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Control and identification of a continuously variable transmission

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Abstract

At the Faculty of Mechanical Engineering a hybrid drive line for passenger cars is being developed. In this concept a flywheel is applied to store the vehicle’s kinetic energy during braking. The stored energy is used to accelerate the vehicle. An IC engine is applied to accelerate the flywheel to its operating speed before drive off and to replenish dissipated energy such as rolling resistance, air drag, etc. Energy exchange between flywheel and vehicle is realized by means of a Continuously Variable Transmission (CVT). This is realized by continuously varying the transmission ratio of the CVT.

The main components of a CVT are two pulleys and a metal V-belt. Each pulley consists of two conical sheaves of which one can be shifted hydraulically in axial direction by means of a hydraulic cylinder. By an oil pressure in the cylinder, the belt is clamped between the sheaves and the CVT is able to transmit power. By shifting the sheaves simultaneously the running radii of the belt and, hence, the transmission ratio is varied. Sheave shifts are realized by pumping oil in, respectively from, the cylinders.

CVT control involves the control of the ratio change and the control of the pulley pressures to prevent belt slip. Spijker [1] accomplished these objectives using two independent parallel controllers. One pulley is controlled to realize the change of transmission ratio, while the other pulley is controlled to realize the clamping forces. Spijker’s controllers are based on first order SISO models. A disadvantage of this approach is the disturbance of the control loops by each other because of pulley interaction. For this reason Ploemen [2] designed a pulley pressure controller based on a model that takes the interaction into account. This model, based on Spijker’s model, includes the interaction in a simple way. This report contains the implementation and testing of the controller on a test-rig. It is seen that the resulting performance is not substantially better than the performance obtained by Spijker’s laws.

The main part of this report involved the identification of the CVT ratio change behaviour. The obtained knowledge of this behaviour will be used to design an improved ratio change controller. During identification an alternative way of CVT steering is developed. This CVT steering enables to change the pulley pressures without disturbing the change of ratio. Using this way of CVT excitation a ratio change model has been estimated. After applying linearizing functions that compensate for non-linearities, the behaviour can be described accurately by low order linear models. Steering the ratio change without disturbing the pulley pressures requires a pressure model that describes the pulley interaction completely. Therefore, a MIMO pressure model has been estimated. Despite some output drift, the pressure behaviour can be modelled adequately by low order linear models.
Chapter 1

Introduction

At the Faculty of Mechanical Engineering a hybrid drive line for passenger cars is being developed. Main objectives are the reduction of fuel consumption and exhaust gas emissions. The system incorporates a high speed flywheel, an Internal Combustion engine and a Continuously Variable Transmission (CVT). The IC engine accelerates the flywheel up to its operating speed before drive off and, during driving, it replenishes dissipated energy such as rolling resistance, air drag, etc. The vehicle is accelerated by decelerating the flywheel. During braking, the kinetic energy of the vehicle is recuperated by accelerating the flywheel, instead of dissipating the energy at the brakes. This way the energy of the drive-line is conserved.

A CVT is employed to exchange energy between the flywheel and the wheels. This is realized by continuously varying the ratio of the CVT. The main components of a CVT are two pulleys and a V-belt. The pulleys consist of two conical sheaves of which one can be shifted hydraulically in axial direction by means of a hydraulic cylinder. By an oil pressure in the cylinder, the belt is clamped between the sheaves and the CVT is able to transmit power. By shifting the sheaves simultaneously the running radii of the belt and, hence, the ratio of input and output pulley shaft speeds are varied. Sheave shifts are realized by pumping oil in, respectively from, the cylinders.

The control objectives are the control of the clamping forces and the change of CVT ratio. The clamping forces have to be sufficiently high to prevent belt slip, but, for reasons of efficiency and durability, the forces should be as low as possible.

Spijker [1] accomplished the control objectives by two independent parallel control loops. One loop realizes the desired rate of ratio change by controlling the oil flow of the driving pulley chamber. The other loop realizes the required clamping forces by controlling the chamber pressure of the driven pulley. The CVT controllers of Spijker [1] are linear adaptive controllers based on simple first order Single Input Single Output (SISO) models. The realized control performance is not satisfactory because the control loops influence each other, i.e. changing the set value of the pressure controller results in a change of ratio, and vice versa. To improve the control performance better models are required, i.e. models that take the interaction between the pulleys into account.

To improve the pressure control performance, Ploemen [2] designed a controller using a pressure model that takes the interaction between the pulleys into account. He used Spijker's law and added the pulley interaction in a simple way. During the first part of the research Ploemen’s control law has been implemented and tested on a test-rig. The results are compared with the law of Spijker.
The main part of the research involved the identification of the CVT ratio change dynamics for the purpose of improving the ratio change model. Furthermore, a CVT pressure model has been identified that describes the pulley interaction completely. System identification is the field of mathematical modelling of systems using experimental data. During an identification experiment, signals, that excite all relevant system dynamics, are applied to the system inputs, while the system outputs are recorded. By means of a computational method a mathematical description of the relation between the inputs and outputs is determined. This input-output description is called a mathematical model. This, so called, black-box modelling is used since it does not require detailed knowledge of the complex physical phenomena of the CVT. Backx and Damen [4] developed a procedure to model Multiple Input Multiple Output (MIMO) systems by linear, time-invariant and finite dimensional models. In Section 4.6 the CVT dynamics are identified using this procedure.
Chapter 2

CVT description

2.1 Function and layout

The main function of the Continuously Variable Transmission (CVT) in a drive line is to transmit power from the input to the output shaft with a continuously variable ratio of the speed of these shafts. The CVT consists of a metal belt composed of thin V-elements and strings, mounted between two V-shaped, adjustable pulleys. One of the conical sheaves of each pulley can move in axial direction. By means of an oil pressure in a hydraulic chamber the belt is clamped between the sheaves of each pulley. The resulting tensile force in the strings, combined with the pushing force between the V-elements, allows the CVT to transmit power. Shifting of the sheaves in axial directions varies the running radii of the belt and, hence, the transmission ratio. Sheave shifts are realized by oil flows to or from the hydraulic chambers.

The pulley chamber pressures to realize the required clamping forces and the oil flows to change the ratio are accomplished by the hydraulic system (see Fig. 2.2). This system

Figure 2.1: CVT schematic layout.

Figure 2.2: Main components of the hydraulic circuit for the CVT in the hybrid drive line
incorporates two servo-valves and a pressure circuit that provides the operating pressure. A pump and an accumulator assure an approximately constant line pressure of 70 bar.

2.2 Definitions

Like in conventionally powered vehicles, the primary pulley, identified by a subscript \( p \), is defined as the pulley at the engine's side of the transmission. The other, secondary pulley is identified by subscript \( s \). This definition is independent of the power flow through the CVT.

The terms driving pulley and driven pulley are used if the power flow in the transmission is relevant. In these cases the pulleys are identified by the subscripts 1 and 2 for the driving and the driven pulley, respectively. The CVT ratio \( i \) follows from \( \omega_s = i \omega_p \), where \( \omega_k (k = p, s) \) is the angular velocity of pulley \( k \). The time derivative \( \frac{d}{dt} \) of this ratio is called the rate of ratio change.

Since, in a hybrid drive-line, a change of CVT ratio implies the CVT to be transmitting power, a relation exists between the rate of ratio change and the pulleys being driven or driving. A positive \( \frac{d}{dt} \) means an acceleration of the vehicle, a deceleration of the flywheel and, from the above definitions, the primary and secondary pulley being the driving and driven pulley respectively. Vice versa, a negative \( \frac{d}{dt} \) implies the primary pulley to be driven and the secondary pulley being driving.

2.3 CVT control objectives.

Two main control objectives for the CVT are:

- control the rate of ratio change. In a hybrid drive-line a change of CVT ratio accelerates or decelerates the flywheel, which introduces a torque in the drive-line. So, by controlling the rate of ratio change the desired output torque can be realized;

- control of the pulley chamber pressures. In the hybrid drive-line, high torques are transmitted by the CVT. In order to prevent belt slip the pulley chamber pressures have to be sufficiently high. On the other hand, high pressures affect the efficiency and the durability of the V-belt negatively, therefore the pressures have to be as low as possible.

2.4 CVT modelling

2.4.1 Hydraulic and geometrical CVT models

To derive the hydraulic and geometrical CVT relations several assumptions are made. In the following these assumptions and the consequences for the relations between valve signal \( u_k \), oil flow \( Q_k \), pulley sheave position and velocity \( x_k \) and \( v_k \), and running radius \( r_k \) are given. (see Spijker [1] pp. 42-44 and Fig. 2.3.)
2.3.1: The electro hydraulic servo-valve

2.3.2: CVT geometry

\[ Q_k = V_{q,k}(\Delta p_k)u_k; \quad V_{q,k} = c_q V_{q,k}^*; \]  \hspace{1cm} (2.1)

Figure 2.3: Definition of valve and CVT geometry quantities

- the valve characteristic is static;
- the valve characteristic is temperature independent;
- the valve response has no time delay;

\[ Q_k = Av_k \]  \hspace{1cm} (2.2)

Constant \( c_q \) depends on the actual line pressure and \( V_{q,k}^* \) is a non-linear function of the pressure drop \( \Delta p_k \) over the valve. For positive flows \( \Delta p_k \) is the difference between the line pressure \( p_{line} \) and the pressure \( p_k \) in the hydraulic chamber of pulley \( k \). For negative flows \( \Delta p_k \) equals the difference between \( p_k \) and the atmospheric pressure \( p_a \) (see Fig. 2.3.1).

