Eindhoven University of Technology

MASTER

Fault voltages and currents in low voltage networks with coupled neutral conductors

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Award date:
1995

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FAUL T VOLTAGES AND CURRENTS IN LOW VOLTAGE NETWORKS WITH COUPLED NEUTRAL CONDUCTORS.

A.M.S. Atmadji

EG/95/797.A.

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SUMMARY

Electricity has infiltrated our industrial and everyday life in such way that we almost can not live without it. The need for electric power transmission facilities has been increased amazingly. As a consequence, numerous problems associated with the reliability, safety and economy of electric power have emerged. This master thesis concerns a safety aspect of low voltage networks, related with the shock danger. Herein, fault voltage and fault current are considered as main parameters. While present investigations mainly represent the network configurations with a resistance network, this work concentrates on the effect of inductive coupling.

The first chapter is an introduction. Chapter 2 is devoted to safety aspects for delivering of electrical energy considering norms and standards. Chapter 3 presents an overview of inductance theory and basic concepts for simply two conductor systems. The interrelationship between the simplified models and an exact model is emphasized. Chapter 4 describes an experimental verification of found theoretical relations, for a three conductor configuration. The model is performed to investigate the effect of earthing electrode and multiple return (neutral) conductors on fault voltage and fault current. Next, on chapter 5 practical low voltage networks are analyzed, measuring of fault voltage and current is performed, and the theoretical model is applied to verify the measurement results. As a separate study, in chapter 6 a current distribution in two conductor system is discussed and the results considering skin effect and proximity effect are visually represented.
# Units and Dimensions

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CHAPTER 1

INTRODUCTION

To provide electrical energy to customers, not only the reliability should be considered also safety aspects are of importance. Faults current must be interrupted within a certain time by protective apparatuses, to limit an overflow of energy through the network. At the same moment, a fault voltage might occur also in the installation which must be kept below a safety level voltage to prevent shock danger until the fault is disconnected.

Generally, to analyze and design a low voltage network one uses resistances to replace the conductors. From then, the fault current and fault voltage may be derived (approximated). The choice of using protective equipments will be installed after considering these parameters and the maximum of delivered power. On a dynamic growing low voltage network, adding extended parallel cables are often used to provide more electrical energy. This may lead a consequence that safety of the network might change. Analyzing of the safety each time this network is expanded, should be performed. Next, the safety of the network must be verified also with measurement. Direct measurements with intended fault on field can be very dangerous. A measuring equipment that can simulate the fault can be very useful to replace an actual fault. Mostly, the measurement results of fault voltage and current can not be simple explained, especially on distribution cable using a resistance model. The magnitude of the fault current there is mostly determined by impedances of the distribution cables. The resistance model may still work if the fault occurs in house installation, which resistances of the house installation cables are much greater compare with the distribution cables.

The aim of the investigation is to get an insight in the phenomenon (effect) of fault voltage and fault current during a fault on low voltage network; in regard with using coupled neutral conductors and placing local earthing electrodes. This work introduces new electrical parameter so called inductance for dealing with fault current and fault voltage. Beginning with summarizing of general definitions and present standard on low voltage network. Followed by basic definition of inductance and solving mathematical expressions. Next, resistances, inductances of the conductors and mutually coupled conductors are applied together to solve simple circuits with 3 conductors and this leads to so called coupling method. Further, this method is completed by introduction a local earthing resistance. Finally, this coupling method is applied on a simple practical network to correlate the measurement results on field with success.

In a complex low voltage network, applying the coupling method to predict fault voltage and fault current may be very difficult or impossible. A measurement is still needed and can be an only way to verify the safety of the low voltage network.

Further, the coupling method is also applied to compute the current distribution on a cross section of a two conductor system. It is successful to demonstrate the skin effect and proximity effect.

All computations are implemented in Matlab language.
CHAPTER 2

PROTECTION METHODS IN LOW VOLTAGE NETWORK

The protection against electric shock is one of the essential aims of the designer of any electrical installation and is the subject of many of the individual requirements embodied in the regulations the designer has to take into account.

Mostly electrical equipments work under rated voltage 220V. The need of safety requirement should be maintained under all work situation including when a fault occurs whereby a human may work with those equipments concerned. It means also that electrical installation has an important aspects beside providing reliable electrical energy and that is the protection of life and limb. An important aspect of these requirements is represented by measures which prevent the occurrence or persistence of a dangerous touch voltage $U_a$.

2.1 DEFINITIONS

According to VDE 0100 Teil 200, follows next the important basic definitions and terms in regard with touch voltage and earthing on low voltage network. [1] and [2]

- **The touch voltage**: voltage appearing during an insulation fault, between simultaneously accessible parts. This touch voltage term is used only in connection with protection against indirect contacts. In certain cases, the value of the touch voltage may be appreciably influenced by the impedance of the person in contact with these parts.
- **Prospective touch voltage**: the highest touch voltage liable to appear in the event of a fault of negligible impedance in the electrical installation.
- **Neutral conductor** (N): a conductor connected to the neutral point of a system and capable of contributing to the transmission of electrical energy.
- **Protective Earth conductor** (PE) or **equipment grounding conductor**: a conductor required by some measures for protection against electric shock for electrically connecting any of the following parts: exposed conductive parts, extraneous conductive parts, main earthing terminal, earth electrode and earthing point of the source or artificial neutral.
- **Protective Earth Neutral** (PEN) **conductor** is an earthed conductor combining the function of both protective conductor and neutral conductor.
- **Live part**: a conductor or conductive part intended to be energized in normal use, including a neutral conductor, but by convention, not a PEN conductor.
- **Exposed conductive part**: a conductive part of electrical equipment, which can be touched and which is not normally live, but which may become live under fault conditions.
- **Extraneous conductive part**: a conductive part not forming part of the electrical installation and liable to introduce a potential, generally the Earth potential.
- **Electric shock**: pathophysiological effect resulting from an electric current passing through a human or animal body.
- **Shock current**: a current passing through a body of a person or animal and having characteristics likely to cause pathophysiological effects.
- **Leakage current**: a current which, in the absence of a fault, flows to earth or to extraneous conductive parts.
- **Residual current**: the algebraic sum of the instantaneous values of current flowing through all live conductors of a circuit at a point of the electrical installation.
- **Direct contact** means contact of person or livestock with electrical parts which are live during operation (active parts)
- **Indirect contact** means contact of person or livestock with conductive parts which do not belong to the operating circuit but which can assume a potential to earth under fault conditions; these parts are the "exposed conductive parts" of the equipment.
- **arm's reach**: a zone extending from any point on a surface where persons usually stand or move about, to the limits which a person can reach with the hand in any direction without assistance.
- **enclosure**: a part providing protection of equipment against certain external influences and, in any direction, protection against direct contact.
- **barrier**: a part providing protection against direct contact from any usual direction of access.
- **obstacles**: a part preventing unintentional direct contact, but not preventing direct contact by deliberate action.
- **Simultaneously accessible parts** are conductors or conductive parts which can be touched simultaneously by a person or where applicable, by livestock. Simultaneously accessible parts may be live parts, exposed conductive parts, extraneous conductive parts, protective conductors and earth electrodes.
- **Earth or ground**: the conductive mass of the Earth, whose electric potential at any point is conventionally taken as equal to zero.
- **Earth electrode**: a conductive part or a group of conductive parts in intimate contact with and providing an electrical connection with earth.
- **Earthing conductor or grounding electrode conductor**: a protective conductor connecting the main earthing terminal or bar to the earth electrode.
- **Main earthing terminal or main earthing bar**: a terminal or bar provided for the connection of protective conductors, including equipotential bonding conductors and conductors for functional earthing if any, to the means of earthing.
- **Equipotential bonding**: electrical connection putting various exposed conductive parts and extraneous conductive parts at a substantially equal potential.
- **Equipotential bonding conductor**: a protective conductor for ensuring equipotential bonding.
- **Functional earthing** is the earthing of a point in the operating circuit which is necessary for the satisfactory operation of equipment or systems. It is said to be
  a. direct, when it introduces no further impedances beyond the earthing impedance
  b. indirect, when it is effected through additional resistance, inductance or capacitance
- **Earthing system** is the total in a defined area of earth electrodes electrically connected together, or metal parts serving the same purpose (e.g. tower footings, armouring, metal cable sheath) and earthing conductors.
- **A natural earth** is a metal part directly connected to earth or water, whose original purpose is not to provide an earth, but which serves the purpose of an earth electrode. This includes pipework, sheet piling, concrete pile reinforcing, metal parts of buildings, etc.
- **A cable acting as an earth electrode** is a cable such that metal sheaths, screens or armouring have a conductance to earth that is of the same order of a strip earth electrode.
- **Short-circuit current** is the current resulting from a fault of negligible impedance between live conductor having a difference in potential under normal operating conditions.

### 2.2 PROTECTION AGAINST ELECTRIC SHOCK

There are two types of shock risk on electrical installation, direct contact and indirect contact. Direct contact means the contact with parts which are live during operation. This affords two kind of protection; complete protection, such insulation, barriers or enclosure and partial protection like obstacles and placing out reach.
- insulation; this must enclose all active parts and may become ineffective as a result of damage. Preferred applications are to conductors and loads.
- barriers and enclosures; a mechanical protective can realize this and the arrangements must be strong enough to prevent their coming into contact with active parts under impact or pressure.
Chapter 2 Protection methods in low voltage networks

- obstacles; these do not, however, prevent the intentional touching of active parts.
- protection by spacing is afforded where active parts are disposed beyond arm's reach.
- additional protection by residual current (earth-leakage) device ($I_{SN} \leq 30\text{mA}$).

To protect against indirect contact to conductive parts that normally this conductive parts can be touched and are not live parts but which may become live under fault condition, the Wiring Rules [IEC Publication 364] recognize five protective measures and these are:

- protection by earthed equipotential bonding and automatic disconnection of supply
- protection by Class II equipment or by equivalent insulation
- protection by nonconducting location
- protection by earth-free local equipotential bonding
- protection by electrical separation

The first of these is the protective measure most commonly encountered and this chapter emphasizes this case. The fundamental requirement protective measure can be stated as follows [3]:

The characteristic of the protective devices for automatic disconnection, the earthing arrangement for the installation and the relevant impedances of the circuits concerned shall be co-ordinated so that during an earth fault the voltages between simultaneously accessible conductive parts occurring anywhere in the installation shall be of such magnitude and duration as not to cause danger.

These voltages are the touch voltage. The simultaneous accessible conductive parts are exposed conductive parts and extraneous conductive parts. Metallic enclosures of equipment, metallic enclosures for cables such as conduit, ducting and trunking and cable armouring are examples of exposed conductive parts. Other metalwork like gas pipes, water pipes, heating equipment and elements of the building structure can be called as extraneous conductive parts obviously, which are defined as conductive parts liable to introduce a potential, generally an earth potential and do not form part of the electrical installation.

An earth fault in installation system can create a touch voltage between:

- the exposed conductive parts of the faulty equipment and extraneous conductive parts.
- the exposed conductive parts of the faulty equipment and similar parts of other circuits in the installation.
- the exposed conductive parts of the faulty equipment and similar parts of other equipment fed by the same circuit.
- the exposed conductive parts of two healthy equipments fed by the same circuit as the faulty equipment.
- the exposed conductive parts of healthy equipment and extraneous conductive parts.

During design those cases must be considered. Verification by testing, must be taken also before the installation operates.

2.3 SAFETY CRITERIA

Generally, the safety criteria of low voltage network, is tested using the following procedure, see fig. 2.1. From a given low voltage network configuration, the significant quantity of fault voltage ($U_f$) and fault current ($I_f$) are determined from computation (if it is conveniently possible). When it is impossible, the measuring should be taken at various place on the network concerned. From the fault current value, the disconnection time of the supply are examined how long the fault might be occurred, considering the data book of the protection equipment (fuse, automatic disconnection equipment etc). It leads into a certain disconnection time as $t_{out}$. And from fault voltage value, using the transition resistance, the touch voltage could be obtained and the current through body can be predicted using the known curve as shown on fig. 2.3 (according to IEC 479), then the maximal allowed duration of this current
leads to $t_{\text{max}}$. Comparing these $t_{\text{out}}$ and $t_{\text{max}}$ gives the protection against indirect contact when $t_{\text{out}} < t_{\text{max}}$. It means that the fault is already disconnected before the electric shock danger occurs on the man at risk.

![Diagram showing safety criteria](image)

**Fig. 2.1 Safety Criteria**

with:
- $U_f$: fault voltage
- $I_f$: fault (short circuit) current
- $R_{\text{trans}}$: transition resistance
- $t_{\text{max}}$: maximal allowed time of the fault
- $R_{\text{body}}$: body resistance
- $I_b$: body current
- $t_{\text{out}}$: disconnection time of the protective device
- $U_a$: touch voltage

The safety of the low voltage network can be achieved when $t_{\text{out}} < t_{\text{max}}$. Otherwise the short circuit occurred too long and cause electric shock danger (in worst case can be deadly) to the person or animal that might be in contact with the exposed conductive part.

Transition resistance ($R_{\text{trans}}$) is a quantity that depends on local circumstances (situation) and may contain the following resistances: footwear-resistance, floor-resistance and surface earthing resistance (to the mass Earth). Practically, these resistances are dynamically changing with time and place, and often unavailable or unknown. The value of $R_{\text{trans}}$ is recommended as 1k$\Omega$, and this value is generally accepted by electricity company. In some case, this resistance value might be so low that the touch voltage could be so close to the fault voltage. It means an extra electric shock danger may be appeared to the person at risk that work on it.

Further, the body resistance is not a constant quantity. It depends on the voltage that may appear between body parts and the current frequency through the body. By low frequency, the body can be assumed as a resistance. The problem is interfered by the fact that each person has actually a different body resistance curve and thus different sensitivity against electric shock. Next figure shows the body resistance between two hands ($R_{\text{bb}}$). [4]
Fig. 2.2 Touch voltage vs body impedance (hand-to-hand)

Now, the body current would be very critical on the person who has the lowest body resistance. How lower the body resistance, how greater the body current will be. It leads that the curve of 5% of fig. 2.2 must be used.

