The onset of liquid loading in inclined tubes

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Abstract

In this investigation the effect of tube inclination on the process of liquid loading has been studied. Knowledge of the effect of inclination is needed to optimize production from deviated wet gas wells and to be able to fully benefit from downhole processing.

By means of experiments performed in an inclinable tube the reversal of the liquid flow direction and the effect of inclination on this process have been studied. The experiments revealed that reversal of the liquid flow direction takes place over a range of gas flow rates, which has been called the reversal zone. This reversal zone is shown to be affected by tube inclination. By inclining the tube from the vertical, the reversal zone appeared to shift to higher gas flow rates. At inclination angles larger than 45°, however, this trend reverses. Furthermore it has been observed that with inclination of the tube the reversal zone becomes smaller.

The traditional Turner criterion was found to give a satisfactory prediction of the onset of liquid loading in vertical tubes. However, the criterion can not accurately predict liquid loading in inclined tube flow.

An alternative model is proposed that accounts for liquid transport both through a film along the tube wall and through drops entrained in the gas core. Liquid loading is now related to the stability of the wall liquid film, taking into account the liquid mass transfer between film and droplets. The model has been validated for vertical pipe flow and also shows good agreement with the experimental observation. In principle this model can be extended to inclined tube flow.
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Chapter 1

Introduction

Gas wells are underground reservoirs that contain gas phase hydrocarbons, usually at high pressure. To transport these gases to the surface a tube is connected to the reservoir. Traditionally the tube is vertical, but since the introduction of deviated well drilling some years ago several wells exist where the tube is inclined.

The bottom part of the tube is perforated and penetrates into the reservoir. Because the pressure in the reservoir is higher than the pressure at the surface the gas enters the tube through the perforations and flows to the surface. Producing gas from the well reduces the pressure in the reservoir. Consequently, the pressure gradient over the tube decreases, which in turn causes a decrease in the gas flow rate.

Wet gas wells contain liquid phase material as well as the gas phase hydrocarbons. This liquid can be water or liquid hydrocarbons from condensation of the gas. The amount of liquid in gas wells is generally low: typically $10^{-3}\%$ of the volume. If the velocity of the gas is sufficient it transports the liquid phase material to the surface.

Because of a phenomenon called liquid loading less gas is produced from a wet gas well compared to the potential production from the well if it were dry: In a depleted wet gas well the gas flow rate may not be sufficient to transport the liquid to the surface. In that case the liquid accumulates downhole. This downhole accumulation of liquid is called liquid loading.

The accumulated liquid blocks off the perforations, so it has to leave the tube before further production of the gas is possible. It is even possible that the production of gas is inhibited completely. In that case the well is said to be killed by the accumulated liquid. Liquid loading is one of the major problems that are encountered during the continued production of depleted wet gas wells.

Liquid loading can be postponed, although it can not be avoided. A common drawback of the methods used to postpone liquid loading is the fact that less gas is produced daily when such a method is applied. One of the methods used for that purpose, the installation of velocity strings, is described in the next section. Thereby liquid loading is postponed, but for economical reasons a high rate of gas production should be maintained as long as possible. To optimize production from a wet gas well a reliable prediction of the onset of liquid loading is needed.
Various models have been proposed to predict the onset of liquid loading, but none of them has been developed for inclined pipe flow. Either explicitly or implicitly the models are based on vertical tubes. To be able to predict the onset of liquid loading in deviated wells it is necessary to study how liquid loading is affected by inclination of the tube.

It is the objective of this investigation to determine the effect of tube inclination on liquid loading. By means of experiments performed in an inclinable tube the process of liquid loading and the effect of inclination on this process are studied. Also an attempt has been made to develop a model based on descriptions of film flow and droplet flow, which can be used to predict the onset of liquid loading in an inclined tube.

The investigation has been initiated by Shell International, Research and Technical Services and it has been carried out in cooperation with the Eindhoven University of Technology as a graduation project for the faculty of applied physics. The project has been sponsored by NAM.

This report consists of four parts. The first part is an introduction. The impact of liquid loading on the production of gas from a wet gas well will be illustrated and a survey will be given on the literature. Also the differences that may be expected when the tube is inclined will be discussed. In the second part the experimental part of this investigation will be discussed and the results of the experiments will be presented. In the third part the model will be presented and it will be compared to the experimental results. The report finishes with a general discussion and some recommendations will be given for further studies.

Figure 1.1: Performance plot for a wet gas well and a dry gas well

1.1 The impact of liquid loading

The effect of liquid loading on the production from a gas well is clearly illustrated by the gas well performance plot of a wet gas well and that of a dry gas well, shown in figure 1.1. For (hypothetical) gas wells these plots show the gas production rate as a function of the pressure in the reservoir.

As has been mentioned in the introduction, the production of gas from the reservoir to the surface causes the reservoir pressure to decrease. The dry gas-curve in the performance plot shows that when the reservoir pressure decreases, the production rate of a dry gas well
Figure 1.2: Performance plot for a wet gas well producing gas through a 7” or a 3.5” tube decreases gradually until it reaches zero. At this point the production of gas stops because the pressure drop over the tube is equal to the hydrostatic pressure.

At a high reservoir pressure the gas production rate of a wet gas well is only slightly lower than that of a dry gas well. Initially the wet gas-curve shows a gradual decrease, like the dry gas-curve, but below a certain reservoir pressure the wet gas-curve suddenly drops to zero. When no gas is produced to the surface anymore, the reservoir still contains gas which would have been produced if the well were dry. The well is said to be killed.

Nowadays methods are used which enable further production of this spare gas. One of these methods is the use of so-called velocity strings. A velocity string is a smaller diameter tube that is inserted into the original tube. The principle this method is based on is illustrated by figure 1.2, which shows the wet gas-curve for both a 7” and a 3.5” tube. It shows that when a 3.5” tube is used liquid loading occurs at a lower reservoir pressure, which means that more gas has yet been produced from the well. However, figure 1.2 shows another effect of using a velocity string: When a smaller diameter tube is used the rate of gas production is significantly lower. Economical considerations therefore require that the installation of a velocity string is postponed until liquid loading occurs, but not until the well is killed. This requires an accurate prediction of the onset of liquid loading.

1.2 Literature survey

Through a lot of research over several decades a considerable knowledge has been gained on the subject of liquid flow reversal in wet gas wells. It is generally agreed that the prevalent flow pattern in a vertical wet gas well is annular dispersed flow. Annular dispersed flow is characterised by a liquid film flowing along the tube wall and liquid drops entrained in the gas that is flowing near the centre of the tube. The liquid in the film and the drops experience an interfacial shear stress that is exerted by the gas flow. As a result of this shear stress the liquid flows upward against gravity. In general the upward velocity of the liquid in the film is much lower than the velocity of the drops and the gas.

The prevalence of this flow pattern was affirmed by a video-recording from a camera that was lowered into a gas well at liquid loading conditions. This recording showed distinctly the existence of a liquid film and a mist of drops. Based on this flow pattern models have been developed for the prediction of liquid loading, some of which are mentioned below.
1.2.1 The Turner criterion

The Turner criterion is probably the best known prediction of the onset of liquid loading. It was presented in 1969 and even today it is used for most of the existing gas wells. In this section the investigation of Turner et al [THD69], which led to the criterion will be discussed shortly.

It was recognised that liquid is transported as a liquid film and as drops. Although there is a continuous exchange of liquid between the film and the drops, the transport mechanisms were treated separately. For each mechanism a model was developed and the separate models were compared with experimental data. From an analysis of these two models it was concluded that the movement of liquid drops is the controlling mechanism for the removal of liquid from gas wells. Furthermore it was found that the minimum flow conditions for liquid removal are those that will provide a gas velocity sufficient to remove the largest drop that can exist.

This is the principle the Turner criterion is based on. The required velocity is calculated using drop break-up and particle mechanics, which is described in more detail in section 3.5.1, and leads to the following equation:

$$V_g = \left( \frac{40 (\rho_l - \rho_g) g \sigma}{C_D \rho_g^2} \right)^{\frac{1}{4}}. \quad (1.1)$$

This equation has been compared with field data, which showed that an upward adjustment of approximately 20 percent is needed to ensure the removal of all liquid from a well. This yields the following equation:

$$V_g T = k_T \left( \frac{(\rho_l - \rho_g) g \sigma}{C_D \rho_g^2} \right)^{\frac{1}{4}}, \quad (1.2)$$

with the constant $k_T = 2.91$. The Turner criterion states that the minimum gas velocity that is needed for the continuous removal of liquid from a gas well is given by equation 1.2.

1.2.2 Further research on liquid flow reversal

With their study Turner et al provided a method to predict the onset of liquid loading which was not entirely empirical. As described in the previous paragraph it was based on a physical analysis of the movement of liquid drops. However, the conditions for the onset of liquid loading according to Turner do not account for tube inclination nor for the water/gas ratio. Therefore the accuracy of the prediction is not sufficient to postpone the killing of inclined wet gas wells optimally nor is it sufficient to fully benefit from so-called downhole processing, such as gas dewatering and gas compression at the bottom of the tube (downhole).

From video-recordings and numerical simulations [P96] it became clear that at the onset of liquid loading the drops were still flowing upward, whilst the liquid in the film flowed downward. In contradiction to the mechanism advocated by Turner, this suggests that liquid loading is initiated by reversal of the flow direction of the film, rather than the drops.
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Other investigators have concentrated on the movement of the liquid in the film. Some considered the process of liquid loading [II81], but special interest was given to the subject of flooding [H96, R81, JN96]. Flooding is the opposite of the process that causes liquid loading: Liquid loading is a result of the transition from total upward liquid transport to downward liquid flow, while flooding is the transition from total downward liquid flow to the occurrence of upward liquid transport. In this investigation the onset of liquid loading is defined as the minimum gas flow rate resulting in total upward liquid transport. The flooding point is defined as the maximum gas flow rate without any upward flowing liquid. In the literature the difference between the onset of liquid loading and the flooding point is not very explicit, but between these two characteristic gas flow rates a range of flow rates exists where part of the liquid is transported upward and part flows downward. This range of gas flow rates is called the liquid flow reversal zone, or simply: reversal zone.

