MASTER

Design of gradient coils to measure diffusion with NMR

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Design of gradient coils to measure diffusion with NMR

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Eindhoven University of Technology
Department of Applied Physics
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Graduation Report

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             mw.ir.C.v.Pul
Abstract

MRI is a non-destructive technique to visualize the inner of the human body. In recent years a new MRI technique to visualize diffusion in human brain has been developed, called Diffusion Weighted Imaging (DWI). To investigate the dependence of this measuring technique on the diffusion itself, a large range of parameters of this method need to be examined. One of these parameters is the gradient strength and the duration of the gradient pulse. Clinical MRI has restrictions on gradient strengths and therefore new gradients to measure in an experimental environment are developed.

A new gradient coil has been designed and developed. This gradient coil can reach very linear magnetic gradient fields up to 300mT/m in a region of interest with a diameter of 8 cm. A new RF coil was designed and developed to transmit and receive the high frequency signals from the sample. Data acquisition and control have been done with the existing NMR software available at the Magnetic Resonance Laboratories (MRL) at the Eindhoven University of Technology (TU/e). New software procedures to measure diffusion have been implemented in this software.

The gradient coil and the RF coil have been calibrated and tested and the first diffusion measurements on water samples with a CuSO₄ solution have been performed.
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# List of abbreviations

<table>
<thead>
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<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>Surface area</td>
</tr>
<tr>
<td>ADC</td>
<td>Apparent Diffusion Coefficient</td>
</tr>
<tr>
<td>B/B₀/B₁</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>D</td>
<td>Diffusion Coefficient</td>
</tr>
<tr>
<td>E</td>
<td>Energy</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>G</td>
<td>Gradient strength</td>
</tr>
<tr>
<td>I</td>
<td>Current</td>
</tr>
<tr>
<td>L</td>
<td>Self-inductance</td>
</tr>
<tr>
<td>M/M₀/Mₓ</td>
<td>Magnetization</td>
</tr>
<tr>
<td>P</td>
<td>Power</td>
</tr>
<tr>
<td>PFG</td>
<td>Pulsed Field Gradients</td>
</tr>
<tr>
<td>R</td>
<td>Radius of a coil</td>
</tr>
<tr>
<td>R</td>
<td>Resistance of the coil</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of Interest</td>
</tr>
<tr>
<td>SJZ</td>
<td>Sint Jozeph Hospital, Veldhoven</td>
</tr>
<tr>
<td>TE</td>
<td>Echo time</td>
</tr>
<tr>
<td>TR</td>
<td>Repetition time</td>
</tr>
<tr>
<td>TU/e</td>
<td>Eindhoven, University of Technology</td>
</tr>
<tr>
<td>b</td>
<td>b-value</td>
</tr>
<tr>
<td>dₖ</td>
<td>distance from the kᵗʰ coil to the origin</td>
</tr>
<tr>
<td>l</td>
<td>length of the wire</td>
</tr>
<tr>
<td>Δ</td>
<td>interval time between the onset of the pulses</td>
</tr>
<tr>
<td>α</td>
<td>flip angle</td>
</tr>
<tr>
<td>δ</td>
<td>duration of one pulsed field gradient</td>
</tr>
<tr>
<td>ε</td>
<td>rise time of one pulsed field gradient</td>
</tr>
<tr>
<td>ϕ</td>
<td>phase shift</td>
</tr>
<tr>
<td>γ</td>
<td>gyromagnetic ratio</td>
</tr>
<tr>
<td>ϕ</td>
<td>phase</td>
</tr>
<tr>
<td>μ</td>
<td>magnetic moment</td>
</tr>
<tr>
<td>μ₀</td>
<td>the magnetic permittivity of vacuum, $4\pi \times 10^{-7} \text{ N/A}^2$</td>
</tr>
<tr>
<td>ρ</td>
<td>resistivity</td>
</tr>
<tr>
<td>ω₀</td>
<td>Larmor frequency</td>
</tr>
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</table>
Chapter 1

Introduction

1.1 Design and development of the gradient coils

Diffusion Weighted Imaging (DWI) is a relatively new Magnetic Resonance Imaging (MRI) technique. Contrast in conventional MR images is based on the difference in relaxation times of various kinds of tissue, whereas DWI is sensitive to microscopic motion of water molecules in a tissue.

Diffusion Weighted Imaging is becoming an important technique for the detection of certain pathologies, for example stroke. These pathologies can be characterized by a change in the diffusion.

An important feature of the in vivo use of DWI is that it needs to be safe and nondestructive. Developing such a technique starts with diffusion measurements on biological tissues with Nuclear Magnetic Resonance (NMR). Clinical NMR systems have restrictions. Some measurement techniques cannot be implemented because they require very fast changes in the magnetic field, which leads to spontaneous nerve stimulation. Besides, the available gradient strengths are low compared to experimental set-ups. Also the manufacturers of these systems keep the operating system inaccessible because of safety reasons. Experimental set-ups have a more open operating system and can deal with larger gradients but are mostly used and build for small objects or animals.

With the arrival of a clinical NMR set-up of 1.5T at the Technical University in Eindhoven (TU/e), a main magnetic field with a large volume became available. To do diffusion measurements with large gradient strengths up to 300mT/m on a large sample up to 10cm, new gradient coils were needed. In this report, the development of new gradient coils is discussed. Those coils are designed to generate large gradient fields on larger volumes than in the existing set-ups at the TU/e for measuring diffusion in biological samples. A set-up was built, using the main magnetic field of the 1.5T system, with the developed gradients coils and RF coil. Software was created to measure diffusion with so-called Pulsed Field Gradients.

The graduation research was performed at the Magnetic Resonance Laboratories (MRL) of the Department of Applied Physics at the Eindhoven University of Technology (TU/e).

1.2 Technology assessment

Disturbance of diffusion in biological tissue can indicate that a tissue is in distress. With the early detection of this disturbance, some tissue can be saved because medicines can be given earlier. Therefore it is necessary that measurement techniques for diffusion are developed and optimized. With the new set-up, the influence of the system parameters, such as gradient strength, on measurements can be investigated. Also the influence of system parameters and external factors (like temperature) on diffusion
measurements can be investigated. With this information, protocols to measure diffusion for clinical use can be improved.

1.3 Structure of this report

In chapter 2, the basics of MRI are discussed together with an explanation of diffusion measurements with MRI. Measurements that are done with existing coils are described. In chapter 3 the development of the gradient coils is explained and in chapter 4 it is described how these coils were tested and calibrated. Also the diffusion measurements with the new coils are discussed here. The last chapter the conclusions are drawn and recommendations are given.
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Chapter 2

Diffusion measurements with MRI

In this chapter the theory of MRI will be explained briefly. The principle of $T_1$ and $T_2$ decay will be explained, and the RF pulses and the sequences necessary to measure these decays. Finally a short introduction on diffusion measurements with MRI will be given.

2.1 Basics of MRI

2.1.1 Magnetic resonance

a. Precession of a magnetic moment

The magnetic moment $\mu$ (spin) of a nucleus is proportional to its mechanic momentum $b$:

$$\mu = \gamma b,$$

(2.1)

where $\gamma$ is the gyromagnetic ratio of the nucleus. For hydrogen $^1\text{H}$:

$$\frac{\gamma}{2\pi} = 42.58 \frac{MHz}{T}.$$

(2.2)

When a nucleus is placed in an external magnetic field $B_0$, the magnetic momentum of this nucleus will experience a torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}_0,$$

(2.3)

and since $\vec{\tau} = \frac{d\vec{b}}{dt}$:

$$\frac{d\vec{\mu}}{dt} = \gamma \frac{d\vec{b}}{dt} = \gamma (\vec{\mu} \times \vec{B}_0).$$

(2.4)

Equation 2.4 describes a precession of the magnetic momentum around $B_0$ with the so-called Larmor frequency:

$$\omega = \gamma B_0.$$

(2.5)

The precession of one spin placed in an external magnetic field $B_0$ is shown in figure 2.1a. Equation 2.5 is called the resonance condition because the nucleus can absorb energy of electromagnetic waves with the Larmor frequency. To explain Magnetic Resonance (MR), a group of spins will be considered because experiments are done with a large en-
semble of nuclei. All spins in the magnetic field $B_0$ will precess with the Larmor frequency, but all with a different phase. The result of all these spins is a net magnetization $M_0$ in the z-direction (because all net transverse magnetization will cancel out). This is schematically shown in figure 2.1b.

