MASTER

Microstrip mixer and antenna design for a 14 GHz FMCW radar system

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Microstrip mixer and antenna design
for a 14 GHz FMCW radar system

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Abstract

This Masters thesis gives the results of a research project of six months, carried out in a collaborative arrangement between a British company, Ogden Safety Systems Ltd., and the University of Bradford (UK). Ogden Safety Systems Ltd. manufacture FMCW radar systems that are used on the rear of large vehicles on building sites to prevent collisions while reversing. A new market for the system is the use inside tanks and silos to measure the level of liquids, powders, etc. in industrial environments.

The most important aim of the project was to make the radar smaller. The approach chosen to achieve this, was to make part of the microwave assembly in microstrip. It was also hoped to reduce the level of background signal arising in the simple existing mixer design, which reduces the sensitivity to close-in reflections. It was decided to make a single balanced mixer in microstrip to replace the single diode mixer that is currently used. Low cost surface mounted diodes (Alpha SMS7621-006) were used as detectors. Two types of hybrid couplers are analysed in this report. Implementations of both types were designed, etched and measured.

Another goal was to improve the radar front end in order to make the system suitable for use inside tanks and silos. An antenna with a low sidelobe level is desired for this application. An 8-patch uniform array was designed and tested. A parallel feeding network of 2-way power splitters is used to feed the array. Matching transformers are included in the feed network to match the input impedance of the patches to the impedance of the feed lines.

A microstrip-to-waveguide transition was made to connect the waveguide parts of the radar system to the microstrip parts. This transition uses a stepped quarter wavelength transformer designed according to the Chebyshev distribution. A good connection between the groundplane of the microstrip substrate and the waveguide is very important. It was found that the performance of the transition depended on how far the dielectric was inserted under the last step of the transformer. This effect is not mentioned in the papers on this subject.

The components designed and tested in this project have not yet been integrated in a complete circuit. However, a proposal for a complete front-end containing the developed components is given and can be used as a start for further work.
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1 Introduction

1.1 INTRODUCTION
This Masters thesis gives the results of a research project of six months, carried out in a collaborative arrangement between a British company, Ogden Safety Systems Ltd. at Doncaster, and the Antennas and Applied Electromagnetics section of the department of Electronics and Electrical Engineering of the University of Bradford (UK).

Ogden Safety Systems Ltd. manufacture FMCW radar systems that are used on the rear of large vehicles on industrial sites to prevent collisions while reversing. A new market for the system is the use inside tanks and silos to measure the level of liquids, powders, etc. in industrial environments. The approved band for the system is between 13.4 GHz and 14.0 GHz.

1.2 ANTI-COLLISION RADAR FOR VEHICLES
In the construction, mining, quarrying and associated industries, many specialised types of vehicle are operated on a variety of sites. Restricted visibility from the driving position, in particular while reversing, is a feature of many of these vehicles. Reversing accidents are responsible for a considerable amount of damage to the vehicles and installations and are a safety hazard to the people working on the site. Several products aimed at the prevention of reversing accidents are commercially available, including audible reversing horns, closed circuit video systems, ultrasonic based sensors and radar sensors.

• Reversing horns
This type of device provides the user with a low-cost and reliable reverse warning system. However, it is a passive system, relying on people taking avoiding action, and does not offer the driver a warning if any obstruction is present. On a congested site there may be many vehicles operating and it is often not obvious to the site users which vehicle is sounding the reversing horn.

• Closed circuit video systems
This system consists of a rearward looking video camera coupled to a monitor screen in the driver’s cabin, allowing the driver to see any obstacle to the rear of the vehicle.
Disadvantages of this system are that the constant attention of the driver is required and that it is often hard to make out the size and distance of objects appearing on the video screen. Another problem is that the camera lens becomes easily obscured by mud.

• Ultrasonic and radar-based sensors

Both ultrasonic and radar devices are based on the detection of the reflection of a transmitted signal by an obstacle. These systems can be used for warning only or can be coupled to an automatic braking system. The main disadvantage of ultrasonic systems is that their performance depends heavily on the size, shape and constitution of the obstacle. Furthermore, contamination of the device influences the signals. Radar systems are capable of accurately detecting small objects and suffer less from contamination of the antenna aperture, but they are more expensive.

1.3 RADAR SYSTEMS FOR LEVEL MEASUREMENTS

A recently discovered market for Ogden’s radar system is the measurement of the level of liquids and powders in industrial environments. Level measurement is a complex subject, as the measured products have different chemical and physical features. Depending on the process conditions, various types of level gauges can be used [1]. Mechanical systems such as float gauges and pressure gauges are more and more replaced by electrical systems, that have higher accuracy and lower maintenance costs. The most common examples of electrical level gauges are:

• Capacitive probes

An electrode that is partly immersed in the product to be measured forms together with the vessel an electrical capacitor. Because the product acts as a dielectric, a change in product level results in a variation in capacitance value. The measured capacitance is thus a measure for the product level.

• Ultrasonic devices

In this system, an ultrasonic transmitter generates a series of sound pressure waves which are reflected by the product surface. The elapsed time between the transmission and the reception of the signal is measured and calculated as a distance. For this system, no contact with the product is required. However, the propagation of ultrasonic signals is influenced by changes in temperature and density of any gas, and ultrasonic devices are not suitable for pressurised vessels.
3 INTRODUCTION

- Radar devices
Whereas ultrasonic systems transmit pressure waves, radar systems transmit electromagnetic waves that are virtually unaffected by any vessel conditions (agitated, pressurised, heated, steamy, foaming, etc.). Radar systems are also non-contact and have the added advantage that measuring through non metallic materials is possible. Radar systems are suitable for all products with dielectric constant greater than 2.

Until recently, ultrasonic level measuring devices dominated the market for non-invasive level measuring, having the advantage over radar systems of being low cost. However, because of the huge growth of the demand for microwave components (especially for satellite TV) in the last decade, mass production has become possible, making the components low cost and radar systems more and more competitive.

1.4 AIM OF THE PROJECT
Market analysis shows that the relatively large size of Ogden's radar system is a limiting factor for the application to smaller vehicles. Therefore, one of the aims of the project was to make the existing system smaller. The approach chosen to achieve this, was to make part of the microwave assembly in microstrip. Another goal was to improve the radar front end in order to make the system suitable for use inside tanks and silos. An antenna with a low sidelobe level is desired for this application. Care had to be taken not to increase the cost of the system, because the price of a radar system is a factor of major importance in the competition with ultrasonic systems.

Chapter 2 gives a review of the theory of FMCW radar. In chapter 3 the radar circuit is described. Chapter 4 describes the design of a microstrip-to-waveguide transition. In chapter 5 the essentials of microstrip circuit design are reviewed. Chapter 6 is about hybrid couplers. Chapter 7 deals with the design of a microstrip antenna. Finally, chapter 8 contains the conclusions of this project and recommendations for further work.
2 FMCW radar

2.1 INTRODUCTION

A radar detects the presence of objects and locates their position in space by transmitting electromagnetic energy and observing the returned echo. Radar systems are based on one of two different principles.

1. A pulse radar transmits a short pulse of electromagnetic energy, after which the receiver is turned on to listen for the echo. An echo indicates that a target is present and the time that elapses between the transmission of the pulse and the receipt of the echo (time-of-flight) is a measure of the distance to the target.

2. A frequency-modulated continuous wave (FMCW) radar uses a continuous signal that changes in frequency as a (known) function of time. The returned signal is compared to the current frequency being transmitted and the phase angle between the two frequencies determines the distance of the target.

The radar systems manufactured by Ogden Safety Systems Ltd. are of the FMCW kind. FMCW radar is by no means a novel concept. It had found applications in the radar altimeter and in proximity fuses as early as the 1940's. In more recent years, however, FMCW has not received as much attention as pulsed radar systems. This is mainly due to the problem of isolating the strong continuously transmitted signal from the weak echo, in order to prevent damage to the receiver. For our application, this is not a big problem since the transmitted power can be relatively small because the targets are all at close range. Below, the general theory of FMCW radar is discussed [2].

2.2 VELOCITY AND RANGE VIA FREQUENCY MEASUREMENTS

In CW radar, the transmitter generates a continuous (unmodulated) oscillation of frequency $f_0$, which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and reflected to the radar antenna. If the target is in motion relative to the radar, the received signal will be shifted in frequency from the transmitted frequency $f_0$ by an amount of $\pm f_d$. This is the so called Doppler effect and is the basis of CW radar. A positive Doppler shift is associated with a closing target and a negative shift with a receding target. Thus, by
observing the received frequency the velocity of the target relative to the radar can be determined.

Distance is measured by the time the signal takes to reach the target, reflect and return to the radar antenna. Since the velocity of the signal is a known constant, the distance can be calculated from the time to travel. To recognise the time of transmission and the time of return, some kind of timing mark must be applied to the carrier wave. A widely used technique to achieve this is to frequency-modulate the CW carrier, resulting in FMCW radar.

Assume that the frequency of the CW signal is made to increase linearly with time, as shown by the solid line in figure 2.1. An echo signal from a reflecting object at a distance R will return after a time \( t = \frac{2R}{c} \), equal to the horizontal separation of the transmitted signal and the received signal represented by the dashed line.

Since the frequency changes linearly with time, the distance to the target can also be calculated from the difference in frequency between the two signals, which is the vertical separation of the lines. This difference signal or beat frequency can easily be obtained via mixing the transmitted signal and the received signal. If the rate of change of the carrier frequency is \( \frac{df}{dt} \), the beat frequency is
The greater the frequency deviation in the time interval, the more accurate the measurement of the transit time can be. Since the operating frequency of the radar cannot increase indefinitely, periodicity in the modulation is necessary. In Ogden’s radar system, the modulating waveform is triangular. As shown in figure 2.2(a), the range frequency is constant throughout the modulation cycle, except at the turn-around region.

\[ f_r = \frac{2R}{c} \frac{df}{dt}. \]  

Figure 2.2 Beat frequency as a function of time for (a) a stationary target and (b) a non-stationary target.

If the target is not stationary, the Doppler frequency shift is superimposed on the range frequency as shown in figure 2.2(b). On one portion of the frequency modulation cycle, the beat frequency is increased by the Doppler shift, while on the other portion, it is decreased. The range frequency \( f_r \) may be extracted by measuring the average beat frequency.
When more than one target is present in the view of the radar, the mixer output will contain more than one difference frequency. If the system is linear, there will be a frequency component corresponding to each target.

### 2.3 VELOCITY AND RANGE VIA PHASE MEASUREMENTS

If the distance to the target is known to be \( R \), the total number of wavelengths \( \lambda \) in the two-way path between the radar and the target is \( 2R/\lambda \) and the total change in phase is \( 4\pi R/\lambda \) radians. If the target is in motion, the distance to the target and thus the phase are continually changing. A very accurate signal processing circuit would be able to detect movements of the target by observing the phase of the beat signal. Since the frequency is the derivative of phase with time, the Doppler angular frequency \( \omega_d \) can be calculated by

\[
\dot{\omega}_d = 2\pi f_d = \frac{d\phi}{dt} = 4\pi \frac{dR}{\lambda} = \frac{4\pi v_r}{\lambda},
\]

where \( f_d \) is the Doppler frequency shift and \( v_r \) is the relative (or radial) velocity of the target with respect to the radar.
3 The radar circuit

3.1 INTRODUCTION

The radar circuit can roughly be divided into two parts: the microwave assembly and the signal processing part. The microwave part is comprised of a voltage controlled oscillator, an antenna and a microwave mixer. The oscillator is briefly discussed in paragraph 3.2 and the mixer is dealt with in paragraph 3.3. The antenna however, is discussed in chapter 7. Although the signal processing part of the radar system is considered a "black box" as far as this project is concerned, it is described in paragraph 3.4 to complete the picture of the radar circuit.