- the deformation of the hydraulic chamber is negligible;
- the oil is incompressible;
- oil leakage is negligible;

\[ x_k = 2 \tan \beta (r_k - r_0); \quad r_o = r_k(i = 1) \]  \hspace{1cm} (2.3)

- the top angle \( \beta \) is constant;
- the belt has a rigid cross-section;
- the belt on each pulley of the pulleys forms an arc with constant radius;

\[ r_p = ir_s \]  \hspace{1cm} (2.4)

- slip between belt and pulley is neglected;

\[ \frac{(\pi - 2\delta)r_p + (\pi + 2\delta)r_s + 2a \cos \delta = L;}{r_s - r_p = a \sin \delta} \]  \hspace{1cm} (2.5)

Eq.'s (2.4) and (2.5) give the relation between \( r_p, r_s, i \) and angle \( \delta \equiv \arcsin \left( \frac{r_s - r_o}{a} \right) \). If any of these variables is given, the other three can be obtained by solving the equations.
The pulley sheave position \( x_k \) as obtained from Eq. (2.3) to Eq. (2.5):

\[
x_k = g_k; \quad g_k = \begin{cases} 2 \tan \beta \left[ \frac{L-2a}{2x} \left( \frac{i-1}{i+1} \right) + L \left( \frac{1}{i+1} \right) \frac{2}{\pi} (1-\cos \delta - \delta \sin \delta) \right] & \text{if } k = p \\ 2 \tan \beta \left[ -\frac{L-2a}{2x} \left( \frac{i-1}{i+1} \right) + L \left( \frac{1}{i+1} \right) \frac{2}{\pi} (1-\cos \delta - \delta \sin \delta) \right] & \text{if } k = s \end{cases}
\] (2.6)

A relation between oil flow \( Q_k \) and rate of ratio change \( \frac{di}{dt} \) can be obtained using Eq. (2.2) and the derivative of Eq. (2.3) to Eq. (2.5). This results in

\[
Q_k = f_k \frac{di}{dt}; \quad f_k = \begin{cases} 2A \tan \beta & \frac{\pi-2\delta}{\pi(1+i)+2(i-1)(i-1)} \sin \delta & \text{if } k = p \\ -2A \tan \beta & \frac{\pi+2\delta}{\pi(1+i)+2(i-1)(i-1)} \sin \delta & \text{if } k = s \end{cases}
\] (2.7)

Hence, the primary and secondary oil flow are related by

\[
Q_s = h_s Q_p; \quad h_s = -\frac{\pi-2\delta}{\pi+2\delta}
\] (2.8)

Spijker [1] derived the following equation to determine \( \delta \) as a function of \( i \):

\[
(1-i) \cos \delta + \left[ \left( \frac{\pi}{2} + \delta \right) + i \left( \frac{\pi}{2} - \delta \right) \right] \sin \delta = (i-1) \frac{L}{2a}
\] (2.9)

With \( \delta \) it is straightforward to determine \( f_k, g_k \) and \( h_s \) as a function of \( i \). A graphical representation of these functions is given in Figure 2.4.

![Graphical representation of functions](image)

**Figure 2.4:** A graphical representation of the functions \( g_{p,s}, f_{p,s} \) and \( h_s \)

### 2.4.2 Clamping forces

Clamping forces at the pulleys have to be sufficiently high to avoid belt slip. On the other hand, high pressures affect the efficiency and the durability of V-belt and pulleys negatively, therefore the pressures have to be as low as possible.

For on-line calculation of the set-point of the pulley pressure controller, Spijker [1] used a Coulomb friction model for the belt-pulley contact. Given an external torque \( T_k \) on pulley \( k \) the set value \( p_{k, set} \) of pressure controller is given by:

\[
p_{k, set} = c_{safe} \frac{\cos \beta T_k}{2\mu A r_k} \quad k = p, s.
\] (2.10)
With $\mu$ is the Coulomb friction coefficient and $c_{safe}$ a safety margin ($c_{safe} = 1.3$).

Note that the required pulley pressure for both primary and secondary pulley is the same, i.e. $\frac{T_p}{r_p} = \frac{T_s}{r_s}$. This is easily seen from $T_p = iT_s$, assuming no power losses, and Eq.(2.4), $r_p = i r_s$.

### 2.4.3 Relation between ratio change and output torque

The relation between the rate of ratio change $\frac{di}{dt}$ and the generated output torque $T_s$, assuming the CVT to transmit power between a flywheel and a vehicle, can be approximated by (Spijker [1]):

$$T_s = \omega_{flyw} \left( \frac{i^2}{\eta J_{flyw}} + \frac{1}{J_{veh}} \right) \frac{di}{dt} \quad (2.11)$$

where $J_{flyw}$ and $J_{veh}$ are the moment of inertia of the flywheel and the vehicle respectively. $\omega_{flyw}$ denotes the angular velocity of the flywheel and $\eta$ the CVT efficiency.
Chapter 3

CVT control

3.1 Introduction

This chapter contains the experimental verification of the pressure controller designed by Ploemen [2]. As stated before, Spijker [1] accomplished the control objectives the using two independent parallel control loops, i.e. the driving pulley is controlled to realize the desired rate of ratio change while the driven pulley is pressure controlled (Section 3.3). The model based controllers have been designed using SISO models. These models describe the pressure and ratio change behaviour of one pulley, i.e. the pulley interaction is not included. Since the pulleys can not be considered to be independent, the control loops disturb each other. For this reason, Ploemen designed a pressure controller based on a pressure model that includes the pulley interaction (Section 3.2 to 3.4). To keep the model simple Ploemen added the interaction making some simplifications. In Section 3.6 Ploemen's controller has been implemented on the test-rig and its performance has been compared with the controller of Spijker.

3.2 CVT control models

3.2.1 Pulley pressure model

To arrive at a dynamic model for control purposes, Spijker [1] carried out experiments on the test-rig described in Section 3.6.1. He added a white noise signal to valve input $u_p$ while $u_s$ was constant and measured the pressure responses $p_p$ and $p_s$. These experiments were executed under the following conditions:

- Rotational speeds of approximately 700 rpm;
- An average pulley pressure of 10 bar;
- A CVT ratio of $i = 1$;

Fig. 3.1 shows the Bode plot of the resulting open-loop pressure transfer functions from valve signal to pulley pressure, i.e.

$$ H_1(s) = \frac{\mathcal{L}\{p_p\}}{\mathcal{L}\{u_p\}}, \quad H_2(s) = \frac{\mathcal{L}\{p_s\}}{\mathcal{L}\{u_p\}}, $$

where $\mathcal{L}$ denotes the Laplace transformation.
Based on these response measurements two pressure models will be derived. We have to realize that these models are only valid for the operating conditions for which the experiments have been performed. Spijker proposed to describe the transfer functions \( H_1 \) and \( H_2 \) by linear first order models. Since the CVT is symmetric for \( i = 1 \) the pressure model can be written as:

\[
\begin{align*}
\mathcal{L}\{p_p\} &= H_1 \mathcal{L}\{u_p\} + H_2 \mathcal{L}\{u_s\} ; \quad H_1 \approx \frac{b_1}{s + a_1}, \quad H_2 \approx \frac{b_2}{s + a_2}.
\end{align*}
\] (3.2)

This model can be rewritten to a state space model of the following form:

\[
\dot{x} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
-a_1 a_2 & 0 & -(a_1 + a_2) & 0 \\
0 & -a_1 a_2 & 0 & -(a_1 + a_2)
\end{bmatrix} x + \begin{bmatrix}
b_1 & b_2 \\
b_2 & b_1 \\
-b_1 a_1 & b_2 a_2 \\
-b_2 a_1 & -b_1 a_2
\end{bmatrix} \begin{bmatrix}
u_p \\
u_s
\end{bmatrix} \] (3.3)

where the state \( x \) equals:

\[
x = \begin{bmatrix}
p_p \\
p_s \\
\dot{p}_p - b_1 u_p - b_2 u_s \\
\dot{p}_s - b_2 u_p - b_1 u_s
\end{bmatrix} \] (3.4)

To arrive at a simple model Spijker neglected the coupling between the pressure responses of both pulleys, i.e. \( H_2 = 0 \) or \( a_2 = b_2 = 0 \). This simplification yields, after zero-pole cancellation, the following model:

\[
\begin{align*}
\dot{p}_p &= a_1 0 \begin{bmatrix} p_p \\ p_s \end{bmatrix} + b_1 0 \begin{bmatrix} u_p \\ u_s \end{bmatrix} \\
\dot{p}_s &= 0 a_1 \begin{bmatrix} p_p \\ p_s \end{bmatrix} + b_1 0 \begin{bmatrix} u_p \\ u_s \end{bmatrix}
\end{align*}
\] (3.5)

From Fig. 3.1 it can be seen that neglecting the pulley pressure interaction is conflicting with reality. For this reason Ploemen [2] proposed a pressure model that takes the interaction into account. To obtain a simple model Ploemen assumed that:

\[
H_2 = \alpha H_1,
\] (3.6)
where $\alpha$ is constant. Fig. 3.2 shows that this assumption is not right since $\alpha$ strongly varies in the relevant frequency band from 1 to 20 Hz. However, if Eq. (3.6) is used then it is easily seen that $a_1 = a_2 \equiv a_p$ and $b_2 = \alpha b_1 \equiv \alpha b_p$. With these assumptions Eq. (3.3) becomes:

$$\begin{bmatrix} \dot{p}_p \\ \dot{p}_s \end{bmatrix} = \begin{bmatrix} a_p & 0 \\ 0 & a_p \end{bmatrix} \begin{bmatrix} p_p \\ p_s \end{bmatrix} + b_p \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} u_p \\ u_s \end{bmatrix}$$  \hspace{1cm} (3.7)

### 3.2.2 Sheave velocity model

Spijker [1] realizes the desired rate of ratio change by controlling the pulley sheave velocity $v_1$ of the driving pulley (Section 3.3). $v_1$ is related to $\dot{\beta}$ by CVT geometry. A pulley sheave velocity $v_1$ is obtained by an oil flow $Q_1$ through the servo valve, which is the consequence of valve signal $u_1$.

