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Spectral characterization of the influence of polarization mode dispersion and crosstalk on optical transmission systems at 40 Gbit/s

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Spectral characterization of the influence of polarization mode dispersion and crosstalk on optical transmission systems at 40 Gbit/s

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Preface

This is the report of my graduation project which finishes my eight years of study in electrical engineering. It treats a study on the influence of impairments on the signal spectrum of an optical system, which I carried out at the Technical University of Catalonia. For an introduction to the study, I refer the reader to chapter two. From chapter three, more theory is treated. To understand this theory, basic knowledge about optical transmission systems and stochastic signal theory is needed.

As with all things in life, it is impossible to achieve success on your own. Therefore I would like to thank Josep Prat for supervising me and Idelfonso Tafur for setting up the project and for his supervision, especially for his help in the last two months. Furthermore I would like to thank Oscar Diaz for introducing me to PMD, for the measurements and for the nice collaboration and talks. I would like to thank Michiel Oostindie for thinking with me on some issues and helping me out with LaTeX and C++ problems, Ivo Willems for arranging facilities by giving me a motherboard and lending me a computer right after my return to Eindhoven and Peter Slegers for helping me with my presentation.

Less related to the actual project work, but certainly important for the success of the project, I would like to thank my friends in Barcelona, especially David and Muntsa for helping me with my Spanish, introducing me to their Spanish culture and for the very nice times.
Abstract

As bit-rates in optical transmission systems increase, the influence of impairments such as polarization mode dispersion (PMD) and coherent homo wavelength crosstalk (CHOC) have a larger impact on the optical signal quality. The reliability of a communication channel can be assessed when it is known to what extent different impairments affect this channel. Methods have been developed to measure various impairments. However, often these methods are intrusive and they are in most cases designed to detect and quantify the effect of just a single impairment. As the influence of impairments change over time, it is of importance to monitor them continuously without taking the communication system out of operation.

The purpose of this report is to find nonintrusive methods that measure the effect of PMD and CHOC on the optical signal quality. The signal spectrum of the electrical field before the photodetection and the signal spectrum of the detected photocurrent will be analyzed to quantify the effects of PMD, CHOC and chromatic dispersion. Mathematical analysis and computer simulation are used to characterize the influence and to test strategies to quantify the impairments. These strategies are then validated on measurements.

The transmission system studied in this thesis operates at 40 Gb/s. The present study shows that the effects of PMD on the signal quality can be detected with an accuracy of 2% - 20% according to simulations and by an average accuracy of 11.1% according to the performed measurements using the spectrum of the detected photo-current. The effect on the signal quality of CHOC can be detected with an accuracy of 0.1% according to simulations using the optical spectrum domain approach. This method is suitable for a maximum crosstalk delay of 0.19 ns. In the electrical domain, the effect of CHOC is detected with an accuracy of 10% by using measurements.

When a communication system is impaired simultaneously by PMD and chromatic dispersion, the accuracy of the detection of PMD decays with 5% according to the performed simulations. CHOC with chromatic dispersion can be detected in the optical domain. The simulation results show that CHOC can be detected with an average accuracy of about 3%. CHOC and PMD can be detected simultaneously, provided that the delay of CHOC is at least two bit times larger than the differential group delay of the PMD present in the channel.

Using the developed spectral methods, the effect of PMD can be detected for values that may slightly impair the optical signal quality, less than 25% bit-time up to total signal degradation. Moreover, the effect of CHOC can be detected for a very wide range of crosstalk values.

I recommend the developed methods to be verified with more exhaustive measurements including a mixture of impairments. The methods should also be checked for accuracy when other types of impairments are present, such as ASE noise and uncoherent crosstalk. Moreover, more accurate results could be accomplished by doing more research into the unique spectral characteristics and influence of each impairment on the signal quality.
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Chapter 1

Introduction

As bitrates in optical transmission systems increase, the influence of impairments such as polarization mode dispersion (PMD) and coherent homo wavelength crosstalk (CHOC) have a larger impact on the optical signal quality. The reliability of a communication channel can be assessed when it is known to what extent different impairments affect this channel. Methods have been developed to measure various impairments. For example by launching monochromatic light and using polarizers to detect PMD [B+03]. However, often these methods are intrusive and they are mostly designed to detect and quantify the effect of just a single impairment. As the influence of impairments change over time, it is of importance to monitor them continuously without taking the communication system out of operation.

The purpose of this report is to find nonintrusive methods that measure the effect of PMD and CHOC on the optical signal quality. As every impairment has its specific influence on the output signal spectrum, clues can be sought using the signal spectrum. This signal spectrum can be measured by the electrical field before photodetection and by the detected photo-current. Both signal spectra will be analyzed to quantify the effects of PMD, CHOC and chromatic dispersion. Mathematical analysis and computer simulation are used to characterize the influence and to test strategies to quantify the impairments. These strategies are then validated by measurements.

This report is organized as follows: firstly in chapter two a literature survey is carried out, from which a working method for this project is described. In chapter three the optical system and the different impairments are modelled. Then in chapter four, analysis and simulation approach are used to describe a method to obtain its parameters for every studied impairment. In chapter five a mixture of these impairments is simulated where methods are being tested if they are able to obtain the parameters in the case when more than one impairment is present simultaneously. Finally in chapter six a conclusion is drawn and recommendations are made.
Chapter 2

Project framework

Before elaborating on the problem stated in the introduction, first the framework must be described, so a clear project description can be formulated.

Therefore in this chapter other work in this area will be treated. After having an overview of what already has been done, a working method for this project can be formulated.

2.1 Overview of research

To be able to assess how the communication is affected by impairments much research has been done. In this paragraph, an overview of methods in the literature is given.

A system can be impaired by many causes, but only some type of impairments seem to gain importance when moving to higher bit rates.

For crosstalk there are different types: heterowavelength, which means that two signals on different carriers interfere and homo wavelength, which means that two signals on the same carrier interfere. According to [Por99] Coherent homo wavelength crosstalk is the most harmful type of crosstalk.

For fiber imperfections two impairments seem to be important in this subject. The first one is polarization mode dispersion (PMD). Since single channel bit rates reached beyond 10 Gbit, it is regarded as a major limitation in optical systems [Sun01]. The other impairment is chromatic dispersion which affects an optical communication system from bit rates higher than 10 Gb/s [W+02]. For the three mentioned impairments, many methods have been designed to compensate for the effects and to measure them.

For PMD for example a polarizer and an analyzer in combination with an OSA can be used to measure PMD [Sun01], [PF94]. Another article claims that it is possible to measure PMD by launching successively, two linear and nonorthogonal polarized monochromatic light waves with the same frequency in the fiber under test and measuring the corresponding output polarization state. [MB01] This allows measurement of PMD in a transmission line without data traffic interruption. In [YSYW03], a method is proposed that can monitor the optical signal to noise ratio and PMD simultaneously. Another method that uses adaptive filter control to mitigate PMD and group velocity dispersion is described in [B+03]. In [EKT03] a system is described that can predict the RF spectrum of a system with PMD knowing the input state of polarization. A more comprehensive literature survey about PMD can be found in appendix C.

For CHOC, in [R+96] a method is described for obtaining the crosstalk level from the RF spectrum.
In [W+02] a method is described to detect chromatic dispersion by analyzing the RF spectrum. The article [WK04] describes the measurement PMD and chromatic dispersion in DPSK systems.

All above methods are just applicable to one or two types of impairment. Most methods require the optical transmission system to be taken out of operation. To be able to monitor a system, a method is needed which with it is possible to monitor more than one impairments without interrupting the data transfer of a transmission system. In the next section a working method will be described which aims at achieving this goal.

2.2 Working method of this project

Methods have already been developed which make successful use of the power spectral density of the signal that arrives at the end of a transmission line. Every impairment will have its own specific effect on the output signal, and will therefore influence the PSD in its unique way. Therefore, in this report the effect of different impairments on the PSDs will be studied.

A method to detect the influence of impairments that does not interrupt the data transmission, can solely make use of the information revealed by the signal at the receiver. The PSD can be measured at two locations as shown in figure 2.1.

![Figure 2.1: Measurement of PSD in two locations in an optical transmission system](image)

- Before the photodetector
  - In this case, the optical signal which arrives at the photodetector is analysed i.e. optical spectrum analysis by an optical spectrum analyser (OSA)

- After the photodetector
  - In this case, the resulting photo-current is analysed, i.e. electrical spectrum analysis by an electrical spectrum analyser (ESA)

The choice to use return-to-zero (RZ) or non-return-to-zero (NRZ) is not that obvious: NRZ is easy to generate at lower, conventional bit rates and has a better spectral efficiency. RZ has a better performance with respect to SNR [Sun01]. To make use of the advantage that already performed research was done in this field with NRZ signaling, in this report NRZ signaling will be used.

Optical systems impaired by polarization mode dispersion (PMD), coherent homoc wavelength optical crosstalk (CHOC) and chromatic dispersion are analysed. The PSD can be obtained by three methods: by mathematical analysis, by simulation and by measurement. These methods are described below.

2.2.1 Mathematical analysis

To calculate the powerspectrum in the optical domain (OSA), the following steps need to be taken:
1. Calculate the electrical field resulting at the photodetector;
2. Calculate the autocorrelation function of this signal;
3. Use Fourier transform to obtain the powerspectrum.

To calculate the powerspectrum in the electrical domain (ESA), first the photocurrent must be calculated. The procedure then becomes:

1. Calculate the electrical field resulting at the photodetector;
2. Calculate its resulting electrical current;
3. Calculate the autocorrelation function of this signal;
4. Use Fourier transform to obtain the powerspectrum.

Mathematical Analysis can be useful to derive an exact formula of the spectrum. The characteristics of this formula can be used to develop a strategy for obtaining the parameters of the impairment under study. A disadvantage of this approach is that when using complex models, the resulting expressions can be extremely long and complicated, or it is impossible to treat the system by analytical means.

2.2.2 Computer simulation approach

To obtain the PSD by simulation the following steps need to be taken:

- Simulate optical system in the time domain to obtain electrical field before the photodetector or the resulting photo-current
- Perform Fast Fourier Transform

Simulation offers a straightforward and fast way to get a reasonable realistic spectrum. Complicated models can be used and more impairments can be mixed with not too much effort. The two limits are the simulation time and the available memory in the used computer. Commercial programs available can be used. The problem of these programs is that usually one does not exactly know the models used and it is difficult (or impossible) to customize features of a system. Moreover they are usually slow.

Therefore a dedicated simulator has been built from C++ code. The setup of the simulator is described in appendix B.

2.2.3 Measurement

For measurement the power spectral densities are obtained by the following steps:

- Setup an optical system in which an impairment is present or emulated
- Measure the signal spectrum using an OSA or an ESA

Carrying out measurements falls beyond the scope of the present project. To be able to test developed methods on measurements, measurements performed by Oscar Diaz have been used.
Chapter 3

Modelling of optical transmission systems

3.1 Introduction

To be able to do mathematical analysis and simulations on optical transmission systems, such a system must first be modelled. There are a variety of possibilities to model a system and its subparts and to be able to make useful mathematical analysis and simulation a good choice is important.

In this chapter different models are treated for optical systems. A general model is presented and for its parts different possible models are treated. For every type of impairment a model is discussed.

First a general model for an optical transmission system will be treated. Next different impairments in the optical link will be modelled.

3.2 General model

In this section a general model is presented for an optical system. In figure 3.1 a schematic diagram is depicted. It consists of a laser source, information source, modulator, link and receiver.

![General model of optical transmission system](image)

The electrical field $E_l(t)$ of the laser output signal is modulated by a signal source $x(t)$. The electrical field after the modulator is denoted by $E_m(t)$. The optical link transports this field to the receiver where it will arrive distorted as $E_d(t)$. The optical receiver produces a photo current $I_d(t)$ as a result of the light upon it.

In the following paragraphs different models will be treated for the laser, signal-source, modulator and receiver.
3.2.1 Laser source

The laser produces a light of frequency \( \omega_0 \). The frequency is not stable and moves around its central point. This can be modelled by a phase \( \phi(t) \) is introduced. The electrical field resulting can be represented by:

\[
E_l(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t))
\]

in which \( P_0 \) is the optical power of the laser. The phase can be modelled as a Gaussian noise. In this case, \( \phi(t) \) is a result of an integral of this Gaussian noise:

\[
\phi(t) = \int_{-\infty}^{t} [\text{Gaussian white noise}] \, dt
\]

3.2.2 Electrical signal source

The signal source represents an information source. This can be represented by a binary signal:

\[
x(t) = \sum_{k=0}^{N-1} I[k] F(t - kT)
\]

in which \( I[k] \) is an information source taking randomly one of the values from the set \( \{ 0, 1 \} \) for each \( k \). It is assumed that \( P(I[k] = 0) = P(I[k] = 1) = \frac{1}{2} \). \( F(t) \) is the pulseform with period \( T \) for which different models can be used:

- **Blockform**
  
  This is the simplest model:
  
  \[
  F(t) = U(t) - U(t - T) = \text{Rect}(t - T)
  \]
  
  in which \( U(t) \) is the step function.

- **Gaussian pulse**
  
  \[
  F(0, t) = F_0 e^{-\frac{1+IC(t)^2}{2}}
  \]

- **Super Gaussian pulse**:
  
  \[
  F(0, t) = F_0 e^{-\frac{1+IC(t)^2}{2\alpha}}
  \]

The super Gaussian pulse looks more like a blockwave. Gaussian pulses are helpful when calculating dispersion fibers as they maintain their form [Agr97]. The parameter \( C \) in the formulas is a linear chirp, which linearly increases or decreases in the carrier frequency of the pulse.