**b. Frame of reference**

Until now only the static (x,y,z) frame of reference is considered. For explanation purposes, a rotating frame of reference $(x',y',z)$ will be introduced. The z-axis is the same in both frames. In the rotating frame, the $x'$- and $y'$-axes rotate with the Larmor frequency with respect to the stationary (x,y,z) frame. In this rotating frame, the individual spins do not precess but the net magnetization is still parallel to $B_0$. For the spins in this rotating frame $B_0$ does not seem to be present.

c. RF-field

In the MR experiment, a second magnetic field $B_1$ is applied, perpendicular to the main magnetic field $B_0$. This $B_1$ field rotates around the z-axis in the (x,y,z) coordinate system with the Larmor frequency and it is a static field in the rotating frame of reference $(x',y',z)$. The net magnetization $M_0$ will then precess around this $B_1$ field, schematically shown in figure 2.1c. (When the frequency of the $B_1$ field is not close to the Larmor frequency, $B_1$ will vary rapidly and average out in the rotating frame of reference, and consequently it will not result in a rotation of $M_0$). The duration $t$ of this applied $B_1$ field determines the angle of rotation $\alpha$ of the total magnetization $M_0$. The flip-angle $\alpha$ can be calculated by:

$$\alpha = \omega \ t = \gamma \ B_1 t \ .$$

(2.6)

The application of the extra $B_1$ field is called the RF (Radio Frequency) pulse or the $\alpha$ pulse.

d. Bloch equations

The magnetization as function of time can be deduced from the Bloch equations:
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\[
\frac{dM_x}{dt} = \gamma M_y B_0 - \frac{M_x}{T_2} + \gamma (\vec{M} \times \vec{B}_1)_x , \quad (2.7a)
\]

\[
\frac{dM_y}{dt} = -\gamma M_x B_0 - \frac{M_y}{T_2} + \gamma (\vec{M} \times \vec{B}_1)_y , \quad (2.7b)
\]

\[
\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1} + \gamma (\vec{M} \times \vec{B}_1)_z , \quad (2.7c)
\]

where \( M_i \) is the net magnetization in the \( i \)-direction of all spins. \( M_0 \) is the initial magnetization of all spins in the \( B_0 \) field. \( T_1 \) is called the spin-lattice relaxation time or longitudinal relaxation time. This is the characteristic time for the total magnetization to recover to the original magnetization \( M_0 \) in the \( z \)-direction. \( T_2 \) is the spin-spin relaxation time, the characteristic time describing the dephasing of the transverse magnetization to zero. The magnetization as function of time after a 90° pulse can be solved from equation 2.7 (\( B_1 \) is then zero):

\[
M_x = M_0 \left( 1 - \exp \left( -\frac{t}{T_1} \right) \right) , \quad (2.8a)
\]

\[
M_y = M_0 \exp \left( -\frac{t}{T_2} \right) \sin(\omega t) , \quad (2.8b)
\]

\[
M_z = M_0 \exp \left( -\frac{t}{T_2} \right) \cos(\omega t) , \quad (2.8c)
\]

where \( t \) is the time since the RF pulse is turned off and \( \omega \) the Larmor frequency.

The rotating transverse magnetization induces a voltage in the RF coil, which is placed perpendicular to the xy-plane. This voltage oscillates with the Larmor frequency and decays with the time constant \( T_2 \) or \( T_2^* \) due to dephasing of all individual spins (figure 2.2).

**Figure 2.2 Rotating magnetization induces a voltage in the RF coil, placed perpendicular to the xy-plane.**

### e. \( T_1 \) and \( T_2 \) decay

Figure 2.3a and b show the longitudinal and transverse magnetization as a function of time in the rotating frame of reference. Usually, if the \( B_0 \) field is not completely homogeneous, the transverse magnetization decays faster than it would decay in a completely homogeneous \( B_0 \) field, because of more spin dephasing, and the characteristic time for this decay is then described by \( T_2^* \). \( T_1 \) is always longer than or equal to \( T_2 \). \( T_1 \) and \( T_2 \) are characteristic for each type of material.
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2.1.2 Free induction decay (FID), spin echo's, and pulse sequences

Different RF pulses with different flip-angles and different interval times can be applied, which will result in so-called spin echoes. Spin echoes, free induction decay and several sequences will be discussed in this section.

a. Free induction decay and spin echo in the Hahn sequence

For an ensemble of spins and only $B_0$ applied, the net magnetization $M_0$ of all spins will be in the z-direction. A $90^\circ$ pulse will force the net magnetization in the x'y'-plane (figure 2.4a). The rotation of the net magnetization around the z-axis induces a signal in the receiving coil and this is called the Free Induction Decay (FID). Immediately after this pulse, the transverse magnetization will decay, due to dephasing of the individual spins. After some time $\tau$, the spins are partly dephased (figure 2.4b), and no net magnetization in the transverse plane is remaining. Then a second RF-pulse, a $180^\circ$ pulse, is given to invert the spins in the transverse plane (figure 2.4c). As a consequence every spin rotates in the other direction (figure 2.4d) and the fast and the slow rotating spins will have the same phase again after time $2\tau$ (figure 2.4e). This is called the spin-echo. The time $TE=2\tau$ is called the echo time. Schematically, this Hahn pulse sequence is presented in figure 2.5. It is a time diagram, in which the RF pulses and the signals are shown on a time axis. At $t=\tau$ after the $90^\circ$ pulse, a $180^\circ$ pulse is applied and at $t=2\tau$, the spin echo occurs. The exponentially decaying line in the figure indicates that the signal decays with a characteristic time $T_2$. The signal intensities of the spin echo is given by:

$$S \propto M_0 \left(1 - \exp\left(-\frac{TR}{T_1}\right)\right) \exp\left(-\frac{TE}{T_2}\right),$$

where $TR$ is the repetition time when the sequence is repeated.
Figure 2.4 Hahn pulse sequence in the rotating frame of reference; (a): The 90° pulse rotates the total magnetization into the \( x'y' \)-plane, (b): due to the magnetic field inhomogeneities the individual spins will dephase, (c): at \( t=\tau \) the 180° pulse is applied, (d): the spins will rephase, (e): spin-echo and maximal signal will occur when the spins are all in phase again at \( t=2\tau \).

Figure 2.5 Hahn pulse sequence. The RF pulses and the induced signals are shown on one axis. First a 90° pulse is applied, the FID occurs and at \( t=\tau \) the 180° pulse is applied. At \( t=2\tau \) the spin echo occurs.

b. Carr-Purcell-Meiboom-Gill sequence (CPMG)

This sequence consists of a 90° pulse followed by a number of 180° pulses (figure 2.6). Because of the multiple 180° pulses, the spins will rephase and dephase after each 180° pulse, creating multiple spin echoes. The intensity of the echoes will decrease as a function of time. This intensity decays with the characteristic time \( T_2 \) (equation 2.8a and b).
2.1.3 Gradients

a. Basics
Gradient fields in MRI are linearly in space varying magnetic fields superimposed on the $B_0$ field and are used to spatially encode the MRI regions of interest. Gradients are usually applied in three directions:

$$G_x = \frac{dB_z}{dx},$$  \hspace{1cm} (2.10a)  
$$G_y = \frac{dB_z}{dy},$$  \hspace{1cm} (2.10b)  
$$G_z = \frac{dB_z}{dz},$$  \hspace{1cm} (2.10c)  

and the total magnetic field becomes:

$$B(\vec{r}, t) = B_0 + G_x(t)x(t) + G_y(t)y(t) + G_z(t)z(t) = B_0 + \vec{G}(t) \cdot \vec{r}(t).$$  \hspace{1cm} (2.11)  

The Larmor frequency $\omega_L(\vec{r}, t)$ of each individual spin becomes position dependent:

$$\omega_L(\vec{r}, t) = \gamma B_0 + \gamma \vec{G}(t) \cdot \vec{r}(t),$$  \hspace{1cm} (2.12)  

with $\gamma B_0 = \omega_L$ is the Larmor frequency. The phase of each spin can be calculated according to:

$$\varphi(\vec{r}, t) = \int_0^t \omega(\vec{r}, t') \, dt',$$

$$\varphi(\vec{r}, t) = \gamma \int_0^t [B_0 + \vec{r}(t') \cdot \vec{G}(t')] \, dt',$$  \hspace{1cm} (2.13)  

$$\varphi(\vec{r}, t) - \varphi(0, 0) = \gamma \int_0^t \vec{r}(t') \cdot \vec{G}(t') \, dt'.$$
If gradients are applied, spins at different positions will have different phases and frequencies. In the received signal, the phase and frequency information can be used to calculate the position of the spins. The intensity of the signal is proportional to the number of spins on that position. In the following paragraphs it will be explained how full 3D spatial encoding can be achieved using 3 gradients; the slice selection gradient, the frequency encoding gradient and the phase encoding gradient.

b. Slice-selection gradient
The total magnetic field, if one gradient $G_z$ in the $z$-direction is applied, is:

$$B = B_0 + G_z z,$$

(2.14)

with $B_0$ the external magnetic field and $z$ the $z$-coordinate of the particle. The frequency of each spin depends on the $z$-position. If the gradient $G_z$ is applied during a $90^\circ$ pulse, only the spins with the Larmor frequency close to the $B_1$ RF frequency will rotate into the $x'y'$-plane. Figure 2.7 shows the linearly dependent $B_z$ field in the $z$-direction as a function of position. Since a finite pulse will always contain a band of frequencies, a finite slice thickness will be excited by the $90^\circ$ pulse.

The width of the frequency band and the gradient strength therefore determine the thickness of the slice.

![Figure 2.7 Slice selection; only the spins between $z_1$ and $z_2$ have the correct Larmor frequency and will rotate into the $x'y'$-plane when a $90^\circ$ pulse is given while the $z$-gradient $G_z$ is applied.]

---

c. Frequency encoding and phase encoding gradient
The frequency and the phase encoding gradient will be explained for completeness but not in extensive detail. More information can be found in [VLA99]. After slice selection, all spins in the selected slice are in the $x'y'$-plane. Although the slice has a non-zero thickness, it will be considered as two-dimensional. Consider in figure 2.8 nine spins in one slice. After the RF pulse accompanied by the slice selection gradient all the spins have the same frequency $\omega$ (figure 2.8a). When an $x$-gradient is applied, the frequencies of the spins will depend on the $x$-position (figure 2.8b). This $x$-gradient is applied during the acquisition of the echo, which will encode the second dimension. The $x$-gradient is usually called the read-out gradient.

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The third dimension is encoded by applying a gradient in the y-direction during a short period of time. This induces an y-position dependent phase shift. The y-gradient is also called the phase encoding gradient.

Figure 2.8 Frequency and phase encoding gradient, (a): after the slice selection all spins have the same frequency $\omega$; (b): x-gradient makes the frequency x-position dependent; (c) the y-gradient is applied during a short period of time which will induce a phase shift in the y-direction (d) to encode the third dimension.

**d. Time diagram and k-space**

Figure 2.9 shows a time diagram for a Hahn pulse sequence if gradients for imaging are used. The first axis shows the RF pulses and the echo as a function of time. On the other three axes, the applied gradients are shown as function of time. The height of the gradient blocks represents the strength of the gradients.

