Holographic interferometry applied to supersonic nozzle flow with condensation

Versteeg, W.

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Holographic interferometry applied to supersonic nozzle flow with condensation

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Wijnand Versteeg
id.nr.: 373877

Applied Physics
Group of Physical Transport Phenomena
Workunit Gas Dynamics

Supervisors:
Prof. Dr. Ir. M.E.H. van Dongen
Ir. G. Lamanna
Summary

In this report, supersonic nozzle flow with condensation is described. Flow properties in supersonic nozzle flow such as density, pressure and temperature are affected by the release of latent heat due to condensation. The relative humidity of the test gas determines the character and amplitude of disturbances in the flow. These disturbances range from small local changes to abrupt discontinuities (i.e. shock waves) in the flow properties.

The supersonic nozzle flow is visualised by means of holographic interferometry. Main objective of this project was to determine the accuracy and reliability of the experimental set-up, consisting of a holographic interferometer and an automated method for the analyses of the interferograms. First, the quality of the interferograms has been optimised.

Secondly, the automated interferogram interpretation, based on the Data Dependent System (DDS) modeling method, has been critically evaluated and tested under simplified conditions. It appeared that the system works under limited conditions, but its reliability for the real case is still questionable.

For three experimental cases of different initial conditions, the holographic interferograms have been evaluated manually using the fringe counting method. The manual method showed satisfying agreement with the corresponding numerical simulations. This method appeared to be more efficient than the automatic method in extracting reliable density fields from the interferograms.
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1. Introduction

In this section, first, a brief overview of supersonic nozzle flow research is presented. Then the dynamics of compressible fluid flow is illustrated in an introductory way, focusing in particular on compressible flow through a duct of non-constant cross section without and with condensation. In this framework, the theory of formation and growth of droplets is also briefly discussed.

1.1 Background

The phenomenon of condensing flows in nozzles was first observed by Prandtl [13] in 1935, who noticed the occurrence of two oblique shock-like disturbances just downstream of the nozzle throat. A year later (1936), Herman [5] published the first systematic experimental investigation of such phenomena and demonstrated that these disturbances are caused by condensation and subsequent release of latent heat in air. Moreover, their location along the nozzle axis depends upon the initial humidity of the mixture. The investigation of the Aachen group also proved the following:

- actual condensation is always observed at supersonic speeds (i.e. water vapour is in a highly supersaturated state)
- the nozzle geometry and size (i.e. temperature gradient) control the water vapour supercooling.
- upon the collapse of this supersaturated state, droplet growth and wall icing may occur as side effects of the condensation.

However, it was not until 1941 that the first unified kinetic and thermodynamic treatment of moist air condensation was presented by Oswatsisch [10]. He proved theoretically that heat in rapid expanding vapours can be supplied only in the transonic/supersonic regime and that, at a microscopic scale, the formation of droplets is due to homogeneous nucleation. This theory, although based on the empirical adjustment of a coefficient inferring the onset of condensation, remained for more than 30 years unsurpassed due to the impossibility to provide accurate experimental data for the nucleation process.

Research on supersonic flow with condensation gained new momentum in 1962 upon the discovery of self-excited oscillations in slender nozzle flow of moist air by Schmidt [17]. Wegener and Cagliostro [27] investigated unsteady condensation of water vapour in nozzles using a Ludwieg tube. Schnerr et al. [22] gave a survey on both steady and unsteady condensation effects in nozzles using Schlieren optics and discovered several new oscillation modes. They combined the experiments with results from numerical simulations with a 2D Euler code (Mündinger (1994) [9]). White et al. (1996) [31] studied condensation in steam turbines and measured droplet sizes with a light extinction method and compared these with numerically obtained values.

The renewed interest of recent years towards the investigation compressible flow with phase transition is motivated by several concurrent and synergetic factors, namely:

1. The increasing possibility to simulate numerically unsteady compressible flows with condensation.
2. The availability of accurate nucleation data.
3. The development of successful semi-phenomenological nucleation theories.
4. Its practical relevance to many different industrial and engineering applications, such as the design of wind tunnels and wet steam turbines, air flow around profiles (for example aeroplane wings and turbine blades/propellers), for safety of high pressure systems in nuclear power plants, for gas cleaning systems and climate control installations, to name a few.

The work described in this thesis is done in co-operation with STORK Special Products in Amsterdam.

In order to manipulate and control the condensation in a flow of a given gas mixture, the underlying principles of such a complex phenomena must be understood. Purpose of the work described in this report is to perform experiments to visualise and determine quantitatively the effects of condensation on the flow. The experiments were performed using a mixture of nitrogen and water vapour in a supersonic nozzle flow created using a Ludwieg tube. The flow through the Laval nozzle is visualised by means of a holographic interferometer, designed by Theeuwes [24]. Data extracted from the experiments are compared with data obtained from simulations performed using a numerical model developed earlier by Prast [15]. A relevant part of this work is also devoted to the critical assessment of the experimental set-up, with particular emphasis on the methods for evaluation of the interferograms, since they represent a key factor in the extraction of accurate quantitative data.

1.2 Nozzle flow

1.2.1 Isentropic flow

As a first step, to get some insight in the complex phenomena occurring in expansion flows, the one-dimensional, adiabatic and frictionless flow of a perfect gas through a Laval nozzle is here presented.

A Laval nozzle is a convergent-divergent duct as shown in figure 1.1. The minimum cross section is known as the throat. The variation in cross-sectional area causes a change in flow properties such as velocity, temperature, pressure and density. The way the properties change depends on the local Mach number, defined as the ratio between the local speed of the gas \( v \) and the local speed of sound \( c \) [2], [16],

\[
M = \frac{v}{c}. \tag{1.1}
\]

The flow is called subsonic when the Mach number is smaller than unity, \( v < c \). The speed of sound is defined as the square root of the derivative of density to pressure at constant entropy:

\[
c^2 = \left( \frac{dp}{d\rho} \right)_s, \tag{1.2}
\]

which for a perfect gas yields

\[
c = \sqrt{\gamma \frac{RT}{\rho}}, \tag{1.3}
\]

with \( \gamma \) the ratio of the specific heats and \( R \) the specific gas constant. For a perfect gas, \( R \) is the difference between the specific heats:

\[\text{1 Carl Gustav Patrick de Laval (1845-1913) was a Swedish inventor who developed such a nozzle as part of a high-speed turbine for driving a cream separator (1883).}\]
Consider a control volume as shown in figure 1.1. Continuity of mass gives us:

\[ \frac{dp}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0, \]

with \( \rho \) the density, \( A \) the cross-sectional area and \( v \) the speed of the flow. The Euler equation holds and can be written as

\[ \frac{1}{\rho} = -v \frac{dv}{dp}, \]

where \( p \) is the pressure. Combining these equations with the definitions of Mach number and speed of sound, yields a relation between speed and cross-sectional area:

\[ \frac{dv}{v} = \frac{1}{M^2 - 1} \frac{dA}{A}. \]

From equation 1.8, it is obvious that \( dA \) and \( dv \) have opposite sign for \( M < 1 \) subsonic flow. This means that subsonic flow accelerates when the cross-sectional area decreases, which is the case in the convergent part of the nozzle. If the flow is supersonic \( (M > 1) \), \( dA \) and \( dv \) have equal sign. Thus, the speed in a supersonic flow decreases when the duct is convergent and supersonic flow will accelerate where the cross-sectional area increases, like in the divergent part of the nozzle. From equation 1.8, it also becomes clear that a positive acceleration from subsonic to supersonic flow, \( dv > 0, M = 1 \), can only occur when \( dA = 0 \), that is, in the throat of the nozzle. The same holds for a flow decelerating from supersonic to subsonic speed. In isentropic flow, the throat is the only place where sonic speed can occur.

\[ Fig. 1.1: \text{Laval nozzle} \]

When a flow is decelerated to zero speed in an isentropic way, the resulting flow properties are called \textit{stagnation properties}. In a steady, isentropic flow, stagnation properties remain constant. By definition, the stagnation enthalpy is written as

\[ h_0 = h + \frac{1}{2} v^2. \]
The stagnation enthalpy of a flow is the sum of the static enthalpy of the fluid \( h \) at a point and the kinetic energy at the same point. The enthalpy of a perfect gas can be expressed as

\[
h = c_p T,
\]

with \( c_p \) the specific heat and \( T \) the temperature. This gives us a relation between the stagnation temperature \( T_0 \) and the static temperature \( T \):

\[
T_0 = T + \frac{v^2}{2c_p}.
\]

And, since for a perfect gas the following relations hold

\[
v^2 = M^2 \gamma RT,
\]

\[
c_p = \frac{\gamma R}{\gamma - 1},
\]

the ratio between static and stagnation temperature can be expressed in terms of Mach number:

\[
\frac{T}{T_0} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2}.
\]

All the other properties can, then, easily be derived making use of the isentropic relations for a perfect gas:

\[
p \propto p^\gamma \propto T^{\gamma - 1}.
\]

We find:

\[
\frac{P}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}},
\]

\[
\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-\frac{1}{\gamma - 1}}.
\]

Equation 1.14 can also be written in differential form as

\[
\frac{dT}{T} = \frac{dT_0}{T_0} - \frac{(\gamma - 1)M^2}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M}.
\]

Considering that for isentropic flow \( dT_0 = 0 \), this equation, together with the differential form of the definition of the Mach number for a perfect gas,

\[
\frac{dM}{M} = \frac{dv}{v} \frac{1}{2} \frac{dT}{T},
\]

can be substituted in equation 1.8 to give a relation between Mach number and cross-sectional area

\[
\frac{dM}{M} = \frac{1 + \frac{\gamma - 1}{2} M^2}{M^2 - 1} \frac{dA}{A}.
\]
Since the cross-sectional area of the nozzle can be expressed as a function of the axial coordinate \( x \), the Mach number can be calculated at every position along the nozzle axis. By using equations 1.14, 1.16 and 1.17, the distribution of temperature, density and pressure can also be calculated along the nozzle as shown in figure 1.2.