3.2.3 Intensity modulator

A modulator is basically a multiplier of two signals. In practice, often Mach-Zehnder modulators are used. The transfer function for a Mach-Zehnder modulator is:

\[
\frac{E_{out}(t)}{E_{in}(t)} = \cos \left( \pi \frac{V_1(t) - V_2(t)}{2V_x} \right)
\]

By altering voltages \( V_1(t) \) and \( V_2(t) \) the ratio between the outgoing and incoming electrical field can be changed [Ber01].
3.2.4 Receiver

The receiver produces a photo current proportional to the intensity of the incident optical signal. The expression for the photo current is given by:

\[ I_d = R|E_d(t)|^2 \]

Where \( R \) is the responsivity of the receiver. Because of the limited rise and fall times, this component may have a lowpass character [Agr97].

3.3 Models of systems with impairments

In this section models for different impairments are treated.

3.3.1 PMD

When light is propagating through an optical fiber, its electrical field can be decomposed in two orthogonal components, both perpendicular to the direction of propagation:

\[ E(z, t) = E_x(z, t) \bar{x} + E_y(z, t) \bar{y} \]

with

\[ E_x(z, t) = E_0 \sqrt{1 - \gamma} e^{i(\omega t - \beta z + \phi_x)} \]
\[ E_y(z, t) = E_0 \sqrt{\gamma} e^{i(\omega t - \beta z + \phi_y)} \]

in which \( E_0 \) is the magnitude of the field, \( \gamma \) is the division of power over the orthogonal components, \( \omega \) the frequency of light, \( \beta \) the propagation constant and \( \phi_x \) and \( \phi_y \) the phases of the \( x \) and \( y \) component respectively [Sun01].

A delay between the two components can arise when a fiber is bended. This causes birefringence, which makes the refraction of the fiber material polarization dependent. In this case, one component of the electrical field may propagate faster than the other component.

The polarization state of the field can be represented by a Jones vector [Sun01]:

\[ \mathbf{j} = \begin{pmatrix} \sqrt{1 - \gamma} e^{i\phi_x} \\ \sqrt{\gamma} e^{i\phi_y} \end{pmatrix} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) e^{-i\varphi} \end{pmatrix} \]

in which the difference in phase \( \varphi = \phi_x - \phi_y \) as well as the division of power is represented. When considering the general model of figure 3.1, the part of the fiber link can be modelled with two attenuators and a delay as is done in figure 3.2. In this figure \( r_0 \) represents the difference in phase \( \phi \).

In table 3.1 the mathematical relations of the components in their time- and frequency domain are shown.
Table 3.1: Relations for a transmission system with PMD

<table>
<thead>
<tr>
<th>Relations</th>
<th>Modulator</th>
<th>Link</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal relation</td>
<td>$E_m(t) = E_i(t)\sqrt{x(t)} = E_i(t)\sum\text{Rect}\left(\frac{t}{\tau}\right)$</td>
<td>$E_d(t) = \sqrt{1 - \gamma} E_m(t)\mathbb{I} + \sqrt{\gamma} E_m(t - \tau_0)\mathbb{I}$</td>
<td>$I_d(t) = R</td>
</tr>
<tr>
<td>Frequency relation</td>
<td>$E_m(\omega) = X(\omega - \omega_0) + X(\omega + \omega_0)$</td>
<td>$E_d(\omega) = E_m(\omega)\left(\sqrt{1 - \gamma^2} + \sqrt{\gamma} e^{-i\omega\tau_0}\mathbb{I}\right)$</td>
<td>$I_d(\omega) = Y_{ph}(E_d(\omega + \omega_0))$</td>
</tr>
<tr>
<td>Frequency response $H(\omega)$</td>
<td>-</td>
<td>$\sqrt{1 - \gamma^2} + \sqrt{\gamma} e^{-i\omega\tau_0}\mathbb{I}$</td>
<td>-</td>
</tr>
</tbody>
</table>
3.3.2 Coherent homowavelength crosstalk (C-HOC)

C-HOC is caused when a replica of the main optical channel has taken a spurious path, and is attenuated and delayed and later interferes with itself. It occurs in optical cross connectors [Por99].

This impairment can be modelled by the following expression:

$$E_d(t) = \sqrt{1 - \gamma} E_m(t) + \sqrt{\gamma} E_m(t - \tau_0)$$

In figure 3.3 this model is implemented in the optical transmission scheme. In table 3.2 the mathematical relations are shown.
Table 3.2: Relations for a transmission system with HOC

<table>
<thead>
<tr>
<th>Relations</th>
<th>Modulator</th>
<th>Link</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal relation</td>
<td>$E_m(t) = E_i(t) \sqrt{x(t)} = E_i(t) \left(\frac{1}{T_i}\right)$</td>
<td>$E_d(t) = \sqrt{1 - \gamma} E_m(t) + \sqrt{\gamma} E_m(t - \tau_0)$</td>
<td>$I_d(t) = R</td>
</tr>
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<td>Frequency relation</td>
<td>$E_m(\omega) = X(\omega - \omega_0) + X(\omega + \omega_0)$</td>
<td>$E_d(\omega) = E_m(\omega) \sqrt{1 - \gamma} + \sqrt{\gamma} e^{-i\omega \tau_0}$</td>
<td>$I_d(\omega) = Y_{pb}(E_d(\omega + \omega_0))$</td>
</tr>
<tr>
<td>Frequency response $H(\omega)$</td>
<td>-</td>
<td>$\sqrt{1 - \gamma} + \sqrt{\gamma} e^{-i\omega \tau_0}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 3.3: Schema of optical system with C-HOC
3.3.3 Chromatic dispersion

Chromatic dispersion causes group delay dispersion, which is pulse broadening that occurs within a single mode. It is caused by the fact that the refractive index is wavelength-dependent. Modulated light has a certain bandwidth. It is possible that the light of lower frequency propagates slower than the part with higher frequency caused by the frequency dependence of the refraction index. This can cause a transmitted pulse to broaden.

According to [Agr97], each frequency component of the optical field propagates in a single-mode fiber as a plane wave and can be written as:

$$E(r, \omega) = \hat{e} F(x, y) \tilde{O}(0, \omega) e^{i\beta z}$$

In which $\beta$ is the propagation constant [Agr97]. Pulse broadening results from the frequency dependence of $\beta$. For quasi monochromatic pulses with $\Delta \omega \ll \omega_0$, $\beta$ can be expanded as:

$$\beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + \beta_1 (\Delta \omega) + \frac{1}{2} \beta_2 (\Delta \omega)^2 + \frac{1}{6} \beta_3 (\Delta \omega)^3$$

In the case of Gaussian input pulses, the transfer function for a single Gaussian input pulse of a fiber with intramodal dispersion can be written as:

$$\frac{E_d(\omega)}{E_m(\omega)} = e^{\frac{1}{2} \beta_2 \omega^2 z + \frac{1}{6} \beta_3 \omega^3 z}$$

The Fourier transform of a Gaussian input pulse is:

$$\mathcal{F} \left( A_0 e^{-\frac{1+iC}{2} \left( \frac{\omega}{\omega_0} \right)^2} \right) = A_0 \sqrt{\frac{2\pi T_0^2}{1 + iC}} e^{-\frac{\omega^2 T_0^2}{2(1 + iC)}}$$

Neglecting the $\beta_3$ term of the transfer function, the output electrical field of a single pulse can be written as:

$$E_d(\omega) = A_0 \sqrt{\frac{2\pi T_0^2}{1 + iC}} e^{-\frac{\omega^2 T_0^2}{2(1 + iC)} + \frac{1}{6} \beta_2 \omega^2 z}$$ (3.1)

In figure 3.4 this model is implemented in the optical transmission scheme. In table 3.3 the mathematical relations are shown.
Table 3.3: Relations for a transmission system with intramodal dispersion

<table>
<thead>
<tr>
<th>Relations</th>
<th>Modulator</th>
<th>Link</th>
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<td>Temporal relation</td>
<td>$E_m(t) = E_i(t)\sqrt{x(t)}$</td>
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<td>$E_m(\omega) = X(\omega - \omega_0) + X(\omega + \omega_0)$</td>
<td>$E_d(\omega) = E_m(\omega)e^{\frac{i}{2}\beta_2 \omega^2 z}$</td>
<td>$I_d(\omega) = Y_{pb}(E_d(\omega + \omega_0))$</td>
</tr>
<tr>
<td>Frequency response $H(\omega)$</td>
<td>$-$</td>
<td>$e^{\frac{i}{2}\beta_2 \omega^2 z}$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Chapter 4

Characterization of impairments

4.1 Introduction

The impairments which are modelled in the last chapters will be treated here as well as a system without impairments. The mathematical expressions for the resulting spectrum in ESA and in OSA will be analytically derived. The results are compared with simulations and different strategies are developed to obtain the parameters of each impairment.

4.2 Optical system without impairments

To be able to distinguish impaired systems, first a system without impairments is treated. For this system the PSD of the optical signal before the photodetector and the PSD of the resulting photo-current will be derived. Next the results will be compared with simulation results.

4.2.1 Calculation of the power density spectrum

To calculate the PSD, first the electrical field resulting at the photodetector is calculated to be performed in the following paragraph. Then the PSD for the optical signal and the photo-current will be calculated.

Electrical field at the photodetector

To calculate the electrical field at the photodetector we start at the beginning (left side) of the scheme in figure 3.1. The laser source emits an electrical field:

\[ E_l(t) = \sqrt{2P_0} \cos (\omega_0 t + \phi(t)) \]  

The equation for a Mach Zehnder modulator is:

\[ \frac{E_{\text{out}}(t)}{E_{\text{in}}(t)} = \cos \left( \pi \frac{V_1(t) - V_2(t)}{2V_x} \right) \]

where \( V_1(t) - V_2(t) \) is the signal source. When it is assumed that the input signal is a block signal, only taking the values 0 and 1, the output signal is not affected by
the cosinus and thus \( E_{\text{out}}(t) = E_{\text{in}}(t) \). In this case, the modulated output signal is:

\[
E_m(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t)) \, x(t)
\]  \hfill (4.2)

It is assumed that the fibre is ideal and the signal is not distorted, so

\[
E_d(t) = E_m(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t)) \sqrt{x(t)}
\]  \hfill (4.3)

**PSD in the optical domain (OSA)**

To calculate the spectrum of the modulated electrical field, first the autocorrelation must be calculated: \( R_{EE}(\tau) = E \{ E_m(t)E_m(t+\tau) \} \). In appendix A.2.1 the calculation of the spectrum of OSA is performed. In the calculations for the autocorrelation, the following assumptions are used:

- The phasenoise is assumed to be Gaussian distributed
- The difference of two autocorrelations of the phasenoise increases linearly with the linewidth of the laser:

\[
R_{\phi}(0) - R_{\phi}(\tau) = \frac{1}{2} W|\tau|
\]

with \( W \) the linewidth of the laser [Cal99].

The result is:

\[
R_{EE}(\tau) = \frac{1}{4} P_0 \cos(\omega_0 \tau) e^{-\frac{1}{2} W|\tau|} \left( 1 + \text{tri}\left( \frac{\tau}{T} \right) \right)
\]  \hfill (A.7)

Neglecting the phasenoise, this result can be simplified to:

\[
R_{EE}(\tau) = \frac{1}{4} P_0 \cos(\omega_0 \tau) \left( 1 + \text{tri}\left( \frac{\tau}{T} \right) \right)
\]  \hfill (A.8)

The power spectrum of OSA is calculated by taking the Fourier transform of the above expression. Neglecting the phasenoise, the result is:

\[
S_{EE}(\omega) = \frac{1}{4} \, P_0 \left( A(\omega - \omega_0) + A(\omega + \omega_0) \right)
\]

\[
A(\omega) = \pi \delta(\omega) + \frac{T}{2} \, \text{sinc}^2 \left( \frac{\omega T}{2} \right)
\]  \hfill (A.9)

If the phasenoise is not neglected, the expression is more complicated and can be found in appendix A.2.1, equation A.11.

**PSD in the electrical domain (ESA)**

The power spectrum in the electrical domain is calculated from the resulting current at the receiver. First the autocorrelation is calculated. In appendix A.2.2 the calculations for the spectrum of ESA have been performed. In the calculations of the autocorrelation, the following assumptions have been used:

- The higher frequency components \( \cos(2\omega_0 t) \) disappear at the receiver because of low pass filtering.
- The signal of the laser \( E_l(t) \) is independent of the signal source \( x(t) \).

The result is:

\[
R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \left( 1 + \text{tri}\left( \frac{\tau}{T} \right) \right)
\]  \hfill (A.13)

The power spectrum is obtained by taking the Fourier transform. The result is:

\[
S_{II}(\omega) = \frac{1}{4} R^2 P_0^2 \left( 2\pi \delta(\omega) + T \, \text{sinc} \left( \frac{T \omega}{2} \right)^2 \right)
\]  \hfill (A.14)
4.2.2 Analysis of spectra

The resulting power spectral densities in the optical and electrical domain are both sinc functions, for which the one of the electrical field is shifted to its carrier frequency $\omega_0$. In this section the results are compared to simulations which have been performed as described in appendix B. Also the influence of linewidth will be treated.

Influence of linewidth

When comparing the graphs of OSA for different linewidth values, only a significant difference is found around the carrier frequency. In figure 4.1 the difference of the power spectral density of a system with three different linewidths with respect to a system with no linewidth is plotted.

![Figure 4.1: Plots of PSD of OSA for three different linewidths](image)

As shown in equation A.9, a Dirac delta function peak arises in this area. Measuring a spectrum, this delta peak will spread over neighbouring frequencies and will be observed around $f_0$ because spectrum analysers have limited resolution and as a consequence of leakage. The relative small difference caused by linewidth around this center frequency will therefore not be perceptible.

Comparison with simulations

In the left graph of figure 4.2 both the analysis result and the simulation result are plotted. The significant difference is visible around the carrier frequency where the Dirac delta function peak is spreaded over the neighbouring frequencies.

In the right part of figure 4.2 the analysis and the simulation result are plotted for the electrical domain (ESA). Again one can see a difference near the zero-frequency as a result of spreading of the Dirac delta function.
4.3 Optical system with polarization mode dispersion (PMD)

As described in paragraph 3.3.1, first order PMD causes a delay in the signal propagation of the two orthogonal axes of a fiber. First order PMD is described by two parameters:

- $\tau_0$, the relative delay between the two orthogonal axes, the differential group delay (DGD);
- $\gamma$, the division of the electric field over the two orthogonal axes.