The transverse magnetization in a slice after the gradients are applied can be written as:

$$M_{xy}(t) = \int_{xy} m(x, y) \exp(-i(\omega t + \varphi)) dx dy = \int_{xy} m(x, y) \exp(-i(k_x x + k_y y)) dx dy, \quad (2.15)$$
where $\omega$ and $\varphi$ are the frequency and phase encoding, and $m(x,y)$ the number of spins as a function of position. It is readily seen in eq.2.15 that $m(x,y)$ can be extracted by Fourier transformation. More details can be found in [VLA99].

![Figure 2.9 Time diagram of a Hahn sequence with gradients.](image)

**Figure 2.9** Time diagram of a Hahn sequence with gradients. The first axis represents the RF pulses and the echo. The other three axis show the gradients in the z-, x- and y-direction.

### 2.2 Diffusion Magnetic Resonance Imaging

#### 2.2.1 Mathematical description of diffusion

Molecular diffusion is the result of thermal, so-called Brownian motion. This type of motion results in random displacements of molecules. In nonuniform systems, the classical first Fick's law describes diffusion [JOS60]. The diffusion coefficient $D$ is introduced as a constant, which characterizes the change of concentration of one type of molecule as a function of time. When the molecules are uniformly distributed, no macroscopic variation can be seen, but there are still random displacements, which is called self-diffusion. In that case other approaches must be used to evaluate the diffusion. The probability of finding a molecule, started at a position $\mathbf{r}_0$, at a position $\mathbf{r}$ at a time $t$ can be calculated. This displacement $\mathbf{r} - \mathbf{r}_0$ follows a Gaussian distribution, which is expressed in Einstein's equation:

$$\left<(r - r_0)^2\right> = 6Dt,$$

(2.16)

where $t$ is the time and $D$ the diffusion coefficient. Because of the microscopic structural heterogeneity of living tissue, the diffusion of water through a tissue is a direction-dependent quantity that can be described by a symmetric tensor $\mathbf{D}$ containing six independent elements [FIN00] ($D_{xy}=D_{yx}, D_{xz}=D_{zx}, D_{yz}=D_{zy}$):
The trace or the so-called isotropic Apparent Diffusion Coefficient (ADC):

\[
ADC = \frac{1}{3} \text{Trace}(D) = \frac{1}{3} (D_{xx} + D_{yy} + D_{zz}).
\]  

In this form the ADC may be used to obtain a so-called isotropic diffusion weighted image.

2.2.2 MR sequences and diffusion imaging

The most widely used method for measuring diffusion with MRI is the Pulsed Field Gradient (PFG) method, incorporated into a Hahn spin-echo sequence. Figure 2.10 shows a time diagram of this PFG sequence. The first axis represents the RF pulses that are given and the echo that will be measured. The second to the fourth axis represent the gradients that are applied in the z, x and y direction. On the fifth axis, the pulsed field gradients are shown which may be applied in any direction.

The purpose of the PFG’s is to magnetically label the spins. Diffusion of labeled spins causes phase dispersion, which leads to signal loss in the echo. As shown in figure 2.10, G is the gradient strength, \(\delta\) the gradient duration and \(\Delta\) the time between the onset of the two pulses. Immediately after the first pulsed field gradient the spins have acquired a phase shift, \(\phi_1\):

\[
\phi_1 = \gamma z_1 G\delta ,
\]  

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where \(z_1\) is the position of the spin along the diffusion encoding axis. Also the second pulsed gradient produce a phase shift, \(\phi_2\):

\[
\phi_2 = \gamma z_2 G \delta .
\]  

(2.20)

Because of the presence of the 180° pulse, the net phase shift is the difference between \(\phi_1\) and \(\phi_2\):

\[
\phi = \phi_1 - \phi_2 = (\gamma G \delta)(z_1 - z_2).
\]  

(2.21)

![Diagram showing the effect of PFG on static, flowing, and diffusing spin](image)

**Figure 2.11** Effect of Pulsed Field Gradients on spins from static, flowing and diffusing particles. If particles are static, the spins in a voxel will defocus during the first lob and completely refocus during the second lob. Spins from moving particles will also defocus and refocus completely, but because of the movement of all particles in the same direction, the particles are subject to another gradient strength during refocusing than during defocusing. Spins of diffusing molecules defocus completely but will not refocus completely because of the random displacement of the molecules.

For spins of static molecules, where \(z_1 = z_2\) the net phase shift in a voxel is zero, for spins from flowing molecules there will be a net phase change unequal to zero. For spins of diffusing particles the phase shift will be zero in a voxel but due to diffusion incomplete refocusing occurs, leading to signal loss. This is illustrated schematically in figure 2.11. Diffusion leads to attenuation of the spin-echo signal. \(S_0\) is the signal of the spin-echo when no gradients are applied, according to equation 2.9, and \(S\) the signal acquired when the PFG’s are applied. The difference in attenuation between \(S_0\) and \(S\) is due to diffusion and can be used to calculate the diffusion coefficient [VLA99]. The attenuation of this signal can be described according to:

\[
\frac{S}{S_0} = \exp(-bD),
\]  

(2.22)

where \(D\) is the diffusion coefficient and \(b\) the gradient factor, a sequence parameter from the set-up. The \(b\)-value is defined as:

\[
b = \int_0^{TE} [k(t)]^2 dt,
\]  

(2.23)
with

\[ k(t) = \gamma \int_0^t G(t')dt' \tag{2.24} \]

where \( G(t) \) is the gradient strength as function of time.

For the PFG's as shown in figure 2.12a; duration \( \delta \) and amplitude \( G \), with a time \( \Delta \) between the onset of the pulses, the b-value is given by [NIC00]:

\[ b = \gamma^2 G^2 \delta^2 \left( \Delta - \frac{1}{3} \delta \right) \tag{2.25} \]

For trapezoidal shape gradients (see figure 2.12.b), the b-value becomes [NIC00]:

\[ b = \gamma^2 G^2 \left[ \delta^2 \left( \Delta - \frac{1}{3} \delta \right) + \frac{1}{30} \varepsilon^3 - \frac{1}{6} \delta \varepsilon^2 \right] \tag{2.26} \]

where \( \varepsilon \) is the gradient rise time for the trapezoidal waveform. These equations (2.22-2.26) are valid only for diffusion in an infinite and homogeneous medium. If diffusion is restricted, e.g. by (im)permeable barriers in the cell membrane in biological tissue, the signal attenuation must be calculated using a more extensive formalism, taking into account each particular situation.

A variation on this technique is the gradient echo technique. In this sequence no 180° pulse is needed to switch the polarity of the gradients. A negative lob immediately follows the positive lob (figure 2.13). In this way much shorter diffusion times can be used. Equations 2.23-2.26 are still valid for the attenuation in the spin echo signal. More information can be found in [LEB89].
Using the gradient echo technique method extra echo attenuation, due to incomplete spin refocusing, can occur due to mismatch between the two gradients. This mismatch easily occurs in the presence of background gradients or Eddy currents. Tissues with short $T_2$ require short TE, which may not allow diffusion times long enough to see signal attenuation. Therefore another technique is used: stimulated echo technique. See [HAH50] for more detail.

Other sequences, like constant field gradient spin echo and stimulated spin echo, are discussed in [CAR54] and [HAH50]. Diffusion weighted imaging is described in [VLA99].

2.2.3 Experimental considerations in diffusion MR

The requirements discussed in this section are problems concerning diffusion measurements. Additional requirements that arise with in vivo diffusion weighted imaging are not discussed in this graduation report.

a. Technical requirements

As shown in the previous sections, the b-value is an important factor in diffusion MRI. This value is proportional to the square of the gradient strength (eq. 2.25 and 2.26). Therefore the gradient strength has to be well-defined and accurate for good diffusion measurements. Rapidly switching of the gradients can induce Eddy currents in the surrounding metals, e.g. the bore of the main magnet system. These Eddy currents produce field gradients that add to the diffusion gradient. To solve this problem is to remove as good as possible all Eddy currents by using actively shielded gradient coils. Also the influence of the Eddy currents can be limited by using gradients with small dimensions (the coil is in this situation not in the neighborhood of any material where Eddy currents can be induced). Conturo et al.[CON95] describes systematically which errors can occur in ADC measurements. One of the important error sources is the shape and the timing of the diffusion gradients. Errors in gradient amplitude, direction and linearity could combine to contribute up to $\sim 10\%$ inaccuracy in the diffusion measurements. Good calibration of the gradient strength therefore is necessary for accurate diffusion measurements.

b. Diffusion in biological systems

Biological systems are heterogeneous and made of multiple subcompartments. The transport between the compartments (microdynamics) should be considered when the MR signals are interpreted. A classical interpretation of the NMR signal may not properly reflect tissue structure or properties. The main difficulty is that the detailed structure of the medium is generally unknown and modeling is required. It would lead too far to go into de-
tail in this report, but for interpreting the measurements one should be aware that restricted diffusion can be present.

On the other hand, a bulk ADC can still be measured.

2.3 Diffusion measurements with existing coils

a. Set-ups used for diffusion measurements

Diffusion measurements were done at two different set-ups: the 6.3T system at the TU/e and the 1.0T system at the SJZ. The 6.3T system is made to perform NMR on small samples in a scientific environment. The 1.0T system is standing in a hospital and is intended to do NMR on humans. The sample used for diffusion measurements with the 6.3T system was water with 0.01M CuSO₄ and diffusion was measured using the PFG technique. For the system at the SJZ the sample was a proportion of the following: 1000 ml deminewater, 770 mg CuSO₄·5H₂O, 2000 mg NaCl, 1 ml arquad (1% solution), 0.05 ml H₂SO₄ - 0.1 N solution. Arquad is a substance that makes sure that no algae will grow in the solution. This reference fluid has the same ADC as water but a shorter T₁ and T₂ time. The method used to measure diffusion was an adapted form of the PFG technique implemented by the manufacturers of the system. Table 2.1 gives an overview of the measurements that are done.

