MASTER

Multivariable control and blind disturbance identification

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Summary

The presence of directionality in multivariable control systems has consequences for the control design for these systems. Multivariable stability measures are required. Also the directions in which disturbances act on a system become important. Both these aspects are discussed in this report.

The multivariable control problem is reviewed and applied in practice, using two different experimental setups. Decentralized controller design, a design based on the diagonal terms of a decoupled plant, is applied. A sequential loopshaping design is also applied, and the stability of these designs is verified.

Identification of disturbance directions and their corresponding sources is achieved on the basis of second and fourth order statistics of data. The closed loop error signal is decomposed to reveal the underlying sources present in the measured signal. Using the found directions, the location of these disturbance sources is determined.

When the disturbance source and direction are determined, this information can be exploited in control design. This is demonstrated by designing a triangular controller which counteracts the disturbance source.
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Chapter 1

Introduction

1.1 Background

The two main subjects of the research presented in this report are control of, and disturbance identification for, multivariable systems.

1.1.1 Multivariable systems

Any system that acts in more than one direction can be considered a multivariable system. "Directionality" is the key difference between a multivariable and scalar system. With "directionality" the direction in which inputs act and outputs are observed is meant. In many cases feedback control is applied to control a multivariable (MIMO) system. The control design process of a MIMO controller also should involve taking directions in account. A multivariable control problem should not simply be broken down into multiple scalar problems without considering the implications of directionality.

From literature different design methods for multivariable controllers are available. In among others [20] and [16], multivariable control is elaborately discussed. The focus in this report will be on loopshaping control design for multivariable systems. A review of manual loopshaping design for multivariable systems is presented in [3]. Multivariable stability measures, as described in [10], [11] and [20], are necessary to evaluate stability of the manual control designs.

Besides stabilization the main reason for applying control is to minimize the influence disturbances have on the system. Each disturbance acts with a certain direction. These disturbances can be compensated for, if their characteristics and locations are known. Therefore an important aspect of the control design process is disturbance identification.

1.1.2 Disturbance identification

The identification of disturbances acting on a MIMO system can be done in many ways. Identification can be split into an identification of the source of the disturbance as well as the direction in which it acts on the system. Ideally the identification would be based on the disturbance itself.
1.1. BACKGROUND

Generally this cannot be achieved and therefore the identification has to be based on observations of the disturbance in the system outputs. These contain a mixture of the disturbances acting on the system. The separation of the observations into sources signals of the mixture is known as blind signal separation (BSS). These source signals should be a representation of the disturbance sources. If it is assumed that these disturbances are independent, obtaining independent sources would correspond to obtaining the disturbance sources.

The separation into independent sources is known as independent component analysis (ICA). Different algorithms are available that can achieve this on the basis of the statistics of data sets. Some use second order statistics (covariance) of the data, others use higher order statistics. In [6], [2], [4], [5], [13], [12], [9] and [15], the principles behind BSS and ICA are explained and illustrated. ICA has of yet not been widely used in control application, in [8] an application on closed loop data is presented. There are several fields in which ICA has already been applied to measurement data; medical applications include source separation of MRI and ECG data. Another well known example is the separation of audio signals, a famous example is the ‘cocktail party problem’, see [21]. In this example different sounds like music and speech are separated according to their sources of origin.

Besides being useful in control design, obtaining disturbance sources can also be useful in system design. The directions found with ICA indicate the location at which disturbances are physically located in the system. This information can be exploited in an adaptation or redesign of the system.

1.1.3 Vibration isolation

The main setup used for this research is a vibration isolation table, described further in chapter 3, [19] and [24]. The vibration isolation problem involves a between low frequent disturbances and active control of disturbances acting on the isolated mass of the system. The vibration isolation problem finds a practical application in systems that require an accuracy with a micrometer or nanometer range such as wafer steppers. At this level of accuracy, disturbances acting on a system through the floor become significant. To reduce them, the system is placed on a large mass which is passively isolated from the floor using a low stiffness suspension, for example air mounts. In this way the eigenfrequency, at which the system is decoupled from the floor, is very low. However the drawback is that any force applied to the isolated mass will cause a large movement as the suspension has a very low stiffness. By adding actuators to the system this disturbance force can be compensated actively, reducing movement of the mass. In the vibration isolation problem, the identification of the disturbances is especially important as the control action to be taken depends on the location of the disturbance. A disturbance originating from the table requires a different control action than a disturbance caused by floor vibration.
1.2 Objectives

The objectives of this research are multivariable control as well as disturbance identification. The active vibration control problem is a good illustration for both disturbance separation as well as multivariable control. Combining blind source separation with multivariable control problems leads to the following problem statement:

Can higher order statistics of sensor observations be used to identify and separate disturbance sources present in multivariable dynamical systems?

To answer this question, a search was conducted on multivariable systems. Several objectives concerning disturbance identification and multivariable control are set:

- Try to improve the decoupling of a system using statistical methods.
- Apply manual multivariable control design methods on a decoupled plant and evaluate stability.
- Identify the source and direction of a disturbance present in more than one controlled axis of a multivariable dynamical system.
- Determine the location of this source using the obtained direction.
- Demonstrate a case in which disturbance knowledge can be exploited in a multivariable control design to improve the system's performance.

1.3 Approach

The approach taken in order to demonstrate MIMO control design as well as the use of blind source separation in control can be summed up in five main steps:

- Identification of two different MIMO setups will be performed.
- Three different decoupling methods will be evaluated for these setups.
- Decentralized control design is done for one of the setups, a sequential loopshaping design for the other.
- Two blind source identification algorithms will be reviewed. Blind source separation will be performed on experimental data from a MIMO setup.
- Finally, a non-diagonal control design based on sources found using BSS is designed and implemented.

1.4 Outline

The approach presented in the previous section is described in the seven chapters of this report. First of all, the second chapter gives some general consideration and the basic configuration of the
control problem at hand. Next, the third chapter reviews the two setups used for the research. Their identification is described. The fourth chapter discusses multivariable control design and three decoupling methods. Specific control design for the vibration isolation problem and multivariable stability criteria are also discussed. The fifth chapter presents the blind source separation problem and two BSS algorithms. These algorithms are applied to measurement data. Additionally allocation of the disturbances is demonstrated based on BSS. In the sixth chapter the design and application of a non diagonal controller are presented. This controller compensates for an identified disturbance using direction information obtained with BSS. In the final chapter, conclusions are drawn from the presented research. Recommendations for future research are also made.
Chapter 2

Disturbances in multivariable feedback systems

In many feedback control systems disturbances are present, which limit the performance of these systems. An important task of the controller is to minimize the effect of these disturbances on the performance of the system. When multivariable (MIMO) systems are considered, these disturbances will generally act on more than one axis of the system.

Linear multivariable systems can be described in the frequency domain by transfer functions from each input variable to each output variable. For a plant \( G(s) \) with \( i \) outputs \( y \) and \( j \) inputs \( u \), the transfers from input \( j \) to output \( i \) are given by the function \( G_{ij}(s) \).

\[
y_i(s) = G_{ij}(s)u_j(s) \tag{2.1}
\]

Throughout this report, this notation will be used for all multivariable systems. For the plant the dependency on Laplace variable is omitted for ease of notation: \( G = G(s) \). In figure 2.1 a basic feedback configuration is given. Both input and output disturbances \( d_i, d_o \) are the inputs of the disturbance models \( G_{d_i} \) and \( G_{d_o} \) respectively. In the output additional sensor noise \( n \) is present.

![Figure 2.1: Feedback configuration](image-url)
The transfers from the input and output disturbances to the closed loop error \( e \) is given by:

\[
e_{d_o} = -S_o G_{d_o} d_o
\]

(2.2)

\[
e_{d_i} = -S_o G G_{d_i} d_i
\]

(2.3)

With the output sensitivity:

\[
S_o = (I + GK)^{-1}
\]

(2.4)

Minimizing the effect of these disturbances will improve the performance of the system. The output sensitivity \( S_o \) and the disturbance model \( G_{d_o} \) determine the direction with which the output disturbance is observed in the error \( e \). Certain directions of the system will be more sensitive than others depending on plant and controller. The effect of the output disturbance \( G_d \) on an open loop \( i \) is given by \( g_{di} \). The relative disturbance gain (RDG) provides a scaling independent measure for the change in gain of a disturbance in direction \( i \) caused by decentralized control.

\[
\beta_i \triangleq \frac{\tilde{g}_{di}}{g_{di}} = \frac{\tilde{G}G^{-1}G_{d_i}}{[G_{d_i}]_i}
\]

(2.5)

In the ideal case the alignment of the disturbance \( G_d \) with plant \( G \) is such that the disturbance is not observable in error \( e \), the disturbance \( G_d \) lies in the nullspace of \( G \).

As an example a system consisting of two input and two outputs with small mutual interactions is considered. The closed loop system has a diagonal lead lag controller \( K \). A disturbance with a frequency of 2 Hz is observed in the outputs of the system with direction: \( G_d = [0.3 \ 0.95]^T \). To reduce the error caused by this disturbance, its direction is taken into account. A notch filter \( N(s) \) with both poles and zeros at 2 Hz is added to the controller \( k_{ii} \rightarrow g_{di}Nk_{ii} \). This locally increases the gains in each loop with the direction of \( G_d \). This corresponds to a locally decreased \( S_o \) and a reduction in the closed loop error. In figure 2.2 the output sensitivity of the original and modified controller are plotted.

An equivalent adaptation of the controller could have been achieved with a SISO approach on each loop separately, not taking \( G_d \) into account. However an advantage of taking direction into account directly is that only one disturbance model, not two, is required, in this case the notch filter. Furthermore if more than one disturbance works on a given system, modeling their projection on each axis can become complex as both models will be of an order equal to the sum of the orders of all disturbance phenomena. An error spectrum can be built up of several (lower order) components. Obtaining these components (sources) is the objective of blind source separation, further discussed in chapter 5.
Figure 2.2: Adapting the output sensitivity taking disturbance directions into consideration
Chapter 3

MIMO Setups

During the research conducted, two setups were used. First of all a system with two inputs and two outputs, called the twindrive. The second system is a vibration isolation table, the Active Vibration Isolation System (AVIS), which has eight inputs and six outputs. Both setups are used to illustrate the application of MIMO control theory and disturbance identification in practice. In the next chapter decoupling and the application of multivariable control on these setups will be discussed. First, both setups will be described briefly and an identification in the frequency domain will be presented, starting with the twindrive system.

3.1 Twindrive

The twindrive setup consists of two identical DC motors, each connected to a small disk. The only coupling between the two motors is an elastic belt which is wrapped around both disks. The positions of the two wheels are measured with encoders, which are connected to the axes of each of the motors. The encoders signals are captured using a TUEdacs device. The motors are powered by two current amplifiers. The input voltages for these amplifiers are also supplied by the TUEdacs device, which is connected to a laptop running realtime Linux. In figure 3.1 a picture of the twindrive is shown.