Fig. 2.3 Body current vs maximal time duration

The body resistance between hand and two feet is generally obtained from this relation: \( R_{jf} = 0.75 \times R_{6f} \). The most simply way to find the touch voltage between hand-to-foot-foot is from voltage divider on fault place to ideal conducting Earth where the contact with the
person to floor is represented by $R_b$ and from floor to infinity point by $R_{trans}$. This relation is written as: $U_a = \frac{R_b}{R_b + R_{trans}} U_f$; for $R_{trans} \to \infty$ $U_a$ tends to zero; at the other extreme $R_{trans} \to 0$ which often represents the practical situation, $U_a \to U_f$, i.e. in this case the touch voltage corresponds approximately to the fault voltage (in worst case likes direct contact this can be a full supply voltage to earth). The touch voltage between hand-to-hand can be determined by the $U_a = I_f \times Z_x$; where $I_f$ is the fault current and $Z_x$ is the impedance between the fault place to main equipotential bonding, where the exposed conductive parts and extraneous conductive parts are connected.

2.4 NORMS AND STANDARDS

The safety norms in regard with the previous figures are shown in the next figure. Depend on kind of the case of body part concerned, that might be hand-to-hand, hand-to-foot-foot with or without footwear, the norms are presented. [4] and [5] (Appendix 1)

![Maximal allowed time vs voltage](image)

**Fig. 2.4 IEC and NEN norms**

where:

- hff: hand-to-foot-foot
- hh: hand-to-hand
- hffR: hand-to-foot-foot plus footwear resistance (1KΩ)

The NEN 1010 norm is very strictly prohibited for high touch voltage while the IEC 479 on lower voltage.
Chapter 2 Protection methods in low voltage networks

2.5 EARTHING OF EQUIPMENT

The most two significant system in electrical installation is discussed below. They have difference point where the one provides neutral potential of the star point transformer (called TN system) to the customer and the other one (TT system) not. According to [1], in a TN system, one point (neutral of the transformer) is directly earthed, called functional earth. The exposed conductive parts of the electrical installation must be connected to this point through the PE or PEN conductor. To limit the voltage to earth of the PE or PEN conductor, and of the exposed conductive parts connected to it, the resultant impedance of all the functional earth electrode should not exceed $2\Omega$. In TT system, one point is also directly earthed; the exposed conductive parts of the electrical installation must be connected to earth electrodes which are separate from the functional earth electrode. Exposed conductive parts which can be touched simultaneously must be connected to the same earth electrode.

Mostly, TN system that often encountered in the practice has a separate conductor which provides an earth potential, so called TN-S. Other TN system has only one conductor which provides simultaneously a neutral and an earth potential, called TN-C, and some other is combination between those two, so called TN-CS.

The basic schematic diagram for a TN-S system comprising an electrical installation fed from an external source of energy, the neutral point of that source being solidly earthed as shown on fig. 2.5. $L, N$ and $E$ are the installation phase (line), neutral and main earthing terminals, respectively. All extraneous conductive parts are connected to the installation main earthing terminal (E), this connection is called the main equipotential bonding and the conductors used are known as main equipotential bonding conductor. The exposed conductive parts of all electrical equipment are similarly connected to that same main earthing terminal by means of the protective conductors of the circuits feeding the various items of equipment. The installation's main earthing terminal is connected via the earthing conductor to the source earth by means of the metallic sheath and/or armouring of the supply cable.

![Fig. 2.5 Basic schematic diagram for a TN-S](image)

For the present let it be assumed that there is no supplementary equipotential bonding. An earth fault has occurred in the current-using equipment 1, the live conductor of the circuit
coming into contact with the metallic enclosure. The fault itself is assumed, has zero impedance and that has been accompanied by an open-circuit in the equipment so that no part of its internal impedance is in the earth fault current path. During the time that the earth fault is allowed to persist, the earth fault current $I_f$ flows and the voltage $U_a$ exists between the exposed conductive parts of the faulty equipment and the extraneous conductive parts. The voltage $U_a$ also exists between the exposed conductive parts of the faulty equipment and those of the 'healthy' equipment 2 fed from another circuit. It should be noted that the latter is not necessary switched on. There is, however, no voltage created between the exposed conductive parts of equipment 2 and the extraneous conductive parts. Thus, $U_a$ is the voltage to which a person at risk would be subjected if that person simultaneously came into contact with the exposed conductive parts of the faulty equipment and similar parts of other equipment or extraneous conductive parts.

Consider fig. 2.5, An earth fault has occurred at the terminal in the current-using equipment and it is assumed that the earth fault has no impedance. The earth fault current ($I_f$) is given by:

$$I_f = \frac{E_a}{Z + Z_{cl} + Z_{c3} + Z_{c4}}$$

where $E_a$ is the induced electromotor force of the source; $Z$ is the internal impedance of the source; $Z_{cl}$ is the impedance of the phase conductor of the supply cable; $Z_{c3}$ is the impedance of the phase conductor of the circuit concerned; $Z_{c3}$ is the impedance of the protective conductors of the circuit concerned; $Z_{c4}$ is the impedance of the supply cable protective conductor.

Because of the nature and covering of the floor on which the person at risk is standing and the probability that he or she would be wearing some type of footwear, it can be assumed that the person is well insulated from the general mass of earth. The shock current through the person would be almost wholly hand-to-hand and as the person's body resistance ($R_b$) is very high compared with the circuit impedances concerned, the shock current can be taken to be $U_a/R_b$. The severity of the electric shock experienced by the person depends on the magnitude and duration of the shock current.

The maximum touch voltage is between the exposed conductive parts of the faulty equipment and extraneous conductive parts bonded to the main earthing terminal. In the event of an extraneous conductive part being omitted from the main equipotential bonding a higher touch voltage could be present between it and the exposed conductive parts of the faulty equipment.

In the event of an earth fault in an installation, touch voltages can be created between the exposed conductive parts of items of healthy equipment and between such parts and extraneous conductive parts bonded to the main earthing terminal.

The basic schematic of a TT system is shown on fig. 2.6 where there are two conductors L(ine) and N(eutral) feed the utilization. The exposed conductive parts of the circuit and extraneous conductive parts entering the location served are required to be connected to the installation earth electrode via main earthing terminal of the installation.
An earth fault has occurred at the terminal in the current-using equipment and it is assumed that is accompanied by an open circuit in that equipment so that no part of its resistance is in the earth fault current path. Neither, it is assumed, has the fault itself any impedance. The earth fault current \( (I_f) \) is given by:

\[
I_f = \frac{E_o}{Z + Z_{cl} + Z_{c2} + Z_{c3} + R_A + R_B} = \frac{U_o}{R_A + R_B}
\]

where \( E_o \) is the induced electromotor force of the source; \( Z \) is the internal impedance of the source; \( Z_{cl} \) is the impedance of the phase conductor of the supply cable; \( Z_{c2} \) is the impedance of the phase conductor of the circuit concerned; \( Z_{c3} \) is the impedance of the protective conductors of the circuit concerned; \( R_A \) is the resistance of the installation earth electrode; \( R_B \) is the resistance of the source earth electrode and \( U_o \) is the nominal voltage to Earth of the supply. \( R_A \) and \( R_B \) may well be significantly larger than the other parameters.

The aim in earthing an item of equipment, i.e., in connecting its exposed conductive parts via the circuit protective conductor to the main earthing terminal of the installation and hence to the earthed point of the source, is to provide an earth fault current path of sufficiently low impedance so that the magnitude of the earth fault current will be sufficiently high to cause rapid disconnection of the device being used to provide protection against indirect contact. In most cases that device will be the fuse or circuit breaker providing overload and/or short circuit protection for the circuit concerned. Alternatively automatic disconnection may be provided by a residual current device. Protective earthing is necessary also to protect people from excessive touch voltage, for this purpose all metal parts of equipment and installations which do not belong to the operating circuit, but which may become connected to live parts under fault conditions, must be connected to an earthing electrode through an earthing conductor. [2]

It must be emphasized that while persons in installations which have been correctly designed using the touch voltage method may, in the event of an earth fault, receive an electric shock, if they are in contact with simultaneously accessible conductive parts, that shock should not be
fatal.

It has been argued that the probability of someone being in contact with an item of equipment which suddenly develops an earth fault, and at that instant also being in contact with an extraneous conductive part or another exposed conductive part, is so remote that it should not be used as the basis for the requirements for this protective measure. That argument has not received general support, the counter argument being that if all extraneous conductive parts are correctly bonded to the main earthing terminal of the installation and all exposed conductive parts also connected to that terminal, then in the event of an earth fault, the person at risk, assuming that he or she is well insulated from the general mass of Earth, is solely at risk from the hand-to-hand touch voltages already described (when considering indirect contact).

One sometimes reads statements to the effect that the main equipotential bonding creates an equipotential zone and it is a common misconception that, in the event of an earth fault, exposed and extraneous conductive parts attain the same potential with respect to earth so that a person within the equipotential zone would not be subjected to a shock risk. Such statement is completely untrue if only main equipotential is present. When, in addition to such bonding there is also supplementary equipotential bonding. It is true that in the event of an earth fault inside the location, the potential between exposed and extraneous conductive parts bonded together can be extremely low. In an installation which is part of a TN-S system, it is also true that even if there is only main equipotential bonding there is no potential between exposed and extraneous conductive parts in the installation if the earth fault occurs in the supply cable to that installation or in another installation being served by the same cable. When such an external earth fault occurs, the installation's main earthing terminal attains some potential above Earth but all the exposed and extraneous conductive parts in the installation will attain that potential. But, if for some reason, an extraneous conductive part "escapes" the main equipotential bonding then, in the event of an external earth fault, fault voltage (and hence touch voltage) will be created between that part and any conductive parts connected to the main earthing terminal, including other extraneous conductive parts which are main bonded to that terminal.

Considering some other condition which give rise to a shock risk. An item equipment is intended to be earthed but that has not been done or if the protective conductor of the circuit concerned becomes open-circuited. If in the equipment there is an insulation breakdown to its enclosure the overcurrent protective device intended to provide protection against indirect contact will not operate for the simple reason that there will be no earth fault current for the protective device to detect. However, the enclosure of the equipment will be at the full voltage($U_o$) to Earth and should the person at risk simultaneously be gripping or touching that enclosure and any other exposed conductive part or extraneous conductive part which is connected to the main earthing terminal, that person will be subjected to a shock current $U_o/R_b$ hand-to-hand. If $U_o=220V$ that magnitude of shock risk will be potentially fatal. The person will also be subjected to a shock current hand-to-feet of $U_o/(R_b+R_{tan})$. The only possible protection against a potentially fatal shock in such circumstances (and in the absence of supplementary equipotential bonding) would be provided by a residual current device with a suitably rated residual current and disconnection time and, of course, operating as intended. It has to be emphasized, that this does not mean that the use of a residual current device permits one to ignore the requirement for a circuit protective conductor.
CHAPTER 3

INDUCTANCE COMPUTATION BETWEEN TWO PARALLEL CYLINDRICAL CONDUCTORS

The phenomenon of inductance can be described through conductor systems which electric current \( i \) flows on it. The current generates a magnetic field around the conductor. When the current is time dependent, the magnetic field is also time dependent. This magnetic field is a quantity that can be used to compute the inductance within certain area occupied by that field.

The inductance between two conductors consists theoretically of two components: self inductance and mutual inductance. The self inductance of a cylindrical straight conductor can be computed when the return conductor lies at far distance, in such way that it practices no influence at this conductor, mathematically this distance is infinity, others it calls a mutual inductance between the two conductors, where one influences one another. In practice these two inductance can be observed (measured) as one total inductance, where the self and mutual inductances are already combined on the measurement. It means also that distinction for it is very difficult (in most case may be impossible to obtain them in certain conductor configuration).

According to [6] and [7], the following definition of self and mutual inductance is given.

**Self inductance**: the property of an electric circuit whereby an electromotive force is induced in that circuit by a change of current in the circuit. The coefficient of self inductance \( L \) of a winding is given by the following expression: \( L = \frac{\partial \Phi}{\partial i} \); where \( \Phi \) is the total flux linkage of the winding and \( i \) is the current in the winding. The term winding here is used to emphasize that the current goes away and back as a closed circuit. The voltage \( e \) induced in the winding is given by the following equation \( e = - \left( L \frac{\partial i}{\partial t} + i \frac{\partial L}{\partial t} \right) \); if \( L \) is constant \( e = -L \frac{\partial i}{\partial t} \). The min sign indicates that the induced voltage exercises against the flowing current. The definition of self inductance \( L \) is restricted to relatively slow changes in \( i \) that is to low frequency. The definitions of self inductances \( L \) is also restricted to case which the branches are small in physical size compared with a wavelength, whatever the frequency.

**Mutual inductance**: the common property of two electric circuits whereby an electromotive force is induced in one circuit by a change of current in the other circuit. The coefficient of mutual inductance \( M \) between two windings is given by the following equation \( M = \frac{\partial \Phi}{\partial i} \), where \( \Phi \) is the total flux linkage of one winding and the current \( i \) in the other winding. The voltage \( e \) induced in one winding by a current \( i \) in the other winding is given the following equation \( e = - \left( M \frac{\partial i}{\partial t} + i \frac{\partial M}{\partial t} \right) \), if \( M \) is constant \( e = -M \frac{\partial i}{\partial t} \). Further, the self and mutual inductance are assumed to be constant and independent of the time. The mutual inductance may also be considered as the number of flux linkages with one circuit.
due to unit current in other circuit. When the roles of the two circuits are interchanged, the change in one of these factors is exactly compensated by the change in the other, and the mutual inductance is the same.

In circuit free from iron, the magnetic induction at any point due to current in other circuit is directly proportional to the current $i$. The total flux linkage is capable of being expressed as a constant $M$ times the current.

To computed the total inductance of an infinitely long tube with circular section carrying current with assuming uniformly distributed, it is convenient to divide this problem into three parts, finding separately the partial self inductance due to the flux inside and outside the wires, adding them to determine self inductance and then find the mutual inductance between the wires. [8], [9] and [10]

The next assumptions are made to obtain both inductances:
- homogeneously current distribution in conductor
- especially for self inductance, the return conductor lies at "infinity" distance
- magnetic field is instantaneous at all time (in stationary state)
- frequency of the current is very low, (in this case 50Hz)

Although the self inductance and mutual inductance of circuit elements not associated with magnetic materials are independent of the value of the current and dependent only on the geometry of the system, it is only in the simplest cases that these constants can be calculated exactly. The most direct method for calculating inductances is based on the definition of flux linkages per ampere. To calculate the flux linkages, it is necessary to write expression for the magnetic field density $B$ at any point of the field and then to integrate this expression over the space occupied by the flux that is linked with the element in question.