<table>
<thead>
<tr>
<th>System</th>
<th>TR (ms)</th>
<th>δ (ms)</th>
<th>Δ (ms)</th>
<th>Gradient (mT/m)</th>
<th>b-value (10⁶ s/m²)</th>
<th>ADC (10⁻⁹ m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SJZ 1.0T</td>
<td>1000</td>
<td>5-20*</td>
<td>75-150*</td>
<td>0-21*</td>
<td>100-6400</td>
<td>1.12-2.96</td>
</tr>
<tr>
<td>TU/e 6.3T</td>
<td>1000</td>
<td>0.5-3</td>
<td>7.5-140</td>
<td>0-250</td>
<td>0-171</td>
<td>1.79 +/-</td>
</tr>
</tbody>
</table>

Table 2.1 Overview of the performed experiments at existing set-ups, TE is the echo time, TR the repetition time and ADC the Apparent Diffusion Coefficient. * The variation of δ, Δ and G at the SJZ was done indirectly by varying the b-value.

The ADC was calculated as explained in section 2.2.2. Figure 2.14 shows a measurement at the SJZ with a maximum b-value of 1600 10⁶ s/m². On the horizontal axis, the b-values are and on the vertical axis the logaritme of the signal strength devide by the zero measurement as a function of the b-values are set. The slope of the interpolated line is the ADC of the sample and is 2.05 10⁻⁹ m²/s.

Figure 2.14 Measurement at the SJZ with a maximum b-value of 1600 10⁶ s/m². The slope of the trendline is the ADC of the measured sample and is 2.05 10⁻⁹ m²/s. The R² value of the interpolation is 0.9994.
b. Conclusions
As shown, diffusion measurements can be performed with the existing coils, however it is not possible to measure with high b-values and high gradient fields on a larger sample. At the SJZ, the following restraints turn up. First, new pulse sequences to measure diffusion can not easily be implemented. Second, above b-values of $3000 \times 10^6$ s/m$^2$, the signal could not be distinguished from the noise. Third, the gradients can only reach up to 21 mT/m.

With the 6.3 T system the parameters can be implemented separately, the gradient can reach up to 380 mT/m. The only disadvantage was the small spherical volume that can be used to insert samples.

In conclusion, we want a gradient coil with a large volume, which can reach high gradients. Also we want to be able to operate the whole system, implementing our own sequences. Considering all possibilities the conclusion was to make a new gradient coil which could fit in the 1.5T set-up, available at the TU/e. The main magnetic field of the 1.5 T will be used, the software of other set-ups at the TU/e could be adapted and used and a new gradient coil and an RF coil should be developed, build and tested.

In the next chapter the development of the new gradient coils will be discussed.
2. Diffusion measurements with NMR
Chapter 3

Development of the gradient coils and the experimental set-up

In this chapter, first the required specifications for the gradient coils will be discussed, including the calculations and the expected performance of these coils. In the last section the total experimental set-up for diffusion measurements will be discussed.

3.1 Specifications

a. Purpose

The new gradient coil will be used to investigate the influence of the PFG parameters on the diffusion measurements for a large range of b-values. Diffusion measurements will be performed with the discussed pulsed field gradients technique (see section 2.11.a). This will be done for small b-values (<2000s/mm²) and for larger b-values. An overview of existing coils is given in Table 3.1. These coils are present at the TU/e at the MRL group and at the Sint Jozeph Hospital (SJJZ) in Veldhoven.

<table>
<thead>
<tr>
<th></th>
<th>G (mT/m)</th>
<th>Δ (ms)</th>
<th>ε (ms)</th>
<th>δ (ms)</th>
<th>directions</th>
<th>Bore diameter (cm)</th>
<th>ROI (diameter in cm)</th>
<th>B₀ deviation in the ROI (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TU/e 1.5T</td>
<td>0-21</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>any</td>
<td>70</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>TU/e 6.3T</td>
<td>0-380</td>
<td>any</td>
<td>0.09</td>
<td>any</td>
<td>any</td>
<td>9.5</td>
<td>8</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>SJZ 1.0T</td>
<td>0-21</td>
<td>9.8-140</td>
<td>0.4</td>
<td>0-65</td>
<td>any</td>
<td>70</td>
<td>30</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.1 Overview of parameters of existing coils at the TU/e and at SJZ.

*: No PFG available at this machine.

b. Available components

For the design of the set-up, several boundary conditions have to be taken into account.

First, the B₀ field of the ACS scanner available at the TU/e will be used, which is 1.5T. The diameter of the opening is 70 cm. The total set-up of the coils, with water-cooling if necessary should fit within the physical space that is available. The most powerful amplifier available is COPLEY 265MS which can deliver a peak current of 300A. A Techron 7700 series was used to do the first measurements. This amplifier can deliver a peak current of 180A and if placed in series with two other 7700 series amplifiers 500A.

c. Parameters G, δ, ε and Δ

To be able to investigate the influence of system parameters on diffusion measurements, a large range of parameters δ, Δ and G has to be used.

• To be able to investigate diffusion dependency on cell membrane permeability, Δ should range from <1.5ms to 200ms for a larger range of b-values.
The minimal rise time $\varepsilon$ is dependent on the load (resistance, inductance, total energy stored in the coils) of the amplifier and will be dependent on the design of the coils. For small pulses, $\varepsilon$ needs to be small. To be able to generate pulses with a pulse width of $1\text{ms}$, $\varepsilon$ should at least be as small as $0.4\text{ ms}$, which is the rise time for the $1.5\text{ T}$ magnet at the SJZ. The rise time $\varepsilon$ can be approximated by:

$$
\varepsilon = \frac{1}{2} \frac{E}{I_{\text{max}} V_{\text{max}}},
$$

(3.1)

where $E$ the total energy stored in the coils, $I_{\text{max}}$ the current running through the coils and $V_{\text{max}}$ the voltage through the coils.

The gradient strength depends on the total amount of current through the coils (section 3.2). The maximum current from the available amplifier is $300\text{ A}$. To be able to use pulses with a gradient strength as high as possible, the maximum current will be used.

d. Dimensions
The dimensions of the sample should be larger than existing high gradient coils and should fit into the ACS scanner, available at the TU/e. This scanner has a stable magnetic field within $5\text{ ppm}$ Diameter Sperical Volume of $30\text{cm}$. The total set-up has to fit into the bore of the magnet, which has a diameter of $70\text{cm}$.

e. Linearity/homogeneity
The linearity of the gradients in the clinical scanner is $99\%$. The linearity of the new gradients is chosen to be at least $97\%$. With this linearity, a well-defined slice selection can still be done. The better this linearity, also the better the ADC can be defined. A deviation of $3\%$ in the gradient results in a deviation of about $10\%$ in the ADC-value (see eq.2.25).

f. Used sequences
The gradients will be used to measure diffusion with PFG implemented in a Spin Echo (Hahn) sequence.

g. Overview
Table 3.2 gives an overview of the specifications.

<table>
<thead>
<tr>
<th>$G$ (m$\text{T/m}$)</th>
<th>$\Delta$ (ms)</th>
<th>$\varepsilon$ (ms)</th>
<th>$\delta$ (ms)</th>
<th>ROI (diameter in cm)</th>
<th>Linearity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0...300$</td>
<td>$\delta+2\varepsilon&lt;1.5$</td>
<td>$&lt;0.4$</td>
<td>$1$</td>
<td>$&gt;5$</td>
<td>$97%$</td>
</tr>
</tbody>
</table>

Table 3.2 Overview of the specifications

3. Development of the gradient coils and the experimental set-up
3.2 Development of the z-gradients

3.2.1 Gradient field: optimization problem

A gradient field is a magnetic field which magnitude depends linearly on the position along the z-axis. The magnetic field can be calculated with the Biot-Savart law:

\[ \vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \vec{r}}{r^2} dl, \]  

(3.2)

where \( \vec{B} \) is the magnetic field, \( \vec{I} \) the current running through the conductor, \( r \) the distance from the current through the conductor to the point where the magnetic field is calculated and \( \mu_0 \) is the magnetic permittivity of vacuum, \( 4\pi \times 10^{-7} \, \text{N/A}^2 \). The gradient field along the z-axis is described by:

\[ G_z = \frac{\partial B}{\partial z}, \]  

(3.3)

For the design of the gradient, one has a large number of parameters that can be varied, such as the number of the coils, distance between the coils and radius of the coils. To get insight the influence of each parameter, several simple designs will be discussed first. Calculations in sections 3.2.1a and 3.2.1b are carried out for a radius R of 10 cm and a current I of 300 A.

a. One coil

Figure 3.1a shows schematically a coil of one loop. The magnetic field on the z-axis is described by:

\[ B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}}. \]  

(3.4)

with \( R \) the radius of the loop and \( z \) the distance from the center of the loop to the point were the magnetic field is calculated. The gradient field on the z-axis is:

\[ G(z) = -\frac{3}{2} \frac{\mu_0 I R^2}{(z^2 + R^2)^{3/2}} \frac{z}{(z^2 + R^2)^{1/2}}. \]  

(3.5)

The magnetic field and the gradient field along the z-axis are shown in figure 3.1b and figure3.1c. A constant gradient can not be reached with one coil.
Design of gradient coils to measure diffusion with NMR

Figure 3.1a Configuration of one coil with radius $R$, and current $I$

Figure 3.1b Calculated magnetic field along the $z$-axis for $I=300$A and $R=0.1$m

Figure 3.1c Calculated gradient along the $z$-axis for $I=300$A and $R=0.1$m

b. Two coils

A second example is a design with two coils placed symmetrically with respect to the origin. The current through the coils is running in opposite directions with the same magnitude. The magnetic field is:

\[ B(z) = \frac{\mu_0}{2} IR^2 \left( \frac{1}{(R^2 + (z-d)^2)^{3/2}} - \frac{1}{(R^2 + (z+d)^2)^{3/2}} \right), \quad (3.6) \]

where $R$ is the radius of each coil and $d$ the distance from the origin to the coils. And the gradient field along the $z$-axis is:

\[ G(z) = -\frac{3}{2} \mu_0 IR^2 \left( \frac{(z-d)}{(R^2 + (z-d)^2)^{5/2}} - \frac{(z+d)}{(R^2 + (z+d)^2)^{5/2}} \right). \quad (3.7) \]
The configuration is shown in figure 3.2a. The magnetic field and gradient field are shown in figure 3.2b. The distance from the coils to the origin is the same as the radius, 10 cm. R.Blok [BLO99] shows that if R=d, a gradient field as high and as linear as possible can be obtained for this configuration. The obtained gradient field does not meet the requirements. The gradient strength is only 20 mT/m and is only 95% linear in a region of 6 cm.