![Figure 1.2: Mach number, temperature, pressure and density distribution along the axis in an isentropic nozzle flow](image)

An other important parameter is the mass flux of the flow, defined as the product of density and flow speed,

\[
\Phi = \rho v.
\]

Making use of the fact that for a perfect gas

\[
\rho = \frac{p}{RT}
\]

and using the equations for the speed of sound and the stagnation properties, the mass flux can be expressed in terms of stagnation properties and Mach number:

\[
\Phi = \frac{p_0}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} M \left( \frac{M}{\gamma - 1} \right)^{1/2} \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/2}. \tag{1.23}
\]

For any fixed values of stagnation temperature and stagnation pressure, the mass flux attains a maximum at Mach number one as shown in figure 1.3. This illustrates again that transonic flow can only occur in the throat of the nozzle, since mass flux attains its maximum value where the cross-section is at its smallest.

The properties of a fluid when the gas is flowing at sonic speed are called critical properties and are denoted by an asterisk (*). The critical mass flux is the maximum mass flux for given stagnation conditions:
Having defined the most relevant properties and expressed them as functions of the Mach number and the gas characteristic parameters, it is now possible to examine the different operating modes for a given nozzle [25]. Consider the nozzle of figure 1.4A, the nozzle flow is determined by the conditions at the entrance and exit, namely the stagnation pressure ($P_0$) and the external pressure ($P_e$). Indicating with $P$ the local pressure in the nozzle, several types of flows can be obtained by varying the external pressure $P_e$ as illustrated in figure 1.4B. When the exit pressure is higher than in point A, the flow will be completely subsonic. The flow in the throat will be critical and maximum mass flux will be attained when the exit pressure reaches point A. An other possible solution is the supersonic solution for point B. Now the gas is accelerated after passing the throat at Mach number one. There is a limited range for $P_e$ values that can be realised isentropically; $P_e \geq P_A$ and $P_e \leq P_B$. For the intermediate values, no possible continuous mathematical solution exists: in this case discontinuous solutions come into existence in the form of shock waves. Across the shockwave, all the flow properties undergo an abrupt change resulting in a loss of kinetic energy and increase of entropy. By lowering the exit pressure, this shock wave will tend to go downstream until the flow in the diverging part is completely supersonic. Further lowering of the exit pressure $P_e$ does not affect the flow in the nozzle. The continuity condition is written as \( \rho v A = \rho^* v^* A^* \), where the asterisk refers to the sonic conditions in the throat. Combining equations 1.21 and 1.23 gives

$$ \frac{A}{A^*} = \frac{1}{M^2} \left( \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2 \right)^{1/2} $$

It appears that the exit Mach number is fixed by the nozzle geometry ($A/A^*$). The nozzle used in this investigation was designed for an exit Mach number of 2.5.
1.2.2 Flow with heat addition

In this sub-section, supersonic flow with heat addition is addressed. The reason for such a choice is that the macroscopic effect of condensation is the release of latent heat to the flow. Therefore the objective of this sub-section is to present the effects of heat addition on the flow characteristics.

\[
\begin{align*}
\frac{dq}{\rho} + \frac{dA}{A} + \frac{dv}{v} &= 0 \\
&= (\text{continuity}), \\
\frac{dp}{\rho} &= dh + vdv = dh_0 \\
&= (\text{energy}),
\end{align*}
\]
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\[ dp + \rho \nu dv = 0 \] (momentum).

In this case, the stagnation enthalpy is not constant: its variation equals the amount of heat added to the flow. It is also worth noticing that the flow can no longer be regarded as isentropic due to the release of heat.

Combining these equations with the definition of Mach number, the equation for the speed of sound and the equation of state of a perfect gas, yields [33]

\[
\frac{dM}{M} = \frac{1 + \frac{\gamma - 1}{2} M^2}{M^2 - 1} \left\{ \frac{dA}{A} - \frac{1 + \gamma M^2}{2} \frac{dh}{h} \right\}.
\]

This equation indicates the change in Mach number as a sum of two terms, one caused by the change in cross-sectional area and the second caused by heat addition. When there is no heat addition, there is no change in stagnation enthalpy and equation 1.29 becomes the equation for isentropic nozzle flow (equation 1.20).

1.2.3 Flow through a constant area duct

In order to get a better understanding of the effects of heat addition to the flow, the case of a constant area duct is discussed in this sub-section. When a constant area duct is considered \((dA = 0)\), equation 1.29 simplifies to

\[
\frac{dM}{M} = -\frac{1}{2} \frac{1 + \frac{\gamma - 1}{2} M^2}{M^2 - 1} \frac{(1 + \gamma M^2)}{h} \frac{dh}{h}.
\]

Because of the appearance of \(M^2 - 1\) in the denominator of this equation, subsonic flow accelerates when heat is added; supersonic flow, instead, decelerates. Consequently, when heat is added to a flow through a constant area duct, the Mach number always tends to unity and no local transition is possible from subsonic to supersonic flow or vice versa. If Mach number one is attained, no more heat can be added without essentially changing the whole flow, including the region upstream of the heat addition. The flow pattern changes completely until more heat addition is possible. If the flow is subsonic, a new flow with a different mass flux will come into existence. If the flow is supersonic, a shock will appear somewhere before the entrance of the constant area duct, making the flow in the duct entirely subsonic.

When the Mach number becomes one and no more heat can be added, the flow is called critical or thermally choked. From equation 1.30, the maximum amount of heat which can be added before thermal choking occurs is calculated. Integration of equation 1.30 yields

\[
\frac{h_{02}}{h_{01}} = \frac{2 M_2^2 (1 + \frac{\gamma - 1}{2} M_2^2)}{(1 + \gamma M_2^2)^2} \frac{(1 + \gamma M_2^2)^2}{2 M_1^2 (1 + \frac{\gamma - 1}{2} M_1^2)} = \frac{\Theta(M_2)}{\Theta(M_1)}.
\]

The heat addition can be expressed in terms of Mach numbers as

\[
\Delta q = h_{02} - h_{01} = h_{01} \left( \frac{\Theta(M_2)}{\Theta(M_1)} - 1 \right).
\]

Substitution of \(M_2 = 1\) gives the maximum or critical heat addition before thermal choking occurs:

\[
\Delta q^* = h_{01} \frac{(M_2^2 - 1)^2}{2(\gamma + 1)(1 + \frac{\gamma - 1}{2} M_2^2) M_2^2},
\]
where subscripts are omitted and $h_0$ and $M$ are properties of the flow before heat addition. When the heat released to the flow exceeds the amount $\Delta q^*$, the flow is called supercritical. In this case, the conditions for a smooth flow can no longer be maintained, the inclusion of a stationary normal shock wave becomes necessary. If the amount of heat released is further increased, the normal shock wave becomes unstable, and an unsteady periodic pattern sets in proximity of the throat.

1.3 Supersonic flow with condensation

Usually when phase changes take place, states of equilibrium between the phases can be observed. During some processes, however, phase changes do not take place at equilibrium conditions, but at rather great departures from these conditions. A typical example is the condensation of the vapour component in a mixture of a condensable vapour and an inert carrier gas when cooled at a very high rate. This sub-section illustrates briefly the major non-equilibrium effects caused by the condensation of an expanding vapour in steady and unsteady nozzle flow. As a first step in this direction, a brief description of the kinematics of phase transitions and nucleation process is necessary for a better understanding of the phenomena.

1.3.1 Saturation

The saturation $S$ indicates the departure from equilibrium conditions. It is defined as the ratio of vapour pressure $p_v$ and saturation pressure $p_s$ at a fixed temperature,

$$S = \frac{p_v}{p_s(T)}.$$  \hfill 1.34

A saturation of unity refers to an equilibrium situation and when plotted in a pressure-temperature state diagram as depicted in figure 1.6, the locus of points $S=1$ is called the liquid-vapour coexistence line. Depending on the initial conditions, the liquid-vapour coexistence line can be crossed by a line representing an isentropic expansion (dashed line). During the expansion, the conditions change very rapidly to a state in the liquid region without condensation having taken place. Here, the saturation is larger than unity and the mixture is called supersaturated or supercooled. At some point C, where $S$ can be as high as 20, condensation of the vapour occurs. Then, relaxation to a point on the coexistence line follows.
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The vapour pressure can easily be expressed in terms of vapour mass fraction $g_v$ and pressure $p$

$$p_v = g_v \frac{R_v}{R}$$

with $R$ and $R_v$ the specific gas constants of the carrier gas and vapour. The equilibrium vapour pressure of vapour coexisting with its liquid phase is described by the Clausius-Clapeyron equation:

$$\frac{dp_s}{dT} = \frac{L(T)p_s}{R_vT^2}$$

where $L$ is the latent heat of evaporation, which, in the case of constant specific heats for the vapour and liquid phase is

$$L = (c_{pv} - c_l)T + L_0,$$

with $c_{pv}$ and $c_l$ the specific heats of respectively the vapour and the liquid, and $L_0$ the latent heat of evaporation at zero temperature. Integrating the Clausius-Clapeyron equation after substituting equation 1.37 gives the Kirchhoff equation,

$$p_s(T) = p_s(T_{ref}) \left( \frac{T}{T_{ref}} \right)^{c_{pv} - c_l} \frac{R_v}{R} \exp \left( \frac{L_0}{R_v} \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \right),$$

where $T_{ref}$ is a reference temperature at which the saturation pressure is known.

Because in an isentropic expansion the temperature can become very low, a different expression for the saturation pressure is used [9]:

$$p_s = \exp \left( A_0 + A_1T + A_2T^2 + B_0 \ln T + C_0 \frac{T}{T} \right),$$

with coefficients determined experimentally [23] and given in Appendix B.

The condensation process can be subdivided into two processes: nucleation and droplet growth. Nucleation is the process of the formation of stable droplets. Droplet growth theory describes the growth of droplets under certain conditions.
1.3.2 Homogeneous nucleation

Many condensation processes follow equilibrium states closely, provided that a liquid (or solid) phase is initially present or other surfaces for condensation are available. In extremely rapid expansions, where the condensate accumulation on foreign nuclei is negligibly small, condensation may be much delayed with respect to equilibrium conditions. In these situations, nucleation is called homogeneous and condensation nuclei are formed by fluctuation in the supersaturated vapour itself. From a heuristic point of view, the process can be explained as follows: random collisions may lead to the agglomeration of a small number of vapour molecules, which is called a cluster. When such a cluster attains a critical size, the impingement of one more molecule on the cluster leads to an unlimited growth and, hence, to the formation of a droplet. The theory of homogeneous nucleation aims to predict the rate of formation of these clusters of critical size.