3.1 Self inductance of a cylindrical straight conductor

To obtain self inductance, the problem is splitted into two parts. First, the determination of partial self inductance inside current leading conductor ($L_{int}$) and second one the determination of partial self inductance outside ($L_{ext}$). Because the magnetic energy is spread over all the space by current leading conductor, the total magnetic energy must be have a finite value. Mathematically, the magnetic field function must be integrateable and convergent all over the space. By addition of the two partials, the self inductance can be acquired. [11], [12] and [13]

The self inductance of a coil is a function of the geometry of the coil and of the permeability of the medium. It has a maximum value if the turns are closely wrapped so that all the flux links all the turns. The forces on all the current-length elements of the coil are so directed as to increase the self energy and the self inductance. Next fig. 3.1.1, (1 shows how to define flux linkage area of a coil with current $I_j$ (contour $C_1$), shaded area $A_j$. To compute magnetic intensity of an arbitrary coil current is mostly difficult, because firstly the flux density $B$ at every place in that linkage area must be known analytically. For a cylindrical straight wire, the linkage area is shaded in fig. 3.1.2, assuming the return conductor lies at infinity distance. With infinitely long wire the magnetic field becomes two dimensional, the end wire effect to be neglected. Inside the wire there is a magnetic field that is readily computed.
Consider an infinitely long straight tube of current of circular cross section of radius $R$, with carrying a current flowing parallel to the axis and uniformly distributed. It is assumed that the current is in steady state. There must, of course, be a return conductor, which is however assumed to be at a distance sufficiently large so that the magnetic its effect may be neglected to define the inductance due to the flux inside wire named as intern inductance. And for extern field (outside wire) it leads to extern inductance. With permissible assumption that the flux lines lie in concentric cylinders.

### 3.1.1 Internal inductance

The flux inside the wire is considered to be difficult to determine the number of flux linkages, so we resort to the energy definition $L_{\text{int}} = \frac{2W_i}{I^2}$, where $W$ equals the energy stored inside one wire, as shown with shaded rectangle OO'BA.

Define $I'$ as the current inside a certain cylinder with radius $r$:

$$I'(r) = \int \int J'dA = J \int \int dA$$

$$= I \left( \frac{r}{R} \right)^2$$

At any internal point P, the magnetic field intensity $H$ is determined only by tube of current that lies inside the radius $r$ of the point, by the current $I' = I \left( \frac{r}{R} \right)^2$. The magnetic field intensity $H$ at arbitrary point P within conductor can be defined and is thus seen to be linearly related to the radius. This is the magnetic field situation for the case of a steady current.

According to the Ampere law given by:
The magnetic field intensity inside the conductor is written as:
\[ H_{\text{int}}(r) = \frac{I}{2\pi} \frac{r}{R^2} \text{ [A/m]} \]  
(3.2)

this magnetic field intensity increases within a cylindrical conductor linear from middle to the outside with current on condition that the conductor assumes a homogeneous medium. A homogeneous medium means that the permeability of the medium is constant on all space, independent of field strength, place and time and this gives permissible assumptions that the stored magnetic energy is linear with magnitude of flowing current.

The stored energy inside the wire can be written as energy per volume unit \( w_i = \frac{1}{2} \int H \cdot dB \)
either the total energy inside is obtained by taking integration over the volume of wire becomes \( W_i = \frac{1}{2} \mu \int \int H_{\text{int}}^2 dV \)

Further, the magnetic energy inside a volume element cylinder is
\( dV = r dr d\theta dz = 2\pi r l dr \)
and fills up gives:
\[ W_i = \frac{1}{2} \mu \int \int \int H_{\text{int}}^2 dV = \frac{1}{2} \int_0^R \left( \frac{I}{2\pi} \right)^2 \left( \frac{r}{R^2} \right)^2 2\pi r l dr \]
\( = \frac{\mu I^2 l_0}{16\pi} \)
\( \text{(3.3)} \)
this follows the partial internal inductance \( L_{\text{int}} \) of self inductance:
\[ L_{\text{int}} = \frac{2W_i}{I^2} = \frac{\mu l_0}{8\pi} \]  
(3.4)

3.1.2 External inductance

The next figure 3.2 describes the Biot-Savart law. The magnetic field intensity \( H \) according to the Biot-Savart law for an arbitrary thinny wire(filaments) in a linear material is given by:
\[ dH = \frac{I}{4\pi} \frac{dl \times \vec{r}}{r^3} \]  
(3.5)
for a straight line, it becomes (using relation \( |dl \times \vec{r}| = dl r \sin \alpha \)):
\[ H = \frac{I}{4\pi} \int \frac{dl \sin \alpha}{r^2} \]  
(3.6)
where filament length \( dl \) has the same direction as current along the z-axis, now \( dl=dz \) en \( H \) has a direction perpendicular through inside this pagina. The angle \( \alpha \) has a reference taking from z-axis
Fig. 3.2 Magnetic intensity of a straight wire

From this figure, the geometrical relations are acquired in the next formulas:

\[ b = r \sin(\pi - \alpha) = r \sin \alpha \]

\[ z = \frac{b}{\tan(\pi - \alpha)} = b \cot(\pi - \alpha) = -b \cot \alpha \]

\[ \Rightarrow dz = b \left(1 + \cot^2 \alpha\right) d\alpha = \frac{b}{\sin^2 \alpha} d\alpha \]

filled up this at formula (3.6) gives next equation:

\[ H = \frac{I}{4 \pi b} \int_{\pi - \alpha_2}^{\alpha_3} \sin \alpha \, d\alpha \]

\[ = \frac{I}{4 \pi b} \left[ \cos \alpha_3 + \cos \alpha_2 \right] \]  \hspace{1cm} (3.7)

It is shown \( \cos \alpha_2 = \frac{a}{\sqrt{a^2 + b^2}} \); \( \cos \alpha_3 = \frac{l-a}{\sqrt{(l-a)^2 + b^2}} \)

The magnetic field intensity outside the wire can be written further as:
This formula indicates that point P may not be too near to the wire, because it makes an infinity magnetic field intensity value that physically never occurred. Besides, practically the thinny wire does not exist (never used). When \( l \gg a \gg b \) the magnetic field intensity can be simplified as \( H = \frac{I}{2 \pi b} \), this gives a known relation that valid for a very long straight wire.

Applied to a practical thick wire and observing the points around the thick wire, the found analytically solution will be checked up of validity of the formula from Biot-Savart law can be still useable. Assuming that the current distributes homogeneously over cross sectional area of the wire. This is permitted because applied current frequency is low, and conductor length is taken very long, so that the skin - and end contact effects can be minimized (neglected).

If the point P is moved to the wire than the next cross-section fig. 3.3 will be seen:

Fig. 3.3 Cross sectional of the conductor

Given an element area \( dA \) in view of point P forms a relation \( dA = r \, dr \, d\theta \). The homogeneously current conductor \( I \) is divided up to current filaments \( (i_f) \), the magnitude of the current in the filament depends on the accompanying area. The relation \( \frac{I}{i_f} = \frac{A}{dA} = \frac{\pi R^2}{r \, dr \, d\theta} \) is determined. Using this current filament permits applying formula found from Biot-Savart law. The magnetic field intensity by current filament \( i_f \) in point P is written as (instead of \( I \) using \( j \) and \( b \) substituted by \( r \)):

\[
H = \frac{i_f}{4 \pi r} \left[ \cos \alpha_2 + \cos \alpha_3 \right]
\]
Chapter 3 Inductance computation of two parallel cylindrical conductors

The magnetic field intensity by all current filaments in point P by introduction of $i_r$ becomes

$$dH_p = \frac{I}{4 \pi^2 R^2} \left[ \cos \alpha_3 + \cos \alpha_2 \right] dr d\theta,$$

this field has direction perpendicular on line PS.

(fig. 3.3.2).

The total magnetic field intensity at P can be determined by taking integration on cross sectional of the conductor where current filaments exist,

$$H_p = \frac{I}{4 \pi^2 R^2} \left[ \cos \alpha_3 + \cos \alpha_2 \right] \int \int dr d\theta$$

because of symmetrical structure between half upside and downside the integration can be simplified. The resultant of two side $H_p$ leads a composition $H_p$ that is perpendicular on line OP and the magnitude becomes with factor $2 \cos \theta$ larger. (see fig. 3.3.3). The integration interval is determined by using next figure 3.4.

![Fig. 3.4 Interval of integration](image)

Integrating of variable $\theta$ is taken from $\theta=\theta_{\min}$ to $\theta=\theta_{\max}$ which this is the maximal angle of the drawn line between point P en R when points R and Q combine, the line PR gets the outside of the conductor. It is shown that $\sin \theta_{\max} = \frac{R}{c} \Rightarrow \theta_{\max} = \arcsin \left( \frac{R}{c} \right)$ (see fig. 3.4.1). The integration of variable $r$ is taken on line QR (PQ to PR) inside the conductor (see fig. 3.4.2).

$$H_p = \frac{I}{2 \pi^2 R^2} \left[ \cos \alpha_3 + \cos \alpha_2 \right] \int_{0}^{\theta_{\max}} \int_{r_{PQ}}^{r_{PR}} \cos \theta dr d\theta$$

The integration to variable $r$ is carried out gives

$$\int_{r_{PQ}}^{r_{PR}} dr = PR - PQ = RQ$$
using relation \( OT = c \sin \theta \rightarrow RQ = 2 \sqrt{R^2 - c^2 \sin^2 \theta} \), fill this up and make use of the standard integral formula\(^1\) [14], gives

\[
H_p = \frac{I}{\pi^2 R^2} \left[ \cos \alpha_3 + \cos \alpha_2 \right] \left[ \frac{\sin \theta}{2} \sqrt{1 - \left( \frac{c}{R} \right)^2 \sin^2 \theta} + \frac{1}{2} \left( \frac{c}{R} \right) \arcsin \left( \frac{c \sin \theta}{R} \right) \right]_{\theta = 0}^{\theta = \arcsin \left( \frac{R}{c} \right)}
\]

The final result is

\[
H_p = \frac{I}{4 \pi c} \left[ \cos \alpha_3 + \cos \alpha_2 \right]
\] (3.10)

It is shown that the result is the same as by using the thin wire (formula 3.7). The relation is still simple because the function depends on center distance \( c \) (the distance between the middle point of the conductor and the point outside compare with distance \( b \) of the current filament formula). Therefore using the Biot Savart formula is still permissible. The magnetic field intensity outside the conductor caused by thick straight cylindrical current is the same if the same current magnitude concentrates at the axis of the straight thin wire.

In a homogeneous material the magnetic field density \( B \) at point \( P \) by a homogeneous current in a round conductor is given by:

\[
B_p = \mu H_p
\]

\[
B_p = \frac{\mu_o I}{4 \pi b} \left[ \frac{a}{\sqrt{a^2 + b^2}} + \frac{l - a}{\sqrt{(l-a)^2 + b^2}} \right]
\] (3.11)

Let us apply the found relation of \( B \) to the problem of external flux. The elemental flux in the shaded area in the fig. 3.1.2 is \( d\Phi_{ext} = B \cdot dA \). The total external flux in a length \( a \) due to the current is \( \Phi_{ext} = \int B \cdot dA \). The integration is taken over rectangle ABCD (outside the conductor), this area is extensive to infinity and it can be solved because all the external flux links all the current.

\[
\int \cos x \sqrt{1 - k^2 \sin^2 x} \, dx = \frac{\sin x}{2} \sqrt{1 - k^2 \sin^2 x} + \frac{1}{2k} \arcsin(k \sin x) + C
\]

\(^1\) Integrated by \( \frac{1}{2} \int \cos x \sqrt{1 - k^2 \sin^2 x} \, dx = \frac{1}{2} \left[ x - \frac{1}{2} \arcsin(k \sin x) + C \right] \)
Chapter 3 Inductance computation of two parallel cylindrical conductors

The magnetic flux is dispersed all over the space. The induced flux is not determined over the whole space but just on part where the flux lies between de lines BC and AD, in other words the area where the flux perpendicular on the main and return current path. This area is called as linkage area. Assuming that the return path lies at infinity far away from main conductor. The integration is taken over the area ABCD where the direction of all magnetic intensity - perpendicular on that area and this with using the standard integral formula \[ \int \] gives a total flux linkage \( \Phi_{\text{ext}} \)

\[
\Phi_{\text{ext}} = \Phi_{\text{ABCD}} = \int_{b=R}^{b=+\infty} \int_{a=0}^{a=+\infty} B_{p} \, db \, da
\]

\[
= \frac{\mu I}{4 \pi} \int_{b=R}^{b=+\infty} \int_{a=0}^{a=+\infty} \left( \frac{a}{\sqrt{a^2+b^2}} + \frac{l-a}{\sqrt{(l-a)^2+b^2}} \right) \, db \, da
\]

\[
= \frac{\mu I}{4 \pi} \left[ \frac{2}{b} \left( \sqrt{l^2+b^2} - b \right) \right] \bigg|_{b=R}^{b=+\infty} = \frac{\mu I}{2 \pi} \left( \sqrt{l^2+b^2} - b \right) - l \ln \left( \frac{l+\sqrt{l^2+b^2}}{b} \right)
\]

\[
\Phi_{\text{ext}} = \frac{\mu I}{2 \pi} \left[ l \ln \left( \frac{l+\sqrt{l^2+R^2}}{R} \right) - \sqrt{l^2+R^2} + R \right]
\]

\[2\]

\[
\int \frac{dx}{\sqrt{a+cx^2}} = \frac{1}{\sqrt{c}} \ln \left( x\sqrt{c} + \sqrt{a+cx^2} \right) + C \quad ; \, c>0
\]

\[
= \frac{1}{\sqrt{-c}} \arcsin \left( x\sqrt{\frac{-c}{a}} \right) + C \quad ; \, c<0, \, a>0
\]

\[
\int \frac{\sqrt{a+cx^2}}{x} \, dx = \frac{\sqrt{a}}{2} \ln \left( \frac{\sqrt{a+cx^2} - \sqrt{a}}{\sqrt{a+cx^2} + \sqrt{a}} \right) + \sqrt{a+cx^2} + C \quad ; \, a>0, \, c>0
\]

\[
= \frac{\sqrt{a}}{2} \ln \left( \frac{\sqrt{a} - \sqrt{a+cx^2}}{\sqrt{a} + \sqrt{a+cx^2}} \right) + \sqrt{a+cx^2} + C \quad ; \, a>0, \, c<0
\]

\[
= \frac{a}{\sqrt{-a}} \arccos \left( \frac{1}{x} \sqrt{\frac{-a}{c}} \right) + \sqrt{a+cx^2} + C \quad ; \, a<0, \, c>0
\]
Now, the external inductance can be obtained by using a linear relation between flux and current $L_{\text{ext}} = \frac{\Phi_{\text{ext}}}{I}$ in respect with linear and homogeneous material. The external inductance of self inductance is written as

$$L_{\text{ext}} = \frac{\mu}{2 \pi} \left[ I \ln \left( \frac{l + \sqrt{l^2 + R^2}}{R} \right) - \sqrt{l^2 + R^2} + R \right]$$ \hspace{1cm}(3.12)

Finally, the total self inductance of a cylindrical straight conductor can be written by:

$$L = L_{\text{int}} + L_{\text{ext}}$$

$$L = \frac{\mu l}{8 \pi} + \frac{\mu}{2 \pi} \left[ I \ln \left( \frac{l + \sqrt{l^2 + R^2}}{R} \right) - \sqrt{l^2 + R^2} + R \right]$$ \hspace{1cm}(3.13)

This function is constant and depends only on the geometrical size of the conductor and the magnetic characteristic of the medium.
3.2 Mutual inductance of two parallel cylindrical conductors

The mutual inductance $M$ of two coils is defined as the incremental flux linkages in one coil per incremental change of current in the other. $M_{12} = \frac{d\Phi_{12}}{dI_2}$ and $M_{21} = \frac{d\Phi_{21}}{dI_1}$. The term coil here emphasizes that there are two different current paths and each current path flows on each coil respectively, which interchanged between them $(I_1$ and $I_2)$ leads to mutual inductance, see fig. 3.5. For equal incremental changes $dI_1$ or $dI_2$ and for constant permeability it leads to $M = M_{12} = M_{21}$. Mutual inductance exists only when there are two coils so placed in space that if a current flows in one of them, some of the flux lines of the resulting magnetic field link the other coil. If coil 1 carries current $I_1$, it sets up a flux $\Phi_1$ of which a part $\Phi_{21}$ links the turns of coil 2, yielding flux linkages through coil 2.