The analyses of liquid film movement provided methods to predict the location of the reversal zone which take the water/gas ratio into account. However, a consistent difference with the predictions of the Turner criterion did not come forth. Furthermore, the influence of tube inclination remained unattended.

The contribution of Oudeman et al (1990) to the knowledge on liquid loading and well killing is worth to be mentioned [O90]. By carrying out specially designed field tests in gas wells all over the world they collected a vast amount of data concerning the critical gas flow rate for liquid loading and the period over which wells load up, until they are killed. These data have been analysed to identify which of the existing methods for predicting liquid loading were most accurate and to improve these if necessary.

1.3 Inclination

The expected effect on liquid flow reversal of tube inclination will be discussed in this section. The main effect of tube inclination is the change of the direction of the tube axis relative to the direction of gravity. In this section two considerations will be given on the way liquid flow reversal is influenced by this effect.

Regarding the forces that are acting on the liquid in axial direction, liquid flow reversal should take place at ever lower gas flow rates when the angle of inclination from the vertical increases. Primarily the liquid in the tube experiences two forces: shear stress exerted by the gas flow in upward direction and a downward force due to gravity. When the tube is inclined the component of gravity along the tube axis, which is the opposing force to the shear stress, becomes smaller. Therefore a lower shear stress will be needed to overcome the gravitational force and "blow" the liquid out of the tube. This consideration suggests that in inclined tubes liquid flow reversal will take place at lower gas flow rates.

Considering the effect of inclination on the flow pattern, liquid flow reversal should occur at higher gas flow rates if the tube is inclined. In a vertical tube the distribution of the thickness of the liquid film is uniform, because the forces acting on the liquid are directed along the tube axis and this symmetry is reflected in the distribution of liquid around the tube. In an inclined tube the distribution of the liquid film will not be uniform. Gravity
is no longer acting axially and this causes the liquid to accumulate at the bottom side of the tube. With inclination of the tube the film thickness at the bottom of the tube will grow and at the top of the tube it will become thinner [V96]. The shear stress that is exerted by the gas acts on the interface between the liquid film and the gas and the liquid in the film is dragged along as a result of the viscosity of the liquid. Therefore a thin film is transported upward more easily. Inclination of the tube causes the film to grow at the lower side of the tube. This means that a higher shear stress is needed for the upward transport of the entire film. Therefore liquid flow reversal may be expected to occur at higher gas flow rates when the tube is inclined.

The experiments described in the next chapter of this thesis are carried out to determine the effect of inclination on liquid flow reversal. The above considerations each lead to an expectation concerning this effect. (Un-)fortunately these expectations are contradictory: The first consideration suggests that liquid flow reversal in inclined tubes will take place at lower gas flow rates, while the second consideration suggests that it will take place at higher gas flow rates.
Chapter 2

Experimental work

At Shell Research and Technical Services in Rijswijk, The Netherlands, an experimental setup has been built, which has been used to study air-water flow in an inclined tube. Special interest was given to the conditions at which the direction of the liquid flow reverses and to the influence of inclination on these conditions.
In this chapter the experimental setup will be described, as well as the experiments that have been performed. Following a description of the equipment the measuring methods and the results of the measurements will be discussed.

2.1 The equipment

The equipment that has been used in this investigation is shown schematically in figure 2.1. The equipment consists of a 12m long cylindrical test section with an inner diameter of 50mm, into which air and water are injected separately. The test section has been constructed of six perspex segments, each having a length of 2m, which have been connected by flanges. The test section has been attached to a rig which can be inclined to any angle between horizontal and vertical. At the top the test section is open to the atmosphere, at the bottom air is injected axially and at 4m downstream of the locus of gas injection water is injected through a porous section of the tube wall. The air leaves the test section at the top, the liquid may leave the test section either at the top or at the bottom.
At the top of the test section a facility has been built to collect the liquid when it is transported upwards. This facility consists of a bucket which is placed on top of the test section and a funnel, which has been mounted upside down in the bucket, as indicated in figure 2.1. Liquid that flows along the wall of the test section flows directly into the bucket. Liquid drops are blown into the funnel and flow along the inside of the funnel towards its brim, where they drip into the bucket. The bucket is drained and the liquid is collected in another bucket which is placed on a scale for weighing.
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Figure 2.1: The setup
The gas section  The air is injected through a specially designed bottom flange, shown in figure 2.2. The air flows through the nozzle at the centre of the flange, which has an inner diameter of 27mm. Before the gas enters the test section, it is transported through a 36mm i.d. flexible tube between the gas supply and the test section. At the gas supply the gas flow rate is controlled using an adjustable valve. As shown in figure 2.1, downstream of the valve the flow rate, the pressure and the temperature of the gas are measured. Details of the measuring instruments are discussed in section 2.2. Besides injection of the air the bottom flange allows liquid to drain from the tube. The air-injection nozzle penetrates 10cm into the test section, so liquid that is flowing downwards can accumulate between the nozzle and the tube wall. Through a hole in the flange the accumulated liquid drains from the tube towards a reservoir. The flange has been mounted in such way that the hole is at the lowest point of the tube, even if the tube is inclined. To enable undisturbed liquid drainage the reservoir should be vented, but care should be taken not to loose any of the gas. Therefore the vented air from the reservoir is re-injected into the test section opposite to the liquid drainage hole.

Physical properties of the gas  The air that has been used in the experiments was supplied at a pressure of 8bar. Since the test section is open to the atmosphere the conditions in the test section are assumed to be atmospheric. The mass density and the dynamic viscosity of the air at atmospheric pressure and temperature ($p_0 = 1.01bar$ and $T_0 = 293K$) are $\rho_g = 1.21kg/m^3$ and $\mu_g = 1.82 \cdot 10^{-5} Pas$.

The liquid section  The water is injected through a porous section of the tube wall, in order to create a liquid film and initially not too many drops. The injection section that has been designed for this purpose is shown in figure 2.3. The water is injected in an annulus on the outside of the porous tube wall and soaks into the test section. In order to obtain an equally distributed film thickness the water enters the annulus through four holes that are equally spaced around the tube.
Figure 2.3: The liquid injection section

Before the water enters the injection section it is transported through a flexible hose. The liquid flow rate is controlled with an adjustable valve. Immediately upstream of this adjustable valve a ball valve has been placed, which can be used to switch the liquid flow on and off without changing the flow rate setting. Upstream of the valves is a flow meter. Technical details of the flow meter will be discussed in section 2.2.

2.2 Measurements

In this section the measurements and the measuring method will be discussed. The location of the measuring instruments is indicated in figure 2.1. The most essential measurements are the gas and the liquid flow rates. These will be discussed first. Thereafter the additional measurements will be discussed, which are mainly used for monitoring. Having discussed all measurements the measuring method is discussed.

The gas flow rate A Foxboro E83W vortex flowmeter is used to measure the volumetric gas flow rate. Since the gas is a compressible medium its volume, and consequently the volume flow, depends on the pressure and the temperature. An increase of the temperature or a decrease of the pressure causes the volume to expand and the volume flow rate to increase. Because the flowmeter is not calibrated for changes in pressure and temperature, the temperature and the pressure are measured as well, to enable correction for the volume expansion. The pressure at the location of the flowmeter is measured with a Translnstruments pressure transmitter. The temperature is measured using a TempControl temperature transmitter. For the correction the following relation has been used:

\[ Q_g = \frac{p_m T_0}{p_0 T_m} Q_{g,m}, \]  \hspace{1cm} (2.1)

where the subscript \( m \) indicates measured values; \( p_0 \) and \( T_0 \) are the atmospheric pressure and temperature.
**Experimental work**

The liquid flow rates The injected liquid flow rate, $m_l$, has been measured using a RS turbine flow sensor. This sensor measures the volume flow of the liquid. Because water is an incompressible medium changes in pressure and temperature do not affect the flow rate, so correction is not necessary.

The flow rates of liquid that is flowing out of the test section have not been measured directly. The liquid which drains from the tube through the bottom flange is collected in a reservoir, which is marked with a level indication. The flow rate of the drainage flow, $m_{ld}$, is determined from the increase of the liquid level in a certain period of time.

The liquid which is collected in the funnel at the top of the test section is transported to a bucket which is placed on a scale. The increase of the total mass on the scale in a period of time is used to calculate the rate of liquid flowing out of the test section at the top, $m_{lu}$. This measurement is used as a consistency check since the sum of the liquid flow rates out of the test section should be equal to the injected liquid flow rate.

**Additional measurements** In the test section two STS-ATM pressure devices have been installed, which have been used for monitoring. One of the pressure devices is placed in a small pipe branching off the test section at the bottom immediately below the brim of the air injection nozzle, the other is placed at the liquid injection section, upstream of the inlet to the annulus.

The sensor at the liquid injection site is placed at the outside of the porous tube wall, so if liquid is flowing into the test section the measured pressure exceeds the pressure in the tube. No information is available on the pressure drop over the porous material, so the quantitative output of this sensor is of limited value.

The sensor at the bottom of the test section is placed in a blind pipe branching off the test section. The liquid accumulated between the tube wall and the air-injection nozzle is likely to accumulate in this pipe branch, which might cause pressure deviations.

Although the quantitative output of both pressure devices is limited, the qualitative information appeared to be useful as an indication of the pressure drop over the tube. In the next section it will be discussed how the behaviour of the pressure drop has been used as an indication of stability of the flow.