It is now clear that the formation of droplets in a gas-vapour mixture depends on several conditions, like saturation and temperature, as it can be found from considering the Gibbs free energy, \( \Delta G \), of formation of a cluster of vapour molecules (with radius \( r \)):

\[
\Delta G = \sigma 4\pi r^2 + \frac{4\pi r^3}{3} \rho_l \Delta \mu,
\]

with \( \sigma \) the surface tension, \( \rho_l \) the density of the liquid condensate and \( \Delta \mu \) the difference in chemical potential between the liquid and vapour phase, which can be approximated by

\[
\Delta \mu = \mu_j - \mu_v = -RT \ln S.
\]

The Gibbs free energy needed to form a droplet is then given by

\[
\Delta G = \sigma 4\pi r^2 - \frac{4\pi r^3}{3} \rho_l R T \ln S,
\]

which can be interpreted as the sum of two terms representing respectively a surface energy and a volume energy. In figure 1.7 the Gibbs energy is plotted as a function of the radius of the cluster. It can be seen that for a saturation smaller than unity, the Gibbs energy has no maximum value. The bigger the cluster, the more energy is needed for its formation. However, for a supersaturated vapour (\( S > 1 \)) there is a maximum value of the Gibbs formation energy. This maximum occurs at a critical radius of

\[
r^* = \frac{2\sigma}{\rho_l R T \ln S}.
\]

The maximum value of the Gibbs energy then becomes

\[
\Delta G^* = \frac{4\pi r^*}{3}.
\]
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Figure 1.7: Gibbs formation energy of a stable droplet

In a supersaturated vapour, a cluster of critical size is in equilibrium with the vapour. However, this equilibrium is unstable. Clusters with a smaller radius will disappear, larger clusters will grow and form droplets. The formation of a droplet is considered to occur by the impingement of a vapour molecule on a cluster of critical size. The nucleation rate can be written as:

\[ J = Z c^* n^* = Z c^* n, \exp\left(\frac{-\Delta G^*}{RT}\right), \]

where \( c^* \) is the impingement rate of vapour molecules on a cluster of critical size and \( n^* \) is the equilibrium number density of clusters of critical size, which is the product of the number density of vapour molecules \( n \), and an exponential distribution. \( Z \) is the so-called Zel'dovich factor that accounts for the nonequilibrium character of the whole process. The final expression for the classical nucleation rate can be written as [1]:

\[ J = \sqrt[3]{\frac{2\sigma m}{\pi \rho_s^2}} \exp\left(\frac{-16}{3} \frac{\sigma^2}{m \rho_s^2 R^3 T^3 \ln 2 S}\right), \]

with \( m \) the mass of the vapour molecule and \( \rho_s \) the density of the vapour.

1.3.3 Droplet growth

When a stable droplet is formed, it will continue to grow. During an expansion in a Laval nozzle, an extremely large number of droplets is formed simultaneously. This means that the droplets remain small, typically a few nanometers, since the vapour soon becomes exhausted. The molecular mean free path

\[ l = \frac{2\eta\sqrt{RT}}{p}, \]

(where \( \eta \) is the viscosity and \( R \) the specific gas constant of the carrier gas), is much larger than the size of the droplets. Consequently, the Knudsen number

\[ Kn = \frac{l}{2r}, \]

with \( r \) the radius of a droplet, is much larger than unity. The droplets grow in a free molecular flow regime. In this regime, interactions between droplets can be neglected and the carrier gas
is not disturbed by the presence of the droplets. Besides, because the droplets remain small, we can assume that the droplets move with the carrier gas at the same speed, therefore no slip between carrier gas and droplets occurs.

The mass growth rate of a spherical droplet is proportional to the surface of the droplet and the difference between the pressure at the surface of the droplet $p_d$ and the vapour pressure of the surroundings $p_v$ [30],

$$\frac{dm_d}{dt} = 4\pi r^2 \cdot \frac{\alpha \cdot (p_v - p_d)}{\sqrt{2\pi R_T}}, \quad 1.49$$

where $\alpha$ is a condensation coefficient that equals the fraction of impinging vapour molecules that actually stick to the liquid surface. Because the mass can be expressed as the product of density and volume, a relation between mass growth rate and the increase in size can be found

$$\frac{dm_d}{dt} = \frac{dV}{dt} = \frac{4\pi}{3} \cdot \frac{d(r^3)}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}.$$ \quad 1.50

Substituting this result in equation 1.49, gives the Hertz-Knudsen law for the growth rate of a droplet

$$\frac{dr}{dt} = \frac{\alpha \cdot (p_v - p_d)}{\rho_i \sqrt{2\pi R_T}}.$$ \quad 1.51

Although at the surface of the droplet liquid and vapour are coexisting, the pressure at the surface of the droplet $p_d$ does not equal the saturation pressure $p_s$. This is due to the Kelvin effect. The surface tension of a small droplet, caused by the curvature, causes an increase in surface pressure

$$p_d(T_d, r) = p_s(T_d) \cdot \exp Ke(T_d, r), \quad 1.52$$

with $r$ the radius of the droplet, $T_d$ the droplet temperature and $Ke$ the Kelvin number given by

$$Ke(T_d, r) = \frac{2\sigma}{\rho_i R T_d r}, \quad 1.53$$

where $\sigma$ is the surface tension.

What remains, is the determination of the droplet temperature $T_d$. An expression for the droplet temperature is given by [21]

$$\frac{T_d}{T} - 1 = (\ln S - Ke) \cdot \left\{ \frac{T}{S} \left[ \frac{1}{p_s(T)} - S \right] + \frac{L}{R_T} \right\}^{-1}. \quad 1.54$$

In this equation, $T$ and $p$ are the temperature and pressure of the surrounding gas-vapour mixture and $S$, $p_s$, $L$ and $Ke$ are the saturation, saturation pressure, latent heat of evaporation and Kelvin number evaluated at this temperature and pressure. The parameter $\theta$ has the dimension of temperature and can be written as:

$$\theta = \frac{D_m L}{\lambda} \cdot \frac{Nu_M}{Nu_H}. \quad 1.55$$

Here, $\lambda$ is the thermal conductivity, $D_m$ the modified diffusion coefficient (Appendix B) and $Nu_M$ and $Nu_H$ the Nusselt numbers of mass transfer and heat transfer given by:

$$Nu_M = \frac{2}{1 + 3.23 \cdot Kn} \quad 1.56$$
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and

\[ Nu_H = \frac{2}{1 + 4.08 \cdot Kn} \]  

1.3.4 Condensation in supersonic flow

Around 1935, when supersonic wind tunnels were first used, a disturbance was observed in the test section of wind tunnels. This weak shock, first visualised by Prandtl, was explained to be an effect of condensation of water vapour present in the atmospheric air. During a fast isentropic expansion in a nozzle, the gas accelerates to supersonic speed and the temperature of the gas drops with a very high cooling rate (1 K/µs). A supercooled state is attained, in which no condensation has taken place yet. This metastable state does not last long. At some point, a sudden peak in the nucleation rate occurs and a large number of small droplets, typically of the size of a few nanometers, is formed instantaneously. Because of the high concentration of small droplets, foreign particles play no role in the condensation process.

Because of its sudden occurrence and its effect on the flow, the process of condensation is also referred to as a condensation shock. These disturbances in the flow properties are caused by the release of latent heat to the flow. When the flow remains entirely supersonic it is called subcritical and oblique shocks appear (X shocks). When a larger amount of vapour is present, however, more heat is released to the flow and a normal shock may appear. Across the shock, there is a discontinuous jump in flow properties and downstream of it, the flow becomes subsonic. This flow is called supercritical. At even higher vapour mass fractions, oscillating shocks may occur.

1.3.5 Unsteady flow

The phenomenon of oscillating shock waves in nozzle flows of vapour-carrier gas mixtures was discovered by Schmidt [17]. After that, many experiments and numerical simulations have been performed. As shown before, temperature will drop while the flow is accelerated in the nozzle. As a result of this, droplets may be formed when there is a condensable component in the gas. The formation of droplets, nucleation, depends on flow properties and, consequently, on Mach number. The situation could occur that the only possible solution for the flow is one with a shock wave, that would have a position somewhere in, or even upstream of the nucleation zone. The temperature increase across the shock reduces the saturation. Therefore it tends to diminish the processes of nucleation and droplet growth. This would mean that the shock could affect the nucleation, or even terminate the nucleation process. The shock then takes away the essential source of its existence, since it was produced by supercritical heat addition of the condensation process. The shock will move upstream and disappear and nucleation will start again. Due to supercritical heat addition a new shock wave that moves towards the throat will be formed and nucleation is ended again. From this, it may be clear that non-stationary solutions or oscillating shocks can occur. These oscillating shocks can occur in different oscillating modes. The mode is determined by the relative humidity of the test gas and the geometry of the nozzle. For an extensive description of these phenomena, the reader is referred to Schnerr [22].