3.2 THE VOLTAGE CONTROLED OSCILLATOR

The frequency modulated microwave signal is generated by a Voltage Controlled Oscillator (VCO). The VCO used in this radar system is a varactor tuned Gunn oscillator, consisting of a Gunn diode and a varactor diode. Gunn diodes are negative resistance devices capable of generating power at microwave frequencies. When a Gunn diode is put in a resonant circuit tuned to its oscillating frequency and excited, the circuit will oscillate. By varying the voltage applied to the varactor diode, the oscillation frequency of the active component can be modulated. It is important that the transmitted carrier frequency is modulated in a linear fashion with respect to time. If the frequency modulation is non-linear, then the spectrum arising due to a single target will cause energy spread over several spectral lines, giving rise to ambiguity concerning the exact position of the target. The approach used in Ogden's radar system is to store digital codes proportional to the voltages required at the varactor diode in an EPROM and to access the codes sequentially.

3.3 THE MICROWAVE MIXER

3.3.1 The square-law detector

Once the signal is modulated, it is transmitted toward the target through the antenna. Part of the transmitted signal is reflected by the target and returns to the radar antenna. In the microwave mixer, the incoming signal from the antenna is down converted to intermediate frequency (IF) by means of a non-linear impedance element which is simultaneously reacted on by the signal and a higher level signal supplied by a local oscillator (LO) [3]. The non-linear element used in the frequency conversion may be any type of rectifying device, in so
far as its characteristics and parasitic components allow efficient rectification at the frequency of interest. In general, a smaller current is induced by a voltage of one sign than by a voltage of the other sign. As shown in figure 3.1, the I-V characteristic of a diode is strongly curved (i.e. non-linear) in the region of the origin. If an alternating voltage, such as shown on the negative current axis, is applied to the diode, the current that is passed through the diode has the form shown on the positive voltage axis. Because there is less current flowing during the negative half-cycles than during the positive ones, there is a net positive current with a magnitude proportional to the applied AC voltage. If the envelope of the AC voltage varies with time, the net current varies in a related fashion and so has components derived from the amplitude modulation of the applied voltage wave.

A non-linear device that is commonly used at microwave frequencies is the Schottky diode. Schottky diodes are metal-semiconductor junctions. Contrary to p-n junction diodes, the operation of this type of device depends on the conduction of majority carriers. Since the semiconductor used is always n-type, the majority carriers are electrons. When a metal is joined to the semiconductor, electrons start to diffuse from the semiconductor in the metal, where their energy is lower. The metal surface will become negatively charged, while in the semiconductor the positively charged donor atoms form a depletion region. The arising electric field between these works to pull the electrons back into the semiconductor and an equilibrium situation will develop. For the current-voltage characteristic an expression similar to that of the p-n junction diode can be derived [4]:

$$I = I_0 \left[ \exp \left( \frac{qV}{nkT} \right) - 1 \right],$$

(3.1)
with the saturation current $I_s$ given by:

$$I_s = AA'kT^2 \exp\left(-\frac{q\phi_B}{kT}\right),$$  \hspace{1cm} (3.2)

where $A$ is the diode area, $A'$ is the Richardson constant and $\phi_B$ is the potential difference between the semiconductor and the metal. The current increases exponentially with voltage for small bias, but is eventually limited by the diode series resistance. For small amplitudes, the diode characteristic around a bias point $(V_0, I_0)$ can be approximated by the first three terms of a Taylor series [5]:

$$I(V) = I_0 + \frac{dI}{dV}(V-V_0) + \frac{1}{2}\left(\frac{dI}{dV}\right)^2 (V-V_0)^2.$$ \hspace{1cm} (3.3)

For a Schottky diode having the standard diode characteristic of equation (3.1), equation (3.3) can be written as

$$I(V) = I_0 + \frac{1}{R_j}(V-V_0) + g(V-V_0)^2,$$ \hspace{1cm} (3.4)

where

$$R_j = \frac{nkT}{q(I_0 + I_s)} \quad \text{and} \quad g = \frac{q^2(I_0 + I_s)}{2(nkT)^2}. \hspace{1cm} (3.5)$$

For very small voltages, the term in the second power of the voltage is large compared with the higher-power terms. Therefore, the rectified current produced from a very small signal is proportional to the square of the applied AC voltage. For this reason, they are often called 'square-law' detectors.

As shown below, the last term of equation (3.4) produces the desired down conversion. If we assume zero bias, then $V_0 = 0$. The small signal variation is the sum of the local oscillator signal $V_{LO} \cos(\omega_{LO}t)$ and the echo signal $V_s \cos(\omega t)$, where the $\omega$ terms are the instantaneous angular frequencies of the signals at time $t$. The output from the heterodyning process is
The fourth term is the baseband beat signal of interest. Ogden's radar uses a so called homodyne receiver: a superheterodyne receiver with zero IF. The function of the local oscillator is replaced by a leakage signal from the transmitter.

3.3.2 The single balanced mixer [6]

The detector of the system currently in production is a single diode mounted in the waveguide. More sophisticated mixer designs are the singly balanced and the double balanced mixer. However, these require more diodes, thus increasing the cost of the system. An advantage of a microstrip mixer is that surface mounted diodes can be used instead of waveguide diodes. Surface mounted diodes are produced in large quantities so they are low cost, making the more complex mixers possible.

A commonly encountered type of mixer is the single balanced mixer. Single balanced mixers consist of a microwave hybrid coupler and two detector diodes. The hybrid coupler is discussed in detail in chapter 6. An advantage of using two diodes for the detection, is that the stationary background reflections can be cancelled out. This improves the sensitivity of the radar system. Furthermore, in a balanced mixer various noise products are cancelled out.

\[
\left[ V_{LO} \cos(\omega_{LO}) + V_s \cos(\omega_s) \right]^2 = \frac{1}{2} V_w^2 \left[ 1 + \cos(2\omega_{LO}) \right] + \frac{1}{2} V_s^2 \left[ 1 + \cos(2\omega_s) \right] + V_{LO} V_s \cos((\omega_{LO} + \omega_s)t) + V_{LO} V_s \cos((\omega_{LO} - \omega_s)t) \tag{3.6}
\]

The fourth term is the baseband beat signal of interest. Ogden’s radar uses a so called homodyne receiver: a superheterodyne receiver with zero IF. The function of the local oscillator is replaced by a leakage signal from the transmitter.

Figure 3.2 LO noise treated as a random phasor added to the constant LO phasor.

The LO signal and its additive noise can be expressed as phasors, as shown in figure 3.2. Since the LO is a sinusoid of fixed amplitude, frequency and phase, the LO voltage is a constant phasor. The noise is a much smaller phasor whose amplitude and phase vary...
randomly. The resultant phasor representing the waveform applied to the mixer through the LO port varies in phase as well as amplitude. Although these variations may be small compared to the LO level, the amplitude noise can be very large compared to the RF level. Another kind of noise is caused by so called spurious responses. Harmonics of the local oscillator signal mix with harmonics of the RF signal and cause interference at the IF frequency:

\[ f_{IF} = mf_{RF} + nf_{LO}, \]

where \( m \) and \( n \) are integers \( 0, \pm 1, \pm 2, \ldots \).

![Diagram](image)

**Figure 3.3** Phase relationships between LO, IF, RF and AM noise voltages in a single balanced mixer using a 180° hybrid: (a) mixer topology; (b) IF current summation; (c) AM noise cancellation.
Figure 3.3(a) shows schematically the balanced mixer in normal operation. In this case, the LO is applied to the Δ-port of the hybrid, so the LO voltage has 180° phase difference at the two diodes. The RF is applied to the Σ-port and is in phase at the diodes. As can be seen in figure 3.3(b), the IF currents combine at the node joining the diodes. The situation is not the same for the AM noise-components, as shown in figure 3.3(c). These enter the mixer at the LO port, are 180° out of phase at the diodes and cancel at the IF output.

Because the mixer subtracts not only the LO and RF frequencies but also their phases, the phase noise on the LO signal is transferred directly to the received signal. Nothing can be done in the mixer to reduce this kind of noise. The spurious response performance of this mixer arrangement is listed below [6]:

1. $k$th order spurious responses, arising from mixing between $m f_{RF} + nf_{LO}$, where $m + n = k$, arise only from the $k$th harmonics,
2. all $(m,n)$ spurious responses, where $m$ and $n$ are even, are eliminated,
3. the $(2,1)$ spurious response $(m = \pm 2, n = \pm 1)$ is eliminated, but not the $(1,2)$ response.

If the LO voltage is applied at the Σ-port and the RF voltage at the Δ-port, then the characteristics are unchanged except that now the $(1,2)$ spurious response is eliminated instead of the $(2,1)$ response.

![Diagram](image)

**Figure 3.4** Phasor diagram illustrating LO, RF and IF voltages; RF and LO phase relationships at each diode.
The single balanced mixer using the 90° hybrid can be analysed in a similar manner. However, in this case the RF and LO signals at the diode ports differ by 90°. This phase relationship is illustrated with a phasor diagram in figure 3.4. As can be seen, at one arm of the hybrid the LO voltage leads the RF voltage, while at the other arm the RF voltage leads. If one of the diodes is reversed, the phase difference between the diode conductance waveform and the applied RF voltage is the same for both diodes and the IF currents will be summed in phase, as shown in figure 3.5. This mixer arrangement can reject the even-order spurious responses, but there is no suppression of either the (1,2) or (2,1) spurious response. The 90° hybrid is completely symmetrical, so the RF and LO ports can be reversed with no changes in the mixer performance.

3.4 SIGNAL PROCESSING [7, 8]

The heart of the radar system is a microprocessor, used not only to periodically send voltage information to the varactor drive, but also for the signal processing. A block diagram of the signal processing circuit is shown in figure 3.6. The input to the signal processing stage is the output (beat) signal of the microwave mixer, containing the distance information. Before the microprocessor can operate upon this signal, it is amplified, filtered against aliasing, sampled and finally converted into digital form by an AD-converter. The method used for analysing the signal is to calculate the Discrete Fourier Transform (DFT).
Figuur 3.6  Block diagram of the signal processing circuit
3.4.1 DFT and windowing

The DFT of a sampled signal $x[n]$, defined over the range $0 \leq n \leq (N - 1)$, is given by

$$X[k] = \sum_{n=0}^{N-1} x[n] \exp^{-j2\pi kn/N}$$ \hspace{1cm} (3.8)

If the input data series $x[n]$ is real then this equation can be simplified and the real and imaginary spectral coefficients are given by the following equations

$$X[k] = \sum_{n=0}^{N-1} x[n] \cos(2\pi kn/N) \hspace{1cm} (Real \ part),$$ \hspace{1cm} (3.9)

$$X[k] = -\sum_{n=0}^{N-1} x[n] \sin(2\pi kn/N) \hspace{1cm} (Imaginary \ part).$$ \hspace{1cm} (3.10)

If $N$ samples of data are collected over a period $T$, the DFT will generate $N$ complex spectral coefficients. The zero order coefficient $F_0$ represents the mean DC level of the signal over the sampling period, the first coefficient $F_1$ represents the spectral energy at the fundamental frequency, $F_2$ the energy at the second harmonic and so on. If the source frequency changes by 150 MHz during the sampling process, then the first Fourier coefficient will represent spectral energy corresponding to targets centred at a distance of one meter. Since the system must be able to detect targets up to eight meters away, a Fourier transform of at least 16 samples must be performed. However, if a length 16 Fourier transform were used in order to calculate spectral coefficients, the anti-aliasing filter would require a very steep response. Since such a filter would be of high order it would be expensive. Therefore a length 32 transform is used in combination with a simpler filter.