### 2.2.1 Measuring method

Having discussed all measuring devices and their location, this section will concentrate on the measuring method.

All experiments have been performed at a fixed angle of inclination and at a constant liquid flow rate. Each measuring session started at a high gas flow rate. In that way the stabilisation of the flow appeared to be faster. Initially the gas flow rate was such that all of the injected liquid was transported upwards and left the test section at the top. During the experiment the gas flow rate was decreased until liquid left the test section both at the top and at the bottom. When that situation was reached several measurements were taken. Between these measurements the gas flow rate was varied with very small differences, both towards higher and lower flow rates. Finally the gas flow rate is decreased until all of the injected liquid left the test section through the bottom flange.

When the gas flow rate is varied, a new equilibrium has to be established before the next measurement can be taken. The pressure drop over the test section is an indication of the
stability of the flow. A change in the gas flow rate causes a change in the pressure drop over the tube: a higher gas flow rate causes a higher pressure drop. The pressure drop is also affected by the presence of liquid in the tube: the more liquid exists in the tube, the higher the pressure drop will be.

As has been told before, the pressure devices in the test section reflect the behaviour of the pressure drop over the tube. The flow is assumed to be stable when the output of these pressure devices is approximately constant. Reaching stability after a change in the gas flow rate lasted typically up to 15 min.

Two considerations might explain why it takes time to reach stability. Entering the reversal zone the tube section below the locus of liquid injection has to be wetted. Near the onset of liquid loading sagging of liquid is slow. The liquid needed to establish the required film thickness is injected at a rather low flow rate (typically \( m_l = 0.7 l/min \) during the experiments). Also within the reversal zone and in case of total upward flow the volume of liquid that is present in the tube strongly depends on the gas flow rate. After a change of the gas flow rate either the liquid volume has to be replenished by the \( 0.7 l/min \) water injection, or the superfluous water has to leave the test section, which is only possible through transportation by the gas or by drainage through the bottom flange.

### 2.2.2 Data acquisition

The measured data are collected and analysed in a computer. The analog signals from the measuring instruments are converted into digital signals in an a/d convertor (adc). A computer monitors the outputs of the adc and the measured data are stored at intervals of 1 s. The stored data are imported into EXCEL for analysis.

The level of the liquid in the reservoir and the output of the scale could not be monitored by the computer, because these measurements are taken visually. These measurements are taken at intervals varying between 30 s and 10 min, but only when the flow is assumed to be stable. The values had to be added to the stored data afterwards.

After the reservoir level and the scale output have been added, the flow rate of the liquid in downward direction is calculated. The outputs of the adc are averaged over the corresponding interval between two measurements.

### 2.3 Results

The experimental setup has been described and the measurements and the measuring method have been discussed. The remainder of this chapter deals with the results which were obtained from the various measurements. It was the objective of the experiments to determine the liquid flow reversal zone and to study the effect of inclination on this reversal zone.

In section 1.2.2 the liquid flow reversal zone (or simply: reversal zone) has been defined as the interval of gas flow rates between the flooding point and the onset of liquid loading, where the liquid flow reverses its direction.

The experimental data will be presented in flow reversal plots: At various values of the superficial gas velocity the flow reversal plot shows the percentage of the injected liquid that is flowing downward. Only in the reversal zone this percentage \( \frac{m_{\text{down}}}{m_l} \) is between 0% and 100%. 

In section 2.3.1 the results of the experiments in a vertical tube will be presented as a typical example of the liquid flow reversal zone. In section 2.3.2 it will be discussed how the flow reversal zone is affected by inclination of the tube.

Figure 2.4: Liquid flow reversal plot for vertical tube

2.3.1 Liquid flow reversal in a vertical tube

A typical example of a flow reversal plot is shown in figure 2.4. The data in figure 2.4 are obtained from experiments in a vertical tube into which liquid is injected at a constant flow rate of $m_l = 0.71/\text{min}$.

The horizontal error bars indicate the standard deviation of the gas flow rate during the experiments. The variation in the gas flow rate is a result of the (in-)accuracy of the flow control. The vertical error bars indicate the observation-uncertainty (the level indication on the reservoir, calibration, time registration).

Irrespective of the size of the error bars the data in the flow reversal zone appear approximately on a straight line. The solid line in figure 2.4 is a straight line fit through these data and it may be observed that this line lies inside the uncertainty limits of all data.

The solid line reaches $\frac{m_{ld}}{m_l} = 100\%$ at a flow rate of $V_{sg} = 12.1 \pm 0.2 \text{m/s}$. This flow rate is identified as the (de-)flooding point in the vertical tube. At lower gas flow rates the tube above the liquid injection section remained dry, since all of the liquid flowed downward.

At a gas flow rate of $V_{sg} = 14.0 \pm 0.1 \text{m/s}$ the solid line reaches $\frac{m_{ld}}{m_l} = 0\%$. This flow rate is identified as the onset of liquid loading. At higher gas flow rates the tube between the air injection and the liquid injection remained dry.

The data in figure 2.4 may be compared to the Turner criterion, which has been discussed in chapter 1. For air-water flow in a 50mm tube the Turner criterion predicts the onset of liquid loading at a gas velocity of $V_{gr} = 14 \text{m/s}$, which is consistent with the experimental data.
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2.3.2 The effect of inclination

With the test section inclined to angles up to 60 degrees from the vertical similar experiments have been performed. The flow reversal plots obtained from these experiments are shown in figure 2.5. The data in figure 2.5(a) are obtained at inclination angles of 0°, 15°, 30° and 45°, figure 2.5(b) shows the data obtained at inclination angles of 45°, 52° and 60°. For a few data error bars are displayed. The plot would be blurred too much if all error bars were included.

The data in figure 2.5 show that the reversal zone shifts towards higher gas flow rates when the angle of inclination is increased up to 45°, but when the inclination angle is increased beyond 45° the reversal zone appears to return to lower gas flow rates. Another effect that may be observed is a change in the width of the reversal zone (i.e. the distance between the onset of liquid loading and the flooding point). The width of the reversal zone becomes smaller when the angle of inclination is increased. At 60° the width of the reversal zone even becomes smaller than the variation of the gas flow rate during the experiments. In that situation a stable flow which enables measurements to be taken was not reached. Therefore it was not possible to obtain data in the reversal zone.
Here a handicap of the equipment should be mentioned. The capacity of the drainage hole at the bottom of the test section appeared to be insufficient. Therefore liquid that reached the bottom of the test section accumulated in the cavity between the air injection nozzle and the tube wall. Although the nozzle penetrates 10cm into the test section, eventually the accumulated liquid even reached to the brim of the nozzle. When that happened this liquid was blown upward again in the form of drops. This effect caused data points to be measured lower than $\frac{\text{meas}}{\text{true}} = 100\%$, at gas velocities below the flooding point. It can be seen in figure 2.5 that the handicap became more prominent at larger inclination angles. Inclination of the tube caused the height of the air injection nozzle to diminish, so the brim could be reached more easily. The handicap did not show up at inclination angles below $\theta = 30^\circ$.

Both effects of tube inclination (reversal zone shift and reversal zone shrinking) are clearly visible when the flooding point and the onset of liquid loading are plotted as a function of the inclination angle (see figure 2.6).

Figure 2.6: The effect of inclination on the reversal zone

The measured data may be compared to the Turner criterion. The Turner criterion does not take the inclination angle into account, so for all measurement series in figure 2.5 it predicts the same gas flow rate at the onset of liquid loading: $V_{gT} = 14 m/s$.

Figure 2.6 shows that the Turner criterion is valid for vertical tubes, but for inclined tubes it is not applicable. At an inclination angle of 45° it even underpredicts the onset of liquid loading by 20%.

2.3.3 Interpretation

The shifting of the reversal zone towards higher gas flow rates could not be explained by a theory based on the movement of liquid drops, such as the Turner criterion. However, the following considerations might explain both the shift towards higher gas flow rates and towards lower gas flow rates. Here the movement of liquid in the form of a film is concerned.

It has been discussed that the liquid in the tube is dragged along by the gas in the form of drops, but in addition to these drops a liquid film exists at the tube wall. In a vertical
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tube this film will be uniformly distributed around the tube. However, when the tube is inclined the film will be thicker at the lower side of the tube and it will be thinner at the top, as a result of gravity. A thin film is more easily dragged upwards than a thick film, so if the film thickness grows a higher gas flow rate is needed for the upward transport of the film.

This is an explanation of the shift towards higher gas flow rates, but inclination beyond 45° causes the reversal zone to shift towards lower gas flow rates. This effect may be explained by the fact that the effective component of gravity (i.e. the component in axial direction) decreases with inclination, according to $\cos \theta$. The effective component of gravity is the main opposing force to the force exerted by the upward flowing gas. If the effective gravity decreases, a lower gas flow rate will be needed for the upward transport of the film.

These two effects are competing with each other and apparently the former is stronger up to an inclination angle of 45°, while beyond 45° the latter is more profound.

In the next chapter a model will be presented which is based on the movement of the liquid in the film. This model has been developed to enable inclination to be taken into account.
Chapter 3

A model for film flow

A model has been developed which can be used to calculate the gas velocity at the onset of liquid loading in vertical tubes. With some adjustments the model may also be used for inclined tubes. This model is based on the movement of a liquid film and differs in that respect from the generally accepted Turner criterion, currently used to calculate the onset of liquid loading.

3.1 Coordinate systems

Throughout this chapter the following coordinates are being used to describe the liquid flow in inclined annular tubes (see figure 3.1):

The inclination angle of the tube is referred to as $\theta$. For a vertical tube $\theta = 0$. The symbol $z$ is used for the position along the tube in axial direction. $\varphi$ and $y$ are used to indicate the position on a cross section of the tube. $\varphi$ is the angular coordinate as shown in figure 3.1, $y$ is the distance to the tube wall.

