Development of a flowmeter for vertical and inclines gas-liquid upward pipe flow

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Development of a flowmeter for vertical and inclined gas-liquid upward pipe flow

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Summary

The object of this investigation is to develop a flowmeter that can measure the superficial velocities of the separate phases in vertical and inclined gas-liquid pipe flow within 10% from their real values. The superficial velocity of a phase is hereby defined as the velocity of that phase in case of a single-phase flow.

An important effect in inclined air-liquid pipe flow is the slip-effect: there is a relative velocity between the different phases because of the buoyancy force. For bubble flow a theoretical slip model, called the FB model, has been developed based on a force balance for a single bubble. The bubble form is a very important input parameter to this model. For elongated bubble flow a semi-empirical slip model from literature, called the EB model, has been used.

For flow rate determination an approximate flow model has been developed. Important input parameters to this model are the slip and the pressure drops across a venturi and a straight pipe. Three versions of this approximate flow model have been used based on a different behaviour of the slip in the pipe and/or in the venturi: homogeneous (hom), constant slip (cs), variable slip (vs) in the venturi. These versions are labelled with FB or EB depending on what slip model has been used as input. Hereby the FB slip model is labelled with (spherical) or (capped) depending on what form of the bubbles has been assumed. Hereby for the case of the vs_FB model the bubbles at the inlet of the venturi are assumed spherically capped and in the throat of the venturi either spherical or spherically capped. These two models are therefore called vs_FB(spherical) and vs_FB(capped) respectively.

Vertical pipe flow experiments show that mainly two flow patterns occur: bubble flow and slug/churn flow. Inclined experiments show that the bubble flow pattern is taken over by the slug/churn flow pattern. Only at large liquid rates dispersed bubble flow appears.

In vertical air-liquid flow measurements the vs_FB(spherical) model predicts superficial liquid velocities within 10% from their reference values. For the slug/churn region (low reference rates) in inclined air-water and air-oil experiments the cs_EB and vs_EB model predict superficial liquid velocities within 10% from their reference values. For the bubble flow region in the inclined experiments all models (hom, cs_FB(capped), vs_FB(spherical) and vs_FB(capped)) predict superficial liquid velocities within 15% from their reference values.

For the superficial gas velocity both in vertical and inclined experiments all models predict the same behaviour. At low gas volume fraction the relative error is very large because then there is almost no gas in the pipe. For increasing gas volume fraction the relative error stabilizes. This stabilizing value increases for increasing reference liquid rate. This makes the approximate flow model not appropriate for determination of the gas flow.
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1 Introduction

The flow of mixtures of two (or more) phases occurs in many industrial processes and is called multi-phase flow. An example of multi-phase flow in the chemical industry is the transport of chemical multi-phase mixtures between two reactor vessels. An example of multi-phase flow in the oil industry is the pumping up of oil-water-gas mixtures from oil wells. It is the object of this investigation to develop a flowmeter that can measure the velocities of the separate phases in multi-phase inclined and vertical pipe flow within 10% from their real values. The practical use of such a flowmeter can be illustrated by the following example. When two oil wells are situated above each other two pipes can be connected to both wells. The production of the separate phases of each well then can be determined at the surface. However when for economical reasons only one pipe is connected to both wells to pump up the multi-phase mixture a multi-phase flow meter has to be placed in between the two wells. From this meter and the total production, that can be determined at the surface, then the production of the separate phases from both wells can be determined.

With respect to single-phase pipe flow multi-phase pipe flow is a complex flow. This complexity manifests itself in the several flow patterns that occur in multi-phase flow and the many effects that occur in these flow patterns. One of those effects is the relative velocity of the different phases in vertical and inclined multi-phase flow as a consequence of the buoyancy force, which is called slip. The occurrence of several flow patterns makes the development of one general flow model very difficult. Hereby it has to be noted that the transition between two flow patterns is not always clear. Therefore different flow models have to be developed for the different flow patterns together with transition criteria that mark the transitions between the different flow regimes. In this investigation two models have been investigated for two different flow patterns. The first model has been derived in literature and is discussed here. The second model is an approximate model which is based on the assumption that the mixture can be considered as one phase with a density that can vary as a consequence of a variable slip-effect. The slip is thus an input parameter to this approximate model. Therefore also models for the slip-effect will be investigated.

In this report the several flow patterns in inclined and vertical multi-phase pipe flow together with the slip-effect are discussed in chapter 2 in which a survey of literature is given. The two multi-phase flow models mentioned above are described in chapter 3. A description of the experimental setup has been given in chapter 4. In the chapters 5 and 6 the results of vertical and inclined multi-phase pipe flow experiments, in which the two flow models have been tested, are discussed. A discussion of the approximate flow model is given in chapter 7. Finally conclusions and suggestions have been given in chapter 8.
2 Literature survey of two-phase flow

2.1 Flow patterns in two-phase vertical and inclined pipe flow

In two-phase pipe flow important quantities, like pressure drop, strongly depend on the geometrical distribution of the phases. Therefore in literature the flow regimes of a two-phase pipe flow are categorized. In this section the flow patterns of a two-phase vertical, horizontal and inclined upward flow will be described.

flow regimes in vertical flow

In literature [3] five different flow patterns have been discerned in the case of vertical upward gas-liquid flow: bubble flow, finely dispersed bubble flow, slug flow, churn flow and annular flow. These patterns are schematically drawn in figure 2.1, except dispersed bubble flow.

![Flow patterns in vertical upward gas-liquid flow, except dispersed bubble flow](image)

**Figure 2.1.** Flow patterns in vertical upward gas-liquid flow, except dispersed bubble flow [3].

Bubble flow and finely dispersed bubble flow are both characterised by a continuous liquid flow in which bubbles are distributed uniformly. The boundary between both flows is indicated by a critical diameter size, as has been determined by Hinze et al. [3]. Above this critical size the bubbles are deformable and move in a zig-zag path. As a consequence some bubbles can coalesce into larger bubbles with a spherical cap (bubble flow). Bubbles smaller than the critical size are rigid spheres moving upward in a straight line (finely dispersed bubble flow).

In slug flow most of the gas is located in large axi-symmetric bubbles with a spherical nose and a flat tail, called Taylor bubbles, that move at a uniform velocity [3]. Their diameter is approximately the same as that of the pipe. Between the Taylor bubbles and the pipe wall a thin film of liquid is falling downwards. This film is free of dispersed bubbles. The Taylor bubbles are
separated by slugs of continuous liquid which bridge the pipe. The liquid slug contains small dispersed bubbles that are distributed uniformly over the slug, except for a region just behind the tail of the Taylor bubble where the gas concentration is much higher than in the bulk of the slug. This is caused by entrainment of gas from the back of the Taylor bubble by the falling liquid film.

The identification of churn flow is difficult since there are several definitions of this flow used in literature. Taitel et al. [3] define churn flow as the pattern where oscillatory motion of the liquid is observed. This occurs when, in a slug flow, the liquid slug is too short to support a stable liquid bridge between two successive Taylor bubbles. The liquid film, that flows around the bubble, then penetrates deeply into the liquid slug and transforms it into an aerated mixture. This mixture then collapses in a chaotic way. The liquid re-collects at the next slug moving upward, which is then doubled in volume (the same holds for the Taylor bubble). If this new formed slug is again too short to support a stable liquid bridge then the process of collapsing and re-collecting of the liquid slug will continue until a stable liquid slug has formed. In this way Taitel et al. [3] see churn flow as an entry region for slug flow. If the pipe is long enough churn flow will eventually go over into slug flow. In literature [3] churn flow is also addressed as annular-slug transition flow or froth flow.

In annular flow most of the liquid is located in a thin liquid film adjacent to the wall of the pipe. The gas phase flows at a large velocity through the center of the pipe with liquid droplets entrained in it. The liquid in the film and the liquid droplets experience an interfacial stress exerted by the gas flow. As a result of this the liquid phase flows upward against gravity. Because of the interfacial stress exerted on the liquid film the interface of the film has a wavy character.

**flow regimes in horizontal flow**

In horizontal flow four flow patterns can be discerned, as described by Taitel&Dukler [5] : dispersed bubble, intermittent (slug, elongated bubble), stratified (smooth and wavy) and annular dispersed flow. These flow patterns have been drawn schematically in figure 2.2.

![Figure 2.2](image-url)  
*Figure 2.2, The flow patterns in horizontal gas-liquid flow as described by Taitel&Dukler [5].*
In dispersed bubble flow the small rigid bubbles move, influenced by gravity, at the upper side of the pipe. Slug flow in horizontal pipes is defined by Taylor bubbles, separated by liquid slugs at the top of the pipe, leaving a liquid layer at the bottom. When the liquid slugs, that separate the gas slugs, contain no dispersed bubbles the slug flow is also addressed to as elongated bubble flow. The stratified pattern is defined by a gas layer at the top and a liquid layer at the bottom of the pipe. The interface between these layers can be smooth or wavy. At large gas rates annular dispersed flow results. In this flow regime the liquid is swept around the pipe wall because of the large gas velocity. The liquid layer at the bottom of the pipe is broader than that at the top because of gravity.

flow regimes in inclined flow

In inclined upward two-phase flow all the flow patterns occur that have been described for the cases of horizontal and vertical flow. These flow patterns however change in character as the inclination angle changes or even occur only for a certain range of inclinations. This is caused by the change of direction of the gravity vector relative to the pipe axis. The main character changes in the flow regimes as the two-phase flow is inclined will now be discussed below.

In inclined (dispersed) bubble flow the bubbles are distributed more uniformly over the cross-section of the pipe with respect to the horizontal case, in which the bubbles are situated at the top of the pipe. In inclined slug and annular flow the liquid layer at the bottom of the pipe decreases relative to the horizontal case while at the top of the pipe a (thin) liquid film starts to grow. The stratified flow pattern appears only for low inclinations from horizontal while the churn flow only appears for very large inclinations from horizontal. This is because gravity starts working as an ordering mechanism at lower inclinations from horizontal.

In general it can be stated that the phases in two-phase flow are distributed more uniformly over the cross-section of the pipe as the flow is inclined from horizontal.

2.2 Flow regime transitions

As already said in section 2.1 important quantities like pressure drop depend strongly on the specific flow regime. Therefore it is important to know when a flow pattern changes into another flow pattern. In literature these transitions are drawn in flow maps. These flow maps are either based on physical mechanisms or on experimental data. The disadvantage of experimental based flow maps is that these are only valid for the experimental conditions. To obtain general validity a flow map must therefore be based on the physical mechanisms that control the pattern transitions.

The coordinates that are used in experimental flow maps, described in literature, are either dimensional or dimensionless coordinates. As dimensional coordinates most of the time the superficial velocities are used. The superficial velocity of a phase is defined as the velocity in case of a single-phase flow. Dimensionless coordinates are used in the hope that the map also holds for other conditions than those which the map is based on. According to Taitel&Dukler [3] however the use of dimensionless coordinates, without a theoretical basis, is as general as that of dimensional coordinates.

In this section theoretical flow maps and the physical mechanisms they are based on will be discussed separately for the case of horizontal, vertical and inclined (upward) flow. These theoretical flow maps will also be compared with experimental flow maps available from literature.

vertical flow map

Taitel&Dukler [3] have developed a model for vertical gas-liquid flow in which the physical mechanisms are described that control the transitions between the flow patterns. In this model 5 different flow regimes are discerned, as described in section 2.1: bubble, dispersed bubble, slug, churn and annular dispersed. The transitions between these flow patterns are drawn in figure 2.3 for the case of water-air vertical upward flow (25°C, 1 atm. and 5 cm pipe diameter) with the superficial liquid and gas velocities $V_{SL}$ and $V_{SG}$ as coordinates. Comparison of this
theoretical flow map with the experimental flow map of Pushkin&Sorokin [3] shows reasonable agreement.

The physical mechanisms that control the transitions in figure 2.3 will be discussed next starting with the bubble flow pattern. Bubble flow exists at relatively low liquid and gas rates where dispersion forces and coalescence are both not dominant so that the bubbles are not too small like in dispersed bubble flow but also not too large like in slug flow. When however the gas rate is increased the frequency of collision of the bubbles increases and therefore also coalescence increases. This process of coalescence is enhanced when more bubbles exceed the critical diameter above which they start to move in a zig-zag path. As a consequence of this process of coalescence bubbles grow and a transition to slug flow occurs. Experimentally [3] it has been seen that this happens at a void fraction, also called gas holdup, of 0.3. Based on a critical gas holdup of 0.25 and an expression for the bubble rise velocity in stagnant fluid, derived by Harmathy et al (see next section), Taitel&Dukler [3] have derived the following expression for the bubble-slug transition, drawn in figure 2.3 as curve A:

\[ V_g = 3.0V_{w} - 1.15 \left( \frac{g \sigma \left( \rho_l - \rho_g \right)}{\rho_i^2} \right)^{1/4} \]  

(2.1)

with \( V_{sl} \) and \( V_{sg} \) as the superficial liquid and gas velocity, \( g \) as the gravitational acceleration, \( \sigma \) as the interfacial tension and \( \rho_l \) and \( \rho_g \) as the liquid and gas density. From this equation it appears that at large liquid rate curve A is linear in log-log scale and that the bubble-slug transition at low liquid rate is approximately independent of the liquid rate.

![Flow Map](image)

**Figure 2.3,** Theoretical flow map by Taitel&Dukler [3] for vertical upward gas-liquid flow (25°C, 1 atm. and 5 cm pipe diameter).

When in bubble and slug flow the liquid flow rate is increased turbulence can overcome coalescence and break up the relatively large bubbles into small discrete bubbles which are typical for dispersed bubble flow. Hinze et al. [3] have derived an expression for the maximum diameter
of these bubbles from a balance between turbulence and interfacial tension. From comparison of
this maximum diameter and the critical diameter, above which zig-zag movement and thus
consequently a higher probability for coalescence occurs, Taitel&Dukler [3] have derived semi­
empirically the following expression for the bubble/slug-dispersed bubble transition, drawn in
figure 2 as curve B:

\[ V_{sl} + V_{sg} = 4.0 \left( \frac{D^{0.429} (\sigma / \rho_l)^{0.089}}{\nu_l^{0.072}} \left[ \frac{g (\rho_l - \rho_g)}{\rho_l} \right]^{0.46} \right) \]  

with \( D \) as the diameter of the pipe, \( \nu_l \) as the kinematic viscosity of the liquid and the factor 4 as
adimensional constant. From this equation it can be seen that transition B is dependent on pipe
geometry as well as on fluid properties.

When the gas rate in dispersed bubble flow is increased at a certain moment maximum
packing of the bubbles is reached. When bubbles are assumed spherical and ordered in a cubical
lattice then it appears that the maximal possible gas holdup is 0.52. At higher gas holdups the
dispersed bubbles are forced into coalescence which causes the transition to slug flow. This
transition is drawn in figure 2.3 as curve C which has a linear slope in log-log scale.

As described in section 2.1 churn flow is considered an entry flow for slug flow.
Taitel&Dukler [3] have derived the following expression for the length \( l_s \) over which churn flow
stabilizes:

\[ \frac{l_s}{D} = 40.6 \left( \frac{V_{sl} + V_{sg}}{gD} + 0.22 \right) \]  

When this stabilizing-length \( l_s \) equals the distance \( l_E \) of the observation point from the inlet then
transition of slug flow into churn flow occurs at that observation point. Equation (2.3), with \( l_s \)
substituted by \( l_E \), thus gives the condition for slug-churn transition which is drawn in figure 2.3 as
curve D. To the left of curve D then slug flow appears (then \( l_s < l_E \)) and to the right then churn
flow (then \( l_s > l_E \)).

For relatively large gas flow rates dispersed bubble and slug flow develop into annular
flow. The fast moving gas then forces the liquid towards the wall of the pipe with some liquid
drops entrained in the gas core, as described in section 2.1. This transition will only be possible
when the liquid film and the entrained drops are lifted against gravity by the shear exerted by the
gas. From the force balance on a single drop Taitel&Dukler [3] have derived the following
expression for the transition to annular flow, drawn in figure 2.3 as E:

\[ V_{sg} = \left( \frac{4k \sigma g (\rho_l - \rho_g)}{3C_D \rho_g^{2/3}} \right)^{1/4} \]  

with \( k \) and \( C_D \) as constants. Equation (2.4) shows that the transition to annular flow is independent
of the liquid flow rate.

When the pipe diameter becomes smaller than a certain critical value bubble flow
disappears completely. This can be understood as follows. As described in section 2.1, bubble flow
consists of discrete deformable bubbles with occasionally a larger Taylor-bubble like bubble. The
rise velocity \( U_{db} \) of these discrete bubbles is independent of the pipe diameter (see next section)
while that of the larger Taylor-bubble like bubbles \( U_{TB} \) depends linearly on the square root of the
pipe diameter (see next section). When the diameter of the pipe is so small that \( U_{TB} < U_{db} \) then the
small discrete bubbles approach the back of the Taylor bubble and coalesce with it. Bubble flow
then develops into slug flow. Taitel&Dukler [3] have derived the following expression for this
transition condition:

6
From the above it follows that for this particular transition experimental data for small pipes cannot be scaled up to larger ones without physical understanding.

**horizontal flow map**

The physical mechanisms that control the transitions in horizontal gas-liquid flow have been discussed in a theoretical model by Taitel & Dukler [5]. Hereby it is assumed that there are 4 flow regimes present in horizontal gas-liquid flow, as described in section 2.1: stratified (smooth, wavy), intermittent (elongated bubble, slug), annular dispersed and dispersed bubble. For the case of horizontal air-water flow (25°C, 1 atm., 2.5 cm pipe diameter) the resulting theoretical flow map is given in figure 2.4 together with an experimental flow map by Mandhane et al. [5]. The coordinates of this flow map are the superficial gas and liquid velocities $U_{sg}$ and $U_{sl}$ given in log-log scale. From figure 2.4 it can be concluded that the theoretical and experimental flow maps show reasonable agreement. The mechanisms that control the different pattern transitions, that are marked by A to D, will now be discussed below.

![Figure 2.4](image)

**Figure 2.4.** Comparison of the theoretical flow map of Taitel & Dukler [5] and the experimental flow map of Mandhane et al. [5] for the case of horizontal air-water flow (25°C, 1 atm., 2.5 cm diameter pipe).

For analyzing the different mechanisms that control the pattern transitions Taitel & Dukler start from the stratified flow pattern. This flow pattern will go over into intermittent or annular dispersed flow when a finite wave on the stratified interface can grow. This condition is met whenever the pressure drop above the wave, due to the acceleration of the gas above the wave, lifts the wave against gravity. In figure 2.4 this condition is given as A. Whether the flow has developed into intermittent or annular dispersed depends on the height of the stratified interface in the pipe. When this height exceeds half the diameter of the pipe intermittent flow is reached. When the stratified interface is beneath the centerline of the pipe annular dispersed flow is reached. This can be explained as follows. When a wave starts to grow it has to be supplied with liquid from the film next to the wave where a trough is formed. When the wave reaches the top
before the trough reaches the bottom of the pipe the flow is blocked and intermittent flow is
developed. In the opposite case blocking of the flow is impossible and the liquid is swept up and
around the pipe. The condition for which the stratified interface is situated at the centerline of the
pipe is marked as B in figure 2.4.

For the transition from stratified smooth to stratified wavy also the generation of waves is
important. These waves are generated by gas flow which is high enough to cause waves but too
low for the transition to intermittent or annular dispersed flow. It is generally assumed that waves
will be developed when shear work on a wave balances the viscous dissipation in the wave. This
transition is marked in figure 2.4 by C.

At high liquid rate and low gas rate the gas bubbles in intermittent flow become small.
Turbulence then can break these bubbles and mix them with the liquid layer by overcoming the
buoyancy forces that try to keep the gas at the upper side of the pipe. Dispersed bubble flow is
then developed. This transition is marked by D in figure 2.4.

Inclined flow map
Barnea et al. [2] have derived flow pattern transitions in inclined upward gas-liquid flow. These
transition criteria will now be discussed and compared with experimental data in a flow map.