In Section 2.4.1 the relation between the oil flow through the valve $Q_k$ and the pulley sheave velocity $v_k$ is given by $Q_k = A v_k$, after making some assumptions. For control purposes, Spijker proposed to use a dynamic relation between $Q_k$ and $v_k$. Since a pulley sheave velocity is introduced by a pressure difference over the hydraulic cylinders the same model structure as the pressure model is used:

$$\dot{v}_k = -a_q v_k + b_q Q_k \quad k = p, s. \hspace{1cm} (3.8)$$

When we compare this dynamic relation with the static relation of Eq. (2.2), $Q_k = A v_k$, we can conclude that $a_q = A b_q$ and so:

$$\dot{v}_k = -a_q v_k + \frac{a_q}{A} Q_k \quad k = p, s. \hspace{1cm} (3.9)$$

### 3.3 Control philosophy

During operation one pulley is under pressure control to prevent belt slip while the sheave velocity of the other pulley is controlled to realize the desired rate of ratio change. To prevent the possibility that the flow to realize the sheave shift results in decreasing pulley pressures, always a positive flow is used. This implies that if $\frac{di}{dt} > 0$ the primary pulley is sheave velocity controlled and the secondary pulley is pressure controlled. Otherwise, the primary pulley is pressure controlled and the secondary pulley is sheave velocity controlled. Besides, the pressure of the sheave velocity controlled pulley is monitored. If the pressure is less than the necessary pulley pressure then this pulley is also pressure controlled. This to prevent belt slip under all circumstances. The strategy can be expressed by the following
switching logic:

if (rate of ratio change $\geq 0$) then
    pressure control on the secondary pulley
    if (primary pulley pressure $\geq$ necessary pulley pressure) then
        sheave velocity control on the primary pulley
    else pressure control on the primary pulley
else
    pressure control on the primary pulley
    if (secondary pulley pressure $\geq$ necessary pulley pressure) then
        sheave velocity control on the secondary pulley
    else pressure control on the secondary pulley
end

In terms of driving and driven this statement can be formulated as: the driven pulley is pressure controlled and the driving pulley is sheave velocity controlled as long as the pulley pressure is sufficiently high, otherwise it is also pressure controlled. This is illustrated by Fig. 3.3.\textsuperscript{1}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cvt_control_block_scheme.png}
\caption{The CVT control block scheme}
\end{figure}

## 3.4 Pressure control

The control law designed by Ploemen \cite{ploemen} adds a simplified version of the interaction between the pulley chamber pressures to the pressure control law of Spijker \cite{spijker}. This law is based on the CVT pressure model, given in Subsection 3.2.1. It is designed to investigate whether this interaction should result in a better control performance. His law is slightly adapted since, in the original form, it is not suited for actual implementation because then the time-derivative $p_1$ of the driving pulley chamber pressure must be known. Although $p_1$ is measured the derivative is not known. Numerical differentiation of $p_1$ might cause problems because of, for instance, measurement noise.

\textsuperscript{1}The used switch blocks connect the outer inputs to the output(s), based on the value of the middle input. If the middle input is greater than or equal to zero the solid line is passed through, else the dashed line is passed through.
The pulley pressure model is only valid under certain conditions, see Subsection 3.2.1. To obtain a controller that will operate under all operating conditions, i.e. independently of the CVT ratio, the oil temperature, etc., an adaptive control strategy is used. The modified control law is based on the following starting-points:

- The pressure model for the pressure controlled pulley is given by Eq. (3.7):
  \[ \dot{p}_2 = -a_p p_2 + b_p u_2 + b_p \alpha u_1, \] (3.10)
  where \( p_2 \) is measured and the input \( u_2 \) is determined by the control law for this pulley. Ploemen obtained a SISO system by elimination of \( u_1 \), using Eq. (3.5). This yields an expression for \( \dot{p}_2 \) which is a function of \( \dot{p}_1 \). As mentioned above estimation of \( \dot{p}_1 \) is not trivial, for instance because of measurement noise.

- The desired pressure behaviour is specified by means of a reference model. This model is designed such that the reference pressure, \( p_r \), in a certain time converges to the desired value \( p_{2,\text{set}} \) which is derived from Eq. (2.10),
  \[ \dot{p}_r = -a_{pr} p_r + a_{pr} p_{2,\text{set}}; \quad a_{pr} > 0. \] (3.11)
  The parameter \( a_{pr} = 40\pi \) to realize the desired system bandwidth of 20 Hz.

- We wish that the output error defined as \( e_p = p_r - p_2 \) is dampened out according the following relation:
  \[ \dot{e}_p = -a_{pr} e_p. \] (3.12)

With these starting-points the control law is derived as follows: from Eq. (3.10) and Eq. (3.11) it can easily be seen that
  \[ \dot{e}_p = \dot{p}_r - \dot{p}_2 = -a_{pr} e_p + [(a_p - a_{pr}) p_2 + b_{pr} p_{2,\text{set}} - b_p \alpha u_1 - b_p u_2]. \] (3.13)
To achieve the desired error dynamics the input \( u_2 \) is chosen as
  \[ u_2 = \frac{\hat{a}_p}{b_p} p_2 + \frac{a_{pr}}{b_p} (p_{2,\text{set}} - p_2) - \hat{\alpha} u_1, \] (3.14)
where \( \hat{a}_p, \hat{b}_p \) and \( \hat{\alpha} \) are the available estimates for \( a_p, b_p \) and \( \alpha \), respectively. The error equation (3.14) then becomes
  \[ \dot{e}_p = -a_{pr} e_p + \vartheta_1 p_2 + \vartheta_2 (p_{2,\text{set}} - p_2) - \vartheta_3 u_1, \] (3.15)
where \( \vartheta_1, \vartheta_2 \) and \( \vartheta_3 \) are given by:
  \[ \vartheta_1 = a_p - \frac{b_p}{\hat{b}_p} \hat{a}_p; \quad \vartheta_2 = \left( 1 - \frac{b_p}{\hat{b}_p} \right) a_{pr}; \quad \vartheta_3 = b_p (\alpha - \hat{\alpha}); \] (3.16)
In order to arrive at a proper adaptation law for the parameters \( \hat{a}_p, \hat{b}_p \) and \( c_p \), such that stability is guaranteed, a candidate Lyapunov function \( V \) is chosen:
  \[ V = \frac{1}{2} e_p^2 + \frac{1}{2} \beta \vartheta_1^2 + \frac{1}{2} \delta \vartheta_2^2 + \frac{1}{2} \gamma \vartheta_3^2; \quad \beta, \delta, \gamma > 0 \] (3.17)
which represents an accumulation of the pressure error and the assumed parameter error. Differentiation of this relation with respect to time yields, after substitution of Eq.(3.15):

\[ \dot{V} = -a_{pr}e_p^2 + \dot{\vartheta}_1[\beta \dot{\vartheta}_1 + e_p p_2] + \dot{\vartheta}_2[\beta \dot{\vartheta}_2 + e_p (p_{2,\text{set}} - p_2)] + \dot{\vartheta}_3[\gamma \dot{\vartheta}_3 - u_1 e_p], \tag{3.18} \]

and therefore \( \dot{V} < 0 \) for each \( e_p \neq 0 \) if \( \dot{\vartheta}_1, \dot{\vartheta}_2 \) and \( \dot{\vartheta}_3 \) satisfy

\[ \begin{align*}
\dot{\vartheta}_1 &= -\frac{1}{\beta} e_p p_2, \\
\dot{\vartheta}_2 &= -\frac{1}{\delta} e_p (p_{2,\text{set}} - p_2), \\
\dot{\vartheta}_3 &= \frac{1}{\gamma} e_p u_1.
\end{align*} \tag{3.19} \]