The axial components of vectorial quantities are defined in the positive $z$-direction.

Figure 3.1: Coordinate systems
3.2 Definitions

A few typical terms for two-phase flow will be used in the next chapter. These terms might need some explanation.

- The **superficial velocity** $V_{sg}$ ($V_{st}$) of the gas (liquid) is a unit for the volume flow. The superficial velocity is defined as the volume flow divided by the cross sectional area of the tube:
  \[ V_{sg} = \frac{4Q_g}{\pi D^2}. \]  

- The **liquid volume fraction** $\lambda_l$ is a fraction of flows. $\lambda_l$ is defined as the liquid fraction of the total volume flow:
  \[ \lambda_l = \frac{V_{sl}}{V_{st} + V_{sg}}. \]  Since the velocity of the gas phase and the liquid phase are not equal, $\lambda_l$ does not denote the instantaneous liquid fraction of a section of the tube.

- The **entrained fraction** $e$ is also a fraction of flows. It is the fraction of the total liquid flow $m_l$ that is accounted for by the drops:
  \[ e = \frac{m_{dr}}{m_l}. \]  

- The **drop holdup** is denoted by the symbol $\alpha_{dr}$. It is the instantaneous fraction of the core volume ($V_{\text{core}}$) that is occupied by the drops:
  \[ \alpha_{dr} = \frac{V_{dr}}{V_{\text{core}}} = \frac{V_{dr}}{\pi \left( \frac{D}{2} - d \right)^2}. \]  Here $d$ is the thickness of the liquid film.

- The **mass concentration of drops**, $C_{dr}$, is the total mass of liquid drops per unit volume of the core. Since $\alpha_l$ is the total volume of liquid drops per unit core volume, $C_{dr}$ is simply given by:
  \[ C_{dr} = \rho_l \alpha_{dr}. \]  

The fraction of liquid in the core is indicated both by $e$ and by $\alpha_l$. The relation between these two quantities can be approximated by the following equation (see appendix A):

\[ \alpha_{dr} = \frac{eV_{sl}}{eV_{sl} + V_{dr}}. \]  

The axial velocity of the liquid drops, $V_{dr}$, depends on the velocity of the gas. In section 3.5.1 the axial motion of the drops will be discussed and an expression will be derived for $V_{dr}$. 
3.3 Assumptions

For the development of the model the following has been assumed:

1. A liquid film exists at the tube wall.

2. This film is thin compared to the diameter of the tube, typically \( \frac{d}{D} < 0.01 \). This is true for low liquid volume fractions \( \lambda_l \). In the experiments \( \lambda_l \) was of the order \( 10^{-3} - 10^{-4} \).

3. The onset of liquid loading is determined by reversal of the flow direction of the liquid in the film.

4. The flow in the film is assumed to be laminar because its Reynolds number is not very large. The critical Reynolds number for laminar flow in a liquid layer is 580. The Reynolds number of the liquid film is typically about 100.

5. The liquid is an incompressible Newtonian fluid.

6. For the liquid film the pressure drop is assumed to be negligible compared with the gravitational force.

7. The flow is assumed to be fully developed.

8. Acceleration of the liquid is assumed to be negligible.

3.4 Vertical tubes, film flow only

In the remainder of this chapter the model will be presented. When gas and liquid phase material are flowing in a tube, a liquid film is observed. The model concentrates on the movement of the liquid in this film. In section 3.5 and in the current section the situation in vertical tubes is considered. Additionally, in the current section it will be assumed that all liquid which is present in the tube flows through the film. This is the easiest situation to describe with a model based on film flow. In section 3.5 the existence of drops will be taken into account as an improvement. In section 3.6 some suggestions will be given to adjust the model for inclined tubes.

3.4.1 The mass flow through a liquid film

The model is based on the force balance for a liquid film. Several forces are acting on the film, such as gravity, pressure drop and shear stress exerted by the flowing gas. Because the fluid is assumed to be an incompressible Newtonian fluid, the balance of forces is given by the Navier-Stokes relation [SHD94]:

\[
\rho_l \frac{D\vec{V}_l}{Dt} = \rho_l \left( \frac{\partial \vec{V}_l}{\partial t} + (\vec{V}_l \cdot \nabla) \vec{V}_l \right) = \mu_l \nabla^2 \vec{V}_l + \rho_l \vec{g} + \nabla p, \tag{3.7}
\]

where \( \rho_l \) and \( \mu_l \) are the mass density and the dynamic viscosity of the liquid.
The onset of liquid loading in inclined tubes

The first equation is the definition of the so-called material derivative of the velocity \( \frac{D\vec{V}}{Dt} \). The material derivative is the rate of change, following an element of the fluid. The term \( \frac{\partial \vec{V}}{\partial t} \) denotes the local acceleration, whereas \( (\vec{V} \cdot \nabla) \vec{V} \) is the convective acceleration.

The right hand side of the second equation summarizes the forces acting on the film. The first term represents the force due to viscous effects in the fluid. The second term is the gravity term and the third term is the pressure drop over the tube. If multiplied with the volume of a small element of the fluid, the Navier Stokes relation can be interpreted as Newton’s second law: The acceleration of the element, multiplied with its mass equals the total force acting on the element.

Steady flow is characterized by the fact that the velocity does not change with time: \( \frac{\partial \vec{V}}{\partial t} = 0 \). It has been assumed that the acceleration of the liquid is negligible, so \( (\vec{V} \cdot \nabla) \vec{V} = 0 \). Together with the assumption that the pressure drop is negligible compared to the gravitational term, the following expression remains for the force balance on the film:

\[
\mu_i \nabla^2 \vec{V}_i = -\rho_i \vec{g}.
\]  

In general the velocity and gravity terms can be split in an axial component and a component perpendicular to \( z \). For vertical tubes though, gravity acts in the axial direction. In that case the direction of gravity (\( \vec{g} \)) is opposite to the direction of the velocity (\( \vec{V}_i \)) and only the axial component of the force balance remains:

\[
\mu_i \frac{\partial^2 V_{i,ax}}{\partial y^2} = \rho_i g. 
\]  

The minus sign of equation 3.8 disappears because gravity acts in the negative \( z \)-direction. This force balance is a differential equation which can be used to calculate the local axial velocity in the film. At the tube wall (\( y = 0 \)) and at the interface between the film and the core (\( y = d \)) boundary conditions should be satisfied: no-slip at the tube wall and continuity of the shear stress at the interface:

\[
V_{i,ax}\big|_{y=0} = 0,
\]

\[
\mu_i \frac{\partial V_{i,ax}}{\partial y} \big|_{y=d} = \tau_i.
\]

The shear stress \( \tau_i \) is imposed by the flowing gas on the liquid film and is assumed to be independent of the film motion. This will be discussed in section 3.4.3. With these boundary conditions the force balance (3.9) can be integrated to obtain the so-called velocity profile in the film:

\[
V_{i,ax}(y) = \frac{\rho_i g}{2\mu_i} \left( y^2 - 2yd \right) + \frac{\tau_i}{\mu_i} y.
\]  

With \( d \) as the film thickness and \( \tau_i \) as the shear stress at the interface (\( y = d \)), the velocity profile gives the local axial velocity of the liquid in the film.
A model for film flow

If the velocity of the liquid is known, the net mass flow can be calculated. Integration of the velocity over the film and multiplication with the mass density yields the mass flow through the film $m_{film}$:

$$m_{film} = \rho_l \pi D \int_0^d V_i \alpha x dy.$$  \hspace{1cm} (3.11)

Substituting equation 3.10 in 3.11 gives the following relation for $m_{film}$:

$$m_{film} = \rho_l \pi D \left( \frac{\tau_l}{2\mu_l} d^2 - \frac{\rho_l g d^3}{3\mu_l} \right).$$  \hspace{1cm} (3.12)

This is a relation between the mass flow in the liquid film and its thickness for any given shear stress.

In their analysis of film flow [MD84] Maron and Dukler have used a dimensionless equivalent of equation 3.12. To obtain this dimensionless equation they used two different reference states, as described subsequently.

The first reference state is that of a free falling film (i.e. no gas flow relative to the liquid flow). The liquid is injected at a mass flow rate $m_l$ and all of the injected liquid flows downward through the film ($m_{film} = -m_l$). For a free falling film the shear stress at the interface is zero. In that case the film thickness can easily be calculated from 3.12:

$$d_N = \left( 3 \frac{m_l}{\pi D \rho_l^2 g} \right)^{\frac{1}{3}}.$$  \hspace{1cm} (3.13)

If upward gas flow takes place counter to the falling film, the velocity profile is distorted in such a way that the local downward velocity near the interface is reduced due to the shear stress exerted by the gas. With sufficient opposing shear stress, the velocity at the interface approaches zero.

The second reference state is the falling film with zero velocity at the interface. Still, in that situation, all of the liquid in the film flows downward. From the velocity profile 3.10 it can be found that the interfacial shear stress in case of zero interfacial velocity, $\tau_{i0}$, has to satisfy:

$$\tau_{i0} = \frac{1}{2} \rho_l g d_0,$$

with $d_0$ the corresponding film thickness.