In this paragraph this phenomenon will be analyzed and characterized. As in the last paragraph, first an analysis is performed, then this analysis is compared to simulations. Finally, a strategy is treated which describes how to obtain parameters of this impairment.

4.3.1 Calculation of the power density spectrum

First the electrical field resulting at the photodetector is calculated. Because the PMD effect can only be seen using an ESA, only the power density spectrum of the resulting photo-current will be calculated.

**Electrical current after the photodetector**

As with the system with no impairments, the electrical field after the modulator is:

$$E_m(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t)) \sqrt{x(t)}$$

The effect of PMD (table 3.1) is:

$$E_d(t) = \sqrt{1 - \gamma} E_m(t) \bar{x} + \sqrt{\gamma} E_m(t - \tau_0) \bar{y}$$

Now the electrical current can be calculated as it has been done in appendix A.3.1. The result is:

$$I_d(t) = RP_0 ((1 - \gamma) x(t) + \gamma x(t - \tau_0))$$

**PSD in the electrical domain (ESA)**

To calculate the spectrum of this current, first the autocorrelation function must be derived: $R_{II}(\tau) = \mathbb{E} \{ I_d(t) I_d(t + \tau) \}$. In appendix A.3.1 the result is calculated. The powerspectrum is obtained by taking the Fourier transform. The result is:

$$S_{II}(\omega) = \frac{1}{4} R^2 P_0^2 \left( 2\pi \delta(\omega) + \left( 1 - 2\gamma(1 - \gamma) + 2\gamma(1 - \gamma) \cos(\omega \tau_0) \right) T \sin \left( \frac{\omega T}{2} \right)^2 \right)$$

(A.17)

4.3.2 Analysis of the spectrum

Part of the formula derived is similar to the formula derived for a system without impairments. The other part modulates the sinc-pulse by $\cos(\omega \tau_0)$. The PMD effect is described by the amplitude and the frequency of this cosine function.

To obtain these parameters one can use different methods. One of them is finding the first minimum of the graph. By knowing at which frequency this minimum
occurs and at which power this minimum is, one can derive the DGD and the power distribution over the two axes respectively. Another way is making use of a second FFT, which is the FFT of the already obtained spectrum. Both methods are described in the next section.

Minimum point method

As the phase of the cosine function is zero, the minimum point occurs at half period, so $\tau_0 = \frac{1}{2f}$ where $f$ is the frequency where the minimum point occurs. From the formula A.17 it can be derived that $\gamma = \frac{1}{2} (1 - \sqrt{p})$, where $p$ is the height of the found minimum.

The ESA is normally readable until 40 GHz. For frequencies bigger than 40 GHz the power is too low to make use of the spectrum. To use the minimal point method, a minimum point must occur at a frequency below 40 GHz. An example of the method is shown in 4.3. The minimum point occurs at half a period of the cosine, this means that the DGD $\tau_0 > 0.5$ Tbit. For bigger $\tau_0$, a second minimum will be visible in the range to 40 GHz and it's not as straightforward to obtain the first minimum. This happens when $\tau_0 > 1.5$ Tbit. So for a simple minimum point method a boundary conditioning is: $0.5$ Tbit $< \tau_0 < 1.5$ Tbit. The procedure is:

- Normalize the measurement with respect to the sinc function
- Find the first minimum
- Obtain $\tau_0 = 1/(2f)$ where $f$ is the frequency at which the minimum occurs
- Obtain $\gamma = \frac{1}{2} (1 - \sqrt{p})$ where $p$ is the height of the normalized measurement

This method is tested on simulations performed as described in appendix B. Parameters such as resolution, number of bits can be found in this appendix. 33 simulations were performed with $\tau_0$ from $0.51$ Tbit to $1.51$ Tbit in steps of $0.03$ Tbit. These were performed for $\gamma = 0.3$ as well as for $\gamma = 0.5$. The results are in table 4.1

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>parameter</th>
<th>Avg. deviation</th>
<th>Max. deviation</th>
<th>Std. deviation (RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>$\tau_0$</td>
<td>6.9 %</td>
<td>67 %</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>2.3 %</td>
<td>12 %</td>
<td>$2.74 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$\tau_0$</td>
<td>6.1 %</td>
<td>67 %</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>10 %</td>
<td>24 %</td>
<td>$2.95 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

The differences for parameter $\gamma$ for $\gamma = 0.5$ are probably caused by the signal processing of the simulator: after simulation the results are filtered by a hanning FIR filter to smoothen the result. This causes extrema to become less extreme. As for large values of $\gamma$ the cosine has a bigger amplitude and sharper tops, the smoothening has a bigger effect on the extrema.

This method could be improved by making use of unfiltered simulation data and using cosine function fitting at the points around the extrema.

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Figure 4.2: Plots of the simulation and analysis of the PSD of OSA and ESA of a non-impaired system

Figure 4.3: Spectrum of ESA with PMD normalized with spectrum of non-impaired system, $\tau_0 = 1.3$, $\gamma = 0.3$
Second FFT method

A FFT can be calculated over the measured (or simulated) power spectral density. An example is shown in figure 4.4. Taking an FFT of a PSD will bring us back into the time domain again. This enables us to obtain the value of $\tau_0$.

![Figure 4.4: Left: simulation and analysis of PMD, right the second FFT. $\tau_0 = 1.84$ and $\gamma = 0.3$](image)

If PMD occurs in the optical system, a cosine function with frequency $\omega \tau_0$ appears in the PSD. Therefore, in the second FFT a peak will appear at the time corresponding to $\tau_0$. The height of the peak will correspond to the division of power over the two axes, $\gamma$.

As shown in figure 4.4, the second FFT doesn’t contain much resolution and has a big peak at $\tau = 0$. There are several reasons for this:

- The PSD contains a Dirac delta function at $f = 0$ GHz which adds power in to "frequencies" in the second FFT;
- The PSD contains a Sinc pulse of frequency $f = 1/T_{\text{bit}}$;
- The spectral reach is only 40 GBit which makes the density of the second FFT very low;
- When calculating an FFT, it is assumed that the PSD is repeated. Repetition of the PSD leads to discontinuities. These discontinuities influence the result by causing leakage: peaks in the resulting spectrum are smeared out over neighbouring frequencies [Cou01].

So some enhancements are needed. To increase the density of the second FFT, zeros can be added. A more elegant way to increase the spectral reach is to mirror the spectrum over the power-axis. Because the phase of the cosinus is zero at $f = 0$ GHz, a mirrored version fits exactly. Moreover, the multiplication with the Sinc-pulse gives us a nice window.

In figure 4.5 this is illustrated. Still this leaves the Dirac delta function in the middle. To remove this peak, a second order polynomial estimate of the surrounding points can be made. The central peak is replaced by this estimate.

In order to remove the Sinc pulse, the second FFT of the sinc pulse can be subtracted from the second FFT of the PSD. Because the height of the Sinc pulse dependents on $\gamma$, this parameter must first be obtained. As the minimal point method showed good results for estimating $\gamma$, this method is used. In figure 4.6 the result is shown. Because the cosinus is multiplied with the Sinc pulse of frequency
Figure 4.5: Left: analysis and simulation of PMD mirrored, right the simulation and its Dirac delta function correction.

Figure 4.6: Resulting second FFT when enhancing the PSD. $\tau_0 = 1.84$ and $\gamma = 0.3$
\( f = 1/T_{\text{bit}} \), the peak is smeared out over three cosinuses at "frequencies": \( \tau_0 \), 
\( T_{\text{bit}} - \tau_0 \) and \( T_{\text{bit}} + \tau_0 \). This causes a bounce-back effect for small \( \tau_0 \) as the cosinus with frequency \( T_{\text{bit}} - \tau_0 \) starts moving towards \( \tau_0 \) for \( \tau_0 < T_{\text{bit}} \). This poses a restriction for this method, so \( \tau_0 > \frac{T_{\text{bit}}}{2} \).

To test the method, simulations were used where \( 0.55 T_b < \tau_0 < 2.50 T_b \) with stepsize \( 0.03 T_b \), for \( \gamma = 0.3 \) and \( \gamma = 0.5 \). The results are in table 4.2.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Avg. deviation</th>
<th>Max. deviation</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.2 %</td>
<td>17.9 %</td>
<td>3.7 ( \cdot 10^{-2} )</td>
</tr>
<tr>
<td>0.5</td>
<td>1.7 %</td>
<td>13.7 %</td>
<td>3.0 ( \cdot 10^{-2} )</td>
</tr>
</tbody>
</table>

The described second FFT method gives good results for \( \tau_0 > 0.5 T_{\text{bit}} \). However, signal degradation already starts at a \( \tau_0 \) of 10\% of \( T_{\text{bit}} \) and a \( \tau_0 \) larger than 30\% \( T_{\text{bit}} \) already is unacceptable, according to the norms of optical communication described in [A+03].

So there is a need for a method that's useable for smaller \( \tau_0 \). Below in fig. 4.7 two plots are shown for the second FFT of two cases of \( \tau_0 \) and \( \gamma \). We can clearly see

![Figure 4.7: Second FFT for two pairs of \( \tau_0 \) and \( \gamma \)](image)

\[ \epsilon = A \left( \frac{\tau_0}{M T_b} \right)^2 \gamma (1 - \gamma) \]  \hspace{1cm} (4.4)

To see if the deviation of the ideal curve, as shown in figure 4.7 is related to the penalty, described in formula 4.4, the dependency is plotted for different \( \tau_0 \) and
different \( \gamma \). As point for deviation measurement \( t = 1 \text{ ps} \) was chosen, because in this region the deviation is maximal. The figures in 4.8 show that by approximation

the deviation is dependent to the square of \( \tau_0 \) and the deviation is linearly dependent to \( \gamma (1 - \gamma) \). This implies that the deviation is linearly dependent to the penalty as described in formula 4.4 and thus, by approximation, it’s possible to retrieve the penalty caused by PMD by using this method.

To deploy this method, the deviation between the ideal second FFT and the one affected by PMD should be calculated in more than one point. Besides, to compensate for an offset in the second FFT, the deviations in the second interval, where \( 22 \text{ ps} < t < 27 \text{ ps} \) can be used. To be able to derive the penalty \( \epsilon \), the ratio between the measured deviation and the resulting penalty should first be known. As this ratio is different for every \( t \), the ratios for all \( t \)'s in these both intervals must be known. The ratios have been calculated by using simulation data for \( \gamma = 0.3 \) and \( \gamma = 0.5 \). In figure 4.9 the plots for the ratios are shown. The plots show that

the ratios are very similar for \( \gamma = 0.3 \) and \( \gamma = 0.5 \).

Now that the ratios are known and seem to be constant with respect to \( \gamma \), the penalty \( \epsilon \) can be derived by calculating the deviations in both intervals. Two intervals are used to be able to compensate for any offset in the second FFT. We call \( \epsilon_1 \) the measured penalty from interval one and \( \epsilon_2 \) the measured penalty from interval

Figure 4.8: Left: deviation as a function of \( \tau_0 \) for \( \gamma = 0.3 \), right: deviation as a function of \( \gamma (1 - \gamma) \) for \( \tau_0 = 0.4 T_{\text{bit}} \)

Figure 4.9: Ratios between penalty and measured deviation to ideal second FFT for left: \( 0 \text{ ps} < t < 4.5 \text{ ps} \) and right: \( 21 \text{ ps} < t < 26 \text{ ps} \).
two. The resulting measured penalty we call $\epsilon_m$. To optimize the compensation, a distribution must be found, such that:

$$\epsilon_m = \alpha \epsilon_1 + (1 - \alpha) \epsilon_2$$

The penalties have been measured using simulations and compared to the penalty that results from formula 4.4. An $\alpha$ of 0.86 was found to be optimal.

Now the system can be tried out on the simulations. The results are in table 4.3. The results show that this method is unsuitable for $\tau_0$ smaller than 0.2 $T_{\text{bit}}$.

Table 4.3: Relative errors in penalties measurement with the second FFT method PMD for small $\tau_0$

<table>
<thead>
<tr>
<th>$\tau_0/T_{\text{bit}}$</th>
<th>$\gamma = 0.3$ (%)</th>
<th>$\gamma = 0.5$ (%)</th>
<th>$\tau_0/T_{\text{bit}}$</th>
<th>$\gamma = 0.3$ (%)</th>
<th>$\gamma = 0.5$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>9681</td>
<td>35398</td>
<td>0.28</td>
<td>8.0</td>
<td>24.6</td>
</tr>
<tr>
<td>0.04</td>
<td>917</td>
<td>315</td>
<td>0.31</td>
<td>25.7</td>
<td>11.2</td>
</tr>
<tr>
<td>0.07</td>
<td>297</td>
<td>297</td>
<td>0.34</td>
<td>1.7</td>
<td>1.3</td>
</tr>
<tr>
<td>0.10</td>
<td>57</td>
<td>144</td>
<td>0.37</td>
<td>16.3</td>
<td>10.7</td>
</tr>
<tr>
<td>0.13</td>
<td>44</td>
<td>82</td>
<td>0.40</td>
<td>12.1</td>
<td>9.2</td>
</tr>
<tr>
<td>0.16</td>
<td>105</td>
<td>44</td>
<td>0.43</td>
<td>1.8</td>
<td>2.2</td>
</tr>
<tr>
<td>0.19</td>
<td>23</td>
<td>46</td>
<td>0.46</td>
<td>2.2</td>
<td>7.3</td>
</tr>
<tr>
<td>0.22</td>
<td>16</td>
<td>26</td>
<td>0.49</td>
<td>4.3</td>
<td>3.3</td>
</tr>
<tr>
<td>0.25</td>
<td>12</td>
<td>97</td>
<td>0.52</td>
<td>0.7</td>
<td>1.9</td>
</tr>
</tbody>
</table>

For the other points the method gives reasonably good results with a relative error of 25 % or less, let alone one exception. The huge errors for small $\tau_0/T_{\text{bit}}$ can be caused by the fact that the resulting penalty is very small, and therefore a small disturbance causes huge errors. Better results could probably be obtained by making use of more exact simulation data, which can be obtained by running with longer simulation time (more bits). Using measurements this would imply a longer measurement time.