The discrete Fourier transform can only produce an exact spectrum for a signal that is exactly periodic over the sampling interval. If this is not the case, a discontinuity will appear in the input signal since the values at the beginning and the end of the sampled data sequence do not coincide. This results in a spectrum with energy distributed over a wide number of spectral coefficients. The method used to reduce the effect is known as windowing and consists of gradually attenuating the signal towards the end of the sampling interval to reduce the amplitude of the discontinuity. The window used in Ogden's radar system is the well known Hamming window.
3.4.2 Averaging the target data

From the calculated spectral coefficients, the system must decide whether or not a target is present. Ideally, if no targets are present, all of the Fourier coefficients are zero, indicating that no reflected radar power has been received. If the smallest target required to trigger the alarm is introduced at a given distance, the value of the appropriate Fourier coefficient can be noted and used as a threshold value for activating the alarm. Drift due to temperature changes and contamination of the radar antenna by mud and water cause the Fourier coefficients to become non-zero. To compensate for this problem, averaging is used. If there is no target movement, the difference between the incoming coefficient and the average value will tend to zero. The drift of the microwave section is slow compared to the rate at which the coefficients are calculated.
4 Microstrip-to-waveguide transition

4.1 INTRODUCTION

One of the aims of the project was to reduce the dimensions of the existing radar system. One way to achieve this, is to make part of the microwave assembly in microstrip, instead of using waveguide. To connect the waveguide parts of the circuit to the microstrip parts, a microstrip-to-waveguide transition had to be designed. As shown in figure 4.1, the microstrip-to-waveguide transition consists of two parts:

1. a ridge-to-microstrip junction, in which the ridge extends to the substrate top of the microstrip line,
2. a ridge to rectangular waveguide impedance transformer.

By reducing the depth of the ridge, the transition converts the dominant mode in the microstrip line to that in the rectangular waveguide. Because the dominant mode field distribution of the first section of the ridge transformer (with the deepest ridge) resembles that of the microstrip line and both ridge waveguide and microstrip line are less dispersive than the rectangular waveguide, a high performance microstrip to waveguide transition with small insertion loss and low return loss over a wide frequency band can be designed [9].

Figure 4.1 Configuration of the microstrip-to-waveguide transition.
4.2 THE QUARTER WAVELENGTH TRANSFORMER [10]

The impedance transformer is a so called quarter-wave transformer. The length $L$ of each section is one-quarter wavelength at the center frequency of the passband and is given by

$$L = \frac{\lambda_{g0}}{4} = \frac{\lambda_{g1} \lambda_{g2}}{2(\lambda_{g1} + \lambda_{g2})},$$  \hspace{1cm} (4.1)$$

where $\lambda_{g1}$ and $\lambda_{g2}$ are the longest and the shortest guide wavelengths in the pass band of the transformer, respectively.

![Diagram](image)

**Figure 4.2** An N-section quarter-wave transformer.

Figure 4.2 shows an N-section quarter-wave transformer. $Z_n$ is the characteristic impedance of the $n$th section of the transformer. $Z_L$ is assumed to be a pure resistance, and may be greater or smaller than $Z_0$. The reflection coefficient at the $n$th junction is

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} = \rho_n.$$  \hspace{1cm} (4.2)$$

For a first approximation, the total reflection coefficient is the sum of the first-order reflected waves only. This is given by

$$\Gamma = \rho_0 + \rho_1 e^{-2j\theta} + \rho_2 e^{-4j\theta} + ... + \rho_N e^{-2NJ\theta},$$  \hspace{1cm} (4.3)$$

where the factors $e^{-2mj\theta}$ account for the phase retardation introduced because of the different distances the various partial waves must travel. If we make the transformer symmetrical, then
\[
\rho_0 = \rho_N, \rho_1 = \rho_{N-1}, \rho_2 = \rho_{N-2}, \ldots \quad (4.4)
\]

In this case (4.4) becomes

\[
\Gamma = e^{-jN\theta} \left[ \rho_0 \left( e^{jN\theta} + e^{-jN\theta} \right) + \rho_1 \left( e^{j(N-2)\theta} + e^{-j(N-2)\theta} \right) + \ldots \right], \quad (4.5)
\]

which is the Fourier cosine series

\[
\Gamma = 2e^{-jN\theta} \left[ \rho_0 \cos N\theta + \rho_1 \cos (N-2)\theta + \ldots + \rho_n \cos (N-2n)\theta + \ldots \right]. \quad (4.6)
\]

In equation (4.6) the last term is \( \rho_{(N-1)/2} \cos \theta \) for \( N \) odd and \( \frac{1}{2} \rho_{N/2} \) for \( N \) even.

### 4.3 THE CHEBYSHEV DISTRIBUTION [10]

By choosing the reflection coefficients \( \rho_n \) (and hence the \( Z_n \)) according to a Chebyshev polynomial, it is possible to obtain the "equal-ripple" characteristic that is often encountered in filter designs. This distribution gives a low VSWR in the passband of the transition. The Chebyshev polynomials \( T_n(x) \) for the first three values of \( n \) are

\[
T_1(x) = x \quad (4.7a)
\]
\[
T_2(x) = 2x^2 - 1 \quad (4.7b)
\]
\[
T_3(x) = 4x^3 - 3x \quad (4.7c)
\]

and the recurrence relation is

\[
T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x). \quad (4.8)
\]

If we change the variable \( x \) in (4.7) to \( \cos \theta / \cos \theta_m \), then the characteristic shown in figure 4.3 is obtained. The equal-ripple section with the low VSWR is confined to the passband of the transition. In the figure, this is the range \( \theta_m \to \pi - \theta_m \), where \( \theta_m \) can be calculated via

\[
\theta_m = \frac{180^\circ}{1 + \lambda_{g1}/\lambda_{g2}}. \quad (4.9)
\]
With the coefficients chosen according to the Chebyshev polynomial, equation (4.6) can be written as

$$\Gamma = A e^{-j\theta_0} T_N \left( \cos \theta / \cos \theta_0 \right)$$  \hspace{1cm} (4.10)

Here, $A$ is a constant which can be determined by solving (4.9) for $\theta = 0$, resulting in

$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N \left( 1 / \cos \theta_0 \right)}.$$  \hspace{1cm} (4.11)

With the help of

$$\cos^n \theta = \frac{1}{2^n} \left[ C^{(n)}_0 \cos n \theta + C^{(n)}_1 \cos (n-2) \theta + \ldots + C^{(n)}_m \cos (n-2m) \theta + \ldots \right],$$  \hspace{1cm} (4.12)

where

$$C^{(N)}_n = \frac{N!}{(N-n)!n!}$$  \hspace{1cm} (4.12a)

are the corresponding binomial coefficients, we can rewrite (4.7) as
\[ T_1(\cos \theta / \cos \theta_m) = \cos \theta / \cos \theta_m \]  
\[ T_2(\cos \theta / \cos \theta_m) = \left(1/\cos \theta_m\right)^2 (\cos 2\theta + 1) - 1 \]  
\[ T_3(\cos \theta / \cos \theta_m) = \left(1/\cos \theta_m\right)^3 (\cos 3\theta + 3\cos \theta) - 3 \cos \theta / \cos \theta_m \]  
\[(4.13a) \quad (4.13b) \quad (4.13c)\]

Using (4.6), (4.10), (4.11) and (4.13) we can solve for the unknown \( \rho_n \). Transformers with more than three sections can be calculated by using more polynomials than the first three only. Finally, the corresponding values of \( Z_n \) are given by

\[ Z_{n+1} = \frac{1 + \rho_n}{1 - \rho_n} Z_n. \]  
\[(4.14)\]

Using these values, the corresponding steps of the ridged waveguide can be found in [11].

Practical junctions however, are non-ideal and cannot be represented by a change of impedance only. A more accurate representation is an ideal junction shunted by a capacitance. The main effect of this capacitance is to move the planes with real \( \Gamma \) out of the plane of the junction. To compensate for this, it is necessary to move the physical junctions as well, in order to make the spacing between adjacent reference planes equal to one-quarter of the wavelength again. The design procedure described in [11] has been used to calculate the positions of the junctions. The dimensions of the transition are shown in figure 4.4. The width is 6.32 mm.

![Figure 4.4 Dimensions of the microstrip-to-waveguide transition](image)

4.4 PERFORMANCE OF THE TRANSITION

The experimental results of the measurements on the transition are shown in appendix B. In figure B.1 the transmission \( S_{21} \) is plotted as a function of frequency for the full range of the
network analyser. The reference level is 0 dB and the vertical scale is 5 dB per division. The horizontal scale is 2.65 GHz per division. The frequency band from 10.6 GHz to 18.55 GHz corresponds to the operating band of waveguide 18, the size of waveguide used. It can be seen that within this band approximately half the input power (-3 dB) is transmitted through the transition into a length of stripline. The other half of the power is partly dissipated, partly radiated and partly reflected.

In figure B.2, the reflection coefficient $S_{11}$ is plotted as a function of frequency. The reference level is 0 dB and the vertical scale is 10 dB per division. The horizontal scale is 400 MHz per division. The band 12.0 GHz to 16.0 GHz is completely within the operating band of waveguide 18. The minimum value for the $S_{11}$ is -37 dB, reached at a frequency of 13.68 GHz. The reflection coefficient is below -20 dB in a 1.5 GHz wide band around this value. To obtain these results, a good connection between the ground plane of the microstrip and the waveguide is essential. It was found that the performance of the transition depended on how far the dielectric was inserted under the last step of the transformer. This effect is not mentioned in the papers on this subject.
5 Microstrip design

5.1 REVIEW OF TRANSMISSION LINE THEORY [12]

5.1.1 Modelling of radio-frequency transmission lines

To transport radio-frequency signals, two types of transmission line are used in the radar system, namely rectangular waveguide and microstrip lines. Regardless of the actual structure, uniform transmission lines can be modelled as shown in figure 5.1.

The constants shown in the figure are:

\[ R = \text{resistance along the line}, \]
\[ L = \text{inductance along the line}, \]
\[ G = \text{conductance shunting the line}, \]
\[ C = \text{capacitance shunting the line}. \]

For radio-frequencies, the effects due to \( L \) and \( C \) are dominant and the transmission line can be approximated as loss-free. The propagation coefficient \( \gamma \) of a wave along the line is characterised by

\[ \gamma = \sqrt{(R + j \omega L)(G + j \omega C)} = \alpha + j \beta, \quad (5.1) \]
where \( \alpha \) is the attenuation coefficient and \( \beta \) is the phase-change coefficient. For high frequencies, equation (5.1) yields

\[
\beta = \omega \sqrt{LC}.
\]  

(5.2)

Because the wave experiences a phase shift of \( 2\pi \) radians per wavelength \( \lambda_0 \) travelled along the line, \( \beta \) can also be written as

\[
\beta = \frac{2\pi}{\lambda_0}.
\]  

(5.3)

Another important parameter is the characteristic impedance of the line \( Z_0 \), defined by

\[
Z_0 = \frac{R + j\omega L}{\sqrt{G + j\omega C}}.
\]  

(5.4)

At high radio frequencies the characteristic impedance can be simplified to \( Z_0 = \sqrt{L/C} \), which is a purely resistive result.

5.1.2 Transmission line impedances

The input impedance to a transmission line varies with the distance progressed along the line. Using the model shown in figure 5.2, the input impedance at \( z = 0 \) is given by the following expression

\[
Z_{in} = Z_0 \left( \frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_L \sinh(\gamma l)} \right),
\]  

(5.5)

for loss-free lines simplifying to

\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right).
\]  

(5.6)
Equation (5.6) can be further reduced for the following three situations:

1. short circuit \((Z_L = 0)\) \(Z_m = jZ_0 \tan(\beta l)\) (5.7)
2. open circuit \((Z_L \rightarrow \infty)\) \(Z_m = -jZ_0 \cot(\beta l)\) (5.8)
3. matched load \((Z_L = Z_0)\) \(Z_m = Z_0\) (5.9)

By choosing the correct line lengths, inductive or capacitive circuit elements are automatically realised in transmission line form. Transmission lines which are a quarter of a wavelength long at the operating frequency of a circuit act as impedance transformers, e.g. transforming short circuits into open circuits and vice versa.

5.2 PROPERTIES OF MICROSTRIP CIRCUITS

5.2.1 Geometry of microstrip and substrate properties

One of the important transmission lines is the microstrip line, used commonly in microwave circuits because they are easily fabricated and connect easily to active devices. The basic microstrip line geometry and field pattern are shown in figure 5.3. The most important physical parameters are the microstrip width \(W\), the thickness of the substrate \(h\) and the relative permittivity of the substrate \(\varepsilon_r\). The important electrical parameters are the characteristic impedance \(Z_0\), the guide wavelength \(\lambda_g\) and the attenuation constant \(\alpha\). The geometry of microstrip involves an abrupt interface between the substrate and the air above it. Because of the non-uniformity of the dielectric surrounding the conductor, the structure cannot support a single
well defined mode of propagation like TEM. However, most of the energy transmitted along microstrip has a field distribution which closely resembles TEM and therefore microstrip is called a quasi-TEM transmission line.

