mixtures and in pure \( \text{N}_2 \).
The results of the analysis are presented in appendix III. Different calculations for one measurement with different starting values which give almost the same minimum show a spread in the parameter values of 10%. The measurements for \( p_{C_3F_6} = 1.5 \) torr with \( p_{N_2} = 375 \) torr were not analysed because the electron current for \( t > T_e \) is too small to derive significant datapoints. The values for \( \eta, \delta \) and \( \beta \) are plotted against \( E/p \) (Fig. 4.22). Because a dependence of \( \eta \) on the partial \( C_3F_6 \) pressure is expected \( \frac{\eta}{p_{C_3F_6}} \) is plotted versus \( E/p \) (Fig. 4.23). In this figure also plots of \( \frac{\delta}{p_{C_3F_6}} \) and \( \frac{\beta}{p_{C_3F_6}} \) are drawn to check if these processes also show such a pressure dependence. Finally two plots have been made for \( \frac{\delta}{p_{N_2}} \) and for \( \frac{\delta}{p_{N_2}\sqrt{E/p}} \) as a function of \( E/p \) in order to find out which representation is the most useful to understand the detachment process (Fig. 4.24).

The values of \( \frac{\eta}{p_{C_3F_6}} \) have been fitted to a function \( f = \frac{c}{\sqrt{E/p}} \) since this relationship is expected for an energy-independent attachment process (see form 4.16). For low \( E/p \) it appears that the pressure of the buffer gas has an influence on the attachment coefficient \( \frac{\eta}{p_{C_3F_6}} \). The coefficient is considerably smaller for \( p_{N_2} = 375 \) torr than for \( p_{N_2} = 750 \) torr. For higher \( E/p \) and higher partial \( C_3F_6 \) pressure the effect disappears. Because of this effect the data for \( \frac{\eta}{p_{C_3F_6}} \) have been fitted twice to the mentioned relationship. Once with the data from measurements with \( p_{N_2} = 750 \) torr (curve 1 in Fig. 4.23a) which gives:

\[
\frac{\eta}{p_{C_3F_6}} = \frac{2.8}{\sqrt{E/p}}
\]  
(4.28)

and once with all the data (curve 2 in Fig. 4.23a) giving:

\[
\frac{\eta}{p_{C_3F_6}} = \frac{2.3}{\sqrt{E/p}}
\]  
(4.29)
The first fit seems to be better because the data for \( pN_2 = 750 \text{ torr} \) shows less spread (\( \sim 20\% \)) than the data for \( pN_2 = 375 \) and because for higher \( E/p \) the data for \( pN_2 = 375 \text{ torr} \) is of the same magnitude as the data for \( pN_2 = 750 \text{ torr} \).

A possible explanation for the fact that \( \frac{\eta}{pC_3F_6} \) depends on the total gas pressure for low electric fields can be given if one looks more closely at the cross-sections for ionization.

In Fig. 4.19 the cross-sections for ionization are given for \( C_3F_6 \) and \( N_2 \). Important is the threshold for ionization which is \( \sim 11 \text{ eV} \) for \( C_3F_6 \) and \( \sim 15 \text{ eV} \) for \( N_2 \).

For high \( E/p \) which means high electron energy the contribution to the average cross-section \( \overline{Q} \) will be in the same order for both gases. For low \( E/p \) the contribution of \( C_3F_6 \) to the average cross-section will be higher than the contribution of \( N_2 \) (\( \overline{Q}_{C_3F_6} \gg \overline{Q}_{N_2} \)) because of the lower threshold of \( C_3F_6 \). The assumption that the ionization coefficient of pure nitrogen can be used in the analysis (\( pN_2 \cdot \overline{Q}_{N_2} \)) \( pC_3F_6 \cdot \overline{Q}_{C_3F_6} \) still holds for high \( E/p \) if \( pN_2 \gg pC_3F_6 \). For low \( E/p \) the product \( pN_2 \cdot \overline{Q}_{N_2} \) is not necessarily larger than the product \( pC_3F_6 \cdot \overline{Q}_{C_3F_6} \).