The inclined transitions of Barnea et al. [2] are modifications of the transitions for
horizontal and vertical gas-liquid flow derived by Taitel&Dukler [3,5]. When modifying these
transitions in horizontal and vertical flow for the inclined case it should always be questioned
whether these modified mechanisms are applicable to inclined flow and if so up to what
inclination range. According to Barnea et al. [2] it appears that the modified vertical mechanisms
can be applied up to 70° inclination from vertical. For angles larger than 70° inclination from
vertical the modified horizontal mechanisms are applicable. Because the modified vertical
mechanisms are applicable to a much broader range of inclinations than the modified horizontal
mechanisms in this investigation only the modified vertical mechanisms will be discussed. The
flow map, in which these modified vertical transition criteria are given as a function of the
superficial liquid and gas velocities $V_{sl}$ and $V_{sg}$ in log-log scale, has been drawn in figure 2.5
together with experimental data for three different inclination angles $\alpha$ from vertical. Comparison
of the experimental data with the theoretical transitions shows reasonable agreement. The
theoretical transition criteria will now be discussed separately below.

![Flow Map](image)

**Figure 2.5.** Comparison of flow maps of the modified vertical (d,e and f) mechanisms with
experimental flow maps by Barnea et al. [2] for air-water flow (25°C, 1 atm., 5cm pipe diameter) for 3 different inclinations (a: $\alpha=70^\circ$, b: $\alpha=40^\circ$, c: $\alpha=20^\circ$).
For the bubble slug transition Barnea et al. [2] have arrived at the following expression, based on the vertical bubble-slug transition criterion derived by Taitel et al.:

\[ V_{sl} = 3V_{sg} - 1.15 \left( \frac{g\sigma(\rho_l - \rho_g)\cos\alpha}{\rho_l^2} \right)^{0.4} \] (2.6)

with \( \alpha \) as the inclination angle from vertical, \( \sigma \) as the surface tension, \( g \) as the gravitational acceleration and \( V_{sl} \) and \( V_{sg} \) as the superficial liquid and gas velocity respectively. This transition has been given in figure 2.5 as line 1. It has to be noted that this transition criterion is obtained from the vertical case by substituting the gravitational acceleration \( g \) by \( g\cos\alpha \). Further it has to be noted that this transition criterion is only valid for inclinations below \( \alpha = 20^\circ \), as has been determined by Barnea et al. [2], because for \( \alpha \) larger than this value the bubble flow region does not exist. This is because then the buoyancy force, that tries to keep the bubbles at the upper side of the pipe, overcomes the turbulent force.

For the transition to dispersed bubble flow in an inclined pipe Barnea et al have arrived at the following criterion:

\[ \left( \frac{\sigma}{(\rho_l - \rho_g)g} \right)^{0.5} \left( \frac{\rho_l}{\sigma} \right)^{0.5} \left( \frac{V_l^{0.2}}{D^{1.2}} \right)^{0.4} \left( V_{sl} + V_{sg} \right)^{0.12} = 1.49 + 8.52 \left[ \frac{V_{sg}}{V_{sg} + V_{sl}} \right]^{0.5} \] (2.7)

with \( D \) as the diameter of the pipe. This transition criterion is given in figure 2.5 as line 2. Though equation (2.7) differs from equation (2.2) for the transition to dispersed bubble flow in vertical pipes, it shows no dependence on the inclination angle. This is surprising because this criterion is based on a balance between turbulence and coalescence, which is influenced by the buoyancy force in inclined situations. Because the influence of the buoyancy force on the coalescence depends on the inclination angle it may be expected that the transition criterion also depends on the inclination angle.

It has to be noted that the transition criterion (2.7) is only valid for gas holdups smaller than 0.52. As for the vertical case then the maximum packing of the (spherically assumed) bubbles is reached so that the bubbles are forced into coalescence despite the turbulent break-up. This dispersed bubble-slug flow transition is therefore also for the inclined case given by the condition \( \alpha_g = 0.52 \). This condition is given in figure 2.5 by 3.

For the transition to annular flow Barnea et al. [2] have arrived at the following expression for the inclined case:

\[ V_{sg} = 3.1 \left( \frac{g\sigma(\rho_l - \rho_g)\cos\alpha}{\rho_l^2} \right)^{0.4} \] (2.8)

As for the bubble-slug transitions this criterion is obtained from the vertical case by substituting the gravitational acceleration \( g \) by \( g\cos\alpha \).

For the slug-churn no quantitative expression has been derived by Barnea et al. [2]. From experiments it appears that even small deviations from vertical already cause the extent of the churn flow region to decrease significantly. This is because then the buoyancy force suppresses the chaotic nature of the churn flow. It appears that the churn flow region disappears completely for inclination angles lower than 70° inclined from vertical.
2.3 Slip effect in two-phase bubble flow

In this section an important effect in multi-phase flow will be discussed, namely the slip-effect. The definition of this effect will be given in subsection 2.3.1 together with the definitions of other quantities of interest. After this the slip-effect for the cases of (dispersed) bubble flow (subsection 2.3.2) and elongated bubble flow (subsection 2.3.3) will be discussed. Hereby both models from literature and a theoretical model based on a force balance on a single bubble, which will therefore be addressed as the FB model, will be considered.

2.3.1 Definitions

Important quantities in gas-liquid pipe flow are the holdup and the volume fraction of a phase. The holdup of the gas (liquid) phase, written as \( \alpha_g (\alpha_l) \), is defined as the local fraction of the gas (liquid) phase in the pipe. The volume fraction of the gas (liquid) phase GVF (LVF) is defined as the ratio of the volume rate of that phase and the total volume rate. For the case of the gas and liquid phase the definitions for the holdups and volume fractions can be written as follows:

\[
GVF = 1 - LVF = \frac{V_{sg}}{V_{sg} + V_{sl}}
\]

(2.9)

\[
\alpha_g = \frac{V_{sg}}{V_{ag}}, \quad \alpha_l = \frac{V_{sl}}{V_{al}}
\]

(2.10)

with \( V_{sg} \) and \( V_{sl} \) as the superficial gas and liquid velocity and \( V_{ag} \) and \( V_{al} \) as the actual gas and liquid velocities.

As said above an important quantity in gas-liquid upward bubble flow is the slip between the phases. This slip effect refers to the physical process in which the light phase rises relative to the heavy phase as a consequence of the buoyancy force. The slip factor \( S \) is defined as follows:

\[
S = \frac{V_{slip}}{V_{ag}} = \frac{V_{al}}{V_{ag} + V_{slip}}
\]

(2.11)

with \( V_{slip} \) as the velocity of the gas bubbles relative to the liquid. As a consequence of the slip effect there is a difference between the holdup of a phase and its corresponding volume fraction. From the definitions given above the following relation between the volume fractions, the holdups and the slip can be derived:

\[
\alpha_g = \frac{S \cdot GVF}{S \cdot GVF + LVF}
\]

(2.12)

From this equation it follows that the gas holdup will always be smaller or equal to the gas volume fraction GVF because the slip factor will always be smaller than 1 (the gas phase always flows faster upward than the liquid phase).
2.3.1 slip-effect in dispersed bubble flow

In this subsection several models for the slip-effect in (dispersed) bubble flow will be discussed. First a theoretical model, called the FB model, based on a force balance on a single bubble will be discussed. After this several empirical and theoretical models, mentioned in literature, will be discussed.

FB-model: force balance on a single bubble

An expression for the slip of a bubble in gas-liquid bubble flow can be derived by considering a force balance on a single bubble (see Appendix B). The forces that appear in this force balance are the buoyancy force, the gravitational force and a drag force. The following expression for the slip velocity of a single bubble then results:

\[ V_{\text{slip}} = \frac{8r|\nabla p|}{3C_D \rho_t \left(1 - \frac{\rho_g}{\rho_t}\right)} \]  

with \( r \) as the radius of the bubble, \( C_D \) as a drag coefficient, \( |\nabla p| \) as the absolute pressure gradient and \( \rho_g \) and \( \rho_t \) as the densities of the gas and liquid phase. An expression for the maximum bubble radius \( r_{\text{max}} \) has been derived semi-empirically by Barnea et al. [17] from a balance between turbulent break-up, coalescence and interfacial tension:

\[ r_{\text{max}} = \frac{1}{2} \left[ 0.725 + 4.15 \cdot GVF^{0.5} \right] \left(\frac{\sigma}{\rho_t}\right)^{0.6} \left(2f_m \frac{V_m^2}{D}\right)^{-0.4} \]

with \( D \) as the diameter of the pipe, \( \sigma \) as the interfacial tension, \( f_m \) as the mixture friction factor (which will be discussed in section 2.4) and \( V_m \) as the mixture velocity (\( V_{\text{slip}} + V_{\text{sg}} \)).

The drag coefficient \( C_D \) for a single bubble moving in liquid is drawn in figure 2.6 as a function of the slip Reynolds number (\( \text{Re} = 2\rho|V_{\text{slip}}/\mu_l| \)) with \( \mu_l \) as the viscosity of the liquid phase. This figure shows a standard drag curve for the case of a solid sphere and two drag curves for the cases of pure and contaminated (containing impurities) liquid. These drag curves can be understood by considering the form of the bubbles and the purity of the liquid.

![Figure 2.6](image-url)  

The drag coefficient \( C_D \) as a function of the Reynolds number [17].
At low slip Reynolds number the bubbles are spherical. When these bubbles can be considered as rigid spheres the drag coefficient is, according to Stokes, $24/\text{Re}$. When the gas in the bubble is assumed to circulate as a consequence of the shear exerted by the liquid on the bubble interface then the drag coefficient is smaller and equals $16/\text{Re}$. When is assumed that the energy for the viscous dissipation is produced by the work done against the drag force on the bubble interface then the drag coefficient is, according to Levich et al. [17], $48/\text{Re}$.

At larger slip Reynolds number ($\text{Re} \approx 500$) the bubbles start to deform. The critical Reynolds number can be derived by considering the pressure variation along the interface, which tends to deform the bubble, against the superficial tension, which wants to keep the bubble spherical. It appears that, because of this deformation, the drag coefficient increases considerably.

At larger slip Reynolds number the bubble has the form of a spherical cap. The flow separates from the edges of the bubble cap, causing a wake to develop. Thus besides the deformation of the bubble from spherical the drag coefficient is also increased by a wake-effect. The drag coefficient finally has a maximum limit of 2.6.

The difference in drag behaviour of bubbles in pure and contaminated liquid can be understood as follows. The impurities in contaminated liquid increase the superficial tension of the bubble interface. At high Reynolds number the impurities will concentrate at the back of the bubble. The superficial tension there will decrease and as a consequence the boundary condition at the interface will be similar to a rigid sphere and the flow will separate from the bubble interface. The drag force will then be increased because of wake-effects.

**Slip models from literature**

Harmathy et al. [14] have semi-empirically derived a relation for the terminal slip velocity $V_\infty$ for dispersed bubbles in a stagnant liquid column, also called the bubble rise velocity. This is done in a similar way as in the FB model. This derivation will now be discussed below.

As in the force balance model, the relationship of Harmathy et al. [14] is based on a balance between the buoyancy force and the drag force that both act on a single bubble. An important quantity that appears in this balance is the volume of the bubble. This volume is determined from a balance between the interfacial tension, which tries to keep the bubble intact, and turbulent break-up. The following relation then results:

$$V_\infty = C \left[ \frac{g \sigma \Delta \rho}{\rho_l^2} \right]^{1/4}$$

with $\Delta \rho$ as the density difference between the phases, $\rho_l$ as the liquid density, $g$ as the gravitational acceleration and $\sigma$ as the interfacial tension. $C$ is a proportionality constant that accounts for the friction coefficient. This constant $C$ appears to range from 1.18 to 1.53, as has been determined experimentally.

It has to be noted that the model of Harmathy is semi-empirical while the FB model is theoretically based. Further in the FB model the influence of the gas holdup on the volume of the bubble is accounted for while this is not the case for the model of Harmathy. Relationship (2.15) is taken as a basis for many empirical and theoretical slip models for dispersed bubble flow. These slip models, that will be discussed next, also take the holdup dependence of the slip-effect along.

Nicolas & Witterholt [1] have empirically found the following relationship for the actual velocity $V_{\text{oil}}$ of oil bubbles rising in upward moving water continuum:

$$V_{\text{oil}} = V_{\text{water}} + \alpha_w n V_\infty$$

with $\alpha_w$ as the water holdup, $n$ as a constant ranging from 0.5 (large bubble limit) to 2 (small bubble limit) and $V_{\text{water}}$ as the actual water velocity. Comparison with experiments of Davarzani & Miller [1] shows, particularly at high water holdup, good agreement for $n=1$. 

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Zuber & Findlay [1] have derived a theoretical model, called drift-flux model, for the actual velocity of the light phase in upward bubble flow. This model however is applicable to any vertical upward two-phase flow. The drift-flux model is based on two slip mechanisms: Firstly because the light phase is assumed to be concentrated in the centre of the pipe where the velocity is highest the light phase will move faster relative to the heavy phase. Secondly because of the density difference the light phase will experience a buoyancy force upward relative to the heavy phase. These two mechanisms then result in the following relation for the actual velocity $V_{light}$ of the light phase:

$$V_{light} = V_{heavy} + V_{slip} = C_o V_m + V_{\infty}$$

(2.17)

with $V_{heavy}$ as the actual velocity of the heavy phase, $C_o$ as a constant ranging from 1.0 to 1.5 and $V_m$ as the mixture velocity. For the case of a turbulent flow profile where a flat velocity pattern exists $C_o = 1.2$. Comparison of this relationship with experiments [1] of vertical upward oil-water bubble flow shows good agreement for low water holdup and a poor agreement for large water holdup.

When in oil-water flow the oil is dispersed as small bubbles in the continuous water phase then a modification to the drift-flux model can be made by substituting the actual water velocity $V_{water}$ for the mixture velocity $V_m$. This model is therefore called the modified drift-flux model. With this model the following expression for the actual velocity $V_{oil}$ of the oil bubbles is then obtained:

$$V_{oil} = C_o V_{water} + V_{\infty}$$

(2.18)

Comparison of this relationship ($C_o = 1.2$) with oil-water bubble flow experiments [1] shows good agreement at large water holdup.

Hasan & Kabir [1] have empirically found the following relation for the slip velocity in oil-water bubble flow based on the drift-flux model:

$$V_{slip} = (C_o - 1) V_m / \alpha_w + V_{\infty} \alpha_w \alpha_o$$

(2.19)

with $C_o$ being 1.2 for bubble flow. Comparison of this relationship with experiments shows poor agreement at large water holdup.

Comparison of all the discussed slip models shows that the Nicolas & Witterholt model gives the best prediction for the slip velocity in oil-water bubble flow experiments at large water holdup. At low water holdup the modified drift-flux model is the best option. When two-phase bubble flow is inclined from vertical the above discussed empirically based models may not be valid anymore. Also the theoretically based (modified) drift-flux model will change. This can be understood as follows. Firstly because of the inclination the component of the buoyancy force normal to the pipe-axis will force the light phase to be concentrated at the upper side and not in the centre of the pipe. Secondly only a component of the buoyancy force is directed along the pipe-axis instead of the buoyancy force being directed completely along the pipe axis. In literature the slip-effect in inclined dispersed bubble flow has not been investigated extensively. This may be because the dispersed bubble flow appears only for a small region of inclinations ($0^\circ$ - $20^\circ$ from vertical, see previous section).

2.3.2 slip-effect in elongated bubble flow

In this subsection the slip velocity of elongated gas bubbles in a stagnant liquid will be discussed for three different cases: vertical, horizontal and inclined elongated bubble flow. After this also the slip velocity of elongated bubbles in a constant liquid flow will be discussed.
From experiments it appears that the drift velocity of large elongated gas bubbles is proportional to the square root of the gravitational acceleration $g$ and the pipe diameter $D$. The empirical slip model that is based on this relationship will be addressed as EB model. As a consequence for the slip factor $S$ the following relation can be written:

$$S = \frac{V_{el}}{V_{ad} + C\sqrt{gD}}$$  \hspace{1cm} (2.20)$$

with $C$ as a proportionality constant. The proportionality constant will now be discussed for three cases: vertical, horizontal and inclined pipe flow.

For the vertical case Davies & Taylor [12] have determined the proportionality constant to be 0.35 based on an analysis of potential flow.

For the horizontal case two different opinions on the drift velocity exist. One opinion is that the drift velocity of the elongated bubble, that is situated at the top of the pipe, must be zero since the buoyancy force is directed normal to the pipe axis. Another opinion is that also a drift velocity exists for the horizontal case caused by an elevation difference along the bubble nose region. Benjamin et al. [12] have derived a theoretical model for the drift velocity in horizontal pipes assuming that the drift velocity equals the penetration velocity of a bubble when liquid is emptied from the pipe. From this model a proportionality constant of 0.542 is determined which is larger than that of the vertical case. Comparison of the drift velocity, determined from the model of Benjamin et al. [12], with experiments shows good agreement. From this agreement it may be concluded that there is a drift velocity of elongated bubbles in horizontal pipes.

<table>
<thead>
<tr>
<th>Data</th>
<th>$\sigma = 0.005$</th>
<th>$\sigma = 0.001$</th>
<th>$\sigma = 0.004$</th>
<th>$\sigma = 0.01$</th>
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<tbody>
<tr>
<td>Hasan &amp; Kabir</td>
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<tr>
<td>Zukoski</td>
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<tr>
<td>Theory</td>
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</table>

**Figure 2.7.** Drift velocity of elongated bubbles in an inclined liquid column determined from experiments (Alves, Zukoski), empirical correlations (Bendiksen, Hasan & Kabir) and an extended model of Benjamin as a function of the inclination angle $\beta$ from horizontal for several values of the surface tension [12].

The drift velocity of elongated bubbles in inclined pipes, determined from experiments (Alves et al. [7,12], Zukoski et al. [12]) and empirical correlations (Bendiksen [12], Hasan & Kabir [12]), is drawn in figure 2.7 as a function of the inclination angle $\beta$ from horizontal for several values of the surface tension. In this figure also the drift velocity determined from a theoretical model, which is extended from the horizontal model of Benjamin et al [12] to include the inclined case, has been drawn as a function of the inclination angle. Comparison of the theoretically and empirically determined drift velocities shows qualitative agreement except that of Hasan & Kabir.
for small inclinations from horizontal. This is because Hasan & Kabir, in contrast to other investigators, assume no drift velocity in horizontal pipes. It also appears from figure 2.7 that the drift velocity first increases when the pipe is inclined from vertical and then decreases. Bonnecaze et al. [18] explain this behaviour by arguing that the hydrostatic head, that forces the bubble to move upward, increases as the pipe inclines from vertical and then decreases.

For the determination of the velocity of elongated bubbles in a constant liquid flow Nicklin et al. [16] assume, similar to the drift-flux model for small dispersed bubbles, that the velocity \( u_{eb} \) of an elongated bubble is the sum of the drift velocity in a stagnant liquid column and the liquid velocity in front of the tip of the bubble nose:

\[
 u_{eb} = C_{eb} u_l + u_{in} \quad (2.21)
\]

with \( u_l \) as the actual velocity of the liquid phase and \( C_{eb} \) as the constant that gives the degree of curvature of the liquid velocity profile in the pipe. From experiments Bendiksen [8] concludes that for upward flow the constant \( C_{eb} \) ranges from 1.0 to 1.2 for Froude numbers \( Fr = u_l / \sqrt{gD} \) lower than 3.5. For Froude numbers higher than 3.5 \( C_{eb} \) approaches 1.2 for all inclinations. Comparison of the conclusions of Bensiksen [8] with other experiments is difficult because of the large spread in reported data.

### 2.4 Friction in two-phase pipe flow

In this section the contribution of the friction to the pressure drop over a two-phase pipe flow will be discussed. This is done because the pressure drops over a venturi, a narrowing of the pipe, and a straight pipe are main input parameters to an approximate flow model which will be discussed in the next chapter.