![Microstrip line geometry.](image)

**Figure 5.3** Microstrip line geometry.

A well established way of making low-cost microstrip circuits is thick film technology. This technology consists of using a printed circuit technique to etch the desired pattern in the copper cladding of a plastic substrate. The tolerance of the etching process used (the smallest gap that can be made) was 0.2 mm. The main problem in microstrip synthesis is to find the linewidth and length corresponding to certain impedance and electrical length. Initially a substrate of thickness $h$ and relative permittivity $\varepsilon_r$ will have to be chosen. The substrate used is RT Duroid 5870 and has the properties shown in table 5.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative permittivity $\varepsilon_r$</td>
<td>2.33</td>
</tr>
<tr>
<td>Thickness of the substrate $h$</td>
<td>0.79 mm</td>
</tr>
<tr>
<td>Thickness of the copper cladding $t$</td>
<td>35 $\mu$m</td>
</tr>
<tr>
<td>Dissipation factor (at 10 GHz)</td>
<td>$\tan \delta = 0.0012$</td>
</tr>
</tbody>
</table>

The linewidth to substrate height ratio ($w/h$) is a strong function of $Z_0$ and of the substrate permittivity. In [12], approximate formulas are given for this relationship, but for our circuit we simply used the values generated by the Hewlett Packard software package LIBRA, resulting in a
line width of 2.353 mm for an impedance of 50 Ohms. As will be described in the following paragraph, the electrical length depends on the relative permittivity of the substrate used.

5.2.2 Electrical length and effective permittivity

The velocity of propagation of an electromagnetic wave depends on the absolute permeability and permittivity of the medium through which the wave passes:

\[ v_p = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\mu, \varepsilon_r}}. \]  

\( (5.10) \)

Most lines are non-ferromagnetic, so that \( \mu_r = 1 \). In free space we have \( c = \lambda_0 \) and in the microstrip \( v_p = f \lambda_g \). Substituting these in equation (5.10) results in

\[ \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r}}. \]  

\( (5.11) \)

From this equation we see that the wavelength is reduced according to the square root of the relative permittivity of the material in which the line is embedded. The electrical length \( \theta \) (in radians) of a line of length \( l \) is now given by

\[ \theta = \beta \cdot l = \frac{2\pi \cdot l}{\lambda_g}. \]  

\( (5.12) \)

A more useful parameter than \( \varepsilon_r \) is the effective permittivity \( \varepsilon_{\text{eff}} \), because part of the field lines is going through the air. For very wide lines, nearly all of the field is confined to the substrate dielectric and the structure resembles a parallel plate capacitor. At this extreme \( \varepsilon_{\text{eff}} \rightarrow \varepsilon_r \). In the case of very narrow lines, the field is almost equally shared by the air (\( \varepsilon_r = 1 \)) and the substrate. At this extreme \( \varepsilon_{\text{eff}} \approx \frac{1}{2}(\varepsilon_r + 1) \). The range of \( \varepsilon_{\text{eff}} \) is therefore

\[ \frac{1}{2}(\varepsilon_r + 1) \leq \varepsilon_{\text{eff}} \leq \varepsilon_r. \]  

\( (5.13) \)
6 Hybrid couplers

6.1 INTRODUCTION

Microwave hybrids are four-port devices having the following three characteristics:

1. all ports are matched
2. RF power applied to any one port is split equally between two of the other ports
3. the remaining port is isolated from the input port

There are two kinds of hybrid coupler: the 90° (quadrature) hybrid and the 180° hybrid. Both types are shown in figure 6.1. The lines between the ports show the phase shift between them.

![Figure 6.1](image)

(a) 180° hybrid; (b) 90° hybrid.

If a signal is applied to port 1 of an ideal 180° hybrid coupler, it appears at ports 4 and 3 with identical phases and at a level 3 dB below the input. No output appears at port 2. Similarly, if port 2 is excited, the outputs are port 3 and 4 and port 1 has no output. The outputs at port 3 and 4 are 180° out of phase and again 3 dB below the input. Port 3 is called the sum (Σ) port and port 4 is called the difference (Δ) port.

If a signal is applied to port 1 of an ideal 90° hybrid coupler, it appears at ports 2 and at a level 3 dB below the input, but the outputs are 90° out of phase. No output appears at port 3. Similarly, if port 2 is excited, the outputs are port 1 and 3 and port 4 has no output. The
outputs at port 3 and 4 are again 90° out of phase and 3 dB below the input. The same reasoning applies to the other ports.

6.2 ANALYSIS

6.2.1 S parameters [13, 14]
According to network theory, linear networks can be described by parameters, which can be determined by measuring the voltage or the current at the open or short circuited terminals. The usual representations for these parameters are the impedance matrix, admittance matrix or chain matrix. However, for high frequency circuits the dimensions of the circuit components are of the same order of magnitude as the wavelength and the wave character of the signals becomes important. In general, many different wave modes exist, making it difficult to determine the total voltage and current. Therefore, in microwave engineering the scattering matrix is used, based on the directly measurable amplitude and phase of the incident and reflected waves.

A general two port network is shown in figure 6.2. The reflection power waves $b_1$ and $b_2$ be represented by the incident power waves $a_1$ and $a_2$:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

(6.1)

or in matrix form

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix},$$

(6.2)
where the incident waves \( a_1 \) and \( a_2 \) are given by

\[
\begin{align*}
    a_1 &= \frac{V_1 + Z_{g1}I_1}{2\sqrt{\text{Re}(Z_{g1})}} \quad \text{and} \quad a_2 = \frac{V_2 + Z_{g2}I_2}{2\sqrt{\text{Re}(Z_{g2})}}.
\end{align*}
\]  

(6.3)

The parameters \( S_{ij} \) are called the S parameters of the network. If \( i = j \) then \( S_{ij} \) is called a reflection coefficient and if \( i \neq j \) then \( S_{ij} \) is called a transmission coefficient. From expression (6.2), \( S_{11} \) and \( S_{21} \) are found to be

\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad \text{and} \quad S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}.
\]  

(6.4)

The condition \( a_2 = 0 \) means there is no incident power coming from port 2. In other words, the S parameters are measured under matched conditions. Using the concept of changing reference plane, the S parameters can be measured on a network located at some distance from the measuring point. Hence, transmission lines are commonly added at the device terminals for the ease of measurement.

### 6.2.2 Scattering matrix for the 180° hybrid [10]

The general form of the scattering matrix for a four-port network such as the hybrid is

\[
[S] = \begin{bmatrix}
    S_{11} & S_{12} & S_{13} & S_{14} \\
    S_{21} & S_{22} & S_{23} & S_{24} \\
    S_{31} & S_{32} & S_{33} & S_{34} \\
    S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}.
\]  

(6.5)

Since the hybrid is a passive network, \( S_{ii} = S_{ji} \). Furthermore, from symmetry we have \( S_{13} = S_{14} \) and \( S_{23} = -S_{24} \). Matching elements that do not destroy the symmetry of the junction may be placed in the arms as to make \( S_{11} = S_{22} = 0 \). Using these relations and the assumption that ports 1 and 2 as well as ports 3 and 4 are uncoupled, the scattering matrix becomes
\[ \begin{bmatrix} S_{13} & S_{13} \\ S_{23} & -S_{23} \\ S_{33} & S_{34} \\ S_{34} & S_{44} \end{bmatrix} \]

The product of the second row with its conjugate gives

\[ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{34}|^2 = 1 \] (6.7)

and the similar expression for row 3 is

\[ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{44}|^2 = 1. \] (6.8)

If we subtract these equations, we obtain

\[ |S_{33}|^2 - |S_{44}|^2 = 0, \] (6.9)

so \( |S_{33}|^2 = |S_{44}|^2 \). From rows 1 and 2 we have

\[ 2|S_{13}|^2 = 1 \quad \text{or} \quad |S_{13}| = \frac{\sqrt{2}}{2}, \] (6.10)
\[ 2|S_{23}|^2 = 1 \quad \text{or} \quad |S_{23}| = \frac{\sqrt{2}}{2}. \] (6.11)

Using these results in (6.7) gives

\[ 1 + |S_{33}|^2 + |S_{34}|^2 = 0. \] (6.12)

This sum can equal zero only if both \( S_{33} \) and \( S_{34} \) are zero. From (6.9) it follows that \( S_{44} \) equals zero also. The reduced form of the scattering matrix becomes
By proper choice of terminal planes in arms 1 and 2, it is possible to make both \( S_{13} \) and \( S_{23} \) real. Thus, using the relations (6.10) and (6.11), the scattering matrix of the 180° hybrid can finally be written as

\[
[S]_{180} = \begin{bmatrix}
0 & 0 & S_{13} & S_{13} \\
0 & 0 & S_{23} & -S_{23} \\
S_{13} & S_{23} & 0 & 0 \\
S_{13} & -S_{23} & 0 & 0 \\
\end{bmatrix}
\] (6.13)

6.2.3 Scattering matrix of the 90° hybrid

The scattering matrix of the 90° hybrid can be derived in the same way. The general four port matrix of (6.5) is reduced to

\[
[S]_{90} = \frac{\sqrt{2}}{2} \begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 0 & 1 & -1 \\
1 & 1 & 0 & 0 \\
1 & -1 & 0 & 0 \\
\end{bmatrix}
\] (6.14)

if we use \( S_{ij} = S_{ji} \) (passive network), \( S_{11} = S_{22} = S_{33} = S_{44} = 0 \) (matching), \( S_{13} = -jS_{12} \) and \( S_{24} = -jS_{34} \) (for the coupled ports) and \( S_{23} = 0 \) and \( S_{34} = 0 \) (for the isolated ports).

From row 2 follows that

\[-|S_{13}|^2 + |S_{34}|^2 = 0 \quad \text{or} \quad |S_{12}|^2 = |S_{34}|^2\] (6.16)

Again, we can choose the terminal planes in arms 1 and 4 to make \( S_{12} \) and \( S_{34} \) real, so the scattering matrix of the 90° hybrid can be written as
6.3 THE RAT-RACE COUPLER

6.3.1 Circuit properties [6, 14]

An implementation of the 180° hybrid that is well known and easy to design is the so-called 'rat-race' or ring hybrid shown in figure 6.3. It consists of a ring of transmission line that is \( 1 \frac{1}{2} \lambda \) in circumference. Its characteristic impedance is \( \sqrt{2} \) times the port impedances. Transmission lines for the four ports are connected to the ring in such a way that two are \( \frac{3}{4} \lambda \) apart and the spacings between the rest are \( \frac{1}{4} \lambda \). Since the port impedances are 50 Ohms, the characteristic impedance of the ring has to be 70.7 Ohms.

![Figure 6.3](image_url)

**Figure 6.3** Topology of the rat-race 180° hybrid coupler.

The operation of the rat-race is illustrated in figure 6.4. If port one is excited, waves can travel around the ring to ports 3 and 4. Because the path from port 4 to port 2 is \( \frac{1}{2} \lambda \) longer than the path from port 3 to port 2, the waves meeting at port 2 are 180° out of phase, so the voltage at this point must be zero. Port 2 is therefore a virtual ground. The parts of the ring from port 2 to port 3 and from port 2 to port 4 are then quarter-wavelength shorted stubs. The terminal impedances of these stubs are infinite, so they have no effect on ports 3 and 4. The remaining parts of the 70.7 Ohm ring act as quarter wavelength transformers, which transform the 50 Ohm loads on ports 3 and 4 to 100 Ohms each at port 1. The parallel
combination of these is 50 Ohms, so the port is matched and the power from the source is split evenly between them.

