MASTER

Electromagnetic-wave scattering by a circular cylinder of arbitrary radius

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Electromagnetic-Wave Scattering by a
Circular Cylinder of Arbitrary Radius

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Summary

This report discusses the scattering of obliquely incident electromagnetic waves by a perfectly conducting circular cylinder of arbitrary radius and finite length. Initially, plane-wave incidence is examined. For that purpose, a model based on the Uniform Theory of Diffraction (UTD) is developed. It can be used to predict the field strength in any arbitrary point in the space surrounding the cylinder. For a cylinder with a small radius compared to the wavelength, an alternative method, viz. the exact solution, is applied to predict the field. This is necessary, since the use of the UTD is limited to objects with surfaces that have radii of curvature which are large in terms of wavelengths.

Results from both theories as a function of cylinder radius and angle of incidence are presented.

Furthermore, a version of the UTD model that suits spherical-wave incidence is described. Results from this model are compared to results from measurements performed at Eindhoven University of Technology as a function of cylinder radius and orientation.

The theory described in the present report is included in a model for calculating the off-axis radiation pattern of a Cassegrain antenna system, taking into account direct strut scattering. Typical results as a function of antenna orientation are given.

Next, the interaction between the scattered fields emanating from multiple parallel cylinders is examined. Simulation and measurement results are presented and compared.

The report finishes with conclusions drawn from the results, and finally, recommendations are given. It is found that the results obtained from UTD calculations agree well with both the exact solution and the measurements when the cylinder radius is 0.3 times the wavelength or more. Furthermore, it is demonstrated, using the developed model, that it is not permissible in general to neglect the influence of direct strut scattering to the radiation pattern of a Cassegrain antenna system. Finally, it is found that the ‘single-cylinder’ model is often a sufficient tool to predict the field strength in multiple-cylinder configurations, as long as single-reflected and single-diffracted rays are not blocked.
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List of abbreviations

2-D         two-dimensional
3-D         three-dimensional
CCIR        Comité Consultatif Internationale des Radiocommunications
DUT         device under test
EC          Telecommunications Division (at EUT)
EM          electromagnetic
EUT (or TUE) Eindhoven University of Technology
FFT         fast Fourier transform
GO          geometrical optics
GTD         geometrical theory of diffraction
IFR         induced-field ratio
MoM         method of moments
OS          optical shadow
PC          personal computer
PTD         physical theory of diffraction
R           receiver
SSF         site shielding factor
T           transmitter
TE          transverse electric
UTD         uniform (geometrical) theory of diffraction
VNA         vector network analyzer
VSAT        very small-aperture terminal
WR          rectangular waveguide
List of symbols

\( a \)
- cylinder radius

\( a \times b \)
- dimensions of the aperture of a rectangular-waveguide antenna \((a \geq b)\)

\( a_b \)
- radius of curvature in the binormal direction

\( a_t \)
- radius of curvature in the tangential direction

\( A(\cdot) \)
- spreading factor

\( \vec{A}, \vec{B} \)
- points that determine the spatial position of the cylinder

\( B \)
- bandwidth

\( \vec{b} \)
- unit binormal vector

\( c \)
- speed of light in vacuum \((2.9979 \cdot 10^8 \text{ m/s})\)

\( \vec{C} \)
- point on the axis of rotation

\( C_n(\cdot) \)
- any arbitrary Bessel function of order \( n \)

\( d_c \)
- distance between the transmitting antenna and the cylinder axis of symmetry

\( d_d \)
- distance between \( \vec{M} \) and the cylinder axis of symmetry

\( d_{ma} \)
- distance between the main aperture and the sub-reflector edges of a Cassegrain antenna system

\( d_T \)
- distance between the axis of rotation and the transmitting antenna aperture

\( d_{TR} \)
- horizontal distance between the transmitting antenna and the receiving antenna

\( D_m \)
- main-reflector diameter

\( D_s \)
- sub-reflector diameter

\( e \)
- eccentricity

\( \vec{E} \)
- electric field

\( E_0 \)
- electric-field amplitude

\( f \)
- frequency

\( f_c \)
- center frequency

\( F \)
- focal distance

\( F(\cdot) \)
- transition function of the UTD

\( \mathcal{F}_n(\cdot) \)
- \( n \)-th term of a sum

\( h_R \)
- height of the receiving antenna above the ground

\( h_T \)
- height of the transmitting antenna above the ground

\( \vec{H} \)
- magnetic field

\( H_0 \)
- magnetic-field amplitude

\( H_n^{(2)}(\cdot) \)
- Hankel function of the second kind of order \( n \)

\( j \)
- \( \sqrt{-1} \)

\( J_n(\cdot) \)
- Bessel function of the first kind of order \( n \)

\( k \)
- wavenumber

\( l \)
- integer
List of Symbols

\( \ell \) integer
\( \ell_{1,2,3} \) lengths needed to determine \( \beta \)
\( M \) point on the axis of rotation of an array with multiple cylinders
\( n, N, N' \) integer
\( \hat{n} \) unit normal vector
\( O \) origin of a coordinate system
\( O(\cdot) \) of order
\( p^*(\cdot), q^*(\cdot) \) Fock scattering functions
\( P^t \) total power
\( \bar{P} \) observation point
\( \bar{P}' \) (projected) observation point with \( z_p = 0 \)
\( \bar{P}_0 \) reference point
\( \bar{P}_{s,h}(\cdot) \) soft/hard Pekeris caret functions
\( \bar{Q}_d \) attachment point on the cylinder surface
\( \bar{Q}_d \) point of departure on the cylinder surface
\( \bar{Q}_r \) reflection point on the cylinder surface
\( r, \theta, \phi \) spherical coordinates
\( R \) Rayleigh distance
\( R_{s,h} \) soft/hard reflection coefficients
\( \mathcal{R} \) dyadic reflection coefficient
\( s, s' \) length of a ray
\( \hat{s} \) direction of propagation of a reflected or diffracted wave
\( \hat{s}' \) direction of propagation of the directly incident wave
\( \text{sgn}(\cdot) \) signum function
\( t \) surface path length
\( \hat{t} \) unit surface tangent vector
\( T_{s,h} \) soft/hard generalized transmission coefficients
\( T \) position of the transmitting aperture center
\( T \) generalized (dyadic) transmission coefficient
\( \hat{u} \) vector
\( \hat{v} \) unit vector (length 1)
\( \hat{v}_\parallel \) normalized parallel component of a vector
\( \hat{v}_\perp \) normalized perpendicular component of a vector
\( V, W \) plane
\( x \) real variable
\( x, y, z \) Cartesian coordinates
\( \hat{x}_1, \hat{x}_2, \hat{x}_3 \) vector triplet
\( X^{r,d} \) parameter used in the calculation of reflection/diffraction coefficients
\( Y_n(\cdot) \) Bessel function of the second kind (Weber’s function) of order \( n \)
\( z \) complex variable
\( Z_0 \) free-space impedance \((120\pi \ \Omega)\)
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\[ \alpha_a \] angle in a plane perpendicular to the cylinder axis of symmetry, corresponding to the attachment point on the cylinder surface
\[ \alpha_d \] angle in a plane perpendicular to the cylinder axis of symmetry, corresponding to the point of departure on the cylinder surface
\[ \alpha_r \] angle in a plane perpendicular to the cylinder axis of symmetry, corresponding to the reflection point on the cylinder surface
\[ \beta \] angle between an incident ray and the cylinder axis of symmetry
\[ \gamma \] reflection angle in the tangential plane
\[ \Gamma(\cdot) \] Gamma function
\[ \delta \] angle that determines the direction of the horizontally polarized field vector
\[ \delta_{abs,rel} \] absolute/relative error
\[ \Delta \] difference
\[ \varepsilon_0 \] permittivity of vacuum \( \left( \frac{1}{36\pi} \cdot 10^{-9} \text{ F/m} \right) \)
\[ \zeta \] angle needed for rotation around the y axis
\[ \eta \] angle used to detect optical blockage for plane-wave incidence
\[ \sqrt{d\eta_a/d\eta_d} \] surface ray spreading factor
\[ \vartheta \] help angle in the plane of rotation used with multiple cylinders
\[ \kappa \] angle in the plane of rotation used with multiple cylinders
\[ \lambda \] wavelength
\[ \mu_0 \] permeability of vacuum \( (4\pi \cdot 10^{-7} \text{ H/m}) \)
\[ \nu \] real variable
\[ \xi \] angle used to detect optical blockage for spherical-wave incidence
\[ \xi_r,d \] Fock parameter
\[ \pi \] 3.14159265...
\[ \rho, \phi, z' \] cylindrical coordinates
\[ \rho_{1,2} \] first and second radius of curvature of a wavefront or a surface
\[ \sigma \] conductivity
\[ \Sigma \] cylinder surface
\[ \tau \] processing time
\[ \varphi \] angle of rotation in the measurement setup
\[ \varphi_a \] azimuth angle
\[ \varphi_e \] elevation angle
\[ \Phi_{OS} \] angle corresponding to the optical-shadow boundary
\[ \chi \] angle by which the cylinder is rotated in the experimental setup
\[ \psi \] reflection angle for spherical waves
\[ \omega \] angular frequency
\[ \nabla \] \( \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \)
Chapter 1

General introduction

Man seeks to live as efficient as possible. He persistently attempts to improve and extend his facilities to communicate over large distances. If such large-distance communication is performed by electromagnetic (EM) means, it is called telecommunication. The kind of telecommunication considered in this report is radiocommunication, i.e. communication via high-frequency EM waves that propagate unguided in space.

For long-distance radiocommunication, satellites are often employed. One can think of point-to-point connections, such as satellite/earth-station communication channels, or point-to-area connections, as used in mobile communication or commercial television.

If the down link of a satellite channel is considered, it is fair to say that the waves arriving at the earth's surface can be considered to be locally plane. The receiving antennas, which can vary from large earth-station antennas to small antennas on top of a moving vehicle, have a directive radiation pattern. This implies that they are to be well-directed towards the transmitting satellite in order to avoid loss and create optimal communication facilities.

However, the off-axis properties of practical antennas are not ideal [VAN DOOREN 1993A], which implies that waves incident from directions other than boresight will influence the wanted signal via the sidelobes of the antenna pattern. This causes (unwanted) interference. Possible interfering signals can be transmitted by communication systems other than the system under consideration, such as transhorizon interference, but they can also be caused indirectly by the satellite signal. Even if the antenna is optimally directed towards the satellite, there will be unwanted off-axis contributions to the received field due to (multiple) reflection and (multiple) diffraction of the waves transmitted by the satellite. These (multipath) phenomena are caused by the presence of conducting edges and surfaces near the propagation path of the waves. One can think of the unavoidable presence of reflector rims, struts, and other antenna support structures, but also environmental obstacles such as buildings or even the earth's surface can be thought of.

For analysis purposes, it is desirable to have a model to predict the impact of these unwanted signals as a function of any antenna orientation with respect to the wanted field transmitted by the satellite. This can be achieved by modeling the antenna system and its vicinity by objects of simple shapes, and calculate the total EM field that is theoretically expected to arrive at the feed of the receiving earth-station antenna, taking into consideration the interaction of the incident wave with the objects. This total field is composed of a directly incident component and a scattered component.
For the purpose of field strength prediction, a number of theories were developed over the past decennia. At Eindhoven University of Technology (EUT), most field strength prediction problems are tackled using the Uniform (geometrical) Theory of Diffraction (UTD) [KOUYOUJMJIAN 1974], which is an extension of the Geometrical Theory of Diffraction (GTD) [KELLER 1962]. This is an asymptotic technique for calculating the scattered-field components, using reflection and diffraction coefficients. At EUT a computer model is under development, that can be used to predict the EM fields at reflector antenna sites (close to or far from obstacles) [VAN DOOREN 1991/1992/1993A, GOVAERTS 1991/1993, VAN DE GRIENDT 1993], and in urban environments [VAN DOOREN 1993B]. It was found that most obstacles are easily modeled by (a combination of) three-dimensional (3-D) objects having eight vertices that may coincide. In this approximation, obstacles with curved surfaces are left out, since they cannot be easily modeled. Except for the rounded top obstacle [VAN DOOREN 1991], the latter were never analyzed completely at EUT, although in [HOEKS 1992] a 2-D self-consistent GTD solution to the scattering by (polygonal) cylinders is discussed. The need for a computer model to predict the EM field near circular cylinders arises, however, from their non-negligible presence in reflector systems and urban environments. Examples are the (often circular) struts of parabolic reflectors, lampposts, high-tension wires, silos, chimneys, and so on.

In this report, a uniform 3-D model for the electromagnetic scattering by a circular cylinder is developed. Initially, the model involves oblique plane-wave incidence, but, in order to be able to compare the simulations with measured results, it must be adjustable for spherical-wave incidence. It was decided to use the UTD, on the one hand to obtain a model compatible with the models used at EUT, and on the other since no suitable alternative is available. The Method of Moments (MoM), which is widely used in electromagnetic computations, and related techniques such as the induced-field ratio (IFR) technique [RUSCH 1989], cannot be used in this context since they need large memory capacity and runtime, from which they cannot be run on a personal computer (PC) [FOSTER 1993].

The restrictions and complications in the use of the UTD for obstacles with cylindrical shapes are examined in this report. It is expected that the UTD fails if the cylinder radius normalized to the wavelength is lower than a certain threshold. This threshold is to be determined, and for the region in which the UTD fails, an alternative solution is to be found in order to obtain a model suitable for arbitrary radius. A number of techniques for scattering by wires are known, but using these in addition to the UTD may leave a region, where the cylinder radius is neither small nor large, uncovered. If so, a solution must be found for this transition region to link the wire method to the UTD.

The model can be verified by comparing simulation results with results from scattering measurements, performed at EUT. The validity of the ‘single-cylinder’ model for multiple-cylinder configurations can also be experimentally verified. An application of the model is found in including the procedure in the off-axis radiation pattern calculation model [VAN DOOREN 1993A] of a Cassegrain antenna system, to calculate strut scattering effects.
Chapter 2

UTD solution of the scattering problem

At the Telecommunications Division of Eindhoven University of Technology, field-strength prediction problems are mostly tackled using the Uniform Theory of Diffraction (UTD), which is an extension of the Geometrical Theory of Diffraction (GTD) proposed by [KELLER 1962]. In this chapter the UTD for curved surfaces is treated for the case in which the obstacle is a circular cylinder.

In order to facilitate the calculation of the EM scattering of a plane wave by a circular cylinder of arbitrary radius $a$ (see figure 2.1), a simple model is constructed to which any arbitrary configuration with one cylinder and one incident plane wave can be reduced. It was found that a relatively simple configuration can be achieved if the whole scenery is rotated and translated such that the cylinder axis coincides with a prescribed axis and that the incident wave propagates parallel to a prescribed plane. It is explained in this chapter that this transformation simplifies the determinations of the shadow boundaries and the diffraction attachment and/or reflection points on the cylinder surface. In the following it is assumed that the cylinder axis coincides with the $y$ axis and that the direction of propagation of the incident field is in the plane $z = 0$ and has an $x$ component that is positive or equal to zero, which is visualized in figure 2.2. If all spatial descriptions are based on Cartesian $(x, y, z)$ coordinates, the solution to the problem of field prediction is probably least laborious. It can easily be verified that any configuration with a single cylinder and an incident plane wave can be reduced to the geometry assumed.

In this chapter the problems are analyzed within the geometry assumed, i.e. after the transformation is carried out. The necessary translations and rotations to obtain this

![Circular cylinder](image)

*Figure 2.1: Circular cylinder*
The obstacle is a circular cylinder of finite length. It is specified by the positions of its top centers and its radius $a$, as depicted in figure 2.1. Its top planes are assumed to be perpendicular to the cylinder symmetry axis. The cylinder starts at $y = 0$ with point $\overline{A}$ and it is stretched along the positive $y$ axis. The direction of propagation of the incident field is denoted by $\hat{s}'$ and must be known. The phase behavior of the incident electric field is determined by the specification of a point in space in which the phase equals zero. All lengths and distances will be given in terms of wavelengths ($\lambda$), from which a frequency specification becomes superfluous. Since it is assumed that the $x$ component of the vector of propagation is positive or equal to zero, this vector can be specified by an angle $\beta$ that is in the $xy$ plane. Its definition is shown in figure 2.2, with $\beta \in [0, \pi]$. It follows that

$$\cos \beta = \hat{s}' \cdot \hat{e}_y,$$

with $\hat{e}_y$ a unit vector pointing into the $y$ direction.

The topic of this chapter is to find the scattered, and subsequently the total field at any observation point $\overline{P}$ in space, taking into account the interaction of the linearly polarized incident wave with the cylindrical obstacle. The total field includes the (possible) contributions of a directly incident field, a reflected field, and multiple diffracted fields. These contributions and the composition of the total field are treated separately in the subsequent sections.

### 2.1 Directly incident field

If the directly incident field at an arbitrary observation point $\overline{P}$ with coordinates $(x_p, y_p, z_p)$ is to be calculated, it should be determined whether this point is in the obstacle shadow.
Figure 2.3: Shadow region in the $xy$ plane

or not. If $\vec{P}$ is outside the cylinder, it might be in the shadow of the cylinder. In practice, this shadow is equivalent to that caused by three plane objects, viz. the cross-section of the cylinder with the $yz$ plane, which is rectangular, and both circular ‘top disks’. This is visualized in figure 2.3, where $\vec{P}$ is presented by its projection on the $xy$ plane $\vec{P}_l$. $\vec{P}$ is in the shadow of the rectangle if

\[(x_p \geq 0) \land (|z_p| \leq a) \land (\beta \leq \eta_1) \land (\beta \geq \eta_2) .\]  

(2.2)

The blockage by the disks is determined using a method described in [RUSCH 1989]. It is found that $\vec{P}$ is in the shadow of the ‘disk’ at $y = |\vec{AB}|$ if

\[a^2 - \left(\left(|\vec{AB}| - y_p\right) \tan \beta + x_p\right)^2 - z_p^2 \geq 0 .\]  

(2.3)

The disk at $y = 0$ casts a shadow on $\vec{P}$ if

\[a^2 - (-y_p \tan \beta + x_p)^2 - z_p^2 \geq 0 .\]  

(2.4)

If none of the expressions (2.2)-(2.4) applies, $\vec{P}$ receives the contribution of the directly incident wave. If it is known that the incident plane wave has a phase equal to zero at the reference point $\vec{P}_0$ and if amplitude and polarization of the electric field at $\vec{P}_0$ is specified by $E$, we calculate the direct field in $\vec{P}$ from

\[\vec{E}^i(\vec{P}) = E e^{-j2\pi(\vec{P}_0 \cdot \hat{s}')},\]

(2.5)

with $\vec{P}_0 \vec{P}$ expressed in terms of $\lambda$. The calculation of the directly incident field is based on straightforward Geometrical Optics (GO). Here and throughout the report, a harmonic time dependence factor $e^{j\omega t}$ is assumed, but it is suppressed in the notations.
2.2 Scattered field

According to UTD, the total field in $\vec{P}$ is composed of a number of contributions. We can take together all contributions other than the directly incident field and call the resulting composition the 'scattered field'. The scattered field is constituted of reflection and diffraction contributions.