Using this, an expression for $d_0$ is obtained from equation 3.12:

$$d_0 = \left( 12 \frac{m_l}{\pi D \rho_l^2 g} \right)^{\frac{1}{3}} = \sqrt[3]{4d_N},$$

so for $\tau_{i0}$ we find:

$$\tau_{i0} = \frac{1}{2} \rho_l g \left( 12 \frac{m_l}{\pi D \rho_l^2 g} \right)^{\frac{1}{3}}.$$  \hspace{1cm} (3.14)
Now the film thickness for an arbitrary mass flow $m_l$ can be defined in terms of $d_N$ and the shear stress at the interface can be normalized with $\tau_{i0}$:

$$R_N = \frac{d}{d_N},$$

$$F = \frac{\tau_i}{\tau_{i0}}.$$  \hspace{1cm} (3.15) \hspace{1cm} (3.16)

With these definitions equation 3.12 can be presented in dimensionless form:

$$R_N^3 - aF R_N^2 + \frac{m_{film}}{m_l} = 0,$$ \hspace{1cm} (3.17)

where $a = \frac{3}{4} \sqrt{4}$. This is the relation that has been used by Maron and Dukler. In practice the ratio $\frac{m_{film}}{m_l}$ varies between $\frac{m_{film}}{m_l} = -1$ (all liquid flows downwards) and $\frac{m_{film}}{m_l} = 1$ (all liquid flows upwards). Again it should be noticed that the reference states $\tau_{i0}$ and $d_N$ correspond with different conditions.

The approach of Maron and Dukler will be discussed in the next section.

### 3.4.2 The analysis of Maron and Dukler

In this section an analysis of film flow will be presented which has been performed by Maron and Dukler. The liquid was assumed to be injected somewhere along the tube length and all of the injected liquid was assumed to flow through the film, either upward or downward. For a given value of the total liquid mass flow $m_l$ they explored the film thickness at various values of the shear stress.

**Figure 3.2:** Solutions of the dimensionless film thickness for upward liquid flow through the film

**Situation A:** upward flow.

If the shear stress exerted by the gas is sufficient, all of the liquid will be lifted upward. No liquid will exist below the injection point, so $m_{film} = m_l$. In that situation the dimensionless film thickness above the injection point can be solved from

$$R_N^3 - aF R_N^2 + 1 = 0.$$ \hspace{1cm} (3.18)
A model for film flow

Figure 3.3: Typical velocity profiles

This equation has one negative real root, which is a non-physical solution for the film thickness. Two positive solutions exist if $F$ exceeds a critical value $F^*$, as shown in figure 3.2. At the critical value of the dimensionless shear stress $F^*$ these solutions come together and for lower values of $F$ there is no positive solution of equation 3.18. Apparently it is impossible to lift all of the injected liquid with a shear stress lower than $F^*$. For lower shear stresses downflow must occur!

The highest value of $R_N$ (the dotted line in figure 3.2) corresponds with a velocity profile showing downward velocities near the tube wall and upward velocities close to the interface (profile (b) in figure 3.3). In that case the shear stress at the tube wall ($\tau_w = \mu \frac{\partial V_{1,xx}}{\partial y} |_{y=0}$) is negative.

This solution is unstable, since at the same value of the shear stress a second solution exists at a smaller film thickness. This lower value of $R_N$ corresponds with a velocity profile showing only upward velocities (profile (a) in figure 3.3). In that case the shear stress at the tube wall is positive $\tau_w > 0$.

At the critical value $F^*$ the two solutions are equal. This is a special situation, because it is the limit for total upward flow through the film. In this situation the shear stress at the wall is zero ($\tau_w = 0$) and the curve in figure 3.2 is vertical $\frac{\partial R_N}{\partial F} \to \infty$. The condition of zero shear stress at the tube wall is satisfied when $\frac{\partial V_{1,xx}}{\partial y} |_{y=0} = 0$. Using the velocity profile (3.10) this leads to the following condition:

$$R_N^* = \frac{2}{3} a F^*.$$ (3.19)

The condition $\frac{\partial R_N}{\partial F} \to \infty$ leads to the same expression. If this is substituted in equation 3.18 a value of $F^*$ is found for the case that drops are absent: $F^* = \sqrt{4}$.

If the dimensionless shear stress is lower than $F^*$ there is no real, positive solution of equation 3.18.
The onset of liquid loading in inclined tubes

Figure 3.4: Solutions of the dimensionless film thickness for upward and downward liquid flow through the film

Situation B: downward flow.
At low shear stresses all of the injected liquid will fall down. In that case no liquid exists above the injection point, so $m_{film} = -m_l$. The dimensionless film thickness below the injection point can be solved from

$$R_N^2 - a F R_N^2 - 1 = 0. \quad (3.20)$$

For this equation only one real root exists, which is positive for all positive values of the dimensionless shear stress. The solution of this equation has been inserted in figure 3.2 (see figure 3.4). For values of $F$ lower than 1 the velocity of the liquid is downward in the entire film. If the dimensionless shear stress is higher than 1 the velocity of the liquid close to the interface is upward. In that situation it can no longer be assumed that all liquid is falling down. For that reason the solutions for $F > 1$ are shown as a dotted line.

Between the two situation described above a range of shear stresses exists ($1 < F < \sqrt[3]{4}$) where upward flow as well as downward flow take place. In that region, previously termed the reversal zone, $m_{film} / m_l$ has a value between $-1$ and 1, depending on the way the injected mass flow is separated into an upward and a downward flowing fraction. Maron and Dukler have presented a solution for this situation as well. Their results are shown in figure 3.5. The arrows in Figure 3.5 indicate the behaviour of the film thickness when the shear stress decreases: The film thickness increases if the shear stress decreases, until the unstable point is reached at $F = F^*$. At $F = F^*$, a jump in the film thickness is predicted. Further decreasing the shear stress causes the film thickness to decrease.

Summarizing the above analysis:

- It is impossible to lift all of the injected liquid with a shear stress lower than $F = F^*$. This point is characterized by $R_N^* = \frac{2}{3} a F^*$.
- If $F$ is lower than 1 all of the injected liquid is falling down.
- If $1 < F < F^*$ part of the injected liquid may flow upward, but downward flow must occur.
Throughout this report the onset of liquid loading has been defined as the lowest possible gas velocity at which all of the liquid is lifted upwards. According to the above analysis liquid loading occurs at the unstable point of the upward flowing liquid film. If all liquid flows through the film, the dimensionless shear stress at the onset of liquid loading is $F^* = \sqrt{4}$.

### 3.4.3 Shear stress and gas velocity

Until now the model has been described in terms of the shear stress at the interface. This shear stress is exerted by the flowing gas and depends on the velocity of the gas. In this section the shear stress will be related to the velocity of the gas.

The properties of the mixture in the core are reflected in the following quantities which will be used as the mixture density $\rho_m$, the mixture viscosity $\mu_m$ and the mixture velocity $V_m$:

\[
\rho_m = \alpha_i \rho_l + (1 - \alpha_i) \rho_g, \\
\mu_m = \alpha_i \mu_l + (1 - \alpha_i) \mu_g, \\
V_m = V_{\phi} + e V_{st}.
\]

If the mixture of the gas and the liquid drops is regarded as a homogeneous mixture, the shear stress at the interface of the film is given by the following relation:

\[
\tau_i = \frac{1}{2} f_{tp} \rho_m V_m^2. \tag{3.21}
\]

Here $f_{tp}$ is a friction factor for two-phase flow. In general the friction in multiphase flow is significantly higher than the friction in single-phase flow. For the case of two-phase pipe flow the American Gas Association have developed an empirical correlation for a normalised friction factor $f_{tp,f}$ as a function of the liquid volume fraction $\lambda_l$. This correlation is shown in figure 3.6.

For very low values of $\lambda_l$ ($\lambda_l < 0.01$) this correlation can be approximated by:

\[
\frac{f_{tp}}{f} = b_1 \lambda_l + b_2, \tag{3.22}
\]

with $b_1 = 667$ and $b_2 = 1.23$. 
The onset of liquid loading in inclined tubes

Figure 3.6: Normalized two-phase friction factor

The (simplified) single-phase friction factor \( f \) which normalizes the two-phase friction factor is determined from the following correlation, developed by Shell Oil for turbulent flow:

\[
f = c_1 R_e_m^c_2 + c_3,
\]

with \( c_1 = 0.636 \), \( c_2 = 0.355 \) and \( c_3 = 0.0072 \). \( R_e_m \) is the Reynolds number of the mixture, defined as:

\[
R_e_m = \frac{\rho_m V_m D}{\mu_m}.
\]

When the existence of drops is not taken into account \( e = 0 \) and \( \alpha_l = 0 \), so \( \rho_m = \rho_g \), \( \mu_m = \mu_g \) and \( V_m = V_{sg} \). In that case for low liquid volume fractions \( \lambda_l \) the shear stress is given by:

\[
\tau_l = \frac{1}{2} (b_1 \lambda_l + b_2) \left[ c_1 \left( \frac{\rho_g V_{sg} D}{\mu_g} \right)^{c_2} + c_3 \right] \rho_g V_{sg}^2.
\]

The experiments have been performed in a \( 50mm \) tube, at a constant liquid flow rate of \( m_l = 0.01kg/s \). For this situation the shear stress has been calculated from equation 3.24 at various values of the gas flow rate. In figure 3.7 the dimensionless equivalent of this two-phase shear stress is plotted as a function of the superficial gas velocity. Notice that the data in this plot are only applicable to air-water flow in a \( 50mm \) tube and they are only valid in absence of drops.

Figure 3.7: Shear stress as a function of the superficial gas velocity
3.4.4 Comparison with the experiments

As has been discussed in section 3.4.2 the onset of liquid loading is predicted at $F = \sqrt{4}$. It is now possible to determine the corresponding gas velocity. Figure 3.7 shows that the corresponding gas velocity is $V_{g,p} = 33 \text{m/s}$. This velocity should be compared to the measured gas velocity at the onset of liquid loading in a vertical tube. The measured gas velocity is $V_{g,m} = 14 \text{m/s}$, and is indicated in figure 3.7.

The predicted gas velocity $V_{g,p}$ is significantly higher than the measured velocity $V_{g,m}$. This might be explained by the fact that the existence of drops has been neglected.

Figure 3.8: The effect of the existence of drops on $F^*$

3.4.5 A consideration to improve the model

In this section it will be shown that the model can be improved if the existence of drops is taken into account. It will be shown that $F^*$ decreases when $e$ increases.