The mutual inductance is a function of the geometry of the two circuits and of the permeability of the medium and has a maximum value if the coils are "closely wrapped" so that the turns are linked by all the flux possible. If the permeability depends on fields strength, so also does the inductance. The inductance is a magnetic quantity comparable with the capacitance in the electric field.

The term turns or coils, in this case of parallel conductor can be assumed as a current path which has return path at infinity distance. It means also that each parallel conductors can be observed as turns/coils.

To compute the mutual inductance of two parallel cylindrical conductors, the previous Biot-Savart relation is still used.

![Fig. 3.5 Linkage flux definition](image)

In fig. 3.5.1 it is shown 2 coils where they are mutually coupled. The current $I_1$ has own current path (current contour C1) and the current $I_2 C2$, respectively. The plane $A_1$ and plane $A_2$ are not necessary parallel. The flux linkage $\Phi_{21}$ is defined as the flux that goes through over the plane $A_2$ with edge boundary by the current contour $C_2$ of $I_2$ generated by current $I_1$. 
with contour C1. Generally, it is not easy to determine this flux linkage \( \Phi_{21} \) because the exactly magnetic field density \( B \) is unknown at all space. For the case of two infinitely long straight tubes of current of circular cross sections, fig. 3.5.2. The next assumptions is made that the each current rise from and to infinity far away, each current has own contour return path at infinity distance, C1 and C2 have a (large) long path. Now, it is possible to compute the flux linkage within linkage area \( A_2 \). Here, \( A_2 \) is shown with the shaded area (ABCD, point C and D at infinity).

The flux linkage \( \Phi_{21} \) due the current \( I_1 \) of contour C2 is defined taking integration of magnetic field density over that area and gives the same time the relation of mutual inductance, which is

\[
\Phi_{21} = \int_{A_2} B \cdot dA_2 = M_{21} I_1
\]

(3.14)

As has been proved before, the current can be assumed lies on the center of the wire, thus, the integration interval of this area \( A_2 \) is taken from the center of wire 2 to infinity and from length of the wire (area ABCD). Using standard integration formula [14] gives:

\[
\Phi_{21} = \Phi_{ABCD} = \int_{b=0}^{b=\infty} \int_{a=0}^{a=l} B_{p} \ dB \ da
\]

\[
= \frac{\mu}{4 \pi} \int_{b=0}^{b=\infty} \int_{a=0}^{a=l} \frac{1}{b} \left( \frac{a}{\sqrt{a^2 + b^2}} + \frac{l-a}{\sqrt{(l-a)^2 + b^2}} \right) \ dB \ da
\]

\[
= \frac{\mu I_1}{2 \pi} \left[ l \ln \left( \frac{l + \sqrt{l^2 + D^2}}{D} \right) - \sqrt{l^2 + D^2} \right]
\]

The mutual inductance \( M_{21} \) between two parallel cylindrical conductors is then defined as follows:

\[
M_{21} = \frac{\Phi_{21}}{I_1} = \frac{\mu}{2 \pi} \left[ l \ln \left( \frac{l + \sqrt{l^2 + D^2}}{D} \right) - \sqrt{l^2 + D^2} \right]
\]

(3.15)

The same way to find \( M_{12} \) can be used and gives the same result \( M_{12} = M_{21} \).

This function can be viewed as shown below:
It is shown that the influence from another conductor with current is the biggest when both conductors lie near each other. How longer the conductors, how bigger they influence to each other. It is also shown that the mutual inductance can be seen almost as linear relation from length parameter and as asymptotically decreasing in relation with distance parameter.

According to [8], the mutual inductance between two circuit can be also computed by using Neumann formula and given by \( M = \int \int \frac{\cos \theta}{r} ds \, ds' \) in which \( \theta \) is the angle of inclination between the two circuit line elements \( ds \, ds' \), \( r \) is the radius vector between them, and the integration is to be taken over the contours of the two circuit. This Neumann formula is the most general expression for finding the mutual inductance. It leads quite simply to a formal expression for the mutual inductance, even though for most cases it is not possible to perform the integrations. However, in such cases also it is possible to obtain a numerical value for a specific case by numerical integration, although this calculation may be tedious.
3.3 INDUCTANCE OF TWO CONDUCTORS (1 PHASE AND 1 RETURN CONDUCTOR)

Consider a linear system with 3 conductors each carrying current, fig. 3.7.1, where the third conductor can be assumed as virtual (help) conductor. It means that both current from the first and second conductors return back through this help conductor. Physically this help conductor does not exist when the first conductor has the same flowing current as in the second conductor but in opposite direction.

The magnetic flux due to each conductor comes from its own current and other currents. The magnetic flux of each conductor can be set up and forms next matrix considering with a linear medium:

\[
\begin{bmatrix}
\Phi_1 \\
\Phi_2 \\
\Phi_R
\end{bmatrix} = \begin{bmatrix}
L_{11} & L_{12} & L_{1R} \\
L_{12} & L_{22} & L_{2R} \\
L_{R1} & L_{R2} & L_{RR}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2 \\
I_R
\end{bmatrix}
\]

Applying Kirchhoff current law from fig. 3.7.2 gives:

\[
E = I_1 Z_{Z1} + V_1 - V_2
\]

The current equation on this network is \( I_1 + I_2 = I_R = 0 \rightarrow I_2 = -I_1 \). This simplifies the matrix and becomes:

\[
\begin{bmatrix}
\Phi_1 \\
\Phi_2
\end{bmatrix} = \begin{bmatrix}
L_{11} & L_{12} \\
L_{12} & L_{22}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

Further, the fall voltages are written in matrix form as:
Chapter 3 Inductance computation of two parallel cylindrical conductors

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_{11} & j\omega L_{12} \\ j\omega L_{12} & R_2 + j\omega L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_1 \end{bmatrix} \]

(3.19)

The total fall voltage over the conductor becomes (vectorially adding)

\[ V_1 - V_2 = \left( R_{tot} + j\omega L_{tot} \right) I_1 \]

(3.20)

The Kirchhoff current law is now simplified as (see fig. 3.7.3)

\[ E = I_1 \left( Z_1 + Z_{kabel} \right) \]

(3.21)

When more than two conductors are used, the simply equation as the last shown is more difficult or can not be set up. The matrix form of the system should be written as his original completely form.

Now, the found exact formula for inductance will be compared with the approximate formula that mostly used in high voltage transmission technique. First, the most general used formula will be derived and the totally inductance will be computed and then examined the difference between them.

According to [15] and [16] where the approximately derivation is built. The totally flux linkage of the first conductor is a scalar addition of the flux from his own current and the flux from the other one.

\[ \Phi_1 = \Phi_{11} + \Phi_{12} \]

(3.22)

The next general formulas for self and mutual inductance are

\[ L_{ii} = \frac{\mu l}{2\pi} \ln \left( \frac{D_{1R}}{r_m} \right) + \ln \left( \frac{D_{12}}{D_{12}} \right) \]

(3.23)

\[ L_{ij} = \frac{\mu l}{2\pi} \ln \left( \frac{D_{ij}}{D_{ij}} \right) \]

applied these for circuit with a main and return conductor (formula 3.20) gives

\[ L_{11} - L_{12} = \frac{\mu l}{2\pi} \ln \left( \frac{D_{12}}{r_m} \frac{D_{1R}}{D_{12}} \right) \]

(3.24)

\[ L_{22} - L_{21} = \frac{\mu l}{2\pi} \ln \left( \frac{D_{21}}{r_m} \frac{D_{1R}}{D_{22}} \right) \]

using the relation \( \lim_{D_{1R} \to D_{2R}} \frac{D_{1R}}{D_{2R}} = 1 \) (the help return conductor lies at far distance), the inductance
The total (approximate) inductance of these two conductors is
\[ L_{\text{tot}} = L_{11} - L_{12} + L_{22} - L_{21} \]
\[ = \frac{\mu l}{2 \pi} \left[ \frac{1}{2} + \ln \left( \frac{D_{12} D_{21}}{r^2} \right) \right] \]  
\[ L_{\text{tot}} = \frac{\mu l}{\pi} \left[ \frac{1}{4} + \ln \left( \frac{D}{r} \right) \right] \] 

This is very known formula for inductance of two conductor system.

When the same way is used to derive the exact self and mutual inductance function, the next equations as has been derived in the chapter 3 can be used
\[ L_{ii} = \frac{\mu l}{8 \pi} + \frac{\mu l}{2 \pi} \left[ l \ln \left( \frac{l + \sqrt{l^2 + R^2}}{R} \right) - \sqrt{l^2 + R^2} + R \right] \] 
\[ L_{ij} = \frac{\mu l}{2 \pi} \left[ l \ln \left( \frac{l + \sqrt{D_{ij}^2 + l^2}}{D_{ij}} \right) - \sqrt{D_{ij}^2 + l^2} + D_{ij} \right] \] 

The total (exactly) inductance of a circuit with a main and a return conductor can be written as
\[ L_{\text{tot}} = \frac{\mu l}{4 \pi} + \frac{\mu l}{\pi} \left[ \ln \left( \frac{D}{R} \left( 1 + \sqrt{1 + \frac{R}{l}} \right) \right) - \sqrt{1 + \left( \frac{R}{l} \right)^2} + \left( 1 + \frac{1}{l} \right)^2 + \frac{R - D}{l} \right] \] 

This function is simplified with first taking \( L_{\text{tot}} \) by length unity and then taking the limit of variable \( l \) to infinity. It leads the same final result as in formula 3.26.

\[ \frac{L_{\text{tot}}}{l} = \lim_{l \to \infty} \left[ \frac{\mu}{4 \pi} + \frac{\mu}{\pi} \left[ \ln \left( \frac{D}{R} \left( 1 + \sqrt{1 + \frac{R}{l}} \right) \right) - \sqrt{1 + \left( \frac{R}{l} \right)^2} + \left( 1 + \frac{1}{l} \right)^2 + \frac{R - D}{l} \right] \right] \]
\[ = \frac{\mu}{\pi} \left[ \frac{1}{4} + \ln \left( \frac{D}{R} \right) \right] \]
In 3-dimensional figure each inductance function (approximate and exact) is represented by length and distance as variables on XY-axis and on Z-axis is the value of the inductance. It can be shown that the difference of view between them can not be noticed. These figures are computed on conductor ($\mu_r=1$) with radius of 5cm.

Fig. 3.8 Approximately inductance function.
It is shown that the approximately function that generally used is not much difference compares with the exactly formula.

To observe the difference between the approximate and the exact inductance function the relative error between them is represented in the next figure using the relative error relation

$$\varepsilon = \left| \frac{L_1 - L_0}{L_1} \right| \times 100\%$$  \hspace{1cm} (3.30)
It is shown that the difference between the two formulas is more significant when the conductors lie far away between them and by small length.
Because of the complexity on the low voltage network system that each return conductors in most of the place may be connected to other return conductor (so much as possible) to maintain the safely network, this means that even in normal situations the current phase not always goes back through the return conductor on the same cable, analyzing simple circuit is carried out to understand occurred fault voltage and fault current using found inductance of the circuit.

A simple circuit is used to examine the found inductance formulas, and then would be implied to a more complex low voltage network system with a number of parallel return conductors.