Thus the validity of the above assumption decreases for

- low \( E/p \)
- low pressure of nitrogen (= high \( C_3F_6/N_2 \) ratio)

\[ Q_{\text{ion}} \]

\[ C_3F_6 \]

\[ N_2 \]

\[ \sim 11 \text{ eV} \]

\[ \sim 15 \text{ eV} \]

\[ pN_2 \gg pC_3F_6 \]

\[ pN_2 \cdot \overline{Q}_{N_2} \]

\[ pC_3F_6 \cdot \overline{Q}_{C_3F_6} \]
In this region we find the dependency of $\eta$ on the total gas-pressure. If $C_3F_6$ contributes significantly to the ionization coefficient then the value for $\alpha = \alpha(N_2)$, used in the analysis, is too low. Since $\alpha - \eta$ is determined from the electron current a higher value of $\alpha = \alpha(N_2) + \alpha(C_3F_6)$ will give a higher value of $\eta$ and this is more in agreement with the values of $\eta$ for $pN_2 = 750$ torr.

How the value of $\alpha$ has to be modified is not that clear since we do not know if the values of $\alpha$ for $pN_2 = 750$ torr are correct. One method is to suppose that the assumption $pN_2 \cdot Q_{N_2} \gg pC_3F_6 \cdot Q_{C_3F_6}$ is right for high $E/p$. This will give a reference point to which the relationship $\frac{\eta}{pC_3F_6} = \frac{C}{\sqrt{E/p}}$ can be fitted (Fig. 4.20)

![Fig. 4.20 Correction of $\eta$ to a reference point](image)

From this approximation the real $\eta$ can be obtained and also the ionization coefficient for $C_3F_6$.

A second method is to assume that the values for $pN_2 = 750$ torr are correct, also for low $E/p$. If these values agree well with the theoretical formula $\frac{\eta}{pC_3F_6} = \frac{C}{\sqrt{E/p}}$ this might very well be the case.

The values of $\eta$ for $pN_2 = 375$ torr can be corrected by fitting them to the $pN_2 = 750$ torr curve. The difference between the two values gives the ionization coefficient of $C_3F_6$ (see Fig. 4.2.1)
Fig. 4.21 Correction of $\eta$ to $p\text{N}_2 = 750$ torr curve

In both cases also values of $\delta$ and $\beta$ will change according to

$$\delta_{\text{new}} = \frac{(\eta \cdot \delta)_{\text{old}}}{\eta_{\text{new}}} \quad (4.30)$$

and

$$\beta_{\text{new}} = \beta_{\text{old}} + \delta_{\text{old}} \left\{ 1 - \frac{\eta_{\text{old}}}{\eta_{\text{new}}} \right\} \quad (4.31)$$

These changes in $\delta$ and $\beta$ are not that large to make the present representation and discussion on these parameters invalid. The question remains if the different ionization coefficients can simply be added. If one looks at the differential equation:

$$\frac{d\eta_e}{dt} = \alpha \eta_e v_e - \eta \eta_e v_e \quad (4.32)$$

then it doesn't matter which molecule will release an electron and the ionization coefficients can be added.

In Fig. 4.23b the large spread in $\frac{d\delta}{p\text{C}_3\text{F}_6}$ as a function of $E/p$ can be seen. No significant trend comes forward and one may conclude that a detachment process involving another $\text{C}_3\text{F}_6$-molecule is unlikely ($F_{\text{nu}}^{\text{det}} \neq f(p\text{C}_3\text{F}_6)$).
Fig. 4.23c gives $\frac{\beta}{pC_3F_6}$ as a function of $E/p$. Because the same relationship to $E/p$ is expected as with $\eta$ these values have also been fitted to a function $f = \frac{c}{\sqrt{E/p}}$ (formula 4.15b). The first curve has been fitted to all the values except for the 1.5/750 data because of the large deviation from the other points (curve 3 in Fig. 4.23c). The relationship becomes:

$$\frac{\beta}{pC_3F_6} = \frac{1.2}{\sqrt{E/p}}$$ \hspace{1cm} (4.33)

The second curve (curve 4 in Fig. 4.23c) has been calculated using all the data:

$$\frac{\beta}{pC_3F_6} = \frac{1.6}{\sqrt{E/p}}$$ \hspace{1cm} (4.34)

For both curves it is found that the data shows a large spread around the curve ($\sim 100\%$). The value of $\frac{\beta}{pC_3F_6}$ tends to decrease more strongly with $E/p$ than can be explained by the above relationship. Probably energy dependent processes have an influence on the conversion coefficient. Because of the large number of processes that can be involved it is hard to indicate which process could be responsible for such a behaviour.