As has been discussed before, the flow direction of the liquid in the film starts to reverse at a critical value of the dimensionless shear stress, $F = F^*$. At this critical shear stress the curve of $R_N$ as a function of $F$ is vertical. The relation between $R_N$ and $F$ has been derived in section 3.4.2:

$$R_N^3 - a F R_N^2 + \frac{m_{film}}{m_l} = 0. \quad (3.25)$$

If drops do exist, the injected liquid mass flow $m_l$ separates into two mass flows: $m_{dr}$ through the drops and $m_{film}$ through the liquid film. Mass conservation requires that

$$m_l = m_{dr} + m_{film}.$$

Since the entrained fraction $e$ has been defined as a fraction of fluxes, this relation can be written as

$$\frac{m_{film}}{m_l} = 1 - e. \quad (3.26)$$

This can be used together with equation 3.17 to obtain the following relation:

$$R_N^3 - a F R_N^2 + 1 - e = 0. \quad (3.27)$$
The onset of liquid loading in inclined tubes

For various values of the entrained fraction $e$ this relation has been plotted in figure 3.8. The curve for $e = 0$ (no drops) is the same as the curve in figure 3.2. Here we find the familiar value $F^* = \sqrt{4}$, but the figure shows that $F^*$ decreases when the entrained fraction increases. This means that the flow direction of the liquid in the film reverses at lower gas flow rates. For that reason it is expected that the model can be improved by taking the existence of drops into account.

3.5 Vertical tubes, film and droplets

The model that has been presented is based on a force balance for the liquid film and on the assumption that all of the injected liquid flows through the film. The analysis of the model led to a prediction of the onset of liquid loading in a vertical tube. However, this model is not satisfactory, since it underpredicts the gas velocity at the onset of liquid loading by a factor 2. In section 3.4.5 it was suggested that introducing drops into the model would be a useful modification.

Liquid drops move in axial direction because they are dragged along with the gas. Radial movement of drops occurs as well, since liquid is extracted from the film to create drops and drops deposit on the film. Therefore it may be expected that the existence of the drops affects the interaction at the interface between the film and the core.

In this section the model will be modified by taking the existence of drops into account. The modified model is also based on the force balance for the film, but it is no longer assumed that all of the injected liquid flows through the film. This assumption will be replaced by the assumption that equilibrium exists between the creation and the deposition of drops. If this equilibrium exists the film will be of a constant thickness.

At first, in section 3.5.1, the axial motion of the drops will be discussed. Then, in section 3.5.2 the interaction of the drops with the film will be examined. The modifications to the description of the liquid flow through the film will be discussed in section 3.5.3.

3.5.1 The axial drop transport

The liquid drops are dragged along by the gas, but their upward velocity is somewhat lower than the velocity of the gas. The motion of liquid drops relative to a gas flow has been treated by Turner et al [THD69] and will be discussed in this section.

By a transformation of coordinates, a drop of liquid being transported by an upward moving gas flow becomes a free falling particle. A free falling drop in a fluid medium will reach a terminal velocity relative to the gas. This terminal velocity, $V_{\text{term}}$, is the maximum velocity it can attain under the influence of gravity and is reached when the drag forces equal the gravitational force:

$$\frac{1}{6} \pi d_{\text{dr}}^3 (\rho_l - \rho_g) g = \frac{1}{4} \pi d_{\text{dr}}^2 \frac{1}{2} C_D \rho_g V_{\text{term}}^2.$$  \hspace{1cm} (3.28)

Here $d_{\text{dr}}$ is the diameter of the drop and $C_D$ is a drag coefficient.

The left hand side of this equation is the gravitational force on a particle falling in a fluid medium of density $\rho_g$. The right hand side of this equation represents the drag force. The drag force depends on the cross sectional area of the drop, rather than on its volume.
The drag coefficient $C_D$ is influenced by the drop shape and the drop Reynolds number. An empirical correlation of $C_D$ for spheres is shown in figure 3.9. It shows that for Reynolds numbers between $10^2$ and $10^5$ the drag coefficient is relatively constant. Based on the drop size prediction of equation 3.35 the drop Reynolds number ranges from $10^4$ to $10^5$. This is the range where the drag coefficient is approximately constant at a value of 0.5. From equation 3.28 a relation is found for the terminal velocity of a drop:

$$V_{\text{term}} = \left( \frac{4(\rho_l - \rho_g)gd_{dr}}{3C_D \rho_g} \right)^{\frac{1}{2}}.$$  

(3.29)

$V_{\text{term}}$ is the maximum velocity of the drop relative to the gas and unhindered by other drops. If the gas velocity is $V_g$, the axial velocity of the drop $V_{dr}$ is given by:

$$V_{dr} = V_g - V_{\text{term}}.$$  

(3.30)

The velocity of the drops is a parameter in the relation between the entrained fraction $e$ and the liquid holdup in the core $\alpha_l$ (equation 3.6). However, $V_{dr}$ depends on the size of the drop. Because the drops vary in diameter there is no unique drop velocity. Therefore a mean drop size will be used in the relation between $e$ and $\alpha_l$.

If the velocity of the gas equals the free fall terminal velocity of the drop, $V_{dr} = 0$: the drop will neither fall nor will it be lifted. Any further increase in the gas velocity would make the drop move upward. Therefore the minimum gas velocity for upward drop movement is equal to the terminal velocity of the free falling drop:

$$V_{g,\text{min}} = V_{\text{term}}.$$  

(3.31)

Equation 3.29 shows that for large drops the terminal velocity is higher than for smaller drops. Hence the larger the drop, the higher the gas flow rate that is needed to lift the drop. The largest drop that exists in the flow therefore determines the minimum gas flow rate that is needed to lift all of the drops.

Now we have found an expression for the velocity of the drops $V_{dr}$ and for the minimum gas velocity that is needed to lift all of the drops that are present in the tube, $V_{g,\text{min}}$. However, the expression for $V_{dr}$ is only useful if a characteristic drop size is known, and for $V_{g,\text{min}}$ the maximum drop size should be calculated.
The characteristic drop size  An empirical relation for the Sauter mean drop diameter in two-phase pipe flow has been developed at KSLA [S95]. The following relation was found from an analysis of drop size measurements in a 50mm vertical tube with water and air:

\[ d_{dr} = d_{dr0} \frac{V_{sg}}{V_{sg} - c_1} \exp \left[ -\frac{V_{sg}}{c_2} \exp \left( -\frac{V_{sl}}{c_3} \right) \right], \]  

with \( d_{dr0} = 250\mu m, c_1 = 4m/s, c_2 = 41.3m/s \) and \( c_3 = 0.09m/s \).

Using this drop size the velocity of the drops is given by:

\[ V_{dr} = V_{sg} - \left( \frac{4(\rho_l - \rho_g)gd_{dr0}V_{sg}}{3CD\rho_g} \frac{V_{sg}}{V_{sg} - c_1} \exp \left[ -\frac{V_{sg}}{c_2} \exp \left( -\frac{V_{sl}}{c_3} \right) \right] \right)^{\frac{1}{2}}. \]  

This drop velocity will be used in the relation between the entrained fraction \( e \) and the drop holdup \( \alpha_{dr} \) (equation 3.6).

The largest drop size  Depending on the gas flow rate a maximum drop size exists for the drops in the flow. The largest drop that exists in the flow determines the minimum gas flow rate that is needed to lift all of the drops. As discussed in section 1.2.1 the Turner criterion is based on the movement of the largest drop that is likely to be present.

Drops moving at a velocity \( V_{rel} \) relative to a gas are subjected to forces that try to shatter the drop. On the other hand the surface tension acts to hold the drop together. An important parameter in this process is the Weber number. The Weber number represents the ratio of the dynamic pressure \( \rho_g V_{rel}^2 \) and the pressure due to surface tension \( \frac{2\sigma}{d_{dr}} \):

\[ We = \frac{d_{dr}\rho_g V_{rel}^2}{\sigma}. \]  

If the Weber number exceeds a critical value a liquid drop will shatter. For free falling drops the value of the critical Weber number is about 30. With this critical value of the Weber number a relation between the maximum diameter of a drop and its velocity relative to the gas is obtained:

\[ d_{dr,m} = \frac{30\sigma}{\rho_g V_{rel}^2}. \]  

If for the relative velocity the terminal velocity is used \( (V_{rel} = V_{term}) \), substituting the expression for the maximum diameter into equation 3.29 yields an expression for the terminal velocity of the largest drop that exists in the flow:

\[ V_{term,m} = V_{g,min} = \left( \frac{40(\rho_l - \rho_g)g\sigma}{CD\rho_g^2} \right)^{\frac{1}{4}}, \]  

which is the same as equation 1.1. As has been discussed, \( V_{g,min} \) is the minimum gas velocity to lift all of the drops in upward direction.

Equation 3.36 was derived by Turner e.a. in 1969 and after calibration with a vast amount of field data it became known as the Turner criterion, as discussed in section 1.2.1.
The interaction of the drops with the film

Besides the axial motion described in the previous section interaction exists of the drops with the film: liquid is extracted from the film to create drops and on the other hand drops deposit on the film. The extraction of liquid from the film is called entrainment.

The rates of entrainment and deposition are expressed as the entrainment flux and the deposition flux. The entrainment (deposition) flux $\Phi_e$ ($\Phi_d$) is the total mass of liquid that is entrained from (deposits on) the film per unit time, per unit area of the tube wall. In this section empirical relations will be presented for entrainment and deposition. Stability of the flow requires that the rates of entrainment and deposition are in equilibrium. At the end of this section this requirement will lead to a dimensionless condition for equilibrium in radial direction.