If the model parameters \( e_p, b_p \) and \( \alpha \) are constant or are slowly varying this results in the following adaptation law for the controller parameters:

\[ \begin{align*}
\dot{\alpha} &= -\frac{1}{\gamma} b_p u_1; \\
\dot{b}_p &= -\frac{1}{\delta} b_p e_p (p_{2,\text{set}} - p_2) \\
\hat{a}_p &= \frac{b_p}{\beta} p_2 - \frac{1}{\delta a_{pr}} (p_{2,\text{set}} - p_2) e_p
\end{align*} \tag{3.20} \]

where \( \beta, \delta \) and \( \gamma \) denote the adaptation speed. The value of parameter \( b_p \) is not known exactly. If the adaptation law for \( \alpha \) is considered, with estimate \( \hat{b}_p \) for \( b_p \) and adaptation speed \( \gamma \), it is easily seen that this law corresponds to an adaptation law with an exactly known value of \( b_p \) and adaptation speed \( \gamma \) if \( \gamma \) is given by \( \gamma = \frac{b_p}{\beta} \gamma_c \). Since \( b_p \) and \( \hat{b}_p \) are assumed to be constant, it can be concluded that the error initiated by parameter uncertainty only affects the adaptation speed. For this reason \( b_p \) and the adaptation speed parameters \( \gamma \), \( \delta \) and \( \beta \) are combined to form new parameters defined as \( \gamma_\beta, \delta_\beta \) and \( \beta_\beta \), respectively. The adaptation laws then become:

\[ \begin{align*}
\dot{\alpha} &= -\frac{1}{\gamma_\beta} e_p u_1; \\
\dot{b}_p &= -\frac{1}{\delta_\beta a_{pr}} e_p (p_{2,\text{set}} - p_2) \\
\hat{a}_p &= \frac{b_p}{\beta_\beta} p_2 - \frac{1}{\delta_\beta a_{pr}} (p_{2,\text{set}} - p_2) e_p
\end{align*} \tag{3.21} \]

The pressure control law of Spijker is obtained by taking \( \alpha = \hat{\alpha} = \hat{\alpha} = 0 \).

### 3.5 Sheave velocity control

During the experimental verification of Ploemen’s interactive pressure controller, the sheave velocity controller of Spijker [1] is used. The sheave velocity controller has to realize the required rate of ratio change. The sheave velocity control law, derived in Appendix A, is based on sheave velocity model Eq.(3.9) combined with valve model, Eq.(2.1):

\[ \dot{v}_1 = -a_q v_1 + \frac{a_{qr}}{A} c_q V_c^* u_1 \tag{3.22} \]

An adaptive control strategy is used to compensate for non-modelled system dynamics. Furthermore, in order to reduce the noise of the measured rate of ratio change, a Kalman filter is employed (see Appendix A). The law is given by:

\[ u_1 = \frac{A}{V_c^*} \left[ \frac{1}{\hat{c}_q} v_1 + \frac{a_{qr}}{\hat{a}_q} (v_{1,\text{set}} - v_1) \right] \quad \hat{c}_q = \frac{1}{\zeta} \left( \frac{v_{1,\text{set}} - V_c^*}{a_{qr}} \right) v_{1,\text{set}} \quad \text{and} \quad \hat{a}_q = 0, \tag{3.23} \]

With \( v_1 \) the Kalman filtered sheave velocity obtained from the measured change of ratio, the set value \( v_{1,\text{set}} \) is obtained from the desired ratio change via Eq.(2.7) and Eq.(2.2). \( \hat{a}_q \) and \( \hat{c}_q \) are the estimates of model parameter \( a_q \) respectively valve parameter \( c_q \). \( a_{qr} = 40\pi \) is a parameter to realize the desired closed-loop bandwidth of 20 Hz.
3.6 Experimental comparison

The control law of Section 3.4 has been implemented in the drive line test-rig. Experiments have been carried out for this control law and for the existing law that does not take the interaction into account. The results of the experiments of both laws have been compared with each other.

3.6.1 Test-rig description

For verification of the CVT controllers a test-rig is used. The test-rig has been designed such that its dynamic behaviour approximates the behaviour of the hybrid drive line. The rig contains two flywheels, a CVT and a flexible shaft (see Fig.3.4). Flywheel 1 and flywheel 2 represent the high speed flywheel and the equivalent vehicle inertia, respectively. The flexible shaft represents the combined drive shaft and tire compliance.

![Figure 3.4: CVT test-rig layout of mechanical components](image)

A electrical DC-motor is used to accelerate the drive line via a toothed belt drive up to the operating speed. Once the desired speed is achieved, the motor is stopped and automatically disconnected through a freewheel unit.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flywheel 1 inertia</td>
<td>25.1 [kgm²]</td>
</tr>
<tr>
<td>Flywheel 2 inertia</td>
<td>1.32 [kgm²]</td>
</tr>
<tr>
<td>Flywheel's maximum operating speed</td>
<td>1800 [rpm]</td>
</tr>
<tr>
<td>Torsional resonance frequency (i = 1.0)</td>
<td>2.18 [Hz]</td>
</tr>
</tbody>
</table>

Table 3.1: CVT test-rig characteristics

3.6.2 Experiments

For both control laws the following experiments were carried out to compare their performance. To investigate the influence of the ratio change control action on the pulley pressures a step function is applied to $\frac{\theta}{\theta_{\text{set}}}$. The set value of the secondary pulley pressure was a block shaped function that switched between 15 and 25 bar. Experiments have been done for different controller parameters and different rotational speeds. At the same time experiments have been performed with and without the flywheel inertia on the secondary shaft, i.e with and without load.

3.6.3 Experimental results

Figure 3.5 shows the experimental results for both pressure control laws. From these figures it can be seen that the pressure performance of the interactive controller is not substantially better than the controller without pulley interaction. Varying the controller parameters or the output shaft inertia does not influence this result. Besides, it can be seen that the
3.5.1: Experimental results of the driven pulley pressure response.

3.5.2: Pressure step responses.

3.5.3: The desired and the measured rates of ratio change.

Figure 3.5: Experimental results

Pressure performance is not disturbed by the ratio change controller. From Figure 3.5.3 it can be concluded that the ratio change performance is not improved when interactive pressure controller is used, compared with the situation with Spijker's pressure controller. However, it can be seen that for both situations, the ratio change performance is disturbed substantial by the pressure control actions. It has to be concluded that the ratio change objectives can not controlled independently from the pulley pressures using this strategy. When the ratio change performance is compared with the pressure performance it can be concluded that the ratio change performance is much worse than the pressure performance. Improving the ratio change performance requires decoupling of the pulley interaction. This can be realized by designing CVT controllers based on models that describe the pulley interaction. So, better models are required. For this reason, the second part of the research, described in the next chapter, involved the identification of the CVT behaviour.
Chapter 4

CVT identification

4.1 Introduction

From Section 3.6.3 it has been concluded that the pressure control performance is much better than the ratio change performance. The ratio change controller is disturbed significantly by the pressure control actions. Improving the closed loop ratio change performance requires decoupling of the ratio change from the pulley pressures. In Subsection 4.3 an alternative way of CVT steering is proposed that completely decouples the ratio change from the pulley pressures, i.e. the pulley pressures can be changed without disturbing the ratio change. To arrive at a better ratio change controller the ratio change dynamics have been identified, using this system excitation. The proposed CVT steering does not decouple the pulley pressures from the ratio change completely (i.e. a change of ratio results in a change of pulley pressures). Decoupling requires a MIMO CVT pressure model. This model can also be used later for controller design. In Section 4.6 a pressure model is derived.

For identification, the IPCOS Computer Aided Control System Design software package has been used. This package, built in the MATLAB 4.0 environment, is a program for MIMO system identification based on the identification procedure developed by Backx and Damen [4].

4.2 System definition

The target of the drive line is to generate an output torque on the driver's requests. In the considered hybrid drive line a torque is generated by varying the CVT ratio, so the torque is directly related to $\frac{di}{dt}$ (see Eq.(2.11)). For this reason the CVT controller directly controls $\frac{di}{dt}$. A disadvantage of this approach is the necessity to determine the current value of $\frac{di}{dt}$. This quantity is obtained by differentiation of the quotient of the measured angular velocities of the primary and secondary pulley shafts, see Appendix B. The differentiating operation causes a phase lag and an amplification of irrelevant high frequency components. Therefore, the obtained signal has to be low pass filtered. This also introduces an undesired phase shift. For these reasons the ratio $i$ itself is used for identification instead of $\frac{di}{dt}$. Transfer of $i$ to $\frac{di}{dt}$ can be approximated later by some filter which is known and taken as a sensor filter in the control design. An other possibility is to control $i$ instead of $\frac{di}{dt}$. The other system outputs are the primary and secondary pulley chamber pressure $p_p$ and $p_s$. The valve signals $u_p$ and $u_s$ are the inputs of the system.
The system to be identified is given in Figure 4.1.1. The angular velocities $\omega_p$ and $\omega_s$

![Diagram of system identification](image)

4.1.1: The system to be identified

4.1.2: The block scheme of the system

Figure 4.1: System definition.

and the pulley pressures $p_p$ and $p_s$ are measured and ratio $i$, is obtained via $i = \frac{\omega_s}{\omega_p}$. The block scheme is given in Fig. 4.1.2.

### 4.3 Alternative CVT steering

Spijker [1] realized the control objectives of the CVT by two independent control loops. See Section 3.3. A disadvantage of this approach is that the control loops influence each other because of the interaction between the pulleys. Especially, the change of ratio is disturbed significantly by a change of $p_{2, \text{act}}$. See Fig 3.5.