### 4.3.3 Parameterisation of measurements

To see how this method performs in real time, the measurements of Oscar Diaz were used. Using a PMD emulator, measurements were taken from an optical system. Because of the low quality of the measurements second FFT analysis was only applied for a DGD bigger than half bittime. The results are in table 4.4. For 26 measurements the method gives an average deviation of 11.1 % and maximum deviation of 41.1 %.

### 4.3.4 Conclusions

In this section the effect of polarization mode dispersion was analyzed and methods for its characterization were treated.

Mathematical analysis showed that the ESA spectrum is modulated by a cosine term. PMD is described by it’s differential group delay ($\tau_0$) and it’s division of power over the two orthogonal axes ($\gamma$). Both these parameters are displayed by respectively the frequency and the amplitude of the cosine.

It is possible to find the parameters of PMD for a DGD bigger than 0.5. For smaller DGD the penalty can be retrieved, which is a function of both the DGD and
Table 4.4: Deviations in DGD measurement with the second FFT method for PMD measurements

<table>
<thead>
<tr>
<th>$\tau_0/T_{\text{bit}}$ measured</th>
<th>$\tau_0/T_{\text{bit}}$</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>0.52</td>
<td>21.2 %</td>
</tr>
<tr>
<td>0.70</td>
<td>0.67</td>
<td>3.9 %</td>
</tr>
<tr>
<td>0.75</td>
<td>0.52</td>
<td>30.7 %</td>
</tr>
<tr>
<td>0.77</td>
<td>0.55</td>
<td>28.5 %</td>
</tr>
<tr>
<td>0.78</td>
<td>1.10</td>
<td>41.1 %</td>
</tr>
<tr>
<td>0.86</td>
<td>1.16</td>
<td>35.1 %</td>
</tr>
<tr>
<td>0.89</td>
<td>0.83</td>
<td>7.2 %</td>
</tr>
<tr>
<td>0.94</td>
<td>1.04</td>
<td>10.6 %</td>
</tr>
<tr>
<td>1.02</td>
<td>1.16</td>
<td>13.9 %</td>
</tr>
<tr>
<td>1.02</td>
<td>0.92</td>
<td>10.1 %</td>
</tr>
<tr>
<td>1.07</td>
<td>1.13</td>
<td>5.7 %</td>
</tr>
<tr>
<td>1.18</td>
<td>1.22</td>
<td>3.7 %</td>
</tr>
<tr>
<td>1.24</td>
<td>1.01</td>
<td>18.6 %</td>
</tr>
<tr>
<td>1.29</td>
<td>1.28</td>
<td>0.4 %</td>
</tr>
<tr>
<td>1.33</td>
<td>1.41</td>
<td>5.8 %</td>
</tr>
<tr>
<td>1.43</td>
<td>1.44</td>
<td>0.5 %</td>
</tr>
<tr>
<td>1.49</td>
<td>1.13</td>
<td>24.1 %</td>
</tr>
<tr>
<td>1.77</td>
<td>1.74</td>
<td>1.5 %</td>
</tr>
<tr>
<td>1.81</td>
<td>1.68</td>
<td>7.1 %</td>
</tr>
<tr>
<td>1.93</td>
<td>1.71</td>
<td>11.3 %</td>
</tr>
<tr>
<td>2.10</td>
<td>2.14</td>
<td>1.9 %</td>
</tr>
<tr>
<td>2.11</td>
<td>2.08</td>
<td>1.4 %</td>
</tr>
<tr>
<td>2.11</td>
<td>2.14</td>
<td>1.4 %</td>
</tr>
<tr>
<td>2.25</td>
<td>2.26</td>
<td>0.6 %</td>
</tr>
<tr>
<td>2.32</td>
<td>2.29</td>
<td>1.1 %</td>
</tr>
<tr>
<td>2.33</td>
<td>2.29</td>
<td>1.6 %</td>
</tr>
</tbody>
</table>

the division of power over the orthogonal axes. This gives reasonably good results
for $\tau_0 > 0.2 T_{\text{bit}}$, the relative error is smaller than 25 %.

Using these methods, the effect of PMD can be retrieved in the range from where
it starts to disrupt the signal to total signal degradation.

4.4 Optical system with C-HOC

In this section a model for an optical system with coherent homowavelength
crosstalk (C-HOC) will be treated. For this impairment, the PSD of the electrical
field before the photodetector and of the resulting photo-current will be derived.

4.4.1 Calculation of the power density spectra

PSD in the optical domain (OSA)

The derivation of the PSD in the optical domain is performed in appendix A.5.1.
The PSD for CHOC can be derived by taking the PSD of a system without impair­
ments and multiplying it with the absolute square transfer function for CHOC:

$$S_{EE-\text{CHO C}} = S_{EE-\text{OS}} \cdot |H_{\text{CHO C}}(\omega)|^2$$
In the appendix it is derived that:

\[ |H_{\text{CHOH}}(\omega)|^2 = 1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega) \]

Using the above approach the following PSD results for an optical system without linewidth:

\[
S_{EE}(\omega) = \frac{1}{4} P_0 \left( A(\omega - \omega_0) + A(\omega + \omega_0) \right) \left( 1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega) \right)
\]

\[
A(\omega) = \pi \delta(\omega) + \frac{T}{2} \sin^2\left( \frac{\omega T}{2} \right)
\]

For a system with linewidth \( W \) the following PSD results:

\[
S_{EE}(\omega) = \frac{P_0}{T} \left( A_W(\omega - \omega_0) + A_W(\omega + \omega_0) \right) \left( 1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega) \right)
\]

\[
A_W(\omega) = -\frac{2 W^2}{(W^2 + 4 \omega^2)^2} + \frac{1 + TW}{W^2 + 4 \omega^2} +
\]

\[
e^{-\frac{TW}{2}} \left( \cos(T\omega) \frac{W^2 - 4 \omega^2}{(W^2 + 4 \omega^2)^2} - \sin(T\omega) \frac{4 \omega}{(W^2 + 4 \omega^2)^2} \right)
\]

**PSD in the electrical domain (ESA)**

In this paragraph the formula of the power spectrum for ESA will be derived. The full calculation can be found in appendix A.5.2. The analysis for for \( \tau_0 < T \) and one for \( \tau_0 \geq T \) are substantially different. Because the delay \( \tau_0 \) in almost all cases is larger than \( T \), only the second case: \( \tau_0 \geq T \) will be treated here. Neglecting the linewidth, the formula for the PSD is:

\[
S_{ff}(\omega) = \frac{P_0^2}{4} R^2 T \left( \text{CH}_1 \sin^2\left( \frac{T \omega}{2} \right) + \text{CH}_2 \cos(\tau_0 \omega) \sin^2\left( \frac{T \omega}{2} \right) + \right.
\]

\[
\text{CH}_3 \sin^2\left( \frac{\tau_0 \omega}{2} \right) + \text{CH}_4 \sin^2\left( \frac{T - \tau_0 \omega}{2} \right)
\]

with

\[
\tau_0 = \tau_0 \mod T_{\text{bit}}
\]

and with coefficients \( \text{CH}_{a1} - \text{CH}_{a4} \):

\[
\text{CH}_1 = 1 + 2 \sqrt{(1 - \gamma) \gamma \cos(\tau_0 \omega_0) + (1 - \gamma) \gamma (\cos(2 \tau_0 \omega_0) - 1)}
\]

\[
\text{CH}_2 = 2 \sqrt{(1 - \gamma) \gamma \cos(\tau_0 \omega_0) + (1 - \gamma) \gamma (\cos(2 \tau_0 \omega_0) + 3)}
\]

\[
\text{CH}_3 = \frac{1}{2} (1 - \gamma) \gamma (\cos(2 \tau_0 \omega_0) + 1) \left( \frac{\tau_0}{T} \right)^2
\]

\[
\text{CH}_4 = \frac{1}{2} (1 - \gamma) \gamma (1 + \cos(2 \tau_0 \omega_0)) \left( \frac{T - \tau_0}{T} \right)^2
\]

The formula with linewidth is even more complicated and can be found in the appendix.
4.4.2 Analysis of the spectrum

Influence of linewidth

Comparing the formulas of ESA with linewidth and without linewidth, the influence of linewidth can be seen to depend on $e^{-W \tau_0}$. For a typical linewidth of 10 MHz there is practically no difference when $\tau_0$ is small. When $\tau_0$ increases to 60 $T_b$ a difference of about 10 % is visible in the region from 0 to 1 GHz. In figure 4.10 the plots of the formulas for the PSD with linewidth and without linewidth are plotted.

![Figure 4.10: Plot of the spectrum of ESA of a system with CHOC with and without linewidth, $\tau_0 = 60 \, T_b$ and $\gamma = 0.1$.](image)

The influence of linewidth on the spectrum of OSA is comparable to the influence on the spectrum of OSA of a system without impairments as the PSD of CHOC is just the multiplication of the PSD of a system without impairments with a cosine function. In figure 4.11 an example is shown.

![Figure 4.11: CHOC OSA with linewidth of 10 MHz, $\tau_0 = 1.99 \, T_b$ and $\gamma = 0.1$.](image)
Second FFT method with OSA

Comparing the expression for the PSD for OSA for CHOC with the expression for the PSD for ESA for PMD, the formulas look very similar. Once again the delay of CHOC \( \tau_0 \) is expressed by the frequency of the cosine and the division of power is expressed as the amplitude of the cosine. This implicates that again a second FFT method may be applicable.

Again a Dirac delta function will give a disturbed FFT, so this peak has to be removed. In figure 4.12 a PSD of CHOC is shown, as well as a zoom to the Dirac delta peak where the replacement can be seen. As by PMD the FFT of the PSD

![Figure 4.12: Plot of the OSA of CHOC and its version with Dirac delta peak replacement. \( \gamma = 0.1 \) and \( \tau_0 = 25T_b \).](image)

can be used to retrieve \( \tau_0 \) and \( \gamma \). In figure 4.13 the second FFT is shown where clearly a peak can be seen at \( \tau_0 = 25T_b \).

![Figure 4.13: Plot of the second FFT of OSA for CHOC. \( \gamma = 0.1 \) and \( \tau_0 = 25T_b \).](image)

The method has been tried out on 27 simulations with \( \tau_0 \) from 1 \( T_b \) to 27 \( T_b \). In table 4.5 the results are shown for \( \gamma = 0.1 \) and \( \gamma = 0.3 \).

The results show that with this method, it is possible to obtain the parameters of this impairments with high accuracy.
Table 4.5: Results approximation of $\tau_0$ with the second FFT method on CHOC in the optical domain

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Avg. deviation (%)</th>
<th>Max. deviation %</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.08</td>
<td>0.51</td>
<td>$7.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.06</td>
<td>0.51</td>
<td>$5.4 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Because the resolution of an optical spectrum analyser is limited, $\tau_0$ can only be measured up to a certain value. The optical spectrum analysers available have a resolution of 0.01 nm at a wavelength of 1538 nm. This corresponds to a resolution of $\Delta f$ of 1.26 GHz. The nyquist theorem tells us that $\tau_0 = 2/\Delta f = 15.9T_b$. So the maximum measurable $\tau_0$ is $15.9T_b$.

**Second FFT method with ESA**

The effect of CHOC on the signal spectrum can be compared to that of PMD: the formula shows a cosine modulation of the Sinc pulse once again. The main difference with PMD is that a factor $\cos(\tau_0\omega_0)$ is present. Because $\omega_0$ is very large, a very small change in $\tau_0$ changes the factor of $\cos(\tau_0\omega_0)$. As the laser frequency is 193 THz, a change in $\tau_0$ of 0.01 ps changes this factor completely. In figure 4.14 four simulation results are shown. The instability is clear. In measurements however,

![Figure 4.14: Plots of simulations of CHOC ESA.](image)

there is no instability. The behaviour could be explained when we calculate the length of fiber that is needed to bridge 0.01 ps. Using the equation $v_p = c/n$, with $n = 1.47$ 0.01 ps corresponds to $2\mu m$. Depending on the setup, in an optical system the length of an optical path may not be that fixed.

In table 4.6 the results are shown using measurements for four different values of $\gamma$, five measurements per $\gamma$.  

32
Table 4.6: Results approximation of $\tau_0$ with the second FFT method on CHOC in the electrical domain

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Avg. deviation (%)</th>
<th>Max. deviation %</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10.3</td>
<td>21</td>
<td>0.63</td>
</tr>
<tr>
<td>0.41</td>
<td>10.4</td>
<td>22</td>
<td>0.65</td>
</tr>
<tr>
<td>0.35</td>
<td>12.0</td>
<td>24</td>
<td>0.68</td>
</tr>
<tr>
<td>0.2</td>
<td>14.3</td>
<td>27</td>
<td>0.78</td>
</tr>
</tbody>
</table>

4.4.3 Conclusions

In this section the effect of coherent homo wavelength crosstalk was analyzed and methods for its characterization were treated.

Analysis showed that the spectrum in the optical domain, which can be measured by an OSA, is the spectrum of a system without impairment, modulated with a cosine function. Detection of CROC in the optical domain gives very good results, but due to the limitations of the OSAs available measurement is bound to a maximum of $\tau_0 = 15.9T_b = 0.19$ ns.

Theory and simulation tells us that CHOC in the electrical domain causes an instable behaviour, but using measurements, this behaviour does not appear and with second FFT analysis CHOC can be detected.
Chapter 5

Mixed impairments

In optical systems it is quite probable that not only one impairment is present. Therefore it is important to find ways to not only distinguish impairments, but also to assess their influence on an optical communication system.

In this chapter the findings of the last chapter will be used to determine if it is possible to distinguish a combination of impairments. Not only the impairments PMD and CHOC will be treated, but chromatic dispersion as well.