**Entrainment** Liquid is extracted from the film as a consequence of the shear stress exerted by the gas. A higher value of the shear stress causes more liquid to be entrained. The entrainment flux also depends on the thickness of the film. Empirical data obtained by Whalley et al [WHH74] show that the entrainment flux increases with increasing film thickness (see figure 3.10). A straight line fit through these data led to the following correlation for the entrainment flux:

$$\Phi_e = k_e \left( \frac{\tau_i d}{\sigma} \right)^2,$$

(3.37)

with $k_e$ the entrainment constant: $k_e = 52.2 \text{kg/m}^2 \text{s}$.

**Deposition** It is commonly accepted that the deposition mass flux depends on the concentration of drops (see for instance [FP78] or [BH79]). Usually the following relation is used for the deposition flux:

$$\Phi_d = k_d C_{dr} = k_d \rho_l \frac{e V_{sl}}{e V_{sl} + V_{dr}}.$$

(3.38)

Here $k_d$ is the so-called deposition constant. For water drops in air at atmospheric pressure $k_d$ has the value of $0.15 \text{m/s}$. The drop velocity, $V_{dr}$, has been discussed in section 3.5.1.
In the literature [FP78] a slightly different formula may be found for the concentration of drops. In these studies it is assumed implicitly that the velocity of the drops is equal to the gas velocity, which is a reasonable assumption in their region of interest, but not at gas velocities close to the reversal zone.

**Equilibrium** Stability of the flow requires that the entrainment flux and the deposition flux are equal. Using relations 3.37 and 3.38 the following relation is obtained for equilibrium:

\[
\frac{k_c}{\sigma} \left( \frac{\tau_i d}{\sigma} \right)^2 = k_d \rho_l \frac{e V_{sl}}{e V_{sl} + V_{dr}}.
\] (3.39)

In section 3.4 the film thickness \(d\) has been expressed in terms of \(d_N\), \(d_N\) being the film thickness in the typical situation of zero shear stress and all liquid flowing through the film:

\[
R_N = \frac{d}{d_N}, \text{ with } d_N = \left( \frac{3 m_f \mu_l}{\pi D \rho_f^2 g} \right)^{\frac{1}{3}}.
\]

The shear stress \(\tau_i\) has been normalized with \(\tau_{i0}\), the shear stress needed to obtain zero interfacial velocity if all liquid flows through the film:

\[
F = \frac{\tau_i}{\tau_{i0}}, \text{ with } \tau_{i0} = \frac{1}{2} \rho_f g \left( \frac{12 m_f \mu_l}{\pi D \rho_f^2 g} \right)^{\frac{1}{3}}.
\]

Using these definitions the dimensionless equivalent of equation 3.39 is obtained:

\[
F^2 R_N^2 = b \frac{e V_{sl}}{e V_{sl} + V_{dr}}, \text{ with } b = \frac{\rho_l k_d}{k_c} \frac{\sigma^2}{\tau_{i0}^2 d_N^2}.
\] (3.40)

This is the dimensionless condition for equilibrium in radial direction. It is a relation between the dimensionless shear stress, the film thickness and the entrained fraction.

### 3.5.3 Modifications to the description of film flow

In the description of film flow only a minor modification has to be made in order to account for the existence of drops. In section 3.4.1 the axial mass flow through the film has been discussed. This led to the following relation:

\[
R_N^3 - a F R_N^2 + \frac{m_{film}}{m_l} = 0.
\]

In section 3.4.5 it has been shown that \(\frac{m_{film}}{m_l} = 1 - e\). Therefore this relation becomes:

\[
R_N^3 - a F R_N^2 + 1 - e = 0.
\] (3.41)

This equation is a second relation between \(R_N\), \(F\) and \(e\).
3.5.4 The modified model

It is useful to summarize what we have obtained so far. We have found two relations (3.40 and 3.41) between $R_N$, $F$ and $e$; in section 3.4.2 a condition at the onset of liquid loading has been derived:

$$R_N^* = \frac{2}{3} a F^*;$$  \hfill (3.42)

In section 3.4.3 the effect of the gas velocity on the shear stress has been discussed and in section 3.5.1 the velocity of the drops $V_{dr}$ is related to the gas velocity. These relations can be used in an iterative procedure to calculate the gas velocity at the onset of liquid loading. But at first a few equations can be combined to reduce the number of equations:

The condition at the onset of liquid loading can be used to eliminate $R_N$ from equations 3.40 and 3.41. Both equations can be rearranged to obtain expressions for the entrained fraction $e$ as a function of the dimensionless shear stress $F^*$ at the onset of liquid loading:

$$e = \frac{V_{dr}}{V_{sl}} \frac{a^2 F^{*4}}{\frac{9}{4} b - a^2 F^{*4}},$$  \hfill (3.43)

$$e = 1 - \frac{4}{27} a^3 F^{*3}. \hfill (3.44)$$

These values of $e$ should be equal, so

$$\frac{V_{dr}}{V_{sl}} \frac{a^2 F^{*4}}{\frac{9}{4} b - a^2 F^{*4}} = 1 - \frac{4}{27} a^3 F^{*3}. \hfill (3.45)$$

Both the shear stress and the velocity of the drops depend on the gas velocity. The shear stress has been discussed in section 3.4.3, the velocity of the drops has been discussed in section 3.5.1. It is now possible to construct an iterative procedure for the calculation of the gas velocity at the onset of liquid loading.

3.5.5 Iteration method and comparison with the experiments

The following iterative procedure has been used to calculate the gas velocity at the onset of liquid loading:

1. Choose $\frac{V_{dr}}{V_{sl}} = 1$.

2. Solve equation 3.45 for $F^*$.

3. Calculate $e$ from equation 3.44

4. Determine the gas velocity corresponding with $F^*$ and $e$.

5. Calculate $\frac{V_{dr}}{V_{sl}}$ for that gas velocity.

6. Repeat steps 2-5 until $\frac{V_{dr}}{V_{sl}}$ does not change anymore.
The onset of liquid loading in inclined tubes

Table 3.1: The relevant physical parameters

<table>
<thead>
<tr>
<th>quantity</th>
<th>symbol</th>
<th>value</th>
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<tr>
<td>gas mass density</td>
<td>( \rho_g )</td>
<td>1.21</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>liquid mass density</td>
<td>( \rho_l )</td>
<td>1000</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>liquid dynamic viscosity</td>
<td>( \mu_l )</td>
<td>( 10^{-3} )</td>
<td>Pas</td>
</tr>
<tr>
<td>liquid mass flow rate</td>
<td>( m_l )</td>
<td>0.01</td>
<td>kg/s</td>
</tr>
<tr>
<td>tube diameter</td>
<td>( D )</td>
<td>0.05</td>
<td>m</td>
</tr>
</tbody>
</table>

Table 3.2: Results of the iteration

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^* )</td>
<td>0.537</td>
</tr>
<tr>
<td>( e )</td>
<td>0.961</td>
</tr>
<tr>
<td>( \frac{V_{se}}{V_{sq}} )</td>
<td>( 2.5 \cdot 10^3 )</td>
</tr>
<tr>
<td>( V_{sq} )</td>
<td>0.84</td>
</tr>
</tbody>
</table>

For the experimental settings the modified model has been evaluated in this way. The experimental settings are summarized in table 3.1. The results of the iteration are shown in table 3.2. In table 3.3 the measured value of the gas velocity at the onset of liquid loading is compared to the predictions of section 3.4 and the current section.

It was found that the onset of liquid loading is predicted by the modified model at a gas velocity \( V_{sq,p} = 15.1 \text{ m/s} \). This value is much closer to the measured gas velocity than the value that was predicted without taking the existence of drops into account.

Table 3.3: Predicted and measured values of \( V_{sq} \) at the onset of liquid loading

<p>| | |</p>
<table>
<thead>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>prediction without drops</td>
<td>33 m/s</td>
</tr>
<tr>
<td>prediction with drops</td>
<td>15 m/s</td>
</tr>
<tr>
<td>measured value</td>
<td>14 m/s</td>
</tr>
</tbody>
</table>

3.6 Inclined tubes

So far the model could only be used for vertical pipes where gravity acts in the axial direction. In inclined pipes the model loses its validity because gravity is not acting in axial direction. This gives rise to a secondary flow in the film: the liquid flows along the wall towards the lower side of the tube cross section. The secondary flow would terminate when all of the liquid in the film were accumulated at the lower side of the tube. However, because of the existence of drops the secondary flow is kept alive: drops that deposit on the wall create a liquid film, even at the top of the cross section. This results in an asymmetric distribution of liquid in the film, in accordance with the effects that were mentioned in section 2.3.3 to explain the shift of the reversal zone towards higher gas flow rates.

In this section some suggestions will be given for adjustments to the model which would extend its validity to inclined tubes. The principles on which the model is based remain the same: The axial mass flow of liquid through the film is determined from a force balance and it is assumed that equilibrium exists between the total entrainment and deposition of drops.
3.6.1 The secondary flow

In an inclined tube the liquid in the film flows along the tube wall towards the lower side of the pipe. This secondary flow is initiated by gravity. As a result of the secondary flow the film thickness will not be uniform around the tube. At the top of a cross section the film will be thinner than at the bottom (see figure 3.11).

In this section the secondary flow will be discussed in more detail. For that reason we will glance back to section 3.4, where the force balance for a sheared liquid film has been derived:

$$\mu_l \nabla^2 \vec{V}_l = -\rho_l \vec{g}$$

The velocity profile of the secondary flow can be obtained from the tangential component of this equation. The tangential component is the component perpendicular to $z$, but tangent to the tube wall.