For detachment no trend could be detected if $\frac{\delta}{pC_3F_6}$ is plotted as a function of $E/p$. Therefore $\frac{\delta}{p}$ is plotted against $E/p$ (Fig. 4.24a) to see if this representation is more consistent and to check whether detachment caused by collision with another molecule might be a dominating process (see form 4.15a). In this figure it can be seen that the values of $\frac{\delta}{p}$ lie on a curve within reasonable spread ($\sim 20\%$). From the definition of $\delta$ one would expect this curve to behave like:

$$\frac{\delta}{p} = \frac{c}{\sqrt{E/p}}$$ \hspace{1cm} (4.35)
No behaviour like this shows up. The value of $\frac{\delta}{p}$ increases with higher $E/p$ which leads us to believe that the curve can be represented by:

$$\frac{\delta}{p} = \frac{f(E)}{\sqrt{E/p}} \tag{4.36}$$

$f(E)$ is a function of the electric field. This would mean that

$$f_{\text{det}}^{\text{nu}} = \left[ \frac{1}{n_{\text{nu}}} \frac{dn_{\text{nu}}}{dt} \right]_{\text{detachment}} \alpha f(E) \cdot p \quad \text{(field enhanced collisional detachment)} \tag{4.37}$$

A graph of this function is given in Fig. 4.24b since rewriting (4.36) gives

$$f(E) = \frac{\delta}{p} \sqrt{E/p} \tag{4.38}$$

From the representation of the data on the detachment coefficient one may conclude that the data is the most consistent if the detachment process is considered as detachment caused by collision with another molecule. In addition this detachment is field-dependent (or energy dependent) according to a function $f(E)$. From the graph of this function (Fig. 4.24b) it may be seen that a possible threshold exists below which no detachment takes place. It is not clear which physical process can explain the field-dependency of the detachment process. The spread in the calculated parameters may be caused by
- inaccuracy of the measurement
- inaccuracy of the datasamples taken from the measurement
- limited accuracy of the fitting procedure.

Another source of error can be a wrong assumption for the ionization coefficient as mentioned in the discussion on the attachment coefficient. If $pC_{3}F_{6} \cdot Q_{C_{3}F_{6}} = pN_{2} \cdot Q_{N_{2}}$ then ionization of both gases will occur with the same probability and the assumption that $\alpha = \alpha(N_{2})$ is not valid. Further analysis of measurements in mixtures with higher partial $C_{3}F_{6}$ pressure is needed to see if the observed trends for $\eta$, $\delta$ and $\beta$ continue. Also experiments that will give information on the physical processes that occur can be useful in order to explain these trends.
Fig. 4.22. $\eta(a)$, $\delta(b)$ and $\beta(c)$ as a function of $E/p$
Fig. 4.23. $\frac{\eta}{PC_3F_6}$ (a), $\frac{\delta}{PC_3F_6}$ (b) and $\frac{\beta}{PC_3F_6}$ (c) as a function of $E/p$
Fig. 4.24. $\frac{\delta}{\sqrt{E/p}}$ (a) and $\sqrt{E/p} \frac{\delta}{p}$ (b) as a function of $E/p$
4.6. Conclusions and recommendations

1. The theoretical model used for the analysis of electron currents in pure $C_3F_6$ and $C_3F_6/N_2$ mixtures explains some of the observed phenomena in $C_3F_6$. Numerical simulation based on this model shows that:
   - the pressure dependency of the apparent electron drift velocity is due to high attachment and detachment processes
   - the existence of a significant electron current for $t > T_e$ is due to detachment

2. Evaluation of the swarmparameters ($\alpha$, $\eta$, $\delta$ and $\beta$) from the electron current is difficult and not straightforward. A fitting program that uses a least squares approximation to fit the data to a nonlinear function gives values for the swarm parameters. Special attention has to be paid to the choice of the data used for fitting and to the choice of the starting values in the iteration.