1.3.6 Numerical model

The experimental results are compared with the numerical simulations performed by Prast [15]. The inviscid flow equations are used to solve the problem of condensation in transonic
flows. The Reynolds numbers of these flows are very high under atmospheric conditions so that viscous friction is restricted to thin boundary layers. Under these conditions, the Navier-Stokes equations reduce to the Euler equations. Following the vapour mass along a streamline the rate of change of the liquid mass fraction $g$ is given by:

\[
\frac{dg}{dt} = \frac{4}{3} \pi \frac{J}{\rho} r^3 + \int_{-\infty}^{t} 4\pi \rho \frac{J(\tau)}{\rho(\tau)} \frac{\partial r(t,\tau)}{\partial \tau} r^2(t,\tau) d\tau.
\]

(1.58)

The first term of the right hand side represents the vapour mass change due to the formation of droplets with critical radius $r^*$ at time $t$ with liquid density $\rho_l$. The number of critical droplets formed per unit volume and time is the nucleation rate $J$. The second term represents the growth of droplets of radius $r$ which were formed at an arbitrary time $\tau$ along the streamline. Since statistical macroscopic properties of the droplet cloud are of interest, such as mean droplet radius and droplet number concentration, the statistical moments of the droplet size distribution $f(r)$ per unit mass of mixture are considered:

\[
Q_n = \int r^n f(r) dr.
\]

(1.59)

This is equivalent to the expression:

\[
Q_n = \int_{-\infty}^{t} r^n(t,\tau) \frac{J(\tau)}{\rho(\tau)} d\tau,
\]

(1.60)

with $n = 0, 1, 2, 3$, in which the integration is carried out over the time at which the droplets are born. The zeroth order moment $\rho Q_0$ equals the number density of droplets, while the third order moment $Q_3$ is related to the liquid mass fraction $g$ according to

\[
\frac{4}{3} \pi \rho_l Q_3 = g.
\]

(1.61)

Introducing the surface average droplet radius $\bar{r}$ according to Hill [6]

\[
\bar{r} = \sqrt[3]{\frac{Q_2}{Q_0}}
\]

(1.62)

and using the assumption that $\frac{\partial r}{\partial t}$ is independent of $r$ (free-molecular regime), it follows that eq. 1.58 is now equivalent to a closed set of four moment equations (Schnerr et al.[22]):

\[
\rho \frac{dg}{dt} = \frac{4}{3} \pi \rho_l \left( J r^{-3} + 3 \rho Q_2 \frac{d\bar{r}}{dt} \right),
\]

(1.63)

\[
\rho \frac{dQ_0}{dt} = Jr^{-2} + 2 \rho Q_1 \frac{d\bar{r}}{dt},
\]

(1.64)

\[
\rho \frac{dQ_1}{dt} = Jr^{-1} + \rho Q_0 \frac{d\bar{r}}{dt},
\]

(1.65)

\[
\rho \frac{dQ_2}{dt} = J.
\]

(1.66)

These differential equations can be solved simultaneously with the flow equations:
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\[
\begin{aligned}
\frac{\partial}{\partial t} \left( \begin{array}{c}
\rho \\
\rho u \\
\rho v \\
\rho E
\end{array} \right) + \frac{\partial}{\partial x} \left( \begin{array}{c}
\rho u^2 + p \\
\rho u v \\
\rho v^2
\end{array} \right) + \frac{\partial}{\partial y} \left( \begin{array}{c}
\rho v \\
\rho u v + p \\
(\rho E + p) v
\end{array} \right) = 0,
\end{aligned}
\]  

where \( \rho, p, u, v \) and \( E \) represent density, pressure, \( x \) and \( y \) velocity components and the total energy, respectively. The released latent heat is accounted for in the internal energy \( e \), represented by

\[
e = E - \frac{u^2 + v^2}{2} = c_v T + g(R_v T - L),
\]

with \( R_v \) the specific gas constant of the vapour. The specific heat at constant volume, \( c_{vo} \), of the mixture is given by

\[
c_{vo} = (1 - g_{max}) c_{vg} + g_{max} c_{vv},
\]

where \( c_{vg} \) is the specific heat of the carrier gas and \( c_{vv} \) of the vapour, respectively. \( g_{max} \) is the vapour mass fraction in the reservoir and consequently the maximum liquid mass fraction that can occur at the end of the nozzle. Assuming a linear dependence of the latent heat \( L \) on temperature \( T \)

\[
L(T) = L_0 + TL_1,
\]

we can write the temperature \( T \) as

\[
T = \frac{e + g L_0}{c_v + g (R_v - L_1)}.
\]

The numerical method developed by Prast [14] is based on the TVD scheme of van Leer, as recommended by Mündinger [9]. This scheme has a good shock capturing capability, which is important when studying the influence of shocks on the condensation process.
2. Holographic interferometry

To acquire quantitative data of the density field in a nozzle flow, holographic interferometry is applied. In this section the principles of holography are explained. The special case of double exposure holography and the holographic set-up, used in this investigation, are evaluated. For processing the interferograms the DDS modelling method is used. This method is here described and evaluated.

2.1 Holography

2.1.1 General introduction

Holography is a technique for recording and reconstructing light waves [4], [26]. The wave, which is to be recorded, is called the object wave. In order to reconstruct, that is to create a facsimile of the object wave, it is sufficient to reproduce its complex amplitude, \( U_0 \), at one plane of the space. The complex amplitude contains both phase and real amplitude of the wave. Once this has been produced, the light propagating away from this plane is identical to the original object wave. To do so, both distributions of real amplitude and phase in the plane must be recorded. However, photographic film or any other detector responds only to irradiance. The object wave irradiance is \( I_0 = U_0 U_0^* \), which is a real quantity, implies that any information relating to the phase distribution is irremediably lost.

To overcome this problem, interferometry can be used to convert a phase distribution into an irradiance pattern, which can be recorded on photographic film. This is the basis of Gabor’s invention of holography in 1949. To attain this, a coherent reference wave is added to the object wave. In this way an interference pattern is formed, which can be recorded on a photographic film. When the film is developed and illuminated properly, it diffracts light in a manner such that the complex amplitude \( U_0 \) is reproduced at the plane of the film, just as the Fresnel zone plate does for one point source. The hologram contains, by way of a complicated fringe pattern, the information corresponding to both the phase and amplitude of the wave scattered by the object. This is illustrated in figure 2.1A. Two light beams strike the holographic plate. One serves as reference beam, the other is the object beam. The complex amplitudes of these waves at the film plane, \( z = 0 \), are as follows:

\[
U_o(x,y) = a_o(x,y)\exp(-i\phi_o(x,y)),
\]

\[
U_R(x,y) = a_R \exp(i2\pi f_y y),
\]

where \( U_o \) is the object wave, \( U_R \) is the reference wave, \( f_y \) is the spatial frequency of the reference wave. The irradiance at the film plane is

\[
I(x,y) = |a_R \exp(i2\pi f_y y) + U_o|^2
\]

\[
= (a_R \exp(i2\pi f_y y) + U_o)(a_R \exp(-i2\pi f_y y) + U_o^*)
\]

\[
= a_R^* + |U_o|^2 + a_R U_o \exp(-i2\pi f_y y) + a_R U_o^* \exp(i2\pi f_y y).
\]
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Assuming that the film is exposed to the irradiance pattern described by equation 2.3, and is developed so that its amplitude transmittance $t(x,y)$ is proportional to $I(x,y)$, it follows:

$$t(x,y) = t_b + \beta \left( |U_0|^2 + a_R U_o \exp(-i2\pi f_y y) + a_R^* U_o^* \exp(i2\pi f_y y) \right), \quad 2.4$$

where the term $t_b$ is proportional to the first term in equation 2.3. If the reconstruction beam is of high quality (i.e. uniform intensity distribution), $t_b$ will be nearly uniform over the film and therefore represent a bias level in the exposure. The constant of proportionality $\beta$ is a property of the film. This can also be written as

$$t(x,y) = t_b + \beta a_o^2(x,y) + 2\beta a_R a_o \cos[2\pi f_y y - \phi(x,y)], \quad 2.5$$

when equation 2.1 is substituted. This equation shows that the hologram consists of a set of “carrier” interference fringes of spatial frequency $f_y$ which are modulated in amplitude by $a_o(x,y)$ and in phase by $\phi(x,y)$.

The developed film with transmittance described by equation 2.5 in fact is a fine diffraction grating and is referred to as a Gabor Hologram. The object wave is reconstructed by illuminating the hologram with a uniform plane wave of laser light travelling in the same direction as the original reference wave, as shown in figure 2.1b. This is referred to as the reconstruction wave; at the plane $z=0$ its complex amplitude is

$$U_e(x,y) = a_e \exp(i2\pi f_y y). \quad 2.6$$

When the hologram is illuminated by this wave, the complex amplitude just to the right of the hologram becomes:

$$U_1(x,y) = tU_e(x,y) = (t_b + \beta |U_o|^2) a_e \exp(i2\pi f_y y) + \beta a_R a_o U_o \beta a_e a_R U_o^* \exp(i4\pi f_y y). \quad 2.7$$

The term $\beta a_e a_R U_o$ is the desired facsimile of $U_o$; therefore the goal of recording and reconstructing the object wave has been attained. This way of holographic recording is called off-axis holography and was developed by Leith and Upatnieks [8].

### 2.1.2 Holographic interferometry

Using holography as described in the former section, three-dimensional images of diffusely reflecting objects can be produced, which appear to be overlaid by interference fringes, that are indicative of deformation, displacement, or rotation of the object. Similarly,
in the case of transparent objects, fringe patterns can be formed which are indicative of changes in refractive index or object thickness. This type of interferometry is possible because a light wave scattered by an object can be holographically recorded and reconstructed with such a precision that it can be compared interferometrically with light scattered by the same object at another time.

When this is done by comparing the holographic recording with the light scattered by the real object, this is called the real-time method, while the fringes can be studied as they evolve in real time. An other way to compare two scenes is exposing the hologram two times in succession. This way the undisturbed and disturbed situation will be recorded in one hologram. This technique is called double exposure holographic interferometry and is applied in this investigation. First, the holographic film is exposed to the scene and reference beam without flow in the nozzle as shown in figure 2.2A. Secondly, the film is exposed with flow, the disturbed scene, as shown in figure 2.2B. These two exposures result in an interference pattern indicative of changes in optical path length due to density variations in the flow.

Figure 2.2: Double exposure holographic interferometry. Undisturbed situation (A), Disturbed situation (B).