Our main interest lies in the influence of the curved surface of the cylinder on EM waves. However, besides the curved surface, the cylinder consists of two plane disks at which also reflection and diffraction occurs. Since we are less interested in the influence of these top disks, we have not taken into account the reflection and diffraction mechanisms occurring at the top surfaces and edges. The interaction of an EM wave with (curved) edges has already thoroughly been examined by the author [GOVAERTS 1991] (with an application on reflector antennas), but including it in the model would be an unnecessary complication. Note that this omission gives rise to spatial discontinuities in the total EM field, but for our purposes we will mostly be able to avoid their occurrence.

2.2.1 Reflected field

In this subsection the field resulting from reflection at the curved surface is treated. We assume that no reflection contributions in $\vec{P} = (x_p, y_p, z_p)$ are found if

$$ (s' \cdot \hat{e}_x = 0) \lor \left( x_p^2 + z_p^2 \leq a^2 \right) \lor \left( (x_p \geq 0) \land (|z_p| < a) \right). \tag{2.6} $$

If equation (2.6) does not hold, it is dependent on $\vec{P}$ and $s'$ whether a reflection contribution is found or not. The possible reflection point at the cylinder surface corresponding to $\vec{P}$ is denoted by $\vec{Q}_r$. If the angle $\alpha_r$ is defined as in figure 2.4, it is found that

$$ \vec{Q}_r = (-a \cos \alpha_r, y_r, a \sin \alpha_r). \tag{2.7} $$

The normal vector to the cylinder surface at $\vec{Q}_r$ is given by

$$ \hat{n} = (-\cos \alpha_r, 0, \sin \alpha_r). \tag{2.8} $$

From figure 2.4 it is derived that

$$ \vec{Q}_r \vec{P} \cdot \hat{e}_z = |\vec{Q}_r \vec{P}| \sin \beta \cos(\pi - 2\alpha_r) = x_p + a \cos \alpha_r, \tag{2.9} $$

$$ \vec{Q}_r \vec{P} \cdot \hat{e}_z = |\vec{Q}_r \vec{P}| \sin \beta \sin(\pi - 2\alpha_r) = z_p - a \sin \alpha_r. \tag{2.10} $$

Note that the figure starts from Snell's law of reflection. If both equations are raised to a square and added it yields

$$ |\vec{Q}_r \vec{P}|^2 \sin^2 \beta = (x_p + a \cos \alpha_r)^2 + (z_p - a \sin \alpha_r)^2. \tag{2.11} $$
With
\[ \overrightarrow{Q_rP} \cdot \hat{e}_y = y_p - y_r = \left| \overrightarrow{Q_rP} \right| \cos \beta \] (2.12)
it is found that
\[ y_r = y_p - \frac{\cos \beta}{\sin \beta} \sqrt{(x_p + a \cos \alpha_r)^2 + (z_p - a \sin \alpha_r)^2} \] (2.13)
A similar result is found in [VAN DOOREN 1991]. The direction of propagation of the reflected ray is represented by
\[ \hat{s} = \frac{\overrightarrow{Q_rP}}{\left| \overrightarrow{Q_rP} \right|} = \frac{\overrightarrow{Q_rP}}{s} \] (2.14)
The single unknown in equation (2.13) is $\alpha_r$. Its value can be found using a simple numerical root-finding procedure on the equation
\[ (\hat{s'} + \hat{s}) \cdot \hat{n} = 0 \] (2.15)
which is a representation of Snell's law. Note that $\alpha_r \in [-\pi/2, \pi/2]$. Once $\alpha_r$ is known, it can be determined whether $\overrightarrow{Q_r}$ is on the cylinder surface or not. $\overrightarrow{Q_r}$ is on the cylinder surface if
\[ 0 \leq y_r \leq |\overrightarrow{AB}| \] (2.16)
and if so, the reflected electric field at $\overrightarrow{P}$ can be calculated using UTD according to [PATHAK 1980]
\[ \overrightarrow{E''}(\overrightarrow{P}) = \overrightarrow{E''}(\overrightarrow{Q_r}) \cdot \mathcal{R} \sqrt{\frac{\rho_1^2 \rho_2^2}{(\rho_1^2 + s)(\rho_2^2 + s)}} e^{-js} \] (2.17)
Here, \( \vec{E}^i(\vec{Q}_r) \) is the incident electric field at \( \vec{Q}_r \) which is calculated according to (2.5), \( \mathcal{R} \) is the dyadic reflection coefficient and \( \rho_{1,2}^r \) are the radii of curvature of the reflected phase front at \( \vec{Q}_r \). The dyadic reflection coefficient is calculated according to

\[
\mathcal{R} = \hat{e}^i_\perp \hat{e}^r_\perp R_s + \hat{e}^i_\parallel \hat{e}^r_\parallel R_h,
\]

in which

\[
\hat{e}^i_\perp = \hat{e}^r_\perp = \frac{\hat{s}' \times \hat{n}}{|\hat{s}' \times \hat{n}|},
\]

\[
\hat{e}^i_\parallel = \hat{e}^r_\perp \times \hat{s}',
\]

\[
\hat{e}^r_\parallel = \hat{e}^r_\perp \times \hat{s}.
\]

In equation (2.18) \( R_{s,h} \) are the soft and hard reflection coefficients, respectively, for a perfect conductor, given by [PATHAK 1980]

\[
R_{s,h} = -\sqrt{-\frac{4}{\xi r}} e^{-j(\xi r)^{3/2}} \left[ \frac{e^{-j\pi/4}}{2\sqrt{\pi \xi r}} (1 - F(X')) + \hat{P}_{s,h}(\xi r) \right].
\]

The functions \( \hat{P}_{s,h}(\cdot) \) are the soft and hard Pekeris caret functions - not to be confused with unit vectors -, respectively, expressed as

\[
\hat{P}_{s,h} = \left\{ \begin{array}{c}
 p^*(x) \\
 q^*(x)
 \end{array} \right\} e^{-j\pi/4} - \frac{e^{-j\pi/4}}{2x\sqrt{\pi}},
\]

where \( p^*(\cdot) \) and \( q^*(\cdot) \) are well-tabulated Fock scattering functions [MCNAMARA 1990]. The parameter \( \xi r \) is called the Fock parameter and is defined according to

**Figure 2.5: Angle \( \gamma \) in the tangent plane**
The function $a_t(Q_r)$ represents the radius of curvature in the direction
\[ \hat{t} = (\sin \alpha_r \sin \beta, \cos \beta, \cos \alpha_r \sin \beta) \] (2.25)
and is defined according to
\[ a_t(Q_r) = \frac{a}{\cos^2 \gamma}, \] (2.26)
with $\gamma$ as shown in figure 2.5. It can be calculated that
\[ \tan \gamma = \frac{\cos \beta}{\sin \beta \sin \alpha_r}. \] (2.27)

The function $F(\cdot)$ in equation (2.22) is the transition function, defined according to [KOUYOUJMJIAN 1974]
\[ F'(x) = 2j\sqrt{x} e^{jx} \int_0^\infty e^{-ju^2} du, \] (2.28)
which can be approximated by a sum of just a few terms for small and large arguments (see for example [GOVAERTS 1991]). The parameter $X^r$ is defined according to [PATHAK 1980] and in this case it can be reduced to
\[ X^r = 4\pi s (\hat{n} \cdot \hat{s})^2. \] (2.29)

From $\hat{n}$ and $\hat{t}$ the binormal vector $\hat{b}$ is found:
\[ \hat{b} = \hat{t} \times \hat{n}, \] (2.30)
with corresponding radius of curvature
\[ a_b(Q_r) = \frac{a}{\sin^2 \gamma}. \] (2.31)

We find that, because of plane-wave incidence, the generally laborious calculations of $\rho_{1,2}'$ [PATHAK 1980] reduce to
\[ \rho_{1,2}' = \frac{a_b(Q_r) a_t(Q_r) (\hat{n} \cdot \hat{s})}{a_b(Q_r) + a_t(Q_r) (\hat{n} \cdot \hat{s})^2} \frac{1}{1 \pm 1}, \] (2.32)
which implies that $\rho_2' \to \infty$. Using equation (2.17), the reflected-field contribution at $\tilde{P}$ can be calculated using the parameters given above.
2.2.2 Diffracted fields

A continuous field distribution in space owing to the presence of the curved surface theoretically requires, besides the direct-incidence and reflection contributions, a third or even more contributions. This necessity will be demonstrated in chapter 4. These contributions are caused by diffraction at the cylinder. The diffraction mechanism involves the rays incident upon the cylinder tangent to its surface. In the chosen configuration they will be in the planes \( z = a \) and \( z = -a \). In these planes they attach to the surface at \( x = 0 \) and then ‘creep’ along the surface. Their paths along the surface must be of stationary optical-path length and it can easily be verified that they can be traced along a helix. The pitch of the helix depends on the angle \( \beta \), the cylinder radius \( a \) and the position of the attachment points. The diffracted field resulting from the incident ray under consideration is constituted by all contributions departing from different positions \( Q_d \) at the corresponding helix. In figure 2.6 the special case situation in which \( \beta = \pi/2 \) is depicted. Note that the helix degenerates into a circle in this case.

We are interested in the rays arriving at the observation point \( \bar{P} \). This implies that a point \( Q_d \) must be found, and from \( Q_d \) the position of the attachment point \( Q_a \) can be derived, because they are both at the same helix. As with reflection, it is assumed that no diffraction occurs if

\[
(\hat{s}' \cdot \hat{e}_x = 0) \lor \left( x_p^2 + z_p^2 \leq a^2 \right).
\]  

(2.33)

If the cylinder is infinitely long, an infinite number of diffraction contributions is found per observation point (while equation (2.33) is false). Each point of departure corresponds to multiple attachment points, viz. all points at which the helix touches either the plane \( z = a \) or \( z = -a \). However, it is permitted to neglect the contributions that have relatively long ‘creeping-wave paths’ and it will be shown in the next chapter that the consideration of a maximum of two diffraction contributions (in total) is sufficient, in general, to determine

\[\]
the total diffraction contribution to the field in \( \hat{P} \).

Analogous to reflection, the points of departure \( \vec{Q}_d \) are found by determining a corresponding angle \( \alpha_d \) (either with a positive or a negative sign), as indicated in figure 2.6. With \( \vec{Q}_d = (-a \cos \alpha_d, y_d, a \sin \alpha_d) \) it follows that

\[
y_d = y_p - \frac{\cos \beta}{\sin \beta} \sqrt{(x_p + a \cos \alpha_d)^2 + (z_p - a \sin \alpha_d)^2},
\]

which shows similarity with equation (2.13). With

\[
\hat{s} = \frac{\vec{Q}_d P}{|\vec{Q}_d P|} = \frac{\vec{Q}_d P}{s}
\]

and

\[
\hat{n} = (-\cos \alpha_d, 0, \sin \alpha_d),
\]

\( \alpha_d \) can be found solving the equation

\[
\hat{n} \cdot \hat{s} = 0
\]

by using a root-finding procedure. It is important to carefully select the appropriate \( \alpha_d \) interval(s). In figure 2.6, for instance, these are given by \( \alpha_d \in [\pi + l2\pi, \pi - \arctan(z_p/x_p) + l2\pi] \) for the ray departing from \( \vec{Q}_d \) and \( \alpha_d \in [\pi - \arctan(z_p/x_p) + l2\pi, 3\pi/2 + l2\pi] \) for that departing from \( \vec{Q}_d' \) \((l \in \{0, \pm1, \pm2, \ldots \})\). The point \( \vec{Q}_d' \) corresponds to the attachment point at \( z = -a \), which is not depicted in the figure. If \( \alpha_d \) has been determined, \( y_d \) can be found from equation (2.34).

From \( y_d, y_a \) can be derived since the displacement in \( y \) direction, which is a function of \( \alpha_d \), is known. Here, a distinction must be made between the contributions that attach at \( z = a \) (upper attachment) and \( z = -a \) (lower attachment), respectively. It will be demonstrated later on that it is sufficient to only consider the contributions with \( \alpha_d \in [\pi/2, 5\pi/2] \) for upper attachment and \( \alpha_d \in (-\pi/2, -5\pi/2] \) for lower attachment. Although the angle range spanned is \( 2\pi \), these intervals deviate from conventional angle intervals for implementation reasons. For convenience the following parameter is introduced:

\[
\Delta \alpha = \left\{ \begin{array}{ll}
\alpha_d - \frac{\pi}{2}, & \text{upper attachment} \\
\alpha_d + \frac{\pi}{2}, & \text{lower attachment}
\end{array} \right. . \tag{2.38}
\]

The \( y \) coordinate of the attachment point \( \vec{Q}_a \) is given by

\[
y_a = y_d - \frac{a \Delta \alpha}{\tan \beta} . \tag{2.39}
\]

The diffraction contribution under consideration actually contributes if both \( \vec{Q}_a \) and \( \vec{Q}_d \) are on the cylinder surface, i.e.

\[
\left(0 \leq y_a \leq |\, \overrightarrow{AB} \,| \right) \land \left(0 \leq y_d \leq |\, \overrightarrow{AB} \,| \right) . \tag{2.40}
\]
If the Boolean expression (2.40) is true, the corresponding diffraction contribution at $P$ is calculated with the UTD according to [Pathak 1980]

$$E^d(P) = E^i(Q_a) \cdot T \frac{e^{-j2\pi s}}{\sqrt{s}}.$$  \hspace{1cm} (2.41)

Here, $T$ is the generalized transmission coefficient. The incident field $E^i(Q_a)$ is determined by (2.5). Note that (2.41) and all following equations are applicable for both upper and lower attachment, unless indicated otherwise.

The generalized transmission coefficient is calculated according to

$$T = T_s \hat{b}_a \hat{b}_d + T_h \hat{n}_a \hat{n}_d,$$  \hspace{1cm} (2.42)

with

$$\hat{b}_a = (\pm \cos \beta, \mp \sin \beta, 0),$$ \hspace{1cm} (2.43)

$$\hat{b}_d = (\sin \alpha_d \cos \beta, \mp \sin \beta, \cos \alpha_d \cos \beta),$$ \hspace{1cm} (2.44)

$$\hat{n}_a = (0, 0, \pm 1),$$ \hspace{1cm} (2.45)

$$\hat{n}_d = (- \cos \alpha_d, 0, \sin \alpha_d),$$ \hspace{1cm} (2.46)

for upper and lower attachment, respectively. In equation (2.42) $T_{s,h}$ are the soft and hard generalized transmission coefficients, respectively, for a perfect conductor, given by [Pathak 1980]

$$T_{s,h} = \frac{-\sqrt[3]{8\pi a_t}}{\sqrt{\pi}} \left[ e^{-j\pi/4} \frac{1 - F(X^d)}{2 \xi^d \sqrt{\pi}} \left( 1 + \hat{P}_{s,h}(\xi^d) \right) \right] e^{-j2\pi t}.$$ \hspace{1cm} (2.47)

The functions $\hat{P}_{s,h}$ were given in equation (2.23). The Fock parameter is defined by

$$\xi^d = \frac{\sqrt[3]{8\pi a_t}}{a_t} t = \sin \beta \sqrt{\frac{\pi a}{\sin \beta}} \Delta \alpha,$$ \hspace{1cm} (2.48)

which is found from the tangential radius of curvature

$$a_t = \frac{a}{\sin^2 \beta},$$ \hspace{1cm} (2.49)

and the path length $t$, which is derived easily:

$$t = \frac{a \Delta \alpha}{\sin \beta}.$$ \hspace{1cm} (2.50)

The argument of the transition function $F(\cdot), X^d$, is given by

$$X^d = \frac{\pi s (\xi^d)^2}{3\sqrt{\pi^2 a_t^2}}.$$ \hspace{1cm} (2.51)
For \( \Delta \alpha \to 0 \) (and thus \( \xi^d \to 0 \)), we approximate \( T_{s,h} \) in order to avoid division by zero:

\[
\lim_{\Delta \alpha \to 0} T_{s,h} = \lim_{\Delta \alpha \to 0} \frac{-\sqrt{\pi} a_t}{\sqrt{\pi}} \left[ \frac{e^{-j \pi/4}}{2 \xi^d \sqrt{\pi}} \left( 1 - F(X^d) \right) + \hat{P}_{s,h}(\xi^d) \right] e^{-j 2\pi t} \approx \\
\approx \lim_{\Delta \alpha \to 0} \frac{-\sqrt{\pi} a_t}{\sqrt{\pi}} \left[ \frac{1}{2 \xi^d \sqrt{\pi}} \left( \frac{\pi \sqrt{s} \xi^d}{\sqrt{\pi} a_t} + \mathcal{O}((\xi^d)^2) \right) - \left\{ \begin{array}{c} p^*(\xi^d) \\ q^*(\xi^d) \end{array} \right\} e^{-j \pi/4} \right] \approx \\
\approx \frac{1}{2} \sqrt{s} - \frac{\sqrt{\pi} a_t}{\sqrt{\pi}} \left\{ \begin{array}{c} p^*(0) \\ q^*(0) \end{array} \right\} e^{-j \pi/4} .
\]

Note that these formulas apply for a plane incident wave only. Substitution of the appropriate parameters introduced in this section yields the diffracted-field contribution from equation (2.41). In the following, the contribution with upper attachment is denoted by \( \hat{E}^{d1}(\vec{p}) \) and that with lower attachment by \( \hat{E}^{d2}(\vec{p}) \).

### 2.3 The total field

If \( \vec{s}^i \) is the vector of propagation in the original coordinate system (which we will call the \( x'y'z' \) system), i.e. before the configuration is transformed to the model of figure 2.2, the polarization vectors are defined as follows. In case of horizontal polarization, the polarization vector is assumed to be in the horizontal \( x'y' \) plane. The unit horizontally polarized electric field vector \( \hat{E}'_{HOR} \) is shown in figure 2.7 and it is defined by

\[
\hat{E}'_{HOR} = (\sin \delta, \cos \delta, 0) ,
\]
in which \( \delta \) is determined by
\[
\delta = - \arccos \left( \left( \hat{s}^i - (\hat{s}^i \cdot \hat{e}_z') \hat{e}_z' \right) \cdot \hat{e}_y' \right) \, \text{sgn} (\hat{s}^i \cdot \hat{e}_y') .
\] (2.54)

Here, \( \text{sgn}(\cdot) \) is the well-known signum function. Hence, the unit vertically polarized electric field vector is found from
\[
\hat{E}_{VER}' = \hat{s}^i \times \hat{E}_{HOR}' .
\] (2.55)

After the rotations, necessary to obtain the model assumed, are applied on the polarization vectors, \( \hat{E} \) in equation (2.5) is found from the rotated polarization vectors \( \hat{E}_{HOR} \) and \( \hat{E}_{VER} \) according to
\[
\vec{E} = \begin{cases} 
E_0 \hat{E}_{HOR} , \text{ horizontal polarization} \\
E_0 \hat{E}_{VER} , \text{ vertical polarization}
\end{cases} ,
\] (2.56)

with \( E_0 \) the field magnitude. The scattered field is composed of the reflection and diffraction contributions:
\[
\vec{E}^s(P) = \vec{E}^r(P) + \vec{E}^{d1}(P) + \vec{E}^{d2}(P) ,
\] (2.57)

and we find the total field \( \vec{E}^t \) at \( P \) by adding all separate contributions treated previously:
\[
\vec{E}^t(P) = \vec{E}^i(P) + \vec{E}^s(P) = \vec{E}^i(P) + \vec{E}^r(P) + \vec{E}^{d1}(P) + \vec{E}^{d2}(P) .
\] (2.58)
Chapter 3

Exact solution of the scattering problem

With the UTD described in the previous chapter some approximations are made. We are interested in the limitations in the use of this theory. According to [KELLER 1962] the agreement between the GTD solution and the exact solution in the case of scattering of a plane wave by a circular cylinder under normal incidence ($\beta = 90^\circ$) is 'quite good' if the wave fits twice or more in the circumference, i.e. $ka \geq 2$, from which it follows that the radius $a \geq \lambda/\pi \approx 0.32\lambda$. This means that we are not able to model cylindrical obstacles with small diameters, theoretically. A number of solutions have been published to plane-wave scattering by a wire, but mostly these depend on the cylinder length. Examples are the Physical Theory of Diffraction (PTD) solution proposed by [UFIMTSEV 1962], and a combination of solutions given in [BOWMAN 1969 (Ch. 12), EINARSSON 1969].