First, using the analysis, expectations will be formulated about the performance of the methods developed in the last chapter on a system with more impairments. Then the methods will be tried on simulations where different impairments are mixed.

5.1 PMD with chromatic dispersion

As shown in the appendix, chromatic dispersion narrows the spectrum in the electrical domain, whereas PMD modulates it with a cosine function. Both impairments could occur in the order PMD/chromatic dispersion, chromatic dispersion/PMD or both at the same time.

Mathematical analysis shows that, because both impairments are writeable as transfer functions (see paragraph 3.3), the order in which they occur does not make a difference and gives the same result. Therefore a simulation setup has been realised in which first the effect of chromatic dispersion is inserted and then the effect of PMD.

In the last chapter it was shown that Gaussian pulses tend to spread when affected by chromatic dispersion. This effect has been used for the realization of this simulation. A realization was made in which the pulsshape of the signal source is a spreaded Gaussian pulse with the following equation [Agr97]:

\[ p(t) = A_0 e^{-\left(\frac{t}{T_0}\right)^2} \]

Two different values for the spreading \( T_0 \) constant have been used: 7 ps and 10 ps, for which 7ps corresponds to little dispersion and 10 ps corresponds to much dispersion. In figure 5.1 the two pulses are shown.

Using the second FFT method to detect the PMD on these simulations would theoretically yield good results, as the influence of chromatic dispersion does not change the modulation effect of PMD. However, for small differential group delay the PSD of PMD tends to narrow the PSD, as the minimum of the cosine function approaches near the 40 GHz point, which is a similar effect as chromatic dispersion.

To see what the effect is, simulations were used where \( 0.55 < \tau_0 < 2.50 \) with stepsize 0.03, for \( \gamma = 0.3 \) and \( \gamma = 0.5 \). In figure 5.2 the results are shown.
Figure 5.1: Two Gaussian pulses with spreading constant $T_0$ of 7ps and 10 ps

Figure 5.2: Deviation in measurement of penalty and $\tau_0$ for two magnitudes of chromatic dispersion with PMD for $\gamma = 0.3$ and $\gamma = 0.5$
In the figures the relative deviation from the actual penalty is shown against the
differential group delay $\tau_0$. For $0.5T_b < \tau_0 < 1.5T_b$ the error is higher than for a
system without chromatic dispersion. For $0.2T_b < \tau_0 < 0.5T_b$ though, the average
error is about as high as for a system without chromatic dispersion.

5.2 CHOC with chromatic dispersion

To retrieve the parameters for CHOC, a second FFT method was successfully applied
on a simulation in the optical domain. In the same manner as above, by using a
spreaded pulse as input, the method is tested on new simulation data. For CHOC
33 simulations were used with $1.51T_b < \tau_0 < 2.5T_b$ for $\gamma = 0.3$ as well as for $\gamma = 0.5$,
this time with a Gaussian input pulsetrain. The results are shown in table 5.1.

Table 5.1: Results approximation of $\tau_0$ with the second FFT method on CHOC
with chromatic dispersion in the optical domain, top: $T_0 = 7ps$, bottom: $T_0 = 10ps$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Avg. deviation (%)</th>
<th>Max. deviation</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.33</td>
<td>21.5</td>
<td>$8.74 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>0.3</td>
<td>2.10</td>
<td>12.5</td>
<td>$5.27 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Avg. deviation (%)</th>
<th>Max. deviation</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.6</td>
<td>14.8</td>
<td>$1.10 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>0.3</td>
<td>2.35</td>
<td>15.3</td>
<td>$6.44 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

Comparing these results with the results without chromatic dispersion, it is clear
that the results are much worse. Nevertheless the average errors are all below the
4% which indicates that this method is still accurate.

5.3 CHOC with PMD

As described before, the effect of CHOC on the spectrum in the electrical domain
is similar to PMD, actually in some cases exactly the same. Therefore theoretically
it is impossible to distinguish CHOC and PMD from the spectrum in the electrical
domain. Because CHOC also has its effect on the spectrum in the optical domain,
a combination of these two gives enough information to distinguish between PMD
and CHOC.

Nevertheless it would be interesting to be able to measure CHOC and PMD just
using the electrical domain, as in this case only one spectrum analyzer is needed.
The differential group delay caused by PMD is typically smaller than $2.5 T_b$ and
the delay caused by crosstalk normally is much larger. Therefore it can be assumed
that a detected delay greater than $2.5 T_b$ will probably be caused by crosstalk and
vice versa.

To see what the second FFT of a simulation with CHOC and PMD looks like,
in figure 5.3 three different situations are plotted.

In every plot we can see a peak at the DGD time and another peak at the
crosstalk delay time. As long as the crosstalk delay and the DGD are two bittimes
different, they can be detected in the second FFT plot.
Figure 5.3: Second FFTs of three different pairs of CHOC and PMD
5.4 Conclusions

Both PMD and crosstalk are well detectable when chromatic dispersion acts on the system. The results are less accurate, though. Also CHOC and PMD can be distinguished in the signal spectrum of the electrical domain, given that the crosstalk delay time is two bittimes larger than the DGD.
Chapter 6

Conclusions and recommendations

In this chapter the conclusions resulting from the work described in the last chapters are drawn and some recommendations are made.

6.1 Conclusions

In this report, transparent signal processing methods have been developed to characterize the impairments CHOC and PMD. The studied methods are mostly based on the Fast Fourier Transform.

The system under consideration operates at 40 Gb/s. The results of these methods are as follows:

- The Differential Group Delay (DGD) of PMD can be characterized in the range from where it starts to impair the signal quality to total signal degradation. The precision is about 20\% for small DGD values and about 10\% for larger DGD using simulations. Using measurements, for larger DGD the average accuracy is 11.1\%.

- CHOC can be characterized by the signal spectrum in the optical domain with a very high precision: less than 1\% according to simulations. The maximum measurable delay is restricted by the resolution of the optical spectrum analyzer. For modern optical spectrum analyzer equipment the maximum measurable delay is 0.19 ns (corresponding to a crosstalk path of 3.8 cm SMF). Characterization in the electrical domain yields an accuracy of about 10\% according to measurements.

A mixture of these impairments can also be detected. The results are as follows:

- CHOC and PMD can simultaneously detected, provided that the delay of CHOC is at least 2T_{bit} larger than the delay of PMD, which in normal situations is the case.

- PMD and chromatic dispersion can be detected simultaneously. The results of the detection do decay about 5\% for PMD as a consequence of the fact that the effect of chromatic dispersion on the signal spectrum resembles the effect of PMD.

- CHOC and chromatic dispersion can be detected simultaneously. In that case the accuracy of the detected CHOC decays to about 3\%.
6.2 Recommendations

Considering the results of this project and my experience in developing the signal processing methods, I make the following recommendations.

- Most of the conclusions of this work result from computer simulations. The developed methods should be tested out using measurements, especially in situations where a mixture of impairments affects the optical channel.

- The methods proved to be reasonably robust for a mixture of impairments. Nevertheless the influence of other impairments, such as ASE noise and other types of crosstalk (heterowavelength, uncoherent crosstalk) on the methods should be simulated and measured.

- Although the developed signal processing methods give quite accurate results, doing more research on the unique characteristic influence of each impairment could probably result in more precise characterization approaches.
Appendix A

Mathematical derivations

In this appendix different mathematical derivations will be treated for optical systems. First a few solutions will be given for some mathematical problems that these derivations have in common. Next different derivations will be given for different optical systems.

A.1 Mathematical solutions

In this section some solutions to mathematical problems will be treated which are used to do mathematical analysis on fibre transmission systems.

A.1.1 Autocorrelation of pulseform

To model a signalsource, often an equiprobable binary signal is used. By using on/of keying, a binary informationsource can be represented by a signal as is shown in figure A.1.

To simplify calculations on blocksignals, especially in calculating expectations, it can be useful to transform to symmetrical blocksignals as is shown in figure A.1. The relation between a blocksignal $x(t)$ taking the values 0 and 1 and a blocksignal $x_a(t)$ taking the values $-1$ and $1$ is:

$$x_a(t) = 2x(t) - 1$$

$$x(t) = \frac{1}{2}(x_a(t) + 1)$$

This transformation has two advantages. Firstly, the expected value of the symmetrical blocksignal $x_a(t)$ is zero:

$$\mathbb{E}\{x_a(t)\} = \frac{1}{2} \cdot (-1) + \frac{1}{2} \cdot 1 = 0$$

Figure A.1: Blocksignal representing a binary source and it's symmetrical equivalent
Secondly, the autocorrelation function of \( x_a(t) \), \( R_{\text{\( x_a x_a \)}}(\tau) = \mathbb{E}\{x_a(t)x_a(t + \tau)\} \) having period \( T \) is:

- For \(|\tau| > T\)
  \[ R_{\text{\( x_a x_a \)}}(\tau) = \mathbb{E}\{x_a(t)\} \mathbb{E}\{x_a(t + \tau)\} = 0 \cdot 0 = 0 \]

- For \( 0 < \tau < T \) one period can be assessed as the product \( x_a(t)x_a(t + \tau) \) is periodical with period \( T \). For part \( T - \tau \) of this period the same block overlaps which result to the expectation \( \frac{1}{2}(1 \cdot 1) + \frac{1}{2}(-1 \cdot -1) = 1 \), for part \( \tau \) both signals are independent, resulting to 0 as shown above.

\[
R_{\text{\( x_a x_a \)}}(\tau) = \frac{1}{T} \left( \int_0^T 0 \, d\tau + \int_\tau^T 1 \, d\tau \right) = \frac{T - \tau}{T}
\]

Because the stochastical process is stationary, the result is symmetrical around \( \tau = 0 \), so:

\[
R_{\text{\( x_a x_a \)}}(\tau) = \begin{cases} 
\frac{T - |\tau|}{T} & \text{if } |\tau| \leq T; \\
0 & \text{if } |\tau| > T.
\end{cases}
= \text{tri}\left(\frac{\tau}{T}\right) \tag{A.2}
\]

Using A.1, the autocorrelation function \( R_{\text{\( xx \)}}(\tau) \) is then:

\[
R_{\text{\( xx \)}}(\tau) = \mathbb{E}\{x(t)x(t + \tau)\} = \frac{1}{4} \mathbb{E}\{(x_a(t) + 1)(x_a(t + \tau) + 1)\} = \frac{1}{4} \left( R_{\text{\( x_a x_a \)}}(\tau) + 1 \right) = \frac{1}{4} \left( \text{tri}\left(\frac{\tau}{T}\right) + 1 \right) \tag{A.3}
\]

### A.1.2 Autocorrelation of multiplied shifted pulseform

When a system processing binary pulses contains delays, the following expression can occur:

\[ y(t) = x_a(t)x_a(t - \tau_0) \]

Taking the autocorrelation of this expression, the following expression arises:

\[ R_{\text{\( yy \)}}(\tau) = \mathbb{E}\{x_a(t)x_a(t - \tau_0)x_a(t + \tau)x_a(t + \tau - \tau_0)\} \]

In the work of Calleja [Cal99] it is assumed that \( \tau_0 \) is very large and thus \( x_a(t)x_a(t + \tau) \) is independent of \( x_a(t - \tau_0)x_a(t + \tau - \tau_0) \). More careful analysis shows though, that the autocorrelation shows periodical behavior with respect to \( \tau_0 \):

\[
R_{\text{\( x_a x_a \)}}(\tau_0) = \frac{\tau_0}{T_b} \text{tri}\left(\frac{\tau}{\tau_0}\right) + \left(1 - \frac{\tau_0}{T_b}\right) \text{tri}\left(\frac{\tau}{T_b - \tau_0}\right)
\]

Where \( \tau_0 = \tau_0 \mod T_b \).
A.1.3 Expectation of waveform with Gaussian phasenoise

When calculating with waveforms, delays and phasenoise, often formulas of the following form arise:

$$\mathbb{E}\{\cos(A + \phi(t))\}$$  \hspace{1cm} (A.4)

If it is assumed that $\phi(t)$ is an independent Gaussian noise source with average zero, this can be solved as:

$$\mathbb{E}\{\cos(A + \phi(t))\} = \mathbb{E}\{\cos(A)\} \mathbb{E}\{\cos(\phi(t))\} - \mathbb{E}\{\sin(A)\} \mathbb{E}\{\sin(\phi(t))\}$$

The second part of this equation can be solved as:

$$\mathbb{E}\{\cos(\phi(t))\} = \frac{1}{\sqrt{2\pi \sigma}} \int_{-\infty}^{\infty} \cos(\phi) e^{-\frac{\phi^2}{2\sigma^2}} d\phi$$

The result of this integral is:

$$\mathbb{E}\{\cos(\phi(t))\} = e^{-\frac{\sigma^2}{2}} = e^{-\frac{\mathbb{E}\{\phi(t)^2\}}{2}}$$

So

$$\mathbb{E}\{\cos(A + \phi(t))\} = \mathbb{E}\{\cos(A)\} e^{-\frac{\mathbb{E}\{\phi(t)^2\}}{2}}$$  \hspace{1cm} (A.5)

A.2 Optical system without impairments

In this section derivations will be given for the spectrum of an optical fiber system without impairments. First the spectrum in the optical domain (before the receiver) will be derived. Thereafter the spectrum in the electrical domain (resulting current after the receiver) will be derived.

The equation for the electrical field emitted by the laser is given by:

$$E_l(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t))$$

with $P_0$ the optical power and a binary equiprobable blocksignal $x(t)$. It is modulated with a signalsource with voltage $x(t)$. When this source is a binary blocksignal, only taking the values 0 and 1, the model of a Mach Zehnder modulator can be simplified by a product modulator. The following equation for the electrical field after modulation is obtained:

$$E_m(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t)) x(t)$$  \hspace{1cm} (A.6)

For an optical system without impairments, the fiber link is assumed to be ideal, so the distorted electrical field $E_d(t) = E_m(t)$.