3. The assumption that in mixtures $C_3F_6/N_2$ with a low partial $C_3F_6$ pressure the drift velocity of the electrons can be taken equal to the drift velocity of electrons in pure nitrogen is supported by our experiments.

4. The assumption that $\alpha = \alpha(N_2)$ in mixtures $C_3F_6/N_2$ with a low partial $C_3F_6$ pressure is not always valid. Since the threshold for ionization for $C_3F_6$ is lower than for $N_2$ the assumption will not be valid for low $E/p$ and low $pN_2$.

5. Better experimental information on the fast processes that happen for $t = +0$ will be helpful in the analysis of electron currents.

6. Calculated values of the swarmparameters $\eta$, $\delta$ and $\beta$ give a good simulation of the electron current. Because of the difficult method and formula's that are used it is hard to give an estimation on the accuracy of these values.
7. From the representation of the swarm parameters $\eta$, $\delta$ and $\beta$ as a function of $E/p$ it follows that the data is most consistent if:

- the attachment process is considered to depend on the partial $C_3F_6$ pressure
- the detachment process is considered to depend on the total gas pressure and also exhibits an energy dependency having a well defined threshold
- the conversion process is considered to depend on the partial $C_3F_6$ pressure and also exhibits an energy dependency.

8. The need for a better fitting program is obvious. Improvements to the current program are:

- use fitting also for the data on $\frac{1}{2}T_e < t < T_e$
- reexamine the choice of the variables $I_0$ and $\eta$ in the minimization. These variables have been chosen for practical reasons. A better convergence of the minimization might be found if variables are chosen on mathematical grounds
- calculation of several minima by choosing different starting values automatically in order to find the best minimum.
5. LITERATURE


[3] Wen Chuan. Spatial distribution of electrons and ions of a single avalanche in which $\alpha, \eta, \delta$ and $\beta$ take place. Private communication (1986).


APPENDIX I

Calculation of $\delta$ and $\beta$ from the electron current approximation with one exponential.

From the constants $\frac{i_0}{A_1-A_2} (A_1+\delta+\beta)$ and $A_1$ (see formula 4.23) $\delta$ and $\beta$ can be calculated if $i_0$, $\alpha$ and $\eta$ are known.

Rewriting (4.8) gives:

$$4A_1(A_1-(\alpha-\eta)) = 4(\alpha-A_1)(\delta+\beta)-4\eta \beta$$

Renaming the following expressions:

$$x = 4A_1(A_1-(\alpha-\eta))$$
$$y = 4(\alpha-A_1)$$
$$z = 4\eta$$

and substituting these in (1) gives a relationship between $\delta$ and $\beta$:

$$\delta = -\left(\frac{y-z}{y}\right) \beta + \frac{x}{y}$$

From formula 4.23 follows:

$$C = \frac{A_1+\delta+\beta}{A_1-A_2} = \frac{A_1+\delta+\beta}{\sqrt{(\alpha-\eta-\delta-\beta)^2-4(\beta\eta-\alpha\beta-\alpha\delta)}}$$

Rewriting (6) yields:

$$\left[\frac{C}{I_0}\right]^2 \{(\alpha-\eta)^2-2(\alpha-\eta)(\delta+\beta)+(\delta+\beta)^2-4\beta\eta+4\alpha(\delta+\beta)\} =$$

$$A_1^2+2A_1(\delta+\beta)+(\delta+\beta)^2$$
Substituting (5) in (7):

\[
\left[ \frac{C}{I_o} \right]^2 \{ (a-\eta)^2 - 2(a-\eta) \left[ \frac{z+2}{y} \right] + \left[ \frac{z+2}{y} \right]^2 - z \beta + 4\alpha \left[ \frac{z+2}{y} \right] \} = \\
A_1^2 + 2A_1 \left[ \frac{z+2}{y} \beta + \frac{x}{y} \right] + \left[ \frac{z+2}{y} \beta + \frac{x}{y} \right]^2
\]

(8)

This expression can be rewritten as a polynomial of second order in \( \beta \):

\[ a \beta^2 + b \beta + c = 0 \]  