![Figure 3.2a Configuration of two coils with radius R and at a distance of d from the origin](image)

![Figure 3.2b Calculated magnetic field along the z-axis for two coils with d=0.10m, R=0.10m and I=300A](image)

![Figure 3.2c Calculated gradient along the z-axis for two coils with d=0.10m, R=0.10m, and I=300A](image)

c. Generalization

For 2N coils, the design for an example for N=7 is shown in figure 3.3. To obtain a symmetric gradient field, the design is such that the left-hand side is the mirror view of the right-hand side. All coils have the same radius and current running through it. The direction of the current is opposite for the left-hand and right-hand sides. The magnetic field can be expressed by:

\[
B(z) = \frac{\mu_0}{2} IR^2 \sum_{k=1}^{N} \left( \frac{1}{\left(R^2 + (z - d_k)^2\right)^{3/2}} - \frac{1}{\left(R^2 + (z + d_k)^2\right)^{3/2}} \right), \tag{3.8}
\]

and the gradient field along the z-axis is:

\[
G(z) = -\frac{3}{2} \mu_0 IR^2 \sum_{k=1}^{N} \left( \frac{(z - d_k)}{\left(R^2 + (z - d_k)^2\right)^{5/2}} - \frac{(z + d_k)}{\left(R^2 + (z + d_k)^2\right)^{5/2}} \right). \tag{3.9}
\]
Figure 3.3 Example of a general design for 2N coils with N=7, with radius R and distance to the origin dk for the kth coil, current I through each coil, the direction of running is opposite for left-hand side and right-hand side.

The parameters used to obtain a gradient field along the z-axis in this configuration are:

1. I: the current through the coils
2. R: the radius of the coils
3. N: the total number of coils on one side
4. dk: the distance of each coil k to the origin

This configuration with all coils with the same current and radius was chosen for several reasons. This relative simple configuration generates the most ideal gradients. Also technically it is convenient to make the coils all of the same size and let the same current run through it.

Using a Maple program, these listed parameters were varied to investigate the influence on the gradient field. The gradient strength at point z=0 is taken as the reference gradient strength. To characterize the linearity of the gradient, the deviation of the gradient strength is introduced. The deviation is the largest relative difference of the gradient strength in the region from z=-d1 until z=d1.

Some rules of thumbs are found by varying these parameters in our model: (Some of these conclusions can also be deducted directly from equation 3.9.)

1. the gradient strength is:
   a. proportional to the current I,
   b. proportional to the radius R,
   c. proportional to the total number of coils 2N,
   d. inverse proportional to the distance of each coil to the origin

2. the linearity is:
   a. inverse proportional to the radius R,
   b. proportional to the distance of each coils to the origin.

In this simplified model, for a first approximation, important parameters were omitted such as:
- the thickness of the coil wires,
- the total energy stored in the coils,
- the resistance of the total set-up,
- the inductance of the total set-up,
- the eddy currents that can be induced in the surrounding material.

All these parameters should be included in the optimization. The purpose of the graduation assignment was not to make a realistic model and therefore literature was searched and Philips Medical System was contacted to help with this optimization.
3.2.2 Solutions from literature

Few articles deal with our requirements: a high gradient field together with a large ROI is an unusual combination. Suits [SU189] describes a configuration of 16 coils to obtain a homogeneous field with a linearity of 97% in a ROI of 14cm. Fig.3.4 shows the magnetic field and the gradient field calculated by Maple with the parameters from [SU189]. The current used is 300 A, the radii of the coils 10 cm. 1 coil is placed at 0.44 times the radius of the coil (4.4 cm) from the origin and 7 other coils at 1.19 times the radius of the coils (11.9 cm) from the origin. The original parameter settings are given in table 3.3. A gradient field of 114 mT/m with a linearity of 97% in the ROI from -0.07 m till +0.07m is obtained.

No literature could be found describing a system with gradient strength of 300mT/m with a linearity of 97% in a ROI of 10cm.

<table>
<thead>
<tr>
<th>Amp-turns - n/n2</th>
<th>solution from [SU189]</th>
<th>values used in Maple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>R</td>
<td>0.1 m</td>
</tr>
<tr>
<td>distance b1</td>
<td>0.44 R</td>
<td>0.044 m</td>
</tr>
<tr>
<td>distance b2</td>
<td>1.19 R</td>
<td>0.119 m</td>
</tr>
<tr>
<td>Current</td>
<td>I1 = I2</td>
<td>300 A</td>
</tr>
</tbody>
</table>

Table 3.3 Solution by [SU189]

3.2.3 Solution from STREAM: working with constraints

Together with Philips Medical Systems we designed a set of gradients, capable of achieving the requirements. A fast iterative computer program, called STREAM, developed by Geran Peeren from Philips Medical Systems in Best, was used to obtain this result. This program can give the current distribution as a function of position when the desired gradient field in the z-direction is given. The number of coils can be obtained by dividing the total current by the 300 A that is available from the amplifier.

The program uses a stream function. A brief introduction in the program follows, for extended information, I like to refer to Philips Medical Systems. The program opti-
mizes a stream function that will lead to the current distribution. Four main constraints are used in the optimization:
1. the gradient field in the central volume,
2. the stored magnetic energy,
3. the resistance and dissipation,
4. the field of currents induced in the surrounding materials.

The optimization is a fast iterative process. The stream function gives a gradient field close to the desired gradient field. As long as the difference of the desired field and that of the stream function is larger than the four constraints, the iteration goes on. The optimization parameters will be varied in such a way that the program is much faster than most other iterative programs.

In our situation, the most important constraint is the gradient field in the central volume. The stored magnetic energy and the resistance and dissipation should be low enough that the coils can still be cooled. The field of induced currents in the surrounding materials was not a big concern because the closed surrounding in the final set-up will be far away from the surrounding magnet and its influence will be negligible.

For the desired gradient field, the program calculated a current distribution of 11300 A. This means that there need to be 38 coils of about 290 A because the available amplifier can only deliver 300 A. The program calculated the position of all these coils and this is listed in appendix A. Table 3.4 gives a short overview of the output of the program.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of imaging sphere</td>
<td>0.0500 m</td>
</tr>
<tr>
<td>Total gradient length</td>
<td>0.387 m</td>
</tr>
<tr>
<td>Number of coils</td>
<td>38</td>
</tr>
<tr>
<td>Radius of coil</td>
<td>0.0995 m</td>
</tr>
<tr>
<td>Amperes running through each coil</td>
<td>290 A</td>
</tr>
<tr>
<td>Thickness wire</td>
<td>0.001 m</td>
</tr>
<tr>
<td>Self-inductance</td>
<td>118 µH</td>
</tr>
<tr>
<td>Resistance</td>
<td>13 mΩ</td>
</tr>
<tr>
<td>Stored energy</td>
<td>5 J</td>
</tr>
<tr>
<td>Dissipation</td>
<td>1100 W</td>
</tr>
</tbody>
</table>

*Table 3.4 Solution from STREAM*

The thickness of the wires taken was assumed 1 mm, which is not sufficiently thick enough to carry 290 A (this will be dealt with in section 3.2.4). The linearity of the solution (97% within the ROI) is shown in figure 3.5 and figure 3.6 shows the ROI in which the gradient is 300 mT/m.
Design of gradient coils to measure diffusion with NMR

Figure 3.5 Linearity of the solution by STREAM. The horizontal axis is the z-axis, the vertical the x-axis. The figure is cylindrical symmetric around this axis. The vertical line through z=0 represents the 0 mT line, the following line to the right is the line where the magnetic field is 10 mT. The line at z=0.02 m is the line where the magnetic field is 60 mT and at z=0.05 m the magnetic field is 150 mT.

Figure 3.6 Gradients contours as a function of z and x. In the inner region the gradient is between 300 mT/m +/- 3% and 290 mT/m +/- 3%.

3. Development of the gradient coils and the experimental set-up
In figure 3.7 the solution calculated with Maple is shown, it shows the magnetic and gradient field along the z-axis. The gradient field in the point \((0,0,0)\) as calculated by the Maple program is 303 mT/m.

\[
\text{Figure 3.7a Calculated magnetic field, with the parameters obtained by STREAM and listed in table 3.4 and appendix A}
\]

\[
\text{Figure 3.7b Calculated gradient with the parameters obtained by STREAM and listed in table 3.4 and appendix A.}
\]

3.2.4 The final design

In this section the final design will be described, together with some expected values for the heat capacity, forces, stored energy and rise times. These values are estimations and they will have to be measured in the final design.

a. Changes on the STREAM design
The current distribution of the STREAM program was used to design the gradient coils. The thickness of the wires has been changed because heat dissipation in the wires can be a problem. Copper wire can be air cooled if the current is less than 3 A/mm\(^2\) and water-cooled if the current is less than 10 A/mm\(^2\). A current of 290 A which is air-cooled needs a wire diameter of 3.57 mm if we assume a duty-cycle of 10\%. We have chosen for a wire thickness of 3.12mm because this is the thickest wire that can be used in the design, otherwise the wires will overlap (see appendix A). This will be no problem because the duty cycle of 10\% can easily be lowered and water cooling can be used if necessary.