![Figure 3.1: The twindrive setup](image-url)
3.1. TWINDRIVE

3.1.1 Identification

The dynamics of the twindrive are identified by performing frequency response measurements. White noise is injected into the first and second input, \( u_1 \) and \( u_2 \) of the system respectively. In each experiment the resulting encoder outputs \( y_1 \) and \( y_2 \) are measured. From this first measurement an open loop FRF of the system is determined. Based on this first FRF a low bandwidth PD controller is designed for each loop based on the diagonal terms. This is done to enable a closed loop FRF identification. This controller is implemented in the first loop and a closed loop input sensitivity measurement is performed, both outputs are measured. The transfer from input \( u_1 \) to output \( y_2 \) can be determined directly. For the other input the same measurement is performed. From these measurements the frequency responses shown in figure 3.2 were obtained. The twindrive system has two main modes, a flexible mode and a rigid body mode. The diagonal terms \( G_{11} \) and \( G_{22} \) consist of a -2 slope i.e. the inertia of motor and wheel, as well as an anti-resonance and a resonance. The resonance is caused by the flexibility of the elastic band. The anti-resonance at 10.5 Hz is caused by the second mass resonating with an amplitude and phase that it nearly cancels out the resonance of the first mass. The cross terms \( G_{12} \) and \( G_{12} \) consist of a -2 slope below the resonance frequency, but for frequencies above the resonance the undriven mass starts lagging behind the driven mass. The undriven mass eventually lags an entire period behind the first, a -4 slope.

The two modes can clearly be identified by performing a singular values decomposition of the FRF matrix \( G(s) \) at each frequency. The found singular values are sorted according to their mode and plotted in figure 3.3 for a frequency range from 4 to 100 Hz. The black line represents the rigid body mode, the gray line the flexible mode of the twindrive. The rigid body mode represents an equal movement of the two engine inertias, combining into one greater inertia, the mass line in figure 3.3. In the flexible mode the systems two disks move in exactly opposite direction. A method to decouple this system in to its two main modes is presented in the next chapter.
3. MIMO SETUPS

3.2 Active Vibration Isolation System

Besides the twindrive, experiments were also carried out on a vibration isolation system manufactured by IDE engineering, a picture of the system is shown in figure 3.4. This setup was also studied in [19] and [24]. The system is referred to as AVIS, short for *Active Vibration Isolation System*. Despite the given name the system combines passive as well as active vibration isolation. The system consists of a steel frame supported on four legs and a rectangular table. On this frame an aluminium plate is mounted, and on this base four vibration isolation modules are placed, see [14]. The upper part of these modules is connected to the table and contains sensors and a part of
the actuators. In figure 3.5 the location of the modules is shown. The isolation modules have three main components, the sensors, the airmounts and the actuators. The rectangular table is placed on these airmounts. Passive vibration isolation is achieved by the low stiffness of pressurized air \( k_{\text{air}} \) in \( z \)-direction. The airmounts are fed from a constant pressure source with an air pressure of 4 Bar. The height of the table can be adjusted through valves on the airmounts. Note that the fourth airmount can not be adjusted in height independently from airmount 3. These two airmounts are interconnected and will level out automatically depending on the valve setting of module 3. Passive vibration isolation in \( x \) and \( y \)-direction is achieved by metal roll-off diaphragms. Their low stiffness \( k_{\text{diap}} \) combined with the mass of the table provides a low eigenfrequency in the \( x \) and \( y \)-directions.

The AVIS has eight actuators, two on each module, that can be used for active vibration isolation. Each actuator consists of a set of two permanent magnets with opposite poles fixed to the frame. And a set of coils fixed on the table, with the center of these coils located between the two permanent magnets. These linear motors can create a force between the base and the table. Two of the actuators work in \( x \)-direction, two in \( y \)-direction and four in the \( z \)-direction. In figure 3.5 the configuration of the actuators is depicted schematically in a side and a top view. Note that

![Figure 3.5: Stiffnesses and actuator forces of the AVIS](image)

the AVIS is not exactly symmetric, the actuators in \( x \)-direction are located at different distances to the center of mass than the \( y \)-direction actuators. Each block of actuators is rotated 90 degrees with respect to the previous model, see figure 3.5. This introduces coupling between the rotational axes and \( z \)-axis. For example for an rotation \( \phi \) the forces of the \( z \)-actuators are chosen equal to achieve such a rotation, \( F_{z2} = F_{z3} = -F_{z1} = -F_{z4} \). Then the moments applied by the \( z \) actuator of modules two and four on the center of mass will have a different magnitude than the moments exerted at modules one and three. As a result the system will have non-zero rotations \( \theta \) and \( \psi \). Six absolute velocity sensors, known as geophones, are used to measure the absolute velocities of the table \( \dot{q} \). These are located in the upper part of module one, two and three, which are rigidly attached to the underside of the table. So the AVIS has eight inputs and six outputs. For more
3. MIMO SETUPS

Details about the actuators and sensors see Appendix A and [19]. Besides the vibration isolation table some extra hardware is necessary to monitor and control the system. The geophone signals are amplified by a current amplifier. The amplified signals are read out with a Quanser Q8 board, see [18]. This board communicates with a PC running xPC Target, see [17]. This target PC is controlled using the host PC, both PC’s are connected to each other using a crosslink cable to transfer the data.

3.2.1 Kinematic decoupling

For control purposes it can be desirable to have a square plant, for instance for applying a diagonal controller. In [19] a kinematic decoupling is derived to obtain a square plant for the AVIS system. Using the location and polarity of the actuators and sensors, a sensor matrix $T_q$ and an actuator matrix $T_u$ are determined that decouple the AVIS in the generalized coordinates $q$ and inputs $u$. With $q$ defined as:

$$q = [x\ y\ z\ \phi\ \theta\ \psi]^T \quad (3.1)$$

The angles $\phi$, $\theta$, and $\psi$ represent the rotations around the $x$, $y$ and $z$-axis respectively. The sensor matrix $T_y$ is square, $T_y \in \mathbb{R}^{(6\times6)}$. The actuator matrix $T_u$ has a dimension $T_u \in \mathbb{R}^{(8\times6)}$. By pre and post-multiplying the non-square plant with these matrices the square plant $G$ is obtained:

$$G(s) = T_y \cdot G_{ns}(s) \cdot T_u \quad (3.2)$$

The matrices $T_y$ and $T_u$ are given in Appendix B. This square plant $G$ is the basis on which further identification and control design is performed. It is identified with six open loop FRF measurements.

3.2.2 Identification

To identify the square system white noise is injected into all six inputs respectively and the six resulting outputs are measured each time. Using the measured time signals a transfer from each input $u_i$ to the six outputs $y$ is determined. The diagonal terms $G_{ii}$ are shown in figure 3.6. For a complete frequency response measurement see appendix C.

Several resonances are observed in the FRF’s of the diagonal terms of the AVIS plant $G$. The combination of the table mass and the stiffness $k_{diap}$ of the diaphragms in $x$ direction causes the low frequent resonance at 1.6 Hz in the $x$ translation of the table $G_{11}$. The first resonance in the $y$ translation $G_{22}$ is caused by the same table mass combined with the stiffness of the roll-of diaphragms $k_{diap}$ in $y$ direction. As these are almost equal the resonance frequency in $G_{22}$ is approximately the same. In $z$-translation, rotation $\phi$ and $\theta$, three low frequent resonances are present. In section 4.1.5, a method to decouple the system in its six main dynamical modes will be presented.

Next the application of active vibration isolation using the available actuators will be discussed. The high frequent modes seen in all the transfers are the components of the table, that all have eigenfrequencies above 100 Hz. The resonance at 121.5 Hz limits the bandwidth of the closed loops of the system. But in the next chapter will be shown that despite static decoupling transformations, residual couplings between the axes are still present which can cause instability of the multivariable closed loop system.
Figure 3.6: Diagonal terms of the openloop FRF of the AVIS
Chapter 4

Multivariable control

In this chapter the application of multivariable feedback control to the twindrive system as well as the AVIS is discussed. First the concept of additive uncertainty is presented. Then static decoupling is discussed and three decoupling methods are presented. The decoupled AVIS plant is then used for control design. Two approaches for multivariable control design are presented in this report:

- Decentralized control, which will be demonstrated on the AVIS system.
- Sequential loopshaping, which is applied to the twindrive system.

Two multivariable stability measures are reviewed. Finally a motivation for disturbance identification and separation of disturbances is given from a control point of view.

4.1 Additive uncertainty

To make manual loop shaping of a multivariable system possible, it has to be broken down into scalar designs based on the diagonal terms of the plant. Interaction can be represented as an additive uncertainty in the plant, see [10]. Even though the cross terms can be identified, they are still treated as uncertainties. This uncertainty is the difference between the diagonal $\tilde{G}$, containing...
only the diagonal terms of $G$, and the full plant $G$. A schematic representation of the concept of additive uncertainty is given in figure 4.1. The uncertainty is scaled with the diagonal plant $\tilde{G}$ to obtain an error system $E$.

$$E = (G - \tilde{G})\tilde{G}^{-1}$$ (4.1)

The system $E$ is the relative error made when assuming a diagonal system, i.e. the cross terms of the plant are taken equal to zero. The closed loop system $\tilde{H}$ on basis of the diagonal plant $\tilde{G}$ and a designed controller $K$ can be defined.

$$\tilde{H} = \tilde{G}K(I + \tilde{G}K)^{-1}$$ (4.2)

The controller $K$ of this system is diagonal and represents the result of a decentralized control design procedure based on $\tilde{G}$, see section 4.2. Using the definitions of $E$ and $\tilde{H}$ the return difference operator of the original system can be factorized as:

$$(I + GK) = (I + E\tilde{H})(I + \tilde{G}K)$$ (4.3)

This factorization is the basis of many multivariable stability criterions.

4.1.1 Decoupling

Decoupling is a commonly used approach in control design for multivariable systems. The goal of decoupling is to minimize the error system $E$, so that the return difference of the full multivariable system corresponds as closely as possible with the decoupled return difference depending on the diagonal open loops $\tilde{G}K$, see equation 4.3. Then, scalar control design techniques can then be used to stabilize a multivariable plant.

In an approximately decoupled plant, interaction remains which can cause instability of the closed loop system, as will be shown in section 4.1.2. To determine if the full system is stable a multivariable stability criterion is required. These stability criteria are based on interaction measures, which will be discussed in the next section. The general decoupling transformation to a diagonal system $G_{\text{diag}}$ can be written as a matrix decomposition:

$$G(s) = T_uG_{\text{diag}}(s)T_y$$ (4.4)

The application of these matrices in the basic control configuration as defined in the introduction, is depicted in figure 4.2. For the decoupling methods discussed in this report, the matrices $T_u$ and $T_y$ are static. To obtain output decoupling matrices $T_u$ and/or input decoupling matrices $T_y$, three methods are given in sections 4.1.3, 4.1.4 and 4.1.5. The quality of decoupling is evaluated using the relative gain array (RGA). This measure provides a tool to study two-sided interaction between elements of the plant $G(s)$, see [20]. For the calculation of the RGA array see Appendix E.
4. MULTIVARIABLE CONTROL

4.1.2 Stability criteria

As stated, closed loop stability of the diagonal terms of the plant does not guarantee stability of the full closed loop system when interaction is present. Several measures for establishing the stability of a multivariable system are available. Two of them, the characteristic loci and the spectral radius will be discussed briefly here.

Characteristic Loci

The characteristic loci of a system are the eigenvalues of an open loop transfer $GK$, evaluated for a range of frequencies $j\omega$. The characteristic loci, also known as eigen loci, can be calculated directly as the eigenvalues of a measured FRF of the open loop. They are based directly on the unfactorized system, the left side of the equality in equation 4.3.

$$\lambda(G(j\omega)K(j\omega)) \quad (4.5)$$

The generalized Nyquist criterion holds for the characteristic loci and is used for stability analysis. A disadvantage of this measure is that the phase and gain margin in the Nyquist plot are only valid margins when a simultaneous change in all parameters is applied, see [20]. For example, multiplying a multivariable controller $K$ by an arbitrary constant $\alpha$. This simultaneous change in all loops will lead to a change $\alpha$ in the eigenvalues:

$$\lambda(\alpha \cdot G(j\omega)K(j\omega)) = \alpha \lambda(G(j\omega)K(j\omega)) \quad (4.6)$$

From this can be concluded that for a simultaneous and equal change of controller gains, the margins as defined for a SISO problem in the Nyquist plot, are also valid for the MIMO case. However an uncertainty of a gain $\beta$ only applied to some of the elements of the open loop will decrease the gain margin with less than $\beta$. This influence of changing the gain in one loop can not be determined directly from the characteristic loci plot.