(9)

with

\[ a = \left[ \frac{C}{I_o} \right]^2 \left( \frac{z}{y} \right)^2 \]  

(10)

\[ b = \left[ \frac{C}{I_o} \right]^2 \frac{1}{y} \left[ -2(a-\eta) - z \frac{2}{y} \right] - \frac{z}{y} \left[ 2A_1 + 2\frac{x}{y} \right] \]  

(11)

\[ c = \left[ \frac{C}{I_o} \right]^2 \left[ (a-\eta)^2 - 2(a-\eta) \frac{x}{y} + \left[ \frac{x}{y} \right]^2 + 4\alpha \frac{x}{y} \right] - A_1^2 - 2A_1 \frac{x}{y} - \left[ \frac{x}{y} \right]^2 \]  

(12)

From the two solutions that follow out of formula (9) and (5) the positive values of \( \delta \) and \( \beta \) should be taken.
This program calculates values for ETA, DELTA, BETA and Io from N data samples taken from the electron current. The method of calculation is described in chapter 4.4. The program uses two standard procedures:

1. INTEGRALTRAPEX: to calculate the integral in formula 4.26
2. MINIQUAD: to fit the theoretical electron current to four data samples on t>Te with a least squares approximation

Information on INTEGRALTRAPEX can be found in RC-information PP 3.1 for MINIQUAD one should look into RC-information PP 5.5.

Variables used:

- ALFA = ionization coefficient
- ETA = attachment coefficient
- DELTA = detachment coefficient
- BETA = conversion coefficient
- TE = electron transit time
- IO = current at t=O
- IX = X-coordinate
- T = time-coordinate
- D = gap-distance
- R = residu calculated by MINIQUAD
- A1 = increment in the one-exponential approximation
- CONST = constant in the one exponential approximation
- AE = absolute error in integral calculation
- RE = relative error in integral calculation
- N = number of data samples
- I, J = counters
- STOPCRIT = stop criterion in iteration procedure
- KMAX = limit in the summation of formula 4.26
- IMAX = maximum number of iterations
- S = dummy variable for INTEGRALTRAPEX
- NAAM = code of measurement: pC3F6/pN2/voltageKV/PAGenumber
- CONV = dummy variable for INTEGRALTRAPEX
- M[1:N] = array with data samples from the electron current
- IT[1:N] = array with values for the electron current calculated from the swarm parameters. Only values for t>Te are calculated.
- TJJD[1:N] = array with the times corresponding to the data samples
- VARC[1:2,1:2] = array used in MINIQUAD
- X[1:2] = array containing the variables for least squares minimization

BEGIN

$INCLUDE"NONLINLIB/ALGOL/DECLARATION ON APPL"
$INCLUDE"NONLINLIB/ALGOL/MINIQUAD ON APPL"
$INCLUDE"NUMLIB/ALGOL/DECLARATION ON APPL"
$INCLUDE"NUMLIB/ALGOL/INTEGRALTRAPEX ON APPL"

FILE INPAR(KIND=DISK,FILETYPE=7);
FILE OUT(KIND=DISK,NEWFILE=TRUE,PROTECTION=SAVE);
REAL ALFA,ETA,DELTA,BETA,TE,IO,IX,T,D,R,A1,CONST,AE,RE;
INTEGER N,I,STOPCRIT,IMAX,KMAX,J,K,S;
STRING NAAM; BOOLEAN CONV;
READ (INPAR,<A25>,NAAM);
READ (INPAR,/,ALFA,ETA,N,IO,D,KMAX,IMAX,AE,RE);
BEGIN
REAL ARRAY M, T, TIJD[1:N], VARCOV[1:2, 1:2], X[1:2];

This procedure calculates the value of the function that has to be integrated (see formula 4.26).

REAL PROCEDURE F(T, K, IX);
REAL T, IX; INTEGER K;

This procedure calculates the value of the electroncurrent at a time t*Te with a summation of KMAX partial sums.