In order to present an alternative method for calculating the scattered field, in this chapter the exact solution to the problem of electromagnetic plane-wave scattering by a circular cylinder of radius $a$ is derived. With the use of the results from this method, the accuracy of the UTD is examined in chapter 4.

The geometrical model used in this chapter is comparable with the one depicted in figure 2.1, except that the cylinder is assumed to be infinitely long. By this assumption, the effects of both cylinder tops are avoided automatically.

We introduce a cylindrical coordinate system ($\rho, \phi, z'$), which is related to the $xyz$ system according to

$$
\begin{align*}
\hat{e}_\rho &= \cos \phi \hat{e}_x + \sin \phi \hat{e}_z \\
\hat{e}_\phi &= -\sin \phi \hat{e}_x + \cos \phi \hat{e}_z \\
\hat{e}_{z'} &= -\hat{e}_y
\end{align*}
$$

(3.1)

In the following the exact solutions for horizontal and vertical polarization are separately derived.

3.1 Horizontal polarization

In the horizontally polarized case the incident electric vector $\vec{E}^i$ is in the plane $\phi = 0$ (which is the $xy$ plane), as shown in figure 3.1.
It is determined by\(^1\)
\[
\vec{E}^i = E_0 (-\cos \beta \hat{e}_x + \sin \beta \hat{e}_y) e^{-jk(x \sin \beta + y \cos \beta)}. \tag{3.2}
\]

From a decomposition of (3.2) it follows that [HARRINGTON 1961 (CH. 5)]

\[
E_{z'}^i = -E_0 \sin \beta e^{jk' \cos \beta} e^{-jk \sin \beta \cos \phi} = -E_0 \sin \beta e^{jk' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n (k \rho \sin \beta) e^{j n \phi}, \tag{3.3}
\]

in which \(J_n(\cdot)\) is the Bessel function of the first kind of order \(n\). Since the incident wave is transverse electromagnetic we find that

\[
H_{z'}^i = 0. \tag{3.4}
\]

We have calculated the other cylindrical components, \(E_{\phi}^i\) and \(E_{\rho}^i\), from (3.3) and (3.4), using Maxwell’s equations

\[
\nabla \times \vec{E}^i = -j \omega \mu_0 \vec{H}^i, \tag{3.5}
\]

\[
\nabla \times \vec{H}^i = j \omega \varepsilon_0 \vec{E}^i, \tag{3.6}
\]

\[
\nabla \cdot \vec{E}^i = 0, \tag{3.7}
\]

\[
\nabla \cdot \vec{H}^i = 0. \tag{3.8}
\]

From equations (3.5) and (3.6) it follows that

\[
-j \omega \mu_0 H_{\rho}^i = \frac{1}{\rho} \frac{\partial E_{z'}^i}{\partial \phi} - \frac{\partial E_{\phi}^i}{\partial z'}. \tag{3.9}
\]

\(^1\)In this analysis we assume that all lengths are known in terms of meters, rather than wavelengths.
Exact Solution of the Scattering Problem

\[ -j \omega \mu_0 H^i_\phi = \frac{\partial E^i_\rho}{\partial z'} - \frac{\partial E^i_{z'}}{\partial \rho}, \]  
\[ (3.10) \]

\[ -j \omega \mu_0 \rho H^i_{z'} = \frac{\partial (\rho E^i_\phi)}{\partial \rho} - \frac{\partial E^i_\phi}{\partial \phi}, \]  
\[ (3.11) \]

\[ j \omega \varepsilon_0 E^i_\rho = \frac{1}{\rho} \frac{\partial H^i_{z'}}{\partial \rho} - \frac{\partial H^i_\phi}{\partial z'}, \]  
\[ (3.12) \]

\[ j \omega \varepsilon_0 E^i_\phi = \frac{\partial H^i_\rho}{\partial z'} - \frac{\partial H^i_{z'}}{\partial \rho}, \]  
\[ (3.13) \]

Substitution of (3.4) and (3.9) into (3.13) yields

\[ \omega^2 \varepsilon_0 \mu_0 E^i_\phi = \frac{\partial}{\partial z'} \left( \frac{1}{\rho} \frac{\partial E^i_{z'}}{\partial \phi} - \frac{\partial E^i_\phi}{\partial z'} \right) \Leftrightarrow \]

\[ k^2 E^i_\phi + \frac{\partial^2 E^i_\phi}{\partial z'^2} = \frac{\partial}{\partial z'} \left( \frac{1}{\rho} \frac{\partial E^i_{z'}}{\partial \phi} \right) \Leftrightarrow \]

\[ \left( k^2 + \frac{\partial^2}{\partial z'^2} \right) E^i_\phi = \frac{E_0 \sin \beta \cos \beta}{\rho} e^{ikz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{in\phi}, \]  
\[ (3.15) \]

the solution of which is given as [WAIT 1955]

\[ E^i_\phi = \frac{E_0 \cos \beta}{k \rho \sin \beta} e^{ikz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{in\phi}. \]  
\[ (3.16) \]

By substituting (3.4) and (3.16) into (3.11) we find

\[ \frac{\partial E^i_\rho}{\partial \phi} = E_0 \cos \beta e^{ikz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J'_n(k \rho \sin \beta) n e^{in\phi}, \]  
\[ (3.17) \]

where \( J'_n \) means \( \partial J_n/\partial \rho \), and it follows that

\[ E^i_\rho = -j E_0 \cos \beta e^{ikz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J'_n(k \rho \sin \beta) e^{in\phi} + f_{\rho}(\rho, z'), \]  
\[ (3.18) \]

in which \( f_\rho(\rho, z') \) can be any arbitrary function of \( \rho \) and \( z' \). In the following it is demonstrated that a solution can be found in which \( f_\rho(\rho, z') = 0 \).

The components of the incident field \( E^i \) need to satisfy Maxwell’s equation (3.7), i.e.

\[ \nabla \cdot \vec{E}^i = \frac{1}{\rho} \frac{\partial (\rho E^i_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial E^i_\phi}{\partial \phi} + \frac{\partial E^i_{z'}}{\partial z'} = 0. \]  
\[ (3.19) \]
By substitution of (3.3), (3.16), and (3.18) into (3.19) and suppression of the common terms, we find

$$\nabla \cdot \vec{E} = \sum_{n=-\infty}^{\infty} c_n J'_n(\nu) + \nu \sum_{n=-\infty}^{\infty} c_n J''_n(\nu) - \frac{1}{\nu} \sum_{n=-\infty}^{\infty} n^2 c_n J_n(\nu) + \nu \sum_{n=-\infty}^{\infty} c_n J_n(\nu),$$

assuming that $f_p(\rho, \zeta') = 0$, $\nu = k\rho \sin \beta$, and $c_n = j^{-n} e^{j n \phi}$. Using the following recurrence formulas and one of their derivatives

$$J'_n(\nu) = -J_{n+1}(\nu) + \frac{n}{\nu} J_n(\nu),$$

$$J'_n(\nu) = J_n(\nu) - \frac{n+1}{\nu} J_{n+1}(\nu),$$

$$J''_n(\nu) = \frac{d}{d\nu} J'_n(\nu) = -J_{n+1}(\nu) - \frac{n}{\nu^2} J_n(\nu) + \frac{n}{\nu} J'_n(\nu),$$

it can be proved that equation (3.19) is satisfied indeed. This implies that we can avail ourselves of a description of the horizontally polarized incident field in cylindrical coordinates. Since both the $\zeta'$ component and the $\phi$ component are tangential to the cylinder surface we can easily apply the boundary conditions in order to obtain the exact description of the resulting scattered field.

The scattered field must also satisfy the wave equation. Since it is outward traveling and must vanish at $\rho \to \infty$ (according to Sommerfeld’s radiation condition) we can seek the solution in a description involving Hankel functions of the second kind [HARRINGTON 1961 (Ch. 5), KORBANSKIY 1966]. First we will consider the $\zeta'$ component of the scattered field. From the periodicity of $E_{\zeta'}$ in $\zeta'$ direction we expect that $E_{\zeta'}$ will also vary according to $e^{j \zeta' \cos \beta}$. The $\zeta'$ component of the scattered field can then be expressed in the form

$$E_{\zeta'} = e^{j \zeta' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} a_n H_n^{(2)}(k\rho \sin \beta) e^{j n \phi}.$$  

In the presence of the cylinder we can compose the $\zeta'$ component of the total field by

$$E_{\zeta'} = E_{\zeta'} + E_{\zeta'}^s.$$  

Since the cylinder is perfectly conducting, the boundary condition $E_{\zeta'} = 0$ applies at $\rho = a$ and thus it follows from (3.25) that

$$e^{j \zeta' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} \left[-E_0 \sin \beta J_n(ka \sin \beta) + a_n H_n^{(2)}(ka \sin \beta) \right] e^{j n \phi} = 0;$$

which is satisfied if

$$a_n = E_0 \sin \beta \frac{J_n(ka \sin \beta)}{H_n^{(2)}(ka \sin \beta)}.$$
The scattered-field components $E^s_{\phi}$ and $E^s_{\rho}$ are found by a procedure similar to the one used to find $E^i_{\phi}$ and $E^i_{\rho}$ from $E^i_{\omega}$. Hence it follows that

$$E^s_{\phi} = -\frac{E_0 \cos \beta}{k \rho \sin \beta} e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n(k \rho \sin \beta)}{H_{(2)}^{n}(k \rho \sin \beta)} H_{(2)}^{n}(k \rho \sin \beta) n e^{jn\phi}, \quad (3.28)$$

$$E^s_{\rho} = j E_0 \cos \beta e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n(k \rho \sin \beta)}{H_{(2)}^{(2)}(k \rho \sin \beta)} H_{(2)}^{(2)}(k \rho \sin \beta) e^{jn\phi}. \quad (3.29)$$

### 3.2 Vertical polarization

For vertical polarization we find the total electric field in the presence of a circular cylinder through the magnetic field. In polar coordinates the incident electric field is specified by

$$\vec{E}^i = E_0 \hat{e}_z e^{-j(k(x \sin \beta + y \cos \beta)).} \quad (3.30)$$

With

$$\vec{H}^i = \frac{1}{Z_0} \hat{s}' \times \vec{E}^i, \quad (3.31)$$

in which $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ ($\approx 120 \pi \Omega$) is the characteristic impedance of the medium of propagation (i.e. air), we can easily derive the incident magnetic field from (3.18), (3.16), and (3.3):

$$H^i_{\rho} = jH_0 \cos \beta e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n'(k \rho \sin \beta) e^{jn\phi}, \quad (3.32)$$

$$H^i_{\phi} = \frac{-H_0 \cos \beta}{k \rho \sin \beta} e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{jn\phi}, \quad (3.33)$$

$$H^i_{z'} = H_0 \sin \beta e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) e^{jn\phi}, \quad (3.34)$$

with $H_0 = E_0/Z_0$. From the incident magnetic field, the incident electric field can be derived using Maxwell’s equations.

It is obvious that $E^i_{z'} = 0$. From (3.12) we find

$$E^i_{\rho} = \frac{H_0 \sin \beta}{\rho \omega \varepsilon_0} e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{jn\phi}$$

$$+ \frac{H_0 \cos^2 \beta}{\rho \omega \varepsilon_0 \sin \beta} e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{jn\phi} =$$

$$= \frac{E_0}{k \rho \sin \beta} e^{jkz' \cos \beta} \sum_{n=-\infty}^{\infty} j^{-n} J_n(k \rho \sin \beta) n e^{jn\phi}. \quad (3.35)$$
Equation (3.13) yields

\[ E^i_\phi = \frac{j k H_0 \cos^2 \beta}{\omega \varepsilon_0} e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n J'_n(k \rho \sin \beta) e^{jn \phi} \]

\[ + \frac{j k H_0 \sin^2 \beta}{\omega \varepsilon_0} e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n J'_n(k \rho \sin \beta) e^{jn \phi} = \]

\[ = j E_0 e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n J'_n(k \rho \sin \beta) e^{jn \phi}. \quad (3.36) \]

It can easily be verified that the vertically polarized incident electric field satisfies Maxwell’s equation (3.7).

The \( \phi \) component of the scattered field can be expressed in the form

\[ E^s_\phi = e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)'}(k \rho \sin \beta) e^{jn \phi}. \quad (3.37) \]

In the presence of the cylinder the \( \phi \) component of the total field is given by

\[ E_\phi = E^i_\phi + E^s_\phi. \quad (3.38) \]

With a perfectly conducting cylinder, the boundary condition \( E_\phi = 0 \) applies at \( \rho = a \):

\[ e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n [j E_0 \sin \beta J'_n(ka \sin \beta) + b_n H_n^{(2)'}(ka \sin \beta)] e^{jn \phi} = 0, \quad (3.39) \]

from which it follows that

\[ b_n = -j E_0 \frac{J'_n(ka \sin \beta)}{H_n^{(2)'}(ka \sin \beta)}. \quad (3.40) \]

The scattered-field component \( E^s_\rho \) is then determined easily by

\[ E^s_\rho = \frac{-E_0}{k \rho \sin \beta} e^{j k z' \cos \beta} \sum_{n=-\infty}^{\infty} j^n \frac{J'_n(ka \sin \beta)}{H_n^{(2)'}(ka \sin \beta)} H_n^{(2)}(k \rho \sin \beta) n e^{jn \phi}. \quad (3.41) \]

The scattered field is completed by its third component \( E^s_z = 0 \), that follows from the perfect conductance of the cylinder.

### 3.3 Résumé

In this section the exact solutions found are lined up and rewritten in the original \( xyz \) coordinates. Thereby we make use of the following relations (see figure 3.2):

\[ \hat{e}_x = \cos \phi \hat{e}_\rho - \sin \phi \hat{e}_\phi \]
\[ \hat{e}_y = -\hat{e}_z \]
\[ \hat{e}_z = \sin \phi \hat{e}_\rho + \cos \phi \hat{e}_\phi \quad (3.42) \]
3.3.1 Horizontal polarization

The incident electric field is specified by

$$\vec{E}^i = \begin{pmatrix} E^i_x \\ E^i_y \\ E^i_z \end{pmatrix} = E_0 \begin{pmatrix} -\cos \beta \\ \sin \beta \\ 0 \end{pmatrix} e^{-jk(x \sin \beta + y \cos \beta)}.$$  \hspace{1cm} (3.43)

By the presence of the cylinder a scattered electric field is generated according to

$$\vec{E}^s = E_0 \begin{pmatrix} \cos \beta \sum_{n=-\infty}^{\infty} p_n \left[ j \cos \phi H_n^{(2)\prime}(\nu) + \frac{n \sin \phi}{\nu} H_n^{(2)}(\nu) \right] \\ -\sin \beta \sum_{n=-\infty}^{\infty} p_n H_n^{(2)}(\nu) \\ \cos \beta \sum_{n=-\infty}^{\infty} p_n \left[ j \sin \phi H_n^{(2)\prime}(\nu) - \frac{n \cos \phi}{\nu} H_n^{(2)}(\nu) \right] \end{pmatrix} e^{-jk \cos \beta},$$ \hspace{1cm} (3.44)

in which

$$\phi = \arctan \left( \frac{z}{x} \right),$$  \hspace{1cm} (3.45)

$$\nu = k \sqrt{x^2 + z^2 \sin \beta},$$  \hspace{1cm} (3.46)

and

$$p_n = j^{-n} \frac{J_n(ka \sin \beta)}{H_n^{(2)}(ka \sin \beta)} e^{j n \arctan(\frac{z}{x})}.$$  \hspace{1cm} (3.47)

3.3.2 Vertical polarization

In the case of vertical polarization, the incident electric field is given by

$$\vec{E}^i = \begin{pmatrix} 0 \\ 0 \\ E_0 \end{pmatrix} e^{-jk(x \sin \beta + y \cos \beta)},$$  \hspace{1cm} (3.48)
which generates the following scattered electric field:

\[
\vec{E}^s = E_0 \left( \sum_{n=-\infty}^{\infty} q_n \left[ \frac{-n \cos \phi}{\nu} H_n^{(2)}(\nu) + j \sin \phi H_n^{(2)'}(\nu) \right] e^{-jky \cos \beta}, \right.
\]

\[
\left. \sum_{n=-\infty}^{\infty} q_n \left[ \frac{-n \sin \phi}{\nu} H_n^{(2)}(\nu) - j \cos \phi H_n^{(2)'}(\nu) \right] \right) e^{-jky \cos \beta}, \quad (3.49)
\]

with

\[
q_n = j^{-n} \frac{J_n'(ka \sin \beta)}{H_n^{(2)'(ka \sin \beta)}} e^{jn \arctan \left( \frac{z}{x} \right)}. \quad (3.50)
\]

### 3.4 Normal incidence

A special case occurs if the angle \( \beta = 90^\circ \). In this situation we speak of *normal incidence* and as a consequence the expressions for the incident and scattered fields can be simplified substantially [HARRINGTON 1961 (Ch. 5)].