Spectral radius criterion

Another way to evaluate the stability of the full system is evaluating the spectral radius of the product of the error system $E$ and the diagonal closed loop $\tilde{H}$. The spectral radius of $H$ is given by the maximum of the absolute value of the eigenvalues $\lambda_i$ of $H$:

$$\rho(H) = \max_{1 \leq i \leq n} |\lambda_i(H)| \quad (4.7)$$

This measure is based on equation 4.3. The multivariable closed loop transferfunction matrix $H$ is guaranteed to be stable if $(I + \tilde{G}K)$ is stable and the spectral radius of this product is smaller than one for all frequencies $j\omega$:

$$\rho(E(j\omega)\tilde{H}(j\omega)) < 1 \quad \forall \ \omega \quad (4.8)$$

Note that this is a very conservative measure of stability as the phase of the eigenvalue is not taken in to account. The system $E(j\omega)\tilde{H}(j\omega)$ can have one or more eigenvalues with an absolute value greater than one, as long as there is a sufficient phase margin. As with the characteristic loci, margins for each control loop cannot be derived directly from the spectral radius plot.
4.1.3 Decoupling of dyadic transfer matrices

For dyadic transfer matrices (DTM) a decoupling as described in [1] and [22] can easily be obtained. An additional application is given in [23]. A square system $G$ of dimensions $n$ by $n$ is dyadic if it has the following properties:

$$\det(G(s)) \neq 0$$  \quad (4.9)

$$G = T_u \text{diag}(g_{11}, \ldots, g_{nn})T_y \quad T_u, T_y \in \mathbb{R}^{n \times n}$$  \quad (4.10)

The approach is based on the eigenvalue decompositions of two data points at frequencies $f_1$ and $f_2$ in the FRF data of the plant $G$, see figure 3.2. To obtain an approximately diagonal plant $\tilde{G} = G_{DTM}$ transformation matrices $T_y$ and $T_u$ as in equation 4.4 are applied to the plant $G$. The constant and real matrices $T_u$ and $T_y$ can be obtained from two eigenvalue problems, based on two points $f_1$ and $f_2$ of a FRF, see appendix D. For the twindrive, the elements of the found matrices have negligible imaginary parts. For a dyadic system $T_u$, $T_y$ are real matrices, so this system is approximately dyadic. The DTM decoupling is implemented in the feedback loop as in the general decoupling configuration, depicted in figure 4.2. To obtain the correct reference the vector $r$ is also multiplied with $T_y^{-1}$.

When DTM-decoupling is applied to the plant $G$ of the twindrive setup the cross terms $G_{12}$ and $G_{21}$ are significantly reduced, see Figure 4.3. The characteristics of the first mode were measured with a constant velocity reference, $r = \begin{bmatrix} v_1 & 0 \end{bmatrix}^T$. The second mode was measured with a sine reference $r = \begin{bmatrix} 0 & \alpha \sin(t) \end{bmatrix}^T$. As visible in Figure 4.3 the first diagonal term, $G_{DTM_{11}}$, is now transformed to a mass-line i.e. the rigid body mode. The diagonal term $G_{DTM_{22}}$ represents the
flexible mode. From the entries in the matrices $T_u^{-1}$ and $T_y^{-1}$ the two modes in which the plant is decoupled are:

\[ T_u^{-1} = \begin{pmatrix} 0.70 & -0.69 \\ 0.71 & 0.73 \end{pmatrix} \quad T_y^{-1} = \begin{pmatrix} 0.71 & 0.71 \\ -0.70 & 0.71 \end{pmatrix} \] (4.11)

Although DTM decoupling was successfully applied here, the DTM decoupling method has some disadvantages. It is not clear beforehand which frequencies $f_1$ and $f_2$ will result in the desired decoupling. It is possible to formulate an optimization problem to find these frequencies, see [22]. Selecting a data point with good coherence is not guaranteed to result in a decoupled plant. For example, for some choices for $f_1$ and $f_2$, the transformation results in a system with cross terms of increased magnitude instead of the desired magnitude reduction. Despite these disadvantages, it turns out to be straightforward to find the frequencies $f_1$ and $f_2$ for the twindrive plant for which a satisfactory decoupling of the plant is obtained.

### 4.1.4 Decoupling based on Joint diagonalization

The AVIS system is approximately decoupled using kinematic decoupling. An attempt is made to further improve the decoupling. To obtain a diagonal plant a transformation has to be applied to the matrix $G(j2\pi f)$, $f \in [f_1, \ldots, f_2]$ such that this matrix is as diagonal as possible in this whole frequency range. This is the objective of the joint diagonalization algorithm presented in [7], see appendix F. More applications of this algorithm will be discussed further in the next chapter in the context of disturbance identification. Here only the results are presented For a range from 1 to 90 Hz the matrices $G(s)$ of an open loop FRF measurement, are jointly diagonalized. The transformation matrices are based on the joint diagonalization matrix $A_{JD}$ found with the algorithm. For the decoupling based on joint diagonalization, the matrices $T_u$ and $T_y$ are given by:

\[ T_u = A_{JD}^T; \quad T_y = A_{JD} \] (4.12)

This results in an improved decoupling of the plant. This is shown in figure 4.4. In this figure the RGA value, a well-known measure of interaction, see [20], is plotted for the original kinematically decoupled plant and this plant jointly diagonalized as in equation 4.4. Some improvement around 4 Hz is observed in the $\phi$ and $\theta$ axes. Below and above the range selected for joint diagonalization the diagonalized plant and the original plant are not well decoupled. This is not of great concern however, as these frequency ranges lie outside of the region of interest for control design. A property of the joint diagonalization of the AVIS plant is the fact that the original coordinates are rotated, a unitary transformation is applied.

### 4.1.5 Decoupling based on JADE

Another approach for decoupling on the basis of the FRF data is decoupling this data into statistically independent components. In this case, these components are determined with the JADE algorithm. A description of the JADE algorithm is given in [2]. The objective of the JADE algorithm is to find a linear decomposition of a signal into statistically independent sources. JADE will also be discussed in a disturbance identification context in the next chapter. The decoupling is a linear decomposition of the data with a matrix $A_{JADE}$. For this decoupling the transformation matrices defined as in the decoupling equation 4.4 are given:

\[ T_y = A_{JADE}; \quad T_u = I \] (4.13)

The six main low frequent modes of the AVIS represent independent sources of the FRF data. To be able to apply the algorithm, the measured FRF matrices for each frequency point are grouped.
Figure 4.4: RGA of $G_{JD}$ compared to RGA of the original plant $G$

according to their outputs.

$$Y = [(G(f_1) \; \ldots \; G(f_n))] \quad (4.14)$$

Matrix $Y$ can now be decomposed with JADE to find a matrix $A_{JADE}$ so that each row of $A_{JADE}^{-1}Y$ constitutes an independent source present in the output of the system. For a chosen frequency range of $f_1 = 0.5 \text{ Hz}$ to $f_2 = 30 \text{ Hz}$ this yields a separation of the low frequent suspension modes. These modes appear on the diagonal of the transformed system $G_{JADE}$. Three suspension mode resonance peaks were present in the $z$-translation, $x$-rotation and $y$-rotation. These modes are separated with the decoupling, see figure 4.5. The remaining diagonal terms are dominated by one of the low frequent modes so they remain almost unaltered after the decomposition. The interaction between the diagonal terms of $G_{JADE}$ is evaluated using the RGA. The RGA for these axes is plotted in figure 4.6. The interaction is negligible for the first, second and sixth loop. The third, fourth and fifth loop show strong mutual interaction.

Clearly the JADE based decomposition is not a suitable method for decoupling the system. However it provides insight in the system. The matrix $A_{JADE}$ provides the directions of the main modes present in the system.

4.2 Decentralized control

The decoupling methods described in the previous sections have the objective to transform a plant to a diagonal form. The aim was to reduce the interaction before a decentralized controller is designed. In decentralized control a diagonal controller $K$ is designed based on each of the diagonal terms of a decoupled plant $\hat{G}$ separately. This follows from the decomposition given in equation 4.3. Stability of the diagonal system is checked taking into account SISO stability margins and bandwidths. As described in section 4.1.2 interaction can cause instability. Therefore
4. MULTIVARIABLE CONTROL

Figure 4.5: Diagonal terms of the plant $G_{JADE}$

the interaction measure needs to be determined for the open loop containing the full system. The decentralized control approach is demonstrated on the AVIS.

4.2.1 AVIS control problem

On the basis of the decoupled AVIS plant, a decentralized controller will be designed. The control objective for the vibration isolation system is split into two main goals:

- Decrease the accelerations of the table resulting from disturbances acting on the table surface.
- Decrease the acceleration of the table due to floor vibrations.

Even though both requirements amount to reducing the acceleration of the table they are split into two categories. A distinction is made between the disturbances originating from the floor and those applied to the table directly. To get an idea of the disturbances a measurement of the velocities is performed without any input signals $u$ and no disturbances artificially applied to the table. In figure 4.7 the spectrum of the velocity measurement of the AVIS is plotted for this measurement. The resonance frequencies of the low frequent suspension modes are clearly visible in the spectrum. The problem of active vibration isolation is explained using a simplified one dimensional problem as depicted in figure 4.8. The mass $m$ representing the table is connected to the floor with a spring with low stiffness $k$ and a damper with damping constant $d$. The transfer
function $H_{\text{pass}}$ from floor movement to the position $x$ of the mass $m$ is given by:

$$H_{\text{pass}}(s) = \frac{X(s)}{Q_0(s)} = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

(4.15)

$$\omega_n = \sqrt{\frac{k}{m}}; \quad \xi = \frac{d}{2m\omega_n}$$

(4.16)

Passive isolation is improved by trying to place the eigenfrequency $\omega_n$ of the mass-spring-damper system as low as possible. For frequencies above this eigenfrequency the magnitude of transfer $H_{\text{pass}}$, decreases, isolating mass $m$ from higher frequent disturbances. Due to physical limitations, there is a limit to which mass $m$ can be increased and stiffness $k$ decreased. Also by designing the vibration isolator as a mass-spring-damper system, a resonance peak in the transfer from floor to mass position is introduced. Increasing the passive damping to damp this resonance is not desired. This will move the zero in the numerator of equation 4.15 to a lower frequency. This in turn will result in deteriorated disturbance rejection at higher frequencies. In figure 4.9 the transfer functions for the system $H_{\text{pass}}$ are plotted for low damping $\xi = 0.4$, and added passive damping $\xi = 2$.

A control strategy that can be used to actively reduce the vibrations of the system is sky-hook damping. Sky-hook damping is negative feedback of the absolute velocity $\dot{x}$ with gain $d_{\text{act}}$ to add damping to the system. The transfer from position then becomes:

$$H_{\text{act}}(s) = \frac{X(s)}{Q_0(s)} = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\omega_n(\xi + \xi_{\text{act}}) + \xi)s + \omega_n^2}$$

(4.17)

$$\xi_{\text{act}} = \frac{d_{\text{act}}}{2\omega_n m}$$

(4.18)

In which $d_{\text{act}}$ is the gain for the velocity feedback $-d_{\text{act}}\dot{x}$ which is applied between the floor and mass, see figure 4.8. The dashed line in figure 4.9 shows the transfer $H_{\text{act}}$ but now with an added
active damping constant $d_{adc} = 5$. The actuators provide a force $F$ between floor and mass $m$ which can be seen as a damper acting on the system from a fixed point, a 'skyhook', see figure 4.8.