The resistance of the coils is described by:

\[
R = \rho \frac{l}{A}, \quad (3.10)
\]

with

\[
\rho = 17 \times 10^{-9} \Omega m, \quad (3.11)
\]
the resistivity of copper, \( l \) the length of the wire and \( A \) the surface area of the wire. The dissipated power can be calculated as follows:

\[
P = I^2 R = 290^2 A^2 \times 0.055 \Omega = 4626 \text{W}.
\]  

(3.12)

For a duty cycle of 10\%, this means that a constant heat dissipation of 463 W should be cooled. Air cooling is possible in this situation (See Appendix B).

b. Expected gradient in the ROI and homogeneity

From the simulation with STREAM can be derived that a gradient of 300 mT/m with a linearity of 97\% in the ROI will be achieved. The ROI is a cylinder with length 8 cm and diameter 8 cm.

c. Forces

The forces on the coils are caused by the interaction of the \( B_0 \) field and the current through the coils. The \( B_0 \) field is 1.5 T in the z-direction with a deviation of 20 ppm in a spherical volume of diameter 50 cm. In order to determine the forces on the gradient set, an approximation can be used:

1. One coil and \( B_0 \)

   Figure 3.8 shows the forces \( F \) on the coil caused by one coil and the \( B_0 \) field. The force can be calculated by:

\[
\vec{F} = l \cdot I \times \vec{B}_0,
\]  

(3.13)

with \( I \) the current running through the wire, \( l \) the length of the wire and \( B_0 \) the main magnetic field. The force acts in- or outwards in radial direction depending on the direction of the current. On 1 cm wire the force is:

\[
F = 4.35 \text{N}.
\]  

(3.14)

2. One coil and the other 37 coils

   The 38 coils are designed to produce a gradient field of 0.3T/m. So the magnetic field is maximum 0.12T for a length of 0.387m. Figure 3.8b shows the total force caused by one coil and the maximum possible \( B \) field caused by the other 37 coils and can be calculated by eq. 3.13, now with \( F \) the force acting on the wire, \( I \) the current running through the wire, \( l \) the length of the wire and \( B \) the maximum possible magnetic field caused by the other 37 coils. The force acts in- or outwards in radial direction depending on the direction of the current. On 1 cm wire the force is:

\[
F = 0.348 \text{N}.
\]  

(3.15)
3. Connection wires

The connection wires are parallel to the main magnetic field and there will be no net force on them (equation 3.13). The connection wires are also parallel to the magnetic gradient field and no net force will occur (equation 3.15).

The maximum force on 1 cm wire that will act on one coil will be the sum of the two forces mentioned above:

\[ F_{\text{max,1 coil}} = 4.698 \, N \, . \]  

(3.16)

This force works on 19 coils inwards and on the other 19 coils outwards. (The current \( I \) is running through 19 coils in one direction and through the other 19 coils in opposite direction.) The holder for the coils has to be designed strong enough to resist the force of 19 coils:

\[ F_{\text{max,19 coils}} = 89.262 \, N \, . \]  

(3.17)

4. Inhomogeneous \( B_0 \), position problems

If the holder with all the coils is not placed exactly parallel to \( B_0 \) but slightly tilted as shown in figure 3.9b, the forces change. The forces on each coil due to the interaction of the current \( I \) and the \( B_0 \) field will be smaller because the angle between \( B_0 \) and \( I \) is not exactly 90°. The connecting wires are not parallel to the \( B_0 \) field anymore but the forces on the incoming and outgoing wires eliminate each other.

\( B_0 \) is homogeneous within 20 ppm in 50 cm diameter spherical volume. Difference in \( B_0 \) will be at most 30 mT on 50 cm. The changes in forces only have an influence on radial components (if the holder is placed parallel to the \( B_0 \) field) and are neglectable.
Design of gradient coils to measure diffusion with NMR

**d. Stored energy/rise time**

The rise time can be calculated using equation 3.1. The stored energy is:

\[ E = \frac{1}{2} L \cdot I^2, \]  

(3.18)

and it can be calculated if the inductance L and the current I are known. The current is 290 A and the inductance calculated by STREAM is 118 \( \mu \)H. The stored energy is:

\[ E = \frac{1}{2} \cdot 118 \cdot 10^{-6} \cdot (290A)^2 = 5J, \]  

(3.19)

as calculated by STREAM (see Table 3.4) and hence the rise time \( \varepsilon \):

\[ \varepsilon = \frac{1}{2} \cdot \frac{E}{I_{\text{max}} \cdot V_{\text{max}}} = \frac{1}{2} \cdot \frac{5J}{300A \cdot 300V} = 55 \mu s, \]  

(3.20)

with \( I_{\text{max}} \) and \( V_{\text{max}} \) the maximal current and voltage the amplifier can deliver.

The specifications of the Copley amplifier indicate a ramp from 0 to 250A within 150\( \mu \)s (accuracy 1% or 2.5A) and within 260\( \mu \)s (accuracy 0.2% or 0.5A). The ramp slope is specified by 250A/1ms. The self-inductance of the coil L can be calculated:

\[ L = \frac{I \cdot R}{dt} = \frac{290A \cdot 0.055\Omega}{250,000A/s} = 63.8 \mu H \]  

(3.21)

The calculated self-inductance differs from the self-inductance from the STREAM program. The stored energy will now be calculated by using the calculated self-inductance:

\[ E = \frac{1}{2} \cdot 63.8 \cdot 10^{-6} \cdot (290A)^2 = 2.7J \]  

(3.23)

**e. Overview of the expected parameters**

Table 3.5 gives an overview of the expected parameter of the design. Parameters concerning the software are omitted here. All parameters are calculated for a duty cycle of 100%. The force is the force on 1cm wire.
Table 3.5 Overview of the expected parameters.

*The force is the maximum force per cm wire acting on one coil.

3.3 Total set-up

For the diffusion measurements not only gradient coils are needed, but a complete MRI console has to be built. In this section the complete set-up will be explained briefly. For further reading on working with NMR systems in the MRL group, I would like to refer to other reports such as Rijniers [RIJ00].

3.3.1 The main magnetic field $B_0$

The main magnetic field of a Philips ACS whole body MRI scanner is used as $B_0$ in our set-up. This magnetic field of 1.5 T is homogeneous within 5 ppm in a Diameter Spherical Volume (DSV) of 30 cm and within 20 ppm in a DSV of 50 cm. In this region the RF coil and the z-gradients coils of our diffusion set-up will be positioned. Figure 3.10 shows a picture of the gradient coils positioned in the ACS scanner.
3.3.2 The gradient field

The z-gradient is discussed in section 3.2. No gradients will be applied in other directions. Figure 3.11 shows the gradient coil.

![Gradient coil](image)

Figure 3.11 Picture of the gradient coil. This picture was made after the wire was fixed. An extra isolation was applied. The isolation can also deal with radial outwards forces.

3.3.3 The RF receiver

The RF-coil is needed for generating the extra $\mathbf{B}_1$ field in the set-up and for receiving the signals from the spin-echos or FID's. This coil is used in the receiving mode and in the transmitting mode. Figure 3.12 shows an illustration of the RF coil in the gradient coil. The power that goes into the coil in the transmitting mode is much larger than the power that the coil will receive from the signals from the sample. A duplexer is used to separate those signals. The coil can be tuned to 64 MHz with an adjustable capacitor. The electronic layout for the duplexer is shown in appendix C.

![RF coil](image)

Figure 3.12 Illustration of the RF coil positioned in the gradient coil. The RF coil exists of three windings and the electronic layout of the RF coil is shown in figure 3.13.
3.3.4 The data-acquisition system

The total set-up is operated and controlled by a PHYDAS system with interface and operating system. Two computers are working, a client and a server. The server is doing the real time tasks and the experimentator can communicate with the PHYDAS system through the client.

The data-acquisition system makes sure that the timing of the programmed experiments is correct. It also saves the data of the experiment.
Chapter 4

Testing of the z-coils

4.1 Measuring the gradient field using a Hall-probe

a. Measurement set-up
The magnetic field was measured with a Hall-probe and a step motor. A Hall-probe works following the Hall effect. If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor. This is most evident in a thin flat conductor as illustrated in figure 4.1. A buildup of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage between the two sides of the conductor. The presence of this measurable transverse voltage is called the Hall effect. Figure 4.2 shows the set-up of the measurement.

Figure 4.1 Working of a Hall-probe.

Figure 4.2 Set-up of the Hall-measurement; the magnetic field was measured with a Hall-probe. A step motor moved the Hall-probe over a total range of 100 by 100 by 100 mm$^3$ in the middle of the coil. The magnetic field in the z-direction was measured every 5 mm in the three directions, while a continuous current of 20 Amperes was running through the coil.
The used Hall-probe has a range of +/- 500 mT and has an output of 50 mV per mT. This output was amplified 10 times. The step motor has a range of 50 by 50 by 100 mm in the x-, y- and z-direction. A continuous current of 20 Amperes is running through the coil. This means that a magnetic field difference of (2.07 +/- 0.06)mT over 10cm will be expected. Measurements of the magnetic field were performed in the middle of the coil, in an area of 100 by 100 by 100 mm³. Every 5mm in all three directions the magnetic field was measured.