4.1 EXPERIMENTAL CIRCUIT WITH REDUCED SOURCE VOLTAGE (1F2N)

The circuit is shown in fig. 4.1. The first return conductor is on the same phase conductor in a cable, and the second return conductor lies on the other cable, away from phase conductor and placed parallel to each other. To know what the influence of the second return conductor to the current distribution overall and the fault voltage at the fault place, the computing by different place of the second return conductor is conducted. The measurement is done with considering that the distance of the second return conductor can be varied. The used voltage source is chosen very low to overcome overheating of the tested cable.

\[ U_0 = I_1(Z_t + Z_q + Z_l) + V_1 - V_2 \]

\[ 0 = V_2 - V_3 \]

The voltage drop on each line conductor is defined in matrix form as

\[ u_1, + \]
\[ v_1, + \]
\[ v_2, + \]
\[ + \]
\[ v_3, + \]
\[ + \]
\[ + \]
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Chapter 4 Fault voltage and current for basic circuits

\[
\begin{align*}
V_1 &= \begin{bmatrix} R_1 + j\omega L_{11} & j\omega L_{12} & j\omega L_{13} \end{bmatrix} \begin{bmatrix} I_1 \end{bmatrix} \\
V_2 &= \begin{bmatrix} j\omega L_{21} & R_2 + j\omega L_{22} & j\omega L_{23} \end{bmatrix} \begin{bmatrix} I_2 \end{bmatrix} \\
V_3 &= \begin{bmatrix} j\omega L_{31} & j\omega L_{32} & R_3 + j\omega L_{33} \end{bmatrix} \begin{bmatrix} I_3 \end{bmatrix} \\
\end{align*}
\]

(4.2)

\[
\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33} \end{bmatrix} = \begin{bmatrix} I_1 \\
I_2 \\
I_3 \end{bmatrix}
\]

(4.3)

With using \( I_3 = -(I_1 + I_2) \) the simply new equation becomes

\[
\begin{align*}
V_1 &= \begin{bmatrix} Z_{11} - Z_{13} & Z_{12} - Z_{13} \end{bmatrix} \begin{bmatrix} I_1 \end{bmatrix} \\
V_2 &= \begin{bmatrix} Z_{21} - Z_{23} & Z_{22} - Z_{23} \end{bmatrix} \begin{bmatrix} I_2 \end{bmatrix} \\
V_3 &= \begin{bmatrix} Z_{31} - Z_{33} & Z_{32} - Z_{33} \end{bmatrix} \begin{bmatrix} I_3 \end{bmatrix} \\
\end{align*}
\]

(4.4)

\[
\begin{align*}
V_1 - V_2 &= \begin{bmatrix} Z_{11} - Z_{13} & Z_{21} - Z_{23} & Z_{22} - Z_{23} + Z_{31} - Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \end{bmatrix} \\
V_2 - V_3 &= \begin{bmatrix} Z_{21} - Z_{23} & Z_{22} - Z_{23} + Z_{31} - Z_{33} \end{bmatrix} \begin{bmatrix} I_2 \end{bmatrix} \\
\end{align*}
\]

(4.5)

At Kirchhoff-equation we need the difference from this voltage drop along the phase and return conductor,

\[
\begin{align*}
U_0 &= \begin{bmatrix} Z_{11} + Z_{13} + Z_{21} - Z_{23} + Z_{22} + Z_{23} \\
2Z_{21} - Z_{23} - Z_{31} + Z_{33} \\
Z_{22} - Z_{23} - Z_{32} + Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\
I_2 \\
I_3 \end{bmatrix}
\end{align*}
\]

The currents distribution now can be calculated with taking the inverse of the system matrix. And then we use these currents to compute the voltage drop and voltage drop difference.

To proof that theory, the next experiment is done, with using two cables each 4X95mm². The phase and first return conductors lay on a cable and the second return conductor lies on the other cable. During this experiment, the cables may not be overloaded so that unnecessary overheating is not happened. The cable ampacity is about 200A and to get this current magnitude at secundary side of the transformator (phase cable), the variac is introduced and the wheel is rotated until this expected current achieved with a given load (in this case the cable self as load). It leads that used source voltage is very low compares with the rated voltage of the cable which the cable is made for. When this current is established there is no more changes done at the primary side of the transformer or at the variac. This way, the steady voltage source is realized. This is done with considering that the cables are lying parallel side by side. But firstly, the inner impedance of the transformator is measured.

The fault voltage is define actually always with regard to a far distance (mass earth), theoretically this is at infinity. But in this case there is no current flows to earth thus the fault voltage here can not be define. The voltage drops along the cables \( V_1, V_2, \) and \( V \) are unmeasurable, because it depends on the path of the measuring (coaxial) wire. The only one
voltage that can be measured and should be verified is the totally voltage drop \((V_1-V_2)\) at phase and return conductor, of course \(V_2-V_1\) will be 0.

4.1.1 EXPERIMENTAL DETERMINATION OF THE TRANSFORMATOR IMPEDANCE

Consider the inner impedance of the transformator for the measurement on cable with low impedance, this measurement is established. The inner impedance of the transformator is measured under symmetrical case with next figure 4.2.

Fig. 4.2 Measurement set up.

The wattmeters, amperemeters and voltmeters are connected on the primary side and just amperemeters only on the secondary side that in 3 phase short circuit situation. To find the inner impedance of the transformator, the variac is slowly rotated until the current on the secondary side the same magnitude has, as rated current given by the fabricant \((I_n=800\text{A})\).

The measurement result is registered on the next table.

Table 1 Current and voltage measuring

<table>
<thead>
<tr>
<th></th>
<th>phase (U)</th>
<th>phase (V)</th>
<th>phase (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Secundary side</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line current ([A])</td>
<td>750</td>
<td>830</td>
<td>830</td>
</tr>
<tr>
<td><strong>Primary side</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line current ([A])</td>
<td>45.2</td>
<td>46</td>
<td>51.2</td>
</tr>
<tr>
<td>Phase voltage ([V])</td>
<td>12.2</td>
<td>10.6</td>
<td>11.0</td>
</tr>
<tr>
<td>Power ([W])</td>
<td>364</td>
<td>382</td>
<td>400</td>
</tr>
</tbody>
</table>

With taking the mean value of the power, currents and voltages 

\[ P_k = 382 \text{ W} \]
\[ U_{f,p,k} = 11.2 \text{ V} \]
\[ I_{f,p,k} = 47.5 \text{ A} \]
\[ I_{f,s,k} = 803.3 \text{ A} \]

and the inner impedance can be calculated:

\[ R_{f,s,k} = \frac{P_k}{I_{f,s,k}^2} = \frac{382}{803.3} = 0.59 \text{ m}\Omega, \quad S_k = U_{f,p,k} \times I_{f,p,k} = 11.3 \times 47.5 = 536.75 \text{ VA} \]

\[ Z_{f,s,k} = \frac{S_k}{I_{f,s,k}^2} = \frac{536.75}{803.3} = 0.83 \text{ m}\Omega, \quad X_{f,s,k} = \sqrt{Z_{f,s,k}^2 - R_{f,s,k}^2} = \sqrt{0.83^2 - 0.59^2} = 0.58 \text{ m}\Omega \]

and found to be \( Z_k = (0.59 + 0.58i) \text{ m}\Omega \) each phase. The transformer is further considered as black box with Y-circuit. During the next test just two phase connection are used and the inner impedance will be 2 times bigger than \( Z_k \) and becomes \( Z_{in} = (1.18 + 1.16i) \text{ m}\Omega \).

The used measurement equipments are written on appendix 2.

4.1.2 EXPERIMENTAL AND SIMULATION RESULTS

Realization of this experiment is conducted with using two cable 4x95mm\(^2\). The phase and first return conductors lie on a cable and the second return conductor lies on the other cable. The measurement is done by each discrete distance of the second return conductor away from the phase conductor. Further, there is demanded that measure current would not be greater than maximal allowed current of the cable (about 200A). This magnitude current can only be realized when the source voltage is variable. Using variac and little transformer, the concerned current is succeeded.

The experiment is done with varying the distance of the second return conductor in relation to the phase conductor at a few discrete values. The simulation and the measurement results are shown in combined figure below. Further, the next computing parameters are used:

\[ Z_{in} = (1.18 + 1.16i) \text{ m}\Omega \]

cable length=50 m
cable type: Al 4x95mm\(^2\)(Alkudia)

The measuring and simulation results are shown in table 2.
Table 2 Measurement and simulation results

<table>
<thead>
<tr>
<th>Distance [cm]</th>
<th>If [A]</th>
<th>In1 [A]</th>
<th>In2 [A]</th>
<th>ΔU [V]</th>
<th>If [A]</th>
<th>In1 [A]</th>
<th>In2 [A]</th>
<th>ΔU [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>220</td>
<td>118</td>
<td>99</td>
<td>5.98</td>
<td>232</td>
<td>128</td>
<td>109</td>
<td>6.7</td>
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<tr>
<td>10</td>
<td>207</td>
<td>122</td>
<td>91</td>
<td>5.91</td>
<td>227</td>
<td>135</td>
<td>102</td>
<td>6.7</td>
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<td>15</td>
<td>203</td>
<td>126</td>
<td>84</td>
<td>5.87</td>
<td>225</td>
<td>139</td>
<td>98</td>
<td>6.7</td>
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<tr>
<td>25</td>
<td>205</td>
<td>131</td>
<td>78</td>
<td>5.87</td>
<td>221</td>
<td>143</td>
<td>92</td>
<td>6.7</td>
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<td>198</td>
<td>135</td>
<td>75</td>
<td>5.94</td>
<td>219</td>
<td>146</td>
<td>89</td>
<td>6.7</td>
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<tr>
<td>60</td>
<td>203</td>
<td>144</td>
<td>72</td>
<td>6.23</td>
<td>216</td>
<td>150</td>
<td>84</td>
<td>6.7</td>
</tr>
<tr>
<td>100</td>
<td>198</td>
<td>146</td>
<td>66</td>
<td>6.16</td>
<td>213</td>
<td>153</td>
<td>79</td>
<td>6.7</td>
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<tr>
<td>160</td>
<td>206</td>
<td>153</td>
<td>63</td>
<td>6.16</td>
<td>211</td>
<td>156</td>
<td>75</td>
<td>6.7</td>
</tr>
</tbody>
</table>

Note: distance is measured between phase and second neutral conductor.

If : the current on phase conductor
In1 : the current on first neutral conductor
In2 : the current on second neutral conductor

The calculation is done with taking by each discrete distance of the second return conductor away from the phase conductor. And composing with the measurement values results next figures 4.3 and 4.4.

![Current distribution as function of distance](image)

Fig. 4.3 Current distribution on each conductors.
4.1.3 DISCUSSION

During the experiment is perceived that the mains voltage varies a little bit. This brings about that the input voltage to the transformer varies also, and finally leads to alternate secondary voltage that we used as source voltage to the cable system. Another difficulty is to set up homogeneously distance along the cable. The cables could not be placed straightly side by side along them and made a little arc because of his length.

Fig. 4.4 Totally voltage drop.
Now with the same fig. 4.1, the practical voltage level is applied to predict (compute) the actual fault voltage and fault current that might be occurred. Firstly, the same circuit is investigated. And then secondly, situation with a little changing at the end of cable, when an earthing electrode is introduced. By this way, the influence of distance of return conductor to phase conductor and placing earthing electrode on fault voltage and fault current could be obtained.

The effect of using earthing electrode on the low voltage network is now analyzed. To proof that fault voltage could be decreased (but not always necessary) without any changing of the short circuit current is investigated with this model. It depends on of course the magnitude of the resistance of functional earth \((R_b)\) and the resistance of the earthing electrode \((R_f)\) at the customer simultaneously. Because this parameters determine the magnitude of the fault voltage at the place where the fault occurs. During this fault an human who (that) can be simulated with body resistance and footwear resistance are introduced as model and connected at the fault place. The internal resistance of human body is determined by IEC 479 that described the allowed maximal current body with regard to time. Here, the current flows to the human body can be calculated with variously values of earth contact resistance, and the safety of the low voltage network can be "maintained/tested."

An another way to figured the problem where the coupling is shown to be clear, the equally simply circuit without earthing electrode as in figure 4.5 is used.

![Figure 4.5](image)

**Fig. 4.5 1 phase with 2 return conductors without earthing electrode**

From this figure, the system equation of Kirchhoff current law can be determined as

\[-U_0 + I_1(Z_k + Z_{11}) + I_2Z_{12} + I_3Z_{13} - I_1Z_{31} - I_2Z_{32} - I_3Z_{33} = 0\]

\[I_2Z_{22} + I_1Z_{21} + I_3Z_{23} - I_1Z_{31} - I_2Z_{32} - I_3Z_{33} = 0\]

(4.6)

with using \(I_j = -(I_1 + I_2)\) and in matrix form written as:

\[
\begin{bmatrix}
U_0 \\
0
\end{bmatrix} =
\begin{bmatrix}
Z_k + Z_{11} - Z_{13} + Z_{33} & Z_{12} - Z_{13} - Z_{32} + Z_{33} & I_1 \\
Z_{21} - Z_{23} - Z_{31} + Z_{33} & Z_{22} - Z_{23} - Z_{32} + Z_{33} & I_2
\end{bmatrix}
\]

(4.7)

The fault voltage at fault place is computed with one of the next equation as respect to the
neutral point of the voltage source.

\[ U_f = U_0 - I_1(Z_{x1} + Z_{11}) - I_2Z_{12} - I_3Z_{13} \]

\[ = -1 * (I_2Z_{22} + I_1Z_{21} + I_3Z_{23}) \]

\[ = -1 * (I_3Z_{33} + I_2Z_{32} + I_1Z_{31}) \]

And fault current is equal as \( I_f \).

The following computing parameters are used:

- \( S \) (rated power) : 630 kVA
- \( U_n \) (rated voltage) : 230 V
- \( Z_x \) (s.c. impedance) : (2.62 + 9.82) mΩ
- \( Z_q \) (net impedance) : 0 (strong net)
- cable type : 95 mm² Al, \( R_{Al} = 365 \) mΩ/km
- cable length : 100 m
- distance between phase and 2nd neutral : varied to 10 m
- \( R_b \) (functional resistance) : 4.8 Ω

The distance of the second neutral conductor is varied and the fault current and fault voltage are presented in fig. 4.6 and 4.7.

![Fault current as function of distance](image)

**Fig. 4.6 Fault voltage without earthing electrode.**

with

- \( I_{f_R} \) : current on phase conductor with resistance model
- \( I_{n_R} \) : current on neutral conductor with resistance model
- \( I_f \) : current on phase conductor with coupling model
- \( I_{n1} \) : current on 1st neutral conductor with coupling model
- \( I_{n2} \) : current on 2nd neutral conductor with coupling model
with

\[ U_{f,R} \] : fault voltage with resistance model
\[ U_f \] : fault voltage with coupling model

It is shown that when the second return conductor lies near the phase conductor, the circuit behaves like resistance (ohmic), which the current distribution on the return conductor is the same and the fault voltage is about a third of the source voltage. A little voltage drop occurs within inner impedance of the source. At large distance the circuit behaves more like inductive, the current distribution on return conductor is not homogene anymore. The phase current becomes less. On the second return conductor flows less current compares with the first return. Further, it is shown also that the fault voltage becomes less.
Then now, the circuit with introducing earthing electrode $R_a$ is analyzed.

![Fig. 4.8 1 phase with 2 return conductors with earthing electrode](image)

The system equation of Kirchhoff current law is adapted and becomes

$$-U_o + I_1(Z_k + Z_{11}) + I_2Z_{12} + I_3Z_{13} - I_1Z_{31} - I_2Z_{32} - I_3Z_{33} = 0$$

$$I_2Z_{22} + I_1Z_{21} + I_3Z_{23} - I_1Z_{31} - I_2Z_{32} - I_3Z_{33} = 0$$

$$I_4(R_b + R_a) + I_3Z_{33} + I_2Z_{32} + I_1Z_{31} = 0$$

(4.9)

with using $I_d = I_1 + I_2 + I_3$ and in matrix form as:

$$\begin{bmatrix} U_o \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_k + Z_{11} - Z_{31} & Z_{12} - Z_{32} & Z_{13} - Z_{33} \\ Z_{21} - Z_{31} & Z_{22} - Z_{32} & Z_{23} - Z_{33} \\ R_b + R_a + Z_{31} & R_b + R_a + Z_{32} & R_b + R_a + Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

(4.10)

The fault voltage is computed with one of the next equation as respect to the voltage at the source (neutral of the transformator).

$$U_f = U_o - I_1(Z_k + Z_{11}) - I_2Z_{12} - I_3Z_{13} - I_4R_b$$

$$= -1*(I_2Z_{22} + I_1Z_{21} + I_3Z_{23} + I_4R_b)$$

$$= -1*(I_3Z_{33} + I_2Z_{32} + I_1Z_{31} + I_4R_b)$$

$$= I_4R_a$$

(4.11)

And fault current is equal as $I_f$.