For horizontal polarization it follows that

\[
\vec{E}^i = \begin{pmatrix} 0 \\ E_0 \\ 0 \end{pmatrix} e^{-jkx}, \quad (3.51)
\]

and

\[
\vec{E}^s = \begin{pmatrix} -E_0 \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n'(ka)}{H_n^{(2)'(ka)}} H_n^{(2)}(kp) e^{jn \phi} \\ 0 \\ 0 \end{pmatrix}, \quad (3.52)
\]

and for vertical polarization we find

\[
\vec{E}^i = \begin{pmatrix} 0 \\ 0 \\ E_0 \end{pmatrix} e^{-jkx}, \quad (3.53)
\]

and

\[
\vec{E}^s = \begin{pmatrix} E_0 \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n'(ka)}{H_n^{(2)'(ka)}} \left[ \frac{-n \cos \phi}{kp} H_n^{(2)}(kp) + j \sin \phi H_n^{(2)'}(kp) \right] e^{jn \phi} \\ 0 \\ E_0 \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n'(ka)}{H_n^{(2)'(ka)}} \left[ \frac{-n \sin \phi}{kp} H_n^{(2)}(kp) - j \cos \phi H_n^{(2)'}(kp) \right] e^{jn \phi} \end{pmatrix}, \quad (3.54)
\]

where

\[
\rho = \sqrt{x^2 + z^2} \quad (3.55)
\]

and

\[
\phi = \arctan \left( \frac{z}{x} \right). \quad (3.56)
\]
Chapter 4

Comparison between the UTD and exact solution

In the previous chapters the UTD and exact solutions for the scattered field from a circular cylinder have been derived. In this chapter the pros and cons of both techniques will be weighed against the other in order to be able to make recommendations on which theory is to be applied to obtain accurate results for a given configuration, also taking into account the time needed. For this purpose we make use of the output obtained from Turbo Pascal computer programs in which the formulas of the theories are implemented. The first section in this chapter considers the composition of the scattered field, obtained from UTD-based calculations. Some typical UTD results are presented. Next, calculated results from both theories are compared as a function of normalized cylinder radius $a/\lambda$ and angle of incidence $\beta$. The chapter ends with a section containing recommendations.

4.1 Typical UTD results

According to Keller [KELLER 1962], the GTD solution is satisfactory when $a > 0.32\lambda$. For radii above this lower limit he found good agreement between the GTD and exact solution. In this section we will start with a cylinder having a radius $a = 3\lambda$, which substantially exceeds Keller’s lower limit. The scattered-field components resulting from plane-wave incidence upon a circular cylinder of the radius chosen have been calculated. The simulations were carried out for far-field observation; the observation points $\vec{P}$ were positioned on a half circle perpendicular to the cylinder axis of symmetry, as depicted in figure 4.1. The corresponding parameters are $\rho = 100000\lambda$ and $0 \leq \phi \leq \pi$.

The UTD solution was calculated for $\beta = 90^\circ$ and $\beta = 45^\circ$, for both horizontal and vertical polarization. In Cartesian coordinates we specify the direction of incident-wave propagation and both linear polarizations by

$$\hat{s'} = (\sin \beta, \cos \beta, 0),$$

(4.1)

$$\hat{E}_{HOR} = (-\cos \beta, \sin \beta, 0),$$

(4.2)

and

$$\hat{E}_{VER} = (0, 0, 1).$$

(4.3)
Figure 4.2 shows the power of the scattered field ('s', solid curve), relative to the power of the field in the absence of the cylinder as a function of φ for β = 90° and horizontal polarization, calculated with UTD. From its continuity we expect that it is composed correctly of all contributions. The importance of the reflected-field contribution ('r', dashed curve) is noted immediately. It represents some kind of average of the total scattered field. In chapter 2 it was mentioned that the number of diffraction contributions contributing to the scattered power is infinitely large. In our software calculations we have taken into account the two most important contributions only, viz. those that have creeping-path lengths (see equation (2.50))

\[ t < \frac{2\pi a}{\sin \beta}. \]  

These contributions are presented by dotted curves in figure 4.2 for upper attachment ('d1') and lower attachment ('d2'). For the φ interval depicted, the latter plays an important role in the path of the scattered power observed near φ = 0°, i.e. 'behind the cylinder'. It is easily explained that its influence decreases with increasing φ, since the creeping-path length t of this contribution increases with increasing φ. This path length is proportional to the number of rays that have already been 'launched' from the cylinder surface, each carrying an amount of the total energy of the diffraction contribution. This implies that the amount of energy left shows an exponential decay with the length t. The fact that this decay does not seem to be exponential in the figure is caused by the numerical approximation of the Pekeris caret functions in the transmission coefficients (2.42).

The optical-shadow region is determined by

\[ |\phi| < \Phi_{OS} = \arcsin \frac{a}{\rho}, \]

with φ ∈ [−π, π]. In this specific case it is found that \( \Phi_{OS} = 0.00172^\circ \). Outside the optical-shadow region it is observed that the upper-attachment contribution does not seem
to influence the scattered field at all. Here, the power increases with increasing $\phi$. Note that if this contribution is mirrored with respect to $\phi = 0^\circ$ or $\phi = 180^\circ$, it will coincide exactly with the lower-attachment contribution, and vice versa.

$$2\pi a \sin \beta \leq t < \frac{4\pi a}{\sin \beta}$$  \hspace{1cm} (4.6)

The creeping-path length of contribution ‘d1’ approaches $2\pi a / \sin \beta$ as $\phi$ approaches $\Phi_{OS}$ from above. At $\phi = \Phi_{OS}$ the curve shows a discontinuity (which is not very clear from the figure since $\Phi_{OS}$ is very small). The ‘new’ contribution ‘d3’ accounts for this discontinuity, but will in its turn give rise to a discontinuity at $\phi = \Phi_{OS}$ when $t = 4\pi a / \sin \beta$. This requires another upper-attachment contribution ‘d5’, etc. The power level of contribution ‘d3’, however, will never exceed the minimum value of the power of ‘d1’.

A similar reasoning applies for lower attachment too because of symmetry. Since we have concluded that the influence of ‘d1’ is negligible for $\phi$ approaching $\Phi_{OS}$ from above, a total number of two diffraction contributions suffices to represent the total diffraction contribution. This conclusion applies to all simulation results, even for very small $a/\lambda$, since no discontinuities in the scattered-field results were found.

In figure 4.3 the scattered power for vertical polarization and normal incidence is shown.
Figure 4.3: Scattered power for normal incidence and vertical polarization; ('s' = scattered, 'r' = reflected, 'd1' and 'd2' = diffracted)

Figure 4.4: Scattered power for oblique incidence and horizontal polarization; ('s' = scattered, 'r' = reflected, 'd1' and 'd2' = diffracted)
It is noted immediately that the reflected-power level lies somewhat lower and that the influence of the power level of the diffraction contributions is more substantial than with horizontal polarization. These effects are caused by the direction of the polarization vector with respect to the cylinder surface $\Sigma$. Theoretically, this originates from the equations $(2.17)$, $(2.18)$, $(2.41)$, and $(2.42)$. These high diffraction-power levels result in a strong influence on the scattered-power curve, which is observed very clearly from the figure. It is, however, still justified to exclude all other diffraction contributions.

For oblique incidence ($\beta = 45^\circ$) we find that the diffraction-power levels are (slightly) higher than for normal incidence, irrespective of the direction of polarization. This is visualized in the figures 4.4 and 4.5. With respect to normal incidence, two major ‘physical’ changes have occurred. Firstly it is found that the creeping-path length $t$ along $\Sigma$ has increased, since it is inversely proportional to $\sin \beta$, which will cause that a smaller amount of energy is left, from which a lowering of the curves would be expected. Secondly it can be made out that the curvature of $t$ has decreased (since $a_t$ has increased), which enables the wave to creep more easily around the cylinder. The latter must be of stronger influence and that will be the explanation for the raise in the diffraction-power levels.

![Figure 4.5: Scattered power for oblique incidence and vertical polarization; ('s' = scattered/ 'r' = reflected/ 'd1' and 'd2' = diffracted)
4.2 Comparison of scattered fields

The scattered fields $\vec{E}^s$ have been calculated using UTD and the exact solution. This was done for both polarization states, for varying radius ($a = 10\lambda$, $a = 3\lambda$, $a = 1\lambda$, $a = 0.3\lambda$, $a = 0.1\lambda$) and varying angle of incidence ($\beta = 90^\circ$, $\beta = 75^\circ$, $\beta = 45^\circ$). Some of the results have been lifted out and these are presented in this chapter. The observation points were on the half circle perpendicular to the cylinder axis of symmetry as shown in figure 4.1 in the previous section.

The least difference between the results obtained with UTD and the exact results is expected for large normalized radii $a/\lambda$. In the figures 4.6 and 4.7 results are presented for a configuration in which $a = 10\lambda$ and $\beta = 45^\circ$, for horizontal and vertical polarization respectively. The curves for both solutions (exact: solid; UTD: dotted) are nearly identical, from which it is concluded that the dependence on $\beta$ is implemented correctly and that the software generates accurate results.

Figures 4.8 and 4.9 contain the results for $a = 1\lambda$ and $\beta = 75^\circ$ for both polarization states.

![Figure 4.6: Scattered power for $a = 10\lambda$, $\beta = 45^\circ$, and horizontal polarization; (solid: exact solution / dotted: UTD solution)](image)

Particularly in the vertically polarized situation small differences between both curves are observed. It should be noted that these differences are not caused by the fact of oblique incidence, since it is found from observing the results that they also exist, although in a less measure, for $a = 1\lambda$ and $\beta = 90^\circ$, with vertical polarization. Note that at $\phi = 180^\circ$ the scattered-power level with $a/\lambda = 1$ lies approximately 10 dB lower than with $a/\lambda = 10$. It is very likely that this is caused by the fact that the amount of (back-)scattered power is proportional to the size of the back-scattering cross section of the cylinder.
Comparison between the UTD and Exact Solution

Figure 4.7: Scattered power for $a = 10\lambda$, $\beta = 45^\circ$, and vertical polarization; (solid: exact solution / dotted: UTD solution)

Figure 4.8: Scattered power for $a = 1\lambda$, $\beta = 75^\circ$, and horizontal polarization; (solid: exact solution / dotted: UTD solution)
For smaller radii \((a/\lambda < 1)\) the simulations for horizontal polarization start showing differences between both curves, but these are still negligible. Figures 4.10 and 4.11 show the results for \(a = 0.3\lambda\) and \(\beta = 45^\circ\). Note that Keller has empirically determined a lower limit \(a = 0.32\lambda\) for the validity of the GTD solution, of which UTD is an extension, at normal incidence [KELLER 1962]. The vertically polarized state shows relatively large differences between both curves, but at a level of approximately -55 dB the scattered-field contribution to the total field is almost negligible. This is visualized in figure 4.12, and from this figure we can conclude that the presence of a cylinder with a small radius does hardly have any influence on the field strength round the cylinder, since the power level has a maximum of less than 0.02 dB. If no cylinder was present, the power level would be exactly 0 dB in any observation point. The total-power result \(P^t(\vec{P})\) as shown in the figure is determined by

\[
P^t(\vec{P}) = 20 \log \left| \vec{E}^t(\vec{P}) \right|.
\]

The oscillations in \(P^t(\vec{P})\) are caused by the interference of the directly incident wave at \(\vec{P}\) with the reflected and diffracted waves coming from the cylinder. It is merely accidental that an apparent null appears at \(\phi = 90^\circ\), owing to the choice of the positions of the observation points.
Comparison between the UTD and Exact Solution

Figure 4.10: Scattered power for $a = 0.3\lambda$, $\beta = 45^\circ$, and horizontal polarization; (solid: exact solution / dotted: UTD solution)

Figure 4.11: Scattered power for $a = 0.3\lambda$, $\beta = 45^\circ$, and vertical polarization; (solid: exact solution / dotted: UTD solution)
Figure 4.12: Total power for $a = 0.3\lambda$, $\beta = 45^\circ$, and vertical polarization; (solid: exact solution / dotted: UTD solution)

Figure 4.13: Scattered power for $a = 0.1\lambda$, $\beta = 90^\circ$, and horizontal polarization; (solid: exact solution / dotted: UTD solution)
The results for the smallest radius used in our calculations \((a = 0.1\lambda)\) are shown in the figures 4.13 and 4.14, for \(\beta = 90^\circ\). The back-scattered power level is approximately 10 dB below that of the cylinder with \(a/\lambda = 1\). It is clearly observed that UTD fails for vertical polarization. This could be expected since UTD is an asymptotic technique that requires some minimum dimensions of the obstacles involved. This failure is not a consequence of the exclusion of less-important diffraction contributions, since the addition of the upper- and lower-attachment contributions ‘d3’ and ‘d4’ has hardly any influence on the scattered power, as visualized in figure 4.15. The solid curve shows the ‘enhanced’ scattered power, which is nearly identical with the ‘poor’ scattered power. Note that the ‘oscillation’ of the scattered power as a function of \(\phi\) decreases with decreasing radius. This is explained by the fact that the differences in propagation distance of the various rays decrease with decreasing radius, which results in a smoother interference pattern of the combined rays.

Figure 4.14: Scattered power for \(a = 0.1\lambda, \beta = 90^\circ, \text{ and vertical polarization; (solid: exact solution / dotted: UTD solution)}\)
Figure 4.15: Scattered power for $a = 0.1\lambda$, $\beta = 90^\circ$, and vertical polarization; (solid: including ‘d3’ and ‘d4’/ dashed: excluding ‘d3’ and ‘d4’/ dotted: diffraction contributions)

4.3 Recommendations regarding the use of the computer programs

In this section the advantages and disadvantages of both theories are considered. These are listed below:

**Exact solution**

**Advantages:**

- value of $a/\lambda$ not restricted
- little geometrical problems

**Disadvantages:**

- provides hardly any physical insight
- infinite number of terms to be calculated (approximation needed)
- processing time relatively high for large $a$ (dependent on the number of terms $N$)
Comparison between the UTD and Exact Solution

- plane-wave incidence only
- requires cylinders of infinite length

**UTD solution**

**Advantages:**
- provides good physical insight
- processing time is not strongly dependent on a particular parameter
- can be extended to cylindrical- and spherical-wave incidence
- top-plane effects can be included
- any (developable) curved surface can be implemented

**Disadvantages:**
- value of $a/\lambda$ limited by a minimum
- complex geometrical problems

From the simulations we can conclude that both computer programs, implementing the UTD and exact solution, respectively, function properly. In this section, recommendations are made on which program is to be used for a given configuration involving cylinder scattering. Important requirements on computer programs are that it should be:

- accurate,
- uniform,
- fast, and
- user-friendly.

The fourth requirement is outside the scope of this chapter, and thus we will focus on the three remaining requirements.

If waves other than plane-waves or cylinders other than ‘infinitely long’ circular cylinders are involved, the UTD solution has to be used. For plane-wave incidence and very long circular cylinders one can avail himself of both theories. For cylinder radii $a \geq 0.3\lambda$ it was found that the accuracy of both solutions suffices. This means that a decision on which program to be used can be made by comparing the processing times needed to calculate the solutions and then choose the fastest program. The processing time of the UTD program is not strongly dependent on any particular parameter. The exact calculation requires the calculation of a sum for $n$ is $-N$ to $N$, from which the processing time is dependent on $N$. The value of $N$ needed to obtain a reasonable accuracy can be determined with
the help of figure B.1 in appendix B as a function of \( a \) and \( \beta \). In the following we start from a relative accuracy of 1 p.p.m. \((\delta_{rel} = 1e-6)\). In dB this yields an absolute error in the scattered-field power with respect to the power in the absence of the cylinder of \( 20 \cdot 10 \log(1.000001/1) = 8.7 \cdot 10^{-6} \) dB. We have recorded the ratio \( \tau_u/\tau_e \) of the processing time needed to calculate the UTD solution and the processing time needed to calculate the exact solution for different values of \( N \). The results are given in table 4.1. From

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \tau_u/\tau_e )</th>
<th>( N )</th>
<th>( \tau_u/\tau_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.13</td>
<td>8</td>
<td>0.68</td>
</tr>
<tr>
<td>3</td>
<td>1.56</td>
<td>9</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>1.24</td>
<td>10</td>
<td>0.56</td>
</tr>
<tr>
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<td>1.02</td>
<td>11</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>0.89</td>
<td>12</td>
<td>0.47</td>
</tr>
<tr>
<td>7</td>
<td>0.77</td>
<td>20</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 4.1: Relative processing time for different \( N \)

the table we find that the exact solution is found to be faster than the UTD solution if \( N \leq 5 \). From \( N = 6 \), corresponding to \( ka \sin \beta \geq 1.3 \) in figure B.1, the UTD algorithm appears to be fastest. This means that it is more efficient to use the exact solution for \( a \sin \beta < 1.3/(2\pi) \approx 0.21\lambda \). (Note that for an angle of incidence \( \beta = 45^\circ \) this yields a maximum \( a = 0.3\lambda \) for which the exact solution is fastest. This maximum corresponds reasonably well to the lower limit of the UTD solution that we found empirically in the previous section.) The UTD solution should be used for \( a \geq 0.3\lambda \).

However, since the scattered-power level for \( a < 0.3\lambda \) is very low, the user can also decide to neglect the scattering effects of any cylinder with a small radius \((a < 0.3\lambda)\). In that case, the UTD program will be a sufficient tool to predict the field strength near circular cylinders.
Chapter 5

Experimental verification

At the Telecommunications Division of EUT we have the ability to perform scattering measurements. An HP8510C Vector Network Analyzer (VNA) is used in this experimental setup, in combination with two antenna probes between which an obstacle can be placed. In this way, the scattering effects of the obstacle can be measured. The basic setup and calibration scheme have been thoroughly described in [KLAASSEN 1992], and a detailed description is therefore not repeated in this report. The setup was slightly altered to enable measurements involving cylindrical obstacles instead of plane rectangular objects. In section 5.1 the geometrical situation is considered.

The results obtained from the measurements can be used to verify simulation results based on UTD (as described in chapter 2). The actual geometry of the setup, however, gives rise to a problem in using the UTD model. Since the space in between the antenna probes is limited by the size of the room that contains the setup, it is not permitted to assume plane-wave incidence. For that reason the UTD model is to be adjusted for spherical-wave incidence. The adjustments needed are treated in section 5.2.

Furthermore, since non-isotropic antenna probes (viz. rectangular waveguides) are used, the software should contain procedures to accomplish this non-ideal antenna behavior. The so-called probe correction, which is in fact the introduction of antenna weight functions, is described in section 5.3.

The chapter is concluded with a number of experimental results from the measurements. These are accompanied by simulation results obtained from the adjusted UTD model. For each measurement a brass circular cylinder was used as Device Under Test (DUT), and the measurements were performed as a function of (normalized) radius $a/\lambda$, polarization, and angle of incidence $\beta$.

5.1 Geometry

The scattering effects are measured in a bistatic way with two antenna probes. A pair of waveguides is used for this purpose, the rectangular apertures of which are vertical with respect to the earth’s surface. The position of the aperture centers with respect to an origin $O$ are given by $\bar{T}$ and $\bar{P}$ for transmitter (T) and receiver (R), respectively. The horizontal
offset (in the center position) mentioned in [KLAASSEN 1992] has been eliminated, from which only a vertical displacement between both boresight directions remains in the center position, since the probes are positioned at different heights \( h_T \) and \( h_R \). By the ‘center position’ we mean the position in which the straight line between \( \overrightarrow{T} \) and \( \overrightarrow{P} \) intersects the cylinder axis of symmetry. In this position, \( \overrightarrow{P} \) is denoted by \( \overrightarrow{P}' \).