![Figure 6.4](image)

**Figure 6.4** Equivalent circuit of the rat-race hybrid with port 1 excited and with ports 3 and 4 as outputs; (a) transmission-line model with port 2 as a virtual ground; (b) equivalent circuit at centre frequency.

The same reasoning can be applied when any of the other ports is excited. This hybrid is relatively narrowband, because its dimensions are frequency dependent. However, for our application, where a bandwidth of only a few percent is needed, it can be used.

### 6.3.2 Experimental results

The dimensions of the rat-race can be calculated as follows. The wavelength in microstrip, using the RT Duroid 5870 substrate (table 5.1) is

$$\lambda_{xy} = \frac{c}{f} \cdot \frac{1}{\sqrt{\varepsilon_r}} = \frac{3.00 \cdot 10^8}{14.0 \cdot 10^7 \cdot \sqrt{2.33}} = 14.0 \text{ mm.}$$
The radius of the $1\frac{1}{2} \lambda$ ratrace can be calculated as

$$r = \frac{1.5 \cdot \lambda}{2\pi} = 3.35 \text{ mm}.$$  

The width of a 70.7 Ohm line is calculated by the LIBRA software as 1.318 mm, making the inner diameter of the ring $d_1 = 5.386 \text{ mm}$ and the outer diameter $d_2 = 8.022 \text{ mm}$. The width of the incoming 50 Ohm lines is 2.353 mm. This structure was etched and tested. It was found that the best performance was at 14.3 GHz. The difference with the design frequency can almost completely be explained by the fact that rather then the relative permittivity $\varepsilon_r$, the effective permittivity $\varepsilon_{eff}$ has to be used, resulting in a slightly bigger structure.

A bigger version of the rat-race was made with dimensions $d_1 = 5.80 \text{ mm}$ and $d_2 = 8.436 \text{ mm}$, aimed at an operating frequency of 13.7 GHz. The experimental results are shown in appendix B. The figures B.3, B.4 and B.5 show the reflection coefficient $S_{11}$ and the transmission coefficient $S_{21}$ as a function of frequency for the configurations shown in figure 6.5, respectively.

Figure 6.5 Measurement configurations for: (a) figure B.3; (b) figure B.4; (c) figure B.5.

For all plots the reference level is 0 dB and the vertical scale is 10 dB per division. The horizontal scale is 400 MHz per division. Ideally, the $S_{11}$ plots should be identical, but it can be seen that it makes a small difference whether we connect the second port of the analyser to the isolation port of the rat-race (figure B.3) or to a coupled port (figures B.4 and B.5). The minimum value of the $S_{11}$ is less than -30 dB at a frequency of 13.35 GHz. The reflection coefficient is smaller than -20 dB in a band of 550 MHz around this value.
The $S_{21}$ plot in figure B.3 shows the isolation between the ports 1 and 2 of the rat-race as a function of frequency. The best isolation is -47.5 dB at 13.6 GHz. The isolation is better than 20 dB in an 800 MHz wide band.

The $S_{21}$ plots in the figures B.4 and B.5 show the transmission to the coupled ports. Ideally, there would be an equal power splitting between these ports and the output power would be -3 dB below the input power level. It can be seen that the transmission constants are fairly constant. However, there is a small imbalance in the power splitting. The total transmitted power to each port is slightly below -3 dB because of losses in the circuit and the reflection of part of the input signal. As expected, the phase difference between the coupled ports of the rat-race was found to be approximately 180°.

6.4 THE BRANCH LINE COUPLER

6.4.1 Circuit properties [15]

As shown in figure 6.6, the branch line coupler consists of a ring that is one wavelength in circumference. Two of the arms have an impedance which is different from the characteristic impedance $Z_0$, for example $Z_a = Z_c = Z_0/\sqrt{2}$, and the other two arms have an impedance of $Z_b = Z_d = Z_0$.

The operation of the branch line coupler is illustrated in figure 6.7. Like in the case of the rat-race coupler, power fed into a port will split and travel both clockwise and anticlockwise around the branches. Hence, the power available at any given port will depend on the phase relationships of these two waves travelling in opposite directions at that port.

![Figure 6.6](image)

Figure 6.6 Topology of the branch line 90° hybrid coupler.
Figure 6.7   Equivalent circuit of the branch line hybrid with port 1 excited and with ports 3 and 4 as outputs.

When power is applied to port 1 then, because of the mismatch by $Z_a$ and $Z_c$, a standing wave is set up on the branch ring. The $\lambda/4$ section between the ports 1 and 2 with impedance $Z_a$ acts as an impedance inverter. The voltage at port 2 will be $V_2 = V_1/\sqrt{2}$. Between the ports 2 and 3 where $Z_b = Z_0$ there will be no standing wave and as a result there will be an output voltage $V_3 = V_1/\sqrt{2}$ at port 3. The phase difference between $V_1$ and $V_2$ is 90°. The power at each port is then

$$(V_2/\sqrt{2})^2 = (V_3/\sqrt{2})^2 = P_1/2$$

(6.18)

The power at port 4 is zero since the waves from port 1 travelling in opposite directions around the branch ring arrive out of phase at port 4 and thus cancel. The same reasoning can be applied when power is applied to ports 2, 3 or 4.

6.4.2 Experimental results

The dimensions of the branch line coupler can be calculated as follows. The wavelength in microstrip, using the RT Duroid 5870 substrate (table 5.1) is

$$\lambda_e = \frac{c}{f \cdot \sqrt{\varepsilon_r}} = \frac{3.00 \cdot 10^8}{14.0 \cdot 10^9 \cdot \sqrt{2.33}} = 14.0 \text{ mm},$$

so the radius of the coupler is

$$r = \frac{\lambda_e}{2\pi} = 2.23 \text{ mm}.$$
If the circuit is made for $Z_0 = 50\Omega$, then there are two arms in the ring with an impedance of 50 Ohms and two with an impedance of $Z_0/\sqrt{2} = 35.4\Omega$. However, the width of a 50 Ohm line is 2.353 mm and the width of a 35.4 Ohm line is even 3.882 mm. If we compare these linewidths with the radius of the circle, then we see that $Z_0 = 50\Omega$ is not a very practical choice. For a higher characteristic impedance, the linewidths become smaller. If we make the ring for $Z_0 = 100\Omega$, then $Z_a = Z_d = 100\Omega$ and $Z_c = Z_e = 70.7\Omega$, corresponding to linewidths of 0.642 mm and 1.318 mm, respectively. To match the 100 Ohm branch line coupler to the 50 Ohm network analyser system, a 70.7 Ohm quarter wavelength transformer was inserted in the incoming lines.

The branch line coupler was found to perform best at a frequency of 14.24 GHz. Again, there is a deviation from the design frequency, because the effective permittivity has to be used instead of the relative permittivity. The structure has to be made a bit bigger to perform well in the band we are aiming for. Values calculated by LIBRA software were used to design a better version of the coupler. The length of a quarter wavelength section of 100 Ohm line was calculated to be 3.940 mm and the length of a quarter wavelength 70.0 Ohm line as 3.851 mm. Hence, the total circumference of the ring is 15.58 mm and the radius is 2.48 mm.

![Figure 6.8](image)

**Figure 6.8** Measurement configurations for: (a) figure B.6; (b) figure B.7; (c) figure B.8.

The experimental results are shown in appendix B. The figures B.6, B.7 and B.8 show the reflection coefficient $S_{11}$ and the transmission coefficient $S_{21}$ as a function of frequency for the configurations shown in figure 6.8, respectively. For all plots the reference level is 0 dB and the vertical scale is 10 dB per division. The horizontal scale is 400 MHz per division. Ideally, the $S_{11}$ plots should be identical, but it can be seen that it makes a small difference whether we connect the second port of the analyser to the isolation port of the rat-race (figure B.6) or to a coupled port (figures B.7 and B.8). The minimum value of the $S_{11}$ is below -30
dB at a frequency of 14.0 GHz. The reflection coefficient is smaller than -20 dB in a band of more than 800 MHz around this value.

The $S_{21}$ plot in figure B.6 shows the isolation between the ports 1 and 2 of the rat-race as a function of frequency. The best isolation is -28 dB at 13.76 GHz. The isolation is better than 20 dB in a 600 MHz wide band. The $S_{21}$ plots in the figures B.7 and B.8 show the transmission to the coupled ports. Ideally, there would be an equal power splitting between these ports and the output power would be -3 dB below the input power level. It can be seen that the transmission constants are fairly constant. However, there is a small imbalance in the power splitting, -3.44 dB and -4.15 dB respectively. The total transmitted power to each port is below -3 dB because of losses in the circuit and the reflection of part of the input signal. As expected, the phase difference between the coupled ports of the branch line coupler was found to be approximately 90°.

The calculations show that the length of a quarter wavelength line depends on the impedance. Therefore, it is likely that a better isolation and a better balance in the power splitting can be achieved if this effect is taken into account. The ports of the ring are then not at 90° exactly, but at 89° and 91°, respectively. Another possibility is to use the square form of the branch line coupler, instead of a ring structure.

6.5 TESTING CIRCUIT WITH DIODES

A testing circuit was made to measure the performance of the diodes. This circuit is shown in figure 6.9. We decided to use the branch line coupler, because the output ports of the branch line coupler are adjacent, making it easier to connect the diode package. The diodes used are Alpha SMS7621-006 surface mounted diodes. This type of diode was recommended for the application by Alpha Industries, Ogden’s supplier of diodes.

Bias can be applied via a radial stub on the end of a quarter wavelength line. The radial stub acts as a parallel plate capacitor to ground and provides a short circuit for RF. Via the quarter wavelength line, the bias point transforms into an open circuit and no RF power will enter the bias circuit. In order to prevent the appearance of standing waves in the radial stub, the width of the stub should not exceed half a wavelength, otherwise the stub will become a microstrip antenna. The gaps in the feedlines act as DC blocks. The common terminal of the diode package was connected to ground. The IF can be obtained from the radial stubs. The maximum diode sensitivity was measured at a bias voltage of 0.9 Volts.
Figure 6.9  Diode testing circuit.
7 Microstrip antennas

7.1 INTRODUCTION
The development of microstrip antennas arose from the idea of using printed circuit technology not only for the circuit components and transmission lines, but also for the radiating elements of an electronic system [17]. Microstrip patch antennas are small, relatively simple and inexpensive to manufacture and are mechanically robust. They can have all kinds of shapes, the most simple one being the rectangular microstrip antenna.

Single microstrip or printed circuit antenna elements have low gain, low radiation efficiency and wide beamwidth [20]. In addition, at high frequencies, the efficiency of microstrip antennas decreases because of conductor and dielectric losses. However, the performance of patch antennas is better if they are combined in an antenna array. The bandwidth required for Ogden’s system is only a few percent and the system is for short range, so a patch antenna array may perform sufficiently well for this application. It was decided to design a patch antenna array to get an idea of the typical performance.

7.2 ANALYSIS [16]
There are many methods of analysis for microstrip antennas. Well known models are the transmission line model and the cavity model. The transmission line model is the easiest, but is suitable only for rectangular patches. In this model, the antenna is represented by two radiating slots, spaced apart by a distance equal to the length L of the patch.

Whereas the transmission line model is suitable only for rectangular patches, the cavity model can (in principle) handle any arbitrary patch shape. The model treats the antenna as a dielectric loaded cavity with two perfectly conducting electric walls (top and bottom), and four perfectly conducting magnetic walls (sidewalls). The four sidewalls represent four narrow apertures (slots) through which the radiation takes place. The main assumption in both models is that the thickness of the substrate is much less than a wavelength.
When the microstrip patch is energised, a charge distribution is established on the upper and lower surfaces of the patch, as well as on the ground plane, as shown in figure 7.1. The charge distribution is controlled by two mechanisms; an attractive and a repulsive mechanism. The attractive mechanism is between the corresponding opposite charges on the bottom side of the patch and the ground plane. The repulsive mechanism is between the equal charges on the bottom surface of the patch, and pushes some charges from the bottom of the patch, around the edges, to the top surface. The movement of these charges creates corresponding current densities $J_b$ and $J_t$ at the bottom and top surfaces of the patch, respectively.