The fault voltage here is computed with regard to infinity point.

The same parameters are used for computation with addition that $R_a$ (earthing resistance) = 5Ω.

The results are shown in fig. 4.9 and 4.10.
It is shown that with using earthing electrode the fault voltage can be reduced without minimal changing of fault current. It means that the protective equipments (fuses and current limiters) respond with this fault current equally as fast as without earthing electrode. By this way, the using of longer cable at the low voltage network can be achieved without any danger of fault voltage. The safety of a low voltage network can be realized with placing earthing
electrodes on each section box.

The effect of earthing electrode for reducing fault voltage can be explained with the definition of fault voltage self, whereby this voltage is defined in regard with infinity point. An accidental fault voltage will be divided on functional resistance $R_b$ on transformer and earthing electrode $R_a$. 
4.3 THE EFFECT OF MULTIPLE RETURN CONDUCTORS (1fkn)

To understand the effect of multiple return conductors on fault voltage and fault current, the following computation with 1, 2, 3, 4 return conductors is performed. With using the same circuit as in chapter 4.1 and a 230V rated voltage, the fault current (on the phase conductor) and the fault voltage (voltage between end of the cable and the neutral of the transformer are computed). The distance of each return conductor(s) to the phase conductor is fixed as follow:

1 return conductor dfn1 = 2 cm
2 return conductors dfn1 = 2 cm, dfn2 = 1 m
3 return conductors dfn1 = 2 cm, dfn2 = 1 m, dfn3 = 3 m
4 return conductors dfn1 = 2 cm, dfn2 = 1 m, dfn3 = 3 m, dfn4 = 8 m

The other important used computing parameters are:

- Rated power \( S \) : 630 kVA
- Voltage \( U_n \) : 230 V
- Transformer impedance \( Z_t \) : \((2.62 + 9.82)\) mΩ (impedance of the transformer)
- Infinity strong net \( Z_q \) : 0
- Cable length : 100 m
- Type cable : 95 mm², Alkudia \( (R_{Al}=365 \text{ mΩ/km}) \)

The following formula corresponds with the number of return conductors.

- 1 return conductor
  \[ U_0 = [Z_r + Z_q + Z_{11} - Z_{12} - Z_{12} + Z_{12}] I_1 \]  (4.12)

- 2 return conductors
  \[ U_0 = \begin{bmatrix} Z_r + Z_q + Z_{11} - Z_{12} - Z_{13} + Z_{13} \\ Z_{21} - Z_{23} - Z_{31} + Z_{33} \\ Z_{22} - Z_{23} - Z_{32} + Z_{33} \end{bmatrix} I_1 \]  (4.13)

- 3 return conductors
  \[ U_0 = \begin{bmatrix} Z_r + Z_q + Z_{11} - Z_{12} - Z_{13} - Z_{14} + Z_{14} \\ Z_{21} - Z_{24} - Z_{41} + Z_{44} \\ Z_{22} - Z_{24} - Z_{42} + Z_{44} \end{bmatrix} I_1 \]  (4.14)

- 4 return conductors
  \[ U_0 = \begin{bmatrix} Z_r + Z_q + Z_{11} - Z_{15} - Z_{15} + Z_{15} \\ Z_{21} - Z_{25} - Z_{51} + Z_{55} \\ Z_{31} - Z_{35} - Z_{51} + Z_{55} \\ Z_{41} - Z_{45} - Z_{51} + Z_{55} \end{bmatrix} I_1 \]  (4.15)
The results are shown in the next figure 4.11.

Fig. 4.11 Fault currents and voltages as function of numbers of neutral conductors

It is shown that with using multiple return conductors the fault voltage is strongly decreasing and the short circuit current is increasing, but weakly. It means also that with using multiple return conductor, safely low voltage network can be achieved. In practice this means, coupling of return conductors as much as possible is generally very recommended. Although with this coupling of neutral conductor, the short circuit current is not highly increasing as expected with using the resistance model so that the fuse might not disconnect the utilization to the feeding source but the low fault voltage can still be hold to the safe level for human being. (As given in chapter 2)
CHAPTER 5

FAULT VOLTAGE AND CURRENT FOR PRACTICAL NETWORKS

The point of this chapter is to simulate and compare the results with the measuring two significant quantities, fault voltage and fault current. First, the using of resistance \((R)\) model is tested as a general and accepted method to compute fault voltage and fault current in low voltage network. And then the next method so called coupling model \((R-L-M\) model) would be introduced. Depends on characteristic and complexity of each network the following computation software will be applied, Microcap or Matlab, with needs necessary assumptions, respectively. Generally, it is not possible to compute fault voltage and fault current exactly because of complexity of low voltage networks where the conductors/cables configuration might be placed arbitrary.

Practically, each low voltage networks may have different standard and norm with regard to the numbers and place of earth electrodes, numbers and coupling place of return conductors, and variety of cable type and protective equipment. These requirements for computation are often unavailable or difficult to get the actual data, because the age of the network itself. This implies that each network has his own safety, protection criteria and complexity. The analyzing of a low voltage network should be examined one by one especially on place where far from transformator house or on other critical place within it. And to make a general solution of this would be impossible or very difficult, considering the variety that practically may be occurred.

Next, there are two low voltage networks discussed, each with different characteristic that often encountered practically on TT-systems.

5.1 ACACIALAAN (STEENBERGEN)

This network is shown on figure 5.1 and has a TT-system. It means that each house (end users) has his own earth electrode. The measurement is done at the section box. At the section box it is noticed that there is mounted no earthing electrode. It is expected that when fault occurred at this section box, the network would behave like a TN-system, it means that all fault currents go back to feeding transformator through the all possible way of return conductors. The numbers of this return ways could be indefinite, because practically all neutral conductors are connected each other at every section box, even when the neutral comes from another transformer. To get this problem conveniently, the nearest return paths are chosen. It is marked here that not all distribution cable is from alkudia 4x95mm². There is GPLK cable between section box D and C.

The data of the low voltage network here is:

\[
\begin{align*}
S \text{ (rated power)} & : 300 \text{ kVA} \\
U_n \text{ (rated voltage)} & : 230 \text{ V} \\
Z_k \text{ (s.c impedance)} & : (4.6 + 21.9i) \text{ m}\Omega \\
type \text{ cable} & : 4x95mm^2 \text{ Alkudia} \ (R_d=365 \text{ m}\Omega/\text{km}) \text{ and } 4x50\text{mm} \text{ Cu} \text{ GPLK with earth sheath} \ (R_{cu}=387 \text{ m}\Omega/\text{km})
\end{align*}
\]
Chapter 5 Fault voltage and current for practical networks

At place A section box D, the fault is made on cable R1. The fault current flows along the phase cable R1 and goes back to the transformer through the neutrals of cable R1 self, cable R2//R3 to section box C then serie with cable R4//R5 and partial of it might go back through cable R6 to section box B series R7//R8 to section box A and then serie with R9//R10 to transformer, as shown in fig. 5.1. This tracing of fault current is a main way to compute it with resistance model in general.

5.1.1 METHODS TO DETERMINE THE FAULT VOLTAGE AND CURRENT

The most general method by design of low voltage network is resistance model. This method until now is still used to predict (compute) the fault current and fault voltage. As shown on chapter 4, the resistance model gives always higher fault current and fault voltage. Measurement indicated that extremely low value of them can not be explained with resistance model. A new method is now introduced and called coupling method. It is expected that this method can explained the measurement as has been shown on chapter 4.

5.1.1.1 Resistance-model (R-MODEL)

This model based on resistance computation where each conductor is substituted with resistance value of the conductor as given by the producent. With R-model, the most possible
flowing return short circuit current is traced to section box far away from the transformer house, (as shown on fig. 5.1). The resistance model is obtained as in fig. 5.2.

The resistance values are obtained from the cable resistance data

<table>
<thead>
<tr>
<th>Cable Type</th>
<th>Resistance Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VG- VMvK 4 × 95 Al</td>
<td>$R_{Al} = 0.365 , \Omega/km$</td>
</tr>
<tr>
<td>GLPK(h) 4 × 50 Cu</td>
<td>$R_{Cu} = 0.387 , \Omega/km$</td>
</tr>
</tbody>
</table>

**Table 3 Resistance values of each cable (see fig. 5.1 and fig. 5.2)**

<table>
<thead>
<tr>
<th>number of cables</th>
<th>length m</th>
<th>cable type</th>
<th>Resistance value mΩ</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>225</td>
<td>95 Al</td>
<td>82,1</td>
</tr>
<tr>
<td>R2</td>
<td>106</td>
<td>95 Al</td>
<td>38,7</td>
</tr>
<tr>
<td>R3</td>
<td>142</td>
<td>50 Cu</td>
<td>55,0</td>
</tr>
<tr>
<td>R4</td>
<td>133</td>
<td>95 Al</td>
<td>48,5</td>
</tr>
<tr>
<td>R5</td>
<td>147</td>
<td>95 Al</td>
<td>53,7</td>
</tr>
<tr>
<td>R6</td>
<td>236</td>
<td>95 Al</td>
<td>86,1</td>
</tr>
<tr>
<td>R7</td>
<td>113</td>
<td>95 Al</td>
<td>41,2</td>
</tr>
<tr>
<td>R8</td>
<td>73</td>
<td>95 Al</td>
<td>26,6</td>
</tr>
<tr>
<td>R9</td>
<td>211</td>
<td>95 Al</td>
<td>77,0</td>
</tr>
<tr>
<td>R10</td>
<td>248</td>
<td>95 Al</td>
<td>90,5</td>
</tr>
</tbody>
</table>

$R_{trafo} = 6.2 \, \text{mΩ}$

$X_{trafo} = 27.5 \, \text{mΩ}$

$R2 \parallel R3 = 22.7 \, \text{mΩ}$
Chapter 5 Fault voltage and current for practical networks

R4 // R5 = 25.5 mΩ
R7 // R8 = 16.2 mΩ
R9 // R10 = 41.6 mΩ

([R7//R8] + [R9//R10] + R6)//(R4//R5) = Rx = 21.7 mΩ

\[
\begin{align*}
\text{Rphase} &= R1 = 82.1 \text{ mΩ} \\
\text{Rreturn} &= (Rx + [R2//R3])//R1 = 28.8 \text{ mΩ} \\
\text{Ztotal} &= Z\text{trafo} + \text{Rphase} + \text{Rreturn} = 117.1 + 27.5i \text{ mΩ}
\end{align*}
\]

\[
I_k = \frac{U_{\text{secondary}}}{|Z_{\text{total}}|} = \frac{229}{0.1203} = 1904A
\]

\[
U_f = I_k \times R_{\text{return}} = 1904 \times 0.288 = 54.8V
\]

5.1.1.2 Coupling model

Coupling method is based on resistance model with addition of magnetic coupling (self inductance and mutual inductance). Because of the complexity of possible flowing return fault current, the network is analyzed with less return paths compared with the computation with resistance model. The main return paths used here are cable R1 self, cable R2//R3 then series with cable R4//R5. Other return paths are neglected, because of their length and far distance offer their influence very little. The coupling of each conductor to others is represented by impedance respectively. This impedance is computed with the given data of the cable type and the lying of the cable. The new model is set up with considering the maximal coupling between them, when the cable lies parallel each other. Other couplings are assumed to be neglected when they make perpendicular position to others.

Fig. 5.3 Coupling model of Acaclalaan.
The new variables are introduced here and correspond with respectively cable (notation i) as follows:

\( Z_{ii} \) : coupling to cable \( j \) from cable \( i \)

- \( Z_{ii} \) : phase of the cable \( R1 \)
- \( Z_{ii} \) : neutral of the cable \( R1 \)
- \( Z_{ii} \) : neutral of the cable \( R4 \)
- \( Z_{ii} \) : neutral of the cable \( R5 \)
- \( Z_{ii} \) : neutral of the cable \( R2 \)

The Kirchhoff voltage law of this network with choosing \( i \) as reference is

\[
-U_a + I_1(Z_{11} + Z_{12}) + I_2Z_{12} + I_3Z_{13} + I_4Z_{14} - \bar{I}_1Z_{24} - \bar{I}_2Z_{23} - \bar{I}_3Z_{21} - \bar{I}_4Z_{22} = 0
\]

\[
I_3Z_{33} + I_4Z_{34} + I_2Z_{32} + I_1Z_{31} + I_5Z_{55} + I_6Z_{56} - \bar{I}_2Z_{44} - \bar{I}_3Z_{43} - \bar{I}_1Z_{41} - \bar{I}_4Z_{42} - \bar{I}_5Z_{55} - \bar{I}_6Z_{56} = 0
\]

Using \( I_2 = -(I_1 + I_5 + I_6) \) and \( I_3 = I_5 + I_6 - I_4 \) relations form the next matrix

\[
\begin{bmatrix}
Z_{11} + Z_{12} + Z_{13} + Z_{14} - Z_{21} - Z_{22} - Z_{23} - Z_{24} + Z_{33} + Z_{34} - Z_{35} - Z_{36} + Z_{44} + Z_{45} - Z_{46} + Z_{55} + Z_{56} - Z_{65} - Z_{66}

0

0

0

0

0

\end{bmatrix}
\]

The fault voltage is computed with one of the next equation as respect to the voltage at the source.

\[
U_f = U_0 - I_1(Z_{11} + Z_{12} - Z_{13} - Z_{14})
\]

5.1.1.3 Coupling model with earthing resistance

Apparently in Acacialaan there is a GPLK cable GPLK used as distribution cable on a branch. This cable has an earth sheath and this earth sheath is always connected with the earthing resistance of each house along that cable. This course gives consequence that at each earthing connection a resistor (\( R_a \)) should be introduced. There is now a new return path of the fault current through \( R_a \), perfectly conducting earth and \( \bar{R} \). The new model is shown in the next figure.
The found algebraic equation for coupling method is now adapted with new variable $R_a$. The Kirchhoff voltage law of this network with choosing $I_2$ as reference is

$$-U_o + I_1(Z_b + Z_{11}) + I_2Z_{12} + I_3Z_{13} + I_4Z_{14} - I_5Z_{24} - I_6Z_{23} - I_7Z_{21} - I_8Z_{22} = 0$$

$$(5.4)$$

Using $I_2 = 1_2 - 1_1 - I_6 - I_8$ and $I_8 = 1_6 - I_7$ relations form the next matrix

$$(5.5)$$

The fault voltage is computed with one of the next equation as respect to the infinity point as fault voltage reference. The fault voltage must be the same with using different path and this can be further used to check up the computation.
Each impedance is computed using exactly self and mutual impedance with the next data:

\[ Z_{11} = Z_{22} \text{ (length=225m,Al)} \]

\[ Z_{33} \text{ (length=133m,Al)} \]

\[ Z_{44} \text{ (length=147m,Al)} \]

\[ Z_{55} \text{ (length=106m,Cu)} \]

\[ Z_{66} \text{ (length=142m,Cu)} \]

\[ Z_{12} = Z_{21} \text{ (length=225m,distance=1.2cm)} \]

\[ Z_{13} = Z_{31} \text{ (length=133m,distance=78m)} \]

\[ Z_{14} = Z_{41} \text{ (length=147m,distance=88m)} \]

\[ Z_{23} = Z_{32} \text{ (length=133m,distance=78m-1.2cm)} \]

\[ Z_{24} = Z_{42} \text{ (length=147m,distance=88m-1.2cm)} \]

\[ Z_{34} = Z_{43} \text{ (length=133m,distance=10m)} \]

\[ Z_{56} = Z_{65} \text{ (length=106m,distance=11.4m)} \]

Functional resistance \( R_b \) is 4.6 \( \Omega \).