The second goal is to reduce the vibrations which are caused by forces acting on the table. In the one dimensional example, a disturbance force $F_d$ is applied to mass $m$. In a vibration isolation system with a very low suspension stiffness $k$, the transfer from a force $F_d$ to the mass acceleration $\ddot{x}$ has a high magnitude. The actuator can be used to create a force between floor and mass that counteracts the disturbance force. To be able to counteract a disturbance the bandwidth should be chosen at least equal to the highest frequency disturbance acting on the system. However for multivariable systems this is not necessarily the case. A way to compensate for a disturbance above the bandwidth is presented in Chapter 6.
4.2.2 AVIS Decentralized control design

Considering the requirements stated in the previous section a decentralized controller is designed. This is done on basis of the decoupled plants. For both the plant $G_{JADE}$ as well as the jointly diagonalized plant, $G_{JD}$, a very similar controller was designed. These designs will therefore not be presented separately. A diagonal controller $K$ is designed, in which each diagonal term in $K_{ii}$ stabilizes the corresponding diagonal term in $G_{ii}$, and achieves the desired bandwidth. This bandwidth is chosen at 20 Hz to allow for a compensation of disturbance on the table surface, from for instance a servo system placed on the table. To achieve this bandwidth the controller gain cannot simply be increased, this will cause instability due to the high gain which is obtained at the first resonances of the system. Therefore the bandwidth has to be achieved by using a lead lag filter. Furthermore for low frequencies the system should not be actively controlled. As low frequent disturbances which originate from the floor cannot be actively reduced. Applying control action for floor disturbances cannot improve the systems performance, only deteriorate it.

As the dynamics of the plant are similar for each of its axes, the controllers $K_{ii}$ have a similar structure. This structure can be summarized as follows:

- At frequencies below the first suspension mode, the open loop has a low magnitude, no significant control force is applied to the system.
- The static gain is chosen in such a way that around the resonance the control loop has a magnitude greater than 0 dB to add skyhook damping to the closed loop system.
- Using a lead lag filter the -1 slope of the plant is lifted to a zero slope up to the desired bandwidth. In this case 20 Hz.
- Above a second order low pass is used to decrease the open loop gain for higher frequencies.
Additional notches are applied to compensate for higher frequent resonances present above the bandwidth. These resonances originate from different parts of the frame, the modules and actuators and can cause instability. The lowest bandwidth limiting resonance of the diagonal systems is located at approximately 121 Hz.

The designed diagonal controller is implemented on the AVIS system. The resulting open loops of the diagonal terms of the controller $K$ and the diagonals of $G_{JD}$ are plotted in figure 4.2.2. After designing this initial diagonal controller $K$, the stability of the entire system, so including the off diagonal terms, has to be checked with a multivariable stability measure. This is necessary as interaction is present in the decoupled system which can cause instability. Both interaction measures discussed in section 4.1.2 where used to evaluate the stability of the designed controller. In the left plot in figure 4.11 the characteristic loci of the open loop $G_{JD}K$ are depicted. In the right plot the eigenvalues of the open loop $G_{JD}K$ are shown. There is clearly a significant difference between the characteristic loci of the diagonal and full plant. Therefore stability must always be checked for the full plant. The main drawback for using the characteristic loci for control design is the absence a useful margin for the stability criterion. As long as the eigenvalues do not encircle the point $[-1, 0]$, the system is stable, but for a change in any parameter of the system the stability is not guaranteed. This makes design for performance difficult. The SISO design can be done with performance in mind, and the resulting performance of the full plant is assumed to approximate this performance.

As for $G_{JD}K$ the characteristic loci of the open loop design $G_{JADE}K$ are also calculated, see figure 4.12. Clearly one eigenvalue passes through the point $[-1, 0]$ indicating that this system is on the edge of stability. The fact that the eigenvalues of this system are different from the ones of the jointly diagonalized system plotted in figure 4.11 is explained by the fact that the JADE decoupling matrix is not unitary and alters the eigenvalues of the system. In this case there is one eigenvalue which differs significantly from the diagonal system. This is due to the interaction. It can be concluded, the coupling observed in the JADE plant $G_{JADE}$, see figure 4.6, between the

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Figure 4.10: Diagonal terms of the open loop $G_{JD}K$
third fourth and fifth loops causes instability of the closed loop system. This despite the fact that all six scalar decentralized designs are stable. Decreasing the gain of the controller of the fourth and fifth loop, moves the eigenvalue away from the point -1. The bandwidth of both these loops is lowered to 10 Hz to obtain a stable system, as the lower controller gain provides a open loop with a greater gain margin. As there is significantly less coupling between the third axis and the fourth and fifth axes the bandwidth of the third loop does not have to be lowered to achieve a stable system.

As stated before another way to check the stability of a multivariable control system is evaluating the spectral radius. This measure is evaluated for the jointly diagonalized open loop, see figure 4.13. This measure clearly indicates instability as there are clear peaks above 0 dB line, when the characteristic loci plot of the same open loop, see figure 4.11, clearly show stable eigenvalues. The fact that phase is not taken to account in the spectral radius criterion means it is not useful in control design for this system. Evaluating stability using the spectral radius can lead to a overconservative design, with an unnecessary low performance.
4.3 Sequential loopshaping

Another control design method for multivariable systems is sequential loop shaping. This method will be demonstrated using the twindrive system, see section 3.1. The basic idea behind this method is the factorization of the determinant of the open loop system as a product of the determinants of the $n$ open loops. This factorization is obtained using Schur’s formula for the determinant of a partitioned matrix, see [20]. For a two dimensional plant, such as the twindrive, this results in the factorization:

$$
\det(I + GK) = \det(1 + g_{11}k_{11}) \cdot \det(1 + g_{22}^*k_{22})
$$

$$
g_2^* = g_{22} - \frac{g_{21}k_{11}g_{22}}{1 + g_{11}k_{11}}
$$

The general configuration for the sequential loopshaping is given in figure 4.14. Designing the controller for the first open loop $g_{11}k_{11}$ can be done with a scalar SISO design method. After this first loop is closed an equivalent plant $g_{eq}^{22}$ from input $u_2$ to output $y_2$ is defined. In figure 4.14 this equivalent plant is indicated with a dashed line. The transfer of $g_{eq}^{22}$ loop is equal to the second factor in the determinant factorization, see equation 4.21, $g_2^* = g_{22}^*$. A stable control design for $g_{eq}^{22}k_{22}$ combined with the stable design for $g_{11}k_{11}$ ensures that the full system $GK$ is also stable. In short the procedure for sequential loopshaping consists of the following steps:

1. Perform an FRF identification of the transfer from the next input to the corresponding output, while all previously closed loops remain closed. The first identification is done with all loops open.

2. Design a controller based on the found FRF.

3. Close the feedback loop by implementing the designed controller.

4. Check the stability of the last closed loop and the margins of all other closed loops.

5. Continue with the first step if any unclosed loops remain.
Although the stability requirement in 4.21 is based on the SISO open loop $g_{11}k_{11}$, the characteristic of the first loop is altered after implementing the controller $k_{22}$. The equivalent plant $g_{11}^{eq}$ is related to the original plant $G$ and controllers $k_{11}$ and $k_{22}$ as:

$$g_{11}^{eq} = g_{11} - \frac{g_{12}k_{22}g_{21}}{1 + g_{22}k_{22}}$$

(4.22)

For this altered first loop characteristic $g_{11}^{eq}$, margins of the open loop can change. Therefore, as stated in step 4 of the procedure, the margins of all the already designed loops have to be checked again as a next loop is closed. The procedure described was followed for the twindrive system. A controller $k_{11}$ is designed based on the first diagonal term $g_{11}$ as shown in figure 3.2. A SISO bandwidth of 20 Hz is chosen. A measurement is carried out to verify this design and most importantly identify the equivalent plant $g_{22}^{eq}$. The open loop $g_{22}k_{22}$ is plotted in figure 4.15. The measured $g_{22}^{eq}$ in figure 4.16, now shows the flexible mode characteristics as seen in the singular
Sequential loopshaping provides a straightforward way to design a stabilizing controller using only scalar design procedures for loop shaping. The drawback of this however, is that the choice of controllers influences the input/output behavior of the loops which are to still to be controlled. For systems of larger dimension, the method can be time consuming as the margins of all closed loops have to be checked after each loop closing.
4.3. SEQUENTIAL LOOPSHAPING

Figure 4.15: Open loop \( G_{11} K_{11} \) of the first loop

Figure 4.16: Identification of the second loop equivalent plant \( G_{22}^{eq} \)
Chapter 5

Blind identification of disturbances

In the previous chapter, general control design for MIMO systems has been applied to two setups. To incorporate a disturbance compensation in control design the disturbances acting on the system have to be identified. The initial step of identification will be the separation of the system output into independent sources. As the disturbances will be present in the output they constitute sources of the output signals. Each source has a direction associated with it in the output coordinates.

The source separation methods presented in this chapter are based on statistical analysis of output observations in time. This analysis can be performed while only making minor assumptions on the disturbance that are to be identified. This type of data analysis is known as blind signal separation (BSS). First of all the application of BSS in control is discussed. Subsequently BSS algorithms and an experimental application are presented.

5.1 Application in control

Before any BSS specifics are presented, the question must be asked which possibilities and limitations BSS has in control. From a control perspective separation of the output into independent sources has some significant benefits.

- It can provide insight in the physical location of the disturbance.
- The direction of a disturbance source can be reconstructed.

Besides these benefits, the BSS algorithms presented in this chapter also have several limitations which must not be overlooked:

- Static mixing of the disturbances is assumed.
- The found sources do not reveal where the disturbance enters the control loop. BSS gives no information about their origin, whether the disturbance is an input or an output disturbance.
5.2 Blind Signal Separation

To identify disturbance sources present in a closed loop system, the error and control signals are measured and analyzed. The idea presented in this chapter is to decompose the error signal into statistically independent source signals. If applied to the servo error $e$ this should yield the independent components of the disturbances in $e$. It is important to realize that statistical independence is not the same as uncorrelatedness. Two variables are uncorrelated when:

$$E\{y_i y_j\} - E\{y_i\}E\{y_j\} = 0, \quad i \neq j$$ (5.1)

While independence in a statistical sense requires that for any measurable function $g_1$ and $g_2$:

$$E\{g_1(y_i)g_2(y_j)\} - E\{g_1(y_i)\}E\{g_2(y_j)\} = 0, \quad i \neq j$$ (5.2)

See [12]. For example, The challenge in BSS is to find components in the error observations that are as independent as possible. This has to be achieved only on the basis of the statistics of these signals. A static linear mixing of the source signals is assumed. The basic linear decomposition obtained by BSS can be formulated as:

$$X = Y + Iw = AS + Iw$$ (5.3)

Matrix $X$ contains a vector with $n$ noisy observations as function of time or frequency and equals the sum of $Y$ and white noise present in the observations $w$. The matrix $S$ contains the $m$ sources, identified from $X$ through statistical analysis. Matrix $A$ is referred to as the mixing matrix. In this work we focus on the case that $m \leq n$. The sources can only be determined up to a scaling factor $\Lambda$ and permutation $P$. So that:

$$X = AS + Iw = \hat{A}\Lambda\hat{S} + Iw$$ (5.4)

$$A = \hat{A}\Lambda\Lambda^{-1}P$$ (5.5)

$$\hat{S} = \Lambda^{-1}PS$$ (5.6)

In order to eliminate the indeterminacy of $\Lambda$ the variance of the components is assumed to be equal to one. The scaling of the components is then contained in the mixing matrix $A$. To obtain a separation as in equation 5.3, many different algorithms are available. Here the discussion is focussed on principle component analysis (PCA) or whitening and ICA algorithms. ICA stands for independent component analysis. In this report PCA and two different ICA algorithms are discussed. Before these algorithms are discussed, it is important to realize there are restrictions on the source signals in $S$ with which successful source separation can be achieved.