A.2.1 Calculation of the PSD before the receiver (OSA)

From formula A.6 first the autocorrelation is calculated, then the spectrum is calculated with neglecting the phasenoise, and finally without neglecting the phasenoise.
Autocorrelation

Assuming $x(t)$ and the cosine part of A.6 are independent, the autocorrelation function is calculated by:

$$R_{EE}(\tau) = 2 P_0 E \{ \cos(\omega_0 t + \phi(t)) \cos(\omega_0 (t + \tau) + \phi(t + \tau)) \} R_{xx}(\tau)$$

This can be rewritten as:

$$R_{EE}(\tau) = 2 P_0 R_{xx}(\tau) \left( \frac{1}{2} \left( E \{ \cos(\omega_0 \tau + \phi(t + \tau) - \phi(t)) \} + E \{ \cos(\omega_0 (2t + \tau) + \phi(t + \phi(t + \tau)) \} \right) \right)$$

The expectation of the term $\cos(\omega_0 (2t + \tau) + \phi(t) + \phi(t + \tau))$ is zero as it is dependent on $t$, so the autocorrelation function becomes:

$$R_{EE}(\tau) = P_0 E \{ \cos(\omega_0 \tau + \phi(t + \tau) - \phi(t)) \} R_{xx}(\tau)$$

Assuming that $\phi(t)$ is a Gaussian, as shown in appendix A.1.3 it can be derived that:

$$E \{ \cos(\omega_0 \tau + \phi(t + \tau) - \phi(t)) \} = \cos(\omega_0 \tau) e^{-\frac{1}{2}E\{\phi(t) - \phi(0)\}^2} \cos(\omega_0 \tau) e^{-\frac{1}{2}E\{\phi(t)^2\} + E\{\phi(0)^2\} - 2E\{\phi(t)\phi(0)\}}$$

Assuming that the difference in autocorrelation $R_{\phi\phi}(0) - R_{\phi\phi}(\tau)$ rises linearly with the linewidth $W$ of the laser as: $R_{\phi\phi}(0) - R_{\phi\phi}(\tau) = \frac{1}{2} W|\tau|$, as demonstrated in [Cal99]:

$$E \{ \cos(\omega_0 \tau + \phi(t + \tau) - \phi(t)) \} = \cos(\omega_0 \tau) e^{-\frac{1}{2} W|\tau|}$$

Using equation A.3 we obtain:

$$R_{EE}(\tau) = P_0 \cos(\omega_0 \tau) e^{-\frac{1}{2} W|\tau|} R_{xx}(\tau)$$

$$= \frac{1}{4} P_0 \cos(\omega_0 \tau) e^{-\frac{1}{2} W|\tau|} \left( 1 + \text{Tri}\left(\frac{\tau}{\tau_s}\right) \right)$$

(A.7)

PSD neglecting the laser phase noise

If it is assumed that the phasenoise of the laser source is zero or constant, with $W = 0$ the autocorrelation function becomes:

$$R_{EE}(\tau) = \frac{1}{4} P_0 \cos(\omega_0 \tau) \left( 1 + \text{Tri}\left(\frac{\tau}{\tau_s}\right) \right)$$

(A.8)

Transforming this the frequency domain, we get:

$$S_{EE}(\omega) = \frac{1}{4} P_0 \mathcal{F}(\cos(\omega_0 \tau)) + \mathcal{F}(\cos(\omega_0 \tau) \text{Tri}\left(\frac{\tau}{\tau_s}\right))$$

$$\mathcal{F}(\cos(\omega_0 \tau)) = \pi (\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$\mathcal{F}(\cos(\omega_0 \tau) \text{Tri}\left(\frac{\tau}{\tau_s}\right)) = \frac{1}{2} (A(\omega - \omega_0) + A(\omega + \omega_0))$$

$$A(\omega) = \mathcal{F}\left(\text{tri}\left(\frac{\tau}{\tau_s}\right)\right) = T \text{sinc}^2\left(\frac{\omega T}{2}\right)$$
So the powerspectrum is:

\[ S_{EE}(\omega) = \frac{1}{4} P_0 \left( A(\omega - \omega_0) + A(\omega + \omega_0) \right) \]  
(A.9)

\[ A(\omega) = \pi \delta(\omega) + \frac{T}{2} \text{sinc}^2 \left( \frac{\omega T}{2} \right) \]  
(A.10)

**PSD including the laser phase noise**

The power density spectrum can be obtained by applying the Fourier transform on the following expression:

\[ R_{EE}(\tau) = \frac{1}{4} P_0 \cos(\omega_0 \tau) e^{-\frac{1}{2} |W| |\tau|} \left( 1 + \text{Tri} \left( \frac{\tau}{T} \right) \right) \]  
(A.7)

This yields:

\[ S_{EE}(\omega) = \frac{1}{4} P_0 \left( \mathcal{F} \left( \cos(\omega_0 \tau) e^{-\frac{1}{2} |W| |\tau|} \right) + \mathcal{F} \left( \cos(\omega_0 \tau) e^{-\frac{1}{2} |W| |\tau|} \text{Tri} \left( \frac{\tau}{T} \right) \right) \right) \]  

\[ \mathcal{F} \left( \cos(\omega_0 \tau) e^{-\frac{1}{2} |W| |\tau|} \right) = \frac{1}{2} \left( A_1(\omega - \omega_0) + A_1(\omega + \omega_0) \right) \]

\[ A_1(\omega) = \mathcal{F} \left( e^{-\frac{1}{2} |W| |\tau|} \right) = \frac{4W}{W^2 + 4\omega^2} \]

\[ \mathcal{F} \left( \cos(\omega_0 \tau) e^{-\frac{1}{2} |W| |\tau|} \text{tri} \left( \frac{\tau}{T} \right) \right) = \frac{1}{2} \left( A_2(\omega - \omega_0) + A_2(\omega + \omega_0) \right) \]

\[ A_2(\omega) = \mathcal{F} \left( e^{-\frac{1}{2} |W| |\tau|} \text{tri} \left( \frac{\tau}{T} \right) \right) \]

\[ \mathcal{F} \left( e^{-\frac{1}{2} |W| |\tau|} \text{tri} \left( \frac{\tau}{T} \right) \right) = \int_{-\infty}^{+\infty} e^{-\frac{1}{2} |W| |\tau|} \text{tri} \left( \frac{\tau}{T} \right) e^{-i\omega \tau} d\tau \]

\[ = \int_{-T}^{+T} e^{-\frac{1}{2} |W| |\tau|} (1 - |\tau/T|) e^{-i\omega \tau} d\tau \]

\[ = \int_{-T}^{+T} e^{-\frac{1}{2} |W| |\tau|} e^{-i\omega \tau} d\tau - \frac{1}{T} \int_{-T}^{+T} e^{-\frac{1}{2} |W| |\tau|} e^{-i\omega \tau} d\tau \]

\[ \int_{-T}^{+T} e^{-\frac{1}{2} |W| |\tau|} e^{-i\omega \tau} d\tau = 2 \int_{0}^{T} e^{-\frac{1}{2} W |\tau|} \cos(\omega \tau) d\tau \]

\[ = \frac{4W}{W^2 + 4\omega^2} + 4e^{-\frac{x}{2}} \frac{2\omega \sin(T \omega) - W \cos(T \omega)}{W^2 + 4\omega^2} \]

\[ \int_{-T}^{+T} e^{-\frac{1}{2} |W| |\tau|} \text{tri} \left( \frac{\tau}{T} \right) e^{-i\omega \tau} d\tau = 2 \int_{0}^{T} e^{-\frac{1}{2} W |\tau| \tau} \cos(\omega \tau) d\tau \]

\[ = \frac{8}{W^2 - 4\omega^2} \left( W^2 - 4\omega^2 \right)^2 + \frac{4 Te^{-\frac{x}{2}}}{W^2 + 4\omega^2} \left( 2 T \omega \sin(T \omega) - T W \cos(T \omega) \right) + \frac{8 e^{-\frac{x}{2}}}{(W^2 + 4\omega^2)^2} \left( 4 W \omega \sin(T \omega) - (W^2 - 4\omega^2) \cos(T \omega) \right) \]
So this yields for $A_2(\omega)$:

$$A_2(\omega) = \mathcal{F}\left(e^{-\frac{1}{2}W^2}\text{tri}\left(\frac{T}{T}\right)\right)$$

$$= \frac{4W}{W^2 + 4\omega^2} - \frac{8(W^2 - 4\omega^2)^2}{T(W^2 + 4\omega^2)^2} - \frac{8e^{-\frac{T^2}{2}}}{T(W^2 + 4\omega^2)^2} \left((W^2 - 4\omega^2)\cos(T\omega) + 4W\omega\sin(T\omega)\right)$$

Combining $A_1(\omega) + A_2(\omega) = A(\omega)$, $S_{EE}(\omega)$ can be written as:

$$S_{EE}(\omega) = \frac{P_0}{T} \left(A_W(\omega - \omega_0) + A_W(\omega + \omega_0)\right)$$

$$A_W(\omega) = \frac{TW}{W^2 + 4\omega^2} - \frac{W^2 - 4\omega^2}{(W^2 + 4\omega^2)^2} + e^{-\frac{T^2}{2}} \left(\sin(T\omega)\frac{4W\omega}{(W^2 + 4\omega^2)^2} - \cos(T\omega)\frac{W^2 - 4\omega^2}{(W^2 + 4\omega^2)^2}\right)$$

(A.11)

### A.2.2 Calculation of the spectrum of ESA

From formula A.6, the induced current by the receiver can be calculated by the formula $I_d(t) = R|E_d(t)|^2$. Using this, we obtain:

$$I_d(t) = R P_0 x(t) \left(1 + \cos (2\omega_0 t + 2\phi(t))\right)$$

If it is assumed that higher frequency terms are filtered out because of the lowpass filter property of the receiver, we obtain:

$$I_d(t) = R P_0 x(t)$$

Now the autocorrelation is calculated by:

$$R_{II}(\tau) = R^2 P_0^2 \mathbb{E}\{x(t)x(t+\tau)\}$$

(A.12)

Rewriting $x(t)$ to a blocksignal which is symmetrical, as is shown in A.1.1, we get:

$$R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \mathbb{E}\{1 + p_a(t) + p_a(t + \tau) + p_a(t)p_a(t + \tau)\}$$

The expectation of $p_a(t)$ is zero, so $R_{II}(\tau)$ becomes (see also appendix A.1.1):

$$R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \mathbb{E}\{1 + p_a(t)p_a(t + \tau)\}$$

(A.13)

Transforming this to the frequency domain, the spectrum is:

$$S_{II}(\omega) = \frac{1}{4} R^2 P_0^2 \left(\mathcal{F}(1) + \mathcal{F}\left(\text{tri}\left(\frac{T}{T}\right)\right)\right)$$

(A.14)

### A.3 Optical system with PMD

In this section derivations will be given for the spectrum of an optical fiber system with PMD. With this impairment, the optical field before the photodetector will be divided into components. It is impossible to measure these components in a running system. Therefore only the spectrum in the electrical domain will be derived.
A.3.1 Calculation of the spectrum of ESA

Formula A.6 shows the resulting electrical field after modulation:

\[ E_m(t) = \sqrt{2P_0} \cos(\omega_0 t + \phi(t)) \sqrt{x(t)} \quad (A.6) \]

Due to polarization mode dispersion, this field will be divided into two field components according to the relation:

\[
E_d(t) = \sqrt{1 - \gamma} E_m(t) x + \sqrt{\gamma} E_m(t - \tau_0) y
\]

\[
= \sqrt{2P_0} (\sqrt{1 - \gamma} \cos(\omega_0 t + \phi(t)) \sqrt{x(t)} x + \sqrt{\gamma} \cos(\omega_0(t - \tau_0) + \phi(t)) \sqrt{x(t - \tau_0)} y)
\]

Now calculating the resulting current from the photodetector with responsivity \( R \):

\[
I_d(t) = R |E_d(t)|^2
\]

\[
= RP_0 ((1 - \gamma)(1 + \cos(2(\omega_0 t + \phi(t)))x(t) + \gamma(1 + \cos(2(\omega_0(t - \tau_0) + \phi(t - \tau_0)))x(t - \tau_0))
\]

Because of the lowpass property of the receiver the terms with \( \cos(2(\omega_0 t + \ldots) \) are filtered out and we obtain:

\[
I_d(t) = RP_0 ((1 - \gamma)x(t) + \gamma x(t - \tau_0))
\]

The autocorrelation is:

\[
R_{II}(\tau) = \mathbb{E}\{I_d(t)I_d(t+\tau)\}
\]

\[
= R^2 P_0^2 \mathbb{E}\{(1 - \gamma)x(t) + \gamma x(t - \tau_0)\}(1 - \gamma)x(t + \tau) + \gamma x(t + \tau - \tau_0))\}\}
\]

Rewriting blocksignal \( x(t) \) to a blocksignal \( x_a(t) \) which is symmetrical, as is shown in A.1.1, we get:

\[
R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \mathbb{E}\{1 + (1 - \gamma)x_a(t) + (1 - \gamma)x_a(t + \tau) + (1 - 2\gamma + \gamma^2)x_a(t) x_a(t + \tau) + \gamma x_a(t - \tau_0) + \gamma (1 - \gamma)x_a(t)x_a(t + \tau - \tau_0) + \gamma^2 x_a(t - \tau_0)x_a(t + \tau - \tau_0)\}\}
\]

The expectation of single \( x_a(\ldots) \) terms is zero, so they can be cancelled. (see also appendix A.1.1) Calculating the expectation of the other terms, \( R_{II}(\tau) \) becomes:

\[
R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \left(1 + (1 - 2\gamma + \gamma^2) R_{x_a x_a}(\tau) + \gamma(1 - \gamma)(R_{x_a x_a}(\tau - \tau_0) + R_{x_a x_a}(\tau + \tau_0))\right)
\]

As is shown in A.1.1, this is equal to:

\[
R_{II}(\tau) = \frac{1}{4} R^2 P_0^2 \left(1 + (1 - 2\gamma + \gamma^2) \frac{1}{T} (p * p)(\tau) + \gamma(1 - \gamma) \frac{1}{T} ((p * p)(\tau - \tau_0) + (p * p)(\tau + \tau_0))\right)
\]

(A.15)

Transforming this to the frequency domain, the spectrum is:

\[
S_{II}(\omega) = \frac{1}{4} R^2 P_0^2 \left(2\pi \delta(\omega) + (1 - 2\gamma + \gamma^2 + 2\gamma(1 - \gamma) \cos(\omega \tau_0)) \frac{1}{T} \mathcal{F}(p(t))^2\right)
\]

(A.16)
Applying a blockform:

\[ p(t) = \begin{cases} 
1 & \text{if } |t| \leq \frac{T}{2}; \\
0 & \text{if } |t| > \frac{T}{2}.
\end{cases} \]

the following spectrum is obtained:

\[ S_{II}(\omega) = \frac{1}{4} R^2 P_0^2 \left( 2\pi \delta(\omega) + (1 - 2\gamma(1 - \gamma) + 2\gamma(1 - \gamma) \cos(\omega T_0)) T \sin(\frac{\omega T}{2}) \right)^2 \]  
(A.17)

### A.4 Optical system with chromatic dispersion

In this section mathematical derivations will be given for the spectrum of an optical fiber system with dispersion. First the spectrum in the optical domain (before the receiver) will be derived. Thereafter the spectrum in the electrical domain (resulting current after the receiver) will be derived.