2.1.3 Interferogram evaluation

During the experiment, variations in density cause differences in optical path length and consequently changes in the phase are introduced. Density and refractive index are related as follows:

\[ n = 1 + K \rho, \]

with \( K \) the Gladstone-Dale constant, which depends only on the medium and the wavelength of the light source. At \( T = 273 \) K and \( p = 1.013 \times 10^5 \) Pa, the refractive index \( n \) of nitrogen for 694 nm light is \( n = 1.000295 \). Now, making use of the perfect gas law, the Gladstone-Dale constant is found to be \( K = 2.36 \times 10^{-4} \).
The speed of light in a medium is related to the refractive index by the following relation:

\[ c = \frac{c_0}{n}, \quad 2.9 \]

where \( c \) and \( c_0 \) are the speed of light in a medium and in vacuum. Because of this, any change in refractive index will result in a difference in time needed to travel through the medium

\[ \Delta t = \Delta \left( \frac{L}{c} \right) = \frac{L}{c_0} \Delta n, \quad 2.10 \]

with \( L \) the distance travelled through the medium, which, in this case, is the width of the tube (10 cm). The resulting phase difference can be expressed as:

\[ \phi = 2\pi \frac{c_0}{\lambda_0} \Delta t, \quad 2.11 \]

where \( \lambda_0 \) is the wavelength of the light source in vacuum. Combining these equations, a relation between the phase difference caused by the flow \( \phi_f \) and a difference in density can be found:

\[ \phi_f = 2\pi \frac{KL}{\lambda_0} \Delta \rho. \quad 2.12 \]

From the interferograms, the actual phase difference \( \phi_a \) between two points can easily be found by counting the fringes between the two points. If the number of fringes is \( m \), then the actual phase difference is given by

\[ \phi_a = \pm m \cdot 2\pi. \quad 2.13 \]

In this case, the phase difference is just the consequence of a difference in density. This is called the case of an initial zero-order setting. There is also the possibility of adding an artificial phase difference upon the existing phase difference caused by density variation. This additive phase difference, or initial phase difference \( \phi_i \), can be induced by tilting the reference beam between the two recordings. The actual phase difference now is produced by two factors: a difference in density and a superimposed initial phase difference \( \phi_i \). To calculate the phase difference caused by the density variations, the initial pattern has to be considered.

\[ 20 \]
where $d$ is the distance between the centres of two neighbouring fringes. The initial phase difference between a reference point $(0,0)$ and a point $\vec{r} = (r_x, r_y)$ can then be expressed as

$$\phi_i(\vec{r}) = 2\pi (\vec{r} \cdot \vec{s}).$$

2.15

In the case of an initial zero order pattern, the distance between two lines is infinite and the length of the vector $\vec{s}$ becomes zero. Obviously, there is no initial phase difference. When an initial pattern with horizontal fringes is used, the initial phase difference relative to the reference point becomes:

$$\phi_i = 2\pi \frac{r_x}{d}.$$ 2.16

The phase difference due to variations in density is the difference between the actual phase difference taken from the interferogram of the experiment and the initial phase difference:

$$\phi_r = \phi_a - \phi_i = 2\pi \{\pm m - (\vec{r} \cdot \vec{s})\}.$$ 2.17

Combined with equation 3.12, this results in a relation for the difference in density between a point and a reference point:

$$\Delta \rho = \frac{\pm m (\vec{r} \cdot \vec{s})}{KL}. 2.18$$

In case of an initial zero-order pattern, this gives

$$\Delta \rho = \pm m \frac{\lambda_0}{KL}. 2.19$$

Consequently, the density difference between points on two neighbouring fringes of an interferogram made with an initial zero-order setting, is $\Delta \rho = 0.029 \text{ kg/m}^3$. For an initial pattern with horizontal fringes at a distance $d$ from each other, it is equal to:

$$\Delta \rho = \left(\pm m \pm \frac{r_x}{d}\right) \frac{\lambda_0}{KL}. 2.20$$

The sign in front of the $r/d$ term is uncertain, because the vector $\vec{s}$ can point in two opposite directions. The proper signs in front of the two terms can be deduced from physical considerations.

2.2 Optical set-up

2.2.1 Description of the holographic interferometer

In figure 2.2 the set-up is drawn schematically. Here the focus is on the optical part of the set-up, the Ludwieg tube is described in section 3.

For holography, a coherent monochromatic light source is needed. Laser light fulfills these requirements. The laser used is a ruby pulse laser (Lumonics HSL2). This laser has an optimum energy output level of 1 Joule in 30 ns. The laser beam comes out of the laser slightly diverging. This divergence is caused by a spatial filtering pinhole inside the laser. In order to make the beam parallel, a positive lens of focal distance of $f=1000\text{mm}$ is put right after the laser at a distance such that the pinhole inside the laser is in the focal point of the lens. Subsequently, the beam is split into by a polarising beamsplitter into a reference beam.
and an object beam. As shown in the figure, the object beam is going through the test-section, the reference beam is diverted under the tube to the other side. The intensity ratio between the two beams can be adjusted by turning the $\frac{1}{2}\lambda$ plate (lpl) in front of the beamsplitter. The other plates are $\frac{1}{4}\lambda$ plates and are used to make the polarisation of both beams circular, in order to match the polarisations of the two beams for optimal interference on the holographic plate. Both beams are expanded by a negative lens ($f = -100$ mm). The focal point of these lenses is positioned in the focal point of the parabolic mirrors ($f = 1500$ mm). This results in two parallel beams with a diameter of 15 cm. The first crosses perpendicularly the test section, before hitting the holographic film. The reference beam strikes the film at an angle different to the one of the object beam. The folding plane mirror, PETpm, can be tilted around a horizontal axes by a high voltage piezo electric transducer (PET). By activating this PET during the first recording, a linear phase distribution is superimposed on the phase difference due to the density field.

2.2.2 Critical assessment on the optical set-up and reconstruction

In the process of evaluating the holograms, several problems, which disturbed the interpretation and translation to quantitative density information, emerged:

- the presence of undesired reflections in the hologram
- the inhomogeneous intensity distribution of the laser light in the plane of recording
- the non-reproducibility of subsequent laser shots
- the intensity of the laser being too high
- vertical interference pattern of unknown origin in the plane of recording.
The undesired reflections in the hologram revealed themselves in the reconstruction as thick black lines along the centre line of the nozzle, reproducing exactly the shape of the nozzle contour. They affected the accuracy of the phase data (i.e. density data) and therefore, had to be removed. The disturbances were caused by unwanted reflection of laser light at the nozzle walls due to misalignment of the laser beam through the test-section. In order to eliminate these reflections, a camera (Nikon F801s) was implemented as a control system for checking the correct alignment of the laser beam. In this way, the correct alignment can be verified prior to the beginning of an experiment.

The inhomogeneous intensity distribution of the laser light resulted in a hologram and a reconstructed interferogram of poor quality. Because of the darker and brighter areas, the fringe visibility reduced and their interpretation became quite troubled. The cause of the non-uniform intensity distribution was the use of the polarising beam splitter which was not suited for the laser light. A new, special coated plate will be inserted.

In double exposure holographic interferometry, the intensities of both exposures have to be of comparable magnitude. The non-reproducibility of subsequent laser shots reduced the reliability of the system. It was caused by inaccurate operational use of the laser. It appeared that the amplifier setting of the laser should be at least 2.00kV, otherwise the laser output is not reliable anymore. By using the laser at 2.00kV however, an other problem occurred: the laser intensity increased too much. The intensity of the laser light in the recording plane had to be reduced considerably in order not to over-expose the photographic material (AGFA 8E75 HD). To reduce intensity, several optical components were mounted in the optical path. This way, the laser output was strained, but the quality of the beam was disturbed by the addition of the optical components. An other attempt to reduce the intensity was made by expanding the beam first, and then cutting out a part of the beam to be used for the recording. This resulted in unwanted reflections from edges of mirrors and lenses which cut off the expanding beam and disturbed its quality to a large extent. An appropriate solution for this problem was, eventually, found by leaving the unnecessary optics out and regulating the intensity with the delay on the laser control panel. This way, the quality of the beam remains, while the intensity can be reduced. The range of delay values, allowed for a correct operation of the laser, varies between 300-500 µs.

The vertical fringes were caused by a damage inside the laser. Technical assistance from the laser manufacturing company (Lumonics Ltd.) was requested to solve this problem. It appeared that a pinhole inside the laser was damaged by misalignment and wrong operation procedure of the laser. Replacement of this pinhole took away the cause of these fringes. However, all holograms in this investigation were made with the damaged pinhole in the laser. It is recommended to check regularly the correct alignment of the laser beam having the power of the pocket cells off.

2.3 Hologram interpretation

2.3.1 Reconstruction

In double exposure holographic interferometry, the reconstruction contains the interferogram between the disturbed and undisturbed scene, as demonstrated in sub-section 2.1.2. In order to reconstruct the object wave, which is in our case the interferogram, the hologram is illuminated with a reconstruction beam, as described in sub-section 1.1.1. For this aim the ruby pulse laser is used. The developed hologram is placed back in the recording position as in the experiment. The hologram is illuminated with the reference beam, which
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now serves as the reconstruction beam, while the object beam is obstructed, as shown in figure 2.5.

![Figure 2.5: Reconstruction of a double exposed hologram](image)

The reconstruction lens in combination with the camera lens focus the image on the CCD (Coupled Charge Device) array of the camera. In this first phase of the project an already available CCD camera (Spindler & Hoyer 260SW black & white 752×582) was used. The image can be acquired on a IBM PC by way of a framegrabber (DT55, 768×512). The framegrabber digitises the intensity in 256 discrete grey values and is triggered on the TTL high to low level transition (HL) produced by the trigger unit. This unit generates a trigger pulse for the laser and the framegrabber. The synchronisation pulses from the camera are used as input for the trigger unit. The image acquiring starts just before a complete exposed video frame is present. The reconstruction set-up is schematically drawn in figure 2.6. It appeared that the framegrabber does not store all the lines. Each second horizontal line is black, so half of the information is lost. It seemed to be a lack of the framegrabber when operating in triggered mode in combination with the camera.
An example of a reconstructed hologram is shown in figure 2.7. The interlace lines, which are clearly visible in this picture, cause serious trouble in the automatic interpretation of the interferogram.

2.3.2 Fringe scanning

Fringe scanning is a manual fringe counting method, in which the co-ordinates of each fringe are stored in a spreadsheet. Each fringe presents a phase shift of $2\pi$ relative to the neighbouring fringe. These co-ordinates in combination with equation 2.20 and the fringe number give the relative value of the density in discrete points in the field. To obtain a continuous density field, an interpolation between the scattered points is done. The density
field can be compared with the calculations. The procedure of scanning is a laborious and time consuming work, but the obtained density field from this procedure is quite reliable because the fringes are recognised by human eye easily, even if they are not continuous or disturbed by noise.