For this reason a different system excitation is proposed. In this approach the pulley pressures needed to prevent belt slip are accomplished by applying a positive voltage $u_{pr}$ to the primary and the secondary pulley valves. A positive change of ratio is obtained by adding a positive signal $u_{st}$ to the primary servo valve and the same but negative signal $-u_{st}$ to the secondary pulley valve. Vice versa, a negative change of ratio is realized by applying a negative signal to the primary and a positive signal to the primary pulley, i.e. for $u_p$, $u_s$ respectively $u_1$, $u_2$:

$$
\begin{align*}
  u_p &= u_{pr} + u_{st} \\
  u_s &= u_{pr} - u_{st}
\end{align*}
$$

and

$$
\begin{align*}
  u_1 &= u_{pr} + |u_{st}| \\
  u_2 &= u_{pr} - |u_{st}|
\end{align*}
$$

for $u_{pr} > 0$ (4.1)

This way of CVT steering is expressed by the block scheme given in Fig. 4.2. Since $u_{pr}$ is applied to both inputs, $u_{pr}$ is not correlated with $i$, i.e. by steering $u_{pr}$ the pulley chamber pressures can be changed without disturbing the change of CVT ratio. Besides, a change of ratio realized by $u_{st}$ does not lead to unacceptable high pulley chamber pressures because of the negative oil flow through the driven pulley valve, initiated by the negative valve signal $-|u_{st}|$.

If the interactive pressure behaviour is known, the interaction effect from $u_{st}$ on $p_2$ can be cancelled completely when $u_2$ is chosen,

$$
\begin{align*}
  u_2 &= u_{pr} - H_{12}(s)|u_{st}|; \\
  H_{12}(s) &= \frac{p_2(s)}{u_1(s)} \\
  H_{22}(s) &= \frac{p_2(s)}{u_2(s)}
\end{align*}
$$

(4.2)
\( H_{12} \) and \( H_{22} \) are transfer functions relating valve signal \( u_1 \), respectively \( u_2 \) to the driven pulley pressure \( p_2 \).

When the method proposed here is compared with the method of Spijker [1] it can be concluded that both realize a change of ratio by adding a positive signal to the driving pulley valve \( u_1 \). To compensate for the increasing pulley pressures both methods decrease \( u_2 \). Spijker [1] determines \( u_2 \) using a pressure feedback controller. The method proposed here, decreases \( u_2 \) with \( u_{St} \), which can be seen as a simple feedforward control action, assuming \( H_{12} = H_{22} \). Since the feedforward controller takes action before the actual rise of pulley pressures, while the feedback controller reacts after the pressure increase is measured, the feedforward controller can be considered faster.

In Section 4.5 the CVT ratio change has been identified using this CVT steering.

### 4.4 Identification procedure

For the identification of the CVT dynamics the procedure developed by Backx and Damen [4] is used. They describe the system behaviour by a discrete time, linear, time invariant, MIMO model of finite order. The identification procedure consists of several steps. Each step is directed to obtain more detailed information about the dynamic behaviour of the system. The information from the previous steps is used as a-priori information for the next step. The procedure covers the following sequential aspects:

1. **Preliminary study:**
   - selection of system inputs and outputs
   - experiments to test the linearity of the system, to find static gains and to get a rough estimate for the largest relevant time constant. Step and sinusoidal input signals are used. From the step responses the static gain, an estimate of the largest time constant and the range of steady state linearity is determined. The sinusoidal responses give an impression of the dynamic linearity.
   - experiments to analyse fast process dynamics and to accurately determine time delay: the Fast PRBNS experiment. A Pseudo Random Binary Noise Sequence (PRBNS) signal is applied to excite all relevant system dynamics. From the measured response the system bandwidth and the smallest relevant time constant can be obtained. The delay between input and output is determined, using correlation analyses.
2. **Experiment for model estimation:** with the information obtained from the preliminary experiments now a final PRBNS experiment can be designed. The collected input/output data will be used to estimate the process model.

3. **Signal processing:** the data from the final experiment is not suitable for immediate use in identification algorithms and has to be pre-processed. The pre-processing involves the following steps:
   - scaling and offset correction;
   - filtering;
   - sample rate reduction;

4. **Model estimation**

To estimate a system model for control purposes the procedure, developed by Backx and Damen [4] is used. This procedure results in a $r$th order MIMO State Space model with $p$ inputs and $q$ outputs:

$$
\begin{align*}
    x_{k+1} &= Ax_k + Bu_k \\
    y_k &= Cx_k + Du_k
\end{align*}
$$

where:
- $y_k \in \mathbb{R}^{q \times 1}$ output vector at time $k$
- $u_k \in \mathbb{R}^{p \times 1}$ input vector at time $k$
- $x_k \in \mathbb{R}^{r \times 1}$ state vector at time $k$
- $A \in \mathbb{R}^{r \times r}$ state matrix
- $B \in \mathbb{R}^{r \times p}$ input matrix
- $C \in \mathbb{R}^{q \times r}$ output matrix
- $D \in \mathbb{R}^{q \times p}$ direct feed through matrix

The input-output relation of a State Space model, assuming an initial state of zero, is:

$$
\begin{equation}
    y_k = \sum_{j=0}^{\infty} CA^{i-1} Bu_{k-j} + Du_k 
\end{equation}
$$

The procedure starts with the estimation of the **Finite Impulse Response** (FIR) or Markov parameter model. A FIR model describes the system input-output behaviour by a finite number $n$ Markov parameters:

$$
\begin{equation}
    y_k = \sum_{i=0}^{n} M_i u_{k-i}; \quad M_i = \begin{cases} 
        D & \text{if } i = 0 \\
        CA^{i-1} B & \text{if } i > 0
    \end{cases}
\end{equation}
$$

where $M_i \in \mathbb{R}^{q \times p}$ the $i$th Markov parameter.

This type of model requires almost no a-priori knowledge of the system. Only the impulse length of the system $n$ has to be determined. This information can be obtained using the largest system constant, determined during preliminary experiments. The drawback of the FIR model is the large number of parameters to be estimated. This makes FIR models not suited for controller design and implementation, because of the computation power required. Therefore, Backx and Damen [4] propose to reduce this model using a so called MPSSM model set\(^1\)

\(^1\)A model set is a selection of models that have similar representations. Model parameters still have to be quantified (Backx [3]).
A MPSSM or Minimal Polynomial and Start Sequence of Markov parameters model can be derived by applying the Caley Hamilton theorem to the state matrix $A$:

$$f(A) = A^r + a_1 A^{r-1} + \ldots + a_{r-1} A + a_r = 0$$  \hspace{1cm} (4.7)

where $f(A)$ is minimal polynomial of $A \in \mathbb{R}^{r \times r}$. With this theorem it is possible to express any power $k > r$ of $A$ as a linear combination $A^0, A^1, \ldots, A^{r-1}$:

$$A^{k-1} = \sum_{i=1}^{r} -a_i A^{k-i-1}$$  \hspace{1cm} (4.8)

Premultiplication of this equation with output matrix $C$ and postmultiplication with input matrix $B$ yields a recurrent relation for the Markov parameters:

$$M_k = \sum_{i=1}^{r} -a_i M_{k-i} \quad \forall \; k > r$$  \hspace{1cm} (4.9)

The resulting input-output relation under the MPSSM model structure is given by:

$$y_k = \sum_{j=0}^{\infty} F_j(\hat{a}, \hat{M}) u_{k-j}; \quad F_j = \begin{cases} \hat{M}_j & j = 0, 1, \ldots, r \\ \sum_{i=1}^{r} \hat{a}_i F_{j-i} & j > r \end{cases}$$  \hspace{1cm} (4.10)

So the structure of the MPSSM model is determined by the degree $r$ of the minimal polynomial. The Section 4.5.9 a method will be given to determine degree $r$. The advantage of the model is found in the decrease in the number of parameters of the model compared to the number of parameters required by the FIR model.

Backx and Damen [4] proposed to estimate the parameters $F_i$ of the MPSSM model from the input and output data using an output error criterion. Because the output error is nonlinear in the parameters of the model, a nonlinear numerical optimization algorithm is used. In order to avoid numerical local minimum problems and speed up the minimization procedure, a good initial estimate of the parameters is essential. This initial estimate is derived by fitting a MPSSM model to the FIR model derived earlier.

Finally, the resulting MPSSM model is translated into a State Space model. A minimal State Space model with matrices $\{A, B, C, D\}$ that satisfies the MPSSM model defined by $\{\hat{a}, \hat{M}\}$ is the canonical observability form (Backx [3]):

$$A = \text{diag}(A_1, A_2, \ldots, A_q);$$

$$A_i = \begin{bmatrix} 0 & & & \\ \vdots & I_{r-1} & & \\ 0 & & & \\ -\hat{a}_r & -\hat{a}_{r-1} & \cdots & -\hat{a}_1 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$$

$$D = \hat{M}_0;$$

The initial value of the state vector is assumed to be zero.

The sequential steps of the described model estimation procedure is given in the block diagram of Figure 4.3.
4.5 Ratio change identification

4.5.1 Preliminary study: Input/output selection

For identification, the system excitation described in Section 4.3 is used, i.e. $u_{pr}$ and $u_{st}$ are the input signals. $u_{pr}$ realizes the required pulley pressures and $u_{st}$ the change of ratio. Since $u_{pr}$ is not correlated with $i$, the identification of the ratio change behaviour can be formulated as the identification of the transfer of $u_{st}$ to $i$. During identification $u_{pr}$ is kept constant, $u_{pr} = 0.4$ volt is used.