For OSA as well as ESA the formulas of an optical system without impairments are used. To see the result of dispersion, as inputsequence is used which has the spectrum:

\[ x(\omega) = A_0 \]  
(A.18)

This is the spectrum of a spreaded Gaussian pulse as result of dispersion.

#### A.4.1 Calculation of the spectrum of OSA

We calculate from the formula after the modulator:

\[ E_m(t) = \sqrt{2F_0} \cos(\omega_0 t + \phi(t)) \sqrt{x(t)} \]  
(A.6)

In this formula two adaptions are made:

1. We call \( X(t) = \sqrt{x(t)} \) which is the (normalized) power of the signal
2. The phaseshift is assumed to be constant: \( \phi(t) = \phi_c \).

Furthermore for now an ideal fiber link is assumed, i.e. \( E_d(t) = E_m(t) \). With these adaptions the electrical field before the receiver comes:

\[ E_d(t) = \sqrt{2F_0} \cos(\omega_0 t + \phi_c) X(t) \]

Now first the autocorrelation is calculated, then the spectrum is calculated.

#### Autocorrelation

Assuming that the lasersource is statistically independent of the signalsource, the autocorrelation is calculated by:

\[ R_{EE}(\tau) = 2 F_0 \mathbb{E} \{ \cos(\omega_0 t + \phi_c) \cos(\omega_0 (t + \tau) + \phi_c) X(t)X(t + \tau) \} \]

\[ = F_0 \mathbb{E} \{ \cos(\omega_0 \tau) \} + \mathbb{E} \{ \cos(2\omega_0 t + \omega_0 \tau + 2\phi_c) \} \] \( R_{XX}(\tau) \)

The expectation of the cosinusterm with \( t \), \( \mathbb{E} \{ \cos(2\omega_0 t + \ldots) \} \) is zero.

\[ R_{EE}(\tau) = F_0 \cos(\omega_0 \tau) R_{XX}(\tau) \]  
(A.19)
Power spectrum

Continuing from formula A.19 the power spectrum is:

$$S_{EE}(\omega) = P_0 \mathcal{F}(\cos(\omega_0 \tau) R_{XX}(\tau))$$

$$= P_0 \mathcal{F} \left( \cos(\omega_0 \tau) \int_{-\infty}^{\infty} x(t) x(t + \tau) d\tau \right)$$

Here is used that the stochastic process is ergodic, so $\mathbb{E}(\tau) = A(\tau)$. Because it's at least wide sense stationair, also $R_{XX}(\tau) = R_{XX}(-\tau)$, so

$$S_{EE}(\omega) = P_0 \mathcal{F} \left( \cos(\omega_0 \tau) \int_{-\infty}^{\infty} x(t) x(t + \tau) d\tau \right)$$

$$= P_0 \mathcal{F} \left( \cos(\omega_0 \tau) \int_{-\infty}^{\infty} x(t) x(t - \tau) d\tau \right)$$

$$= P_0 \mathcal{F} \left( \cos(\omega_0 \tau) (X(\tau) * X(-\tau)) \right)$$

$$= \frac{P_0}{2} \left( |\tilde{X}(\omega - \omega_0)|^2 + |\tilde{X}(\omega + \omega_0)|^2 \right)$$

When taking equation A.18, $|\tilde{X}(\omega)|$ can be calculated:

$$|\tilde{X}(\omega)| = \frac{\sqrt{2\pi} A_0 T_0}{\sqrt{1 + \frac{C^2}{A_0^2}}} e^{-\frac{\omega^2 \tau^2}{2(1 + \frac{C^2}{A_0^2})}}$$

A.4.2 Calculation of the spectrum of ESA

To calculate the spectrum of ESA, we use formula A.12 which shows the autocorrelation of the resulting current in a system without impairments:

$$I_d(t) = RP_0 |X(t)|^2$$

The autocorrelation of a Gaussian pulse is again ergodic, and so $R_{xx}(\tau) = R_{xx}(-\tau)$. The powerspectrum can be derived as:

$$R_{11}(\tau) = R^2 P_0^2 R_{xx}(\tau)$$

Transforming to the frequencydomain:

$$S_{II}(\omega) = R^2 P_0^2 \mathcal{F}(R_{xx}(\tau))$$

$$= R^2 P_0^2 \mathcal{F}(R_{xx}(-\tau))$$

$$= R^2 P_0^2 \mathcal{F} \left( \int_{-\infty}^{\infty} x(t) x(t - \tau) d\tau \right)$$

$$= R^2 P_0^2 \mathcal{F} \left( x(\tau) * x(-\tau) \right)$$

$$= |I_d(\omega)|^2$$

Transforming expression A.18 to the time domain, calculating the absolute value and transforming back to the frequency domain, the following equation results for $S_{II}(\omega)$:

$$S_{II}(\omega) = R^2 P_0^2 \pi A_0^4 T_0^2 e^{-\frac{\omega^2 (\tau_0^4 + 2C^2 \tau_0^2 + (1 + C^2) \tau_0^2)}{2\tau_0^2}}$$
A.5 Optical system with coherent HOC

In this section mathematical derivations will be given for the spectrum of an optical fiber system with coherent HOC.

Formula A.6 shows the resulting electrical field after modulation:

$$E_m(t) = \sqrt{2P_0 \cos (\omega_0 t + \phi(t))} \sqrt{x(t)}$$  \hspace{1cm} (A.6)

Due to coherent HOC the electrical field will be distorted according to the following relation:

$$E_d(t) = \sqrt{1 - \gamma} E_m(t) + \sqrt{\gamma} E_m(t - \tau_0)$$

$$= \sqrt{2P_0} \left( \sqrt{1 - \gamma} \cos (\omega_0 t + \phi(t)) \sqrt{x(t)} + \sqrt{\gamma} \cos (\omega_0 (t - \tau_0) + \phi(t)) \sqrt{x(t - \tau_0)} \right)$$  \hspace{1cm} (A.20)

First the spectrum in the optical domain (before the receiver) will be derived, next the spectrum in the electrical domain (resulting electrical current after the receiver) will be derived.

A.5.1 Calculation of the spectrum of OSA

In this section two calculations are performed: one with neglection of the phaseshift $\phi(t)$ and one without neglection of the phaseshift.

The spectrum of OSA neglecting phaseshift

$$S_{EE-CHOC} = S_{EE-OS} \cdot |H_{CHOC}(\omega)|^2$$

It can be derived that:

$$|H_{CHOC}(\omega)|^2 = 1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega)$$

Using the above approach the following PSD results for an optical system without linewidth:

$$S_{EE}(\omega) = \frac{1}{4} P_0 (A(\omega - \omega_0) + A(\omega + \omega_0)) \left(1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega)\right)$$

$$A(\omega) = \pi \delta(\omega) + \frac{T}{2} \sin^2\left(\frac{\omega T}{2}\right)$$

For a system with linewidth $W$ the following PSD results:

$$S_{EE}(\omega) = \frac{P_0}{T} (A_W(\omega - \omega_0) + A_W(\omega + \omega_0)) \left(1 + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega)\right)$$

$$A_W(\omega) = -\frac{2W^2}{(W^2 + 4\omega^2)^2} + \frac{1 + TW}{W^2 + 4\omega^2} + e^{-\frac{2\pi}{T}} \left(\cos(\omega_0) \frac{W^2 - 4\omega^2}{(W^2 + 4\omega^2)^2} - \sin(\omega_0) \frac{4W\omega}{(W^2 + 4\omega^2)^2}\right)$$

A.5.2 Calculation of the PSD of the photo-current

In this paragraph the PSD of the photo-current is derived with and without the influence of linewidth.
Neglecting the linewidth

Neglecting the linewidth, the following autocorrelation function can be derived:

\[ R_{II}(\tau) = \frac{1}{8} R^2 P_0^2 \left( A_1 + A_2 R_{x_a x_a}(\tau) + A_3 \left( R_{x_a x_a}(\tau - \tau_0) + R_{x_a x_a}(\tau + \tau_0) + A_4 R_{x_a x_a x_a x_a}(\tau, \tau_0) \right) \right) \]

\[ A_1 = 2 + 2\gamma(1 - \gamma) \left( 1 + 2 R_{x_a x_a}(\tau_0) + \cos(2\tau_0 \omega_0) \left( 1 + 2 R_{x_a x_a}(\tau_0) \right) \right) + 4\sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \left( 1 + R_{x_a x_a}(\tau_0) \right) \]

\[ A_2 = 2 \left( 1 - \gamma(1 - \gamma) + 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) + \gamma(1 - \gamma) \cos(2\tau_0 \omega_0) \right) \]

\[ A_3 = 2 \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) + \gamma(1 - \gamma)(3 + \cos(2\tau_0 \omega_0)) \]

\[ A_4 = \gamma(1 - \gamma)(1 + \cos(2\tau_0 \omega_0)) \]

Transforming this to the frequency domain, the following PSD results:

\[ S_{II}(\omega) = \frac{R^2 P_0^2}{2 T} \left( \frac{T^2}{2} \sin^2 \left( \frac{\omega T}{2} \right) \left( 1 - 4 \gamma(1 - \gamma) + A_3 \cos(\omega \tau_0) + 1 \right) + A_4 \left( \frac{(T - \tau_0)^2}{4} \sin \left( \frac{\omega}{2}(T - \tau_0)^2 \right) + \frac{\tau_0^2}{4} \sin \left( \frac{\omega \tau_0^2}{2} \right) \right) \right) \]

With:

\[ \tau_0 = \tau_0 \mod T_0 \]

Including the linewidth

When the linewidth is included in the calculations, the following autocorrelation function results:

\[ R_{II}(\tau) = \frac{1}{8} R^2 P_0^2 \left( A_1 + A_2 R_{x_a x_a}(\tau) + A_3 \left( R_{x_a x_a}(\tau - \tau_0) + R_{x_a x_a}(\tau + \tau_0) + A_4 e^{-\frac{w x_T}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \right) + R_{x_a x_a}(\tau - \tau_0) + R_{x_a x_a}(\tau + \tau_0) + R_{x_a x_a x_a x_a}(\tau, \tau_0) \right) + A_5 e^{-\frac{w x_T}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \left( 1 + 2 R_{x_a x_a}(\tau_0) + 2 R_{x_a x_a}(\tau) + R_{x_a x_a}(\tau - \tau_0) + R_{x_a x_a}(\tau + \tau_0) + R_{x_a x_a x_a x_a}(\tau, \tau_0) \right) \]

\[ A_1 = 2 + 4 e^{-\frac{w x_T}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \]

\[ A_2 = 2 - 4 \gamma(1 - \gamma) + 4 e^{-\frac{w x_T}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \]

\[ A_3 = 2 \gamma(1 - \gamma) + 2 e^{-\frac{w x_T}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0) \]

\[ A_4 = \gamma(1 - \gamma) \]

\[ A_5 = \gamma(1 - \gamma) \cos(2\tau_0 \omega_0) \]
Transforming this to the frequency domain, the following PSD results:

\[
S_{ff}(\tau) = \frac{1}{4} R^2 P_2^2 \left( -\frac{B_1}{T(W^2 + \omega^2)^2} + \frac{B_2}{T(W^2 + \omega^2)^2} + \right.
\]
\[
B_3 \cos(\omega(T - \tau_0)) \frac{(W^2 - \omega^2)}{T(W^2 + \omega^2)^2} - B_4 \cos(\omega(T + \tau_0)) \frac{(W^2 - \omega^2)}{T(W^2 + \omega^2)^2} +
\]
\[
B_5 \cos(T \omega) \frac{(W^2 - \omega^2)}{T(W^2 + \omega^2)^2} + B_6 \cos(\omega(T - \tau_0)) \frac{(W^2 - \omega^2)}{T(W^2 + \omega^2)^2} +
\]
\[
B_7 \cos(\omega \tau_0) \frac{(W^2 - \omega^2)}{T(W^2 + \omega^2)^2} + B_8 T \sin^2 \left( \frac{T \omega}{2} \right) +
\]
\[
B_9 \left( \frac{\cos(\omega \tau_0)}{T \omega^2} + \frac{2 W^2 \cos(\omega \tau_0)}{T(W^2 + \omega^2)^2} \right) +
\]
\[
B_{10} 2 W \cos(\omega \tau_0) \frac{W^2 + \omega^2}{T(W^2 + \omega^2)^2} + B_{11} 2 T \cos(\omega \tau_0) \sin^2 \left( \frac{T \omega}{2} \right) -
\]
\[
B_{12} \frac{\omega \sin(T \omega)}{T(W^2 + \omega^2)^2} - B_{13} \sin(\omega(T - \tau_0)) \frac{\omega}{T(W^2 + \omega^2)^2} -
\]
\[
B_{14} \frac{\sin(\omega \tau_0)}{T(W^2 + \omega^2)^2} - B_{15} \sin(\omega(T - \tau_0)) \frac{\omega}{T(W^2 + \omega^2)^2} -
\]
\[
B_{16} \frac{W \sin(\omega \tau_0)}{T(W^2 + \omega^2)^2} - B_{17} \frac{\omega \sin(\omega \tau_0)}{T(W^2 + \omega^2)^2}.
\]