The transmitting probe and the DUT are mounted on a support structure that can be rotated in a horizontal plane. In the center position, a number of geometrical parameters are defined. The horizontal distance between the axis of rotation and \( \overrightarrow{T} \) is denoted by \( d_T \), and \( d_c \) corresponds to the horizontal distance measured between the cylinder axis of symmetry and \( \overrightarrow{T} \). The aperture planes are separated by \( d_{TR} \). The cylinder can be rotated in the vertical plane containing \( \overrightarrow{T} \) and \( \overrightarrow{P} \) by an angle \( \chi \). Figure 5.1 shows a side view of the measurement setup in center position. Note that the cylinder is depicted non-realistically, viz. slantly towards the viewer, in order to make it recognizable.

Since part of the setup is rotatable, the measurements can be performed as a function of observation angle. The practical rotation of the arm that contains \( \overrightarrow{T} \) and the cylinder can be modeled by taking \( \overrightarrow{P} \) on a circular arc, which yields a substantial simplification of the simulation model. The observation points will then become a function of the angle of rotation \( \varphi \). At \( \varphi = 0 \) (which is the center position), \( \overrightarrow{P} \) is denoted by \( \overrightarrow{P}' \). A schematic top view of the measurement setup is given in figure 5.2.

The measurements are performed over a frequency range \( f \in [46, 54] \) GHz (bandwidth \( B = 8 \) GHz) with 801 samples per observation point in order to provide reasonable resolution. This resolution is needed because fast Fourier transformations (FFT) are applied in the signal processing carried out by the VNA to go from frequency to time domain and back. The FFTs provide the ability to control the measurements as a function of time, from which we are able to select the time interval we are interested in. The data are stored for
the center frequency $f_c = 50$ GHz only. A bandwidth $B = 8$ GHz requires a measurement time of approximately 120 s per observation point.

5.2 UTD for spherical-wave incidence

The measurements performed involve spherical-wave incidence. As a consequence, the use of the model described in chapter 2 is not justified. In this section we will first discuss the adjustments to the reflection and diffraction procedures of the UTD model necessary to suit spherical-wave incidence, followed by a reconsideration of the scattering geometry.

5.2.1 Reflected field

In this subsection, we assume that the cylinder axis of symmetry coincides with the $y$ axis and that $\hat{s}'$ is parallel to the $xy$ plane, similar to the plane-wave incidence UTD model. The $xy$ plane in the model corresponds to the plane that contains $\hat{T}$ and $\hat{P}'$ in the experimental setup. The incident ray has a finite length $s'$, since it emanates from a (nearby) probe. This length $s'$ is measured between $\hat{T}$ and the reflection point $Q_r$. Since the incident wavefront is spherical, the radii of curvature of the reflected wave cannot be calculated with the simplified equation (2.32). It is obvious that a pair of finite radii of curvature are to be obtained. In the adjusted model, we have calculated $\rho_{1,2}'$ from a procedure described in [GOVAERTS 1991]. This procedure requires the input of an incident vector triplet and a surface vector triplet, both with corresponding radii of curvature. For the present geometry, $\rho_{1,2}'$ are calculated from the procedure with the following input:

$$
\begin{align*}
\hat{x}_3^i &= \hat{s}' , \\
\hat{x}_1^i &= \hat{e}_z , \quad \rho_1^i = s' , \\
\hat{x}_2^i &= \hat{x}_3^i \times \hat{x}_1^i , \quad \rho_2^i = s' ,
\end{align*}
$$

(5.1)
and

\[
\begin{align*}
\hat{x}_3^E &= \hat{n}, \\
\hat{x}_1^E &= \hat{e}_y, \quad \rho_1^E \to \infty, \\
\hat{x}_2^E &= \hat{x}_3^E \times \hat{x}_1^E, \quad \rho_2^E = a.
\end{align*}
\] (5.2)

Besides the calculation of \(\rho_{1,2}^E\), the calculation of the parameter \(X^r\) according to chapter 2 (equation (2.29)) needs to be adjusted. Here, the distance parameter \(s\) in (2.29) needs to be replaced by \(ss'/(s + s')\) for spherical-wave incidence, which yields

\[
X^r = 4\pi \frac{ss'}{s + s'} (\hat{n} \cdot \hat{s})^2.
\] (5.3)

### 5.2.2 Diffracted fields

In this subsection, we will again assume that \(\hat{s}'\) is parallel to the \(xy\) plane. For plane-wave incidence the spreading factor of the diffracted field has the simple form \(1/\sqrt{s}\), which is easily derived from the general form

\[
\sqrt{\frac{\rho^d}{s(\rho^d + s)}},
\] (5.4)

in which it is assumed that the point of departure of the ray \(\tilde{Q}_d\) is a caustic. For plane-wave incidence, \(\rho^d \to \infty\) and the spreading factor reduces to \(1/\sqrt{s}\). For spherical-wave incidence, \(\rho^d\) is finite, and it is easily found that

\[
\rho^d = s' + t,
\] (5.5)

with \(s'\) the length between \(\tilde{T}\) and the attachment point \(\tilde{Q}_a\) and \(t\) the creeping-wave path length. Since the rays have a diverging character, a factor is required to accomplish conservation of energy at the cylinder surface \(\Sigma\). On a power basis, this so-called *surface ray spreading factor* is determined by the ratio of the distance between two diffracting rays near \(\tilde{Q}_a\) and the distance between the same rays near \(\tilde{Q}_d\). On a field strength basis, the square root should be taken. Hence, the factor is defined by

\[
\sqrt{\frac{d\eta_a}{d\eta_d}} = \sqrt{\frac{s'}{s' + t}},
\] (5.6)

illustrated in figure 5.3. If this factor is also taken into account in the calculation of \(\tilde{E}^d\), equation (2.41) should be replaced by

\[
\tilde{E}^d(\tilde{P}) = \tilde{E}^i(\tilde{Q}_a) \cdot T \sqrt{\frac{s'}{s (s' + t + s)}} e^{-j2\pi s}.
\] (5.7)
As with reflection, the distance parameter $s$ in equation (2.51) is to be replaced by $s/s' + (s + s')$, which yields

$$X^d = \frac{\pi s s' (\xi^d)^2}{(s + s')\sqrt{\pi^2 a^2_t}},$$  \hspace{1cm} \text{(5.8)}$$

with $\xi^d$ and $a_t$ defined as in chapter 2.

### 5.2.3 Reconsideration of the geometry

In the original model discussed in chapter 2, the whole scenery is transformed such that the cylinder axis of symmetry coincides with the $y$ axis and $s'$ has a positive $x$ component and is parallel to the $xy$ plane. It is examined how this simple model can be achieved for spherical-wave incidence. It is found that a more intensive approach is needed, since the scenery needs to be transformed per observation point, and separately for the reflection and diffraction contributions. This implies that the number of transformations is determined by three times the number of observation points in case of spherical-wave incidence. For plane-wave incidence one complete transformation is sufficient. This yields a reduction in speed of the computer model under consideration.

Firstly, the center position is considered. It is assumed that the cylinder axis of symmetry coincides with the $y$ axis and that the origin $O$ of the system is positioned at this axis. In the practical situation, $O$ is at a height $h_T$ above the earth's surface. In the model, the plane containing $\vec{T}$ and $\vec{P}$ is the $xy$ plane, which is depicted in figure 5.4. The position of the transmitter in the model is easily determined:

*Figure 5.3: Divergence of the rays on $\Sigma$*
The position of the observation points is determined by introducing a number of additional points, such as

\[ O' = \begin{pmatrix} (h_T - h_R) \sin \chi \\ -(h_T - h_R) \cos \chi \\ 0 \end{pmatrix}, \]

(5.10)

from which it is easily found that

\[ \tilde{C} = \begin{pmatrix} (h_T - h_R) \sin \chi - (d_c + d_T) \cos \chi \\ -(h_T - h_R) \cos \chi - (d_c + d_T) \sin \chi \\ 0 \end{pmatrix}. \]

(5.11)

With the help of (5.11) the observation points are found from

\[ \tilde{P}(\varphi) = \begin{pmatrix} (h_T - h_R) \sin \chi + ((d_T + d_{TR}) \cos \varphi - (d_c + d_T)) \cos \chi \\ -(h_T - h_R) \cos \chi + ((d_T + d_{TR}) \cos \varphi - (d_c + d_T)) \sin \chi \\ -(d_T + d_{TR}) \sin \varphi \end{pmatrix}, \]

(5.12)

with center observation point

\[ \tilde{P}' = \tilde{P}(0) = \begin{pmatrix} (h_T - h_R) \sin \chi + (d_{TR} - d_c) \cos \chi \\ -(h_T - h_R) \cos \chi + (d_{TR} - d_c) \sin \chi \\ 0 \end{pmatrix}. \]

(5.13)
Figure 5.5: Side view of the shadow region for spherical-wave incidence

Reflection geometry
For spherical-wave incidence, a slight adjustment is to be made to the procedure used to find $\vec{Q}_r$ for plane-wave incidence, since the source has a fixed position $\vec{T}$. The observation point is in the shadow of the cylinder if

$$\xi = \arctan \left| \frac{z_p}{x_p - x_T} \right| < \arcsin \left| \frac{a}{x_T} \right| = \xi_0, \quad (5.14)$$

as shown in figure 5.5. Consequently, the procedure to determine $\vec{Q}_r$ is considered, which shows much similarity with that described by the equations (2.7) up to (2.17). Figure 5.6 shows a side view of the reflection mechanism. The angle of incidence $\beta$ is measured between $\xi'$ and $\hat{e}_y$ in the $W$ plane. The reflection point is determined by

$$\vec{Q}_r = (-a \cos \alpha_r', y_r, a \sin \alpha_r') , \quad (5.15)$$

and it is accompanied by the normal vector

$$\hat{n} = (- \cos \alpha_r', 0, \sin \alpha_r') . \quad (5.16)$$

From the figure it is derived that

$$\vec{Q}_r \vec{P} \cdot \hat{e}_x = | \vec{Q}_r \vec{P} | \sin \beta \cos (\pi - 2\alpha_r + \psi) = x_p + a \cos \alpha_r', \quad (5.17)$$

$$\vec{Q}_r \vec{P} \cdot \hat{e}_z = | \vec{Q}_r \vec{P} | \sin \beta \sin (\pi - 2\alpha_r + \psi) = z_p - a \sin \alpha_r'. \quad (5.18)$$

Subsequently, it is easily found that the equations (2.11), (2.12), and (2.13) apply also for the new situation. However, since $\beta$ depends on $y_r$ according to

$$\tan \beta = \frac{\sqrt{(-x_T - a \cos \alpha_r')^2 + a^2 \sin^2 \alpha_r'}}{y_r - y_T} , \quad (5.19)$$
Figure 5.6: Side view of the reflection mechanism

(2.13) can be replaced by

\[ y_r = \frac{y_p + y_T \sqrt{\frac{(x_p + a \alpha_r' \cos \alpha_r')^2 + (z_p - a \sin \alpha_r')^2}{x_T^2 + 2ax_T \cos \alpha_r' + a^2}}}{1 + \sqrt{\frac{(x_p + a \alpha_r' \cos \alpha_r')^2 + (z_p - a \sin \alpha_r')^2}{x_T^2 + 2ax_T \cos \alpha_r' + a^2}}} \]

With (2.14) and

\[ \hat{s}' = \frac{\overrightarrow{TQ_{r}}}{|\overrightarrow{TQ_{r}}|} = \frac{\overrightarrow{TQ_{r}}}{\overrightarrow{s'}}, \]

\( \alpha_r' \) can be determined from equation (2.15). The reflection point \( \vec{Q}_r \) can be found from substitution of \( \alpha_r' \). Furthermore, \( \beta \) can be calculated from equation (5.19).

A geometry with \( \hat{s}' \) parallel to the \( xy \) plane is obtained from rotating the whole scenery around the \( y \) axis by an angle \( \zeta = -\psi \) (see appendix A), with

\[ \psi = \arctan\left(\frac{a \sin \alpha_r'}{-x_T - a \cos \alpha_r'}\right), \]

which is easily found from the figure. The following vectors are to be rotated: \( \hat{s}, \hat{s}', \vec{T}, \vec{P}, \) and \( \vec{Q}_r \). After that, the reflected field can be calculated from (2.17), using the adjusted parameters \( \rho_{1,2} \) and \( X^r \).

**Diffraction geometry**

For the determination of \( \vec{Q}_a \) and \( \vec{Q}_d \), in this subsection it is tried to avoid the numerical root-finding procedure employed in chapter 2. For simplicity, we assume that \( \vec{P} \) is outside the space given by

\[ (-a \leq x \leq a) \land (-a \leq z \leq a), \]
which is the case in the measurements. In figure 5.7, this space is blackened. It is obvious that \( \vec{P} \) is expected to be outside of the cylinder. The attachment angle is easily derived:

\[
\alpha_a = \pm \arccos \left( \frac{a}{-x_T} \right),
\]

for upper and lower attachment, respectively. The point of departure \( \vec{Q}_d \) will always be found on the straight line stretched in \( y \) direction that is in the plane tangent to \( \Sigma \) and through \( \vec{P} \). Four spatial areas are defined, which are different for upper and lower attachment. A cross section of these areas is also given in figure 5.7. If \( \vec{P} \) is in the area with the number \( l \), \( \alpha_d \) is found from

\[
\alpha_d = \frac{l \pi}{2} \pm \arccos \left( \frac{a}{\sqrt{x_p^2 + z_p^2}} \right) - (-1)^l \arctan \left( \frac{z_p}{x_p} \right)^{(-1)^l}.
\]

Consequently, \( \Delta \alpha \) is found from

\[
\Delta \alpha = \alpha_d - \alpha_a + n2\pi \in [0,2\pi),
\]

with \( n \in \{0, \pm 1, \pm 2, ..\} \). The angle of incidence \( \beta \) is easily determined, once \( \Delta \alpha \) is known. In figure 5.8, the diffracted-ray path is depicted as a straight line, as if the cylinder surface is ‘unfolded’ to become plane. It is easily found that

\[
\ell_1 = \sqrt{x_T^2 - a^2},
\]

\[
\ell_2 = a \Delta \alpha,
\]

and

\[
\ell_3 = \sqrt{x_p^2 + z_p^2 - a^2},
\]
from which $\beta$ is determined according to

$$\tan \beta = \frac{\ell_1 + \ell_2 + \ell_3}{y_p - y_T}. \quad (5.30)$$

Next, the positions of the diffraction points are determined by

$$\tilde{Q}_a = \begin{pmatrix} -a \cos \alpha_a \\ y_p - \frac{\ell_1}{\tan \beta} \\ a \sin \alpha_a \end{pmatrix}, \quad (5.31)$$

$$\tilde{Q}_d = \begin{pmatrix} -a \cos \alpha_d \\ y_T + \frac{\ell_3}{\tan \beta} \\ a \sin \alpha_d \end{pmatrix}. \quad (5.32)$$

The incident-ray path length is defined as $s' = |\overrightarrow{TQ_a}|$. The geometry assumed is obtained from rotating the whole scenery around the $y$ axis by an angle

$$\zeta = \pm \left( |\alpha_a| - \frac{\pi}{2} \right), \quad (5.33)$$

for upper and lower attachment, respectively. The vectors to be rotated are: $\overrightarrow{T}$, $\overrightarrow{P}$, $\overrightarrow{Q_a}$, and $\overrightarrow{Q_d}$. Furthermore, $\alpha_d$ should be diminished with $\zeta$.

Hence, the diffracted field is calculated from equation (5.7), using the adjusted $X^d$ from equation (5.8).
5.3 Probe correction

In order to illuminate the cylinder as uniformly as possible, antennas with low-directivity properties are needed. Antenna probes with isotropic radiation patterns are desired, but it is well known that these are not available. This problem was examined in [KLAASSEN 1992] and a suitable solution was found in using rectangular-waveguide antennas of the WR19 type. These have apertures of $a \times b = 4.775 \times 2.388$ mm$^2$, resulting in single TE$_{10}$-mode propagation at $f = 50$ GHz, which is the center frequency used. Note that the parameter $a$ used here does not denote the cylinder radius. It can be readily verified that the Rayleigh distance $R$ of the WR19 probe is limited to a maximum of a few centimeters. The far-zone field of a rectangular waveguide in a spherical $(r, \theta, \phi)$ coordinate system as depicted in figure 5.9 is determined by [SILVER 1965 (CH. 10)]

\[
E_r = 0 ,
\]

\[
E_\theta = -\sqrt{\frac{\mu}{\epsilon}} \frac{\pi a^2 b}{2\lambda^2} \sin \phi \left[ 1 + \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \cos \theta + \frac{1 - \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}{1 + \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \left(1 - \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \cos \theta\right) \right] e^{-jkr} \frac{r}{r} ,
\]

\[
E_\phi = -\sqrt{\frac{\mu}{\epsilon}} \frac{\pi a^2 b}{2\lambda^2} \cos \phi \left[ \cos \theta + \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} + \frac{1 - \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}{1 + \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \left(\cos \theta - \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}\right) \right] e^{-jkr} \frac{r}{r} ,
\]

if $r > R$. In air, which is the medium the EM waves propagate in, we can approximate the permeability and the permittivity from which $\sqrt{\mu/\epsilon} \approx 120\pi$ and $\lambda = \frac{1}{f\sqrt{\mu \epsilon}} \approx 3.10^{11}$ mm. For $f = f_c = 50$ GHz this yields $\lambda = 6$ mm.

The following vector relations apply:
Figure 5.9: Spherical coordinate system for the probe

\[
\mathbf{\hat{e}_\theta} = -\sin \theta \mathbf{\hat{e}_{WR}} + \cos \theta \cos \phi \mathbf{\hat{e}_{yWR}} + \cos \theta \sin \phi \mathbf{\hat{e}_{zWR}}, \\
\mathbf{\hat{e}_\phi} = -\sin \phi \mathbf{\hat{e}_{yWR}} + \cos \phi \mathbf{\hat{e}_{zWR}}. \quad (5.37)
\]

In the measurement setup, the Cartesian coordinate system of the transmitter is given by

\[
\mathbf{\hat{e}_{xWR}} = \cos \chi \mathbf{\hat{e}_x} + \sin \chi \mathbf{\hat{e}_y}, \\
\mathbf{\hat{e}_{yWR}} = -\mathbf{\hat{e}_z}, \\
\mathbf{\hat{e}_{zWR}} = \mathbf{\hat{e}_{xWR}} \times \mathbf{\hat{e}_{yWR}}, \quad (5.38)
\]

for 'horizontal' polarization, and

\[
\mathbf{\hat{e}_{xWR}} = \cos \chi \mathbf{\hat{e}_x} + \sin \chi \mathbf{\hat{e}_y}, \\
\mathbf{\hat{e}_{zWR}} = \mathbf{\hat{e}_z}, \\
\mathbf{\hat{e}_{yWR}} = \mathbf{\hat{e}_{zWR}} \times \mathbf{\hat{e}_{xWR}}, \quad (5.39)
\]

for 'vertical' polarization. Note that for \(\chi = 0\) the 'horizontal' polarization is in fact polarization parallel to the cylinder axis of symmetry, and 'vertical' polarization is 'perpendicular' polarization.