4.5.2 Preliminary study: System linearization

The outlined identification method assumes the system to be linear. However, the system obtained in Section 4.3 is highly non-linear. The non-linearities of the system are caused by the actuators, (the servo valves) and CVT geometry. Compensation for these non-linearities is done by application of linearizing functions.

The non-linear behaviour of valve $k$ is given by Eq.(2.1), i.e. $Q_k = V_{q,k} u_k$. Provided that $V_{q,k}$ is known, the valve behaviour can be linearized by replacing $u_k$ by $V_{q,k}^{-1} Q_{st,k}$, where $V_{q,k}^{-1}$, the inverse valve characteristic, is a so called linearizing function and $Q_{st,k}$ is the steer flow. The resulting valve behaviour is $Q_k = Q_{st,k}$. This linearizing function is
used to write the system input $u_{st}$ in terms of the primary pulley flow $Q_{st,p}$:

$$u_{st} = \begin{cases} \frac{V_{q,p}^{-1}Q_{st,p}}{h_s} & \text{if } Q_{st,p} \geq 0 \\ h_s V_{q,s}^{-1}Q_{st,p} & \text{if } Q_{st,p} < 0 \end{cases} \tag{4.12}$$

If $Q_{st,p} > 0$, $Q_{st,p}$ is realized by flow steering of the primary pulley, otherwise flow steering is applied to the secondary pulley valve. $Q_{st,p}$ has to be $h_s(i)Q_{st,p}$, as shown in Section 2.4.1.

To obtain a linear input-output behaviour the geometrical non-linearities have to be compensated. A linearizing function is designed to write the system output in terms of primary pulley sheave position $x_p$. The function that describes the transfer from output $i$ to $x_p$ is given by $g_p(i)$ (Eq. 2.6). The block scheme of the linearized system is given in Fig. 4.4.

![Block scheme of steady state linearized system](image)

Figure 4.4: The block scheme of steady state linearized system

4.5.3 Preliminary study: Linearity tests

The first test is the test for steady state linearity of the system. The input signal for this test is the step signal. For interpretation of the results a fancy input signal, defined as $x_{st,p} = \frac{1}{A} \int Q_{st,p} dt$, is used. As a consequence input and output have the same unit and can be compared easily.

Apart from the steady state non-linearities, also the dynamic non-linearities have to be investigated. Sinusoidal signals have been used to test this non-linearity.

Steady state linearity test

To investigate whether the transformed system of the previous section really behaves steady state linear, several step signals are applied to $Q_{st,p}$. Positive as well as negative steps with different heights have been used. The input signal, written in terms of $x_{st,p}$, and the system response of two experiments, are given in Fig. 4.5.1 and Fig. 4.5.2 for a positive, respectively a negative step in $Q_{st,p}$. From these figures it can be concluded that the response reasonably corresponds with the steer input. For the positive step it can be seen that the response is degressive, which can be explained by the oil leakage. v.d. Leest [6] measured the oil leakage flow of the primary and secondary pulley for several static pulley chamber pressures. He found a linear relation between the oil leakage flow and the pulley chamber pressure. Besides he found the primary oil leakage flow to be ten times the oil leakage flow of the secondary pulley.
These conclusions correspond to the step responses of Fig. 4.5. The oil leakage of the secondary pulley is negligible and the oil leakage of the primary pulley is increasing because of the degressive pulley sheave displacement. This can be explained by the increasing pulley pressure during the experiment, see Fig. 4.5.3.

In order to verify the used inverse valve characteristic a straight line is fitted through the response. The slope of the fitted line corresponds to the sheave velocity $v_p$. The flow $Q_p$ is obtained by multiplying the pulley sheave velocity $v_p$ by the pulley piston area $A$. Fig. 4.5.4 shows $Q_p$ versus the steering flow $Q_{p, st}$ for different experiments.

From this figure it can be concluded that the used inverse valve characteristic realizes a linear system response.

**Dynamic linearity test**

To test the system for dynamic non-linearities sinusoidal signals have been used. Experiments are done for different amplitudes as well as different frequencies. The input and the measured output, written in terms of primary pulley sheave position, are given in Figure 4.6. From these Figures the following conclusions can be drawn:

- All output signals have a trend. This phenomenon may be related to the difference in oil leakage between the primary and secondary pulley.
The 1 Hz signals are corrupted with a 10 Hz signal. The lower the amplitude of the signal the more this corruption is reduced. This corruption may be caused by a resonance in the drive-line. During experiments it was seen that its intensity is also related to the clutch pressure: a low pressure results in a high intensity. Nevertheless, the corruption can not be eliminated by applying the maximum clutch pressure.

- The frequency of the output signals correspond to the frequency of the input signals.
- Figure 4.6.2 shows that the amplitudes of the output signals reasonably correspond to the amplitude of the input signals responses.

The conclusion is that the system can be said to be dynamic linear. Furthermore, it can be concluded that the system output is corrupted by drift and a 10 Hz signal. In the next section an output feed-back controller will be designed to correct for the drift, because signals that contain drift are not suited for identification. It is expected that the 10 Hz corruption will not cause problems during identification because of its low power.

4.5.4 Preliminary study: Drift correction controller design

Trends or drifts are changes of the outputs caused by not modelled dynamics. They are not correlated to the system inputs. The results of the dynamic linearity test in the previous section show that the drift has a low frequency and a relatively large amplitude. Because of the low-frequency behaviour and, for this frequency, the short duration of the experiment, the trend will not average out during the identification experiment. Therefore the trend can not be considered as independent output noise. As a consequence, the correlation between input signal and trend will not equal zero and even can be large because of the large amplitude of the trend. This will considerably deteriorate the estimation results.

To eliminate the drift, a simple feedback controller is applied. In Appendix A it is shown that the obtained ratio \( i \) and so the system output \( x_p \), are corrupted with measurement noise proportional to the rotational frequencies of the CVT shafts. A low pass filter is applied to filter out the measurement noise and to let the low frequency drift pass. The
chosen filter is of first order to keep the order of the closed-loop system as low as possible. A filter time constant \( \tau_f \) of \( \frac{1}{4\pi} \) is used to obtain a sufficiently high attenuation of the noise. The filter transfer function \( H_f \) can be written as:

\[
H_f = \frac{K_f}{\tau_f s + 1}
\]

(4.13)

The chosen value of the static gain \( K_f \) is \( \frac{1}{80} \) \([m^2/s]\).

For identification the closed-loop system is used. Afterwards the open-loop transfer function \( H_o \) can be obtained using:

\[
H_o = \frac{H_c}{1 - H_c H_f}
\]

(4.14)

where \( H_c \) is the transfer function of the closed-loop system.

4.5.5 Preliminary study: Fast PRBNS experiment

The Fast PRBNS experiment is directed to determine the system bandwidth and the system’s largest time constant. The information obtained is used to design the PBRNS experiment for the final data acquisition. This experiment involves the application of a Pseudo Random Binary Noise Sequence (PRBNS) signal. (see Appendix B). The definition of the signal is based on the system characteristics. Length, sampling time and minimum switching time and magnitude of the fast PRBNS are determined based on this characteristics:

Experiment duration: The length of the experiment is related to the slowest settling time of the process. Backx and Damen [4] state that the experiment duration has to be chosen ten to twenty times the expected largest time constant. A rough estimate of the largest time constant of the open-loop can be obtained from the step experiment. From this experiment no significant transient response can be seen, therefore the open-loop transfer function can be estimated by:

\[
H_o = \frac{x_p(s)}{Q_{st,p}(s)} \approx \frac{K_o}{s}
\]

(4.15)

where the static gain \( K_o \) equals \( \frac{1}{A} \), with \( A \) the piston area of the hydraulic cylinder. Using Eq.(4.13) the transfer function of the closed-loop system \( H_c \) can be written as:

\[
H_c = \frac{H_o}{1 + H_o H_f} \approx \frac{K_o(\tau_f s + 1)}{\tau_f s^2 + s + K_o K_f}
\]

(4.16)

Now, an estimate of the largest relevant time constant for the closed-loop system can be calculated. This yields a largest time constant of about 0.5 s. As a result an experiment duration of 25 seconds is chosen.

PRBNS switching time: The minimum switching time \( \theta \) of the PRBNS, i.e. the shortest duration in which the signal stays constant, determines the bandwidth of the signal (Appendix B), and thus the frequency range over which the system is excited. Since this experiment is aimed to investigate all relevant dynamics present in the system, the switching time is chosen relatively high, i.e. a factor 10 to 20 faster than the expected fastest time constant. In Appendix B it is also shown that \( \theta \) is inverse proportional to the power contents of the signal. To keep the system persistent exciting, \( \theta \) must not be chosen to fast. Hence, \( \theta = 12.5 \) ms is chosen.
**PRBNS amplitude:** The power of the signal is proportional to the square of the amplitude of the signal. The amplitude of the Fast PRBNS signal has to be chosen such that the signal to noise ratio is sufficiently high. An amplitude of 4.5 l/min. is chosen.

**Influences of drive-line dynamics**

From Table 3.1 it is known that the resonance frequency of the drive-line is 2.18 Hz, which is caused mainly by the flexible shaft. These drive-line dynamics influence the measured CVT behaviour. To investigate this influence, Fast experiments were done with and without secondary flywheel inertia. By disconnecting the secondary flywheel, the CVT does not transmit power.