In the cavity model, the microstrip patch is represented by the equivalent electric current density $J_t$ at the top surface of the patch. The current density $J_b$ at the bottom of the patch is not needed for this model. The four slots are represented by the equivalent electric current density $J_s$ and equivalent magnetic current density $M_s$ as shown in figure 7.2, each represented by

$$J_s = \hat{n} \times H_a$$  \hspace{1cm} (7.1)

and

$$M_s = -\hat{n} \times E_a$$  \hspace{1cm} (7.2)

where $E_a$ and $H_a$ represent, respectively, the electric and magnetic fields at the slots. For microstrip antennas with very small height-to-width ratio, the current density $J_t$ at the top of the patch is much smaller than the current density $J_b$ at the bottom of the patch and will assumed to be zero. Since the tangential fields along the edges of the patch are very small, the corresponding equivalent electric current density $J_s$ will be very small and will assumed to be zero as well.
Thus the only non-zero current density is the equivalent magnetic current density $\mathbf{M}_s$ along the side periphery of the patch radiating in the presence of the ground plane. The presence of the ground plane can be taken into account by image theory, as shown in figure 7.3. The final current density on the sides of the patch is

$$\mathbf{M}_s = -2\hat{n}\times\mathbf{E}_a$$  \hspace{1cm} (7.3)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.2}
\caption{Equivalent current densities on the sides of a rectangular microstrip patch; (a) $\mathbf{J}_s, \mathbf{M}_s$ with ground plane; (b) $\mathbf{M}_s$ with no ground plane.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.3}
\caption{Equivalent models for magnetic source radiation near a perfect electric conductor; (a) $\mathbf{M}_s$ with groundplane; (b) $\mathbf{M}_s$ and image equivalent; (c) summation of the two sources.}
\end{figure}
In the transmission line model, the microstrip antenna is represented by two radiating slots along the length of the patch. While there are four slots in the cavity model, the obtained results are comparable, because only two of the slots are radiating. The fields of the other two slots have the same magnitude but opposite phase and cancel each other, as illustrated in figure 7.4.

![Figure 7.4](image)

**Figure 7.4** Current densities on the non-radiating slots of a rectangular patch antenna.

To obtain the radiated field of a microstrip patch, first the radiated fields of each slot are derived. Then the two radiated fields are superposed to obtain the complete radiation pattern of the antenna. To solve the problem, the vector potential method described in [16] (see Appendix A) is used.

The vector potential $A$ for an electric current source $J$ is given by

$$A = \frac{\mu}{4\pi} \iiint_V \mathbf{J} \frac{e^{-jkr}}{R} \, dv'$$

(7.4)

and the vector potential $F$ for a magnetic current source $M$ is given by

$$F = \frac{e}{4\pi} \iiint_V \mathbf{M} \frac{e^{-jkr}}{R} \, dv'$$

(7.5)
For the case of a patch antenna, integration must be performed over a surface \( s \). For the far field, we can approximate \( R \) by

\[
R \equiv r - r' \cos \psi \quad \text{for phase variations} \\
R \equiv r \quad \text{for amplitude variations}
\]

(7.6a) (7.6b)

Using this approximation, we can rewrite the vector potentials as

\[
A = \frac{\mu}{4\pi} \int_J \frac{e^{-jkr}}{R} ds' \equiv \frac{\mu e^{-jkr}}{4\pi r} N
\]

(7.7a)

with

\[
N = \int_J J_s e^{jkr'\cos \psi} ds'
\]

(7.7b)

and

\[
F = \frac{\varepsilon}{4\pi} \int_J M_s \frac{e^{-jkr}}{R} ds' \equiv \frac{\varepsilon e^{-jkr}}{4\pi r} L
\]

(7.8a)

with

\[
L = \int_J M_s e^{jkr'\cos \psi} ds'
\]

(7.8b)

As explained, the current densities of a rectangular aperture mounted on an infinite groundplane may be written as

\[
M_s = \begin{cases} 
-2\hat{n} \times E_s + \hat{n} \times 2E_0 & -a/2 \leq x' \leq a/2 \\
0 & -b/2 \leq y' \leq b/2 \\
& \text{elsewhere}
\end{cases}
\]

(7.9a)

\[
J_s = 0 \quad \text{everywhere}
\]

(7.9b)

If we substitute (7.9b) in equation (7.7), it can be seen that \( N = 0 \) everywhere and hence \( A = 0 \) everywhere. Substituting (7.9a) in equation (7.8) and using the integral
\[
\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} e^{i\alpha z} dz = \frac{\sin\left(\frac{\alpha}{2} c\right)}{\frac{\alpha}{2} c}
\]

results in

\[
L_\theta = 2abE_0 \cos \theta \cos \phi \left(\frac{\sin X}{X} \frac{\sin Y}{Y}\right)
\]

(7.11a)

and

\[
L_\phi = -2abE_0 \sin \phi \left(\frac{\sin X}{X} \frac{\sin Y}{Y}\right)
\]

(7.11b)

where

\[
X = \frac{ka}{2} \sin \theta \cos \phi
\]

and

\[
Y = \frac{kb}{2} \sin \theta \sin \phi.
\]

Combining (7.8), (7.9a) and (7.11) gives the field radiated by the aperture

\[
E_r = 0
\]

(7.12a)

\[
E_\theta = j \frac{abkE_0 e^{-jkr}}{2\pi r} \left[\sin \phi \left(\frac{\sin X}{X} \frac{\sin Y}{Y}\right)\right]
\]

(7.12b)

\[
E_\phi = j \frac{abkE_0 e^{-jkr}}{2\pi r} \left[\cos \theta \cos \phi \left(\frac{\sin X}{X} \frac{\sin Y}{Y}\right)\right]
\]

(7.12c)

The array factor for the two elements of the same magnitude and phase separated by a distance equal to the effective length of the patch \( L \) is

\[
AF = 2 \cos \left(\frac{kL}{2} \cos \theta\right).
\]

(7.13)

Thus, the total field for the two slots representing the microstrip antenna is:
• for the E-plane ($\theta = 90^\circ$, $0^\circ \leq \phi \leq 90^\circ$ and $270^\circ \leq \phi \leq 360^\circ$)

$$E_\theta = j \frac{abkE_0 e^{-jkr}}{\pi r} \left\{ \frac{\sin \left( \frac{k_0 h}{2} \cos \phi \right)}{\sin \left( \frac{k_0 h}{2} \cos \phi \right) \cos \left( \frac{kL}{2} \cos \theta \right)} \right\} (7.14)$$

• for the H-plane ($\phi = 0^\circ$, $0^\circ \leq \theta \leq 180^\circ$)

$$E_\phi = j \frac{abkE_0 e^{-jkr}}{\pi r} \left\{ \frac{\sin \left( \frac{k_0 h}{2} \sin \theta \right) \sin \left( \frac{k_0 h}{2} \cos \theta \right)}{\cos \left( \frac{kL}{2} \cos \theta \right)} \right\} (7.15)$$

7.3 ELEMENT WIDTH AND LENGTH

For a dielectric substrate of thickness $h$, an antenna operating frequency $f_r$, a practical value for the width of the patch is given in [16, 17, 18]:

$$W = \frac{c}{2f_r} \left[ \frac{\varepsilon_r + 1}{2} \right]^{\frac{1}{2}} (7.16)$$

Smaller widths result in a lower radiation efficiency, while for larger widths higher order modes cause field distortions. Because the dimensions of the patch are finite along the length and width, the fields at the edges of the patch undergo fringing. Because of this effect, electrically the patch of the microstrip antenna looks greater than its physical dimensions. This is illustrated in figure 7.5, where the dimensions of the patch along its length have been extended on each side by a distance $\Delta L$, which is a function of the effective dielectric constant $\varepsilon_{eff}$ and the width-to-height ratio $W/h$. An approximation for the length extension is given by

$$\Delta L = h \cdot 0.412 \left( \frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} \right) \left( \frac{W}{h} + 0.264 \right) \left( \frac{W}{h} + 0.8 \right) (7.17)$$
Because the dimensions of the patch are finite along the length and width, the fields at the edges of the patch undergo fringing. Because of this effect, electrically the patch of the microstrip antenna looks greater than its physical dimensions. This is illustrated in figure 7.5, where the dimensions of the patch along its length have been extended on each side by a distance $\Delta l$, which is a function of the effective dielectric constant $\varepsilon_{\text{eff}}$ and the width-to-height ratio $W/h$. An approximation for the length extension is given by

$$\Delta l = h \cdot 0.412 \left[ \frac{\varepsilon_{\text{eff}} + 0.3(W/h + 0.264)}{\varepsilon_{\text{eff}} - 0.258(W/h + 0.8)} \right]. \quad (7.17)$$

The effective dielectric constant, discussed in chapter 5, is approximated by

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ 1 + \frac{12h}{W} \right]^{\frac{1}{2}}. \quad (7.18)$$
Finally, for a patch of effective dielectric constant $\varepsilon_r$ and length extension $\Delta l$, the length will be

$$L = \frac{c}{2f_r \sqrt{\varepsilon_{\text{eff}}}} - 2\Delta l$$  \hspace{1cm} (7.19)$$

For the RT Duroid 5870 substrate we used (see table 5.1), and for an operating frequency of 14.0 GHz the following values are obtained: $W = 8.30$ mm, $L = 6.56$ mm, $\varepsilon_{\text{eff}} = 2.12$ and $\Delta l = 0.40$ mm.

### 7.4 ANTENNA ARRAYS

As said before, patch antenna array configurations must be employed in applications where reasonably high gain and narrow beamwidth are required. In a linear array, the antenna elements are located finite distances apart along a straight line. A linear array is called uniform when the elements are equally spaced. For practical reasons, the antenna elements are in most cases identical. An array of identical elements all of identical magnitude and each with a progressive phase is referred to as a uniform array. The array factor can be obtained by considering the elements to be point sources. If the actual elements are not isotropic sources, the total field can be formed by multiplying the array factor of the isotropic sources by the field of a single element. This is the pattern multiplication rule [16, 19], and it applies only for arrays of identical elements.

For an array of an even number of isotropic elements $2M$ (where $M$ is an integer), with the separation between the elements $d$ and $M$ elements placed on each side of the origin and assuming that the amplitude excitation is symmetrical about the origin, the array factor for a non-uniform amplitude array can be written as

$$AF_{2M} = \frac{a_1 e^{j(1/2)kd \cos \theta} + a_2 e^{j(3/2)kd \cos \theta} + \ldots + a_M e^{j(M-1/2)kd \cos \theta}}{a_1 e^{-j(1/2)kd \cos \theta} + a_2 e^{-j(3/2)kd \cos \theta} + \ldots + a_M e^{-j(M-1/2)kd \cos \theta}}$$  \hspace{1cm} (7.20)$$

or

$$AF_{2M} = 2 \sum_{n=1}^{M} a_n \cos \left( \frac{2n-1}{2} kd \cos \theta \right)$$  \hspace{1cm} (7.21)$$
where $a_n$'s are the excitation coefficients of the array elements. If the total number of isotropic elements in the array is odd, $2M + 1$ (where $M$ is an integer), the array factor can be written as

$$AF_{2M+1} = 2a_1 + a_2 e^{jkd \cos \theta} + a_3 e^{2jkd \cos \theta} + \ldots + a_{M+1} e^{jMkd \cos \theta}$$

or

$$AF_{2M+1} = 2 \sum_{n=1}^{M} a_n \cos((n-1)kd \cos \theta).$$

The amplitude of the centre element is $2a_1$.

The array factor is a function of the geometry of the array and the excitation phase. By varying the separation and/or the phase between the elements, the characteristics of the array factor and of the total field of the array can be controlled. The assumption made for pattern multiplication is that the current in each element is the same as that of the isolated element. However, this is usually not the case. Because of mutual coupling the antenna elements will influence each other. This effect is stronger if the elements are more closely spaced.