For the receiver, the system configuration depends on the position of the observation point \(P\). It is found that

\[
\mathbf{\hat{e}_{xWR}} = -\mathcal{A} \cos \chi \mathbf{\hat{e}_x} - \mathcal{A} \sin \chi \mathbf{\hat{e}_y} - \frac{z_p}{d_T + d_{TR}} \mathbf{\hat{e}_z}, \\
\mathbf{\hat{e}_{zWR}} = \sin \chi \mathbf{\hat{e}_x} - \cos \chi \mathbf{\hat{e}_y}, \\
\mathbf{\hat{e}_{yWR}} = \mathbf{\hat{e}_{xWR}} \times \mathbf{\hat{e}_{zWR}}, \quad (5.40)
\]

for parallel polarization, and

\[
\mathbf{\hat{e}_{xWR}} = -\mathcal{A} \cos \chi \mathbf{\hat{e}_x} - \mathcal{A} \sin \chi \mathbf{\hat{e}_y} - \frac{z_p}{d_T + d_{TR}} \mathbf{\hat{e}_z}, \\
\mathbf{\hat{e}_{yWR}} = \sin \chi \mathbf{\hat{e}_x} - \cos \chi \mathbf{\hat{e}_y}, \\
\mathbf{\hat{e}_{zWR}} = \mathbf{\hat{e}_{xWR}} \times \mathbf{\hat{e}_{yWR}}, \quad (5.41)
\]
for perpendicular polarization, with

\[ A = \sqrt{\frac{(d_T + d_{TR})^2 - z_p^2}{d_T + d_{TR}}} \]  

(5.42)

Note that these vectors that belong to the probes should be rotated around the \( y \) axis (in the model) with \( \vec{T} \) and \( \vec{P} \), if such a rotation is necessary.

### 5.4 Results

The cylinders used in the measurements are straight ‘tubes’ with walls of thickness 1 mm and open ends. The surrounding medium is air. The cylinders are made of brass and have not been specially prepared. This implies that they might suffer from small irregularities of the surface, slight curvature in axial direction, and possibly inhomogeneities, all of which are invisible imperfections. A perfect conductivity \((\sigma \to \infty)\) is assumed, although the conductivity of brass is finite, naturally. The cylinders are assumed to be infinitely long. In practice, the cylinders have lengths of approximately 700 mm \((\equiv 116.75\lambda)\). It is expected that oscillation effects owing to reflections of the induced currents at the cylinder ends are negligible.

The measurement results are processed using gating techniques that should eliminate the unwanted reflections at all obstacles in the vicinity of the DUT. These gating procedures also reduce the (additional) scattering effects caused by the cylinder tops [KLAASSEN 1992]. The gate creates an ellipsoid, outside of which all scattering effects are eliminated. The ellipsoid increases in size with the gate stop time, which was carefully set, taking into account the cylinder length, for the measurements employed.

The geometrical parameters \( d_T, d_{TR}, h_T, \) and \( h_R \) are constant for all measurements. They were measured with an accuracy of \( \pm 0.5 \) mm \((\equiv 0.08\lambda)\). In table 5.1 these lengths are given in mm plus a conversion to wavelengths \((\lambda \approx 6 \) mm). The angle span of the observation-

<table>
<thead>
<tr>
<th></th>
<th>( d_T )</th>
<th>( d_{TR} )</th>
<th>( h_T )</th>
<th>( h_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>in mm</td>
<td>452</td>
<td>699</td>
<td>1498</td>
<td>1443</td>
</tr>
<tr>
<td>in ( \lambda )</td>
<td>75.39</td>
<td>116.58</td>
<td>249.84</td>
<td>240.67</td>
</tr>
</tbody>
</table>

**Table 5.1: Non-variable parameters in the setup**

point range was taken \( \varphi \in [-12^\circ, 12^\circ] \), with a \( 0.1^\circ \) step. This implies that per configuration a total measurement time of approximately eight hours is required.

Five cylinders of different radius \((a/\lambda = 0.5-5)\) are used as a DUT, with \( \chi = 0^\circ \). To verify the results for oblique incidence, the cylinder with \( a/\lambda = 1.5 \) was also mounted in a slanted position, with \( \chi = 22.1^\circ \) \((\pm 0.1^\circ)\). Since we want to perform all measurements for both
parallel and perpendicular polarization, a total number of twelve measurements suffices. The distance between the transmitter and the cylinder \( d_c \) varies with varying radius and varying angle \( \chi \). It was measured with an accuracy of ±0.5 mm, the results of which are found in table 5.2, again with a conversion to wavelengths.

<table>
<thead>
<tr>
<th>( \chi ) [°]</th>
<th>( a ) [mm]</th>
<th>( d_c ) [mm]</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.50</td>
<td>248</td>
<td>41.36</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>253</td>
<td>42.20</td>
</tr>
<tr>
<td>9</td>
<td>1.50</td>
<td>256</td>
<td>42.70</td>
</tr>
<tr>
<td>12</td>
<td>2.00</td>
<td>259</td>
<td>43.20</td>
</tr>
<tr>
<td>30</td>
<td>5.00</td>
<td>285</td>
<td>47.53</td>
</tr>
<tr>
<td>22.1</td>
<td>9</td>
<td>435</td>
<td>72.55</td>
</tr>
</tbody>
</table>

Table 5.2: Variable parameters in the setup

Figures 5.10 up to and including 5.19 show the results of the measurements, accompanied by the results obtained from UTD calculations (for spherical-wave incidence). The optical shadow boundaries are depicted by dashed lines. The optical-shadow region, denoted by ‘OS’ lies between the shadow boundaries. Each dot corresponds to a measurement point, and it is observed that the measurements agree very well with the UTD simulation results, represented by the solid curves. The small deviations will probably be caused by the inaccuracies of the geometrical parameters. Note that an absolute error of ±0.5 mm corresponds to a ±30° phase shift. Furthermore, an error is introduced by the rotation of the setup in \( \varphi \) direction. An inaccuracy of ±0.1° should be taken into account, equivalent to a worst-case boresight displacement of 0.1 \((2\pi/360) (d_T + d_{TR}) = 2.0 \text{ mm (} \approx \lambda/3)\) at the receiver site. From figure 5.13 it is seen that an error in rotating the setup was made near \( \varphi = -7° \). This position is marked by a dashed line.

A number of remarkable features are observed from the figures. It is clearly seen that the parallel-polarization results show larger global maxima and minima than those for perpendicular polarization. In the vicinity of the center position \( \varphi = 0° \), the power of the horizontally polarized state is substantially lower. This implies that the site shielding factor (SSF), which is a measure for the shielding properties of an obstacle, is best if the field component parallel to the cylinder axis of symmetry is relatively large. It is obvious that the SSF increases with increasing radius, which is readily observed from the figures. The course of the measurement results seems to be least smooth in the os regions. In these regions, the results are provided only by the waves that creep along the surface \( \Sigma \), and since \( \Sigma \) is not ideal, deviations are introduced.
Figure 5.10: The relative total received power for $a = 0.5\lambda$, $\chi = 0^\circ$, and parallel polarization (solid: UTD result / dots: measurement results)

Figure 5.11: The relative total received power for $a = 0.5\lambda$, $\chi = 0^\circ$, and perpendicular polarization (solid: UTD result / dots: measurement results)
Figure 5.12: The relative total received power for $a = 1\lambda$, $\chi = 0^\circ$, and parallel polarization (solid: UTD result / dots: measurement results)

Figure 5.13: The relative total received power for $a = 1\lambda$, $\chi = 0^\circ$, and perpendicular polarization (solid: UTD result / dots: measurement results)
Figure 5.14: The relative total received power for $a = 1.5\lambda$, $\chi = 0^\circ$, and parallel polarization (solid: UTD result / dots: measurement results)

Figure 5.15: The relative total received power for $a = 1.5\lambda$, $\chi = 0^\circ$, and perpendicular polarization (solid: UTD result / dots: measurement results)
Figure 5.16: The relative total received power for $a = 2\lambda$, $\chi = 0^\circ$, and parallel polarization (solid: UTD result / dots: measurement results)

Figure 5.17: The relative total received power for $a = 2\lambda$, $\chi = 0^\circ$, and perpendicular polarization (solid: UTD result / dots: measurement results)
Figure 5.18: The relative total received power for $a = 5\lambda$, $\chi = 0^\circ$, and parallel polarization (solid: UTD result / dots: measurement results)

Figure 5.19: The relative total received power for $a = 5\lambda$, $\chi = 0^\circ$, and perpendicular polarization (solid: UTD result / dots: measurement results)
The results for $\chi = 22.1^\circ$ are given in figures 5.20 and 5.21. If these are compared with figures 5.14 and 5.15 it is found that the agreement between the measured and simulated results is less for the oblique incident case. This is probably caused by the additional inaccuracy introduced by the measurement of $\chi$. Note that the creeping-ray path length $t$ is proportional to $\chi$, which in this case will also make the measurement results worse, since the surface is not ideal. The oscillations in the curves are of a higher frequency with oblique incidence than with normal incidence (see figures 5.14 and 5.15). This is readily explained by the fact that the slant position of the cylinder reduces the path lengths $s$, since $d_c$ is larger in this case. Despite the fact that the surface path length $t$ is increased, this implies that the phase shifts of the different contributing fields caused by one observation point step differ more than with $\chi = 0^\circ$. This yields an increased interference frequency.

![Figure 5.20](image)

*Figure 5.20: The relative total received power for $a = 1.5\lambda$, $\chi = 22.1^\circ$, and 'parallel' polarization (solid: UTD result / dots: measurement results)*
Figure 5.21: The relative total received power for \( a = 1.5\lambda, \chi = 22.1^\circ \), and 'perpendicular' polarization (solid: UTD result / dots: measurement results)
Chapter 6

Direct strut scattering in a Cassegrain antenna system

A Cassegrain antenna system basically consists of two reflectors and a feed. The main reflector has a paraboloidal shape, and the sub reflector is hyperboloidal. In this chapter an (axi)symmetrical Cassegrain antenna is examined. A method for calculating its off-axis radiation pattern, taking into account the presence of both reflectors, was presented recently [GOYAES 1991, VAN DOOREN 1993a]. In this approach, the UTD is used and contributions up to and including double diffraction at the reflector rims are included. In the model used, the sub reflector is supposed to ‘float’ in the air. In practical situations, however, a sub-reflector support structure is needed to keep the reflector in its fixed position, and the influences of gravity and wind impose mechanical requirements to this support structure. Its unavoidable presence gives rise to additional blockage of the antenna aperture, from which it is desirable to reduce the dimensions of the structure to a minimum. A commonly used solution is found in constructing so-called ‘struts’, fixed at the reflector rims (or surfaces). It is obvious that these struts give rise to additional scattering contributions to the transmitted or received field. In the following section a number of important strut scattering mechanisms is treated.

6.1 Strut scattering

Figure 6.1 shows a Cassegrain antenna system, including a so-called quadripod sub-reflector support structure consisting of four struts. In the figure, the struts are mounted at the sub- and main-reflector edges, but it also occurs that the struts are constructed at the main-reflector surface. If the latter is the case, the mechanisms involving strut scattering are threefold; considering the situation in which the antenna is transmitting these are:

#1 the plane wave from double reflection (subsequently at the sub and the main reflector) interacts with the struts;

#2 the spherical wave from single reflection at the sub reflector interacts with the struts before reflection at the main reflector, and

#3 the spherical wave from the feed interacts directly with the struts.
In figure 6.2 these mechanisms are illustrated. In the following we assume that the struts are straight circular cylinders.

The first attempts to take into account the effects on the radiation pattern caused by the presence of the struts involved aperture blockage caused by the mechanisms #1 and #2 [Gray 1964, Dijk 1967, Ruze 1968]. This method is called the zero-field approach, since the field strength in the aperture is assumed to be zero in the blocked regions. The strut shadow cast on the plane \( V \) by mechanism #1 is rectangular, and the blocked part of the aperture cast by mechanism #2 is enclosed by cylindrical arcs. The latter can be approximated by a trapeze-shaped shadow, as depicted in figure 6.3, yielding negligible errors in the calculation of the far-field radiation pattern [Dijk 1967]. Note that the effect from mechanism #2 can be avoided by mounting the struts at the main-reflector edge. The zero-field method is used in calculating the mainlobe of the antenna from the aperture field, and other methods should be addressed to calculate the off-axis radiation pattern.

The actual scattering involved with mechanism #2 is not reported in the literature as far as it is known by the author; all analysis techniques are confined to the zero-field approach when it comes to blockage from mechanism #2. The first mechanism (#1), however, has widely been examined, and several techniques have been presented for calculating the
scattering effects caused by the plane wave emanating from the aperture and impinging on the struts [Rusch 1976, Lee 1979, Brachat 1980, Rusch 1982, Kildal 1988, Sletten 1988 (Ch. 4)]. It was found that the presence of the straight struts gives rise to substantial sidelobes, since all contributions result from an equal angle of incidence. The sidelobes are found at angles determined by the tilt of the strut(s), and they hamper satisfaction of the CCIR-recommended reference diagrams. A number of solutions to this problem have been presented. One possible solution is to change the scattering surface of the strut, for instance by constructing a scattering structure at the strut surface, such as sawtooth-like structures [Satoh 1984], by attaching small metal pieces to the strut surface [Thielen 1981], or by applying struts with triangular cross sections [Landecker 1991]. It was found that a relative sidelobe level reduction of several dBs can be obtained. Another solution is to bend the struts outward, also in order to obtain spreading of the scattered field, and this method is found to be as efficient as the solution mentioned previously [Thielen 1981]. With a combination of both solutions, an optimal benefit from the spreading of the scattered field is obtained [Schindler 1985]. A further recommendation is to direct the greater part of the scattering cross section of the sub-reflector support structure towards the ‘cold’ sky, rather than towards the ‘warm’ earth, in order to minimize the antenna noise temperature [Dijkstra 1967]. Especially with ‘tripod’ support structures, i.e. structures with three struts, this method bears fruit, since optimal performance can be achieved by using an unconventional Y-shaped tripod [Landecker 1991]. Note that aperture blockage caused by support structures can be disposed of by making use of offset reflector antenna systems.

Mechanism #3 is merely neglected in analyses of double-reflector antenna systems. In general, it is expected that the direct strut scattering does not substantially influence the total radiation pattern of the antenna, from which the mechanism is omitted. In the following section it is demonstrated that this omission is not always permissible, using
6.2 A numerical example of mechanism #3

In this section the strut scattering according to the third mechanism is examined for a chosen Cassegrain antenna geometry. A quadripod \( \bigotimes \) strut structure is assumed, which is symmetrical with respect to the planes with zero elevation (\( \varphi_e = 0^\circ \)) and zero azimuth (\( \varphi_a = 0^\circ \)), respectively. The antenna orientation is depicted in figure 6.4. A Cartesian \( x_1 x_2 x_3 \) coordinate system is introduced, the origin of which is in the main aperture center. The Cassegrain antenna geometry is unambiguously specified by four independent parameters, viz. the main-reflector diameter \( D_m \), the focal distance of the paraboloid \( F \), the sub-reflector diameter \( D_s \), and the eccentricity of the hyperboloid \( \epsilon \). A thorough description of the geometrical relations within the antenna system is given in [GOVAERTS 1991]. The feed model used is the representation of the radiation pattern of a corrugated horn with the polarization properties of a Huygens source. Its gain function is defined by

\[
G(\theta) = \begin{cases} 
G_0 (a_{feed} + \cos^{m_{taper}} \theta) & \text{for } 0 \leq \theta \leq \frac{\pi}{2} \\
G_0 a_{feed} & \text{for } \frac{\pi}{2} < \theta \leq \pi 
\end{cases},
\]

in which \( \theta \) is the angle that incoming or outgoing rays make with respect to the symmetry axis of the reflectors (i.e. \( \hat{x}_1 \)). The feed gain \( G_0 \) is a function of the parameters \( a_{feed} \) and \( m_{taper} \):

\[
G_0 = \frac{1}{\sqrt{a_{feed}^2 + \frac{a_{feed}}{m_{taper}+1} + \frac{1}{4m_{taper}+2}}}.
\]

The angle by which the incoming and outgoing rays are rotated around \( \hat{x}_1 \), measured from \( \hat{x}_3 \) in counter-clockwise direction, is denoted by \( \phi \). Hence, the polarization vector is determined by

\[
\hat{e}_{pol} = \begin{cases} 
\sin \phi \hat{e}_\theta + \cos \phi \hat{e}_\phi & \text{for horizontal polarization} \\
\cos \phi \hat{e}_\theta - \sin \phi \hat{e}_\phi & \text{for vertical polarization}
\end{cases}.
\]
The struts are expected to be mounted at the reflector rims. To determine their influence on the total off-axis radiation pattern of the antenna, each strut is separately modeled according to the method described in chapter 2. The cylinder end points $\vec{A}$ and $\vec{B}$ for the struts, numbered according to figure 6.4, are found from

$$\vec{A} = \begin{cases} 
-d_{ms}\hat{x}_1 - \frac{1}{4}\sqrt{2}D_s\hat{x}_2 + \frac{1}{4}\sqrt{2}D_s\hat{x}_3 & \text{(strut 1)} \\
-d_{ms}\hat{x}_1 + \frac{1}{4}\sqrt{2}D_s\hat{x}_2 + \frac{1}{4}\sqrt{2}D_s\hat{x}_3 & \text{(strut 2)} \\
-d_{ms}\hat{x}_1 + \frac{1}{4}\sqrt{2}D_s\hat{x}_2 - \frac{1}{4}\sqrt{2}D_s\hat{x}_3 & \text{(strut 3)} \\
-d_{ms}\hat{x}_1 - \frac{1}{4}\sqrt{2}D_s\hat{x}_2 - \frac{1}{4}\sqrt{2}D_s\hat{x}_3 & \text{(strut 4)} 
\end{cases}$$

(6.4)

with $d_{ms}$ the distance between the sub-reflector edge and the main aperture, and

$$\vec{B} = \begin{cases} 
-\frac{1}{4}\sqrt{2}D_m\hat{x}_2 + \frac{1}{4}\sqrt{2}D_m\hat{x}_3 & \text{(strut 1)} \\
\frac{1}{4}\sqrt{2}D_m\hat{x}_2 + \frac{1}{4}\sqrt{2}D_m\hat{x}_3 & \text{(strut 2)} \\
\frac{1}{4}\sqrt{2}D_m\hat{x}_2 - \frac{1}{4}\sqrt{2}D_m\hat{x}_3 & \text{(strut 3)} \\
-\frac{1}{4}\sqrt{2}D_m\hat{x}_2 - \frac{1}{4}\sqrt{2}D_m\hat{x}_3 & \text{(strut 4)} 
\end{cases}$$

(6.5)

The antenna is expected to be receiving a plane wave from a direction specified by $\varphi_e$ and $\varphi_a$. In this first analysis, the radiation pattern in the plane $\varphi_e = 0^\circ$ is examined. Note that the axial symmetry of the reflector systems vanishes when the struts are taken into account, which implies that the radiation pattern is no longer axial symmetric. The observation point is the feed. The radiation pattern is determined as a function of azimuth, and the direction of propagation of the incident wave is defined by

$$\hat{s}' = \cos \varphi_a \hat{x}_1 + \sin \varphi_a \hat{x}_2,$$

(6.6)

with corresponding polarization vector for horizontal polarization

$$\hat{E}_{HOR} = -\sin \varphi_a \hat{x}_1 + \cos \varphi_a \hat{x}_2.$$  

(6.7)

The polarization vector for vertical polarization is found according to

$$\hat{E}_{VER} = \hat{s}' \times \hat{E}_{HOR}.$$  

(6.8)

The direction of incidence upon the struts thus varies with varying azimuth. This implies that with each azimuth step the cylinder and its environment should be translated and rotated in its own $xyz$ system to obtain a configuration in which the cylinder is parallel to the $y$ axis and $\hat{s}'$ is in the $xy$ plane. Note that the polarization vectors of equation (6.3) need to be redefined in the $xyz$ system. Using the model, the resulting scattered field (reflection and diffraction contributions) from each individual strut is calculated. Note that the weighting with the feed gain function $G(\theta)$ and the polarization vector $\hat{e}_{pol}$ should take place before the reflection and diffraction contributions are combined, since $\theta$ and $\phi$ differ per contribution.