Variables used:
- DEELSOM = partial sum
- SOM = total sum
- FAC = multiplication factor for each partial sum

REAL PROCEDURE STROOM(KMAX, T);
REAL T; INTEGER KMAX;
BEGIN
REAL FAC, SOM, DEELSOM;
K := 0; FAC := 1;
SOM := FAC*INTEGRALTRAPEX(F(T, K, IX), IX, 0, D, AE, RE, 6, S, CONV);
DEELSOM := SOM; K := 1;
WHILE K <= KMAX AND DEELSOM = 0.01*SOM DO
BEGIN
FAC := FAC*DELTA*X[1]/(K*(K+1));
DEELSOM := FAC*INTEGRALTRAPEX(F(T, K, IX), IX, 0, D, AE, RE, 6, S, CONV);
SOM := SOM + DEELSOM;
K := K + 1;
END;
STROOM := X[2]*DELTA*X[1]*EXP(-(DELTA + BETA)*T*D/TE)*SOM;
END;

The procedure PARBER calculates values for DELTA and BETA from data on 1/2Te<t<Te (see chapter 4.4 and appendix 1).

Variables used:
- A, B, C = coefficients of a second order polynomial
- W = solution of this polynomial that gives positive values
- for DELTA and BETA
- A2, B1, B2, C1, C2 = components of coefficients A, B and C
- X, Y, Z, P, Q = variables used to simplify expressions

PROCEDURE PARBER(ALFA, ETA, DELTA, BETA, A1, CONST, IO);
REAL ALFA, ETA, DELTA, BETA, A1, CONST, IO;
BEGIN
REAL A, B, C, A2, B1, B2, C1, C2, P, Q, X, Y, Z, W;
X := 4*A1*(A1 - ALFA + ETA); Y := 4*(ALFA - A1); Z := 4*ETA;
P := (Z - Y)/Y; Q := X/Y;
A2 := (Z/Y)**2;
B1 := (2*(ETA - ALFA)*Z + 2*Z*X/Y - Z*Y + 4*ALFA*Z)/Y;
B2 := 2*Z*(A1 + X/Y)/Y;
C1 := (ALFA - ETA) * (ALFA - ETA - 2*X/Y + (X/Y)**2 + 4*ALFA*X/Y);
C2 := A1*(A1 + 2*X/Y + (X/Y)**2);
A := ((CONST/IO)**2 - 1)*A2;
B := ((CONST/IO)**2)*B1 - B2;
C := ((CONST/IO)**2)*C1 - C2;
W := SORT(2**2 - 4*A/C)/(2*A);
BETA := -(B/(2*A)) - W; DELTA := P*BETA + Q;
END;

The procedure RESIDU specifies the function that has to be
% fitted to the data. The fitting is done for samples 5 to 8  
% ( t>T_e ).  

PROCEDURE RESIDU(X,R,J) ;  
BEGIN  
    PARBER(ALFA,X[1],DELTA,BETA,AL,CONST,X[2]) ;  
    T:=TJJD[J+4] ;  
    R:=M[J+4]-STROOM(KMAX,T) ;  
END ;  

PROCEDURE GRADRES(X,GRAD,J) ;  
BEGIN  
END ;  

The main program starts here :  

% - data is read from the file INPAR  
% - the constants for the one exponential approximation are  
% calculated from samples 1 to 4  
% - starting values for MINI QUAD are assigned to array X  
% - MINI QUAD is called  
% - with the calculated swarmparameters the electroncurrent is  
% calculated for the sampletimes t>T_e . The values are stored  
% in array IT  
% - output is written to file OUT an example of this file is  
% included in this appendix  