2.3.3 DDS modeling

Theeuwes [24] applied and described one of the methods of automatically extracting the phase from an interferogram. He used a linear autoregression; the Data Dependent System (DDS) method proposed by Pandit [12] to model the digitised scan lines. This methodology uses discrete data to derive a model that represents a real system. The minimal requirement imposed on the analysed data set is a dynamic dependency between neighbouring data values. As part of the assignment, this DDS method was critically investigated.

A digitised interferogram with horizontal initial phase pattern of period \( Y \) is considered. This phase pattern is imposed by tilting the PET-mirror between the recording of the reference scene and the disturbed scene. Suppose the mirror was tilted in the recording of the reference scene. The complex amplitude of the reconstructed scene beam can be written as

\[
U_s(x, y) = A_s(x, y)e^{-i\phi_s(x, y)s+2\pi \frac{x}{Y}}. \tag{1.21}
\]

The complex amplitude of the reconstructed disturbed scene, recorded with the PET-mirror not tilted, can be written as

\[
U_d(x, y) = A_d(x, y)e^{i\phi_d(x, y)}. \tag{1.22}
\]

The superposition of these complex amplitudes result in an intensity distribution which can be written as:

\[
I(x, y) = |U_s(x, y) + U_d(x, y)|^2 + \text{noise}(x, y)
= A_s^2 + A_d^2 + A_s A_d \left[ e^{i(\phi_s(x, y)+2\pi \frac{x}{Y})} + e^{-i(\phi_s(x, y)+2\pi \frac{x}{Y})} \right] + \text{noise}(x, y)
= \Gamma_s(x, y) + \Gamma_d(x, y) \left[ e^{i(\phi_s(x, y)+2\pi \frac{x}{Y})} + e^{-i(\phi_s(x, y)+2\pi \frac{x}{Y})} \right] + \text{noise}(x, y)
= \Gamma_s(x, y) + \Gamma_d(x, y) + \Gamma^*(x, y) + \text{noise}(x, y).
\]

Where \( \Gamma \) is called the self-coherence function, the function which represents the intensity distribution obtained in a reconstruction, given by

\[
\Gamma(x, y) = A_s A_d e^{-i(\phi_s(x, y)+2\pi \frac{x}{Y})}. \tag{2.24}
\]

\( \Delta\phi(x, y) = \phi_s-\phi_d \) is the phase difference between the disturbed scene beam and the reference scene beam caused by the density distribution:

\[
\Delta\phi(x, y) = \frac{2\pi KL}{\lambda}(\rho_R - \rho(x, y)). \tag{2.25}
\]

The objective is to extract the phase distribution from the recorded intensity pattern. For this, the self-coherence function has to be identified. The phase mod \( 2\pi \) can be obtained by

\[
\Delta\phi(x, y) + 2\pi \frac{Y}{Y} \pm 2k\pi = \text{arg}(\Gamma(x, y)). \tag{2.26}
\]
Next, a phase unwrapping algorithm is required to add or subtract \(2\pi\) to obtain a continuous phase distribution. Finally, the linear phase distribution can be subtracted to obtain the desired phase difference.

In the description of the Data Dependent System (DDS) modeling method, the main formulas are given here, without going into details. For an extensive description of this method, the reader is referred to Pandit [11], [12]. The DDS modeling method uses autoregression of order \(p\) \(AR(p)\) to express the digitised interferogram along vertical scan lines. The current pixel intensity value \(I(y)\) at location \(y\) is expressed as function of its predecessors \((I(y-1), I(y-2))\):

\[
I(y) = \phi_1 I(y-1) + \phi_2 I(y-2) + \ldots + \phi_p I(y-p) + a(y),
\]

where \(\phi_i\) are the autoregression coefficients and residuals \(a(y)\) which should be normally distributed with zero mean \(\langle a(y) \rangle = 0\) and standard deviation \(\langle a^2(y) \rangle = \sigma_a^2\). Primary aims of the modeling are to get a correct estimate of the parameters \(\phi_i\) and to minimise the residuals \(a(y)\). The coefficients and residuals can be estimated by minimisation of the residual sum of squares. The model order \(p\) should be chosen in such a way that the residuals are mutually independent and the residual sum of squares does not decrease significantly for increasing model order.

An alternate representation of the \(AR(p)\) difference equation (2.27), after subtraction of the mean signal, is given as the convolution of an impulse response function or Green’s function \(G_v\) with the stochastic residuals:

\[
I(y) = \sum_{v=0}^{p} G_v a(y-v).
\]

The Green’s function has the form

\[
G_v = w_1 \mu_1^v + w_2 \mu_2^v + \ldots + w_p \mu_p^v,
\]

where \(\mu_m\) are the roots of the autoregressive polynomial, given by

\[
\mu^p - \phi_1 \mu^{p-1} - \phi_2 \mu^{p-2} - \ldots - \phi_p = 0
\]

and for distinct \(\mu_m\) the significance or weighting factor of each term \(\mu_m^v\) is determined by

\[
w_m = \frac{\mu_m^{p-1}}{\prod_{k=1, m\neq k}^{p} (\mu_m - \mu_k)}.
\]

The model decomposes the observed intensity data in modes of various wavelengths. Physically meaningful interferogram features are captured in the significant modes, whereas the modes corresponding to background and noise are insignificant. By selection of the significant modes, noise and background are filtered out. Periodic phenomena are revealed by complex conjugate roots, whereas the real roots represent exponential modes. For a complex pair of roots, the typical form is

\[
w_k (\mu_k)^v + w_k^* (\mu_k^*)^v = r_k^v A_k \cos(\omega_k v + \beta_k),
\]

where \(r_k = |\mu_k|; \omega_k = \text{arg}(\mu_k); A_k = 2 |w_k|; \beta_k = \text{arg}(w_k)\). The phase difference caused by the density distribution, as given in equation 2.25, is expected to contribute strongly to the self-coherence function. Since it is a complex function, the strongest complex mode \(m\) of the modal decomposition represents the major part of the distribution.
\[ \Gamma_m(y) = \sum_{\nu=0}^{y} (\mu_\nu)^{y} w_\nu a(y - \nu). \]  

2.33

The self-coherence function can be computed by the use of its recursive form given by 2.27 for \( p=1 \):

\[ \Gamma_m(y) = \Gamma_m(y-1) \mu_m + w_m a(y). \]  

2.34

2.3.4 Unwrapping

In the DDS modelling system, the self-coherence function is found. The phase of interest is wrapped in the argument of the self-coherence function:

\[ \Phi_d = 2\pi \frac{y}{Y} + \Delta \varphi(y) = \tan^{-1} \left( \frac{\text{Im}(\Gamma_m(y))}{\text{Re}(\Gamma_m(y))} \right) \pm 2\pi. \]  

2.35

This function is wrapped between \( -\pi \) and \( \pi \) and has to be corrected to a continuous function. This is done automatically by the unwrapping module in AVS, written by Theeuwes. When jumps are detected above a certain threshold \( \tau \), \( 2\pi \) is subtracted or added (\( \phi_0 \)), according to the direction of change. After the subtraction of the linear phase of the carrier fringes, the desired phase distribution is obtained:

\[ \Delta \varphi = \Phi_d(x - x_0, y - y_0) + \phi_0(x - x_0, y - y_0) - \frac{2\pi}{Y} (y - y_0). \]  

2.36

A schematic overview of the used method is shown in the block diagram of figure 2.8. This model was implemented in modules which were integrated in the Application Visualisation System (AVS), a software package that runs under UNIX operating system.

**Figure 2.8:** Block diagram of the used method
Since the unwrapping algorithm is rather naive and not all steps in phase are detected correctly, implementing more sophisticated way of detecting phase steps have been investigated. The adjustment, which was tried on the existing algorithm, is described below. When a phase step is detected, the next phase step is not expected in the direct vicinity of the first. Therefore, a number of pixels can be skipped after a step detection, in order to reduce the chance of inaccurate step detections. This adjustment was implemented, but, however, did not work as expected. In the first place because the mean period in some areas is rather low (3-7 pixels), which means that skipping three pixels each time a step is detected would already cause problems. An other reason is the high rate of noise in the reconstructions (i.e. spikes) which cause illegal phase steps when skipping neighbours. In the original case, spikes do not cause extra problems in unwrapping, because subsequent addition and subtraction of $2\pi$ nullifies the effect. Improvement in resolution of the reconstruction is the first aim to improve the automatic interpretation.

1.3.5 Evaluation of DDS modeling system

To get some insight in the performance of the DDS modeling system, several tests on this system have been accomplished:

- perfect interferogram interpretation
- extreme case test function processing

First, simulated density fields computed by Prast [15] are artificially converted into interferograms for different values of humidity. The quality of these interferograms is perfect, compared to the experimental ones, as shown in figure 2.9.

Figure 2.9: Perfect interferogram
No noise or any other disturbances due to imperfections in the beam or hologram are present. These perfect interferograms are used as input for the DDS system. Results are convincing for a smooth interferogram, as long as there is no shock, as shown in figures 2.10A-C. When a shock occurs, the interpretation gets troubled and the correspondence between the input and output in the 2D density field disappears. Some irregularities are added with respect to the original ones.
Figure 2.10: Density contour plots for perfect interferogram interpretations (upper) compared to input (lower) for humidities 25.5% (A), 37.9% (B), 57.1% (C).
The modelling has been tested further with several test functions representing the density along a certain vertical scan line, as shown in figures 2.11 A-D. These functions are chosen symmetric as we would expect them to be in the real case. The functions are not very realistic representatives of the density distribution, but they give an insight in what would happen in the extreme case. The real density functions are smoother, so we may expect the system to work for the real case if it works for the test functions. The interferometric signal for a certain density distribution is calculated by adding a certain background period \( Y \) to the density profile and taking the cosines of this total:

\[
\cos\left(\text{testfunction}(y) + \frac{2\pi y}{Y}\right).
\]  

It appeared that for most of the cases the system extracts the best representation of the input function when the background frequency \( Y \) lies around 6-12 pixels per period. The sign of the test function is lost in the argument of the cosinus. The model order \( p \) is chosen for each test as low as possible \((p < 5)\). The sharp edged linear functions show the best response to their input functions. They are both symmetric, as the input functions, which is not the case for the parabolic and gaussian functions. The parabolic function shows a loss of sign in the phase and some strange behaviour, the system does not succeed in extracting a resembling reproduction. The gaussian function also shows some abnormalities. These strange response functions give rise to serious concern.