With as constants \( B_1 - B_{17} \):

\[
B_1 = 8 W^2 \gamma (1 - \gamma) (1 + e^{-2 W \tau_0} \cos(2 \tau_0 \omega_0))
\]
\[
B_2 = 4 \gamma (1 - \gamma) ((1 + T \omega) + e^{-2 W \tau_0} (1 - T \omega) \cos(2 \tau_0 \omega_0))
\]
\[
B_3 = \gamma (1 - \gamma) \left( e^{W(T - \tau_0)} + e^{-W(T + \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_4 = e^{-W \tau_0} \gamma (1 - \gamma) \cos^2(\tau_0 \omega_0)
\]
\[
B_5 = 2 \gamma (1 - \gamma) \left( e^{-W} + e^{W(T - 2 \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_6 = \gamma (1 - \gamma) \left( e^{-W(T - \tau_0)} + e^{W(T - \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_7 = \gamma (1 - \gamma) \left( e^{-W \tau_0} + e^{W(2 \tau_0 - \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_8 = 1 - 2 \gamma (1 - \gamma) + 2 e^{-\frac{\tau_0}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0)
\]
\[
B_9 = e^{-W \tau_0} \gamma (1 - \gamma) \left( \cos(2 \tau_0 \omega_0) + 1 \right)
\]
\[
B_{10} = e^{-W \tau_0} \gamma (1 - \gamma) \left( 2 W \left( \cos(2 \tau_0 \omega_0) - 1 \right) + \frac{1}{T} \left( \cos(2 \tau_0 \omega_0) + 1 \right) \right)
\]
\[
B_{11} = \gamma (1 - \gamma) + e^{-\frac{\tau_0}{2}} \sqrt{\gamma(1 - \gamma)} \cos(\tau_0 \omega_0)
\]
\[
B_{12} = 4 \gamma \gamma (1 - \gamma) \left( e^{-W} + e^{W(T - 2 \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_{13} = 2 W \gamma (1 - \gamma) \left( e^{-W(T - \tau_0)} + e^{W(T - \tau_0 - 2 \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_{14} = 2 W \gamma (1 - \gamma) \left( e^{-W \tau_0} - e^{W(2 \tau_0 - \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_{15} = 2 W \gamma (1 - \gamma) \left( -e^{-W(T - \tau_0)} + e^{-W(T + \tau_0)} \cos(2 \tau_0 \omega_0) \right)
\]
\[
B_{16} = 2 e^{-W \tau_0} \gamma (1 - \gamma) \left( \cos(2 \tau_0 \omega_0) + 1 \right)
\]
\[
B_{17} = 2 e^{-W \tau_0} \gamma (1 - \gamma) \left( \cos(2 \tau_0 \omega_0) - 1 \right)
\]
Appendix B

Simulations

To simulate an optical system, a simulator was designed using the C++ compiler. The target was to obtain the PSD before the optical receiver and after. Of course building a simulator is not something that can easily be explained in an appendix: lots of know how is needed, not only about the programming language, but also about numerical computations and how to improve algorithm speed.

To give an impression on how the simulator for this project was developed, the development, as well as the used models is treated. Finally the used parameters for the simulations are treated.

B.1 General setup

Basically every system can be modelled by nodes and edges between them. An optical system for example contains the nodes laser, electrical signal source, modulator, fiber, and receiver which can be connected by edges in this model. A simulation is performed by numerically calculating data which passes different nodes in a system using the mathematical model of every node.

The first decision to make when building a simulator is in which domain to simulate: the time domain or the frequency domain. To simulate in the frequency domain seems fairly easy: of most components the frequency response is known. Simulating would then imply passing the frequency response through all nodes. Because not all nodes can be modelled in the frequency domain, it was needed to simulate in the time domain. The steps are:

- Calculate for each node the realization until reaching the last node
- Compute the FFT to retrieve the PSD

The realizations of simulations were very irregular, which is a common phenomenon when taking FFTs over simulated data. To improve the clarity a FIR filter was used which took an average of every point and the neighbouring points.

After this operation, the realizations were very regular, but when doing more simulations using the same parameters, the results were somewhat different. This was caused by the fact that the realizations did not include sufficient bits. Changing the simulation time to simulate with a larger number of bits was not an option because of limited computer memory. To alleviate this problem more results were generated over which an average was taken, which solved the problem. Finally the steps taken by the simulator are:

- Calculate for each node the realization until reaching the last node
- Compute the FFT to retrieve the PSD
• Repeat the above steps to generate a number of realizations of the PSD
• Take an average over the different PSDs
• Apply FIR filtering to smoothen the result

B.2 Used models

B.2.1 Laser
\[ E_l(t) = A \cos(w_0 t + \phi(t)) \]
\( \phi(t) \) is the result of a Gaussian process.
\[ \phi(t) = \int_{-\infty}^{t} [\text{Gaussian process}] dt \]

B.2.2 modulator
\[ E_m(t) = E_l(t) x(t) \]

B.2.3 electrical signal source
The electrical signal source generates a NRZ (none return to zero) pattern of a set of pulses \( p(t) \), which can be chosen to be a blockwave or Gaussian:
\[ p(t) = A_0 e^{-\left( \frac{t}{\sigma_P} \right)^2} \]

B.2.4 receiver
\[ I_d(t) = R |E_d(t)|^2 \]

B.3 Used parameters

B.3.1 Resolution
To comply with nyquists sampling theorem, sampling a signal must be done at a frequency at least two times the largest frequency of the source. The source in an optical transmission system is the laser. Using the following relation:
\[ f = \frac{c}{\lambda} \]
with \( \lambda = 1538 \text{ nm} \), one can calculate the frequency of the laser is 195 THz. After modulation the frequency spreads. Taking into account different pulseforms, it can be calculated that less than 1% of the signal power falls after 200 THz, which makes a resolution of 400 THz acceptable.

B.3.2 Simulation time
To get a realization which coincides with reality, a realization should be generated with as many bits as possible. A boundary is formed by the memory of the machine on which this realization is generated as the memory storage needed depends on the simulation time and the resolution. Moreover, when performing a FFT, even more memory is needed which grows exponentially with the size of realization.
Tests revealed that a simulation time of 2.5 ns (100 bits at 40 GHz) approximately 50 MB was needed. As commercial simulators usually simulate with 64 bits, this is a reasonable simulation time.
B.3.3 Number of simulations

Doing several tests using above parameters, 50 simulations (which implies a simulation time of 5000 $T_b$) resulted in a stable PSD.

B.3.4 FIR filter

For the smoothing FIR filter a hanning filter was used with a length of five points.
Appendix C

Literature survey: methods for monitoring PMD in optical fibers

Introduction

Last July I started a research project in optical communication. As with most research project it is needed to perform a literature survey to obtain information about the treated subject. Performing this literature survey is not an easy task, as there are many information sources which yield numerous results, most useless and just a few useful.

To improve the effectiveness of a literature survey, in the library practical at the department of electrical engineering students are trained to obtain the basic knowledge about information sources, to do a literature survey using different methods and to set up a literature search project. The trained skills are then exercised by doing a literature survey for a project. My project is a graduation project on impairments in optical systems. This appendix gives an overview of the literature survey I performed and the yielded results.

The appendix contains a brief description of the graduation project, the assignment for the literature survey and the setup. Next methods such as the snowball method and the citation method are used. The resulting information is linked to the contents of the report of the graduation project and a conclusion is drawn.

Assignment

In this section both the assignment for the graduation project as the assignment for the literature survey will be treated. A table of contents of the resulting report is shown at the end of this section.

Abstract of the graduation assignment

Theoretical development and simulation of a supervision scheme to discriminate, identify and quantify the impact of several impairments and faults in a Wavelength Division Multiplexing (WDM) optical network/system in a transparent manner. In such a system, signal degradation due to dispersion, Polarization Mode Dispersion (PMD), non-linearities, crosstalk, Amplifier Spontaneous Emission noise (ASE), etc. are mixed in the optical fiber during propagation and for monitoring purposes
and determination of the optimum operation of the network the origin of such degradations must be located. By combining the mathematical analysis of the optical signal in both, optical domain and electrical domain without reaching the digital level, the performance of the supervision method is transparent to the bit rate and digital format of the transmission. In this project, three main phenomena are selected for study: PMD taking into account the contributions in first and second order, chromatic dispersion (CD) and homo and hetero-wavelength crosstalk (HOC, HEC).

**Assignment for the literature survey**

Purpose of the literature survey is to obtain information about how PMD is modelled and to learn about methods to retrieve the parameters of first order PMD in optical systems. The emphasis of these methods is on using spectral analysis which can be used to monitor such a system.

Look for publications in the last 20 years. Look for conference articles, articles in scientific journals and books. Material about specific measurement techniques is beyond the scope of the assignment.

**Table of contents of graduation report**

The table of contents for the graduation project report is the following:

1. Preface
2. Abstract
3. Introduction
4. Project framework
5. Modelling of optical transmission systems
6. Characterization of impairments
7. System with more impairments
8. Conclusion

**Setup of literature survey**

The first step in doing a literature survey is to use common information sources. These sources are mentioned in this section as well as the used keywords.

**Information sources**

The following information sources were used to obtain material:

- Online catalogue of TU/e library (VUBIS)
- IEEE-IEE electronic library online (IEEE Xplore)
- INSPEC (online)
- conference proceedings ECOC 2001 - 2003 (paper version)
- Science citation index, Science Citation Index Expanded, 1994 till june 2004
- Colleagues at the university
Used keywords

I used the following keywords:

- polarization mode dispersion, polarisation mode dispersion
- spectrum, spectral, RF spectrum
- DGD, differential group delay
- OSA, ESA
- monitoring

Selection criteria

The criteria for selecting publications were that the title must include a method for detecting PMD. Furthermore, the abstract must include spectral analysis or another new method for detection of this impairment. For the choice of selection, I used fulltext in which I checked if the method was non-intrusive or if the method could detect more impairments at the same time.

Results

In table C.1 the number of results per information source are shown. Using the search engines 11 results were depicted. Five of those I selected for further use: [PF94], [Sun01], [YSYW03], [EKTT03] and [B+03]. The other seven were more focusing on measurement techniques, or their emphasis was not on spectral analysis.

Table C.1: Number of results per information source (deduplicated in order of appearance)

<table>
<thead>
<tr>
<th>Info source</th>
<th>No. of results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vubis</td>
<td>0</td>
</tr>
<tr>
<td>IEEE Xplorer</td>
<td>5</td>
</tr>
<tr>
<td>INSPEC</td>
<td>2</td>
</tr>
<tr>
<td>proceedings ECOC</td>
<td>3</td>
</tr>
<tr>
<td>Science citation index</td>
<td>3</td>
</tr>
<tr>
<td>Colleagues</td>
<td>1</td>
</tr>
</tbody>
</table>

Snowball method

The results of the snowball method can be found in figure C.1. To start, I choose four publications, namely [YSYW03], [EKTT03], [B+03] and [Sun01]. The first three works treat the detection of impairments by spectral analysis. The last work treats PMD elaborately.

It clearly shows that all publications written before 2003 link back to [PW86]. The newer publications link back to papers which treat more detail like parts of their method and thus are not relevant for this research. The diagram illustrates how research in PMD detection seems to be having a revival in which newer authors don't seem to use the traditional literature.
Citation method

Most papers resulting from the performed searches are very recent. To do an effective citation method I used one of the more older, yet interesting papers. The paper [PF94] treats a method which uses spectral analysis and is linked to by the other found papers. Moreover the author Poole was coauthor of [PW86] which is linked to many times. Nevertheless this paper treats PMD more general and is therefore of less interest than [PF94]. The result of the citation method is in figure C.2.

It yields three papers. As [Cin04] is more focussing on the physical side of PMD measurement and [ST04] treats temporal behaviour I deselected them for further use. The paper [MB01] though, treats a different spectral method for PMD monitoring.
Figure C.1: Scheme of results of the snowball method

Figure C.2: Scheme of results of the citation method
Relationships found literature and contents report

From the last searches six papers are selected for further use. The relationships between the found literature and the contents of the report can be found in table C.2

Table C.2: Relations of found literature and contents report

<table>
<thead>
<tr>
<th>Chapter</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>[PF94]</td>
</tr>
<tr>
<td>Project framework</td>
<td>[PF94]</td>
</tr>
<tr>
<td>Modeling of optical transmission systems</td>
<td>[Sun01]</td>
</tr>
<tr>
<td>Characterization of impairments</td>
<td></td>
</tr>
<tr>
<td>System with more impairments</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td></td>
</tr>
</tbody>
</table>

In table C.3 the relations between the different subjects is shown.

Table C.3: Relations of found literature and subjects

<table>
<thead>
<tr>
<th>Chapter</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modelling of PMD</td>
<td>[PF94]</td>
</tr>
<tr>
<td>Methods for detection</td>
<td>[PF94]</td>
</tr>
<tr>
<td>Spectral analysis</td>
<td>[PF94]</td>
</tr>
</tbody>
</table>
Conclusions

In this appendix a literature survey was carried out to obtain information about PMD and methods using spectral analysis to be able to monitor it. A search using the conventional search engines yielded five useable results. The snowball diagram showed that only the older publications link together. In the last years more publications in this subject are published which do not have the usual connection to the traditional work.

The citation method yielded another useful result which sums up to six usable papers. Selecting the relevant publications there doesn’t seem to be a most important author. We can say though, that from Poole ([PF94], [PW86], [FP91]) are the most referenced papers related to PMD.
Bibliography