FOR I:=1 STEP 1 UNTIL N DO READ (INPAR,I,M[I],TJJD[I]) ;  
Al:=(LN(M[2]/M[1])+LN(M[3]/M[2])+LN(M[4]/M[3]))/0.375  
CONST:=(M[1]/EXP(Al*0.625)+M[2]/EXP(Al*0.75)+  
M[3]/EXP(Al*0.875)+M[4]/EXP(Al))/4 ;  
TE:=TJJD[4] ; X[1]:=ETA ; X[2]:=IO ; STOPCRIT:=2 ;  
MINIQUAD(N-4,2,X,GRADRES,0,42,STOPCRIT,1E-15,1E-3,  
IMAX,1,VARCOV,OUT) ;  
ETA:=X[1] ;  
FOR I:=5 STEP 1 UNTIL N DO BEGIN  
    T:=TJJD[I] ; IT[I]:=STROOM(KMAX,T) ;  
END ;  
WRITE(OUT,<"De code van de meting is ":A25,/>,NAAM) ;  
WRITE(OUT,<"Iteratiesstartstroom IO is ":E12.4,/>,IO) ;  
WRITE(OUT,<"Beginstroom IO is ":E12.4,/>,X[2]) ;  
WRITE(OUT,<"De meetpunten zijn t (nsec) m(t) (A) 
BEREKEND (A) ",/>);  
FOR I:=1 STEP 1 UNTIL N DO WRITE(OUT,<X20,E12.4,X3,E12.4,  
X3,E12.4>,TJJD[I],M[I],IT[I]) ;  
WRITE(OUT,<>);  
WRITE(OUT,<"ALPHA",X9,"ETA",X7,"DELTA",X8,"BETA"  
/>);  
WRITE(OUT,<4E12.4,/>,ALFA,ETA,DELTA,BETA) ;  
END;  
END.
The inputfile for ITERATION is called INPAR and it is made up as follows:

100  NAAM
200  ALFA,ETA,N,IO,D,KMAX,IMAX,AE,RE
300  M[1],TIJD[1]
400  M[2],TIJD[2]
500  M[3],TIJD[3]
600  M[4],TIJD[4]
700  M[5],TIJD[5]
800  M[6],TIJD[6]
900  M[7],TIJD[7]
1000 M[8],TIJD[8]

ETA and IO are starting values for the iteration. The other variables are specified in the comment to ITERATION. Below an example of an inputfile is given. The outputfile OUT on the next pages is generated with this input.

100  6/750/23KV/PAG1
200  1.087,2,8,146@-6,1,7,8,3,8,-3,8,-3,200
300  165 @-6 , 65 ,
400  180 @-6 , 78 ,
500  195 @-6 , 91 ,
600  206.3@-6 , 104 ,
700  135 @-6 , 110,5 ,
800  103.5@-6 , 117 ,
900  60 @-6 , 130 ,
1000 34.5 @-6 , 143 ,
**MINIQUAD: INPUT REPORT**

**METHOD**
- MARQUARDT

**DERIVATIVES**
- NUMERICAL

**LINEMIN METHOD**
- WEAK QUADRATIC SEARCH

**NUMBER OF VARIABLES**
- 2

**NUMBER OF RESIDUALS**
- 4

**MAXIMUM NUMBER OF ITERATIONS**
- 8

**OUTPUT PARAMETER**
- 1

**EPS FOR ABSOLUTE ACCURACY**
- 1.00E-15

**EPS FOR RELATIVE ACCURACY**
- 1.00E-03

**STOPCRIT**
- 2 FROM $1 = \text{NORMDELTAX} < \text{EPSABS} + \text{EPSREL} \times \text{NORM(XK)}$
- $2 = \text{DELTAFX} < \text{EPS} + \text{EPSREL} \times \text{ABSFX(K)}$
- $3 = \text{NORMGRAD} < \text{EPSABS} + \text{EPSREL} \times \text{NORMGRAD(X0)}$

*** VARIABLES ***

<table>
<thead>
<tr>
<th>X</th>
<th>STARTING</th>
</tr>
</thead>
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**FX**
- 1.44406332E-09

*** VARIABLES ***

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Solution

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De code van de meting is: 6/750/23KV/PAG1

Iteratiestartstroom IO is: 1.4600E-04

Beginstroom IO is: 1.3550E-04

De meetpunten zijn

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ALPHA ETA DELTA BETA

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APPENDIX III: CALCULATED SWARM PARAMETERS FOR MIXTURES C$_3$F$_6$/N$_2$

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<th>Voltage (kV)</th>
<th>$E/p$ (V/cm torr)</th>
<th>$v_e(N_2)$ ($10^7$ cm/sec)</th>
<th>$v_{app}(C_3F_6/N_2)$ ($10^7$ cm/sec)</th>
<th>$\alpha(N_2)$ (1/cm)</th>
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