### 7.5 FEEDING NETWORKS

The microstrip antenna can be fed by a microstrip line or by a coaxial line. These two configurations are shown in figure 7.6. In the case of a coaxial feed, the connector is attached to the back of the substrate and the centre conductor is connected to through the substrate to the patch.

![Microstrip line](image)

**Figure 7.6(a) Patch with microstrip feed**
In our application however, a microstrip feed is the logical choice, because the microstrip mixer will be etched on the same substrate as the antenna. An array of microstrip patches can be fed by a single line, as shown in figure 7.7(a), or by multiple lines, shown in figure 7.7(b). The first is referred to as a series feed network while the second is referred to as a parallel or corporate feed network.

The series network has smaller line lengths than the parallel network and hence has a lower insertion loss. Furthermore, there is a lower number of discontinuities, resulting in a better control of the phase [20]. The corporate or parallel feed system occupies a larger area, but maintains equal path lengths from the input to the output ports. The antenna elements can be frequency scaled with the feed lines unchanged. We decided to use the parallel feed network. Parallel feed networks can take the form of an n-way power splitter or a combination of m-way
splitters. For our feeding network, it was decided to use two-way splitters. This minimises the ratio of maximum to minimum impedance required in the feed structure. In general, the input impedance of the patches differs from the impedance of the feed line, and a matching transformer is required. Usually matching is accomplished by using either tapered lines or quarter wavelength lines, as shown in figure 7.8.

Figure 7.8 Tapered and $\lambda/4$ impedance transformer lines to match 100 Ohm patches to a 50 Ohm line.

For the matching in our array we used quarter wavelength transformers. The input impedance of the patches was measured to be approximately 150 Ohms, so an extra quarter wavelength
A transformer of $\sqrt{100 \cdot 150} = 122\Omega$ was included just before the patch. The final version of the array is shown in figure 7.9.

Figure 7.9 Final version of the 8-patch uniform array

7.6 EXPERIMENTAL RESULTS

7.6.1 Basic parameters
To describe the performance of an antenna, definitions of various parameters are necessary. In this paragraph, the definitions for bandwidth, directivity and gain and for the 3 dB beamwidth are given. However, these and more parameters are described in detail in most standard textbooks, for instance [16].

- **Bandwidth**
The bandwidth of a microstrip antenna is usually defined as the frequency range over which the value of the input voltage standing wave ratio (VSWR) is smaller than a certain value S. Usually, $\text{VSWR} \leq 2$ is taken.

- **3 dB Beamwidth**
According to [16] the half power beamwidth is defined as: "In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half the maximum value of the beam."
• **Directivity and gain**

The antenna directivity $D$ may be defined as the ratio of the maximum radiation intensity (power density) at the peak of the main lobe to the radiation intensity of an isotropic radiator with the same radiated power. The power gain $G$ of an antenna in a given direction is defined as

$$G(\theta, \phi) = 4\pi \frac{P_{in}}{P_{rad}}. \quad (7.24)$$

Since the radiated power $P_{rad}$ is the input power minus the losses in the system, the relation between gain and directivity is given by

$$G(\theta, \phi) = \eta \cdot D(\theta, \phi), \quad (7.25)$$

where the efficiency $\eta$ is defined as the ratio of the total radiated power to the total input power.

### 7.6.2 Single patch

The experimental results of the measurements on the single patch are shown in appendix B. In figure B.9 the reflection coefficient $S_{11}$ is plotted as a function of frequency on a logarithmic scale. The reference level is 0 dB and the vertical scale is 10 dB per division. The horizontal scale is 400 MHz per division. The minimum value reached is -16.3 dB for a frequency of 13.4 GHz. The real and imaginary value of the input impedance can be seen in the Smith chart in figure B.10. The patch is not completely in resonance for the frequency of 13.56 GHz, since the impedance has an imaginary part of -4.3 Ohms. The matching could also be better, since a value of 36.4 Ohms is measured where 50 Ohms would be expected.

The VSWR is plotted as a function of frequency in figure B.11. The reference level is 1 and the vertical scale is 1 per division. The horizontal scale again is 400 MHz per division. It can be seen that the bandwidth (the frequency range where VSWR≤2) is approximately 600 MHz. The (non-normalised) far-field radiation patterns of the single patch are shown in figure B.12 for a frequency of 13.4 GHz. The co-polarization shows a broad main lobe and the cross-polarization level is uniform. The characteristics are rather messy, partly due to the losses in the measurement system. The main reason to include these patterns is to show the improvement in the radiation pattern when an antenna array is used.
7.6.3 8-Patch uniform array

The results for the 8-patch uniform array are shown in appendix B. In figure B.13 the reflection coefficient $S_1$ is plotted as a function of frequency on a logarithmic scale. The reference level is 0 dB and the vertical scale is 10 dB per division. The horizontal scale is 400 MHz per division. As can be seen, the reflection coefficient is lower for the array than for the single patch. The minimum value reached is -24.2 dB for a frequency of 13.48 GHz. The real and imaginary value of the input impedance can be seen in the Smith chart in figure B.14. The behaviour is more complicated than for the single patch, as could be expected. It can be seen that the array is in resonance for the frequency of 13.48 GHz. The matching could be better, but is close to 50 Ohms. In figure B.15, the VSWR is plotted as a function of frequency. The reference level is 1 and the vertical scale is 1 per division. The horizontal scale again is 400 MHz per division. It can be seen that the bandwidth is quite large: almost 1 GHz.

The current distribution on the patches was measured by probing the near field at the positions shown in figure B.16. The results are shown in figure B.17, where the phase and amplitude are plotted in a polar plot for a CW frequency of 13.8 GHz. Ideally, the phase and amplitude should be identical for all the patches. As can be seen, the phase difference between the patches is only very small, since all the measured values are in a small section of the circle. However, the amplitude distribution shows a different result. The patches 2, 3, 4, 5, 6 and 7 have almost equal amplitudes, but the patches 1, and 8 radiate considerably more power. This can be explained by the fact that, since the elements are relatively closely spaced, they will influence each other. The patches 1 and 8 have only one ‘nearest neighbour’ and therefore behave in a different way. The amplitudes and phase angles of the points shown in figure B.3 are listed in table B.1. For the single patch, an amplitude of -19.8 dB was measured. If we add the power radiated by the 8 patches, a value of -24.3 dB is obtained. Hence, it can be concluded that there is approximately 4.5 dB loss in the splitting network.

The far-field radiation patterns of the patch array are shown in figure B.18, B.19, B.20 and B.21 for frequencies of 13.4, 13.6, 13.6 and 13.8 GHz, respectively. The co-polarization shows that the width of the main lobe is much smaller than for the single patch, resulting in a higher gain. It can be seen that the array has about 8 dB more gain than the single patch. The 3 dB beamwidth is approximately 15 degrees. The sidelobe level is between -10 dB and -15 dB. The cross-polarization level is uniform and about -15 dB.
8 Conclusions and recommendations

8.1 CONCLUSIONS

The most important aim of the project was to make Ogden Safety Systems' anti-collision radar smaller. The approach chosen to achieve this, was to make part of the microwave assembly in microstrip. It was also hoped to reduce the level of background signal arising in the simple existing mixer design, which reduces the sensitivity to close-in reflections. It was decided to make a single balanced mixer in microstrip to replace the single diode mixer that is currently used. A single balanced mixer consists of two detecting diodes and a hybrid coupler. An additional advantage of a mixer circuit in microstrip is that low cost surface mounted diodes can be used. Alpha Industries, Ogden's supplier of diodes, recommended their SMS7621-006 type detector diode for this application.

There are two kinds of hybrid coupler, namely the 180° hybrid and the 90° hybrid. Both types are analysed in this report. Implementations of both types were designed, etched and measured, namely the ‘rat-race’ 180° hybrid, based on a ring with a circumference of 1.5 wavelengths and the branch line 90° hybrid, based on a ring with a circumference of one wavelength. The performance (isolation as well as balance in power splitting) of the rat-race was measured to be slightly better than the performance of the branch line coupler. However, the rat-race has the disadvantage that one of the input lines has to be crossed to bring the output lines together. The output ports of the branch line coupler are adjacent, making it easier to connect the diode package. For this reason, we decided to use the branch line hybrid. A testing circuit was made to measure the performance of the diodes. Besides the branch line coupler and the diode package, this circuit included radial stubs and DC-blocks to apply bias to the diodes. The diodes were found to be detecting. The maximum sensitivity of the diodes was measured to be at a bias of 0.9 Volt.

Another goal was to improve the radar front end in order to make the system suitable for use inside tanks and silos. An antenna with a low sidelobe level is desired for this application. An 8-patch uniform array was designed and tested. This linear array is only the first step in a design process. A two-dimensional array is probably needed to obtain a concentrated beam in more than one plane.
A parallel feeding network of 2-way power splitters is used to feed the array. Two-way power splitters minimise the ratio of maximum to minimum impedance required in the feed structure. A parallel network was chosen rather than a series network, because in the parallel network the distance from the feed point to the patches is the same for all the patches. Generally, this results in a wider bandwidth. However, the parallel network occupies a larger area than the series network, resulting in larger Ohmic losses. The feed network radiates part of the power as well, resulting in a deterioration of the antenna performance. Matching transformers are included in the feed network to match the input impedance of the patches to the impedance of the feed lines.

The bandwidth of the antenna was found to be nearly 1 GHz, which is sufficient for this application. The measured sidelobe level is between -10 dB and -15 dB. A lower sidelobe level can be achieved by tapering the amplitude distribution. At the moment the outer patches are radiating more power than the inner patches. This is due to the fact that the patches are relatively closely spaced and are influencing each other. The half power beamwidth was found to be approximately 15 degrees.

A microstrip-to-waveguide transition was made to connect the waveguide parts of the radar system to the microstrip parts. This transition uses a stepped quarter wavelength transformer designed according to the Chebyshev distribution. The transmission from the waveguide to the microstrip was found to be -3 dB. The reflection coefficient is less than -20 dB in the approved band for the radar system. A good connection between the groundplane of the microstrip substrate and the waveguide is very important. It was found that the performance of the transition depended on how far the dielectric was inserted under the last step of the transformer. This effect is not mentioned in the papers on this subject.

8.2 RECOMMENDATIONS FOR FURTHER WORK

The most effective direction in which the work can be taken would be to include the developed components into a complete front-end in which the background response has been further minimised by incorporating a balanced diplexer. Figure 8.1 shows a proposal for a complete layout in which the diplexer is realised by a second hybrid. A coupler has been included to provide a reference signal. Using a reciprocal network as a diplexer implies 6 dB signal reduction, but this system is not believed to be noise limited and the advantage of reducing the background signal would outweigh this. Further areas of improvement would be to realise harmonic rejection filters in microstrip, to develop an antenna with tapering to reduce sidelobes and possibly eventually to produce a microstrip version of the oscillator.
Figure 8.1 Proposal for a complete radar front-end containing the components developed in this project.


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## List of symbols

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<tr>
<th>Symbol</th>
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<tr>
<td>(A)</td>
<td>diode area, vector potential for electric current source</td>
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<tr>
<td>(A^*)</td>
<td>Richardson's constant</td>
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<tr>
<td>(c)</td>
<td>velocity of light</td>
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<tr>
<td>(C)</td>
<td>capacitance, coupling factor</td>
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<td>(C_N)</td>
<td>binomial coefficient</td>
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<td>directivity</td>
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<td>(J)</td>
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<td>(M)</td>
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<td>(n)</td>
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<td>(T)</td>
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<td>$T_n(x)$</td>
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<td>$\Gamma$</td>
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<tr>
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<td>$\lambda_\gamma$</td>
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<td>$\eta$</td>
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<td>$\pi$</td>
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<td>angle</td>
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<td>$\theta$</td>
<td>angle</td>
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<td>$\omega$</td>
<td>angular frequency</td>
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<tr>
<td>$\psi$</td>
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A Vector potentials [16]

While it is possible to calculate the \( \mathbf{E} \) and \( \mathbf{H} \) fields directly from source-current densities \( \mathbf{J} \) and \( \mathbf{M} \), it is common practice in the analysis of radiation problems to introduce auxiliary functions, known as vector potentials, which will aid the solution of the problems. The most common vector potential functions are the \( \mathbf{A} \), the magnetic vector potential, and \( \mathbf{F} \), the electric vector potential.