Earthing resistance \( R_a \) is 2 \( \Omega \).

Taking the frequency of the source as 0Hz results the DC computation of the network.

Because of the complexity of low voltage network can be, it is difficult and can be impossible to apply this method on arbitrarily low voltage network configuration.

5.1.2 MEASUREMENT AND SIMULATION RESULTS

Next, the measurement value is compared with different ways of computation model.

Table 4 Measurement and computation results.

<table>
<thead>
<tr>
<th></th>
<th>Measurement</th>
<th>R-model</th>
<th>Coupling model</th>
<th>R and ( R_a ) model</th>
<th>Coupling and ( R_a ) model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault voltage [V]</td>
<td>20.1</td>
<td>54.8</td>
<td>45.3</td>
<td>27</td>
<td>20.6</td>
</tr>
<tr>
<td>Fault current [A]</td>
<td>1520</td>
<td>1920</td>
<td>1400</td>
<td>1932</td>
<td>1406</td>
</tr>
</tbody>
</table>

The resistance model gives the value of fault voltage and fault current that higher is than the measuring value.

With only coupling model, the currents distribution on each conductor are computed with formula 5.1-5.3 (acacia_a.m)

\[ I_1 = 1402A, I_2 = 1178A, I_3 = 230A, I_4 = 205A, I_5 = 260A, I_6 = 174A \]

It shows that coupling model now seems to explained the fault current but the fault voltage just decreases about 10V compare with resistance model. Further it is shown also that the fault current goes back to the transformer through the neutral on the same cable. Just. a partial of it goes back through other neutral.

Now, the computation with using resistance model (including the earthing resistance) is carried on to test the usefulness of resistance model. It results in decreasing of fault voltage but almost no effect to the fault current compare to resistance model without \( R_a \).
Next, the complete circuit is now computed with formula 5.4-5.6 and gives the results that explain the measuring results of fault voltage and fault current. With coupling and earthing resistance (\( R_a = 2 \Omega \)) model, the currents distribution are (aca_ra.m)

\[
\begin{align*}
I_1 &= 1406A, \quad I_2 = 1174A, \quad I_3 = 233A, \quad I_4 = 207A, \quad I_5 = 263A, \quad I_6 = 177A, \quad I_7 = 5A, \quad I_8 = 179A.
\end{align*}
\]

The fault voltage now is 20.6V. If now the earthing resistance is changed to \( R_a = 4 \Omega \). The fault voltage would be 26V and fault current still 1406A. And the current distribution would be (aca_ra.m)

\[
\begin{align*}
I_1 &= 1406A, \quad I_2 = 1174A, \quad I_3 = 233A, \quad I_4 = 207A, \quad I_5 = 263A, \quad I_6 = 177A, \quad I_7 = 4A, \quad I_8 = 179A
\end{align*}
\]

The differences of the currents are very little and lie on decimal. The influence of the earthing resistance to the current is very minimum but provides strong decreasing to the fault voltage.

5.1.3 DISCUSSION

It can be marked that resistance model as most used general way to compute fault voltage and fault current can not explain the measurement values. And using inductance model, the reducing of fault voltage is not low enough, although the fault current now is approximated. It is apparently clear later that the earth sheath of the GPLK cable on conductor 6 is connected with earthing electrodes of the house, because this earth sheath is connected to the neutral of the cable, it leads a voltage reducing at the neutral when a fault occurred. Using inductance model and earthing resistance, the measurement values are verified (explained). It is shown also that with earthing resistance the fault current increases a little bit but reduces the fault voltage extremely. The computation results indicate that the fault current flows back mostly through the neutral conductor that lies near phase conductor (on the same cable). A little fraction of the fault current flows on conductor 3 and 4. Using R-model, most fault currents would flows in these conductors because the short length of these conductors. Unfortunately, all paths of fault currents could not be measured to verify these computation results.

Analyzing low voltage network with TN systems using the coupling method will be continued in the future.
5.2 HILDEBRANDTSTRAAT (DEN BOSCH)

This low voltage network has its own unique characteristic, the neutral conductors at each house along the cable are connected to the water pipe system and this is connected again with cable sheaths. Here, the used distribution cable is GPLK copper with an earth sheath. The water pipe system can be seen as common earthing electrodes to an ideally far earth potential. Because of this complexity it is impossible to compute the fault voltage and fault current exactly. As it will be shown that the measurement of the fault voltage quantity indicates that the network is very safe, the interesting question is: how a very low voltage of the measuring value can be explained, when there are so much earthing points are introduced.

The data of the low voltage network here is:

$$\text{Hildebrandstr. HSR 90}$$

$$S=400\text{KVA}, 10\text{KV}/380\text{V}$$

$$U_k=3\%$$

$$R_a=2\Omega$$

$$\text{Trafo}$$

$$\text{F1}$$

$$\text{F2}$$

$$\text{Hildebrandstr.}$$

$$\text{F3}$$

$$\text{F4}$$

$$\text{GPLK}$$

$$4x70 \text{ Cu}$$

$$494 \text{ m}$$

$$25 \text{ woningen}$$

$$2 \text{ flats}$$

$$24 \text{ won.}$$

$$\text{s.k. 1275}$$

$$R_a=3.6\Omega$$

$$\text{Messen}$$

$$\text{F5}$$

$$\text{F6}$$

$$\text{Ra}=3.5 \Omega$$

$$\text{v.k. 56}$$

Fig. 5.5 Schematic low voltage network.
5.2.1 METHOD TO DETERMINE THE FAULT VOLTAGE AND CURRENT

This distribution cable is ended at the section box and before the measuring, the coupling of neutral conductor is broken up with other neutrals. It means that there is no influence to the measuring fault voltage and fault current anymore with other earthing system, except the earthing along the GPLK cable self.

The model used to simulation the fault is shown below:

![impedance model](image)

- node 1: rated voltage
- node 2: fault voltage
- node 3: voltage at 1st house
- node 4: voltage at the middle house
- node 5: star point voltage of the transformer
- Zf: inner impedance of the transformer
- Zci: impedance of part i of the copper

The complete electric equivalent scheme is found on appendix 4.

The computing data

- $S$ (rated power): 400 kVA
- $R_b$ (functional resistance): 2 $\Omega$ (at transformer)
- $R_c$ (earthing resistance): 3.5 $\Omega$ (at section box)
- cable length: 500 m
- type cable: 4x70 Cu, GPLK, $R_{Cd} = 270$ m$\Omega$/km, $L_{Cd} = 0.261$ $\mu$H/m
- Numbers of houses: 30

Before measuring, the coupling of the neutral with other neutral conductors from other circuit in section box is disengaged to avoid the effect of the earthing through these neutrals. During the measuring, just the effect of earthing resistance along the cable self would be significant.
5.2.2 MEASUREMENTS AND SIMULATION RESULTS

Table 5 Measurement and computation results.

<table>
<thead>
<tr>
<th></th>
<th>Measurement</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault voltage [V]</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>Fault current [A]</td>
<td>936</td>
<td>989</td>
</tr>
</tbody>
</table>

NB: With using a mean earthing resistance each house \( R_a = 0.1 \Omega \).

Fig. 5.7 Voltages and time simulation curves

where

\( U(1) \): the rated voltage

\( U(2) \): fault voltage

\( U(3) \): fault voltage at 1st house

\( U(4) \): fault voltage at the middle
Chapter 5 Fault voltage and current for practical networks

Currents occurred at fault situation

Fig. 5.8 Currents and time simulation curves

where

- $I_f$ : fault current
- $I_n$ : current on neutral conductor
- $I_{Ra}$ : current on earthing electrode

5.2.3 DISCUSSION

Using Microcap as computation software, the very low fault voltage and fault current this network can be explained when the earthing resistance of each house has a value of $R_h=0.1\Omega$ in respect with using GPLK cable. So low earthing resistance value is possible, considering the present fact where the connection of cable sheath to the metal water pipe system is fixed and the parallel earthing electrodes from each house to perfect conducting earth are established.
CHAPTER 6

CURRENT DISTRIBUTION FOR A TWO CONDUCTOR SYSTEM

The phenomena affecting the distribution of electric current inside conductors is studied (discussed). The interaction between electric current and the alternating magnetic flux generated by electric current determines current distribution inside conductor. Depending on the geometry of the problem examined, this phenomena are known as skin effect, proximity effect and eddy current.

**Skin effect**: the time-varying magnetic field produced by the flow of ac current inside a conductor results in an uneven distribution of electric current in the cross section of the conductor. The electric current tends to concentrate near the surface of the conductor (skin), away from the center.

**Proximity effect**: the time-varying magnetic field produced by the flow of ac current in a conductor results in an uneven electric current distribution in the cross section of a nearby conductor.

**Eddy currents**: an alternating magnetic field in an conducting medium of arbitrary shape causes the flow of electric currents. These currents are called eddy currents.

The skin effect is present in every conductor. The intensity of the phenomenon depends on the frequency of the electric current, the properties of the material and the diameter (geometrical size) of the conductor. The proximity effect can be noticed on conductors very close to each others. This effect is not substantial by large separation distances among conductors. Eddy current are important in transformers, electric motors and other devices that generated localized alternating magnetic fields in conducting media (magnetic core like iron etc.)

Now, the self and mutual inductance formulas are applied to compute the current distribution on cross sectional of two conductors. The (center) distance between the two conductors is maintained at the same distance along the conductor. The length of the conductor is chosen very long so that the end effect can be neglected. Also, the thermal effect is not considered.

The used computation parameters are:

- cycle frequency : 100π rad/sec
- conductor length : 60 m
- type conductor : Al ($R_{20}=157E-6$ Ωm and $\alpha_{20}=0.004$ 1/C)
- cross sectional : 1000 mm$^2$ (radius=1.8 cm)
- variously center distance : 4.4 cm and 11 m

Each cross sectional of the conductor is divided (split up) into small conductor elements and the algebraic equation of current distribution is then set up and computed.

![Fig. 6.1 Splitting up the cross sectional of the conductor into element](image)
The numbers of elements conductor are 845 each conductor. It means that matrix equation would be as big as 1690X1690, where all elements conductor are brought into computation. Each element conductor has an unique coordinate on the cross-sectional of the conductor. It means also that the distance between these element should be first computed.

Taking all these 1690 elements, next equation system will be set up:

\[
\begin{align*}
E_1 &= (Z_k + Z_q + Z_l)(I_1 + I_2 + \ldots + I_{845}) + V_1 - V_{1690} \\
E_2 &= (Z_k + Z_q + Z_l)(I_1 + I_2 + \ldots + I_{1689}) + V_2 - V_{1690} \\
&\vdots \\
E_{1690} &= (Z_k + Z_q + Z_l)(I_1 + I_2 + \ldots + I_{1689}) + V_{845} - V_{1690} \\
0 &= V_{846} - V_{1690} \\
0 &= V_{1689} - V_{1690} \\
\end{align*}
\]

(6.1)

where:
- \( Z_q \): net impedance at the middle voltage side
- \( Z_k \): inner impedance of the transformer
- \( Z_l \): load impedance

This equation can be rewritten in matrix relation: \([E] = [A][I]\); The vector \( I \) can be solved by taking the inverse of \( A \) and then multiply it with voltage vector \( E \): \( I = \text{inv}(A) \times [E] \), where matrix \( A \) contains significant variables, corresponding with new matrix notation of \( Z_q, Z_k \) and \( Z_l \) respectively.; \([A] = [Z_{\text{trafo}}] + [Z_{\text{net}}] + [Z_{\text{load}}] + [N]\)

The matrix equation for fictive voltage drop in the conductor is written as:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_{1690}
\end{bmatrix} =
\begin{bmatrix}
R_1 + j\omega L_{1,1} & j\omega L_{1,2} & \cdots & j\omega L_{1,1689} & j\omega L_{1,1690} \\
R_2 + j\omega L_{2,1} & R_2 + j\omega L_{2,2} & \cdots & j\omega L_{2,1689} & j\omega L_{2,1690} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
R_{1690} + j\omega L_{1690,1} & j\omega L_{1690,2} & \cdots & R_{1690} + j\omega L_{1690,1689} & R_{1690} + j\omega L_{1690,1690}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{1690}
\end{bmatrix}
\]

(6.2)

This is a matrix with size of 1690X1690. Taking \( I_{1690} \) as reference current, the next equation can be substituted: \( I_{1690} = -\sum_{k=1}^{1689} I_k \) because \( \sum_{k=1}^{1690} I_k = 0 \)