To discuss the contribution of the friction to the pressure drop over a two-phase pipe flow a straight pipe with length \( L \) and diameter \( D \) will be considered. Hereby it will be assumed that the two-phase flow is steady and incompressible. According to Aziz & Govier [4] then the contribution \( \Delta p_{Fr} \) of the friction to the pressure drop over this pipe segment can be written as follows:

\[
 \Delta p_{Fr} = \frac{1}{2} f_p \alpha_l \rho_l V_m^2 \frac{L}{D} \quad (2.22)
\]

with \( \alpha_l \) as the liquid holdup, \( V_m \) as the mixture velocity and \( f_p \) as a two-phase friction factor. For the case of a two-phase pipe flow AGA has developed a correlation for a normalised two-phase friction factor \( f_{tp}/f \) as a function of the gas volume fraction GVF which is drawn in figure 2.8. The (simplified) single-phase friction factor \( f \) which normalizes the two-phase friction factor is determined from the following correlation, developed by Shell Oil for turbulent liquid flow:

\[
f = 0.0072 + 0.636 / Re^{0.355} \quad (2.23)
\]

with Re as the Reynolds number of the liquid flow. Figure 2.8 shows that the two-phase friction factor and the above defined single-phase friction factor are approximately related by the following expression:

\[
f_{tp} = f(1 + GVF)^2 \quad (2.24)
\]

with GVF as the gas volume fraction.
The single phase friction factor mentioned above is a fit for several other correlations which are only valid for a certain kind of flow. In general three types of flow can be discerned: laminar flow (Re<2000), turbulent flow (Re>3000) and a laminar-turbulent transition flow (2000<Re<3000). These regions can be seen in figure 2.9, in literature addressed to as a Moody diagram, where several relations for the friction factor have been drawn as a function of the Reynolds number and the pipe roughness $\varepsilon/\delta$.

Figure 2.8, Normalised friction factor for two-phase pipe flow (AGA)

Figure 2.9, The single-phase friction factor as a function of the Reynolds number (Moody diagram).
3 Velocity models

In this chapter an approximate flow model will be discussed that predicts the superficial velocities of the separate phases of a gas-liquid mixture flowing upward through a vertical or inclined pipe. Further a flow model will be discussed which is developed by Fernandes et al. [11] for vertical slug flow. Besides these models also the effect of compressibility on the prediction of the pressure drop over a venturi, which is an input parameter to the approximate flow model, will be discussed.

3.1 Approximate flow model

In this section an approximate flow model will be discussed which gives predictions for the superficial liquid and gas velocities in vertical and inclined liquid-gas pipe flow. Hereby both the gas and liquid phase are assumed to be incompressible. Further the gas-liquid flow is assumed to be steady.

In the approximate model a combination of a venturi, a narrowing of the pipe, and a straight pipe is used. The pressure differences between the inlet and the throat of this venturi and over the straight pipe, also called gradiometer (see chapter 4), are, next to the slip between the phases, the input parameters to this approximate flow model. After a general equation has been derived for the superficial velocities different slip behaviours will be assumed corresponding to three different versions of the approximate flow model: the homogeneous (no slip), the constant slip and variable slip models.

The approximate flow model is based on the assumption that the gas-liquid mixture can be considered as one phase for which the Bernoulli-equation is applicable. Hereby it is assumed that the density and velocity of this single phase are given by the mixture density \( \rho_m \) and the mixture velocity \( V_m \). According to this assumption the following relation can be written for the pressure difference \( \Delta p_v \) over a venturi, through which a gas-liquid mixture flows:

\[
\Delta p_v = \Delta \left( \frac{1}{2} \rho_m V_m^2 \right)
\]

(3.1)

From this equation then the mixture velocity \( V_m \) can be determined. Hereby use is made of the equations for conservation of mass for the separate phases as the mixture flows through the venturi. Considering the assumption that the densities of the separate phases are constant these equations then can be written as follows:

\[
\alpha_{gi} V_{ag,i} A_t = \alpha_{gi} V_{ag,t} A_i \\
\alpha_{li} V_{al,i} A_t = \alpha_{li} V_{al,t} A_i
\]

(3.2)

with \( V_{ag,i} \) and \( V_{al,i} \) as the actual gas and liquid velocities at the inlet of the venturi, \( V_{ag,t} \) and \( V_{al,t} \) as the actual gas and liquid velocities in the throat of the venturi, \( \alpha_{gi} \) and \( \alpha_{li} \) as the gas and liquid holdup at the inlet of the venturi, \( \alpha_{gt} \) and \( \alpha_{lt} \) as the gas and liquid holdup in the throat of the venturi. Hereby the upper equation gives the conservation of gas flow and the lower one the conservation of liquid flow through the venturi. Further for the determination of \( V_m \) the mixture density in the throat and at the inlet of the venturi have to be known. It must be noted that the slip-effect influences the holdups of the phases and thus also the mixture density \( \rho_m \). For the relation between this mixture density and the holdups the following expression can be written:
\[ \rho_{mi} = \alpha_h \rho_l + \alpha_g \rho_g \]
\[ \rho_{mt} = \alpha_h \rho_l + \alpha_g \rho_g \]

(3.3)

with \( \rho_{mi} \) and \( \rho_{mt} \) as the mixture densities at the inlet and in the throat of the venturi, \( \rho_l \) and \( \rho_g \) as the liquid and gas density respectively. With the help of the equations (3.1), (3.2) and (3.3) the mixture velocity \( V_{mi} \) at the inlet of the venturi can be expressed in terms of \( \Delta p_v \) and the holdups in the throat and at the inlet of the venturi:

\[
V_{mi} = \sqrt{\frac{2 \Delta p_v}{A_t^2 \left( \alpha_h \rho_l + \alpha_g \rho_g \right) - \left( \alpha_h \rho_l + \alpha_g \rho_g \right)}}
\]

(3.4)

with \( A_t \) and \( A_i \) as the cross-sectional areas at the inlet and in the throat of the venturi, \( \alpha_h \) and \( \alpha_g \) as the liquid and gas holdups respectively in the throat of the venturi and \( \alpha_{il} \) and \( \alpha_{gi} \) as the liquid and gas holdups at the inlet of the venturi.

The superficial velocities of the separate phases \( V_{sli} \) and \( V_{sgi} \) at the inlet of the venturi can be determined from \( V_{mi} \) by multiplying \( V_{mi} \) with the liquid volume fraction \( LVF \) and the gas volume fraction \( GVF \) respectively. For these volume fractions an expression can be derived, based on equation (2.12), as a function of the holdups and slip \( S_i \) at the inlet of the venturi (see Appendix C):

\[
LVF = 1 - GVF = \frac{\alpha_h S_i}{1 + \alpha_h (S_i - 1)}
\]

(3.5)

With the help of the equations (3.4) and (3.5) then the following relations can be obtained for \( V_{sli} \) and \( V_{sgi} \) as a function of the slip at the inlet and the local holdups at the inlet and in the throat of the venturi:

\[
V_{sli} = LVF \cdot V_{mi} = \frac{\alpha_h S_i}{1 + \alpha_h (S_i - 1)} \sqrt{\frac{2 \Delta p_v}{A_t^2 \left( \alpha_h \rho_l + \alpha_g \rho_g \right) - \left( \alpha_h \rho_l + \alpha_g \rho_g \right)}}
\]

(3.6)

\[
V_{sgi} = GVF \cdot V_{mi} = \frac{\alpha_g S_i}{1 + \alpha_h (S_i - 1)} \sqrt{\frac{2 \Delta p_v}{A_t^2 \left( \alpha_h \rho_l + \alpha_g \rho_g \right) - \left( \alpha_h \rho_l + \alpha_g \rho_g \right)}}
\]

(3.7)

From these equations it can be seen that the superficial velocities depend on the pressure drop \( \Delta p_v \), the geometry of the venturi, the local slip-effect and the holdups at the inlet and in the throat of the venturi. The determination of these holdups will now be discussed next.

The holdups at the inlet of the venturi can be determined in the following way from the hydrostatic head that is given by the pressure difference \( \Delta p_{\text{gradio}} \) over the gradiometer (see chapter 4):

\[
\alpha_h = 1 - \alpha_g = \frac{\Delta p_{\text{gradio}}}{\rho_l h_{\text{gradio}}}
\]

(3.8)

with \( h_{\text{gradio}} \) as the height of the gradiometer and \( g \) as the gravitational acceleration. Hereby it is assumed that the gas density can be neglected with respect to the liquid density.
The holdups in the throat of the venturi can be determined from mass balances for the separate phases (see Appendix C and equation 3.12). It follows that these holdups depend on the holdups at the inlet and on the slip factors \( S_i \) and \( S_t \) at the inlet and in the throat of the venturi respectively. For the determination of the superficial velocities it is thus necessary to know the slip factor at the inlet as well as that in the throat of the venturi. In the following two subsections therefore three different assumptions on the slip behaviour in the pipe and/or the venturi will be discussed. In the first subsection a constant slip is assumed in the pipe, which equals the slip in the venturi. Further also a special case of constant slip behaviour will be discussed, namely no slip. This kind of flow is called homogeneous flow. In the second subsection a constant slip will be considered in the pipe with an increased slip behaviour in the venturi due to an extra pressure drop caused by the acceleration of the mixture.

### 3.1.1 Constant slip

In this subsection it is assumed that in the whole pipe, also in the venturi, there is a constant slip between the phases of a gas-liquid pipe flow. This means that the holdups of the separate phases are constant throughout the whole pipe. However from equation (3.5) it also means that they don't equal their corresponding volume fractions \( LVF \) and \( GVF \). Schematically this can be written as follows:

\[
S_i = S_t = \text{constant} \neq 1 : \quad \alpha_{yi} = \alpha_{y} \neq GVF \\
\alpha_{yi} = \alpha_{y} \neq LVF
\]  

(3.9)

Inserting (3.9) into the equations (3.6) and (3.7) for the superficial velocities at the inlet of the venturi gives:

\[
V_{sl} = \frac{\alpha_y S}{1 + \alpha_y (S - 1)} \left( \frac{2 \Delta p_v}{\frac{A_i^2}{A_r^2} - 1} \left( \alpha_{yi} \rho_f + \alpha_y \rho_g \right) \right)
\]

(3.10)

\[
V_{sg} = \frac{\alpha_y S}{1 + \alpha_y (S - 1)} \left( \frac{2 \Delta p_v}{\frac{A_i^2}{A_r^2} - 1} \left( \alpha_{yi} \rho_f + \alpha_y \rho_g \right) \right)
\]

(3.11)

A special form of the constant slip model is when no slip is assumed \((S=1)\). The flow for which the no-slip condition holds is called the homogeneous bubble flow. In that case the holdups in the pipe equal their corresponding volume fractions \( LVF \) and \( GVF \).

### 3.1.2 variable slip bubble flow

In this subsection it is assumed that there is a constant slip between the phases in the pipe but that the slip-effect increases in the venturi due to an extra buoyancy force, next to that caused by the gravitational force, due to the extra pressure drop caused by the acceleration of the mixture. Further it is assumed, despite the fact that the mixture is not accelerated anymore in the throat, that the slip in the throat of the venturi equals that in the converging part of the venturi because of relaxation of the slip-effect.

As a consequence of the increased slip-effect in the venturi relative to that at the inlet the holdups in the throat of the venturi may differ from those at the inlet. As said before the holdups at the inlet of the venturi can be determined from the pressure drop over the gradiometer. The holdups in the throat of the venturi can be determined from mass balances considered from the inlet to the throat of the venturi for the separate phases (see Appendix C):
\[ \alpha_{hl} = 1 - \alpha_{gr} = \frac{\alpha_{y}S_{y}}{\alpha_{y}S_{y} + \alpha_{gr}S_{y}} \]  

(3.12)

Inserting equation (3.12) into (3.6) and (3.7) then gives for the superficial velocities at the inlet of the venturi:

\[ V_{gh} = \frac{\alpha_{y}S_{y}}{1 + \alpha_{y}(S_{y} - 1)} \sqrt{\frac{2\Delta p_{y}}{\frac{A_{y}^{2}}{\alpha_{y}S_{y} + \alpha_{gr}S_{y}} - \left(\alpha_{y}\rho_{y} + \alpha_{gr}\rho_{g}\right)}} \]  

(3.13)

\[ V_{gw} = \frac{\alpha_{g}}{1 + \alpha_{y}(S_{y} - 1)} \sqrt{\frac{2\Delta p_{y}}{\frac{A_{y}^{2}}{\alpha_{y}S_{y} + \alpha_{gr}S_{y}} - \left(\alpha_{y}\rho_{y} + \alpha_{gr}\rho_{g}\right)}} \]  

(3.14)

It has to be noted that a variable slip-effect in gas-liquid flow through a venturi has been discussed recently by Boyer et al. [19].

3.2 Slug flow model

In this section a mechanistic model will be discussed for slug flow in vertical pipes that has been derived by Fernandes et al. [11]. In this slug-flow model an idealized slug unit is considered as has been drawn in figure 3.1. This idealized slug unit consists of a Taylor bubble, a free-falling film between the Taylor bubble and the pipe wall followed by a cylindrical liquid slug in which gas bubbles are dispersed. The falling liquid film is assumed to be free of dispersed bubbles. It also will be assumed that the slug flow is one-dimensional, fully developed and in steady-state. The characteristic quantities that appear in this slug unit are defined in Table 1.

The (simplified) slug-flow model of Fernandes et al. [11] consists of a set of 8 equations that can be derived by considering the physical mechanisms that occur in slug flow. The derivation of these equations will now be discussed one by one starting with considering the relation between the average gas holdup \( \alpha_{SU} \) in the slug unit and the average gas holdups \( \alpha_{TB} \) and \( \alpha_{LS} \) in the Taylor bubble and liquid slug respectively.

average gas holdups in a slug unit

The ratio of the length \( L_{TB} \) of the Taylor bubble section and that of the whole slug unit \( L_{SU} \) is defined as \( \beta \). For the average gas holdup in a slug unit \( \alpha_{SU} \), then the following relation with the average gas holdups in the Taylor bubble \( \alpha_{TB} \) and liquid slug \( \alpha_{LS} \) can be written:

\[ \alpha_{SU} = \beta \alpha_{TB} + (1 - \beta) \alpha_{LS} \]  

(3.15)

mass balances over whole slug unit

When the gas and liquid in a slug unit are assumed incompressible then the mass balances of these phases over the slug unit transform into volume balances. Then the mass balance for the gas phase over a slug unit can as follows be expressed in terms of the superficial gas velocity \( U_{SG} \) and the ratio \( \beta \):

\[ U_{SG} = \beta U_{TB} + (1 - \beta) \alpha_{LS} U_{GLS} \]  

(3.16)

with \( U_{TB} \) and \( U_{GLS} \) as the gas velocity in the Taylor bubble and the liquid slug respectively. The mass balance for the liquid phase can be written similarly in terms of the superficial liquid velocity \( U_{SL} \):
$$U_{SL} = (1 - \beta)(1 - \alpha_{LS})U_{LLS} - \beta(1 - \alpha_{TB})U_{LTB}$$  \hspace{1cm} (3.17)

with $U_{LLS}$ and $U_{LTB}$ as the liquid velocity in the liquid slug and the liquid film next to the Taylor bubble.

Figure 3.1, Idealized slug unit in the vertical slug flow model of Fernades et al. [11].

mass balances relative to slug unit

The slug unit moves upward at a velocity $U_{TB}$ which is larger than the velocities of the separate phases. In other words a downward flow of liquid and gas occurs relative to the Taylor bubble and liquid slug. The relative liquid flow occurs because liquid flows downwards through the liquid film next to the Taylor bubble. The mechanism of the relative gas flow can be understood as follows. As liquid at the bottom of the liquid slug drains into the free-falling film the gas bubbles, dispersed in this liquid, are forced into coalescence with the Taylor bubble because they are too large to fit into the film. At the back of the Taylor bubble gas bubbles are torn off by the liquid film that flows into the underlying liquid slug.

Mass balances can be developed considering the gas and liquid flow relative to the upward moving slug unit. The mass balance for the gas phase can be derived by considering that the gas flow relative to a liquid slug must equal that through a following Taylor bubble. This can be expressed by the following relation:

$$(U_{TB} - U_{GLS})\alpha_{LS} = (U_{TB} - U_{GTB})\alpha_{TB}$$  \hspace{1cm} (3.18)

Similarly the liquid flow relative to a liquid slug must equal that being drained in the liquid film. This can be expressed by the relation:
\[(U_{TB} - U_{LTS})(1 - \alpha_{LS}) = (U_{TB} - U_{LTS})(1 - \alpha_{TB})\]

(3.19)

**Bubble rise-velocity**

The velocity of the Taylor bubble (and thus that of the slug unit) can be derived using the drift-flux approach as has been used by Zuber&Findlay [1] (see section 2.3). The following relation is then obtained:

\[U_{TB} = C_{0}(U_{SG} + U_{SL}) + 0.35\sqrt{gD}\]

(3.20)

with \(C_{0}\) as a constant that is approximately 1.2, \(g\) as the gravitational acceleration and \(D\) as the diameter of the pipe (see section 2.3).

The velocity of dispersed gas bubbles in the liquid slug can be expressed in terms of the liquid velocity in the liquid slug and the bubble rise velocity due to buoyancy. For the bubble rise velocity Fernandes et al. [11] have used an expression as has been derived by Nicolas&Witterholt [1]. The following relation for the gas bubble velocity in the liquid slug is then obtained:

\[U_{GLS} = U_{LLS} + 1.53\left(\frac{g\sigma(\rho_{l} - \rho_{g})}{\rho_{l}^{2}}\right)^{\frac{1}{4}}(1 - \alpha_{LS})^{\frac{1}{2}}\]

(3.21)

**Falling film**

Fernandes et al. [11] assume that the liquid film next to the Taylor bubble experiences no interfacial shear from the pipe wall nor from the free interface with the Taylor bubble. Brotz et al. [11] have developed an empirical expression which relates the thickness \(\delta_{L}\) of such a free falling film to the velocity \(U_{LTB}\) of the liquid in this falling film. The thickness of the falling film can also be expressed in terms of the gas holdup \(\alpha_{TB}\) in the Taylor bubble section considering that the liquid in this section is only present in the liquid film at the wall. When both relations for the film thickness are set equal the following relation is obtained:

\[\left[\frac{1}{2}\right]^{1/2} = 9.916 gD\left[1 - \alpha_{TB}\right]^{1/2}\]

(3.22)

The equations (3.15) to (3.22) form the simplified model of Fernandes et al. [11]. In these equations however eleven unknowns appear: \(U_{SL}, U_{SG}, \beta, \alpha_{TB}, \alpha_{LS}, \alpha_{SL}, U_{TB}, U_{GLS}, U_{GTB}, U_{LLS}, U_{LTS}\). Therefore to determine the superficial velocities of the separate phases three of the remaining unknowns have to be used as input parameters to the model. In this investigation the gas holdups in the slug unit are used as these input parameters.

In their model Fernandes et al. [11] have used the superficial velocities of the separate phases as input parameters. To provide for closure Fernandes et al. [11] have assumed the gas holdup \(\alpha_{LS}\) in the liquid slug to equal the value which exists at the transition from bubble to slug flow. For this transition value Fernandes et al. [11] have used the value 0.25 which has been suggested by Taitel et al. [3] (see section 2.2). Several other investigators have made different assumptions for \(\alpha_{LS}\). For example Griffith&Wallis [11] have assumed that the liquid slug is free of gas bubbles \((\alpha_{LS}=0)\). Barnea et al. [12] have developed a physical model which predicts the gas holdup in the liquid slug. This model will now be discussed below.

Barnea et al. [12] have investigated the gas holdup in liquid slugs of the slug flow pattern. They assume that the transition value of the gas holdup for the bubble-slug transition at a certain turbulent level is the maximum holdup of the liquid slug at that turbulent level. Barnea et al. [12] assume the turbulent level to be constant at constant mixture velocity \(U_{M}(=U_{SL}+U_{SG})\). Therefore, in the slug flow region in a flow pattern map, the gas holdup in the liquid slug is constant along a line of constant \(U_{M}\) and equals the transition gas holdup at the point where this line crosses a flow regime transition line. In figure 3.2 a flow pattern map with lines of constant \(U_{M}\) is drawn for the
case of vertical upward gas-liquid flow through a pipe with diameter D=5 cm. This figure shows that the lines of constant $U_M$ in the slug flow regime cross three flow regime boundaries: AB for transition to bubble flow, BC and CD for transition to dispersed bubble flow. For the determination of the maximum gas holdup in the liquid slugs the slug flow regime therefore can be divided into three subregions that correspond to these three transition lines.

![Diagram](image)

**Figure 3.2.** Flow pattern given by Barnea et al. [12] for determination of the gas holdup in the liquid slug in vertical gas-liquid slug flow.