A.1 THE VECTOR POTENTIAL \( \mathbf{A} \) FOR AN ELECTRIC CURRENT SOURCE \( \mathbf{J} \)

The vector potential \( \mathbf{A} \) is useful in solving for the electromagnetic field generated by a harmonic electric current \( \mathbf{J} \). The magnetic flux \( \mathbf{B} \) is always solenoidal, that is, \( \nabla \cdot \mathbf{B} = 0 \). Therefore, it can be represented as the curl of another vector because it obeys the vector identity

\[
\nabla \cdot \nabla \times \mathbf{A} = 0
\]

(A.1)

where \( \mathbf{A} \) is an arbitrary vector. Thus we define

\[
\mathbf{B}_A = \mu \mathbf{H}_A = \nabla \times \mathbf{A}
\]

(A.2)

or

\[
\mathbf{H}_A = \frac{1}{\mu} \nabla \times \mathbf{A}
\]

(A.2a)

where subscript \( A \) indicates the field due to the \( \mathbf{A} \) potential. Substituting (A.2a) into Maxwell's curl equation

\[
\nabla \times \mathbf{E}_A = -j\omega\mu \mathbf{H}_A
\]

(A.3)

reduces it to

\[
\nabla \times \mathbf{E}_A = -j\omega\mu \mathbf{H}_A = -j\omega \nabla \times \mathbf{A}
\]

(A.4)
which can also be written as

\[ \nabla \times [E_A + j\omega A] = 0 \]  \hspace{1cm} (A.5)

From the vector identity

\[ \nabla \times (-\nabla \phi_e) = 0 \]  \hspace{1cm} (A.6)

and (A.5), it follows that

\[ E_A + j\omega A = -\nabla \phi_e \]  \hspace{1cm} (A.7)

or

\[ E_A = -\nabla \phi_e - j\omega A \]  \hspace{1cm} (A.7a)

The scalar function \( \phi_e \) represents an arbitrary electric scalar potential which is a function of position. Taking the curl of both sides of (A.2) and using the vector identity

\[ \nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \]  \hspace{1cm} (A.8)

reduces it to

\[ \nabla \times (\mu \mathbf{H}_A) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \]  \hspace{1cm} (A.8a)

For a homogeneous medium, (A.8a) reduces to

\[ \mu \nabla \times \mathbf{H}_A = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \]  \hspace{1cm} (A.9)

Equating Maxwell’s equation

\[ \nabla \times \mathbf{H}_A = \mathbf{J} + j\omega \epsilon \mathbf{E}_A \]  \hspace{1cm} (A.10)

to (A.9) leads to

\[ \mu \mathbf{J} + j\omega \mu \epsilon \mathbf{E}_A = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \]  \hspace{1cm} (A.11)
Substituting (A.7) into (A.11) reduces it to

\[ \nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{I} + \nabla(\nabla \cdot \mathbf{A}) + \nabla(\omega \mu \mathbf{e}_e) \]
\[ = -\mu \mathbf{I} + \nabla(\nabla \cdot \mathbf{A} + j\omega \mu \mathbf{e}_e) \]
\[ \text{(A.12)} \]

where \( k^2 = \omega^2 \mu \varepsilon \). In (A.2), the curl of \( \mathbf{A} \) was defined. Now we are at liberty to define the divergence of \( \mathbf{A} \), which is independent of its curl. In order to simplify (A.12), let

\[ \nabla \cdot \mathbf{A} = -j\omega \mu \mathbf{e}_e \Rightarrow \mathbf{\phi}_e = -\frac{1}{j\omega \mu \varepsilon} \nabla \cdot \mathbf{A} \]
\[ \text{(A.13)} \]

which is known as the Lorentz condition. Substituting (A.13) into (A.12) leads to

\[ \nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{I} \]
\[ \text{(A.14)} \]

In addition, (A.7a) reduces to

\[ \mathbf{E}_A = -\nabla \mathbf{\phi}_e - j\omega \mathbf{A} - j\frac{1}{\omega \mu \varepsilon} \nabla(\nabla \cdot \mathbf{A}) \]
\[ \text{(A.15)} \]

A solution to the inhomogeneous Helmholtz equation of (A.14) is

\[ \mathbf{A} = \frac{\mu}{4\pi} \iiint \mathbf{J} e^{-jkr} \frac{1}{R} dv' \]
\[ \text{(A.16)} \]

Once \( \mathbf{A} \) is known, \( \mathbf{H}_A \) can be found from (A.3a) and \( \mathbf{E}_A \) from (A.15). \( \mathbf{E}_A \) can also be found from Maxwell's equation (A.10) with \( \mathbf{J} = 0 \).

**A.2 THE VECTOR POTENTIAL F FOR A MAGNETIC CURRENT SOURCE M**

The fields generated by a harmonic magnetic current in a homogeneous region, with \( \mathbf{J} = 0 \) but \( \mathbf{M} \neq 0 \), must satisfy \( \nabla \cdot \mathbf{D} = 0 \). Therefore, \( \mathbf{E}_F \) can be expressed as the curl of the vector potential \( \mathbf{F} \) by
\[ E_F = -\frac{1}{\epsilon} \nabla \times F \]  
(A.17)

Substituting (A.17) into Maxwell’s curl equation

\[ \nabla \times H_F = j \omega \epsilon E_F \]  
(A.18)

reduces it to

\[ \nabla \times (H_F + j \omega F) = 0 \]  
(A.19)

From the vector identity of (A.6), it follows that

\[ H_F = -\nabla \phi_m - j \omega F \]  
(A.20)

where \( \phi_m \) represents an arbitrary magnetic scalar potential which is a function of position.

Taking the curl of (A.17)

\[ \nabla \times E_F = -\frac{1}{\epsilon} \nabla \times \nabla \times F = -\frac{1}{\epsilon} \left[ \nabla \nabla \cdot F - \nabla^2 F \right] \]  
(A.21)

and equating it to Maxwell’s equation

\[ \nabla \times E_F = -M - j \omega \mu H_F \]  
(A.22)

leads to

\[ \nabla^2 F + j \omega \mu \epsilon H_F = \nabla \nabla \cdot F - \epsilon M \]  
(A.23)

substituting (A.20) into (A.23) reduces it to

\[ \nabla^2 F + k^2 F = -\epsilon M + \nabla (\nabla \cdot F) + \nabla \left( j \omega \mu \epsilon \phi_m \right) \]  
(A.24)

By letting
\[ \nabla \cdot \mathbf{F} = -j \omega \mu \varepsilon \phi_m \Rightarrow \phi_m = -\frac{1}{j \omega \mu \varepsilon} \nabla \cdot \mathbf{F} \quad (A.25) \]

reduces (A.24) to

\[ \nabla^2 \mathbf{F} + k^2 \mathbf{F} = -\varepsilon \mathbf{M} \quad (A.26) \]

and (A.20) to

\[ \mathbf{H}_F = -j \omega \mathbf{F} - \frac{j}{\omega \mu \varepsilon} \nabla (\nabla \cdot \mathbf{F}) \quad (A.27) \]

A solution to the inhomogeneous Helmholtz equation of (A.26) is

\[ \mathbf{F} = \frac{\varepsilon}{4\pi} \iiint_{\nu} \mathbf{M} \frac{e^{-jkR}}{R} d\nu' \quad (A.28) \]

Once \( \mathbf{F} \) is known, \( \mathbf{E}_F \) can be found from (A.17) and \( \mathbf{H}_F \) from (A.27) or (A.22) with \( \mathbf{M} = 0 \).
Measurement results

All the measurements were made using a Hewlett Packard HP8510C Network Analyser System, comprised of:

- HP8515A S-parameter test set (45 MHz - 26.5 GHz.),
- HP83631B synthesized sweeper (10 MHz - 26.5 GHz.)
Figure B.1  Transmission coefficient of the microstrip-to-waveguide transition as a function of frequency.
Figure B.3 Reflection coefficient and transmission coefficient of the ratrace hybrid in the measurement configuration of figure 6.5(a).
Figure B.4 Reflection coefficient and transmission coefficient of the ratrace hybrid in the measurement configuration of figure 6.5(b).
Figure B.5   Reflection coefficient and transmission coefficient of the ratrace hybrid in the measurement configuration of figure 6.5(c).
Figure B.6  Reflection coefficient and transmission coefficient of the branch line hybrid in the measurement configuration of figure 6.8(a).
Figure B.7  Reflection coefficient and transmission coefficient of the branch line hybrid in the measurement configuration of figure 6.8(b).
Figure B.8  Reflection coefficient and transmission coefficient of the branch line hybrid in the measurement configuration of figure 6.8(c).
Figure B.9  Reflection coefficient of the single patch as a function of frequency.
Figure B.10 Smith chart of the input impedance of the single patch as a function of frequency.
Figure B.11
VSWR of the single patch as a function of frequency.

**MARKER 1**
13.4 GHz
point 71

START
12.0000000000 GHz

STOP
16.0000000000 GHz

23 JUL 98
10:09:06
Figure B.12  Far-field radiation patterns of the single patch at 13.4 GHz; (a) co-polar pattern; (b) cross-polar pattern.
Figure B.13  Reflection coefficient of the uniform array as a function of frequency.
Figure B.14 Smith chart of the input impedance of the uniform array as a function of frequency.
Figure B.15

VSWR of the uniform array as a function of frequency.

START
12.0000000000 GHz

MARKER 1
13.46 GHz
1.1274

point 74

STOP
16.0000000000 GHz
23 JUL 98
10:05:20

APPENDIX B
Table B.1  Measured values for the current distribution on the patches of the array

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<th>$S_{ii}$ Amplitude</th>
<th>$S_{ii}$ Angle</th>
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<tr>
<td>1</td>
<td>-30.8 dB</td>
<td>-152°</td>
</tr>
<tr>
<td>2</td>
<td>-34.0 dB</td>
<td>-146°</td>
</tr>
<tr>
<td>3</td>
<td>-34.6 dB</td>
<td>-158°</td>
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<tr>
<td>4</td>
<td>-34.7 dB</td>
<td>-148°</td>
</tr>
<tr>
<td>5</td>
<td>-34.5 dB</td>
<td>-146°</td>
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<tr>
<td>6</td>
<td>-35.4 dB</td>
<td>-151°</td>
</tr>
<tr>
<td>7</td>
<td>-35.1 dB</td>
<td>-145°</td>
</tr>
<tr>
<td>8</td>
<td>-30.7 dB</td>
<td>-135°</td>
</tr>
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</table>

Figure B.16  Positions for the current distribution measurement
Figure B.17
Polar plot of the near-field measurement of the uniform array.

\[ \text{P.S.} \]
\[ \text{REF} \ 35.0 \ \text{mUnits} \]
\[ \text{1} \ 7.0 \ \text{mUnits} \]
\[ \downarrow \ -30.764 \ \text{dB} \ -151.94^\circ \]
\[ \ \text{hp} \]

\[ \text{SCALE:} \ 7.0 \ \text{mUnits/div} \]

\[ \text{C.W.} \ 13.800000000 \ \text{GHz} \]

\[ \text{MARKER 1} \]
\[ 13.8 \ \text{GHz} \]
\[ -30.764 \ \text{dB} \]
\[ -151.94^\circ \]

22 JUL 98
14:30:09
Figure B.18  Far-field radiation patterns of the 8-patch uniform array at 13.4 GHz; (a) co-polar pattern; (b) cross-polar pattern.
Figure B.19  Far-field radiation patterns of the 8-patch uniform array at 13.6 GHz; (a) co-polar pattern; (b) cross-polar pattern.
Figure B.20  Far-field radiation patterns of the 8-patch uniform array at 13.8 GHz; 
(a) co-polar pattern; (b) cross-polar pattern.
Figure B.21  Far-field radiation patterns of the 8-patch uniform array at 14.0 GHz; (a) co-polar pattern; (b) cross-polar pattern.