5.2.1 Required signal properties

To obtain a decomposition assumptions are made regarding the class of disturbance sources. In [2] the assumption is made that the covariance matrix of the source signals is either a deterministic ergodic sequence with a covariance function as equation 5.7, or a stationary multivariate process with covariance as in equation 5.8:

$$\lim_{T \to \infty} T^{-1} \sum_{t=1,T} s(t+\tau)s(t) = E(s(t+\tau)s(t)^*) = diag[\rho_1(\tau), \ldots, \rho_n(\tau)]$$ (5.7)

$$E^\#(s(t+\tau)s(t)^*) = diag[\rho_1(\tau), \ldots, \rho_n(\tau)]$$ (5.8)
The $E$ operator in the first equation indicates the deterministic average, while $E^\#$ in the second stands for the ensemble average. The parameter $\tau$ indicates a time shift. These requirements imply that the sources have to be mutually uncorrelated to be able to separate them successfully based on their covariance matrix.

For ICA stronger requirements apply. These signals have to be statistically independent. The definition in 5.2 has to hold to be able to separate them into independent sources.

5.2.2 Principle component analysis

An important step in most ICA algorithms is PCA. PCA is also known as whitening. Whitening is a statistical method that tries to eliminate redundancy from a data set. The redundancy of the set is evaluated using the correlation of the variables. Whitening can be performed on any matrix $X$ of which the first and second order statistics are known, or can be calculated. Define a matrix $X(t)$ with rows containing the $n$ vectors with variables $x_1(t) \ldots x_n(t)$. The principal components can be found by searching for a transformation that results in an identity covariance matrix based on the observations $X(t)$. The first step required is to center the data by removing the mean of matrix $X$:

$$X \leftarrow X - E\{X\}$$ (5.9)

The covariance matrix of $X$, $R_X$ of dimension $n \times n$ is then computed. The correlation matrix $R_X$ has the variances on the diagonal and the covariances on the off-diagonal.

$$R_X(0) = E\{XX^T\}$$ (5.10)

To obtain white signals from $X$ an eigenvalue or singular value decomposition of the covariance matrix $R_X$ is constructed:

$$R_X = U\Sigma U^T$$ (5.11)

The eigenvalue decomposition and singular value decomposition are equal as matrix $R_X$ is symmetric, $U = V$. The matrix $\Sigma$ can then be partitioned into the $n - m$ highest singular values $\Sigma_S$ and $m$ singular values representing negligible components $\Sigma_N$, $\sigma_{Si} \gg \sigma_{Ni}$:

$$R_X = \begin{bmatrix} U_S & U_N \end{bmatrix} \begin{bmatrix} \Sigma_S & 0 \\ 0 & \Sigma_N \end{bmatrix} \begin{bmatrix} U^T_S \\ U^T_N \end{bmatrix}$$ (5.12)

If less sources than sensors are taken into account an error is introduced in the decomposition. These costs of the whitening procedure are given by norm of the reduced space between the complete singular value decomposition $\Sigma$ and the reduced singular value decomposition with singular values $\Sigma_s$.

$$\text{costs} = \frac{||U_S\Sigma_S U^T_S||_2}{||U\Sigma U^T||_2}$$ (5.13)

Note that the error made by prewhitening cannot be compensated by further steps in the ICA algorithm. The matrix $\Sigma_S$, and corresponding eigencolumns in $U_S$ define the decomposition of the covariance matrix $R_S$ as:

$$R_S = U_S\Sigma_S U^T_S$$ (5.14)

The covariance matrix $R_S$ can be written as in equation 5.10, and can be transformed to the identity matrix:

$$\Sigma_S^{-\frac{1}{2}} U^T_S R_S U_S \Sigma_S^{-\frac{1}{2}} = I$$ (5.15)
From this equation the whitening matrix $\hat{W}$ is obtained:

$$\hat{W} = \Sigma_S^{-\frac{1}{2}} U_S^T$$  (5.16)

Applying this matrix to the observations $X$ results in whitened signals $Z$:

$$Z = \hat{W} X$$  (5.17)

Signals $Z$ are white as their covariance matrix $R_Z$ equals the identity matrix. This whitening matrix $\hat{W}$ has dimension $m \times n$. This results in a dimension reduction from observations to sources if. An example of whitening is given in figure 5.1. Four observations are a linear mix of three independent source signals, as seen in the plots on the left. As the four observations are based on three sources, the fourth singular value of the covariance matrix equals zero. This singular value $\sigma = 0$ is therefore discarded. In the whitening step the data is reduced to three signals. After applying whitening to the mixed signals, these three signals are transformed to have small covariance according to equation 5.17. The resulting signals are shown in the four central figures. But the signals obtained by this transformation are not independent. Maximal variance and therefore minimal covariance of the signals which is the objective of whitening methods is not equivalent to independence, but an important step to achieve separation into independent source signals. The goal of the two algorithms presented next, is to find a rotation of this whitened data that results in independence of the sources.

![Figure 5.1: Whitening performed on observations of linearly mixed source signals](image)

### 5.2.3 SOBI

Here we adopt the approach of [2]. A straightforward algorithm for BSS is SOBI. SOBI stands for Second Order Blind Identification, i.e. it blindly identifies the sources based on second order statistics of the data. It is based on the assumption that the sources will have a low correlation,
5. BLIND IDENTIFICATION OF DISTURBANCES

i.e. a low covariance. As the covariance matrix of the observations can be calculated this can be used as a basis for BSS. If a linear operation on the covariance matrix yields a new more diagonally dominant covariance matrix, the same operation applied to the observations will yield less correlated signals. To obtain a covariance matrix with low off-diagonal terms SOBI makes use of joint diagonalization. The purpose of joint diagonalization is finding a matrix which minimizes sum of the of the non-diagonal terms of a matrix. For a description of the joint diagonalization algorithm see appendix F.

SOBI algorithm

The joint diagonalization technique can be used to obtain a transformation which diagonalizes the covariance matrix $C_x$. The SOBI algorithm consists of four main steps:

1. Obtain the whitened data $z(t)$ by performing whitening as performed in section 5.2.2
2. Generate sample estimates $\hat{R}(\tau)$ by calculating the covariance matrices for a fixed set of time lags $\tau \in \{\tau_j | j = 1, \ldots, K\}$.
3. A unitary matrix $\hat{U}$ can then be found which is a joint diagonalizer of the set $\{\hat{R}(\tau_j) | j = 1, \ldots, K\}$.
4. The estimate of the source signals is then: $\hat{s}(t) = \hat{U}^H\hat{W}x(t)$, the mixing matrix $A$ estimate is given by: $\hat{A} = \hat{W}^\dagger\hat{U}$. Where $\dagger$ denotes the Moore-Penrose pseudo inverse.

5.2.4 JADE

Besides the SOBI algorithm another BSS algorithm of interest is the JADE algorithm, see among others [6] and [5]. JADE is short for Joint Approximate Diagonalization of Eigen-matrices. As the name suggest this algorithm also uses the same joint diagonalization concept as SOBI. However the matrices which are diagonalized in this algorithm are not the covariance matrices. The JADE algorithm is based instead on the fourth order cumulants of the observations. Cumulants provide another possible measure for independence that can improve the source separation compared to the use of the covariance matrix as a measure of independence.

Cumulants

The JADE algorithm finds independent sources on the basis of the cumulants of the observed variables. Cumulants based on more than one variable are known as cross-cumulants. The second order and fourth order cross-cumulants for variables $X_i$ are defined as:

$$\text{Cum}(X_1, X_2) = E\{\hat{X}_1\hat{X}_2\} \quad (5.18)$$

$$\begin{align*}
\text{Cum}(X_1, X_2, X_3, X_4) &= E\{\hat{X}_1\hat{X}_2\hat{X}_3\hat{X}_4\} - E\{\hat{X}_1\hat{X}_2\}E\{\hat{X}_3\hat{X}_4\} \\
&\quad - E\{\hat{X}_1\hat{X}_3\}E\{\hat{X}_2\hat{X}_4\} - E\{\hat{X}_1\hat{X}_4\}E\{\hat{X}_2\hat{X}_3\} \quad (5.19)
\end{align*}$$

in which the data is centered: $\hat{X}_i = X_i - E\{X_i\}$. On the basis of cumulants a JADE contrast function is defined. This function reflects a measure of statistical independence. When this
function is equal to zero the variables are perfectly independent. This function will be minimized using the joint diagonalization method as described in [7], see Appendix F. This function amounts to a summation of squared cross-cumulants:

$$\Phi_{JADE}(X) = \sum_{ijkl \neq iikl} (Q_{ijkl}^X)^2$$  \hfill (5.21)

With $Q_{ijkl}^X = \text{Cum}(X_i, X_j, X_k, X_l)$ the cross-cumulant of variables $X_iX_jX_kX_l$.

![Whitened signals](image1)

![Source signals found with JADE](image2)

**Figure 5.2: JADE performed on observations of whitened signals**

**JADE algorithm**

Source separation within the JADE algorithm can be summarized in the following five steps:

1. Form the whitened data $Z$ by performing whitening on the observation. (Analogous to first step in SOBI)

2. Estimates a maximal set $\{\hat{Q}_1^Z\}$ of cumulant matrices.

3. Find unitary rotation matrix $\hat{V}$ that results in cumulant matrices which are as diagonal as possible. This is achieved by joint diagonalization of the cumulant matrices.

4. The mixing matrix estimate is given by: $\hat{A} = \hat{V}\hat{W}^{-1}$, the components can then be computed: $\hat{S} = \hat{A}^{-1}\hat{X} = \hat{V}^+Z$.

When JADE is applied to the whitened signals of figure 5.1 the sources shown in figure 5.2 are found. These indeed correspond to the sources used to generate the observations.
5.3 Experiments

To establish whether the presented algorithms are able to aid in control design, some practical cases were tested with experiments. Experiments are carried out on the AVIS system, described in the second chapter of this report. To determine whether the two discussed BSS algorithms can separate disturbances acting on a dynamical system in practice, disturbances are applied to the AVIS system.

5.3.1 Disturbance application

The AVIS system is subject to continuous excitation by disturbances from the environment, such as floor vibrations, sound etc. and by artificially added disturbances. In the source separation problem, these artificial disturbances are assumed to be unknown. The objective is to recover the sources of these artificial disturbances from the observations of the output or servo error of the system. Because the location, direction and amplitude of the applied disturbances are known, they can be compared to the recovered sources and mixing matrix found with ICA.