The test series show that the function response sometimes looks quite good, but is not reliable in all cases. The departures are not unambiguously explained and the general confidence in the system has to be reconsidered.
Figure 2.11A: Input test functions and responses from the DDS system as function of background period $Y$. 
Figure 2.11B: Input test functions and responses from the DDS system as function of background period $Y$. 
Figure 2.11C: Input test functions and responses from the DDS system as function of background period $Y$. 
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Figure 2.11D: Input test functions and responses from the DDS system as function of background period $Y$. 

Y = 3

Y = 6

Y = 12

Y = 24

Y = 48
3. Ludwieg tube set-up & experimental results

In this section the Ludwieg tube set-up is evaluated. The parts of the set-up are explained on the basis of drawings and figures. A typical experiment is discussed, as well as the procedures of preparing the test mixture before doing the experiment. The results obtained from experiments and simulations are presented and discussed.

3.1 Ludwieg tube set-up

3.1.1 Description

The Ludwieg tube is a relative simple set-up to create a supersonic flow for a short period of time (typically 50 ms) under well defined conditions. The tube consists of a pipe of 12 meter in length and an inner cross-sectional area of 10×10 cm². On one side, the tube is separated from the vessel by a polyester membrane (Melinex polyester film, 20 μm). On the other side the tube is closed, as shown in figure 3.1. The membrane lies against cross wires and can be ruptured instantaneously by applying a high voltage (40 Volt) on the wires. The La val nozzle is mounted in the test section, which is situated 50 cm in front of the membrane (upstream). Two glass windows on both sides of the test section allow the visualisation of the flow. The circulation system consists of a pipe which is connected to both ends of the Ludwieg tube. The circulation pump CP (Verder VDE 0530) enables the mixing of the gas in the tube. The humidity is measured with the relative humidity meter H (Humicap 239033), which is situated in the circulation system. The dynamic pressure sensors K1 (Kistler 5011A-10) and K2 (Kistler 5001SN) are situated respectively upstream and downstream of the nozzle. Calibrations of the measurement devices are found in Appendix C. The nozzle was designed by Prast [13], the characteristics of the geometry are listed in Appendix A.

3.1.2 Preparation

Prior to the start of a new experiment, first a new membrane is mounted. (An extensive operational checklist is given in Appendix D.) This should be done carefully to avoid leakage and disturbances during the experiments. Then the tube and vessel are evacuated by vacuum pumps VP1 (Pfeiffer duo 016B) and VP2 (Speedivac ISC 450B). Remnant water from previous experiments and other pollution is removed this way. When the pressure in the tube is below 0.1 torr, indicated by static pressure meter P (Barocel 600A), the vacuum pump VP1 is switched off. Distilled water is added by means of a syringe through a rubber membrane. The amount of added water lies in the order of several millilitres, according to the desired humidity. The nitrogen carrier gas is added until pressure in the tube is atmospheric. The pressure in the vacuum vessel is also made atmospheric in order to reduce leakage of vapour through the membrane. To obtain a stable, homogeneous vapour-gas mixture, the gas is circulated by applying the circulation pump CP. The relative humidity is measured by the relative humidity meter H. This is connected to the x,t-writer, in order to judge if the humidity is stable. When the humidity is stabilised, the valves between the circulation line and the Ludwieg tube are closed and the circulation pump is switched off. The vacuum vessel is pumped at low pressure again.
Meanwhile, the alignment of the optics is checked. The laser used for the alignment is a continuous He-Ne laser ($\lambda=633$ nm; 20 mW), which is situated right behind the ruby pulse laser. The position and direction of the alignment laser beam should be exactly the same as the pulse laser. Therefore, the alignment laser is positioned such, that the beam goes the same path through spatial filtering part of the ruby laser. Because the location of the spot from the pulse laser is hard to identify, burn paper is used to verify that the beams hit the same spot at a larger distance. When the continuous laser is aligned with the pulse laser, the beam is directed through the centre of the beam splitter. Reference beam and object beam are aligned through the centre of the mirrors and lenses. The concave side of the small lenses should be directed from the laser, in order to avoid ionisation of the air by focusing of the reflected beam. The object beam is directed perpendicular through the test section, the absence of reflections is checked with the infrared camera. Polarisation of both beams is checked and set circular by turning the $\frac{1}{4}\lambda$ plates. The intensity ratio between the object and reference beam is set at one. The correct illumination in the plane of the holographic plate is checked visually with the pulse laser after the alignment procedure, paying attention to the uniform light distribution on the holographic plate.
3.1.3 Typical experiment

Upon completion of all this preparatory work, the room is obscured and the hologram casing is opened. The piezo mirror is switched on \((V=600)\) for adding reference fringes and the laser is fired once. The piezo mirror is switched off again and the membrane is ruptured while the triggering of the laser is active. To trigger the pulse laser (Lumonics HLS2) during the experiment, the signals from the dynamic pressure sensors \(K_1\) and \(K_2\) are used. When \(K_2\) indicates a rapid drop in pressure, the Le Croy generates a trigger pulse on which the delay unit is triggered. After 30 ms, the flow through the nozzle becomes stationary and a trigger pulse is given to the laser. The laser fires a short (30 ns) pulse, and the film is exposed. After the second exposure the hologram is shut again. The signals from \(K_1\), \(K_2\), the trigger pulse and the laser pulse are recorded in the ADC (Le Croy Waveform Recorder 6810).
Figure 3.2 shows typical pressure signals from K1 and K2. During a period of time of 50 ms, the pressure at the nozzle entrance is constant and the flow is stationary. The pressure value at the first plateau, which lies around $8.5 \times 10^4$ Pa, can be taken from these signals to calculate stagnation properties and throat conditions.

![Graph showing typical pressure signals from K1 and K2.](image)

**Figure 3.2:** Typical pressure signals before (K1) and after (K2) the nozzle during an experiment.

When the membrane is ruptured, an expansion wave travels towards the nozzle and further into the tube as shown in figure 3.3. The flow will become *choked*, that is, the mass flux attains its maximum value at the throat of the nozzle where the Mach number reaches unity. The expansion wave moves to the closed end of the tube where it is reflected. Until the reflected expansion wave returns to the nozzle entrance, the conditions at the entrance, like pressure, temperature, density and Mach number are constant.
3.2 Experimental results

3.2.1 Conducted experiments

A series of experiments was performed at room temperature and atmospheric pressure. The range of humidities is between 9 and 85%. The experiments and the conditions are listed in table 3.1.

<table>
<thead>
<tr>
<th>experiment number</th>
<th>relative humidity (%)</th>
<th>temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31203</td>
<td>29.7</td>
<td>20.7</td>
</tr>
<tr>
<td>02404 *</td>
<td>9.03</td>
<td>20.7</td>
</tr>
<tr>
<td>03206</td>
<td>8.77</td>
<td>20.1</td>
</tr>
<tr>
<td>11410</td>
<td>14.9</td>
<td>20.2</td>
</tr>
<tr>
<td>25513</td>
<td>25.5</td>
<td>20.2</td>
</tr>
<tr>
<td>42014 *</td>
<td>37.9</td>
<td>20.6</td>
</tr>
<tr>
<td>30916</td>
<td>29.5</td>
<td>20.7</td>
</tr>
<tr>
<td>57318 *</td>
<td>49.3</td>
<td>20.7</td>
</tr>
<tr>
<td>58719</td>
<td>50.4</td>
<td>20.6</td>
</tr>
<tr>
<td>89520</td>
<td>73.5</td>
<td>19.7</td>
</tr>
<tr>
<td>78721</td>
<td>65.4</td>
<td>20.1</td>
</tr>
<tr>
<td>67622</td>
<td>57.1</td>
<td>20.3</td>
</tr>
<tr>
<td>10224</td>
<td>83.1</td>
<td>21.6</td>
</tr>
</tbody>
</table>

Table 3.1: Experiment numbers, corresponding relative humidities and temperatures. The marked experiments are evaluated.
The water vapour fraction in these mixtures is quite low. This means that the droplets formed remain small. To reach higher fractions and accordingly bigger droplets, the tube will be heated in the future. Droplet size measurements (3-wavelength light extinction method) is in preparation and will be applied soon to determine the size of the formed droplets.

3.2.2 Results

In the series of experiments, three representative experiments (02404, 42014, 57318) have been evaluated manually. This is done by manual scanning of the interference lines. The interferograms are showed below in figure 3.4.

Figure 3.4: Experimental interferograms which were scanned manually. Relative humidities respectively: 9.0% (A), 37.9% (B) and 49.3% (C).
The values of relative humidity, temperature and pressure at the first plateau in the experiment are used as input for the corresponding numerical simulation. The comparisons of the density fields calculated respectively by the numerical simulations and the fringe scanning of the experimental interferograms are shown in figure 3.5-7.

Figure 3.5: Experiment 02404. Initial conditions: RH=9\%, T_0=279.8 \text{ K}, P_0=8.56\times10^4 \text{ Pa}. Left: 2D-density field, numerical (upper), experimental (lower), \Delta \rho between contour lines is 0.02 kg\text{ m}^{-3}; Right: density value along the nozzle axes.

The numerical and experimental case without condensation is shown in figure 3.5. Here, the density contour lines from the numerical simulation show very good correspondence with the experimentally obtained results. The density value along the nozzle axes shows the same fine correspondence. In the case of higher initial relative humidity a shock occurs, as shown in figure 3.6, the correspondence in the contour plot becomes slightly less evident, however, the density development along the nozzle axes is still quite reasonable. The experimental density increase as a consequence of condensation occurs at the same location and the strength is in the same order of magnitude as in the simulation. Even if the shock becomes stronger, as shown in figure 3.7, the correspondence in density along the axes is still evident. The density value obtained from the experiment shows a slightly less abrupt path compared to the numerical one, but the location and strength of the shock are quite good. Scanning runs into problems when the shocks become stronger. In this case, the interference lines are not continuous anymore and phase information is lost over the shock.
Holographic interferometry applied to supersonic nozzle flow with condensation

Figure 3.6: Experiment 42014. Initial conditions: RH=37.9%, $T_0=280.3$ K, $P_0=8.52 \cdot 10^4$ Pa. Left: 2D-density field, numerical (upper), experimental (lower), $\Delta \rho$ between contour lines is 0.02 kg·m$^{-3}$; Right: density value along the nozzle axes.