The first subregion contains lines of constant $U_M$ that cross the transition line AB belonging to the bubble-slug transition. The gas holdup at this transition is supposed to have a constant value of 0.25, as suggested by Taitel et al. [3]. The maximum gas holdup in the liquid slugs corresponding to this first subregion therefore also has a constant value of 0.25.

The second subregion is related to the transition line CD for dispersed bubble-slug transition based on the mechanism of maximum packing of the dispersed bubbles. The gas holdup at this transition line CD has a constant value of 0.52. Therefore the maximum gas holdup in the liquid slugs corresponding to this second subregion also has a constant value of 0.52.

The third subregion of the slug flow regime is situated between the two former discussed subregions. The gas holdup at the transition line BC, that corresponds to this third subregion, ranges from 0.25 at point B to 0.52 at point C. The maximum gas holdup in the liquid slugs corresponding to this third subregion therefore also ranges from 0.25 (along the constant $U_M$ line that crosses B) to 0.52 (along the constant $U_M$ line that crosses C).

The maximum gas holdup of the liquid slugs in the subregion, that corresponds to the transition line AB, agrees with the assumption of Fernandes et al. [11] for closure of the model. However it has to be taken into account that as the turbulent level is increased the maximum gas holdup in the liquid slugs can increase whenever another sub-region of slug flow is reached.

The slug flow model described in this section is developed for vertical slug flow. Felizola et al. [6] have also developed a slug flow model for inclined flow, which is a modification of the vertical slug flow model of Fernandes et al. [11]. In the development of the inclined slug flow model Felizola et al. [6] assume the inclined slug flow also to consist of idealized slug units. As for the vertical case, also for the inclined case then a set of eight equations with eleven unknowns can be derived. However to solve the superficial velocities this set of equations has to be solved numerically. Therefore this inclined model will not be discussed in this investigation.
3.3 Influence of compressibility on bubble flow

In this section the influence of compressibility of a gas-liquid bubble mixture on the pressure drop over a venturi, through which this mixture flows, will be discussed. Hereby it will be assumed that the fluid phase is incompressible and that the bubble flow is steady. Further the bubble flow will be considered to behave isothermal.

The compressibility of the gas-liquid mixture is expressed by the speed of sound $c$, which is defined by van Wijngaarden [10] as:

$$c^2 = \frac{dp}{dp}$$

(3.23)

with $p$ as the pressure in the mixture, neglecting the pressure difference between the gas bubbles and the continuous liquid caused by interfacial tension, and $\rho$ as the mixture density. From this equation it follows that the speed of sound is infinitely large for incompressible phases.

For an isothermal gas-liquid flow in which no slip occurs between the phases Van Wijngaarden [10] has given the following approximation for the speed of sound $c_T$, based on equation (3.23), as a function of the pressure $p$, the gas and liquid holdups $\alpha_g$ and $\alpha_l$ and the liquid density $\rho_l$:

$$c_T^2 = \frac{p}{\rho_l \alpha_g \alpha_l}$$

(3.24)

This equation is a good approximation for the speed of sound unless the gas holdup is very low (bubble flow) or very large (annular flow). At 1 bar and a gas holdup of 4 percent it follows from this approximation that the speed of sound of the gas-liquid mixture is about 50 m/s, which is much lower than that of the separate phases (for example, $c_{\text{air}} = 300 \text{m/s}$ and $c_{\text{water}} = 1500 \text{m/s}$). This means that the bubble mixture is more compressible than the separate phases it consists of.

When a slip-effect between the phases is assumed, Van Wijngaarden et al [10] give the following approximation for the speed of sound, provided again that the gas holdup is very low:

$$c_T^2 = \frac{(1 + 2\alpha_g)p}{\rho_l \alpha_g \alpha_l}$$

(3.25)

Assuming a slip-effect between the phases thus leads to a larger predicted value of the speed of sound and thus also to a lower compressibility of the mixture.

The influence of the compressibility of the mixture on the pressure drop over a venturi can be understood qualitatively as follows. Because of the expansion of the gas phase in the venturi the mixture will experience an extra acceleration of the velocity. This extra acceleration will invoke an extra increase in the absolute value of the pressure gradient as can be seen from equation (3.1) (see section 3.1). This increase of the pressure gradient can also be understood quantitatively by considering an equation, derived by Van Wijngaarden [10], for an isothermal bubble flow (very low gas holdup) through a venturi in which no slip-effect is assumed:

$$\frac{dp}{dx} = \frac{1}{A} \frac{dA}{dx} \rho_m V_m^2 \frac{1}{1 - M^2}$$

(3.26)

with $A$ as the cross-sectional area, $V_m$ as the mixture velocity and $M$ as the Mach number which is the ratio of the the mixture velocity and the speed of sound. This equation shows that as the speed of sound of the mixture decreases (decreasing Mach number), which means an increase of the
compressibility of the mixture, the pressure gradient in the (converging part of the) venturi must indeed increase in absolute sense.
4 Experimental setup

In this chapter the experimental setup will be discussed which has been used for vertical and inclined oil-air and water-air velocity measurements. First the general setup and then the setup of the flowmeter will be discussed. After this a description of the experiments will be given.

General setup

The general experimental setup that has been used in this investigation for the above described inclined liquid-air flow measurements has been drawn schematically in figure 4.1. This figure shows a circuit through which a mixture of maximum three phases can flow: water, oil and air. These phases are initially stored in reservoirs from which the phases flow separately towards a mixing point, where the phases are mixed into a pipe with a diameter of 10.8 cm. Hereby the separated flows are sustained by two pumps and a compressor and they are controlled by three valves situated in the three flow lines. Between the control valves and the mixing point the separate flows are measured by three single-phase flow meters. The readings of these single-phase flow meters have been used as reference for the multi-phase flow meter.

![Diagram of experimental setup](image)

Figure 4.1, The general experimental setup

From the mixing point the mixture flows towards an inclination point from where the pipe is inclined upward under a certain angle $\alpha$ from horizontal for 13 meters. At the top the pipe is curved into downward direction so that the mixture flows downward under the same inclination angle back to the inclination point. From this point on the mixture flows into a separator where the different phases of the mixture are separated. These separated phases then flow into the above mentioned reservoirs closing the circuit.
As the mixture flows upward through the inclined pipe it can be observed with a camera through a transparent piece of pipe. This camera is linked to a video for analyse afterwards. After having passed this transparent piece of pipe the mixture flows further upward through a flowmeter that will be discussed later on in this chapter. This flow meter is situated 2 meters under the top of the inclined pipe.

When the air is mixed with the liquid at the mixing point then it will orden itself at the top of the cross-section of the horizontal pipe, that is situated between the mixing point and the inclination point, as a consequence of the buoyancy force. In this way a continuous liquid with large gas voids is produced. This configuration thus works as a slug generator. To avoid this effect the air can also be mixed with the gas at the inclination point. A more homogenised mixture is then obtained. Whether the air is mixed with the liquid at the mixing point or at the inclination point depends on the kind of flow pattern that has to be investigated.

setup of the flowmeter

Now the setup of a flow meter will be discussed that has been used in this investigation to measure quantities that are input parameters to the approximate flow model (see section 3.1) and the vertical slug flow model by Fernandes et al. [11] (see section 3.2). Further this setup gives data for flow pattern identification as will be seen in the chapters 5 and 6. The setup of this flow meter has been drawn schematically in figure 4.2.

![Figure 4.2](image)

**Figure 4.2, The setup of the flowmeter**

Figure 4.2 shows a venturi, a short gradiometer (0.20 m) and a long gradiometer (1.54 m). A gradiometer is a straight pipe over which a pressure difference is measured. Also over the venturi a pressure difference has been measured. This pressure difference has been measured between the inlet and the throat of the venturi, which are 0.17 m separated. Both over the venturi and the gradiometers the pressure differences have been measured by two types of pressure meters which will be discussed below. The measured pressure differences are converted by an ADC into digital signals which are stored on a PC that is linked to the ADC. Hereby the ADC is controlled by a program that is written in "lotus measure".
For measuring the pressure differences two types of pressure meters have been used: pressure difference meter (dp-cell) and an absolute pressure meter. The dp-cell consists of a large membrane and two pipes connected to each side. These pipes are filled with Silicone oil whose density is well-known. At the end of each pipe a small membrane is attached that contacts the gas-liquid flow through a hole in the pipe wall. The large membrane thus gives the pressure difference between the points where the small membranes are attached to the pipe wall minus the hydrostatic head of the Silicone oil in the two pipes. Because the density of the Silicone oil is well-known this hydrostatic head can be calculated. In this way from the reading of the dp-cell the pressure difference between the points, where the small membranes are attached, can be determined.

The absolute pressure meters also consist of a membrane. One side of this membrane is connected to the atmosphere while the other side is connected to the gas-liquid flow through a hole in the pipe wall.

Qualitatively it must be noted that the dp-cells are more accurate than the absolute meters but on the other hand the absolute pressure meters have a shorter reaction time to pressure changes than the dp-cells. Therefore both meters have been used in the setup of the flowmeter. Finally it must be noted that quantitatively no information is available about the frequency response of both the dp-cells and the absolute pressure meters.

description of experiments

In this investigation inclined and vertical oil-air and water-air experiments have been done. This means that during the experiments either the control valve of the oil flow line or that of the water flow line is closed. As said above three different inclinations have been investigated: 30°, 60° and 90° inclined from horizontal. The air-liquid experiments have been performed for three different reference liquid rates which are kept constant during an experiment. The procedure of measuring during an experiment will be discussed below.

At the beginning of each experiment the single-phase flow meters are read. After this the ADC is commanded by the lotus measure program to read the pressure meters over the venturi and the gradiometers for 20 seconds at a frequency of 10 Hz. This procedure is repeated at the same reference liquid superficial velocity at a larger gas volume fraction until data are gathered over the whole range of GVF.
5 Vertical pipe flow experiments

5.1 Flow regime transitions

For determination of the superficial velocities of the separate phases in vertical gas-liquid flow measurements have been done with oil-air and water-air mixtures in vertical pipes which have been described in chapter 4. Which kind of flow model has to be applied for the determination of these superficial velocities depends on the occurring flow regime of the two-phase mixture. The determination of the flow regimes of the gas-liquid flow measurements has been done by using two different methods: visual observation and measuring the pressure difference over a venturi as a function of time.

It is concluded from visual observation of the vertical gas-liquid flow experiments that three flow regimes occur: bubble flow, slug/churn flow and annular flow. Bubble flow is encountered at low gas and liquid rates where the bubbles show a zig-zag movement. As the liquid rates are increased (dispersed) bubble flow also occurs at larger gas rates. As, at constant remaining low liquid rate, the gas rate is increased bubble flow transforms into slug/churn flow which can be detected by its oscillatory motion of the liquid film. The transition bubble-slug/churn is very difficult to detect because at low gas rates the Taylor bubbles are very short and occur at a very low frequency which makes the detection of these Taylor bubbles difficult. At increasing gas rate the slug frequency increases and the Taylor bubbles become longer. The liquid film that is seen to flow downwards along the Taylor bubble is pushed upward by the next upward moving liquid slug. In this action vortices occur in the liquid slug. As the gas rate reaches large values the liquid is swept around the pipe wall. This flow is considered as annular flow.

The second method for flow pattern determination is to consider the pressure drop $\Delta p_v$ over a venturi as a function of time $t$. From these $\Delta p_v/t$-plots the same three flow regimes have been observed as from visual observation: bubble, slug/churn and annular flow. Characteristic $\Delta p_v/t$-plots of these flow regimes are given in figure 5.1. The behaviour of the pressure drop as a function of time for these three flow regimes will now be discussed below.

The pressure drop over a venturi, through which a gas-liquid mixture flows, depends on the mixture density and on the actual liquid velocities at the inlet and in the throat of the venturi. In slug flow dynamic variations occur in both the mixture density and the actual liquid velocity. The change in the mixture density is due to separation of the phases with respect to the pipe axis in the form of liquid and gas slugs. The change in actual liquid velocity appears when the upward liquid velocity in the liquid slug changes into the downward liquid velocity in the liquid film next to the Taylor bubble. These two variations combined indicate that the pressure drop over a venturi, through which a slug flow flows, oscillates as a function of time as can be seen in figure 5.1b.
Figure 5.1, Characteristic $\Delta p_v/t$-plots for bubble, slug/churn and annular flow in vertical air-water flow.
In bubble and annular flow the phases are distributed more homogeneously with respect to the pipe axis than in slug flow. The actual liquid velocity in bubble and annular flow can approximately be assumed to be constant, which is acceptable considering the large fluctuations in the actual liquid velocity in slug flow. However in bubble flow large Taylor bubble-like bubbles occur which cause a temporary low mixture density. In annular flow large droplets may be entrained in the gas core which cause a temporary large mixture density. Therefore in both flows a constant pressure drop over the venturi is expected with peaks to zero (bubble flow) and to much larger values (annular flow) which can be seen in the figures 5.1a (bubble flow) and 5.1c (annular flow). Hereby it has to be noted that the frequency of the peaks in the $\Delta p_{v}/t$-plot, which characterises annular flow, is very large because of the large rates of the separate phases.

The occurring flow regimes that have been observed using the two methods mentioned above have been marked in a flow map, given in figure 5.2, with the superficial velocities $V_{sl}$ and $V_{sg}$ of the liquid and gas phase respectively as coordinates in log-log scale. Also in this figure the transitions derived by Taitel&Dukler [3] have been drawn. Considering the difficulty in both methods to determine the flow regime transition both methods show reasonable agreement with each other and with the theoretically derived transition boundaries of Taitel&Dukler [3]. From this flow map it can be concluded that the bubble flow and the slug/churn flow regime are the main occurring flow regimes. Therefore in this chapter models will be used that are based on these flow regimes.

![Flow Map](image)

**Figure 5.2.** Experimental flowmap based on visual observation and $\Delta p_{v}/t$-plots of water-air and oil-air experiments together with a theoretical flow map by Taitel&Dukler [3].
5.2 Approximate flow model

In this section vertical water-air experiments will be discussed. For these experiments the superficial velocities of the separate phases, given by the approximate flow model (see section 3.1), are discussed with respect to their reference values. Three versions of the approximate flow model will be investigated: the homogeneous, constant slip and variable slip model. The input parameters to these models are the pressure drops over a gradiometer and a venturi and the slip factor at the inlet and possibly, for the case of the variable slip model, in the throat of the venturi. In the following subsections first several models for the slip-effect will be discussed. After this the predictions for the superficial liquid and gas velocities will be discussed relative to their reference values.

5.2.1 Slip models

In this subsection the results of several slip models for the slip-effect at the inlet and in the throat of the venturi in vertical water-air flow experiments ($V_{SL_{ref}}=1$ m/s) will be discussed. These models can be divided in models available from literature and the FB model, which is based on a force balance on a single air bubble in a continuous liquid (see section 2.3). The models from literature which will be discussed here are the model of Nicolas&Witterholt [1] (NW), the drift-flux model [1] (DF), the modified drift-flux model [1] (MDF) and the model of Hasan&Kabir [1] (HK) which are described in section 2.3.

![Slip at inlet venturi: Comparison of slip models from literature](image)

**Figure 5.3.** Comparison of several slip models available from literature for water-air experiments ($V_{SL_{ref}}=1$ m/s).

In figure 5.3 the slip factor $S$ at the inlet of the venturi in vertical water-air experiments ($V_{SL_{ref}}=1$ m/s), determined from the models available from literature, is given as a function of the gas volume fraction GVF. Although these slip models are developed for bubble flow (GVF<0.3) the predictions of the slip factor $S$ will be given for the whole range of GVF. This is done to obtain predictions for the slip factor in flow regimes other than bubble flow. Figure 5.3 shows that the Nicolas&Witterholt [1] model and the modified drift-flux model predict an increasing slip-factor as a function of GVF up to 1 at GVF=1. The drift-flux model and the model of Hasan&Kabir on the other hand predict a decreasing slip factor as a function of GVF down to 0.5 at GVF=1. Comparison of all
the slip-models shows that there is no consistency in the prediction of the slip factor. However it has to be noted that an increase of the slip factor to 1 for increasing GVF may be expected considering the fact that the actual rates of the phases increase at increasing GVF. A constant remaining slip velocity then gives an increasing slip factor (see equation 2.11). From these considerations it may be concluded that the Nicolas&Witterholt model [1] and the modified drift-flux model [1] are acceptable over the whole range of GVF despite the fact that they are mainly developed for bubble flow.

The slip factor S at the inlet of the venturi has also been determined from the theoretical FB slip model, which is based on a force balance for a single bubble (see section 2.3). In figure 5.4 this slip factor S has been given as a function of GVF for two limits of the friction coefficient Cd which correspond to spherical bubbles (Cd=48/Re) and to spherical capped bubbles (Cd=2.6). For comparison also the empirical slip model of Nicolas&Witterholt (NW) [1] has been given.

![Figure 5.4](image)

Figure 5.4, Slip S at the inlet of the venturi as a function of GVF determined from the slip model FB and the model of Nicolas&Witterholt [1] for water-air experiments (Vs_l_ref=1m/s).

Figure 5.4 shows that the slip factor at the inlet of the venturi, predicted by the FB model for the spherical bubbles (Cd=48/Re), is very low for a large range of GVF and does not agree with the empirical model of Nicolas&Witterholt [1]. This very low slip factor is physically not acceptable. Figure 5.4 shows further that the slip factor, predicted by the FB model for the spherical capped bubbles (Cd=2.6), shows a reasonable agreement with that of the empirical model of Nicolas&Witterholt [1], which is originally developed for oil-water bubble flow. This agreement in its turn agrees with a slip Reynolds number Re_s of approximately 1000, which is valid at the inlet of the venturi for the considered water-air experiments, from which, with the help of figure 2.6, it indeed appears that the friction factor Cd is approximately 2.6. From the above it may be concluded that in the considered water-air experiments the bubbles at the inlet of the venturi are spherical capped.

Finally figure 5.4 shows that the slip, given the FB model for both bubble forms, approaches one as GVF approaches one. As said above this is acceptable, despite the fact that the FB model is only developed for bubble flow (GVF<0.3), because at large GVF the liquid and gas phases have, relative to low GVF, large actual velocities.
The slip factor $S$ in the throat of the venturi has also been determined from the FB model and the Nicolas&Witterholt [1] model. For the case of the FB model this has been done by assuming a constant pressure gradient in the converging part of the venturi. For the case of the Nicolas&Witterholt [1] model an effective gravitational acceleration has been used that takes the acceleration of the mixture in the venturi into account. Further it has been assumed in both models that the slip-effect in the throat of the venturi equals that in the converging part of the venturi despite the fact that the mixture is not accelerated anymore in the throat. This assumption is valid when the distance between the converging part and the measurepoint in the throat, where the pressure is measured, is small compared to the distance over which the slip stabilises. Further the converging part of the venturi is assumed to be long enough for the slip-effect to stabilize.

In figure 5.5 the slip factor $S$ in the throat of the venturi in vertical water-air experiments, determined from the FB model for two different values of Cd and the model of Nicolas&Witterholt [1], has been given as a function of GVF. The two different Cd factors, which are used in the FB model, correspond to spherical bubbles (Cd=48/Re) and to spherical capped bubbles (Cd=2.6).

![Slip in the throat of the venturi: dependence on Cd](image)

Figure 5.5, Slip in the throat of the venturi in water-air experiments ($V_{SL_{ref}}=1m/s$) given by the FB model for two different Cd factors and the Nicolas&Witterholt [1] model.

Figure 5.5 shows that the FB model gives a lower slip factor $S$ for the spherical bubbles than for the spherical capped bubbles, which are in better agreement with the results of the Nicolas&Witterholt [1] model. This better agreement for spherical capped bubbles in the venturi however does not agree with a slip Reynolds number of approximately 100, which is valid for the water-air experiments in the throat of the venturi. As from this Reynolds number it appears (with the help of figure 2.6) that the friction coefficient equals that for spherical bubbles. Hereby it has to be noted that the prediction for the slip for spherical bubbles is not so low as for spherical particles at the inlet of the venturi. A minimum of $S=0.5$ (the air bubbles move twice as fast as the liquid) is physically acceptable. Further it has to be noted that the NW model is an empirical model derived for oil/water flow. Its application to gas/liquid flow may therefore be questionable. From the above it may be concluded that the form of the bubbles in the throat of the venturi is not clear. Both Cd factors (for spherical and spherical capped bubbles) will therefore be used in the determination of the superficial velocities in the next subsections.
Comparison of the slip models shows that both the FB(spherical) and the FB(capped) models predict a larger slip-effect in the throat of the venturi relative to that predicted by the FB(capped) model at the inlet of the venturi. This agrees with the fact that the buoyancy force in the converging part of the venturi is larger than that in the pipe due to the extra pressure drop over the venturi caused by the acceleration of the mixture.