The Cassegrain antenna system under consideration is a VSAT system that was also used in the calculations in [GOVAERTS 1991]. The corresponding parameters are: $D_m = 20\lambda$, 


$F = 8\lambda$, $D_s = 2.66\lambda$, $\epsilon = 1.65$, $a_{\text{feed}} = 0.00316$, and $m_{\text{taper}} = 36.75$. For a 10 GHz satellite channel it would be a 60 cm antenna, which would probably be accompanied by struts of a 2 cm diameter, i.e. $a = \lambda/3$. This implies that $a$ is just above the threshold recommended for the use of UTD, and that struts with smaller radii should be approached with the model based on the exact solution.

The observation point is never blocked by the struts for the given configuration. Because of azimuthal symmetry, the radiation pattern is calculated for $\varphi_a \in [1^\circ, 179^\circ]$, with an angular step of 0.2° for both polarization states. Figures 6.5 and 6.6 show the contributions from the struts (dashed curves) and the radiation patterns excluding strut scattering (solid curves) for horizontal and vertical polarization, respectively. Since the total strut contribution is composed of the scattered fields from the two struts that are between the source and the observation point (i.e. the feed) that are in phase, it has a smooth course. The influence of the feed gain function is clearly observed from the shape of the strut contribution. The fact that this contribution is present in a certain azimuth interval only is caused by the presence of blocking reflectors and by the finite extent of the cylinder. The discontinuities in the strut-contribution curve can be readily explained by the fact that for certain azimuth angles reflection may occur, while diffraction is no longer possible, or vice versa. For example, the discontinuity at $\varphi_a \approx 110^\circ$ is caused by the disappearing of the reflection contribution for increasing azimuth. In figures 6.7 and 6.8 the radiation patterns that exclude (solid curve) and include (dashed curve) direct strut scattering are given. For the present configuration substantial sidelobes are created in the plane $\varphi_e = 0^\circ$ by the presence of the sub-reflector support structure in the region $\varphi_a \in [20^\circ, 40^\circ]$, for both polarization states. Furthermore, a sidelobe reduction is created at $\varphi = 90^\circ$ for horizontal polarization only.

The time available for the research of strut scattering was insufficient to consider configurations with strut structures other than those presented in the foregoing. Because of this, the preliminary results are presented with some reservation, since they are not completely verified. It is fair to say, however, that the curves have plausible courses. In future research, the software needs to be adjusted and tested to obtain a uniform model for calculating the strut scattering from any arbitrary strut configuration. Special cases of interest, and thus subjects for future research, are the radiation patterns in planes containing one or more struts. Further investigation should also point out whether the orientation examined in the example is the worst-case strut scattering orientation or not; the already mentioned situation in which the radiation pattern in a plane containing one or more struts is examined may give rise to even more substantial strut-scattering contributions.

Once the model is adjusted and tested, it may be a valuable tool for the design or analysis of any reflector antenna system involving cylindrical support structures. Furthermore, it may be used to optimize the shape of the struts within a given reflector configuration.
Figure 6.5: Received powers in a Cassegrain antenna system; total power excluding strut scattering (solid) and the direct strut scattering contribution (dashed) for horizontal polarization

Figure 6.6: Received powers in a Cassegrain antenna system; total power excluding strut scattering (solid) and the direct strut scattering contribution (dashed) for vertical polarization
Figure 6.7: Received powers in a Cassegrain antenna system; total power excluding strut scattering (solid) and including direct strut scattering (dashed) for horizontal polarization

Figure 6.8: Received powers in a Cassegrain antenna system; total power excluding strut scattering (solid) and including direct strut scattering (dashed) for vertical polarization
Chapter 7

Scattering by multiple parallel cylinders

In this chapter the EM scattering caused by multiple cylindrical obstacles is discussed. The presence of multiple obstacles introduces *mutual-interaction terms* that contribute to the total field received in any observation point $\vec{P}$. It is likely that the influence of the mutual interaction vanishes if the separation between the obstacles becomes large, but it was shown recently that it is not always negligible [Elsherbeni 1992].

Oloafe found an analytical solution for the scattered field from two equal parallel cylinders for plane-wave incidence [Oloafe 1970]. His theory is based on the boundary-value method. Ragheb and Hamid developed a suchlike theory for plane-wave incidence upon $N$ parallel circular cylinders, assuming either thin cylinders or large separation [Ragheb 1985]. Elsherbeni and Hamid proved that these restrictions can be eliminated if the Method of Moments (MoM) is used to calculate the mutual-interaction terms [Elsherbeni 1987], and Elsherbeni and Kishk presented a model for multiple parallel dielectric cylinders [Elsherbeni 1992].

Scattering by multiple parallel cylinders has thus been a widely examined topic over the years, but it is remarkable that the constraint of plane-wave incidence is introduced unvariably. In chapter 5 the geometrical complications in modeling the scattering from spherical-wave incidence were pointed out. It is trivial that the presence of multiple cylinders will enlarge the complexity of the model. In the present chapter *multiple reflection and diffraction are neglected in the model*. The multiple cylinders are treated separately, and their individual complex scattered fields are simply added. The UTD simulation results obtained are compared with results from measurements performed at EUT, in order to draw conclusions on the applicability of the ‘single-cylinder’ model for multiple-cylinder configurations. It is expected, although not proved here, that it is possible to also model the multiple scattering with UTD. The ray tracing based upon the laws of reflection and diffraction will yield a system of equations that can be solved numerically, from which the reflection and diffraction points can be found.

### 7.1 Geometry

For the measurements, the setup described in chapter 5 is used, and the parameters given in table 5.1 apply again. The DUT is a rectilinear array of parallel circular cylinders with symmetry axes perpendicular to the earth’s surface (i.e. $\chi=0$, see figure 5.1). The array is
rotatable around an axis through $\vec{M}$ that is also perpendicular to the earth’s surface and that is at a distance $d_c=17.8$ mm (≈ 29.67λ) from $\vec{T}$, as shown in the top view in figure 7.1.

It is obvious that the UTD for spherical-wave incidence is to be used to simulate these measurements. Since the mutual-interaction terms are not taken into account, the scattering by each cylinder can be treated individually. The orientation of the probes with respect to the cylinder is to be reconsidered. As with all simulations performed in this report, we start from a geometry in which the cylinder axis of symmetry coincides with the $y$ axis. Furthermore, it is assumed that $\vec{T}$ is in the $xy$ plane (similar to section 5.2.3).

The geometry given in figure 7.1 shows that the orientation of the array is determined by an angle $\kappa$, and the distance between the cylinder axis of symmetry and $\vec{M}$ is denoted by $d_d$. Note that both $\kappa$ and $d_d$ can have a negative value. For convenience, an angle $\vartheta$ is introduced according to

$$\vartheta = \arctan \left( \frac{d_d \cos \kappa}{d_c - d_d \sin \kappa} \right).$$ \hspace{1cm} (7.1)

From the figure we find that

$$\vec{T} = \begin{pmatrix} -\sqrt{d_c^2 + d_d^2 - 2d_dd_c \sin \kappa} \\ 0 \\ 0 \end{pmatrix},$$ \hspace{1cm} (7.2)
and

$$\vec{P}(\varphi) = \begin{pmatrix} -\sqrt{d_x^2 + d_z^2} - 2d_xd_z \sin \kappa - d_T \cos \vartheta + (d_T + d_{TR}) \cos(\vartheta - \varphi) \\ -(h_T - h_R) \\ -d_T \sin \vartheta + (d_T + d_{TR}) \sin(\vartheta - \varphi) \end{pmatrix}. \tag{7.3}$$

The Cartesian coordinate system of the transmitter is given by

$$\begin{align*}
\hat{e}_{xWR} &= \cos \vartheta \ \hat{e}_x + \sin \vartheta \ \hat{e}_z, \\
\hat{e}_{zWR} &= \hat{e}_z, \\
\hat{e}_{yWR} &= \hat{e}_{zWR} \times \hat{e}_{xWR},
\end{align*} \tag{7.4}$$

for parallel polarization, and

$$\begin{align*}
\hat{e}_{xWR} &= \cos \vartheta \ \hat{e}_x + \sin \vartheta \ \hat{e}_z, \\
\hat{e}_{yWR} &= -\hat{e}_x, \\
\hat{e}_{zWR} &= \hat{e}_{xWR} \times \hat{e}_{yWR},
\end{align*} \tag{7.5}$$

for perpendicular polarization.

For the receiver, the system configuration depends on the position of the observation point $\vec{P}$. It is found that

$$\begin{align*}
\hat{e}_{xWR} &= -\cos(\vartheta - \varphi) \ \hat{e}_x - \sin(\vartheta - \varphi) \ \hat{e}_z, \\
\hat{e}_{zWR} &= \hat{e}_y, \\
\hat{e}_{yWR} &= \hat{e}_{zWR} \times \hat{e}_{xWR},
\end{align*} \tag{7.6}$$

for parallel polarization, and

$$\begin{align*}
\hat{e}_{xWR} &= -\cos(\vartheta - \varphi) \ \hat{e}_x - \sin(\vartheta - \varphi) \ \hat{e}_z, \\
\hat{e}_{yWR} &= -\hat{e}_y, \\
\hat{e}_{zWR} &= \hat{e}_{xWR} \times \hat{e}_{yWR},
\end{align*} \tag{7.7}$$

for perpendicular polarization.

### 7.2 Two identical cylinders

A practical example of a rectilinear array of identical parallel circular cylinders is a number of lampposts along a straight road. A road-user employing mobile communication might suffer from the presence of these obstacles. A typical mobile-communication frequency is 1.5 GHz. Since the center frequency of the measurements, i.e. 50 GHz, is much higher, the lampposts can actually be replaced by cylinders with manageable diameters in the measurement setup. The cylinders used in the 50 GHz measurements have radii of 2.5 mm, which yields $a/\lambda = 5/12 (> 0.3)$. At 1.5 GHz this corresponds to a cylinder diameter
of 167 mm, which is near the diameter of commonly used lampposts.

In this section, the situation with two ‘lampposts’ is examined. The angle $\kappa$ is chosen to be $0^\circ$, and the cylinders are positioned symmetrically with respect to $\bar{M}$. An observation point range of $\varphi \in [-15^\circ, 15^\circ]$ was set up with an angle step of 0.1°, yielding a measurement time of approximately ten hours. Intuitively it can be expected that the practical separation of the lampposts (25 m or more) causes negligible interaction. The separation within the DUT is restricted by the geometrical and mechanical properties of the measurement setup. The setup allows a physical separation of approximately 400 mm, but it should be noted that the stability decreases with increasing separation, since the cylinders are fairly thin and they start ‘swishing’ very easily. A further restriction is imposed by the

![Graph](image)

**Figure 7.2:** The relative total received power for two equal cylinders separated by 200 mm (solid: UTD result / dashed: measurement result)

(trapeze-shaped) time gate used. The rays from multiple reflection and diffraction have longer path lengths than the direct and single-reflected and single-diffracted rays, and thus they require a larger amount of time between transmission and reception. This may cause that the mutual-interaction contributions are filtered by the gate. A solution would be to enlarge the gate time, but since the risk is incurred that spurious reflections from obstacles other than the DUT are included, this idea was abandoned. It was calculated that a maximum separation of approximately 242 mm is allowed to avoid filtering of the double-reflection and double-diffraction contributions by the gate for all measurement positions taken into account. For that reason, a DUT with a separation of 200 mm ($\equiv 33.3\lambda$) was set up. The separation, which was measured between the cylinder axes of symmetry in the measurement setup, corresponds to 6.7 m at 1.5 GHz. The distance $d_q$ is $\pm 100$ mm, respectively. The results presented in the whole chapter are for parallel polarization only.
Figure 7.2 shows the results; the solid curve represents the UTD simulation result, and the measurement result is dashed. From observation it is found that the measurement result shows some asymmetry, which is probably due to the poor mechanical stability of the DUT, but it can be concluded that the influence of the multiple reflection and diffraction is not very large, since both curves agree rather well, taking into account the inaccuracies of the measurement setup.

The total mutual-interaction field is expected to be merely determined by the double reflection contribution. The influence of the separation and the cylinder radius on the interaction is approached theoretically, considering the simple case of normal plane-wave incidence and assuming double reflection. The amplitude of the field impinging on the second cylinder after being reflected at the first cylinder is proportional to a spreading factor, as defined in equation (2.17). For normal plane-wave incidence, the spreading factor is defined by

$$A(s) = \sqrt{\frac{\rho^*}{\rho^* + s}}$$

on a field strength basis, in which $\rho^* \propto a$, and $s$ is approximately equal to the separation between the cylinders. Hence, the field amplitude of the double-reflection contribution increases with decreasing $s$ or, since $s$ is positive (and mostly much greater than $\rho^*$), with increasing $a$. This implies that the mutual interaction is largest with small separation or thick cylinders. From a separation decrease to 120 mm ($\equiv 20\lambda$, corresponding to 4 m at 1.5 GHz) the results presented in figure 7.3 were obtained. Here, the influence of the multiple reflection and diffraction, which is expected to be larger than with a 200 mm separation, is noted immediately from the course of the curves. From the influence of the multiple reflection and diffraction contributions, unavoidably included in the measurement result, the values and positions of some maxima and minima differ substantially from those in the UTD result, which does not contain multiple-reflection and multiple-diffraction contributions. Next, the cylinders are replaced by cylinders with a doubled radius, i.e. $a/\lambda = 5/6$. Note that the separation of 120 mm between the cylinder axes of symmetry yields a slightly smaller path length $s$ in equation (7.8) for the double-reflection contribution with the thicker cylinders! The results of the measurement and the simulation are found in figure 7.4. Although large mutual-interaction effects are expected, they seem to be absent or canceling each other. The single-reflection and single-diffraction terms, which have larger amplitudes then with the thin cylinders, are obviously determining the total power received. The multiple scattering should be investigated more thoroughly to attach conclusions to these results, for the results do not agree with the (theoretical) expectations. Since the amount of time available for the present research is insufficient, this recommended investigation is not carried out in this report.

In the following section, scattering from four equidistantly spaced parallel cylinders as a function of $\kappa$ is examined.
Figure 7.3: The relative total received power for two equal cylinders \((a = 2.5 \text{ mm})\) separated by 120 mm (solid: UTD result / dashed: measurement result)

Figure 7.4: The relative total received power for two equal cylinders \((a = 5.0 \text{ mm})\) separated by 120 mm (solid: UTD result / dashed: measurement result)
7.3 Four cylinders

The scattering from four cylinders in a rectilinear array with equal separation is treated in this section. Firstly, cylinders with equal diameters are considered. The configuration corresponds to a street with four lampposts within four meters (if scaled to 1.5 GHz), and the effect of the angle of incidence of the mobile-communication signal is investigated in the following. Scaled to a 50 GHz frequency this yields $a/\lambda = 5/12$ and $d_{d}/\lambda = 10, 5, -5, \text{ and } -10$, respectively. The measurements were carried out for five different geometries, with $\kappa = 0^\circ, 20^\circ, 40^\circ, 60^\circ, \text{ and } 80^\circ$, respectively, as depicted in figure 7.5. In the figure, the vertical lines represent the respective center positions. The corresponding results are shown in the figures 7.6 through 7.10. If figure 7.6 is compared with figure 7.3, corresponding to a DUT containing the outer two of the present four cylinders, a comparable influence of the multiple reflection and diffraction is observed, taking into account the different power scales. Note that the increase in total scattering cross section yields a lowering of the minima in the curves. Figure 7.7 shows the results for $\kappa = 20^\circ$. The asymmetry can be readily explained by the introduction of an angle $\kappa \neq 0^\circ$. The results seem to agree better than with $\kappa = 0^\circ$. The same conclusions can be attached to the results in figures 7.8 and 7.9, with $\kappa = 40^\circ$ and $\kappa = 60^\circ$. From these results it appears that the need to take into account mutual-interaction terms is very small, since this interaction is almost negligible. However, if an angle $\kappa = 80^\circ$ is set up, substantial influences of the multiple reflections and diffractions are found, as demonstrated in figure 7.10. The superposition of the individual results obtained from the UTD yields a very small power level (relative to the directly received field) near the center position, which is approximately 10 dB less than the measured power level. The difference is caused by the fact that the blockage of single-reflected and single-diffracted rays in the real situation with $\kappa = 80^\circ$ (see figure 7.5), are not taken into account with the UTD simulations. The differences in the region $\varphi < -5^\circ$ are probably caused by the constructive and destructive interference of the actual mutual-interaction contributions, that are not taken into account with the UTD result. To check the credibility of the measurement result, the measurement was repeated. In the meantime not a single part of the setup was touched, but despite of that a (slightly) different result, represented

![Figure 7.5: Orientation of the array with respect to the center position, for the different values of $\kappa$ used](image)

...
Figure 7.6: The relative total received power for four equal cylinders and $\kappa = 0^\circ$ (solid: UTD result / dashed: measurement result)

Figure 7.7: The relative total received power for four equal cylinders and $\kappa = 20^\circ$ (solid: UTD result / dashed: measurement result)
Figure 7.8: The relative total received power for four equal cylinders and $\kappa = 40^\circ$ (solid: UTD result / dashed: measurement result)

Figure 7.9: The relative total received power for four equal cylinders and $\kappa = 60^\circ$ (solid: UTD result / dashed: measurement result)
by the dotted line in figure 7.10, was obtained. The difference is a good indication for the inaccuracy of the present setup.

The last geometry examined involves four cylinders with different radii. The DUT employed is oriented with $\kappa = 0^\circ$. It has two thin cylinders ($a/\lambda = 5/12 = 0.42$) at $d_d/\lambda = 5$ and -10 and two cylinders with a doubled radius ($a/\lambda = 5/6 = 0.83$) at $d_d/\lambda = 10$ and -5. The results are shown in figure 7.11. As with the situation involving two cylinders, the presence of cylinders with relatively large radii does not give rise to substantial mutual-interaction influences, and the results appear to be similar to or even better than those obtained from a configuration with relatively thin cylinders.