Figure 3.7: Experiment 57318. Initial conditions: RH=49.3%, $T_0=279.7$ K, $P_0=8.54 \cdot 10^4$ Pa. Left: 2D-density field, numerical (upper), experimental (lower), $\Delta \rho$ between contour lines is 0.02 kg·m$^{-3}$; Right: density value along the nozzle axes.
The automatic evaluation does not yet provide reliable density fields. The interferogram quality should be improved first to make automatic interpretation possible. It is demonstrated before (section 2.3.5) that the system works to a certain extent if the input interferogram is of high quality and no shocks occur. It is expected that, when the interferograms are less disturbed by noise and if resolution is higher, this will result in a more accurate density field. At this moment, because of the relative fine correspondence between calculated and hand scanned fields, the manual scanning procedure method is preferred for obtaining quantitative density information from experiments.
Holographic interferometry applied to supersonic nozzle flow with condensation
4. Conclusions

In this section concluding remarks on the research are presented. Recommendations on how to proceed are done.

4.1 Remarks on the holographic interferometer

As stated in the introduction, this research is mainly focused on analysing the following issue: to what extent can the experimental set-up, consisting of a holographic interferometer and an automated method for the analysis of the interferograms, supply detailed and accurate data on the density field during supersonic nozzle flow with condensation?

For a systematic and coherent analysis, two major areas of investigation were identified:

- A critical assessment of the influence of the optical components (i.e. laser, lenses, mirrors and beam splitter) on the quality of the experimental holograms and their digitisation.
- A critical evaluation of the automated method based on a non-linear regression technique for the extraction of density data from the holograms.

The evaluation of the holographic interferograms was complicated by disturbances. The causes of these disturbances have been indicated and attempts in removing them were made. In the first place, it appeared that the reproducibility of subsequent laser shots and beam quality of the laser was insufficient. With technical assistance from the manufacturer of the laser, the performance of the laser was optimised. Secondly, it emerged that the polarising beam splitter was not suited for this application. This component will be replaced by a special coated glass plate. Thirdly, it appeared that the spatial resolution and the performance of the CCD camera were insufficient. During the digitisation process, the spatial resolution of the reconstructed holograms was reduced to half of its value due to the interlaced scanning of the CCD camera. This made the automatic interpretation quite troublesome. In addition, the procedures of optical alignment and preparation of the tube were optimised.

Attention was paid to the possibilities of automatic interpretation of the interferograms by means of the Data Dependent System (DDS) modeling method. The method proved not to give reliable results except for some specific conditions. Attempts to improve the unwrapping procedure (i.e. the process of determining the number of $2\pi$ steps to be added) did not result in the expected improvement. Presently, the results obtained from the automatic evaluation of experimental interferograms are not satisfying yet.

The method of holographic interferometry has been applied to supersonic nozzle flow with condensation. Three experimental cases of different initial conditions have been evaluated manually and compared with corresponding numerical cases. The manual method of interpreting the experimental interferograms proved to be successful and showed satisfying agreement with the corresponding numerical simulations.

Summarising, the following has been achieved:

- The laser beam quality has been improved and consequently the quality of the hologram and its reconstruction have been optimised.
• The correct operational procedure has been established.
• It is demonstrated that the DDS method did not work properly for experimental interferograms, since the method is very sensitive to noise.

4.2 Recommendations

As noted before, the quality of the interferograms is of crucial importance to the DDS method. Therefore, the quality of the digitisation process has to improve, i.e. the spatial resolution of the camera has to increase in order to facilitate the automated evaluation.

The DDS method as applied in this set-up is a complicated and concealing method, consisting of many uncontrollable steps. Therefore, it might be sensible to look for a more straightforward automatic method to interpret the interferograms. The method of counting the interference lines has proven itself to be a simple but reliable and efficient method. The possibilities of automatic line tracing should be investigated, since these kind of processes already were successfully applied in holographic interferometry and medical science [3]. Other techniques applied in holographic interferometry, such as phase stepping [6] or digital holography [18], might lead to more reliable density information.
References


Holographic interferometry applied to supersonic nozzle flow with condensation


Appendix A

Nozzle geometry

The nozzle used in experiments is a Laval nozzle designed by Prast. Its characteristics are:

- Half throat height \( y^* \) = 0.01 m
- Curvature upstream \( R_{up} \) = 0.08 m
- Curvature downstream \( R_{down} \) = 0.06 m
- Half exit height \( y_{exit} \) = 0.0265 m
- Length from throat \( x_{exit} \) = 0.1167 m
- Design Mach number \( M_{exit} \) = 2.5076

A detailed description of this design is found in [13].
Appendix B

Material properties

Here, properties of nitrogen and water (liquid and vapour) used in calculations and simulations are listed.

Nitrogen

Molar mass \( M = 0.028013 \ \text{kg} \)
Specific gas constant \( R = 296.8 \ \text{J kg}^{-1} \text{K}^{-1} \)
Specific heat \( c_p = 1041 \ \text{J kg}^{-1} \text{K}^{-1} \)
Ratio of specific heats \( \gamma = 1.40 \)
Viscosity \( \eta = 17.6 \times 10^{-6} (T/295)^{0.767} \ \text{kg m}^{-1} \text{s}^{-1} \)

Water (liquid and vapour)

Molar mass \( M_v = 0.018016 \ \text{kg} \)
Specific gas constant \( R_v = 461.5 \ \text{J kg}^{-1} \text{K}^{-1} \)
Specific heat of the vapour \( c_{pv} = 1813 \ \text{J kg}^{-1} \text{K}^{-1} \)
Specific heat of the liquid \( c_l = 4187 \ \text{J kg}^{-1} \text{K}^{-1} \)
Density of the liquid \( \rho_l = 997 \ \text{kg m}^{-3} \)
Surface tension

\[
\sigma = \begin{cases} 
D_0 + D_1 \cdot (273.15 - T) & T \geq 249.39 \text{ K} \\
D_2 + D_3 T + D_4 T^5 & T < 249.39 \text{ K}
\end{cases}
\]

\[
D_0 = 7.61 \times 10^{-4} \\
D_1 = 1.55 \times 10^{-4} \\
D_2 = 1.1313 \times 10^{-10} \\
D_3 = -3.7091 \times 10^{-13} \\
D_4 = -5.6464 \times 10^{-6}
\]

Latent heat of evaporation \( L = 2.452 \times 10^{-6} - 2373 \cdot (T-295) \ \text{J kg}^{-1} \)
Saturation pressure

\[
p_s = \exp \left\{ A_0 + A_1 T + A_2 T^2 + B_0 \ln T + C_0 / T \right\} \ \text{Pa}
\]

\[
A_0 = 2.1125 \times 10^1 \\
A_1 = -2.7246 \times 10^2 \\
A_2 = 1.6853 \times 10^5 \\
B_0 = 2.4576 \\
C_0 = -6.0944642 \times 10^3
\]

Nitrogen water-vapour mixture

Modified diffusion coefficient \( D_m = 18.0 \times 10^{-6} (T/295)^{1.085} \ \text{kg m}^{-1} \text{s}^{-1} \)
Holographic interferometry applied to supersonic nozzle flow with condensation
Appendix C

Calibrations of the measurement devices used

Here, the calibration fits of the relative humidity meter and the dynamic pressure sensors are given.

Humicap 239033 relative humidity meter:

\[ S = D_0 + D_1 V_{rh} \]

with

\[ D_0 = 6.3766 \times 10^{-2} \]
\[ D_1 = 7.4982 \times 10^{-2} \]

Kistler1 5011A-10:

\[ P_{K1} = -3.3015 \times 10^{-3} + 9.7078 \times 10^{-4} \cdot V_{K1} \ \text{Pa} \]

Kistler2 5001SN:

\[ P_{K2} = 5.6916 \times 10^{-4} + 9.486 \times 10^{-4} \cdot V_{K2} \ \text{Pa} \]
Appendix D

Checklist holographic interferometry

Preparation

0. Change membrane.
1. Close shocktube.
2. Empty expansion vessel.
3. Empty shocktube and circulation system.
4. Close valve to shocktube pump.
5. Inject water.
7. Fill tube with nitrogen.
8. Release overpressure.
9. Open valve expansion vessel.
10. Start x-t writer and wait until humidity is stationary.
11. Meanwhile check the alignment of the optics.
   - align alignment laser
   - block reference beam, align object beam
   - check absence of reflections in nozzle with Nikon camera
   - block object beam, align reference beam
   - check polarisation
   - check intensity ratio object/reference=5/6

Experiment

12. Switch off the circulation pump.
13. Evacuate the expansion vessel and switch off the pump.
14. Write down
   - Experiment number, date, time
   - Film identification (A1, A2, B1, B2, C1, C2)
   - Po=Pressure
   - To=Temperature
   - Vrh=Humidity voltage of the Humidity sensor
   - VT =Temperature voltage of the Humidity sensor
   - Vpiezo=Voltage on piezo mirror
   - Vamp=Voltage to laser amplifiers
   - Phi=angle of filmholder relative to shocktube
15. Close the valves to the shocktube.
16. Switch on
   - pressure sensors
   - piezo mirror
triggerunits, (connect output delay with in vd camera)
Lecroy
acquisition PC

17. Start catalyst on the PC with:
   cat L 6 read thomas space
   Set trigger settings
t1 pk1
t2 pk2
t3 hls
t4 trg

18. Obscure the room
19. Open the hologram
20. Charge laser
21. Fire once for reference shot
22. Switch piezo mirror off
23. Set pressure sensors on operate
24. Set triggerunits
25. Lecroy program on single trigger
26. Switch on the current supply
27. Press set on laser triggerunit
28. Close the hologram
29. Switch off the current supply
30. Dump the charged laser
32. Save the pressure/trigger signals
   kistler nozzle exit   pk1HHNNN.raw
   kistler nozzle entrance pk2HHNNN.raw
   fire trigger laser    hlsHHNNN.raw
   trigger out lecroy    trgHHNNN.raw