5.2.2 Superficial liquid velocity

In this subsection the predictions for the liquid superficial velocity $V_{\text{sl}}$ in water-air experiments, given by the approximate flow model discussed in section 3.1, will be discussed with respect to their reference values $V_{\text{sl\_ref}}$. Hereby it will be considered if the goal of this investigation, predicting the liquid rate within 10% relative to its reference rate, is reached.

Three versions of the approximate flow model will be investigated: the homogeneous, constant slip and the variable slip model. For the case of the constant slip model that slip is used as is given by the FB model for spherically capped bubbles at the inlet of the venturi. For the case of the variable slip model the bubbles are also assumed spherically capped at the inlet of the venturi. In the throat of the venturi however two limits of the bubble form will be investigated: spherical bubbles ($C_d=48/Re_5$) and spherically capped bubbles ($C_d=2.6$).

In figure 5.6 the relative error in the superficial liquid velocity $V_{\text{sl}}$ with respect to their reference values $V_{\text{sl\_ref}}$ in air-water experiments, given by the above mentioned three versions of the approximate flow model, are given as a function of GVF for three different values of $V_{\text{sl\_ref}}$ (a: $V_{\text{sl\_ref}}=0.5$ m/s, b: $V_{\text{sl\_ref}}=1$ m/s, c: $V_{\text{sl\_ref}}=2$ m/s). Hereby the relative error in the superficial liquid velocity, also called the relative liquid error, is defined as:

$$\text{rel. error} \quad V_{\text{sl}} = \frac{V_{\text{sl}} - V_{\text{sl\_ref}}}{V_{\text{sl\_ref}}} \quad (5.1)$$

The slip factor, that is used in the constant slip model, is given by the FB model for the case of spherically capped bubbles ($C_d=2.6$) at the inlet of the venturi. This model is therefore called $c_s_{FB}(\text{capped})$. In the variable slip model the slip factor is also given by the FB model in which the bubbles in the throat of the venturi are assumed spherical ($C_d=48/Re_5$) and at the inlet of the venturi spherically capped ($C_d=2.6$). This model is called $v_s_{FB}(\text{spherical})$. Finally the homogeneous model (no slip) will be called hom.

From figure 5.6 now first the region GVF=0-0.3 will be considered, which corresponds to bubble flow (see section 5.1), because this is the region where the FB model is developed for. For this region figure 5.6 shows for $V_{\text{sl\_ref}}=0.5$ m/s and 1 m/s that all three models give relative errors in $V_{\text{sl}}$ that range within 10% from zero, which is the goal of this investigation. This result is satisfying considering the fact that the flow model is approximate.

For $V_{\text{sl\_ref}}=2$ m/s the relative liquid error in the bubble flow region, predicted by all three models, increases very rapidly up to 20% at increasing GVF and then remains constant (hom and $c_s_{FB}(\text{capped})$) or decreases back within 10% from zero ($v_s_{FB}(\text{spherical})$), which is the goal of this investigation. A possible explanation for the rapid increase may be that the mixture velocity in the throat of the venturi approaches the speed of sound which is relatively low for gas-liquid mixtures. This effect will be discussed in section 7.4.

Further figure 5.6 shows for the bubble flow region (GVF<0.3) that the $v_s_{FB}(\text{spherical})$ model gives the lowest values for the relative error in $V_{\text{sl}}$ and that the homogeneous model predicts the largest relative errors for all three values of $V_{\text{sl\_ref}}$. The difference between the homogeneous model and the two other slip models can be understood as follows. When a slip-effect is assumed in gas-liquid flow the liquid holdup increases. As a consequence the local mixture density increases and
from the approximate equation (3.1) it then follows that the mixture velocity $V_m$ must decrease. Because, by definition, the liquid volume fraction LVF remains constant this means that the superficial velocity must decrease leading to a lower relative liquid error. The difference between the cs_FB(capped) model and the vs_FB(spherical) model can be understood as follows. From considering the FB model it appears that the slip factor $S$ for spherical bubbles in the throat of the venturi decreases with respect to that of spherically capped bubbles at the inlet of the venturi for all three reference liquid rates (see also previous subsection). From this behaviour it can be concluded that the liquid holdup and thus also the mixture density in the throat of the venturi increases relative to that at the inlet (see equation 2.12). The approximate equation (3.1) then shows that the mixture velocity must decrease leading to a lower relative liquid error as described above.

Comparison of the reference superficial velocities with the flow map discussed in section 5.1 shows that the main flow regime in the region for GVF larger than 0.3 is churn flow (except for a small region for $V_{sl\, ref}=0.5$ m/s where slug flow occurs). It appears for this region that the homogeneous slip model predicts the largest relative errors, that range within 30% from zero, whereas the vs_FB(spherical) model predicts the lowest relative errors that range within 10% from zero, which is the goal of this investigation. These results are surprisingly well not only because the flow model is based on an assumption but also because the FB model, which is an input model to this approximate model, is developed for bubble flow and not for churn flow. A possible explanation for this satisfying result may be that in the approximate flow model opposing effects cancel out. These effects will be considered in chapter 7 where the assumption of the approximate flow model will be discussed.

Comparison of the bubble flow region (GVF<0.3) and the slug/churn flow region (GVF>0.3) shows for the cases $V_{sl\, ref}=0.5$ and 1 m/s that the absolute relative error increases in the slug/churn region relative to that in the bubble flow region. This agrees with the fact that the FB model, which is an input model to the approximate flow model, is developed for bubble flow. However for the case $V_{sl\, ref}=2$ m/s no difference between the two mentioned regions can be observed. It may be that at this large reference liquid velocity other effects start playing an important role such as compressibility (see chapter 7).

Finally figure 5.6 shows that the relative errors in the superficial velocities given by all three models become zero at GVF=0 for which single-phase flow occurs. This agrees with the fact that the approximate equation on which the approximate flow model is based reduces to the Bernoulli-equation which is indeed valid for single-phase flow.
Figure 5.6, Relative error \( \left( \frac{V_{sl} - V_{sl,ref}}{V_{sl,ref}} \right) \) in the superficial liquid velocity determined from three versions of the approximate flow for three different reference liquid velocities:
- **hom**: homogeneous flow model
- **cs_FB(capped)**: constant slip model with the slip given by the FB slip model for spherically capped at the inlet of the venturi.
- **vs_FB(spherical)**: variable slip model with the slip given by the FB slip model for spherically capped bubbles at the inlet of the venturi and spherical bubbles in the throat of the venturi.
In contrast to above now the bubbles both in the throat and at the inlet of the venturi will be assumed to be spherically capped ($C_d=2.6$). For this case the variable slip model will be called $v_{	ext{FB}}(\text{capped})$. The relative errors in the superficial liquid velocity, given by both the $v_{	ext{FB}}(\text{capped})$ and $c_{	ext{FB}}(\text{capped})$ models, in water-air experiments ($V_{\text{sl,ref}}=0.5\text{m/s}$) have been given in figure 5.7 as a function of GVF. For the whole range of GVF it appears that the variable slip model predicts larger relative errors than the constant slip model. This is because the slip factor in the throat of the venturi is larger than that at the inlet. As a consequence the liquid holdup in the throat of the venturi will be smaller than that at the inlet causing a lower mixture density in the throat. Equation (3.1) then shows that the mixture velocity must increase relative to the case for which the mixture density remains constant, as is the case for the constant slip model.

**Figure 5.7.** Relative error ($=\left(V_{\text{sl}}-V_{\text{sl,ref}}\right)/V_{\text{sl,ref}}$) in the superficial liquid velocity in air-water experiments ($V_{\text{sl,ref}}=0.5\text{m/s}$) determined from the variable slip and constant slip models with the FB model as input model. The bubbles are assumed spherically capped both in the pipe and in the venturi.

It has to be noted that the relative error predicted by the variable slip model for any bubble form must be lower than that predicted by the $v_{\text{FB}}(\text{capped})$ and larger than that predicted by $v_{\text{FB}}(\text{spherical})$. This can be understood as follows. When the spherical capped bubbles in the throat of the venturi become more spherical then the bubbles experience a lower friction force and as a consequence the slip velocity increases (see section 2.3). Consequently the slip factor $S$ and thus also the gas holdup must decrease (see equation 2.12). A decreasing gas holdup in its turn means an increasing mixture density. The approximate equation (3.1) then shows that the mixture velocity $V_m$ and thus also the superficial liquid velocity $V_{\text{sl}}$ must decrease so that the relative error in $V_{\text{sl}}$ becomes smaller. This may continue until the bubbles become spherical and thus reach the lowest friction coefficient.
5.2.3 Superficial gas velocity

In this subsection the superficial gas velocities \( V_{sg} \) in vertical water-air experiments, given by different versions of the approximate flow model, will be discussed with respect to their reference values \( V_{sg\text{ref}} \). As in the previous subsection the same three versions of this flow model will be investigated: the homogeneous (hom), the constant slip (cs) and the variable slip (vs) model. For the slip in these models also here the FB model is used.

As in the previous section, the slip in the constant slip model is given by the FB model for spherically capped bubbles (\( C_d=2.6 \)) at inlet conditions. This model will therefore also here be called cs_FB(capped). For the case of the variable slip model the slip at the inlet of the venturi is given by the FB model for spherically capped bubbles while the slip in the throat of the venturi is given by the FB model for spherical bubbles. This variable model will therefore be called vs_FB(spherical) as in the previous section. It must be noted, also in this subsection, that the FB model is developed for the case of a single bubble. Using this model as an input model to the approximate model therefore makes this model only applicable to bubble flow (\( GVF<0.3 \)). However to obtain also predictions for other flow regimes the relative error in the superficial gas velocity will be considered over the whole region of GVF.

In figure 5.8 the relative error in the superficial gas velocities \( V_{sg} \) relative to the reference values \( V_{sg\text{ref}} \) have been given as a function of GVF for water-air experiments for three different values of \( V_{sl\text{ref}} \) (a: \( V_{sl\text{ref}} = 0.5 \text{ m/s} \), b: \( V_{sl\text{ref}} = 1 \text{ m/s} \), c: \( V_{sl\text{ref}} = 2 \text{ m/s} \)). Hereby the relative error in the superficial gas velocity, also called the relative gas error, is defined similar to the relative liquid error (see also equation (5.1)):

\[
\text{rel error } V_{sg} = \frac{V_{sg} - V_{sg\text{ref}}}{V_{sg\text{ref}}} \tag{5.2}
\]

From figure 5.8 first the general behaviour of the relative gas error as a function of GVF will be discussed for all reference liquid rates. Further the dependence of the relative gas error on the reference liquid rate will be discussed. Finally the predictions of the different versions of the approximate model will be compared with each other.

Figure 5.8 shows that the relative gas error is very large for low GVF for all reference liquid rates. This is because at low GVF the gas flow is very low. A small absolute difference between the predicted and the reference velocity then already gives a large relative error. Further figure 5.8 shows that the relative error decreases very rapidly as a function of GVF and stabilizes at large GVF. This is because at increasing GVF the gas rate also increases so that the above described effect becomes less strong.

Figure 5.8 shows further that the stabilized relative gas error at large GVF increases at increasing reference liquid rates. This may be because at large liquid rates the compressibility of the gas phase in the throat of the venturi starts playing a role. At low reference liquid rates the relative gas error stabilizes within 30% from zero whereas at a large reference liquid rate the relative error remains larger than 100%. The stabilizing range of 30% may be acceptable but the very large stabilized error at large reference rates leaves room for future work.

Finally figure 5.8 shows that the variable slip model gives lower relative errors than the constant slip model. This is because, for the case of the variable slip model, the slip-effect (for the assumed bubble forms) in the throat of the venturi is larger than at the inlet so that the mixture velocity must decrease to predict the same pressure drop over the venturi (see equation 3.1) as has been explained in the previous sub-section. This means that also the superficial gas velocity must decrease because GVF remains constant by definition.
Relative error \( \frac{(V_{sg} - V_{sg \_ref})}{V_{sg \_ref}} \) in the superficial gas velocity as a function of GVF determined from three versions of the approximate flow model for three different reference liquid superficial velocities:

- **hom**: homogeneous flow model
- **cs\_FB(capped)**: constant slip model with the slip given by the FB slip model for spherically capped bubbles at the inlet of the venturi.
- **vs\_FB(spherical)**: variable slip model with the slip given by the FB slip model for spherically capped bubbles at the inlet of the venturi and spherical bubbles in the throat of the venturi.
5.3 Slug flow model

In this section the modified vertical slug flow model of Fernandes et al. [11], which predicts the superficial velocities of the separate phases in vertical slug flow, will be discussed for water-air experiments. First the input parameters to this model will be discussed and after this the relative errors in the superficial velocities (see subsection 5.2.2) of both phases with respect to their reference values will be considered.

In this investigation the following three quantities are considered as input parameters to the vertical slug flow model: the gas holdup \( \alpha_{TB} \) in the Taylor Bubble, the gas holdup \( \alpha_{LS} \) in the liquid slug and the gas holdup \( \alpha_{SLU} \) in the entire slug unit. These gas holdups have been determined from considering the dynamic behaviour of the pressure drops over a short gradiometer (0.2m) and a long gradiometer (1.6m). The way this has been done can be explained as follows. From the dynamic pressure drops over the gradiometers the corresponding hydrostatic head can be determined. From these hydrostatic heads the corresponding dynamic gas holdups in both gradiometers can be determined. In figure 5.9 these dynamic gas holdups have been given as a function of time \( t \) for the case of a vertical oil-water experiment (\( V_{sl_{ref}}=0.5 \text{m/s}, \ GVF\approx0.6 \)).

![Dynamic holdup behaviour](image)

Figure 5.9, The dynamic liquid holdups in the short and long gradiometer as a function of time \( (V_{sl_{ref}}=0.5 \text{m/s}, \ GVF\approx0.6) \).

It will now be assumed that the slug units are longer than the short gradiometer and much shorter than the long gradiometer. This assumption agrees with visual observation of the considered experiments. The peaks in the gas holdups in the short gradiometer then indicate Taylor bubbles flowing by whereas the minima indicate liquid slugs. The gas holdups in the Taylor bubbles and the liquid slugs flowing by are thus by approximation given by the maximum values and the minimum values of the waves respectively. In the long gradiometer several slug units are assumed to be present. The average gas holdup over the slug units is thus approximately given by the average gas holdup over this long gradiometer. Hereby it should be noted that the number of Taylor Bubbles and liquid slugs in the long gradiometer are not always the same causing also here a dynamic behaviour in the holdup.

Figure 5.9 shows that the holdup in the short gradiometer varies between 0.7 and 0.55 for oil-air experiments \( (V_{sl_{ref}}=0.5 \text{m/s}) \). For these experiments the holdups in the Taylor bubbles and in the liquid slugs respectively are thus given by these extremes according to the above explained procedure. Comparing these values with literature shows no agreement. For example Fernandes et al. [11]
predicts an $\alpha_{TB}$ of 0.88 from his experiments while assuming an $\alpha_{LS}$ of 0.25. This disagreement may be because the frequency of the slug flow is so high that the short gradiometer cannot measure the pressure difference accurately. In this way peaks of the waves are cut off and therefore show such a relatively small amplitude.

Because of the large discrepancy between the Taylor bubble and liquid slug holdup values determined from experiments and values given in literature these experimental determined holdups have not been used in the slug flow model. To investigate if the slug model is applicable to the vertical gas-liquid experiments the holdups have been fitted from the slug model by setting the relative error in the superficial velocities to zero. From considering whether these holdups are physically acceptable conclusions may be drawn about the applicability of the slug flow model. These fitted holdups will now be discussed next.

For the considered water-air slug flow experiments it appears that the superficial water velocities match with their reference values over the whole range of slug flow conditions for $\alpha_{TB}=0.93$ and $\alpha_{LS}=0.25$. These values show a good agreement with those given by Fernandes et al. [11]. However, hereby it has to be noted that a slight change in these parameters causes a very large relative error in the superficial liquid velocity. For matching the superficial gas velocities to their reference values over the whole range of slug flow conditions no pair of constant values for the holdups in the Taylor bubble and the liquid slug have been found. It thus can be concluded that the vertical slug flow model of Fernandes et al. [11] is not appropriate for determination of superficial velocities of the separate phases in vertical slug flow.
5.4 Vertical pipe flow: Conclusions

In this section some conclusions and recommendations will be given concerning the determination of the superficial velocities of the separate phases in vertical air-liquid flow experiments.

As shown in section 5.1 in the considered vertical air-liquid flow experiments mainly two flow patterns occur: bubble flow at low gas rates and slug/churn flow at large gas rates. For the slip the force balance (FB) model, that corresponds to bubble flow, has been used. Here it has to be noted that the bubble form is an important input parameter to the FB model. Therefore two limits of the bubble form have been investigated: spherical bubbles and spherically capped bubbles.

For flow rate determination the approximate flow model has been used with the FB slip as input parameter. Three different versions of the approximate flow model have been investigated: homogeneous model (hom), constant slip (cs) and variable slip (vs) model. Depending on the bubble form these different versions are labelled FB(spherical) or FB(capped).

For the bubble flow region (GVF<0.3) it appears that the vs_FB(spherical) predicts relative errors of the superficial liquid velocity relative to their reference values, also called relative liquid errors, within 10% from zero. This is the goal of this investigation. For the slug flow region it appears that the vs_FB(spherical) model also predicts relative liquid errors within 10%. Also for this region thus the goal of this investigation is reached. This is surprising because the flow model is not only approximate but also because the FB slip model has been developed for bubble flow. As a suggestion for future work the EB slip model, that corresponds to slug flow, can be used as input model to the approximate model for the slug/churn region (GVF>0.3).

For the case of the superficial gas velocity all flow models give the same behaviour. The relative error in the gas rate is very large at low GVF, decreases at increasing GVF and eventually stabilises at large GVF. For low reference liquid rates the relative gas error stabilizes within 30% from zero, which may be acceptable. However, at large reference liquid rates the relative gas error remains larger than 100%, which leaves room for further work.
6 Inclined pipe flow experiments

6.1 Flow regime transitions

As for vertical flow, inclined air-liquid (air-oil and air-water) flow experiments have been done for the determination of the superficial velocities in inclined air-liquid flow. Also here, as for the vertical case, the determination of these superficial velocities depends on the occurring flow regimes. Therefore in this section the flow regimes and their transitions, that occur in the considered inclined air-liquid flow measurements, will be discussed. This will be done for two different inclinations: \( \alpha = 30^\circ \) and \( \alpha = 60^\circ \) with \( \alpha \) as the inclination angle from vertical. For these two inclinations two corresponding flow maps are given in which the observed flow regimes are mapped for all considered experiments.

For the determination of the flow regimes the method of visual observation has been used. Visual observation shows that for both inclinations three flow patterns have been observed: (dispersed) bubble, slug/churn and annular flow. These flow patterns will now be discussed below for both inclinations starting with that for \( \alpha = 30^\circ \). At both low liquid and gas rates bubble flow appears. This bubble flow is detected by deformed bubbles that move in a zig-zag path at the top of the pipe. At larger liquid rates, with still a low gas rate, dispersed bubble flow, characterised by small spherical bubbles at the top of the pipe, is observed. At low liquid rates and increasing gas rates bubble flow goes over into intermittent (slug/churn) flow which can be detected by the oscillatory motion of the liquid phase at the bottom side of the pipe. Finally at very high gas rates the liquid, which mainly is concentrated in a liquid film at the bottom of the cross-section of the pipe, is occasionally swept around the pipe wall. This pattern is defined as annular flow.

At low liquid and gas rates for the inclination of \( \alpha = 60^\circ \) instead of bubble flow slug flow is detected. As said in section 2.2 this is because then the buoyancy force overcomes the turbulent force so that gas voids are formed at the top of the pipe. Also here dispersed bubble flow is observed as the liquid rate is increased at a constant gas rate. Annular flow is also encountered at large gas rates. The main character change of these flow patterns with respect to those for \( \alpha = 30^\circ \) is that the gas phase is now more concentrated at the top of the cross-section of the pipe.