Disturbances are applied to the table using two shakers. These are rigidly connected to the table, see picture 5.3. There are 30 positions on the table surface where these shakers can be placed. Different types of current signals are applied to the shakers to exert a force on the table. The acceleration of the moving masses of the shakers is measured using accelerometers. At the same time the closed loop error signal of the AVIS is measured. For these measurements a low bandwidth controller is used, with crossovers slightly above the first resonance frequencies. First the disturbance path from shaker to table output is considered.
5.3. EXPERIMENTS

5.3.2 Disturbance path

To establish whether the static mixing is a valid assumption, the disturbance path is evaluated. This is done by determining the transfer from the force exerted by the shaker to the velocities of the table. A shaker is placed in the center of the table and a FRF measurement is done. The resulting FRF for the velocity in \(\dot{z}\) and its coherence are plotted in figure 5.4. The coherence shows that this measurement is not reliable below 18 Hz. This frequency is the eigenfrequency of the shaker. Above this frequency the transfer shows a fluctuating magnitude and phase. Some transfers from the shaker to the other axis show different magnitude and phase. Also the disturbance path is different for every location selected for the shaker. Despite this, static mixing of disturbances is assumed. As will be shown in the next section, it is still possible to separate disturbances although the mixing contains some dynamics.

5.3.3 Experimental results

Several experiments were carried out according to the approach described. Here four cases of source separation are presented. Each case represents a different configuration of the shakers. A different location and/or different angle on the table. The data is analyzed off line by calculating the covariance matrix, as well as applying the SOBI and JADE algorithms. For each case:

- The magnitude of the singular values \(\sigma_i\) of the covariance matrix \(R_x\) are reviewed to establish how many sources should be selected. One or more of the smallest components are discarded,
if they do not represent a significant part of the error signal.

- The recovered sources are compared with the measured acceleration of the shakers.
- The absolute correlation coefficients based on the covariance matrix are depicted using a color map. This is done to visualize the amount of correlation between servo errors and disturbance and between the servo errors themselves.

Next the results of the experiments are presented.

**Case 1: A single sinusoidal disturbance**

To start with, a single disturbance is applied to the table. The shaker is placed on position $(0.29 \, m, 0.34 \, m)$ and a sine input signal with a frequency of $30 \, \text{Hz}$ and peak acceleration of approximately $1 \, \text{ms}^{-2}$ is applied. The resulting sinusoidal force causes a disturbance of the table. The resulting closed loop error is measured. The covariance matrix $R_e$ of the measured data is calculated. As in equation 5.14 a singular value decomposition of $R_e$ is determined. The found singular values of $R_e$ are plotted in figure 5.3.3. These values represent the gains of the six main components in the measured error signal $e$. On basis of this plot an initial choice for the number of sources in $S$ can be made. There is one main component in the singular values. But in the time signals, see left plots in figure 5.3.3, a second disturbance is clearly present, therefore a two source decomposition is made, $S \in \mathbb{R}^2$. In the left of figure 5.3.3 the measured error is plotted. The JADE and SOBI algorithm are applied to the error signals to obtain a decomposition into two statistically independent sources. The resulting source signals $s$ are plotted to the right of the errors.

Below these two sources, the integrated acceleration of the moving shaker mass $\int a(t)dt$ is shown. The integrand is shown to be able to compare the error and disturbance on a velocity level.
5.3. EXPERIMENTS

Figure 5.6: Measured error, separated sources and measured applied velocity for case 1

No numerical values are shown on the y-axis for this velocity as its values contain drift due to numerical integration of the acceleration signal. Two sources are identified from the error data, two sinusoidal signals. Clearly the applied sine disturbance was identified up to a permutation and constant scaling factor.

The frequency of the other source signal corresponds to the second resonance in the diagonal terms of the FRF of the axis, see figure 3.6. The sources found with SOBI and JADE, are almost identical up to a permutation. The JADE source combined with the mixing matrix $A_{JADE}$ however has the correct sign.

To verify the mixing matrices which gives the direction of the disturbance, the spectra of the error signals are analyzed. If it is assumed that the main component in the error spectrum at 30 Hz is related to the sine disturbance, the gains of these spectra give a measure for the direction of the disturbance. In table 5.1 the gains of the spectra and the mixing matrix columns $A_{SOBI}, A_{JADE}$ are scaled with their largest entry. The direction of $A_{JADE}$ as well as $A_{SOBI}$ are approximately equal to the direction found from the error spectra as seen in table 5.1.

Of course the identification of the sine disturbances could have been done using directions derived directly from the error spectra. But when disturbances with more than a single relevant frequency are considered this becomes more difficult. Also two narrow band disturbances with similar fre-
5. BLIND IDENTIFICATION OF DISTURBANCES

<table>
<thead>
<tr>
<th>axis</th>
<th>$|\sqrt{\Phi_e}|_\infty$</th>
<th>$|A_{SOBI}|_\infty$</th>
<th>$|A_{JADE}|_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{x}$</td>
<td>0.0196</td>
<td>-0.0193</td>
<td>0.0195</td>
</tr>
<tr>
<td>$\dot{y}$</td>
<td>0.0165</td>
<td>0.0162</td>
<td>-0.0163</td>
</tr>
<tr>
<td>$\dot{z}$</td>
<td>0.9591</td>
<td>-0.9596</td>
<td>0.9605</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>0.0848</td>
<td>-0.0824</td>
<td>0.0837</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>0.2686</td>
<td>0.2678</td>
<td>-0.2642</td>
</tr>
<tr>
<td>$\dot{\psi}$</td>
<td>0.0068</td>
<td>0.0068</td>
<td>-0.0066</td>
</tr>
</tbody>
</table>

Table 5.1: Directions obtained from the spectrum, SOBI and JADE decompositions for case 1

Frequencies are difficult to identify. The spectra of these signals can overlap and therefore no accurate direction can be obtained. For further analysis the covariance matrix $R_c$ is visualized using a

![Gray scale map of the matrix](image)

Figure 5.7: Correlation coefficients of the errors and identified sources, case 1

Gray scale map of the matrix, see figure 5.7. This plot shows the absolute value of the correlation coefficients between the errors and independent components found with JADE. Dark indicates strong correlation, lighter less coherence. The independent components show a small correlation with each other. The second component which corresponds to the applied sine disturbance shows a strong correlation with the $\dot{z}$, $\dot{\phi}$ and $\dot{\theta}$. 
Case 2: Two sine signals

As second set of disturbances is applied to the system to evaluate the performance of JADE and SOBI when two disturbances are applied. Two shakers are placed on the table, one located at position (0.25 m, 0.23 m), the other at position (-0.19 m, -0.26 m). To the first shaker a sine signal of 30 Hz is applied with a peak acceleration of approximately 2.5 ms\(^{-2}\). The input of the second shaker is a sine of 32 Hz with a peak acceleration of 5.2 ms\(^{-2}\). An extra weight is also added to this shaker, i.e. the moving mass is increased compared to the first one.

As in the previous case, a singular value decomposition is made of the covariance matrix \(R_e\), see figure. The 4-th singular value is not significant compared to the largest singular value, \(\sigma_4 \leq 0.01\sigma_1\). Three singular values compose the main part of the error, therefore this is the number of sources that is taken into account. The data is then decomposed into three components using the JADE and SOBI algorithm, see figure 5.9. The decomposition reveals the two sinusoidal disturbances and the higher frequency source found in case 1. As in case 1 the decomposition made by SOBI and JADE seem similar up to permutations in the JADE decomposition. However, the 30 Hz disturbance, the first component shown for the JADE and SOBI decomposition has distinctly different directions in both decompositions: The \(y\) component of the SOBI direction vector for the 30 Hz disturbance has a different permutation compared to the JADE vector. The \(z\) and \(\phi\) components of the SOBI vector are significantly different from the corresponding JADE and spectrum components as well. The lower magnitude of the force exerted by the 30 Hz disturbance, compared to the 32 Hz disturbance makes it harder to identify as the signal to noise ratio of this disturbance source is lower.

From the covariance matrix of the errors and JADE components in figure 5.10 the correlation between the 32 Hz and the \(z\), \(\phi\) and \(\theta\) axes is very clear. The correlation coefficients of the 30 Hz disturbance with all error signals are lower than 0.4. This explains why the SOBI algorithm does not obtain the same source signal as JADE. The second order statistics are less clear, and the JADE algorithm is able to obtain a better decomposition based on fourth order statistics of
Figure 5.9: Measured error and found sources for case 2

the same data.
5.3. EXPERIMENTS

Figure 5.10: Correlation coefficient of the errors and identified sources, case 2

<table>
<thead>
<tr>
<th>axis</th>
<th>$|\Phi_e|_1$</th>
<th>$|A_{SOBI}|_\infty$</th>
<th>$|A_{JADE}|_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{x}$</td>
<td>0.0185</td>
<td>0.0112</td>
<td>-0.0126</td>
</tr>
<tr>
<td>$\dot{y}$</td>
<td>0.0117</td>
<td>0.0146</td>
<td>-0.0092</td>
</tr>
<tr>
<td>$\dot{z}$</td>
<td>0.3017</td>
<td>0.8279</td>
<td>-0.2903</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>0.7820</td>
<td>0.1869</td>
<td>-0.7300</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>0.5443</td>
<td>-0.5283</td>
<td>0.6185</td>
</tr>
<tr>
<td>$\dot{\psi}$</td>
<td>0.0258</td>
<td>-0.0150</td>
<td>0.0046</td>
</tr>
</tbody>
</table>

Table 5.2: Directions obtained from the spectrum, SOBI and JADE decompositions for case 2

Case 3: Sine and step signal

In this case the shakers are placed on different positions on the table, one at the location (0.22 m, 0.19 m) and the other at location (0.0 m, 0.0 m), the center of mass of the table. A sine signal of 30 Hz is applied to the shaker in the location (0.22 m, 0.19 m), with 2.4 ms$^{-2}$ peak acceleration. A block signal of 1 Hz is applied to the shaker in the center of mass of the table. This step signal results in a signal consisting of repeated step responses of the shaker, with 1.3 ms$^{-2}$ peak acceleration. The difficulty in separating this type of signal is the fact that it has a wide spectrum. It is impossible to separate the disturbances based on the spectra of the errors of the six axes, as there is not a single peak that can be identified as the gain of the step response signal in that axis. As the transfer from force to velocity of the AVIS is not constant for all frequencies, the assumption of static mixing is not fulfilled. This will lead to a less accurate source approximation.

The singular value decomposition of the covariance matrix of the measured error signal, shown in figure 5.3.3, shows two main components in the covariance matrix. A decompositions into two sources is determined with JADE as well as SOBI, see figure 5.12. The separation of these signals is achieved, however a part of the sinusoidal component remains in the first component of the SOBI decomposition. The JADE algorithm achieves a more accurate decomposition in this case.
The covariance matrix shown in figure 5.13 indicates a very strong correlation between the second independent component and the $\dot{\phi}$ axis. This corresponds to our expectations as the shaker in position (0.22 m, 0.19 m) will cause a moment around the $y$-axis.

The correlation between the second component and the $\dot{\theta}$ axis is considerably smaller as the distance in $y$ direction from the center of mass is also smaller. The first component has largest correlation coefficients with the $z$ axis, as it works in the center of mass it contributes less to rotations $\phi$ and $\theta$.