Figure 4.3 shows the interpolation of the magnetic field of 121 measuring points at the position y=50mm. The x and z coordinates go from 0 to 50 mm, where (x,z)=(0,0) is the center of the coil, and (x,z)=(50,0) is near the left side of the coil. The figure can be compared with figure 3.5 with the expected values from STREAM. The interpolated lines are slightly tilted, which is caused by the incorrect positioning of the step motor referred to the coil.

**Figure 4.3 Magnetic field in the y=50 mm plane in the middle of the coil. The point (x,z)=(0,0) is the middle point of the coil and the point (x,z)=(50,0) is near the left side of the coil. This graph is an interpolation of 121 measurement points (every 5mm in each direction). The reason why the lines are not really vertical is because of the incorrect positioning of the coil with respect to the positioning system.**

**b. Calculating the gradient field from the Hall-probe measurement**
The gradient field was calculated from the obtained magnetic field. For every 5mm in the x- and y-direction, the gradient was calculated over the total z-range. Figure 4.4 shows the interpolation of the magnetic field in the middle of the coil. The 21 measurement points are interpolated in the z-range. The gradient is calculated as the slope of the interpolated line and is (20.73 +/- 0.03)mT/m. This is repeated for 441 points in the (x,y) plane. The gradient fields at the other points looked the same and are not shown. At most points the gradient field was in the range of (20.6 +/- 0.6) mT/m.
c. Conclusions
The first measurements of the magnetic field of the coil show a very linear gradient field over the measured z-range (100mm) in the middle of the coil. For low current (20 Ampere) the magnitude and the linearity of the gradient field is as expected from the design. The reason why at some points the gradient field is lower than expected is because of the unstable set-up of the step motor in the coil and the unstable fixing of the Hall probe to the step motor. NMR measurements will be done to check the gradient field at higher currents.

4.2 Measuring the gradient field using NMR

a. Remark
The gradient coil is designed to reach a gradient of 300 mT/m. The gradient will be tested up to 50 mT/m. The part of the NMR measuring system that will operate the input of the gradient amplifier can reach +/-2.5 V and the transductance of the amplifier is 20 A/V. In the future the set-up will be adapted so that it will become possible to reach higher gradient fields.

b. Calibration software-hardware
Before measuring the gradient field, the output current of the gradient was calibrated with the software. The input gradient (mT/m) was varied, while the output of the gradient amplifier was measured. Figure 4.5 shows the result. The measured current was 0.9814 times the input (mT/m) from the software. This attenuation of the current was taken into account in the transductance for the software to the gradient amplifier.
c. Measurement set-up

To check the homogeneity of the gradient, NMR is used. A one-dimensional image of a sample phantom (figure 4.6a) has been made with the z-gradient coil. A Hahn spin-echo sequence (see figure 2.9) without the slice selection encoding gradient is used. The phantom sample is made of Perspex and contains seven tubes (diameter of 5mm) with an intermediate distance of 2.5mm. The tubes are 1 cm deep. These tubes are filled with the water and a 0.01M CuSO$_4$ solution. A second sample is shown in figure 4.6b. The phantom has the same shape as the first one but now without any tubes. There is reference fluid all over the measurement area, 50mm long, 5mm width and 10mm deep. Measurements at this sample were done to see the characteristics of the RF coil.

Figure 4.7 shows the set-up of the measurements. The RF coil is positioned on a table that can be fixed in the gradient coil. The base of the RF coil is 60mm under the middle of the gradient coil. The RF coil is described in chapter 3. The gradient was varied from 5 mT/m to 50 mT/m. The sample was imaged by the z-gradient and the frequency where the reference fluid in the 7 tubes gave an echo was measured. The gradient can be calculated, following:

$$G_z = \frac{2\pi \Delta f}{\gamma \Delta z} \quad (4.1)$$

The sample is positioned at different heights in the RF coil, from 3.5 cm above the base up to 9.5cm above the base of the RF coil. This means that the gradient coil was measured in its center.
Figure 4.6a Phantom with 7 tubes (diameter 5mm) with an intermediate distance of 2.5mm. The tubes are 1 cm deep and filled with a CuSO₄ solution.

Figure 4.6b Reference sample to measure the characteristics of the RF coil.

Figure 4.7 Set-up for the calibration measurements. The sample was moved upwards the RF coil. The first measurement was done when the reference fluid was at 3.5cm above the base of the RF coil and the last when the tubes with the reference fluid was 9.5 cm above the base of the RF coil.

**d. Results**

Figure 4.8 shows the calibration measurements at 3.5 cm in the RF coil when a lower and a higher gradient was applied.
The frequency shift between the maxima of the curves corresponds with the positions of the reference fluid in the tubes. For 15mT/m the frequencies were the maxima of the signals occurs could easily be obtained. For 50mT/m the frequencies were taken at half of the width of the peak. Figure 4.9 shows the frequency shift as function of the spatial coordinates of the tubes. The gradient has been calculated from the slope of the interpolated line. The results are listed in table 4.1. Figure 4.10 and 4.11 shows the results where the sample was positioned at 6.5 cm from the base of the RF coil and figure 4.12 and 4.13 where the sample was positioned at 9.5 cm.

Figure 4.9 Plots of the frequency shift as function of the spatial coordinate, when the sample is 3.5 cm above the base of the RF coil. The gradient, calculated from the slope of the interpolated line is listed in table 4.1.
4. Testing of the z-coils
Figure 4.13 Plots of the frequency shift as function of the spatial coordinates, when the sample is 9.5 cm above the base of the RF coil. The gradient, calculated from the slope of the interpolated line is listed in table 4.1.

<table>
<thead>
<tr>
<th>Height above the base of the RF coil (cm)</th>
<th>Software input gradient (mT/m)</th>
<th>Software input gradient (Hz/mm)</th>
<th>Calculated gradient from frequency shift (Hz/mm)</th>
<th>Calculated gradient from frequency shift (mT/m)</th>
<th>R² value of calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>15</td>
<td>638.7</td>
<td>627.8</td>
<td>14.7</td>
<td>0.9992</td>
</tr>
<tr>
<td>50</td>
<td>2129</td>
<td>1999.6</td>
<td>47.0</td>
<td>0.9999</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>15</td>
<td>638.7</td>
<td>599</td>
<td>14.1</td>
<td>0.9981</td>
</tr>
<tr>
<td>50</td>
<td>2129</td>
<td>1981</td>
<td>46.5</td>
<td>0.9999</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>15</td>
<td>638.7</td>
<td>618</td>
<td>14.5</td>
<td>0.997</td>
</tr>
<tr>
<td>50</td>
<td>2129</td>
<td>1953.1</td>
<td>45.9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Calculated gradient fields

Some of the interpolations in figures 4.9, 4.11 and 4.13 are made with less than 7 points because the software that was used to calculate the positions of the peaks could not detect every peak. Also some peaks were not present at all, for example peak 4, 5 and 6 at 9.5 cm above the base of the RF coil with a gradient of 50 mT/m. The absence of those peaks is due to the characteristics of the RF coils. To illustrate this an image of the second reference sample (figure 4.6b) was taken under the same circumstances. The measurements from this sample are shown in figure 4.14 and 4.15. At 3.5 cm and 9.5 cm above the base of the RF coil one can see that the RF coil is not uniformly picking up the signal. At 6.5 cm above the base of the reference coil, the RF field is most uniform. (The dip, which can be seen is due to the fact that there was a little air bubble in the sample.)
3.5 cm, grad 30mT/m

9.5 cm, grad 50mT/m

Figure 4.14 Reference measurement at 3.5 cm above the base of the RF coil with a gradient of 30mT/m and at 9.5 cm above the base of the RF coil with a gradient of 50mT/m

6.5 cm, grad 50mT/m

Figure 4.15 Reference measurement at 6.5 cm above the base of the RF coil with a gradient of 50 mT/m. The dip in the curve is a little air bubble in the sample.

Figure 4.16 shows the calculated gradient from the frequency shifts versus the input gradients from the software, for the sample positioned at 6.5 cm above the base of the RF coil. This was done for all heights and the results are shown in table 4.2.

Figure 4.16 Calculated gradient from the frequency shift versus input gradient from the software. The interpolated line is calculated with a $R^2$ value of 0.9997. The measured gradient is 0.9335 times the input gradient from the software. The sample was positioned at 6.5 cm above the base of the RF coil.

4. Testing of the z-coils
Table 4.2 Ratio between the calculated gradient strength from the measured frequency shifts and the gradient strength imposed by the software.

<table>
<thead>
<tr>
<th>Height above the base of the RF coil (cm)</th>
<th>Ratio between calculated gradient and input gradient</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>1.0194</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>0.946</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>0.9284</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>0.9335</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>0.9664</td>
<td></td>
</tr>
<tr>
<td>8.5</td>
<td>0.9307</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>0.956</td>
<td></td>
</tr>
</tbody>
</table>

e. Conclusions
• The best position of the sample to measure was between 5.5 cm and 7.5 cm above the base of the RF coil, which is the center of the gradient coil.
• The calculated gradient is lower than expected.
• The calculated gradient is very linear.

4.3 rise times/ stored energy/ forces/ heat dissipation

a. Heat dissipation
During the measurement with the Hall-probe every 20 seconds the temperature was monitored. The current running through the coil was 20 Amperes continuously. After 1.5 hours the temperature was raised 2°C. The conclusion is that no extra cooling will be needed with a gradient of 300 mT/m and a duty cycle of 10%.

b. Forces
Outside the main magnet field, no forces were present. When the gradient coil was placed inside the holder in the main magnet and current was applied, no detectable forces were seen. The expected radial forces on the loop were well captured by the synthetic tube and the holder. No movement of the total gradient could be detected. Therefore we did not measure the forces on the coil, because we concluded that they were not large enough to have considerable influence.

c. Stored energy/rise times
The rise time of the amplifier was measured. For up to 52 mT/m the rise time was 34μs. Also with lower gradients the rise time is 34μs. Figure 4.17 shows the rise time for a gradient of 52mT/m.