The flow regimes observed in the inclined flow experiments have been mapped in two flow maps, which are drawn in figure 6.1, corresponding to the two considered inclinations. In these flow maps also the theoretical transitions, derived by Barnea et al. [2] (see section 2.2), have been given for comparison. Both figures show, considering the difficulty in visual observation to detect the transitions, qualitative agreement between the experimental and theoretical transitions. Comparing both flow maps with each other and with that for vertical flow (see figure 5.2) a very important aspect is that when the inclination from vertical increases the slug flow pattern takes over the bubble flow pattern. Therefore for modelling air-liquid flow in inclined pipes slug flow is a very important flow regime.
Another method for flow pattern determination, which next to visual observation has also been used for vertical air-liquid flow, is measuring the pressure drop over a venturi through which the mixture flows. However for the inclined case this method is not used because the differences between the $\Delta p/v$-plots of the different flow regions are not that clear as in the vertical case. This can be understood by considering inclined slug flow relative to vertical slug flow. As slug flow is inclined the liquid film becomes (much) thicker. As a consequence the oscillations in mixture density as a function show a smaller amplitude. Also because of this relatively increasing liquid film the variations in the liquid velocity become smaller. As a result the oscillation in the pressure difference over the venturi will have a smaller amplitude than in the vertical case. It is thus much more difficult to discern slug flow from the other observed flow regimes. Only at reasonably large gas rates occasionally an oscillatory behaviour of the pressure difference over the venturi as a function of time can be observed. For these conditions only little liquid is present to form a thick liquid film at the bottom side of the pipe.
6.2 Approximate flow model

In this section water-air and oil-air experiments for two different inclinations, 30 and 60 degrees from vertical, will be discussed. For these experiments the superficial velocities, given by three versions of the approximate flow model, will be discussed with respect to their reference values. As said in section 3.1 the input parameters to this approximate model are the pressure drops over a gradiometer and a venturi. Also the slip at the inlet and in the throat of the venturi have to be known. What kind of slip models have been used in the flow model will be discussed in subsection 6.2.1. After this the relative errors in the superficial liquid velocities for both inclinations will be considered. Finally also the relative errors in the superficial gas velocities will be discussed.

6.2.1 Slip models

In this sub-section the slip models will be discussed that have been used as input models to the different versions of the approximate flow model which gives the superficial velocities for the separate phases in inclined water-air and oil-air experiments. The kind of slip model that has been used depends on the occurring flow regimes.

The flow maps given in figure 6.1 for inclined water-air experiments show that the slug flow regime is the main occurring flow pattern for both inclinations (30 and 60 degrees from vertical). Only at large liquid rates dispersed bubble flow occurs. Therefore at low liquid rates the EB slip model for elongated bubbles will be used as has been discussed in section 2.3.2. At large liquid rates the FB slip model, which is developed for bubble flow, will be used. Because of similarity with vertical flow the FB slip model will not be discussed here. The results of the EB slip model for water-air inclined experiments will now be discussed below.

As discussed in section 2.3.2, the EB slip model predicts the slip velocity of an elongated bubble in a continuous liquid which is proportional to the square root of the gravitational acceleration and the diameter of the pipe. The corresponding slip factors $S$ both at the inlet and in the throat of the venturi have been given in figure 6.2 for a water-air experiments ($V_{sl\_ref}=1m/s$) for both considered inclinations. For the case of the slip $S$ in the throat an effective gravitational acceleration has been determined from the constant assumed pressure gradient over the venturi. Hereby it is further assumed, as in the vertical case, that the slip in the throat of the venturi equals that in the converging part of the venturi because of relaxation of the stabilisation of the slip-effect.

Figure 6.2 shows that the slip $S$ at the inlet increases up to 1 for increasing GVF. As said in the previous chapter for vertical flow this is physically correct because for increasing GVF the actual rates of the phases increase. A constant remaining absolute difference between these rates then results in a larger slip factor.

Further figure 6.2 shows that the slip in the throat of the venturi remains constant as a function of GVF. This is because the effect of the increasing actual rates at increasing GVF is compensated by the effect of the increasing effective gravitational acceleration in the venturi which is due to the increasing pressure drop over the venturi. Further it can be seen that the slip $S$ in the venturi is lower than that at the inlet. As a consequence the liquid holdup and thus also the mixture density in the venturi must be larger relative to that at the inlet.

Comparison of figures 6.2a and 6.2b shows that both the slip at the inlet and in the throat of the venturi are slightly larger for the case of 30 degrees inclination with respect to that of 60 degrees inclination. This is because the buoyancy force has a larger component along the pipe axis for 30 degrees inclination from vertical than for 60 degrees inclination.
Figure 6.2, The slip in the throat and at the inlet of the venturi in inclined water-air experiments (Vsl_ref=1 m/s) given by the EB slip model.
6.2.2 Superficial liquid velocity

In this subsection the superficial liquid velocities $V_{sl}$ in inclined water-air and oil-air experiments, given by the approximate flow model, are discussed with respect to their reference values $V_{sl\,ref}$. Hereby the goal of this investigation, which is to predict the liquid rates within 10% from their real values, will be considered.

As for the vertical case, three versions of the approximate flow model will be discussed: the homogeneous (hom), constant slip (cs) and variable slip (vs) model. For the slip in this models both the EB slip for slug flow and the FB slip for bubble flow are used as input parameters. Hereby it must be noted that slug/churn flow appears at low reference liquid rates ($V_{sl\,ref}$=0.5m/s and 1m/s) while dispersed bubble flow occurs at large reference liquid rates ($V_{sl\,ref}$=2m/s) as can be seen in the flow maps in section 6.1.

For the low reference liquid rates the constant slip model and the variable slip model, that use the EB slip as input parameter, are called cs_EB and vs_EB respectively. The homogeneous model is called hom. For the large reference liquid rates ($V_{sl\,ref}$=2 m/s) the slip in the constant slip model is given by the FB model for spherically capped bubbles at the inlet of the venturi. This model is therefore called cs_FB(capped). For the variable slip model the slip is also given by the FB model. Hereby the bubbles both at the inlet and in the throat of the venturi are either assumed spherically capped or spherical. The two corresponding versions of the variable slip model are therefore called vs_FB(capped) and vs_FB(spherical).

The superficial liquid velocities, determined from the approximate flow model, in water-air and oil-air experiments, 30 degrees and 60 degrees inclined from vertical, will now be discussed below. Hereby the angle $\alpha$ is defined as the angle of inclination from vertical. In the figures 6.3 and 6.4 the relative errors in these superficial liquid velocities relative to their reference values, also called relative liquid error, have been given as a function of GVF for three different reference liquid rates. Hereby the relative liquid error is defined as in equation (5.1). The results for the low reference liquid rates and large reference liquid rates will be discussed separately because for these conditions different flow patterns have been observed: bubble flow at large rates and slug flow at low rates (see section 6.1). Now first the labels of the used flow models will be discussed.

For the low reference liquid rates ($V_{sl\,ref}$=0.5m/s and 1m/s) bath figures 6.3 and 6.4 show over the whole range of GVF that the homogeneous flow model predicts the largest relative liquid errors and that the vs_EB model gives the lowest relative liquid errors. This can be explained in the same way as has been done for the vertical case. Further both figures show that the cs_EB model predict relative errors within 15% over the whole range of GVF while the vs_EB model predicts relative errors within 10%, which is the goal of this investigation. This result is satisfying considering the fact that the flow model is approximate.

Both figures 6.3c and 6.4c show, for large reference liquid rates ($V_{sl\,ref}$=2m/s), that all the different versions of the approximate flow model, except the vs_FB(spherical) model at $\alpha$=30°, predict about the same relative error in the superficial liquid velocity. The relative error first increases rapidly up to about 15% and then remains constant (or decreases back to zero at large GVF as is the case for $\alpha$=30°). This behaviour may be because the mixture velocity approximates the speed of sound, which is very low in multi-phase mixtures, so that compressibility of the gas phase starts playing a role. This compressibility effect will be discussed in chapter 7.

The vs_FB(spherical) model at 30 degrees inclination from vertical in figure 6.3c shows a total different behaviour for the relative error as a function of GVF than the other models. After a short increase the error decreases gradually down to -15%. Hereby it has to be noted that the prediction for the relative error by the vs_FB model for a certain bubble form must be larger than that of vs_FB(spherical) and lower than that of vs_FB(capped), as has been explained in chapter 5. In other words the relative error predicted by the variable slip model for $\alpha$=30° must be between -15% and 15% from zero. To obtain predictions for the relative liquid error within 10% from zero therefore the form of the bubbles both at the inlet and in the throat of the venturi have to be known. This leaves room for future work.
Comparison of the vs_FB(spherical) model at \( \alpha=30^\circ \) in figure 6.3c and that at \( \alpha=60^\circ \) in figure 6.4c shows a different behaviour. In contrast to the vs_FB(spherical) model for \( \alpha=30^\circ \), the vs_FB(spherical) model for \( \alpha=60^\circ \) shows only a small difference with respect to the other models at \( \alpha=60^\circ \). This large discrepancy may be because for \( \alpha=30^\circ \) a water-air experiment has been investigated while for \( \alpha=60^\circ \) an oil-air experiment has been investigated. The large difference in viscosity of the liquid phase combined with the fact that the viscosity is a very important parameter in the vs_FB(spherical) may give an explanation for the different behaviour. Hereby it also has to be noted that the viscosity of the oil is only known in a very wide range, which makes the prediction by the vs_FB(spherical) model less reliable. The influence of viscosity on the slip and thus the superficial velocity must be investigated in future work.

Finally both figures 6.3 and 6.4 show for all reference liquid velocities that the relative error is zero at GVF=0. As has been said for the vertical case, this agrees with the fact that for GVF=0 the approximate equation, which the approximate flow model is based on, reduces to the Bernoulli-equation.
Figure 6.3, Relative error in the superficial liquid velocity ($\frac{V_{sl} - V_{sl\, ref}}{V_{sl\, ref}}$), determined from the approximate flow model, in water-air and oil-air experiments 30 degrees inclined from vertical for three different reference liquid superficial velocities as a function of GVF.

- **hom**: homogeneous model
- $cs_{\, EB}/vs_{\, EB}$: constant slip model/variable slip model with EB slip as input parameter.
- $cs_{\, FB}$ (capped): constant slip model with FB slip for spherically capped bubbles at inlet conditions.
- $vs_{\, FB}$ (capped/spherical): variable slip model with FB slip for either spherically capped or spherical bubbles throughout the whole pipe.
Figure 6.4. Relative error in the superficial liquid velocity \((\text{Vsl-Vsl_ref}/\text{Vsl_ref})\), determined from the approximate flow model, in water-air and oil-air experiments 60 degrees inclined from vertical for three different reference liquid superficial velocities as a function of GVF.

- **hom**: homogeneous model
- **cs_EB/vs_EB**: constant slip model/variable slip model with EB slip as input parameter.
- **cs_FB(capped)**: constant slip model with FB slip for spherically capped bubbles at inlet conditions.
- **vs_FB(capped/spherical)**: variable slip model with FB slip for either spherically capped or spherical bubbles throughout the whole pipe.
6.2.3 Superficial gas velocity

In this subsection the superficial gas velocities $V_{sg}$, given by the approximate flow model, are discussed relative to their reference values $V_{sg_ref}$ for inclined water-air and oil-air experiments. As in the previous subsection here the same three versions of the approximate flow model will be investigated: the homogeneous (hom), constant slip (cs) and variable slip (vs) model. Also the same slip models will be used for the same conditions, namely the EB slip model for slug flow at low reference liquid rates $(V_{sl_ref}=0.5 \text{m/s and } 1 \text{m/s})$ and the FB model for bubble flow at large reference liquid rates $(V_{sl_ref}=2 \text{m/s})$. Therefore the same labels, that have been used for the different flow models in the previous subsection, will now also be used here.

The superficial gas velocities, given by the approximate model for water-air and oil-air experiments, 30 and 60 degrees inclined from vertical, will now be discussed below. Hereby the angle $\alpha$ will, as in the previous subsection, be defined as the inclination angle from vertical. In the figures 6.5 and 6.6 the relative error in the superficial gas velocities relative to their reference values, also called relative gas error (see equation (5.2)) have been given as a function of GVF for three different reference liquid rates $(V_{sl_ref}=0.5, 1, 2 \text{m/s})$. Hereby it must be noticed that the EB slip model is used for low reference liquid rates while the FB slip model has been used for large reference liquid rates. From both figures first the general behaviour of the relative gas error, given by all versions of the approximate flow model, will be discussed for both inclinations and for all reference liquid rates. Further the behaviour of the relative gas error, given by all versions of the approximate flow model, will be discussed as a function of the reference liquid rate and of the inclination. Finally the predictions of the several flow models will be compared with each other. Hereby, as in the previous subsection, a distinction will be made between the models for large reference rates $(V_{sl_ref}=2 \text{m/s})$, that correspond to bubble flow, and those for low reference rates $(V_{sl_ref}=0.5 \text{m/s and } 1 \text{m/s})$, for which slug flow occurs (see section 6.1).

Both figures 6.5 and 6.6 show for all three different reference liquid rates that the absolute value of the relative gas error, given by all the considered models, is very large at low GVF. This can be explained in the same way as has been done for the vertical case (see section 5.2.3). Further both figures show for all models that the relative error decreases rapidly at increasing GVF. Also this behaviour can be explained similar to the vertical case. At large GVF all considered models eventually give relative errors within 20% except for the case of $V_{sl_ref}=2 \text{m/s}$ at $\alpha=30^\circ$ where the relative error remains larger than approximately 100% and even increases at larger GVF. This may be because at large liquid rates the compressibility of the gas phase in the throat of the venturi starts playing a role. The stabilizing range of 20% is acceptable but the large stabilized error at $V_{sl_ref}=2 \text{m/s}$, $\alpha=30^\circ$ leaves room for further work. From comparison of the figures 6.5 and 6.6 no clear behaviour of the relative gas rate error can be observed as a function of the inclination angle.

Comparison of the different flow models shows that the homogeneous flow model gives the lowest absolute values for the relative gas error for all reference liquid rates and for both inclinations except for $V_{sl_ref}=0.5 \text{ m/s at } \alpha=60^\circ$. This is surprising since the homogeneous model is the simplest version of the approximate flow model. Further comparison shows that the considered models give about the same relative errors for all reference liquid rates and for both inclinations both for bubble flow conditions $(V_{sl_ref}=2 \text{ m/s})$ and slug flow conditions $(V_{sl_ref}=0.5, 1 \text{ m/s})$. One exception is the vs_FB(spherical) model which gives much larger relative gas errors at low GVF. The agreement between all the considered models may be caused by the fact that the relative errors, given by these models, are so large that the mutual differences between the models are much smaller.
30 degrees inclined from vertical

a) Vsl_ref=0.5m/s, water-air

![Graph](image1)

b) Vsl_ref=1m/s, water-air

![Graph](image2)

c) Vsl_ref=2m/s, water-air

![Graph](image3)

**Figure 6.5.** Relative error in the superficial gas velocity \((V_{sg} - V_{sg \_ref})/V_{sg \_ref}\), determined from the approximate flow model, in water-air and oil-air experiments 30 degrees inclined from vertical for three different reference liquid superficial velocities as a function of GVF.

- hom: homogeneous model
- cs_EB/vs_EB: constant slip model/variable slip model with EB slip as input parameter.
- cs_FB(capped): constant slip model with FB slip for spherically capped bubbles at inlet conditions.
- vs_FB(capped/spherical): variable slip model with FB slip for either spherically capped or spherical bubbles throughout the whole pipe.
60 degrees inclined from vertical

a) \( V_{sl\_ref}=0.5\text{ m/s, water-air} \)

b) \( V_{sl\_ref}=1\text{ m/s, oil-air} \)

c) \( V_{sl\_ref}=2\text{ m/s, oil-air} \)

Figure 6.6, Relative error in the superficial gas velocity \( (V_{sg}-V_{sg\_ret})/V_{sg\_ret} \), determined from the approximate flow model, in water-air and oil-air experiments 60 degrees inclined from vertical for three different reference liquid superficial velocities as a function of GVF.

hom: homogeneous model

cs\_EB/\textit{vs\_EB}: constant slip model/variable slip model with EB slip as input parameter.

cs\_FB(\text{capped}): constant slip model with FB slip for spherically capped bubbles at inlet conditions.

\textit{vs\_FB(capped/spherical)}: variable slip model with FB slip for either spherically capped or spherical bubbles throughout the whole pipe.
In this section some conclusions and recommendations will be given concerning the determination of the superficial velocities of the separate phases in inclined air-liquid flow.

As shown in section 6.1 in inclined air-liquid experiments mainly slug/churn flow occurs at low reference liquid rates while at large reference liquid rates mainly (dispersed) bubble flow occurs. It has to be noted that the transition between these flow patterns is not very clear. For the slip different models have been used corresponding to each flow regime. For the slug flow pattern the elongated bubble (EB) model has been used while for the bubble flow pattern the force balance (FB) model has been used. Hereby it has to be noted that the form of the bubbles is an important input parameter to the FB model. Therefore two limits of the bubble form have been investigated: spherical bubbles and spherically capped bubbles.

For flow rate determination the approximate flow model has been used with the above mentioned slip models as input. Three different versions of the approximate models have been investigated: homogeneous model (hom), constant slip model (cs) and variable slip model (vs). These last two models (cs and vs) are labelled with EB or FB depending on what slip model has been used as input.

For slug/churn flow (low reference liquid rates) the vs_EB model gives relative errors in the superficial liquid velocity within 10% from zero, which is the goal of this investigation. For dispersed bubble flow (large reference liquid rates) all models predict relative errors in the superficial liquid velocity within 15% from zero. Further work thus has to be done to obtain predictions within 10% from zero. Hereby for the case of the vs_FB(spherical) model the form of the bubbles both at the inlet and in the throat of the venturi has to be investigated. Further also the viscosity of the liquid phase in the vs_FB(spherical) model (see subsection 6.2.2) may play an important role.

For the case of the superficial gas velocity all flow models give the same behaviour. The relative error in the gas rate is very large at low GVF and stabilizes at increasing GVF. For low reference liquid rates the relative error stabilizes within 10% from zero, which is acceptable. However for increasing reference liquid rates the relative error remains larger than 25% (even more than 75% for 30 degrees inclined from vertical), which leaves room for further work.
7 Discussion of approximate flow model

In this chapter the approximate equation (3.1) will be discussed on which the approximate flow model, discussed in section 3.1, is based. This will be done by considering a momentum equation derived by Biesheuvel and Van Wijngaarden [9] for dilute bubble flow which will be discussed in subsection 7.1. In this momentum equation several effects have been taken into account such as dynamic behaviour and the slip. By neglecting this dynamic behaviour and by assuming the slip \( S \) to be constant a flow model based on this momentum equation can be derived. This constant slip model will be compared with the approximate constant slip model in section 7.2. Further the influence of the dynamic behaviour of the bubble flow on the prediction of the superficial velocity will be investigated in section 7.3. After this a constant slip model, which takes the compressibility of the gas phase into account will be compared with the constant slip model of Biesheuvel and Van Wijngaarden [9] in section 7.4.

7.1 Momentum equation for dilute bubble flow

In this section the momentum equation, derived by Biesheuvel and Van Wijngaarden [9] for dilute bubble flow, will be discussed. From this equation an expression equivalent to the approximate equation (3.1) will be derived. The constant slip model that can be derived from this expression will then be compared with the the approximate constant slip model in the next section.

In this section a gas-liquid bubble pipe flow is considered in which a slip-effect between the phases is assumed. Despite this slip-effect however the surface tension of the bubbles is assumed large enough to keep the bubbles spherical. It is however possible that because of compressibility effects the bubble radius changes. Finally it is assumed that the bubble mixture is so dilute that interactions between the bubbles can be neglected. This approximation is, according to Biesheuvel and Van Wijngaarden [9], accurate in the first order of the gas holdup \( \alpha_g \) (the effect of bubble-bubble interaction is proportional to the square of the gas holdup).