**Case 4: A disturbance applied to the frame**

In the final, fourth case one of the shakers is placed on the frame of the AVIS system. A sine signal input with a period of 30 Hz, and 3.9 ms$^{-2}$ peak acceleration is applied to this shaker. The other is placed on the table and a step signal is applied to it, with 1.8 ms$^{-2}$ peak acceleration. The singular value decomposition of the covariance matrix, figure 5.14 shows two main components in the covariance matrix. However these do not include the disturbance from the frame. A third source has to be assumed to retrieve this disturbance. It turns out there is another disturbance which is more significant than the disturbance applied to the frame. A decompositions into three sources is made in order to include this disturbance. In figure 5.9 the sources found with SOBI and JADE are plotted. The first source is most likely a vibration introduced by the step signal directly applied to the table, as it can also be observed in the acceleration. The second source resembles closely the integrated acceleration signal of the shaker on the table. The third source represents the disturbance applied to the frame of the AVIS. The correlation coefficient matrix shows that the applied step, the second independent component, is strongly correlated with $\phi$.
5.3.4 Allocation of disturbance sources

To find the location of a disturbance source \( d(t) \) the mixing matrix \( A \) is used. To enable allocation of the disturbance sources the table is modelled as a rigid body in the relevant frequency range. As stated before it cannot be determined from the sources found with the BSS algorithms if the sources found are input or output disturbances. In the disturbance allocation approach presented here, the disturbances are assumed to be input disturbances. In figure 5.17 a disturbance force \( F \) acting on the table surface is depicted. The point on the table where it is located is defined as the vector \( \mathbf{r} = [r_x, r_y, r_z]^T \). The forces \( F \) acting on the table can be written as:

\[
F(t) = m\ddot{v}(t) \tag{5.22}
\]

\[
\int F(t)dt = mv(t) + v_0 \tag{5.23}
\]

To determine \( r_x \) and \( r_y \) using the decomposed velocities the force is integrated with respect to time and zero stationary speeds are assumed \( v_0 = 0 \). The disturbance force \( F \) causes translation.
The moments around the center of mass of the table can be expressed as:

\[
\frac{\mathbf{J}\dot{\mathbf{\omega}}}{r} = \mathbf{r} \times \mathbf{F} \Rightarrow \begin{bmatrix}
J_x & 0 & 0 \\
0 & J_y & 0 \\
0 & 0 & J_z
\end{bmatrix} \begin{bmatrix}
\ddot{\phi}_d \\
\ddot{\theta}_d \\
\ddot{\psi}_d
\end{bmatrix} = \begin{bmatrix}
0 & F_z & -F_y \\
-F_z & 0 & F_x \\
F_y & -F_x & 0
\end{bmatrix} \begin{bmatrix}
r_x \\
r_y \\
r_z
\end{bmatrix}
\] (5.27)

Note that matrix containing the forces \( \mathbf{F} \) is singular in this three dimensional case, the vector \( \mathbf{r} \) cannot be determined.

To be able to determine the \( r_x \) and \( r_y \) component of \( \mathbf{r} \) the assumption that the location of the disturbance lies in the plane \( z = 0 \) is made. This is a valid approximation as the disturbances are applied to the table surface. The allocation problem then reduces to the non singular 2 dimensional problem. Note that this allocation of disturbances cannot be considered blind in a strict sense as prior knowledge about the disturbance source is used.

Suppose the disturbance source \( d(t) \) is found with the JADE from the closed loop error signal. Then the corresponding mixing \( A_d \) is a column of the mixing matrix \( A \) mapping the disturbance \( d(t) \) onto the coordinates.

\[
\begin{bmatrix}
v \\
\omega
\end{bmatrix} = A_d d(t)
\] (5.28)

With elements \( a_i \) of \( A_d \) for axes \( i \). Source \( d(t) \) then appears on both sides of the equality 5.27. The source can now be eliminated from the allocation problem. The location of the disturbance

---

Figure 5.13: Correlation coefficient of the errors and identified sources for case 3

speeds and angular velocities defined with respect to the center of mass of the table:

\[
v = \begin{bmatrix}
\dot{x}_d \\
\dot{y}_d \\
\dot{z}_d
\end{bmatrix}^T
\] (5.25)

\[
\omega = \begin{bmatrix}
\dot{\phi}_d \\
\dot{\theta}_d \\
\dot{\psi}_d
\end{bmatrix}^T
\] (5.26)
source is then found:

\[
\begin{bmatrix}
  r_x \\
  r_y \\
\end{bmatrix} = \begin{bmatrix}
  0 & ma_z \\
  -ma_z & 0 \\
\end{bmatrix}^{-1} \begin{bmatrix}
  J_x & 0 \\
  0 & J_y \\
\end{bmatrix} \begin{bmatrix}
  a_\phi \\
  a_\theta \\
\end{bmatrix}
\]  

(5.29)

This is verified experimentally on the basis of several measurements of an error signal for different disturbance cases. A JADE and SOBI decomposition is made for each measurement. Equation 5.29 is then used to determine a location on the plane \( z = 0 \). First of all a basic decomposition of a single sine disturbance is shown.
Single shaker with sine input

Analog to the first case, a sine is applied to a shaker at location near the corner of the table (0.29 \( m \), 0.34 \( m \)). Additionally this experiment is repeated on 12 other locations on the table. For each location the error signal is decomposed into two sources in the same way as presented before. The allocation of the sine disturbance is done for all experiments and the found coordinates are plotted in figure 5.18. The large symbols indicate the actual location of the shaker, the smaller ones the estimated location based on the JADE mixing matrix \( A_{\text{JADE}} \).

The locations approximate the actual shaker locations. Except for the three locations in the bottom right of figure 5.18, all approximations lie within a distance of 0.05 \( m \) of the actual location. Note that only the locations based on \( A_{\text{JADE}} \) are plotted here as \( A_{\text{SOBI}} \) and \( A_{\text{JADE}} \) are almost equal.

Two shakers with sine inputs

In section 5.3.3 a case separation of two sinusoidal disturbances was discussed. The SOBI and JADE decompositions resulted in different mixing matrices \( A_{\text{JADE}} \) and \( A_{\text{SOBI}} \). The allocation of these two sinusoidal disturbances, confirms that the \( A_{\text{JADE}} \) mixing matrix represents the actual
mixing more closely than $A_{SOBI}$. In figure 5.19 the actual and estimated locations of the 30 Hz sine source signal, $d_1$ and the 32 Hz sine source signal $d_2$ are indicated for $A_{SOBI}$ and $A_{JADE}$. The smaller signal to noise ratio of the observations of disturbance $d_2$ can explain the worse performance of SOBI in this case. The second order statistics are more sensitive to noise than the fourth order cumulants.

Two shakers with sine and block input

In this case the separation of a sinusoidal disturbance and a repetitive step response is considered. The sinusoidal signal is a 25 Hz sine signal which is applied to a shaker in the position (0.25 m,0.08 m) and a 10 Hz block signal applied to a shaker in position (-0.085 m,-0.155 m). Again a JADE decomposition is calculated, and the locations extracted from the mixing matrix. This experiment is now done four times with an increasing measurement duration. Each measurement is divided into 10 parts of equal length. In figure 5.20 the found locations are shown with a ‘+’ sign, for four lengths of the data vector. Based on the shortest 30 second measurement the location of the sinusoidal disturbance is already identified correctly. For the step type disturbance, caused by the block signal, the 10 points calculated on the basis of only a 3 second interval vary in a wide range around the actual location. Taking more data points into account results in the same location for the sine, and an increasingly improved approximation for the location of the second disturbance. The fact that the allocation improves significantly for a longer measurement duration is explained by the fact that the estimate of the cumulant improves when more samples are taken into account. This can be evaluated by taking the norm over the cumulant matrix used in the JADE algorithm. This matrix $Q$ is the basis for the JADE objective function see equation 5.21. For a data vector of 350 seconds with sample time $T_s = 0.001s$ $Q$ is calculated. For increasing lengths of the data vector the difference between the norm of $Q$ for longer data vectors decreases. The cumulant estimation becomes better for more evaluated samples.
Figure 5.17: Disturbance force acting on the table surface

Figure 5.18: Localization of a single sinusoidal disturbance
Figure 5.19: Simultaneous localization sinusoidal disturbances $d_1$ and $d_2$
Figure 5.20: Localization of a step type disturbance and a sinusoidal disturbance for increasing number of samples
Figure 5.21: The norm of fourth order cumulant $Q$ seems to converge to a constant value for increasing data length.
Chapter 6

Control design incorporating disturbance knowledge

In the previous chapter the possibility of source separation using ICA algorithms were demonstrated. This decomposition into independent sources can be exploited in control design. The identified sources can be used to obtain a model of the disturbance. Together with the identified directions $A$ a compensation for the disturbance can be designed. An example of such an approach is shown on the AVIS system.

6.1 Triangular control

Consider a decomposition of the error in statistically independent output disturbance sources as presented in the previous chapter, $e = AS$. Assume that the blind identification of this disturbance was successful and we take $G_d = A$ although $A$ is only equal to $G_d$ up to a gain and permutation. To minimize the gain from a disturbance in $d_o$ to the error $e$ the output sensitivity $S_o$ has to be minimized taking into account to the direction and frequency of the disturbance.

A way to achieve this is triangular control, see [3]. Cross terms can be added to a controller $K$ to compensate for a disturbance. If a only the second column of the controller $K$ with controller input $e_2$ is considered for triangular control $S_o$ becomes:

$$
S_o = \begin{bmatrix}
    s_{11} & -g_{11}k_{12}s_{22}s_{11} & 0 & \cdots & 0 \\
    0 & s_{22} & 0 & \cdots & 0 \\
    \vdots & -g_{ii}k_{i2}s_{22}s_{ii} & s_{ii} & \ddots & \vdots \\
    \vdots & \vdots & 0 & \ddots & \vdots \\
    \vdots & \vdots & \vdots & \ddots & 0 \\
    0 & -g_{nn}k_{n2}s_{22}s_{nn} & 0 & \cdots & 0 & s_{nn}
\end{bmatrix}
$$

(6.1)

In which the sensitivity of loop $i$:

$$
s_{ii} = \frac{1}{1 + g_{ii}k_{ii}}
$$

(6.2)

The cross terms $k_{i2}$ in the first columns of equation 6.1 can be used to compensate for a disturbance
in $d_o$ in the $i$-th loop. The error $e_i$ caused by a disturbance in $d_o$ then becomes:

$$e_i = (-g_i k_{i2}a_2 s_{22} s_{ii} + s_{ii} a_i) d_o$$  \hfill (6.3)

In which $a_i$ is an element of mixing matrix $A$ representing the gain of the $n$-th disturbance $G_d d_o$ in the $i$-th loop. To obtain zero $e_i$ the controller $k_{i2}$ has to be chosen as:

$$k_{i2} = \frac{1}{g_{ii} s_{22}} \cdot \frac{a_i}{a_2}$$  \hfill (6.4)

This control action should cancel the error caused by a disturbance $d_o$ acting on the system. This kind of feedback term in $K$ is similar to a feedforward action. Only in this case the control action is not based on a known reference but instead on an observation of the disturbance in a different element of the closed loop error vector, in this case $e_2$.

Stability is guaranteed for a perfectly decoupled system $\tilde{G}$. If interaction is present in the plant the triangular control action can cause instability due to interaction. It is therefore preferable to choose an input $e_n$ for the triangular controller $k_{in}$ which has no or small interaction with loop $i$.

An advantage of this triangular control method is that the diagonal terms $g_{ii} k_{ii}$ do not have to be modified to compensate for the disturbance $d_o$. This makes it possible to compensate for a disturbance at a higher frequency than the bandwidths of $g_{ii} k_{ii}$. A practical application of this kind of triangular controller is given in the next section.

### 6.2 Application on the AVIS

To illustrate the triangular control method, a triangular controller is designed based on disturbance sources found with JADE. Here a case similar to case 1 is used as a basis for the triangular design. A sine disturbance with a frequency of 30 Hz is applied to the table, with a shaker at location $(0.20 \ m, 0.10 \ m)$. The resulting closed loop error $e$ is measured and a JADE decomposition into four components is made. This results in the source signals shown in figure 6.1. The fourth source signal corresponds to the 30 Hz disturbance. The direction associated with this disturbance source is given by the fourth column of the matrix $A_{\text{JADE}}$.