Outside the main magnet the resistance of the gradient was 75 mΩ. This was measured with 20 Amperes running through it and with very short connection wires from the amplifier to the gradient coil. With the resistance known and the rise time from previous section, the inductance of the coil can be calculated following Eq. 3.23:

$$L = \frac{IR}{\frac{dl}{dt}} = 2.6 \mu H,$$  \hspace{1cm} (4.2)
the stored energy with $I=300\,A$ is (Eq. 3.20):

$$E = \frac{1}{2} L \cdot I^2 = 0.234\,J.$$  \hfill (4.3)

\textbf{d. Pulsed field gradients}

The area under the two pulsed field gradients was measured with an oscilloscope (figure 4.18). The difference between both areas is about 0.3%.

Figure 4.17 Rise time of the gradient amplifier when 52 mT/m was applied.

Figure 4.18 PFG pulse shape measurements. The current is been plot as a function of time. The area under the pulses is calculated and the difference between the two plots is about 0.3%.
4.4 Diffusion measurements with the new coil

a. Set-up of the measurements
Diffusion measurements were done with the same reference fluid as has been used to measure diffusion at the SJZ (see section 2.3). The fluid was placed in a region where the RF field is most homogeneous, being between 5.5 and 8.5 cm above the base of the RF coil.

Diffusion was measured, using the PFG sequence (see section 2.2.2). The b-value was varied in three different ways. First the gradient G was varied from 0 to 50 mT/m, with constant Δ and δ times. Second the b-value was varied by δ variation with a constant gradient G and a constant Δ time. The echo time was chosen to be 24 ms to obtain the best Signal to Noise Ratio. Table 4.4 gives an overview of the performed experiments.

<table>
<thead>
<tr>
<th>TE (ms)</th>
<th>TR (ms)</th>
<th>δ (ms)</th>
<th>Δ (ms)</th>
<th>Gradient (mT/m)</th>
<th>b-value (10^6 s/m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>900</td>
<td>0.9</td>
<td>12.5</td>
<td>20</td>
<td>0-22</td>
</tr>
<tr>
<td>24</td>
<td>900</td>
<td>3.498</td>
<td>12.5</td>
<td>0-50</td>
<td>0-6</td>
</tr>
</tbody>
</table>

Table 4.4 Overview of performed experiments with the new gradient coils. TE is the echo time and TR the repetition time.

b. Results
Figure 4.19 shows the result where the δ time has been varied. The calculated ADC is 6.59 \times 10^{-8} m^2/s. Figure 4.20 shows the result when the gradient G has been varied. The calculated ADC is 4 \times 10^{-8} m^2/s.

4. Testing of the z-coils
Design of gradient coils to measure diffusion with NMR

**Figure 4.20** Diffusion measurements where b-values were varied by varying the gradient strength \( G \). The slope of the interpolated line is the ADC value and is \( 4 \times 10^8 \text{ m}^2/\text{s} \).

c. Conclusions
This result seems to be very poor (the expected ADC value is \( 2 \times 10^{-9} \text{ m}^2/\text{s} \) for water at room temperature [LEB93]). The reason why in the first measurement the signal attenuate more then expected when gradients were applied is still under investigation. Possible reasons why the measured ADC values are higher then expected are:
- difference in area of the two pulsed field gradients,
- eddy currents that are present nevertheless,
- 90° and 180° condition could not be perfect,
- distortion of the RF field on the gradient during RF sending.
The reason for the high ADC values is still under investigation.
4. Testing of the z-coils
Conclusions and recommendations

The possibility to do advanced diffusion measurements on existing coils was investigated. These systems however have a number of limitations, such as: a small volume, small gradient, a closed operation system, a low signal to noise (SNR) level at different b-values. None of the tested systems was equipped in such a way that diffusion measurements could be performed with high gradients on large samples. The conclusion was to build a new set-up with new specifications: a gradient which can reach to 300mT/m, short pulses should be possible (short rise times will be needed), relative large volume and with an 'open' operating system.

A new gradient was designed and developed. A computer model is used to find the parameters for the gradient coils in such a way that it matches the aims for the gradient: linear in a sperrical volume of 8cm and which can reach up to 300mT/m in the region of interest. Afterwards this coil was built. Together with a new RF coil, the main magnetic field of the ASC scanner and an existing operating system a new set-up to measure diffusion with NMR is developed. The total set-up is calibrated and tested and the following conclusions are made:

- the gradient field is linear in the expected area,
- maximum of the gradient that can be reached is 52mT/m, but will reach in the future 300mT/m,
- and the operating system allows free pulse programming.

One main problem that is still under investigation is the high values for the measured ADC values.

At the end, some recommendations can be made:

- a new RF coil which can produce a larger homogeneous RF field should be developed and implemented,
- and the SNR of the measurements will be better with a new RF coil.

Finally, I hope lots of measurements with these new coils will be performed to test theoretical models on diffusion in biological tissues.
Design of gradient coils to measure diffusion with NMR

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[LEB93] D.Le Bihan, R.Turner, Diffusion and perfusion nuclear magnetic resonance, boek MRA Potchen, p.326


Appendix A

Positions of the coils

The output of the STREAM program is given here. In the list the width, z-center, radius and thickness are in meters. The z-center is the position of the center of the coil referred to the origin, the radius is that of the coil, the thickness is the thickness of the wire and the Amp-turns the amount of Amperes running through the coil.

| Number of coil section pairs | 38 |
| Radius of imaging sphere | 0.0500 m |
| Operating current | 289.568 Amp |
| Total magnet length | 0.387 m |
| Self-inductance | 0.00012 H |

<table>
<thead>
<tr>
<th>No. of turns</th>
<th>Z-center</th>
<th>Radius</th>
<th>Thickness</th>
<th>Amp-Turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.190959</td>
<td>0.099500</td>
<td>0.001000</td>
<td>-290</td>
</tr>
<tr>
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<td>-0.175876</td>
<td>0.099500</td>
<td>0.001000</td>
<td>-290.</td>
</tr>
<tr>
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Appendix B

Air cooling

Characteristic heat of copper: 8960 kg/m³

Total amount of copper used in the gradient:

\[ m_{Cu} = \rho_{Cu} \cdot V \]

\[ V = l \cdot A = [(38 \cdot 2 \cdot \pi) + 2 \cdot L] \cdot \pi \cdot R^2 = 1.88 \cdot 10^{-4} m^3 \]

\[ m_{Cu} = 8960 \frac{kg}{m^3} \cdot 1.88 \cdot 10^{-4} m^3 = 1.68 kg \]

The total amount of air needed to cool the amount of copper can be found by solving the next equation:

\[ m_{Cu} \cdot c_{Cu} = m_{air} \cdot c_{air} \]

with \( c_{Cu} \) and \( c_{air} \) the specific heat capacity of respectively copper and air, and \( m_{Cu} \) and \( m_{air} \) the mass of respectively copper and air. The total amount of air needed per second is:

\[ m_{air} = \frac{1.68 kg \cdot 387 \frac{J}{kg \cdot K}}{1000 \frac{J}{kg \cdot K}} = 0.720 kg \]

which takes a volume of:

\[ V_{air} = \frac{m_{air}}{\rho_{air}} = \frac{0.720 kg}{1.293 \frac{kg}{m^3}} = 0.557 m^3 \]

This amount of air need to be refreshed every second to be able to cool down the gradient, which can easily be done with air cooling.
Appendix C

The duplexer

The duplexer is a device which routes the incoming pulses and the outgoing NMR signal. The incoming pulses are supplied by the NMR measurement system via a power amplifier and have to go into the RF coil. The NMR signal will leave the RF coil and has to be directed towards the receiver in the NMR measurement system.

The duplexer for the hydrogen signal is made by a double pi network. Figure D.1 shows the scheme of the duplexer. The sample has impedance of $50\Omega$ and the output is directed to a pre-amplifier with impedance of $450\Omega$. The working of the duplexer in the transmitting and receiving mode is represented in figure D.2 and D.3.

In the transmitting mode the second pair of diodes are in block mode, in the receiving mode the first two pairs of diodes are in block mode.

![Figure D.1 Scheme for the duplexer, $L_1$ is 135nH, $L_2$ is 190nH, $C_1$ is 47pF and $C_3$ is 22pF.](image1)

![Figure D.2 Working of the duplexer in the transmitting mode. $R_1$ is 50Ω and represents the impedance of the RF amplifier. $L_1$ is 135nH and $C_1$ is 47pF.](image2)

A computer program is used to simulate the behavior of the duplexer. Figure D.4 is a plot of the voltage transfer as a function of time for the duplexer in the transmitting mode. The input was set as 1V. At 63.9MHz the input signal is not attenuate. Figure D.5 shows a plot of the voltage transfer of the input (which comes now from the sample) and output signal of the duplexer as a function of time in the receiving mode. At 63.9 MHz the input signal (0.5V) is amplified 3 times (1.5V).
Figure D.3 Working of the duplexer in the receiving mode. $L_1$ is 135nH, $L_2$ is 190nH, $C_1$ is 47pF, $C_2$ is 10pF and $C_3$ is 22pF. $R_1$ is 50Ω and represents the impedance of the RF coil and $R_2$ is 450Ω and represents the impedance of the pre-amplifier.

Figure D.4 Transfer of the voltage of the input signal as a function of frequency in the sending mode of the duplexer. At 63.9MHz, the input signal of 1V is not attenuate.
Figure D.5 Plot of the voltage transfer in the receiving mode. At 63.9MHz the signal of the output (1.5V) is three times the signal coming from the sample (0.5V).