The equation 6.2 would be simplified and can be written as:
\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_{1690}
\end{bmatrix} =
\begin{bmatrix}
R_1 + j\omega(L_{1,1} - L_{1,1690}) & j\omega(L_{1,2} - L_{1,1690}) & \cdots & j\omega(L_{1,1689} - L_{1,1690}) \\
R_2 + j\omega(L_{2,1} - L_{2,1690}) & R_1 + j\omega(L_{2,2} - L_{2,1690}) & \cdots & j\omega(L_{2,1689} - L_{2,1690}) \\
\vdots & \vdots & \ddots & \vdots \\
-R_{1690} + j\omega(L_{1,1690} - L_{1,1690}) & -R_{1690} + j\omega(L_{2,1690} - L_{1,1690}) & \cdots & -R_{1690} + j\omega(L_{1,1690} - L_{1,1690})
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{1690}
\end{bmatrix}
\]

\[
[M] = [M][I]; \quad n=1690; \text{ total numbers of element conductor.}
\]

\[
\dim \{V\} = 1690 \times 1, \quad \dim \{M\} = 1690 \times 1689, \quad \dim \{I\} = 1689 \times 1
\]

The elements of the matrices are fulfilled by the next algorithm

\[
n=1690;
\]

\[
\text{FOR } ii = 1 \text{ TO } n \text{ DO}
\]

\[
\text{FOR } jj = 1 \text{ TO } (n-1) \text{ DO}
\]

\[
\text{IF } jj = ii \text{ DO}
\]

\[
M_{ii,jj} = R_n + j\omega(L_{ii,jj} - L_{ii,n})
\]

\[
= R_n + j\omega \frac{\mu_n}{2\pi} \left[ \frac{1}{4} + \ln \left( \frac{D_{ii,n} \left( 1 + \sqrt{1 + \left( \frac{r}{l} \right)^2} \right)}{\left( r + 1 + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} \right)^2} \right) \right] - \sqrt{1 + \left( \frac{r}{l} \right)^2} + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} + \left( \frac{r - D_{ii,n}}{l} \right);
\]

\[
\text{ELSEIF } ii = n \text{ DO}
\]

\[
M_{ii,jj} = -R_n - j\omega(L_{ii,n} - L_{ii,jj})
\]

\[
= -R_n - j\omega \frac{\mu_n}{2\pi} \left[ \frac{1}{4} + \ln \left( \frac{D_{ii,n} \left( 1 + \sqrt{1 + \left( \frac{r}{l} \right)^2} \right)}{\left( r + 1 + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} \right)^2} \right) \right] - \sqrt{1 + \left( \frac{r}{l} \right)^2} + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} + \left( \frac{r - D_{ii,n}}{l} \right);
\]

\[
\text{ELSE DO } (jj \neq ii \neq n)
\]

\[
M_{ii,jj} = j\omega(L_{ii,jj} - L_{ii,n})
\]

\[
= j\omega \frac{\mu_n}{2\pi} \ln \left( \frac{D_{ii,n} \left( 1 + \sqrt{1 + \left( \frac{D_{ii,jj}}{l} \right)^2} \right)}{\left( r + 1 + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} \right)^2} \right) - \sqrt{1 + \left( \frac{D_{ii,jj}}{l} \right)^2} + \sqrt{1 + \left( \frac{D_{ii,n}}{l} \right)^2} + \left( \frac{D_{ii,jj} - D_{ii,n}}{l} \right);
\]

\[
\text{END; } \{\text{IF}\}
\]

\[
\text{END; } \{\text{FOR}\}
\]

\[
\text{END; } \{\text{FOR}\}
\]
Chapter 6 Current distribution for a two conductor system

Next, the matrix $N$ of the totally voltage drop along the conductor can be computed and this has a size of 1689X1689.

\[
V_1 - V_{1690} = (M_{1,1} - M_{1,1690}) I_1 + (M_{1,1} - M_{2,1690}) I_2 + \ldots + (M_{1,1689} - M_{1689,1690}) I_{1689}
\]
\[
V_2 - V_{1690} = (M_{1,2} - M_{1,1690}) I_1 + (M_{2,2} - M_{2,1690}) I_2 + \ldots + (M_{1,1689} - M_{1689,1690}) I_{1689}
\]
\[
V_{1689} - V_{1690} = (M_{1,1689} - M_{1,1690}) I_1 + (M_{2,1689} - M_{2,1690}) I_2 + \ldots + (M_{1689,1689} - M_{1689,1690}) I_{1689}
\]

(6.4)

with next algorithm:

\[
\text{n}=1690;\\ 
\text{FOR} \ ii=1 \ \text{TO} \ (n-1) \ \text{DO}\\ 
\quad \text{FOR} \ jj=1 \ \text{TO} \ (n-1) \ \text{DO}\\ 
\quad \quad \text{N}_{\ ij} = M_{\ ij} - M_{\ nj}\\ 
\quad \text{END; \{FOR\}}\\ 
\text{END; \{FOR\}}\\ 
\]

Finally, the matrix $Z_{\text{trafo}}$ and $Z_{\text{load}}$ will be set up:

\[
\text{m}=n-1;\\ 
\text{Znet}=\text{zeros}(m,m); \quad \% \ \text{Initialization with 0 matrix}\%\\ 
\text{Ztrafo}=\text{zeros}(m,m); \quad \% \ \text{Initialization with 0 matrix}\%\\ 
\text{Zload}=\text{zeros}(m,m); \quad \% \ \text{Initialization with 0 matrix}\%\\ 
\text{FOR} \ ii=1:845,\\ 
\quad \text{FOR} \ jj=1:845,\\ 
\quad \quad \text{Znet}(ii,jj) = Zq;\\ 
\quad \text{Ztrafo}(ii,jj) = Zk;\\ 
\quad \text{Zload}(ii,jj) = Zl;\\ 
\quad \text{END; \{FOR\}}\\ 
\text{END; \{FOR\}}\\ 
\]

After the computation of vector $I$, the element currents can be prepared to 3-dimensional view and each of them will be placed into the cross-sectional of the conductors, considering with the origin place where the coordinates is already known. Then a 3-dimensional view of the current distribution can be obtained.

To know the proximity and skin effect on current distribution, the conductors are placed on two different center distance. The result of the computation is shown in the next figures.
Current distribution of two straight cylindrical conductors

![Current distribution of two straight cylindrical conductors](image)

Fig. 6.2 Current distribution of two conductors close distance.

Current distribution contour of two straight cylindrical conductors

![Current distribution contour of two straight cylindrical conductors](image)

Fig. 6.3 Contour of two conductors close distance.

Total current : 705 A
center distance : 4.4 cm

It is shown that the current distribution is not homogene but symmetrical along horizontal axis perpendicular of the conductor length. Fig 36 and 37 show both skin and proximity effect on current distribution on each conductor. The highest current density occurs on the conductor parts facing each other and the lowest lies within the conductor. Generally, the current is concentrated to outside of the conductor.
Chapter 6 Current distribution for a two conductor system

Current distribution of two straight cylindrical conductors

Fig. 6.4 Current distribution of two conductors far distance

Total current : 643 A
center distance : 11 m

As shown on fig. 6.4 and 6.5, the distance between the two conductor is made very large. The proximity effect can not be observed anymore and remains the significant skin effect only on each conductor. It leads to (almost) symmetrical current distribution with regard to conductor center on each conductor. The current is concentrated also to outside of the conductor. Although, it is remarkable that the total current is now lower than when the distance is closely near each other.
The basic skin-depth is found using a known formula \( \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \). For Aluminium, the relative permeability \( \mu = 1 \) and conductivity \( \sigma = 3.85 \times 10^7 \) mho/m. At frequency 50Hz the skin-depth will be at \( \delta = 1.15 \text{cm} \). It means that the current at this distance from outside of the conductor will remain so low as 37% compares the current at the outside. Figure 6.4 and 6.5 show that the decreasing of the current within conductor is stronger than that of 37% at the same distance of \( \delta \). The explanation of this difference is not studied yet. In the future, this work might be continued.
7 CONCLUSIONS AND RECOMMENDATIONS

From the foregoing project the following conclusions can be made.

1. An exact inductance formula is derived and compared with commonly used approximations. Approximation leads to a relative large error when the conductors are separated by a big distance and are of small length.

2. During the measuring of 1 phase with 2 equal return conductors, it was observed that both return conductors carry different currents.

3. By coupling of the neutrals, the fault voltage can be decreased strongly and the fault current will be increased weakly.

4. Using earth electrodes on different places in low voltage networks can reduce the fault voltage extremely and has almost no effect for the fault current.

5. For low voltage networks with couples neutrals the resistance model is not valid to explain the fault voltage and current. Using the coupling model is helpfull for intelligently simplified configurations. Measuring those quantities is another way to verify the safety of the low voltage network.

6. The coupling model still needs to be verified with a purely TN system.

7. The current distribution of a two conductor system is computed. The skin- and proximity effect are presented. However no thermal effects were considered.
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September 1994.
ACKNOWLEDGEMENT

I would like to thank Ir. Joop Sloot for conducting this project, Mr. Hans Vossen for helping and assistance during measurement at High Current Laboratory at Eindhoven University of Technology (TUE) and Ir. Wim Ploem from Electricity Company of North Brabant (PNEM) for disposal of measuring equipment on field, cables and maps of the analyzed low voltage network. Also, I would thank to my colleague Alfred Arts and the other people from PNEM that helped us to carry out the measurement on field.
APPENDIX 1  PROGRAM FOR NORMS AND STANDARDS

% program name: normen.m
% IEC and NEN 1010 norms
clear;
% axis([-1.65 2.465 -1.7 0.7]);
% aanraakcurve NEN 1010, hand-hand en hand-voet-voet (IEC479-1)
% hand-voet-voet
Unen = [50, 60, 75, 90, 110, 150, 220, 250 ];
ten = [5, 2, 1, 0.5, 0.2, 0.1, 0.05, 0.03 ];
% hand-hand zonder schoeisel
Uhh = [45, 50, 55, 60, 80, 100, 120, 140, 160,...
      180, 200, 220, 240, 260 ];
thh = [100000, 5, 2, 1.49, 0.76, 0.58, 0.455, 0.42, ...
      0.4, 0.37, 0.35, 0.34, 0.29 ];
% hand-voet-voet zonder schoeisel
Uhv = [20, 40, 45, 50, 55, 60, 80, 100, 120, 140, 160,...
      180, 200, 220, 240, 260 ];
ths = [100000, 5, 2, 1.39, 1, 0.85, 0.59, 0.435, 0.40, 0.365, ...
      0.335, 0.305, 0.27, 0.24, 0.205];
% hand-voet-voet-schoeisel R=1kOhm
UhvR = [65, 70, 75, 80, 100, 120, 140, 160, 180, 200, 220,...
       240, 260 ];
thsR = [100000, 5, 2.2, 2, 1, 0.76, 0.64, 0.57, 0.5, 0.485, 0.455,...
       0.435, 0.42 ];
% plotten
figure,loglog(Unen,ten,Unen,ten,'ro',Uhh,thh,Uhh,thh,'gx'),hold on,
loglog(Uhv,ths,Uhv,ths,'b*'),UhvR,thsR,UhvR,thsR,'y+'),grid;
title('Afschakeltijd als functie van de spanning');
xlabel('U [V]'),ylabel('tmax [s]');
print -dmeta normen1.wmf
% color and b&w
figure,semilogy(Unen,ten,'k',Unen,ten,'ro',Uhh,thh,'k',Uhh,thh,'kx'),hold on,
semilogy(Uhv,ths,Uhv,ths,'b*'),UhvR,thsR,UhvR,thsR,'y+'),grid;
figure,semilogy(Unen,ten,'k',Unen,ten,'ro',Uhh,thh,'k',Uhh,thh,'kx'),hold on,
semilogy(Uhv,ths,'k',Uhv,ths,'b*'),UhvR,thsR,'k',UhvR,thsR,'k*'),grid;
axis([-20 290 -1.7 0.7]);
text(150,1000,'o : NEN 1010 hff');
text(150,500,'+ : IEC 479 hffR');
text(150,200,'x : IEC 479 hff');
text(150,100,'* : IEC 479 hffR');
title('Maximal allowed time vs voltage');
xlabel('V [V]'),ylabel('t [max [s] ]');
print -dmeta normen.wmf
## APPENDIX 2 MEASUREMENT EQUIPMENT AND APPARATUS

<table>
<thead>
<tr>
<th></th>
<th>Name</th>
<th>Merk</th>
<th>Error</th>
<th>Registration number</th>
<th>Last calibrating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Voltmeter/Amperemeter</td>
<td>Multivib</td>
<td>1.5%</td>
<td>920557,920546,920556,920552,920551,920550,920549,920545,920554</td>
<td>03-06-1992,01-06-1992</td>
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<td>2</td>
<td>Wattmeter</td>
<td>Hartman &amp; Braun</td>
<td>0.5%</td>
<td>840717</td>
<td>03-09-1984</td>
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<td>3</td>
<td>Micro-Ohmmeter</td>
<td>Keithley 580</td>
<td>1%</td>
<td>EG30540M1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Current transformer</td>
<td>Goerz, KL 0.2, GE 4461</td>
<td></td>
<td>EO130L,EO134L,EO129L</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Current piers</td>
<td>H &amp; B, nr 71552, 2%, 600A, 50Hz</td>
<td></td>
<td>EA1291</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Universal-Anleger</td>
<td></td>
<td>2%, 600A, 50Hz</td>
<td>EH219UM05E</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Zanger stromwandler</td>
<td>GE4453, 0.5/3kV- 50-60Hz</td>
<td></td>
<td>EA2172</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Variac transformer</td>
<td>Transforma Amsterdam, type 185636, No. 379300, prim. 380V sec. 0-380V, 24A, 50Hz</td>
<td></td>
<td>EH477TR-02</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Transformer YD11</td>
<td>Transforma Amsterdam, type 033699, No. 379301, prim. 380V, sec. 20-40V, 800/400A 50Hz</td>
<td></td>
<td>EH48TZ01</td>
<td></td>
</tr>
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<td>10</td>
<td>Durchsteck-Stromwandler TIL05</td>
<td>H &amp; B Elim</td>
<td>2%, Reihe 0.5-500/5-10VA, 50Hz, KL. 0.2</td>
<td>EA1284,EA1288,EA1283,EA1287</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Digital scope</td>
<td>Vuko, VKS460C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Fault voltage and fault current measuring coffer</td>
<td>PANENSA MIC-11</td>
<td>1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX 4  THE COMPLETE EQUIVALENT CIRCUIT AT HILDEBRANDT (DEN BOSCH)

.model Vl Sin(F=50 A=311 DC=0)
.DEFINES lengte 500
.DEFINES RCu 270E-6*1.2
.DEFINES Rf RCu*lengte
.DEFINES Rc Rf/30
.DEFINES Lf0 2.61E-7
.DEFINES Lf Lf0*lengte
.DEFINES Lfc Lf/30