In obtaining an expression for the equation of motion for the above described dilute bubble flow Biesheuvel and Van Wijngaarden [9] have derived expressions for the average bulk stress tensor and the average momentum flux tensor (see Appendix A). Hereby it is assumed that the contribution of the gas phase to the momentum flux can be neglected. In the derivation of both tensors use is made of an averaging method in which averaged quantities are constant on a certain scale, called the mesoscale. This scale is large with respect to the distance between two bubbles but small relative to the distance over which quantities, such as the averaged gas and liquid velocity \( V_{ag} \) and \( V_{al} \), vary significantly. The equation of motion for the dilute bubble flow is obtained by equating the rate of change of momentum to the sum of the divergence of the stress tensor and external forces. Only those forces are permitted that are constant on the mesoscale, such as gravity. The following equation then results in vector notation:

\[
\frac{\partial}{\partial t}\left(\alpha_i \rho_i V_{si}\right) + \nabla \cdot \left(\alpha_i \rho_i V_{si} V_{si}\right) = -\nabla(p) + \alpha_i \rho_i g - \nabla \cdot \left(\alpha_g \rho_g \left(\frac{dR}{dt}\right)^2 I + \frac{1}{2} V_{slip} V_{slip}\right) + O(a_g^2) \tag{7.1}
\]

with \( p \) as the bulk pressure which is corrected for the friction with the pipe wall, \( g \) as the gravitational acceleration, \( V_{slip} \) as the slip velocity between the phases \((=V_{ag} - V_{al})\), \( R \) as the radius of the bubble and \( I \) as the unit tensor. Hereby it has to be noted that \( V_{slip} V_{slip} \) denotes a tensor.
of the square of the actual liquid velocity \( V_{al} \). In equation (7.1) the terms on the left hand side are the local and convective acceleration terms related to the liquid phase. On the right hand side the third term corresponds to the volume changes of the bubbles and to the slip effect.

In this investigation the dilute bubble flow is considered to be quasi-one dimensional, as in the case of a flow through a venturi. By integrating equation (7.1) over a volume \( W \) in the converging part of the venturi, bounded by two cross-sections \( A \) which are an infinitesimal distance \( dx \) apart, and by applying the theorem of Gauss then the following momentum equation is obtained for this quasi-one dimensional flow (see Appendix A):

\[
\alpha \beta \left( \frac{\partial}{\partial t} V_{al} + V_{al} \frac{\partial}{\partial x} V_{al} \right) = \frac{1}{A} \frac{\partial}{\partial x} \left( \alpha \rho \frac{1}{2} V_{slip}^2 A \right) + O(\alpha^2)
\]

This momentum equation for quasi-one dimensional pipe flow will now be simplified by assuming that the slip term, the dynamic term and the term due to volume changes of the bubbles can be neglected. This is done to obtain an equivalent expression for the approximate equation (3.1). The neglected terms are very difficult to estimate, which leaves room for further work. The above mentioned assumptions then give the following simplified momentum equation:

\[
\alpha \beta V_{al} \frac{\partial}{\partial x} V_{al} = \frac{1}{A} \frac{\partial}{\partial x} \left( \alpha \rho \frac{1}{2} V_{slip}^2 A \right) + O(\alpha^2)
\]

where the pressure gradient has already been corrected for the hydrostatic head and the friction with the pipe wall. This simplified momentum equation has to be integrated over the converging part of the venturi to arrive at an expression equivalent to equation (3.1), which the approximate flow model is based on. This integration can be performed when the liquid holdup \( \alpha_1 \) and thus the slip factor \( S \) (see equation (2.12)) is assumed to be constant. The following expression is then obtained:

\[
\Delta p = \Delta \left( \frac{1}{2} \rho_m V_{al}^2 \right)_{throat} - \Delta \left( \frac{1}{2} \rho_m V_{al}^2 \right)_{inlet}
\]

with \( \rho_m \) as the mixture density. Comparison of this equation with the approximate equation (3.1) shows that the mixture velocity \( V_m \) in equation (3.1) is substituted by the actual liquid velocity \( V_{al} \). The difference between the constant slip models derived from these two equations will be considered in the following subsection.

When the liquid holdup is assumed to be variable the integration of equation 7.3 over the converging part of the venturi can not be done analytically. Therefore no variable slip model has been derived from equation 7.3.

7.2 Comparison of constant slip models

From the simplified BVW (Biesheuvel and Van Wijngaarden) momentum equation (7.4) together with the assumption of constant slip and mass conservation the following relation for the superficial liquid velocity can be derived:
\[ V_{sf} = \alpha_i \left( \frac{2\Delta p_v}{\alpha_i \rho_i \left( \frac{A_i^2}{A_t^2} - 1 \right)} \right) \]  \hspace{1cm} (7.5)

with \( A_i \) and \( A_t \) as the cross-sectional area at the inlet and in the throat of the venturi. Hereby it has to be noted that this expression is only valid for dilute bubble flow because this is the flow pattern that the BVW momentum equation is derived for.

The relation (7.5) for the superficial liquid velocity will now be compared with that given by the approximate constant slip flow model, as has been given in equation 7.6:

\[ V_{sf} = \frac{\alpha_i S}{1 + \alpha_i (S - 1)} \sqrt{\frac{2\Delta p_v}{\alpha_i \rho_i \left( \frac{A_i^2}{A_t^2} - 1 \right)}} \]  \hspace{1cm} (7.6)

with \( S \) as the constant slip factor. Hereby it has to be noted that the gas density is neglected relative to the liquid density. The ratio of both relations for the superficial velocity can be written as follows:

\[ \frac{V_{sf, \text{appr. model}}}{V_{sf, \text{BVW}}} = \frac{S}{1 + \alpha_i (S - 1)} = \frac{1}{\alpha_i \frac{S}{\alpha_i} + \alpha_i} = \frac{LVF}{\alpha_i} \]  \hspace{1cm} (7.7)

This ratio shows that, because \( S \) is always smaller than one, the superficial liquid velocity given by the approximate constant slip model is always smaller than that given by the BVW constant slip model.

For the case of a vertical water-air experiment \((V_{sl, \text{ref}}=0.5\text{m/s})\) the relative errors in the superficial liquid velocity relative to their reference values, determined from both relations (7.5) and (7.6), have been given in figure 7.1 as a function of GVF. Hereby the slip in the approximate constant slip model has been given by the FB model for spherically capped bubbles at the inlet of the venturi. Despite the fact that the BVW momentum equation has been derived for dilute bubble flow the whole range of GVF has been given.

**Figure 7.1.** The relative error \((=(V_{sl}-V_{sl, \text{ref}})/V_{sl, \text{ref}})\) in the superficial liquid velocity in a vertical water-air experiment \((V_{sl, \text{ref}}=0.5\text{m/s})\) given by the cs_FB(capped) and BVW constant slip models as a function of GVF.
Figure 7.1 shows for the dilute bubble region (GVF<0.3) that the relative error of the approximate constant slip model is about 10% smaller than that of the BVW constant slip model. This agrees with the fact that the superficial velocity given by the approximate model must be smaller than that given by the BVW model as is predicted by the ratio in (7.7). On the other hand it is very surprising that the approximate model, which is based on an assumption, gives lower errors than that based on the BVW momentum equation, which is derived from physical mechanisms. A possible explanation for this surprising result may be that the neglected terms in the BVW momentum equation do play an important role.

It is interesting to note for GVF>0.3, which corresponds to slug or churn flow (see section 5.1), that also here the approximate constant slip model predicts about 10% lower relative errors than the BVW constant slip model. Figure 7.1 further shows that for this slug/churn region the relative error is much larger than for the bubble flow region. This is not surprising because both models have not been developed for slug/churn flow but for bubble flow.

Finally it is interesting to note also here that for GVF=0 the relative errors given by both models become zero. This is because for these single-phase conditions both equations, which both models are based on, transform into the Bernoulli-equation.

### 7.3 Influence of dynamic behaviour

In this subsection a dilute dynamic gas/liquid bubble pipe flow will be considered to investigate the influence of dynamic behaviour on the prediction of the superficial velocity. This will be done by again considering the momentum equation of Biesheuvel, which will be integrated over the converging part of a venturi. The dynamic behaviour of the dilute bubble flow is supposed to consist of an oscillation of the liquid holdup $\alpha_l$ and the actual liquid velocity $V_l$ around their average values. Schematically this can be written as follows:

$$\alpha_l(t) = \bar{\alpha}_l + \tilde{\alpha}_l(t)$$

$$V_l(t) = \bar{V}_l + \tilde{V}_l(t)$$

(7.8)

with the average values denoted by a bar and the oscillatory part denoted by a tilde. Further it will be assumed that the mixture is incompressible (no volume changes of the bubbles) and that there is no slip. Further the pressure will be assumed to be corrected for the friction with the wall and the hydrostatic head.

Considering the assumptions described above the momentum equation of Biesheuvel then becomes with an inaccuracy of $\alpha_g$ :

$$\frac{1}{\rho_l} \frac{\partial p}{\partial x} = (\bar{\alpha}_l + \tilde{\alpha}_l) \frac{\partial}{\partial t} (\bar{V}_l + \tilde{V}_l) + (\bar{V}_l + \tilde{V}_l) \frac{\partial}{\partial x} (\bar{V}_l + \tilde{V}_l)$$

(7.9)

To determine the average pressure drop $\Delta p$ over a venturi this momentum equation has to be integrated over the converging part of this venturi and averaged in time. The following expression for the averaged pressure drop then results :

$$\frac{1}{\rho_l} \Delta p_v = \int_{\text{throat}} \left( \int_{\text{inlet}} \left( \bar{\alpha}_l \frac{d\bar{V}_l}{dt} + \tilde{\alpha}_l \frac{d\tilde{V}_l}{dx} \right) dx + \right.\left. \int_{\text{inlet}} \left( \bar{\alpha}_l \frac{d\bar{V}_l}{dt} + \tilde{\alpha}_l \frac{d\tilde{V}_l}{dx} \left( \frac{1}{2} V_{al}^2 \right) + \bar{\alpha}_l \frac{d}{dx} \left( \frac{1}{2} V_{al} V_{al} \right) \right) dx \right)$$

(7.10)
The first term on the right hand side of this equation gives the pressure drop in case the dilute bubble flow is steady. The second term gives the dynamic contributions to the average pressure drop. It is very difficult to estimate these dynamic contributions quantitatively because both the variations in the holdup and the actual liquid velocity are difficult to determine from bubble flow experiments. Therefore it is difficult to say quantitatively whether the dynamic terms can be neglected relative to the steady term or not.

It appears from equation (7.10) that the first and the third dynamic term can both be positive and negative so that their influence can not be estimated qualitatively. However the second term is always positive. Taking this positive dynamic term into account means that the prediction for the superficial liquid velocity, determined from equation (7.10), will decrease.

7.4 Influence of compressibility

In this section the behaviour of the compressibility of an oil-air flow as a function of GVF will be investigated by considering the speed of sound of the mixture. Also the influence of the compressibility of the gas phase on the prediction of the superficial liquid velocity in a vertical oil-air experiment will be investigated. This will be done by comparing an incompressible constant slip model of Slijkerman et al. with a constant slip model, also derived by Slijkerman et al., which takes the compressibility of the gas phase into account. Both models of Slijkerman et al. are based on the simplified BVW equation (7.4).

The behaviour of the compressibility of the mixture is directly given by the speed of sound of the mixture (see section 3.3). For isothermal conditions the speed of sound, which can be determined from the equations (3.24) and (3.25), in a vertical oil-air experiment ($V_{sl\_ref}=2m/s$) has been given as a function of GVF in figure 7.2 for both slip and no-slip conditions. Hereby it must be noted that the oil-air mixture is assumed to in constant equilibrium, which means that the pressure difference between the bubbles and the surrounding continuous liquid phase remains constant.

Figure 7.2 shows that the speed of sound is much smaller for the oil-air mixture than for the oil phase itself over about the whole range of GVF. As said in section 3.3 the mixture is thus much more compressible than the oil phase. For GVF=0 the speed of sound is 480 m/s. This does not agree with the reference value of about 1200 m/s for the oil phase. This discrepancy agrees with the fact that the equations (3.24) and (3.25) are a good approximation for the speed of sound unless the gas holdup is very low or very large. Further figure 7.2 shows at large GVF that the speed of sound is smaller when no slip is assumed. Finally it is noteworthy that the speed of sound given in figure 7.1 is determined under isothermal conditions. For the case of adiabatic conditions the speed of sound will be larger and the mixture thus more incompressible.
Now the influence of the compressibility of an oil-air flow on the prediction of the superficial oil velocity will be investigated. This will be done by considering a constant slip model, derived by Slijkerman et al. based on the simplified Biesheuvel momentum equation, that takes the compressibility of the gas phase into account. Hereby it is assumed that the gas phase expands adiabatically as the gas-liquid mixture flows through a venturi. Further it is assumed that the liquid phase is incompressible and that the two-phase flow is steady. The following relation for the superficial liquid velocity then results:

\[ V_{sl} = \frac{\alpha_i}{\gamma - 1} \rho_i \left[ 1 - \frac{1}{\beta_1} \right] + \alpha_i \rho_i \left[ 1 - \frac{1}{\beta_1} \right] + \alpha_i \rho_i \left[ 1 - \frac{1}{\beta_1} \right] \cdot \frac{1}{g h} \]  

with \( \gamma \) as the isentropic exponent, \( \beta \) as the ratio of the pressure at the inlet and in the throat of the venturi, \( S \) as the slip, \( g \) as the gravitational acceleration and \( h \) as the difference in height between the inlet and the throat of the venturi.

The relative errors in the superficial liquid velocity, given by the compressible (cs_comp.) and the incompressible (cs_incomp.) constant slip flow models, with respect to their reference values are given in figure 7.3 for an oil-air experiment (V_{sl_{ref}}=2m/s) as a function of GVF. Hereby it is assumed that the slip is given by the FB model for spherically capped bubbles at the inlet of the venturi. Although this slip model is developed for bubble flow, the relative error in figure 7.3 is given for the whole range of GVF.

Figure 7.3 shows that the compressible flow model gives lower relative errors in the superficial velocity than the incompressible flow model. This agrees with the fact that part of the pressure drop over the venturi is due to the expansion of the gas phase, as has been discussed in section 3.3. Further figure 7.3 shows that the difference between both models becomes larger at increasing GVF. This is because at increasing GVF the mixture velocity increases, as a consequence of the constant superficial liquid velocity (see chapter 4), so that also the pressure drop over the venturi increases. As a consequence the gas phase in the venturi shows a larger
expansion so that a larger part of the pressure drop over the venturi is due to the compressibility effect.

At large GVF the difference between the compressible and the incompressible flow model is maximal 5%. Implementing the compressibility effect in the approximate flow model therefore may decrease the relative liquid error at large GVF for inclined experiments at large reference liquid rates.

Further, figure 7.3 shows that there is almost no difference between the compressible and the incompressible flow model at low GVF. From this it may be concluded that the large increase of the relative liquid error at low GVF both in vertical and inclined experiments (see chapters 5 and 6) is not due to the compressibility effect. To understand this rapid increase of the relative liquid error at low GVF therefore further work has to be done.

![influence of compressibility](graph.png)

Figure 7.3, The relative error in the superficial oil velocity \(=(V_{sl}-V_{sl\_ref})/V_{sl\_ref}\) in a vertical oil-air experiment \(V_{sl\_ref}=2\text{m/s}\), given by both the compressible and the incompressible constant slip models of Slijkerman et al.
8 Conclusions and suggestions

In this chapter the conclusions will be given concerning the flow models and the slip models that have been discussed in this investigation. Further also suggestions for future work will be given.

Conclusions

The conclusions of this investigation, that will be discussed here, will be divided into several parts starting with the flow regimes that have occurred in the considered air-liquid experiments:

Flow regimes

In vertical air-liquid pipe flow experiments mainly two flow patterns occur: bubble and slug/churn flow. In inclined air-liquid pipe flow experiments also these two flow patterns occur. At increasing inclination from vertical the bubble flow pattern is taken over by the slug/churn flow pattern. Comparison of the experimental flow maps, which give the transitions between these flow patterns, show good agreement with theoretical flow maps by Taitel&Dukler [2] (vertical flow) and Barnea et al. [3] (inclined flow). This result is satisfying considering the difficulties to determine the transitions between the several flow patterns.

Slip models

An important effect that occurs in inclined air-liquid pipe flow is the slip-effect: the gas in inclined air-liquid flow moves faster than the liquid because of the buoyancy force. For the slip several models have been investigated that correspond to bubble flow or to elongated bubble flow. These slip models, that correspond to these different flow patterns, will now be discussed here separately starting with that for bubble flow.

In literature several slip models for bubble flow have been investigated. However, these models show no consistency as a function of GVF. The empirical Nicolas&Witterholt (NW) model [1] for oil-water bubble flow is used for comparison with a Force Balance model (FB) slip model that has been developed in this investigation from a force balance on a single bubble. In this FB model the bubble form is a very important input parameter. Therefore in this investigation two limits of the bubble form have been investigated, which correspond to two limits of the friction factor $C_D$ for the single bubble: spherical ($C_d=48/Re$) and spherically capped ($C_D=2.6$). The FB model that assumes the bubbles to be spherical or spherically capped is labelled FB(spherical) or FB(capped) respectively.

The slip at the inlet and in the throat of a venturi is investigated because the approximate flow model, that has been developed in this investigation, uses such a venturi. At the inlet of the venturi the FB(capped) model shows a good agreement with the NW model. The slip Reynolds number, which is measure for the bubble form, predicts also a spherically capped form of the bubbles at the inlet of the venturi. It thus may be concluded that the form of the bubbles at the inlet of the venturi is spherically capped. In the throat of the venturi the FB(capped) model also shows good agreement with the NW model. However, the slip Reynolds number predicts the bubbles in the throat of the venturi to be spherical. Hereby it also has to be noted that the NW model is an empirical model.
that has been derived for oil-water bubble flow (and thus not for gas-liquid flow). From this it may be concluded that the form of the bubbles in the throat of the venturi is not clear. Comparison of the slip models shows that both the FB(spherical) and the FB(capped) models predict a larger slip-effect in the throat of the venturi relative to that predicted by the FB(capped) model at the inlet of the venturi. Finally it must be noted that the FB(spherical) model predicts a very large slip-effect at the inlet of the venturi, which is physically not acceptable.

For elongated bubble flow, a special form of slug flow, the elongated bubble (EB) slip model has been investigated for inclined air-liquid pipe flow experiments. Also this slip model predicts a larger slip-effect in the throat of the venturi relative to that at the inlet.

Approximate flow model

Three versions of the approximate flow model, which is based on the assumption that the mixture can be considered as one phase, have been investigated: homogeneous (hom), constant slip (cs) and variable slip (vs). First the conclusions for the determination of the liquid flow rate will be given. After this conclusions concerning the determination of the gas flow rate will be given. For the liquid flow rate the vertical and inclined case will be treated separately.

The different versions of the approximate flow model are either labelled FB or EB depending on what slip has been used as input parameter. In the vs_FB model the bubbles at the inlet of the venturi are assumed spherically capped while in the throat of the venturi both limits of the bubble form have been investigated. Depending on the bubble form in the throat of the venturi the vs_FB will be labelled vs_FB(spherical) or vs_FB(capped). In the cs_FB model the slip is given by the FB model for spherically capped bubbles at the inlet of the venturi.

For the bubble flow region (GVF<0.3) in vertical water-air experiments the vs_FB(capped) model predicts the lowest relative errors in the superficial liquid velocity relative to their reference values, also called relative liquid error. These errors range within 10% from zero. This is satisfying because this is the goal of this investigation. The homogeneous model predicts the largest relative liquid errors within -10% and 30% from zero. This is not surprising since this model is the simplest version of the approximate flow model. The cs_FB(capped) model predicts lower relative errors than the homogeneous model but larger relative errors than the vs_FB(capped) model.

For the slug flow region (GVF>0.3) in vertical water-air experiments also the vs_FB(spherical) model predicts the lowest relative liquid errors. Also these errors range within 10% from zero. This result is surprising not only because the flow model is approximate but also because the FB slip model has been developed for bubble flow and not for slug/churn flow. Also here the homogeneous model predicts the largest relative liquid errors.