The objective will be to compensate for this sinusoidal disturbance, $d_{\sin}$, in the third, fourth and fifth axis of the AVIS. As the disturbance is applied in $z$ direction, the disturbance has the highest gain in these three axes.

The disturbance $d_{\sin}$ has a maximal magnitude at a frequency of $f_{d_{\sin}} = 30 \ Hz$. As the openloop has a small magnitude at that frequency the approximation is made:

$$s_{nn} \approx 1$$  \hfill (6.5)

The element $k_{i2}$ in 6.4 now reduces to:

$$k_{i2} = \frac{1}{g_{ii}} \cdot \frac{a_i}{a_2}$$  \hfill (6.6)

With $a_i$ the $i$-th element of the fourth column of matrix $A_{\text{JADE}}$. As shown in [3] this control action can be applied as a scalar gain. However the blind identification has provided the disturbance source signal. This knowledge is exploited by introducing a second order notch filter $N(s)$ to fit the main component in the spectrum of that source. In this way the control action $k_{i2}$ is based on a filtered error signal containing only the relevant frequencies around $f_{d_{\sin}}$. This has the added benefit that interaction which might be present in the plant at other frequencies than $f_{d_{\sin}}$ is not excited.
The term $g_{ii}$ in the control terms $k_{i2}$ is approximated by a gain representing the magnitude of $g_{ii}$ and a second order low pass filter $LP(s)$ in series with the notch $N(s)$ is used to approximate the phase of $g_{ii}$. By choosing the eigenfrequency of a low pass filter with unity static gain, the phase of the filters $N(s)LP(s)$ in series can be altered to match the phase of $g_{ii}$.

To enable tuning of the gain of the control action a parameter $\alpha$ is added. The resulting cross terms in controller $K$; $k_{i2}$ are given as:

$$k_{i2} = \frac{\alpha}{|g_{ii}|} \frac{a_i}{a_2} N(s)LP(s)$$  \hfill (6.7)

Cross terms $k_{i3}, k_{i4}, k_{i5}$ were added to a low bandwidth controller for the kinematically decoupled system. A value of $\alpha = 1.4$ was found to produce the best results. In figure 6.2 the achieved error with triangular control and the error obtained with the unaltered diagonal controller are plotted. The disturbance is significantly reduced in the $\dot{\phi}, \dot{\theta}$ and $\dot{\psi}$ axes. Using triangular control enables a compensation of this error above the bandwidths of the $\dot{\theta}$ axis.
Figure 6.2: Disturbance compensation with a triangular controller
Chapter 7

Conclusions and recommendations

7.1 Conclusions

The problem statement in the introduction asked whether statistics of the observations of multi-variable dynamical systems can be used to identify disturbance sources and separate them into independent components. To answer this question, research was conducted which has led to the main conclusion:

Separation and identification of independent disturbance sources can be achieved using statistical analysis of the observations of multivariable dynamical systems.

This is conclusion is based upon the experiments done with the JADE and SOBI algorithms on the AVIS system. Additional conclusions regarding both multivariable systems and blind source separation can be drawn.

7.1.1 Multivariable systems

- Decoupling of a dyadic system based solely on frequency response data was successfully applied in practice. However the selection of the frequencies in the frequency response function for the calculation of the decoupling matrices strongly affects the quality of decoupling.

- Decoupling using the joint diagonalization algorithm can result in an improved decoupling. The transformation matrices are only based on the numerics of the matrices of the FRF measurement, the transformation is therefore hard to interpret.

- In multivariable control design, the characteristic loci are preferred over the spectral radius criterion for the evaluation of stability. The spectral radius does not take into account phase and only the largest eigenvalue of the error system at each frequency.

- Using the JADE algorithm to transform the system into independent sources does reveal the main modes of the system. For decoupling it is in general not applicable, as the modes of the system will generally not correspond to the chosen coordinate system and coupling is introduced.
7.1.2 Blind source separation

Successfull separations of disturbances present in the closed loop error of a dynamical system were achieved. The allocation of disturbances based on these decompositions was successfully demonstrated for several cases. Regarding the source separation and allocation problem the following conclusions can be drawn:

- **Source separation**
  - Independent disturbance sources in a dynamical system can be successfully separated using both the SOBI as well as the JADE algorithm.
  - The signal to noise ratio of the source signals affects the quality of the source separation. For disturbances with a small signal to noise ratio, the JADE algorithm performs better than the SOBI algorithm. The SOBI algorithm is based on second order statistics, which are more sensitive to noise. For measurement data with a low signal to noise ratio, use of the JADE algorithm is therefore preferable above the SOBI algorithm.
  - Static mixing can be assumed on the AVIS system for a limited frequency range. The validity of the static mixing assumption will depend on the properties of systems under consideration.
  - Increasing the number of samples for calculation improves the quality of the decomposition. This can be explained by the averaging out of random components in the measurement.

- **Disturbance allocation**
  - Allocation of disturbances can be achieved in a two dimensional case. In the three dimensional case the problem is singular and the location can only be determined up to a line.
  - The accuracy of disturbance allocation is affected by the presence of dynamics in the mixing of sources. Because of this disturbances with a narrow spectrum are easier to localize than disturbances with a broader spectrum.

7.2 Recommendations

Based on this research a set of recommendations can be made regarding both multivariable control as well as blind source separation.

- The directions found with blind source separation algorithms can be used to define weighting filters in an multivariable $H_\infty$ control problem. Weighting filters can then be designed which incorporate disturbance compensation.
- Investigate in which way(s) the interaction, especially one sided interaction, present in MIMO systems can be identified and quantified.
- The interpretation of fourth order statistics, for example the cumulants which are part of the JADE objective function, is difficult. If possible, linking the properties of fourth order cumulants to systems and disturbance properties provides more insight into the ICA decomposition.
- Study online versions of the ICA algorithm and establish whether these are suitable for identifying disturbances with time dependent mixing.
Bibliography


Appendix A

Geophone characteristics

The velocity of the AVIS system is measured using geophones. These sensors contain a permanent magnet which passes through a number of coils. This generates a voltage in the coils. This voltage is a measure for the velocity, but only above the resonance frequency of the permanent magnet. To obtain velocity In [19] the design of a compensation for the low frequent region is presented. To calibrate the geophone white noise was injected into an avis input. The resulting geophone signal was measured, together with a laser vibrometer measurement of the table at approximately the location of the geophone. In figure A.1 the measured frequency response from geophone to velocity is shown. The transfer shows that the geophone is compensated adequately at low frequencies at that the relation from voltage to velocity can be approximated by a constant.

Figure A.1: Frequency response geophone
Appendix B

Kinematic decoupling

The kinematic decoupling of the AVIS system is based on [19]. The locations of the actuators and sensors are expressed in the base coordinate frame. Also the polarity of the sensors and actuators is taken into account. This led to the following actuator and sensor matrices:

Actuator matrix:

\[
T_u = \begin{pmatrix}
0.0000 & -0.5000 & 0.0000 & -0.0000 & -0.6485 \\
0.0974 & 0.1233 & -0.2500 & -0.6667 & 0.5263 & -0.0000 \\
-0.5000 & 0.0000 & -0.0000 & -0.0000 & -0.5119 \\
0.0974 & -0.1233 & -0.2500 & 0.6667 & 0.5263 & -0.0000 \\
0 & 0.5000 & -0.0000 & 0.0000 & -0.0000 & -0.6485 \\
-0.0974 & -0.1233 & -0.2500 & 0.6667 & -0.5263 & -0.0000 \\
0.5000 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.5119 \\
-0.0974 & 0.1233 & -0.2500 & -0.6667 & -0.5263 & 0.0000
\end{pmatrix}
\]  

Sensor matrix:

\[
T_y = \begin{pmatrix}
-0.3947 & 0 & 1.0000 & 0.1158 & -0.3947 & -0.1158 \\
0.5000 & 0.1467 & 0 & -0.1467 & -0.5000 & 0 \\
0 & -0.5000 & 0 & -0.0000 & 0 & -0.5000 \\
0 & -1.3333 & 0 & 1.3333 & 0 & 0 \\
0 & 0 & 0 & 1.0526 & 0 & -1.0526 \\
1.0526 & 0 & 0 & 0 & 1.0526 & 0
\end{pmatrix}
\]
Appendix C

Complete openloop FRF measurement
<table>
<thead>
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<th>Frequency [Hz]</th>
<th>Magnitude [dB]</th>
<th>Phase in degrees</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>$G_{12}$</td>
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<tr>
<td>$G_{16}$</td>
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</table>
Appendix D

Calculation decoupling matrices for dyadic systems

In [22] decoupling input and output transformation matrices $T_y$ and $T_u$ are derived for dyadic systems. The calculation of these matrices is based directly on frequency response data of a dyadic plant $G$.

$$G_{diag}(j\omega) = T_y^{-1}G(j\omega)T_u^{-1}$$

(D.1)

If constant and real matrices $T_u$ and $T_y$ exist, the system $G$ is dyadic. These transformation matrices can be obtained directly from the data. The columns of $T_y$ are calculated as the eigenvectors of $G(2\pi f_2)G(2\pi f_1)^{-1}$:

$$G(2\pi f_2)G(2\pi f_1)^{-1} = T_yD_1T_y^{-1}$$

(D.2)

With $D_1$ a diagonal matrix. The columns of $T_u^{-1}$ are equal to the eigenvectors of $G(2\pi f_1)^{-1}G(2\pi f_2)$:

$$G(2\pi f_1)^{-1}G(2\pi f_2) = T_u^{-1}D_2T_u$$

(D.3)

With $D_2$ diagonal. The choice of the frequencies $f_1$ and $f_2$ is critical for the quality of the decoupling.
Appendix E

Relative gain array

The relative gain array provides a measure for the two sided interaction, present in a $n \times n$ plant $G$, see [20]. The RGA array is defined for frequency $s = j\omega$ as:

$$RGA(G(j\omega)) = \Lambda(G(j\omega)) \triangleq G(j\omega) \times (G(j\omega)^{-1})^T$$ (E.1)

In which $\times$ denotes the element by element multiplication, or Schur product. The RGA array provides a convenient way to determine a measure of interaction in the plant, directly from the frequency response data. The summation of column and row elements of the RGA array should in both cases be equal to 1. If the diagonal elements of the RGA array have magnitude 1 and the cross terms zero, there is no two sided interaction between the first input and the second output and vice versa. The matrix $G$ is then upper or lower triangular or diagonal.
Appendix F

Joint diagonalization

In [7] the joint diagonalization algorithm is described. The goal of the joint diagonalization algorithm is to minimize the terms $m_{ij}$ in a matrix $M$, for $i \neq j$. The sum of these terms is defined as the 'off' of a matrix $M$:

$$\text{off}(M) = \sum_{1 \leq i \neq j \leq n} |M_{ij}|^2 \quad (F.1)$$

For all the matrices in a particular set $\mathcal{M} = \{M_1, \ldots, M_K\}$ a joint diagonality criterion can be defined for a unitary matrix $V \in \mathbb{C}^{(n \times n)}$:

$$\mathcal{C}(\mathcal{M}, V) = \sum_{k=1,K} \text{off}(V^H M_k V) \quad (F.2)$$

The matrix $V$ has to be found in order to obtain a matrix that is as diagonally dominant as possible. A complex Givens rotation parameterizes $V$. The unitary matrix $V$ is then of the form:

$$V = \begin{bmatrix} \cos \theta & e^{j\Phi} \sin \theta \\ -e^{-j\Phi} \sin \theta & \cos \theta \end{bmatrix} \quad (F.3)$$

The optimal diagonalization is found by rotation such that the joint diagonality criterion in F.2 has a minimal value.