A quick impression of the difference in dynamic behaviour for both situations is obtained by computing the *Transfer Function Estimate* (TFE) from the Fast PRBNS input/output data. The TFE estimates the system transfer function by

\[ \hat{H}_c(f) = \frac{S_{Q_{st,p}x_p}(f)}{S_{Q_{st,p}Q_{st,p}}(f)} \]

where \( S_{Q_{st,p}x_p} \) is the estimated cross power spectrum between input \( Q_{st,p} \) and output \( x_p \) and \( S_{Q_{st,p}Q_{st,p}} \) the auto power spectrum of \( Q_{st,p} \). Figure 4.7.1 shows the Bode plots of \( \hat{H}_c(f) \) for the situation with and without secondary flywheel. From this figure it can be concluded that the drive-line dynamics significantly influence the measured CVT behaviour. Since the assignment is to identify the CVT dynamics, in the sequel the situation without secondary flywheel inertia is used.

**Determination of the system bandwidth**

The system bandwidth \( f_{bw} \) is the highest significant frequency passed by the system and is defined here as the frequency for which the amplitude of the transfer function has a 40 dB attenuation.
Knowledge of the system bandwidth is important for the determination of the PRBNS switching time $\theta$ for the final experiment and, related to this quantity, the sample frequency. In Appendix B it is shown that the switching time $\theta$ determines the bandwidth of the PRBNS signal, and consequently the frequency range over which the system is excited. Since for identification it is desired that all relevant system dynamics are excited equally, the choice of the minimum switching time of the PRBNS signal depends on the system bandwidth.

Fig. 4.7.1 shows the estimated $|\hat{H}_c(f)|$ from the Fast PRBNS data. From this figure it can be seen that $f_{bw} = 20$ Hz.

**Determination of the largest relevant time constant.**

Knowledge of the largest relevant time constant of the system is important for the determination of the minimum duration of the final PRBNS experiment. In Appendix C it is shown that when the input of a linear system is white noise, the cross correlation function between input and output is the impulse response of the corresponding input/output transfer. Figure 4.7.2 shows the cross correlation between $Q_{st,p}$ and $x_p$ (both normed). The figure shows that the largest relevant time constant is about 0.5 s.

### 4.5.6 Experiment for model estimation: PRBNS experiment

This experiment is directed to collect the input/output data that will be used for the actual identification of the system. The experiment involves the application of the PRBNS signal, designed with the system information obtained from the preliminary experiments. The signal should excite the process over a frequency range larger than the bandwidth of the process to ensure that the model can simulate both fast and slow input variations. This requirement determines the following PRBNS characteristics:

**PRBNS experiment duration:** Backx and Damen [4] state that the choice of the length of the final experiment has to be approximately 10 to 20 times the largest relevant time constant. This choice of the length of the experiment ensures a robust estimation, also of the large time constants. The estimated largest relevant time constant, obtained from the Fast PRBNS experiment is 0.5 s: an experiment duration of 25 s is chosen. Half of the resulting data set is used for model estimation, the rest for model validation.

**PRBNS minimum switching time:** This parameter determines the minimum time the signal has a constant value. From Fig. C.4 of Appendix C it is easily seen that if the maximum switching frequency $\frac{1}{\theta}$ is chosen more than 4 times the bandwidth the input spectrum has an almost flat power spectrum. In Eq.(C.1) of Appendix C it is shown that a decrease of the switching time also results in a decrease of the energy content of the signal. The switching time should therefore not be chosen too low. As a consequence maximum switching frequency is chosen 4 times the system bandwidth, i.e. $\theta = 12.5$ ms.

**PRBNS amplitude:** In order to maximize the signal to noise ratio it desirable to maximize the energy in the inputs, i.e. the amplitude of the inputs. In principle this maximizes the accuracy of the model. But during normal operation the amplitudes will never be higher than about 5 l/min. An amplitude of 4.5 l/min. has been chosen.
The sampling frequency is chosen higher than the frequency essentially required for identification. The sample frequency has to be chosen such that the sample time is 5 to 10 times smaller than the minimum switching time. The redundancy in these signals is used for signal conditioning, such as:

- removal of trends and subtraction of offsets for both input and output signal;
- filtering of the input and output signal to prohibit aliasing effects and reduce influences of disturbances on the measured signals;

If these high-rate sampled signals are used for parameter estimation, problems occur at high frequencies. The input signal has only small power at these frequencies, while the recorded output signal may still have relatively high power, e.g. due to aliasing effects and disturbances. This results in high gains at high frequencies (see Fig. 4.8.1).

![Power spectra before signal processing](image1)

4.8.1: Power spectra before signal processing

![Power spectra after signal processing](image2)

4.8.2: Power spectra after signal processing

Figure 4.8: Power spectra of input and output data.

The chosen sample frequency is 400 Hz. This is five times the switching frequency of the PRBNS. Higher sample frequencies do not contain more information since the ratio signal used for the determination of output \( x_p \) is sampled with a frequency equal to 20 times the rotational frequency of the primary pulley shaft.

### 4.5.7 Signal processing

#### Scaling and offset correction

The mechanism used for the estimation of the model parameters is based on minimization of the quadratic cost criterion. When the order of magnitude of the inputs and outputs differ significantly, the signal with the largest numerical values will automatically get highest priority in the quadratic criterion. Besides the differences in amplitudes of the signals also the nominal values or offsets of the signal cause problems. This problem is solved by correcting the signals for offset and by scaling them afterwards (Backx and Damen [4]). The procedure followed is:
average signals are subtracted in order to allow the use of a linear model without
offset to describe the dynamic behaviour of the system;

both input and output signals are scaled with respect to the power contents after
offset correction. In this respect the power of the dynamic part of the signals is equal
to the variance of the signals. So signal $x$ is scaled using:

$$x_s = \frac{x}{s_x^2}; \quad s_x^2 = \frac{1}{l} \sum_{j=0}^{l} (x_j - \bar{x})^2,$$  \hspace{1cm} (4.17)

where $s_x^2$ denotes an estimate for the variance of $x$ and $\bar{x}$ the average value of $x$ over
all $l$ samples.

As a consequence all outputs will be equally important for the identification algorithms.
Note that the resulting model describes the transfer from the scaled input to the scaled
output. Using Eq. (4.17) it is easy to obtain the true, unscaled input-output model.

Filtering

To make sure that the collected system data represent the true system dynamics, filtering
of the data is required. The main functions of filtering is to prohibit aliasing effects caused
by sampling of the signals and to reduce influences of disturbances of the measured signals.
On the other hand filtering of the data may not result in significant loss of relevant system
data.

From Appendix A it is known that the output signal $x_p$ is corrupted with noise with
a frequency equal to the rotational frequency of the CVT shafts. To keep the corruption
out of the system pass band the rotational frequency of the CVT shafts has been chosen
as high as possible for the final experiment. The power spectra of the obtained input and
output signals of the final experiment are given in Fig. 4.8.1. In this figure the corruption
can be perceived as a peak in the power spectrum of output $x_p$ at about 28 Hz.

This measurement noise is filtered using a non-causal digital 6th order Chebychev filter
with a cut-off frequency of 28 Hz and a stopband ripple 20 dB down. This filter has no
significant attenuation in the system pass band and since it is non-causal it does not cause
a phase shift.

The measure to prohibit folding or aliasing effects involves the application of a 3rd order
Butterworth low-pass filter on both input and output signal. To obtain a sufficient decline
at half the sampling rate a cut-off frequency of 100 Hz has been used.

Sample rate reduction

The input and output signal have been sampled with 400 Hz, a much higher frequency than
strictly needed for the actual parameter identification. The redundancy in the sampled
signals is used for accurate scaling and filtering. However, these high-rate sampled signals
are not suited for parameter estimation because, at high frequencies, the output signal
contains more power than the input signal which results in high gains at these frequencies.
To overcome this problem the sampling rate is reduced such that the sampling time equals
the minimum switching time of the PRBNS, i.e. a reduction factor of 5. As a result the
spectrum of the input signal will have equal power over the full frequency range up to half
the reduced sampling frequency. The power spectra of the scaled, filtered and decimated input and output signals are given in Fig 4.8.2. The attenuation of the spectrum of \( x_p \) beyond 25 Hz is caused by the Chebychev filter.

### 4.5.8 Model estimation: FIR model estimation

Assuming the system behaviour fits into the class of Markov parameter models, defined in Eq.(4.6), its behaviour can be described by:

\[
Y = M\Omega + N
\]

with:

\[
Y = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+l} \end{bmatrix} : \text{output signal matrix}
\]

\[
\Omega = \begin{bmatrix} u_k & u_{k+1} & \cdots & u_{k+l} \\
 u_{k-1} & u_k & \cdots & u_{k-1+l} \\
 \vdots & \vdots & \ddots & \vdots \\
 u_{k-m} & u_{k-m-1} & \cdots & u_{k-1+l} \end{bmatrix} : \text{input signal matrix}
\]

\[
M = \begin{bmatrix} M_0 & M_1 & \cdots & M_m \end{bmatrix} : \text{Markov parameter matrix}
\]

\[
N = \begin{bmatrix} n_k & n_{k+1} & \cdots & n_{k+i} \end{bmatrix} : \text{noise signal matrix}
\]

In this description only the output of the system is assumed to be corrupted with noise.