Concluding, we can state that the interaction contributions to the scattered field caused by the presence of multiple (parallel) cylinders are negligible in many cases. It is expected, although not proved here, that a first improvement of the model involving multiple cylinders will be found by taking into account the optical blockage of single-reflected or single-diffracted rays. Note that this adjustment may give rise to discontinuities in the resulting curves. A further investigation should point out why the results show less mutual-interaction influence for cylinders with larger radii than theoretically expected. It is suggested to make use of cylinders with radii well above the threshold of $0.3\lambda$, since the UTD may introduce substantial errors when thinner cylinders are used. Finally, it should be mentioned that the use of the UTD model is not restricted to parallel cylinders, since it also suits oblique incidence.

![Figure 7.10: The relative total received power for four equal cylinders and $\kappa = 80^\circ$ (solid: UTD result / dashed and dotted: measurement results)](image)
Figure 7.11: The relative total received power for four cylinders with different radii and $\kappa = 0^\circ$ (solid: UTD result / dashed: measurement result)
Chapter 8

Conclusions

A model for calculating the electromagnetic plane-wave scattering by a circular cylinder of arbitrary radius was developed and implemented in software. With the model, the scattering behavior of any perfectly conducting cylinder, arbitrarily oriented with respect to the propagation direction of the incident wave, can be analyzed. For this purpose, the configuration is transformed to a prescribed geometry before the calculations are carried out.

It was found that for a normalized cylinder radius $a/\lambda \geq 0.3$ a procedure based on the UTD can be used. It appears that it is sufficient to take into account two diffracted rays per observation point with this method. It is demonstrated that all other diffraction contributions have negligible influence, caused by the energy loss of the diffracted wave as it propagates along the surface.

For radii $a < 0.3\lambda$ it was found that the UTD solution fails in predicting the correct scattered field. An alternative method, i.e. the exact solution of the scattered field, is suggested, derived, and implemented. Its numerical calculation provides accurate results, and the computing time needed with small radii is approximately equal to the time needed for the UTD calculations. From comparisons between the results from the exact solution and the UTD model for radii $a \geq 0.3\lambda$, it is concluded that the UTD model is accurate in this region and needs less computing time. This implies that both theories complement each other, since there is no cylinder radius region left uncovered, and the computing time is minimized.

Furthermore, a UTD model accounting for spherical-wave incidence was developed and implemented in software. In this case, a more complex model is required compared to the model for plane-wave incidence. It is shown that results obtained from measurements performed at EUT involving spherical waves agree well with the simulation results from the UTD model. The small differences observed can be readily explained by the (geometrical) imperfections of the current measurement setup and the devices under test. To reduce some of these inaccuracies, a more accurate setup should be constructed. In the near future an improved setup will be available at EUT.

From both the measurement and simulation results it was found that the shielding effectiveness of circular cylinders is best if the field is polarized parallel to the cylinder axis of symmetry. As expected, the site shielding factor increases with increasing cylinder radius, independent of the polarization state.
An application of the model, viz. the calculation of direct strut scattering in reflector antenna systems, is presented. Preliminary, but plausible, results of direct strut scattering in a given Cassegrain antenna system demonstrate that in general it is not permissible to neglect its influence on the off-axis radiation pattern of the antenna. With the inclusion of strut scattering and a method to calculate the field strength at boresight, the model to calculate the radiation pattern of a Cassegrain antenna system is completed. The strut scattering mechanisms other than the direct strut scattering should also be taken into account, and for that purpose, the models presented in this report can be applied. Future research on strut scattering is recommended.

Finally, it is demonstrated that the 'single-cylinder' model can be used for a large number of configurations involving multiple cylinders. From comparison between measurements performed at EUT and simulations it is found that the effects of multiple reflection and diffraction are negligible in most practical situations, such as the presence of multiple lamp-posts in a mobile-communication environment. In situations where the single-reflected and single-diffracted rays are blocked by the presence of another (cylindrical) obstacle, the 'single-cylinder' approach fails. A simple adjustment to the model that accounts for this optical blockage may improve the accuracy of the simulation results.
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A special word of thanks goes out to all of the EC cohabitants of the eleventh floor for colorful conversations, competent cooperation in case of complex computations, collaborative coffee consumption, comic coincidental combat, comprehension, constructive comments, and common concern. (I hope this sentence hasn’t driven you COConuts!)

Finally, I owe my parents Ans and Goof Govaerts a lot for their unbounded love and care.

[The Beatles 1963] Twist and shout
B. Russell and P. Medley
from Please, please me, Music For Pleasure/EMI-Records Limited, Holland, 1963

One last remark: I think that even the most brilliant engineer isn’t any good if he doesn’t care about putting his findings into words unambiguously and correctly!
Appendix A

Translation and rotations

The configuration in which a plane wave is incident upon a circular cylinder is specified by the following parameters:

- \( \vec{A} \) and \( \vec{B} \): two points at the intersections of the cylinder symmetry axis and the top planes;
- \( a \): the cylinder radius;
- \( \vec{P}_0 \): the reference point in which the phase of the incident field equals zero;
- \( \vec{s}' \): the direction of propagation of the incident field;
- \( \vec{E} \): the (linearly polarized) electric field;
- \( \vec{P} \): the observation point in which the total field is to be calculated.

In the uniform model we expect \( \vec{s}' \) to be in the \( xy \) plane with its \( x \) component \( s'_x \geq 0 \). Further we want \( \vec{A} \) to be at the origin \( O \) of the \( xyz \) system, with \( \vec{B} \) located somewhere on the positive \( y \) axis, as visualized in figure 2.2. To accomplish these requirements some translations and rotations must be made. The necessitated procedures are subsequently treated in the following sections.

A.1 Translation

If \( \vec{A} \neq O \) the whole scenery is to be translated, which means that the vector \( \vec{A} \) should be subtracted from \( \vec{B}, \vec{P}, \) and \( \vec{P}_0 \).

A.2 First rotation

If \( \vec{B} \) is not on the \( y \) axis after the translation is performed, the whole scenery is to be rotated. First a rotation around the \( z \) axis is carried out (which is not necessary if \( \vec{B} \) is positioned at the same \( z \) axis) and next, if \( \vec{B} \) is not already found on the \( y \) axis, the whole is rotated around the \( x \) axis. This procedure thus involves two subsequent rotations from which we will find \( \vec{B} \) on the positive \( y \) axis.

In the following the software listing of the rotation procedure is given. It should be applied four times in a row, with \( \text{vec1} = \vec{A} \vec{B} \) and \( \text{vec2} = \vec{s}', \vec{E}, \vec{P}_0, \) and \( \vec{P} \). We expect that the software listing itself is sufficiently clarifying the procedure and will therefore not give a wordy description of the steps to be taken.
procedure rotatetoy( vec1 : vector;
        var vec2 : vector);

var len,ang1,ang2 : real;

begin
{rotate around z axis:}
    normalize(vec1,vec1);

    if abs(vec1[2])<1e-10 then ang1:=pi/2
    else ang1:=abs(arctan(vec1[1]/vec1[2]));

    if ((vec1[1]<0) and (vec1[2]>=0)) then ang1:=-ang1;
    if ((vec1[1]<0) and (vec1[2]<0)) then ang1:=ang1-pi;
    if ((vec1[1]>=0) and (vec1[2]<0)) then ang1:=pi-ang1;

    vec1[2] :=sqrt(sqr(vec1[1])+sqr(vec1[2]));
    vec1[1] :=0;

    if abs(vec2[2])<1e-10 then ang2:=pi/2
    else ang2:=abs(arctan(vec2[1]/vec2[2]));

    if ((vec2[1]<0) and (vec2[2]>=0)) then ang2:=ang2;
    if ((vec2[1]<0) and (vec2[2]<0)) then ang2:=ang2-pi;
    if ((vec2[1]>=0) and (vec2[2]<0)) then ang2:=pi-ang2;

    len:=sqrt(sqr(vec2[1])+sqr(vec2[2]));
    vec2[1] :=sin(ang2-ang1)*len;
    vec2[2] :=cos(ang2-ang1)*len;

{rotate around x axis:}
    if abs(vec1[2])<1e-10 then ang1:=pi/2
    else ang1:=abs(arctan(vec1[3]/vec1[2]));

    if ((vec1[3]<0) and (vec1[2]>=0)) then ang1:=ang1;
    if ((vec1[3]<0) and (vec1[2]<0)) then ang1:=pi-ang1;
    if ((vec1[3]>=0) and (vec1[2]<0)) then ang1:=pi-ang1;

    if abs(vec2[2])<1e-10 then ang2:=pi/2
    else ang2:=abs(arctan(vec2[3]/vec2[2]));

    if ((vec2[3]<0) and (vec2[2]>=0)) then ang2:=ang2;
    if ((vec2[3]<0) and (vec2[2]<0)) then ang2:=ang2-pi;
    if ((vec2[3]>=0) and (vec2[2]<0)) then ang2:=pi-ang2;

    len:=sqrt(sqr(vec2[3])+sqr(vec2[2]));
    vec2[3] :=sin(ang2-ang1)*len;
    vec2[2] :=cos(ang2-ang1)*len;
end;

A.3 Mirror

Next \( \vec{s}' \) should be directed in accordance with the model. Since \( s'_x \) is expected to be positive or zero, the whole configuration is to be mirrored with respect to the \( yz \) plane if \( s'_x < 0 \). This is easily done by changing the sign of the \( x \) components of \( \vec{E}, \vec{P}_0, \vec{P} \), and \( \vec{s}' \) itself.
A.4 Second rotation

Figure A.1: Definition of the angle $\zeta$

The final requirement is that of $\hat{s}'$ being positioned in the $xy$ plane. If it is not, the whole scenery should be rotated around the $y$ axis by an angle

$$\zeta = \arctan \left( \frac{-s_z'}{s'_x} \right). \quad (A.1)$$

Figure A.1 shows how $\zeta$ is defined in the coordinate system.

If the vector that is to be rotated is denoted $\vec{v}$, we compose the rotated vector $\vec{v}'$ using equations (A.2), (A.3), and (A.4):

$$v'_x = \begin{cases} -\sqrt{v_x^2 + v_z^2} \cos \left( \zeta + \arctan \left[ \frac{v_x}{v_z} \right] \right), & v_x < 0 \\ -v_z \sin \zeta, & v_x = 0 \\ \sqrt{v_x^2 + v_z^2} \cos \left( \zeta + \arctan \left[ \frac{v_x}{v_z} \right] \right), & v_x > 0 \end{cases} \quad (A.2)$$

$$v'_y = v_y, \quad (A.3)$$

$$v'_z = \begin{cases} -\sqrt{v_x^2 + v_z^2} \sin \left( \zeta + \arctan \left[ \frac{v_x}{v_z} \right] \right), & v_x < 0 \\ v_z \cos \zeta, & v_x = 0 \\ \sqrt{v_x^2 + v_z^2} \sin \left( \zeta + \arctan \left[ \frac{v_x}{v_z} \right] \right), & v_x > 0 \end{cases} \quad (A.4)$$

In this way, the vectors $\vec{E}, \vec{P}_0, \vec{P}$, and $\hat{s}'$ itself are to be rotated and the desired configuration is obtained.
Appendix B

Bessel functions

In this appendix the accuracy of the algorithms used to determine the functions needed for the exact solution of the scattered field is examined. These functions are the solutions $w(z)$ of the differential equation

$$ z^2 \frac{d^2 w}{dz^2} + z \frac{dw}{dz} + (z^2 - n^2)w = 0, \quad (B.1) $$

in which $z$ is complex. The solutions are called Bessel functions, but to obtain a less-confusing nomenclature the Bessel functions of the second and third kind are sometimes called Weber and Hankel functions, respectively. The Bessel function of the first kind of order $n$ is defined according to [Abramowitz 1965 (Ch. 9), Press 1989 (Ch. 6)]

$$ J_n(x) = \left( \frac{1}{2}x \right)^n \sum_{k=0}^{\infty} \frac{(-\frac{1}{4}x^2)^k}{k! \Gamma(n + k + 1)}, \quad (B.2) $$

which reduces to

$$ J_n(x) = \left( \frac{1}{2}x \right)^n \sum_{k=0}^{\infty} \frac{(-\frac{1}{4}x^2)^k}{k! (n + k)!}, \quad (B.3) $$

since $n$ is assumed to be an integer. The complex variable $z$ is replaced by the real variable $x$, since we do not have to deal with complex variables. An important property for our purpose is the fact that $\lim_{x \to 0} J_n(x)$ exists.

The Hankel function of the second kind of order $n$ is defined by

$$ H_n^{(2)}(x) = J_n(x) - jY_n(x), \quad (B.4) $$

in which $Y_n(x)$ is the Bessel function of the second kind (Weber's function) of order $n$ defined by

$$ Y_n(x) = \lim_{\nu \to n} \frac{J_\nu(x) \cos(\nu \pi) - J_{-\nu}(x)}{\sin(\nu \pi)}. \quad (B.5) $$

Here, it should be noted that $\lim_{x \to \infty} H_n^{(2)}(x)$ exists. The Hankel function of the second kind is therefore suitable to present outward-traveling waves.
For all Bessel functions (denoted by $C$) the following recurrence relations can be derived [ABRAMOWITZ 1965 (Ch. 9)]:

\[
C_n(x) = \frac{2(n-1)}{x} C_{n-1}(x) - C_{n-2}(x), \quad (B.6)
\]

\[
C'_n(x) = C_{n-1}(x) - \frac{n}{x} C_n(x), \quad (B.7)
\]

\[
C'_n(x) = -C_{n+1}(x) + \frac{n}{x} C_n(x). \quad (B.8)
\]

Further we the following relation will prove to be useful:

\[
C_{-n}(x) = (-1)^n C_n(x). \quad (B.9)
\]

Once $J_0(x)$, $Y_0(x)$, $J_1(x)$, and $Y_n(x)$ are known, we can easily calculate any arbitrary Bessel function needed. To calculate these functions, the software described in [PRESS 1989 (Ch. 6)] is used for a basis. In [PRESS 1989 (Ch. 6)], algorithms are presented to numerically determine $J_n(x)$ and $Y_n(x)$ for $n \geq 0$.

We have empirically determined the absolute and relative errors introduced by these algorithms. The exact value $C_n(x)$ can be written in terms of the calculated value $\tilde{C}_n(x)$ and the absolute error $\delta_{abs}$:

\[
C_n(x) = \tilde{C}_n(x) \pm \delta_{abs}, \quad (B.10)
\]

with $\delta_{abs} \geq 0$. The relative error is then found by

\[
\delta_{rel} = \frac{\delta_{abs}}{|C_n(x)|}. \quad (B.11)
\]

The exact values are tabulated in [ABRAMOWITZ 1965 (Ch. 9)] as a function of $n$ and $x$. We have examined the software by [PRESS 1989 (Ch. 6)] to calculate $J_0(x)$, $J_1(x)$, $J_n(x)$ ($n \geq 2$), $Y_0(x)$, $Y_1(x)$, and $Y_n(x)$ ($n \geq 2$) for different combinations of $n$ and $x$ in order to obtain insight into the magnitude of the errors made. The results are tabulated in table B. Although a fairly large absolute error occurs in the calculation of $Y_n(x)$, it is obvious that the algorithms are still very accurate since $\delta_{rel}$ never exceeds $10^{-8}$.

<table>
<thead>
<tr>
<th>function</th>
<th>max $\delta_{rel}$</th>
<th>max $\delta_{abs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_0(x)$</td>
<td>$3 \cdot 10^{-7}$</td>
<td>$9 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$J_1(x)$</td>
<td>$6 \cdot 10^{-6}$</td>
<td>$6 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$J_n(x)$</td>
<td>$6 \cdot 10^{-7}$</td>
<td>$2 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>$Y_0(x)$</td>
<td>$3 \cdot 10^{-6}$</td>
<td>$4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$Y_1(x)$</td>
<td>$4 \cdot 10^{-7}$</td>
<td>$5 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$Y_n(x)$</td>
<td>$3 \cdot 10^{-8}$</td>
<td>$6 \cdot 10^{-1}$</td>
</tr>
</tbody>
</table>

*Table B.1: Maximum errors in the numerical calculation of Bessel functions*
The calculation of the scattered field (using either (3.44) or (3.49)) requires sums of an infinite number of terms. However, since the absolute value of these terms decreases with increasing order, these terms can be neglected for \( n \) greater than a certain value \( N \). In the following we each time consider either the \( x \), \( y \), or \( z \) component of the scattered field to be calculated. If all terms independent of \( n \), \( ka \sin \beta \), \( k \rho \sin \beta \) and \( \phi \) are suppressed we can present the scattered-field component \( E^s_i \) (\( i = x \), \( y \), or \( z \)) by

\[
E^s_i \propto \sum_{n=-\infty}^{\infty} \mathcal{F}_n(ka \sin \beta, k \rho \sin \beta, \phi).
\]

If a maximum required \( \delta_{rel} \) is given, \( N \) is the smallest value of \( \ell \) for which equation (B.13) applies:

\[
\left| \mathcal{F}_{-\ell}(ka \sin \beta, k \rho \sin \beta, \phi) + \sum_{n=-\ell+1}^{\ell-1} \mathcal{F}_n(ka \sin \beta, k \rho \sin \beta, \phi) \right| < \delta_{rel},
\]

with \( \phi \) constant. It is obvious that (B.13) should apply for all components to obtain the required accuracy of the solution.

Since we are interested especially in the scattered far-field component, we will assume that \( k \rho \sin \beta \) is very large. From this assumption it is found that variations in the scattered field are strongly dependent on variations in \( ka \sin \beta \), and that \( N \) increases with increasing \( ka \sin \beta \).

The user can acquire a desired accuracy in the solution for the scattered field if equation (B.13) is implemented in the software and \( \delta_{rel} \) is specified by him on forehand. He should also specify a value \( N \) that determines the initial number of elements being summed (\( n \) varying from \(-N\) to \( N \)). If \( N < N \), \( N \) should be increased automatically in the calculations (by a loop in the software), until equation (B.13) holds. In figure B.1 we have generated a graph of the relation between \( ka \sin \beta \) (denoted by ‘\( ka \sin (\beta) \)’), \( N \) (denoted by ‘\( \text{minimum } N \)’), and \( \delta_{rel} \). This was accomplished by determining the validity of equation (B.13) for all six components (viz. three components per polarization state) at \( \phi = \frac{\pi}{180} \), with \( l \) an integer, \( l \in [0, 180] \). Hereby, we chose \( k \rho \sin \beta = 2\pi \cdot 10^5 \).

The results shown in figure B.1 were found using curve fitting with a polynomial of the fifth degree. It is obvious that the accuracy of the calculation of the ‘exact’ solution is improved by increasing the maximum order \( N \) of the summation. It is easily seen that an increase of \( N \) with 2 or 3 will yield a tenfold increase of the accuracy.

From figure B.1 the user can determine a proper combination of the input parameters \( a \), \( \beta \), \( \delta_{rel} \), and \( N \).
Figure B.1: $N$ ('minimum $N$') as a function of $ka \sin \beta$ for different values of $\delta_{\text{rel}}$
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