Since the gas flows in axial direction the secondary flow is perpendicular to the gas flow, which means that in tangential direction the shear stress is zero. Keeping this in mind, the velocity profile of the secondary flow is easily obtained analogous to section 3.4.1:

$$V_{l,\text{tan}} = \frac{\rho_l}{2\mu_l} g \sin \theta \sin \varphi \left(y^2 - 2yd\right). \quad (3.46)$$

In this expression $g \sin \theta \sin \varphi$ is the tangential component of gravity (see figure 3.12).
The onset of liquid loading in inclined tubes

Integration of the velocity profile over the film thickness and multiplication with the mass density yields an expression for the secondary mass flow \( m_{sec} \):

\[
m_{sec} = \frac{\rho_l^2}{3\mu_l} g \sin \theta \sin \varphi d^3(\varphi). \tag{3.47}
\]

\( m_{sec} \) is the mass flow in tangential direction per unit tube length. As can be seen in equation 3.47 \( m_{sec} \) depends on the inclination angle and on the film thickness. Both explicitly (\( \sin \varphi \)) and implicitly (through the film thickness) \( m_{sec} \) depends on \( \varphi \).

### 3.6.2 Suggestions for adjusting the model

The model is based on a force balance for the liquid film. Analogous to section 3.4.1 the axial velocity profile is obtained from this force balance:

\[
V_{i,ax} = \frac{\rho_l g \cos \theta}{2\mu_l} (y^2 - 2yd) + \frac{\tau_i}{\mu_l} y. \tag{3.48}
\]

In this expression \( g \cos \theta \) is the axial component of gravity in a tube that is inclined to an angle \( \theta \). By integration of the velocity profile over the film and multiplication with the liquid mass density the mass flow through the film \( m_{film} \) is found, which by definition is equal to \((1 - e)m_l\):

\[
m_{film} = (1 - e)m_l = \rho_l \int_0^{2\pi} \frac{D}{2} \left[ \frac{\tau_i}{2\mu_l} d^2(\varphi) - \frac{\rho_l g \cos \theta}{3\mu_l} d^3(\varphi) \right] d\varphi. \tag{3.49}
\]

However, integration over \( \varphi \) requires knowledge on the film thickness distribution \( d(\varphi) \).

The film thickness distribution might be determined from a mass balance on a section of the liquid film: Liquid flows contributing to this mass balance are the local rates of entrainment and deposition, the secondary flow and the axial motion of the liquid in the film:

\[
\frac{1}{D/2} \frac{dm_{sec}}{d\varphi} + \frac{dm_{film}}{dz} = (\Phi_d - \Phi_e). \tag{3.50}
\]

If the flow is assumed to be fully developed, \( m_{film} \) does not change in the axial direction, so \( \frac{dm_{film}}{dz} = 0 \).

As a first approximation, correlations 3.37 and 3.38 can be used for the local entrainment and deposition fluxes. In that case the deposition flux is still uniform around the tube, but the entrainment flux depends on \( \varphi \) through the film thickness:

\[
\Phi_e = k_e \left( \frac{\tau_i d(\varphi)}{\sigma} \right)^2,
\]

\[
\Phi_d = \rho_l k_d \frac{1}{1 + \frac{V_{le}}{V_{st}}}. \]

This means that the rate of entrainment is higher at the bottom of the tube cross section than at the top. Deposition is also affected by gravity. Since the drops fall towards the
lower tube wall the deposition flux at the bottom of the tube cross section will exceed the
deposition flux at the top of the cross section. However, as a first assumption this effect
might be neglected.
The secondary flow has been discussed in section 3.6.1, where it is shown that $m_{sec}$ is given
by:

$$m_{sec} = \frac{\rho_l^2}{3\mu_l} g \sin \theta \sin \varphi d^3(\varphi).$$

These relations can be substituted into equation 3.50 to obtain:

$$\frac{d}{d\varphi} \left[ \frac{\rho_l^2}{3\mu_l} g \sin \theta \sin \varphi d^3(\varphi) \right] = \frac{D}{2} \left[ \rho_l k_d \frac{1}{1 + \frac{\tau}{\varepsilon \mu_t}} - k_e \left( \frac{r_i d(\varphi)}{\sigma} \right)^2 \right]. \quad (3.51)$$

These are the changes that appear when inclination is introduced into the model.
In a vertical tube a force balance and mass conservation in axial direction led to equation
3.41. Equilibrium in radial direction led to equation 3.40. Together with the condition at
liquid loading (equation 3.42) these equations could be solved in an iterative procedure.
In an inclined tube the force balance and mass conservation in axial direction lead to
equation 3.49. Equilibrium in radial direction leads to equation 3.51. There is no general
condition at the onset of liquid loading, because at different values of $\varphi$ the liquid flow
direction will not be equal. A more complex iterative procedure with equations 3.49 and
3.51 is needed to obtain a solution for $m_{film}$.
The above suggests how a liquid loading model may be developed for inclined tubes. A lot
more work is needed to complete such a model and to validate against measured data.
Chapter 4

Conclusions and recommendations

This chapter contains some concluding remarks on the investigation and recommendations for future work.

It has been shown that reversal of the liquid flow direction takes place over a range of gas flow rates, the so-called reversal zone, limited by the onset of liquid loading and by the flooding point.

The reversal zone is shown to be affected by tube inclination. Experiments revealed that the reversal zone shifts to higher gas flow rates by inclining the tube. This trend reverses at inclination angles larger than 45°. Furthermore it has been observed that the width of the reversal zone becomes smaller with inclination of the tube.

The traditional Turner criterion gives a satisfactory prediction of the onset of liquid loading in vertical tubes. However, this criterion can not accurately predict liquid loading in inclined tube flow.

The observations have been explained qualitatively by a model that accounts for liquid transport both through a film along the tube wall and through drops entrained in the gas core. In this model liquid loading is related to the stability of the liquid film, taking into account the liquid mass transfer between the film and the droplets. The model has been validated for vertical pipe flow and shows good agreement with the experimental observations. In principle this model can be extended to inclined tube flow. Some suggestions for the necessary adjustments have been given.

Further work is needed to develop a quantitative predictive method for liquid loading in inclined tubes.
The onset of liquid loading in inclined tubes
Bibliography


[S95] Schellekens, C. J., "Drop size distribution in annular dispersed up-flow" draft report.


The onset of liquid loading in inclined tubes
Appendix A

The relation between $\alpha_{dr}$ and $e$

In this appendix a relation will be derived between the drop holdup $\alpha_{dr}$ and the entrained fraction $e$.

The drop holdup in a section of the tube is defined by equation 3.4:

$$\alpha_{dr} = \frac{V_{dr}}{V_{core}} = \frac{V_{dr}}{V_{dr} + V_{g}},$$

(A.1)

where $V_{dr}$ is the volume occupied by the drops and $V_{g}$ is the volume occupied by the gas. When the volumes are divided by the length of the section and multiplied with the velocity of the drops, this equation can be rewritten in terms of the volumetric flow rates:

$$\alpha_{dr} = \frac{Q_{dr}}{Q_{dr} + V_{g}Q_{g}},$$

(A.2)

or, using the definition of the entrained fraction (equation 3.3):

$$\alpha_{dr} = \frac{eQ_{l}}{eQ_{l} + \frac{V_{g}}{V_{g}Q_{g}}}.$$  

(A.3)

For low liquid volume fractions $\lambda_{l}$, the gas velocity $V_{g}$ may be approximated by the superficial gas velocity $V_{sg} = \frac{Q_{g}}{\pi D_{f}}$. In that case the expression for the drop holdup becomes:

$$\alpha_{dr} = \frac{eQ_{l}}{eQ_{l} + \pi D^{2}V_{dr}}.$$  

(A.4)

Which can be rewritten to obtain equation 3.6:

$$\alpha_{dr} = \frac{eV_{sl}}{eV_{sl} + V_{dr}}.$$  

(A.5)
Appendix B

List of variables

Greek symbols

- $\alpha_{dr}$: drop holdup
- $\theta$: inclination angle
- $\lambda_l$: liquid volume fraction
- $\mu_g$: dynamic viscosity of the gas $\text{Pas}$
- $\mu_l$: dynamic viscosity of the liquid $\text{Pas}$
- $\rho_g$: gas mass density $\text{kg/m}^3$
- $\rho_l$: liquid mass density $\text{kg/m}^3$
- $\sigma$: surface tension $\text{N/m}$
- $\tau_i$: interfacial shear stress $\text{Pa}$
- $\tau_{i0}$: shear stress causing zero interfacial velocity $\text{Pa}$
- $\tau_w$: shear stress at the tube wall $\text{Pa}$
- $\varphi$: angular coordinate $\text{rad}$
- $\Phi_d$: deposition mass flux $\text{kg/m}^2\text{s}$
- $\Phi_e$: entrainment flux $\text{kg/m}^2\text{s}$

Other variables

- $a$: constant in film thickness relation
- $b$: constant in equilibrium condition
- $C_D$: drag coefficient for spheres
- $C_{dr}$: mass concentration of drops $\text{kg/m}^3$
- $d$: film thickness $\text{m}$
- $d_0$: film thickness at $\tau_i = \tau_{i0}$ $\text{m}$
- $d_{dr}$: drop diameter $\text{m}$
- $d_{dr0}$: reference drop diameter $\text{m}$
- $d_N$: thickness of a free falling film $\text{m}$
- $D$: tube diameter $\text{m}$
- $e$: entrained fraction
- $F$: dimensionless shear stress $\frac{\tau_i}{\tau_{i0}}$
- $F^*$: dimensionless shear stress at the onset of liquid loading -
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<th>Variable</th>
<th>Description</th>
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<td>$g$</td>
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<td>$k_d$</td>
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</tr>
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</tr>
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<td>$R_{N}^*$</td>
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</tr>
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<tr>
<td>$V_{dr}$</td>
<td>axial velocity of the drops</td>
<td>m/s</td>
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<tr>
<td>$V_g$</td>
<td>velocity of the gas</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_{T}$</td>
<td>gas velocity at liquid loading according to Turner e.a. [THD69]</td>
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</tr>
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