For inclined pipe flow experiments, both 30° and 60° inclined from vertical, the flow models for bubble flow (low reference liquid rates) and slug/churn flow (large reference liquid rates) have been treated separately. For the slug/churn flow region the cs_EB and the vs_EB model predict relative liquid errors within 10% from zero, which is the goal of this investigation. For the (dispersed) bubble flow region all models (hom, cs_FB(capped), vs_FB(capped) and vs_FB(spherical)) predict relative liquid errors within 15% from zero. Further work has to be done to obtain predictions within 10% from zero for this dispersed bubble region. Hereby for the vs_FB(spherical) model the form of the bubbles, the viscosity of the liquid phase and the compressibility of the gas phase in the venturi have to be investigated. The conclusions concerning the influence of the compressibility on the relative liquid error will be given further on in this chapter.

For the relative error in the superficial gas velocity both in vertical and inclined experiments all the considered versions of the hypothetical flow model show the same behaviour for the whole range of GVF. At low GVF the relative error is very large and it decreases at increasing GVF. This is
because at low GVF the gas rate is so low that a small absolute error in the gas rate may give a very large relative error. Finally the relative gas error stabilizes at large GVF. At increasing reference liquid rate the relative gas error stabilizes at a larger relative error. For $V_{\text{SL, ref}}=0.5$ m/s the relative error stabilizes in a range of 10% from zero, which is acceptable, whereas for $V_{\text{SL, ref}}=2$ m/s the error stabilizes between 50% and 100%, which may be caused by compressibility effects. These compressibility effects for gas flow determination have to be investigated in future work. Finally no dependence of the relative error in the superficial gas velocity on the inclination angle has been found.

Biesheuvel and Van Wijngaarden momentum equation

The assumption of the approximate flow model has been compared to an equivalent expression developed from a momentum equation for dilute bubble flow as has been derived by Biesheuvel and Van Wijngaarden [9]. Hereby several terms corresponding to dynamic behaviour, volume changes of the bubbles and the slip have been neglected. Further a constant slip is assumed. The constant slip model, based on the BVW momentum equation, predicts about 5 to 10 percent larger relative liquid errors than the cs-FB(capped) model for vertical water-air experiments. This difference may be because the neglected terms in the BVW momentum equation do play a role. Therefore these terms have to be investigated in future.

Slug flow model of Fernandes et al.

The slug flow model of Fernandes et al. is not recommendable for velocity measurements in vertical slug flow. This is because already a slight change in the input parameters of the slug flow model gives very large changes in the predicted superficial velocities.

Influence of compressibility

It appears that the speed of sound of the considered gas-liquid mixtures is much lower than that of the separate phases. This means that the mixture is much more compressible than the separate phases. The constant slip model of Slijkerman et al., that takes the adiabatic expansion of the gas phase into account, gives lower predictions for the relative error in the liquid superficial velocity than the incompressible slip model of Slijkerman et al. The maximum difference between both models is 5%, which occurs at large GVF. The goal to predict the relative liquid error within 10% may thus be reached at large GVF for experiments at large reference liquid rates in inclined pipes by implementing the compressibility-effect into the approximate flow model.

Suggestions

Now some suggestions will be given for future work:

- For flow rate determination in slug flow the dynamic pressure drops over two similar venturis may be correlated to each other. (idem for two short gradiometers or even two fast pressure sensors)

- Estimate the neglected terms in the Biesheuvel momentum equation.
- Use the EB slip model as an input model to the slug/churn region in vertical air-liquid experiments.

- Use a Multi-capacitance flow meter to determine the input parameters to the slug flow model of Fernandes et al. [11]. Hereby also other input parameters can be measured than those chosen in this investigation.

- Study the behaviour of bubble break-up and coalescence in a venturi. Study the behaviour of deformation of bubbles.

- Study the effect of a large gas density, which cannot be neglected relative to the liquid density, and the role of the viscosity of the liquid phase on the slip-effect.

- Implement the compressibility-effect of the gas phase in the venturi into the approximate flow model.
List of references


Appendix A

Momentum equation for dilute bubble flow

In this appendix a momentum equation for dilute bubble flow will be derived, as has been done by Biesheuvel et al. In this derivation first expressions will be derived for the average stress tensor \( \langle \sigma \rangle \) and the momentum flux tensor \( \mathbf{M} \). The momentum equation is then obtained by equating the rate of change of momentum to the sum of divergence of the stress tensor and the external forces.

The dilute bubble flow considered in this derivation will be assumed to be unsteady, inhomogeneous and incompressible. Further the bubbles will be assumed to be spherical. Also a large slip-effect is assumed between the bubbles and the liquid.

Finally in this derivation quantities are considered on a certain mesoscale. This scale is very large with respect to the distance between two bubbles but very small relative to the distance over which quantities like the averaged actual liquid and gas velocities, \( \mathbf{V}_{al} \) and \( \mathbf{V}_{ag} \), vary.

bulk stress tensor

In the derivation of an expression for the averaged bulk stress tensor a volume \( W \) is considered on the mesoscale. For this volume Biesheuvel et al have considered the components \( \langle \sigma_{ij} \rangle \) of the averaged bulk stress tensor in terms of the contribution of the liquid phase and that of the gas phase and the interfaces combined. This can be written as follows:

\[
\langle \sigma_{ij} \rangle = \frac{1}{W} \int_{W_1} \sigma_{ij} dW + n \left( \int_{W_{ij}} \sigma_{ij} dW \right)
\]  

with \( n \) as the density of the bubbles in the volume \( W \), \( W_1 \) as the volume occupied by the liquid and \( W_{ij} \) as the volume occupied by a test sphere and its interface region in the volume \( W \). Hereby it has to be noted that the integral in the second term on the right hand side depends on the positions of other spheres. The averaging of this integral is done over all possible realizations of the test spheres in volume \( W \).

The integral in the second term on the right hand side of equation A1 can be determined by applying the theorem of Gauss to. When the inertia of the gas bubbles are neglected then the following relation is obtained:

\[
\int_{W_{ij}} \sigma_{ij} dW = \int_{A_i} \sigma_{ik} n_k dA = - \int_{A_i} \rho r_j n_j dA
\]

with \( A_i \) as the interface that lies completely in the liquid, \( r \) as the position vector of a point relative to the centre of volume \( W \), \( \rho \) as the pressure in the liquid in which no hydrostatic head is included and \( n \) as the unit vector normal to the surface \( A_i \).

The first term on the right hand side of A1, which gives the contribution of the liquid to the bulk stress tensor, can be written as follows:
\[
\frac{1}{W} \int_{W} \sigma_{ij} \, dW = \alpha_i \langle \sigma_{ij} \rangle = -\alpha_i \langle p \rangle \delta_{ij} \quad (A.3)
\]

with \(\delta_{ij}\) as the Kronecker delta and \(\alpha_1\) as the liquid holdup.

The equations A1, A2 and A3 combined give the following relation for the bulk stress tensor:

\[
\langle \sigma_{ij} \rangle = -\alpha_i \langle p \rangle \delta_{ij} - n \left( \int_{W} p_r n_r \, dA \right) \quad (A.4)
\]

To determine the integral for the test sphere in this equation Biesheuvel et al. have assumed that the bubble flow is so dilute that interactions between the bubbles can be neglected. As a first approximation then the test sphere can be considered to be situated in an infinite uniform flow. As a consequence the integral is the same for every realization of the test spheres in the volume \(W\).

For the approximation of a single test sphere in an infinite uniform flow Biesheuvel et al. have derived an expression for the local pressure \(p\). This is done by applying the theorem of Bernoulli to an expression for the hydrodynamic potential, which is derived from mass conservation of the single test sphere whose radius \(R\) may change in time. Substituting this expression for the pressure in equation A4 then gives the following expression for the averaged bulk stress tensor \(\langle \sigma \rangle\):

\[
\langle \sigma \rangle = -\alpha_i \langle p \rangle + \alpha_g \langle p \rangle + \frac{3}{2} \alpha_g \rho_g \left( \frac{dR}{dt} \right)^2 + \alpha_g \rho_g R \frac{d^2 R}{dt^2} - \frac{1}{4} \alpha_g \rho_g \left( \left| V_{ag} - V_{al} \right| \right)^2 \mathbf{I} \quad (A.5)
\]

with \(\mathbf{I}\) as the second order unit tensor, \(R\) as the radius of the test bubble, \(V_{ag}\) as the actual gas velocity, \(V_{al}\) as the actual liquid velocity, \(\rho_g\) as the gas density and \(\rho_l\) as the liquid density. Hereby it must be noted that \(\left( V_{ag} - V_{al} \right) \) is approximated by \(\left( V_{ag} - V_{al} \right)\). This is accurate in the order \(\alpha_g\).

Because the bulk pressure \(\langle p \rangle\) is related to \(\langle \sigma_{ij} \rangle\) by:

\[
\langle p \rangle = -\frac{1}{3} \langle \sigma_{ii} \rangle \quad (A.6)
\]

the bulk stress tensor can be written as follows:

\[
\langle \sigma \rangle = -\langle p \rangle \mathbf{I} + \alpha_g \rho_g \left[ \frac{3}{20} \left( \left| V_{al} - V_{ag} \right| \right)^2 \mathbf{I} - \frac{9}{20} \left( V_{al} - V_{ag} \right) \left( V_{al} - V_{ag} \right) \right] \quad (A.7)
\]

**Momentum flux tensor**

Biesheuvel et al. assume that the contribution of the gas phase to the momentum of the mixture can be neglected relative to that of the liquid phase. The following expression for the momentum flux tensor \(\mathbf{M}\) can then be written as follows:
with \( v \) as the local velocity. Biesheuvel et al divide this local velocity into the average velocity \( V_0 \) and a fluctuating part \( v' \). Inserting this into equation A8 gives:

\[
\mathbf{M} = \alpha_i \rho_{,i} V_0 V_0' + \alpha_i \rho \left[ V_0 (V_{g} - V_0) + (V_{g} - V_0) V_0 \right] + \frac{\rho_i}{W} \int v' v' dW
\]

(A.9)

To determine the integral in this equation Biesheuvel et al have also here assumed that the bubble flow is so dilute that interactions between the bubbles can be neglected. As a first approximation then also here the bubbles in the volume \( W \) can be considered as single bubbles in an infinite uniform flow. The integral in equation A.9 can then be written as follows:

\[
\frac{1}{W} \int v' v' dW = n \int_{r \leq R(t)} v' v' dr + O(\alpha_g^2)
\]

(A.10)

with \( n \) as the bubble density in the volume \( W \), \( r \) as the position vector relative to the centre of a test sphere and \( R \) as the radius of this test sphere. The integral on the right hand side corresponds to one test sphere and is therefore integrated over all space outside this sphere.

For the case of a test sphere in an infinite uniform flow Biesheuvel et al have derived an expression for \( v' \) from the hydrodynamic potential, derived from mass conservation of a single bubble whose radius changes in time. Substituting this expression in equation A10 then the following relation is obtained by Biesheuvel et al from the equations A10 and A9:

\[
\frac{\mathbf{M}}{\rho_i} = \alpha_i V_{al} V_{al} + \alpha_g \left[ \frac{dR}{dt} \right]^2 + \frac{3}{20} \left| (V_{ag} - V_{al}) \right|^2 I + \frac{1}{20} \alpha_g (V_{ag} - V_{al}) (V_{ag} - V_{al})
\]

(A.11)

Hereby it again must be noted that \((V_{ag} - V_0)\) is approximated by \((V_{ag} - V_{al})\). This is accurate in the order \( \alpha_g \).

**momentum equation for dilute bubble flow**

Biesheuvel et al have obtained a momentum equation for a dilute bubble flow by equating the rate of change of the momentum flux tensor to the sum of the divergence of the bulk stress tensor and external forces. Based on the equations A7 and A11 this momentum equation then can be written as follows:

\[
\frac{\partial}{\partial t} \alpha_i \rho_{,i} V_{al} + \nabla \cdot \left( \alpha_i \rho_{,i} V_{al} V_{al} \right) = -\nabla \langle p \rangle + \alpha_i \rho g - \nabla \cdot \left( \alpha_g \rho \left[ \frac{dR}{dt} \right]^2 I + \frac{1}{2} (V_{ag} - V_0) (V_{ag} - V_0) \right) + O(\alpha_g^2)
\]

(A.12)

with the same variables as defined above.
momentum equation for quasi-one dimensional dilute bubble flow

For the case of a dilute bubble flow through a venturi with its axis in the x-direction the vector equation A12 can be transformed into a scalar equation for the x-component by integrating over a volume V which is bounded by two cross-sections which are a small distance dx apart. This will now be discussed below. Hereby it will be assumed that the bubble flow is steady. Performing the above mentioned integration of equation A12 over the mentioned volume V gives:

\[
\oint\oint\oint \nabla \cdot (\alpha_i \rho_i \mathbf{V}_i) dV = \oint\oint\oint (-\nabla \langle p \rangle + \alpha_i \rho_i \mathbf{g}) \cdot \nabla \cdot \left( \alpha \rho_i \left[ \frac{dR}{dt} \right]^2 \mathbf{I} + \frac{1}{2} \left( \mathbf{V}_{ag} - \mathbf{V}_i \right) \left( \mathbf{V}_{ag} - \mathbf{V}_i \right) \right) dV
\]  

(A.13)

By applying the theorem of Gauss this volume integral transforms into the following surface integral:

\[
\oint\oint \left( \alpha_i \rho_i \mathbf{V}_i \cdot \mathbf{n} \right) dA = \oint\oint \left( -\langle p \rangle + \alpha_i \rho_i \left( \frac{dR}{dt} \right)^2 \mathbf{I} \cdot \mathbf{n} - \frac{1}{2} \left( \frac{dR}{dt} \right)^2 \left( \mathbf{V}_{ag} - \mathbf{V}_i \right) \left( \mathbf{V}_{ag} - \mathbf{V}_i \right) \right) dA + \alpha_i \rho_i \mathbf{g} \cdot \mathbf{V}
\]  

(A.14)

with \( \mathbf{A} \) as the surface that encloses the volume V. It will now be assumed that the velocity of the mixture flows parallel to the pipe wall. For the x-component of equation A14 then the following relation can be written:

\[
\left. \left( \alpha_i \rho_i \mathbf{V}_i \right) \right|_{x+dx}^x = \left. \left( \left( \oint\oint \left( p + \alpha_i \rho_i \left( \frac{dR}{dt} \right)^2 \right) \mathbf{n} dA \right) - \left( \frac{1}{2} \alpha \rho_i \left( \mathbf{V}_{ag} - \mathbf{V}_i \right)^2 \right) dA \right) \right|_{x+dx}^x + \alpha_i \rho_i \mathbf{g} \cdot \mathbf{A} d\mathbf{x}
\]

(A.15)

with \( \mathbf{V}_{ag} \) and \( \mathbf{V}_i \) as the actual liquid and gas velocities in the x-direction. The integral for the pressure \( p \) in equation A15 can be calculated as follows:

\[
\left( \oint_{\mathbf{A}} p dA \right)_{x+dx}^x = (pA)_{x+dx}^x - p \frac{\partial A}{\partial x} \frac{dx}{x} = A \frac{\partial p}{\partial x} \frac{dx}{x}
\]

(A.16)

The same procedure can be done for the integral which accounts for the change of the bubble radius. Substituting this equation into A15 and then dividing by \( \partial x \) then gives:
Using the law of conservation of mass for the liquid phase and dividing A16 by A then gives the momentum equation for quasi-one dimensional steady dilute bubble flow:

\[
\frac{\partial}{\partial x} \left( \alpha_1 \rho_l V_{al}^2 A \right) = -A \frac{\partial p}{\partial x} - A \frac{\partial}{\partial x} \left( \alpha_1 \rho_l \left( \frac{dR}{dt} \right)^2 \right) \hat{x} \\
+ \alpha_1 \rho_l g_x A - \frac{\partial}{\partial x} \left( \frac{1}{2} \alpha_1 \rho_l \left( V_{ag} - V_{al} \right)^2 A \right)
\]

(A.17)

\[
\alpha_1 \rho_l V_{al} \frac{\partial}{\partial x} V_{al} = -\frac{\partial}{\partial x} (p) - \alpha_1 \rho_l g_x - \frac{\partial}{\partial x} \left( \alpha_1 \rho_l \left( \frac{dR}{dt} \right)^2 \right) \\
- \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{1}{2} \alpha_1 \rho_l \left( V_{ag} - V_{al} \right)^2 A \right)
\]

(A.18)
Appendix B

Derivation of the FB slip model

In this appendix the FB model will be discussed which gives an expression for the slip of a bubble in bubble flow, based on a force balance for a single bubble. Hereby it is assumed that the slip is stabilized at any moment.

In the FB model three forces are assumed to work on the bubble: the pressure force, the drag force and the gravity force. Further it is assumed that the bubble drags a liquid mass along, called the virtual mass, whose volume is half that of the bubble. The force balance on a single bubble then can be written as follows, as has been done by Auton et al. [15]:

\[
|\nabla p| V_{\text{bubble}} - \rho_b V_{\text{bubble}} g - \frac{1}{2} C_D \rho_l \pi r^2 V_{\text{slip}}^2 = \rho_b V_{\text{bubble}} a_b + \frac{1}{2} \rho_l V_{\text{bubble}} (a_b - a_l)
\]  

(B.1)

with \(|\nabla p|\) as the absolute pressure gradient, \(V_{\text{bubble}}\) as the volume of the bubble, \(\rho_b\) as the density of the bubble, \(\rho_l\) as the liquid density, \(C_D\) as a friction coefficient, \(r\) as the radius of the bubble, \(V_{\text{slip}}\) as the slip velocity, \(a_b\) as the acceleration of the bubble and \(a_l\) as the acceleration of the continuous liquid. It is noteworthy that the second term on the right hand side gives the acceleration of the virtual mass by the bubble relative to the continuous liquid.

When the slip is stabilized then the acceleration of the continuous liquid and the bubble are equal:

\[a_b = a_l\]  

(B.2)

The acceleration \(a_l\) of the continuous liquid can be determined from a force balance for a cubic volume of the continuous liquid. This force balance gives:

\[a_l = \frac{|\nabla p|}{\rho_l} - g\]  

(B.3)

Substituting the equations B2 and B3 into equation B1 then gives the following relation for the slip velocity \(V_{\text{slip}}\) as a function of the bubble radius, friction coefficient \(C_D\) and the absolute pressure gradient:

\[V_{\text{slip}} = \sqrt{\frac{8 |\nabla p| r}{3 C_D \rho_l} \left(1 - \frac{\rho_b}{\rho_l}\right)}\]  

(B.4)
Appendix C

Approximate flow model relations

In this appendix the derivation of the relations 3.5 and 3.12 as have been used in the Appendix B flow model will be discussed. Equation 3.5 gives the relation between the volume fractions and the holdups and slip at inlet conditions of the venturi. Equation 3.12 gives the relation between the holdups at the inlet of the venturi and those in the throat of the venturi.

Equation 3.5 is arrived at when the following equation, based on equation 2.12, is rewritten:

\[
\alpha_{gi} = \frac{(1 - LVF)S_i}{(1 - LVF)S_i + LVF}
\]  

(C.1)

with \( \alpha_{gi} \) as the gas holdup at the inlet of the venturi, \( S_i \) as the slip factor at the inlet of the venturi and \( LVF \) as the liquid volume fraction.

Equation 3.12 is derived from mass balances for the liquid and gas phases respectively over the converging part of the venturi. These two mass balances are written as follows:

\[
\begin{align*}
\alpha_{gi} \rho_y V_{ag,i} & = \alpha_{gi} \rho_y V_{ag,i} \\
\alpha_{li} \rho_l V_{al,i} & = \alpha_{li} \rho_l V_{al,i}
\end{align*}
\]

(C.2)

with \( V_{ag,i} \) and \( V_{al,i} \) as the actual gas and liquid velocities at the inlet of the venturi, \( \alpha_{li} \) as the liquid holdup at the inlet of the venturi, \( \alpha_{gi} \) and \( \alpha_{lt} \) as the gas and liquid holdup in the throat of the venturi. Hereby the first gives the mass balance for the gas phase and the second one that for the liquid phase. These two equations can be substituted into each other considering the fact that sum of the holdups at the same positions must be one. Rewriting the resulting equating then gives equation 3.12:

\[
\alpha_{li} = 1 - \alpha_{gi} = \frac{\alpha_{li} S_i}{\alpha_{gi} S_i + \alpha_{li} S_i}
\]

(C.3)
Thanks!

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