The model used for the estimation of the system behaviour can be written as:

\[
\hat{Y} = \hat{M}\Omega
\]

with the model output vector \( \hat{Y} \) and the estimated Markov parameters \( \hat{M} \). A Least Squares estimation of the parameters in an output error criterion is used. This implies the minimization of the following criterion:

\[
\min_{\hat{M}} V_\alpha = \min_{\hat{M}} \left\| Y - \hat{Y} \right\| = \min_{\hat{M}} \left\| Y - \hat{M}\Omega \right\|
\]

(4.20)

The resulting Least Squares solution is:

\[
\hat{M} = Y\Omega^T(\Omega\Omega^T)^{-1}
\]

(4.21)

It can be proved that for a white noise input signal the FIR model estimate is statistically unbiased (the expected value of the estimate equals the true value) and consistent (the estimate equals the true value, when the data sequence length tends to infinity). See Backx and Zhu [5].

Fitting a FIR model with 180 Markov parameters on the final input-output data results in an impulse response given in Fig. 4.9.1. Fig. 4.9.2 shows the model validation. The model output and the output error signal (difference between measured output and the model output) are given versus time. The standard deviation of the error signal is 0.0625.

### 4.5.9 Model estimation: Reduction of the FIR model to an MPSSM model

**Determination of the degree of the minimal polynomial**

From Eq.(4.9) it can be seen that the minimal polynomial degree \( r \) is the number of independent Markov parameters. Backx and Damen [4] propose a method to determine an
appropriate degree from a given set of Markov parameters available from the FIR model based on the singular value decomposition of a finite block Hankel matrix $H$ build from these Markov parameters:

$$H = \begin{bmatrix}
\text{col}(M_1) & \text{col}(M_2) & \cdots & \text{col}(M_j) \\
\text{col}(M_2) & \text{col}(M_3) & \cdots & \text{col}(M_{j+1}) \\
\vdots & \vdots & \ddots & \vdots \\
\text{col}(M_i) & \text{col}(M_{i+1}) & \cdots & \text{col}(M_{j+i-1})
\end{bmatrix}$$

(4.22)

$\text{col}(M_k)$ indicates that all columns of $M_k$ are put below one another into one column vector. $H$ has to contain more rows and columns than the expected degree of the minimal polynomial. Degree $r$ corresponds with the number of nonzero singular values of matrix $H$.

Figure 4.9: FIR model estimation results.

Figure 4.10: MPSSM plots.

i.e. if no noise is present on the Markov parameter entries. In practice this situation does
not exist, due to estimation error noise all singular values will be greater than zero. An
appropriate value for the degree \( r \) is the number of singular values significantly greater than
the noise level of the singular values. Figure 4.10.1 shows a plot of the singular values of
the block Hankel matrix of the estimated FIR model. The estimated minimal polynomial
degree \( r \) is 2.

**Determination of a MPSSM model from a FIR model**

In the next section a MPSSM model will be determined directly from measured input/output
data. Because the output error is nonlinear in the parameters of the MPSSM model, a non-
linear optimization method has to be used. In order to avoid numerical local minima and
to speed up the minimization procedure an initial estimate of the parameters obtained by
fitting a MPSSM model to the FIR model, is determined.

Fitting the FIR model to a MPSSM model involves the minimization of the criterion:

\[
\min_{\hat{a}, \hat{M}} \min_{\hat{a}, \hat{M}} \| \hat{M} - F(\hat{a}, \hat{M}) \|
\]  

(4.23)

where \( \hat{a} \) and \( \hat{M} \) are the MPSSM model parameters, \( \hat{M} \) is the Markov parameter sequence of
the FIR model. Different methods have been developed to solve this minimization problem
(Backx [3]).

**4.5.10 Model estimation: Direct estimation of MPSSM parameters**

Direct estimation from input/output data of the MPSSM model parameters involves the
computation of:

\[
\min_{\hat{a}, \hat{M}} \min_{\hat{a}, \hat{M}} \| Y - F(\hat{a}, \hat{M})\Omega \|
\]  

(4.24)

The initial estimate of \( \hat{a} \) and \( \hat{M} \) is obtained from the initial MPSSM model derived in the
previous section. Figure 4.10.2 shows the output error model validation. It can be seen
that the output error is small, the standard deviation of the error signal is 0.0685. Besides,
the figure shows that the error is not correlated with the output signal, i.e. the error is
caused by output noise.

**4.5.11 Model estimation: Final model**

Using Eq.(4.11) the obtained MPSSM model can easily be written in (discrete-time) State
Space form. This model describes the closed loop transfer from scaled input \( Q_{st,p} \) to scaled
output \( x_p \). By means of Eq.(4.17) the true, unscaled input-output relation is obtained.
Then, the influence of the drift correction controller is eliminated by calculating the open-
loop transfer using Eq.(4.14). After discrete time to continuous time conversion yields the
following model:

\[
H_o = \frac{x_p}{Q_{st,p}} = 2.36 \left[ \frac{\text{mm}}{\text{s} \left[ \frac{\text{mm}}{\text{s}} \right]} \right].
\]  

(4.25)

This model, a pure integrator, describes the linearized ratio change behaviour in terms of
\( x_p \) and \( Q_{st,p} \). The real system input \( u_{st} \) and output \( i \) can be obtained using Eq.(4.1) and
Eq.(2.6) respectively.

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Table 4.1: The obtained models

<table>
<thead>
<tr>
<th></th>
<th>Transfer function</th>
<th>Poles</th>
<th>Zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
<td>( \frac{s^2 + 11s + 14347}{s(s^2 + 136.1s + 7220.3)} )</td>
<td>( s = 0 )</td>
<td>( s = -5.39 + 117.94i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( s = -68.1 + 50.86i )</td>
<td>( s = -68.1 - 50.86i )</td>
</tr>
<tr>
<td>1.00</td>
<td>( \frac{2.36}{s} )</td>
<td>( s = 0 )</td>
<td>--</td>
</tr>
<tr>
<td>1.50</td>
<td>( \frac{4s^2 + 118s + 49022}{s(s^2 + 343s + 22470)} )</td>
<td>( s = 0 )</td>
<td>( s = -16.89 + 117.1i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( s = -88.11 )</td>
<td>( s = -16.89 - 117.12i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( s = -255.0 )</td>
<td></td>
</tr>
</tbody>
</table>

4.5.12 Influences of the working point

In Eq. (2.3) output \( x_p \) is defined as \( x_p = 2 \tan \beta (r_p - r_0) \) where \( r_0 = r_p (i \leq 1) \). Because of this definition and because of the feedback control of \( x_p \) (Section 4.5.4) to prevent drift, the experiments were always done around ratio \( i = 1 \). To investigate the dynamic behaviour around other ratios, experiments were done for different definitions of \( x_p \). By choosing \( r_0 \), every desired working-point for ratio \( i \) can be obtained.

Final PRBNS experiments were done and models were estimated for \( i \) around 0.67 and 1.5. The Bode plots of the estimated models for these working-points, together with the model obtained in the previous section, are given in Figure 4.11. Table 4.1 gives the transfer functions of the obtained models and their poles and zeros. The models for \( i = 0.67 \) and
i = 1.5 are of third order and contain a pure integrator. The model for i = 0.67 has, besides the pole of the integrator at 0 Hz, a pair of underdamped poles at 13.4 Hz. The model for i = 1.5 has the integrator pole and two overdamped poles at 14.0 and 40.6 Hz. Both models have a pair of complex conjugated minimum phase zeros at 18.8 Hz.

Note that the Chebychev filter used to filter the measurement noise has a cut-off frequency of 28 Hz, which means that the obtained models are unreliable for frequencies higher than about 24 Hz.

4.6 CVT pressure identification

4.6.1 Preliminary study: Input/output selection

For identification of the CVT pulley pressures, u_p and u_s are chosen as the system inputs. The system outputs are the primary and secondary pulley pressure p_p and p_s. The primary and secondary pulley valve signals are chosen as system inputs instead of u_{pr} and u_{st}, the inputs used during ratio change identification, because the excitation of the pulley pressures by u_{st} is very poor. Using u_p and u_s as inputs results in better estimation results.

To maintain sufficiently high pulley pressures a constant signal has been applied to both inputs. The CVT is kept around a certain ratio by feedback sheave position controller designed in Section 4.5.4. A static gain is chosen small (K_f = \frac{1}{300} [m^2/s]) to reduce the influence of the feedback loop on the pulley pressures.

4.6.2 Preliminary study: Linearity test

To test the range of linearity a so called staircase signal is applied to one input while the other input is kept constant. This experiment can also be used to estimate the largest time constant. The stair input signal and the measured output are given in Figure 4.12. From this figure the following conclusions can be drawn:

- at high pressures the pressure responses contain drift;
• the steady state pressure response on a step depends on the actual pressure level, i.e. the pressure response is *steady state non-linear*.

• the steady state step response is reached after about 2 s, i.e. the largest time constant is about 0.5 s.

The steady state non-linearities can be compensated using linearizing functions (see Subsection 4.5.2). These functions can be designed by means of the steady state responses of this experiment. The drift can be eliminated by means of a feedback loop (see Subsection 4.5.4). Due to a lack of time, another approach is followed: during the experiment for model estimation the pressures are excited in a range that can be considered steady state linear. Since the operating pressures of the CVT are between 5 and 25 bar this range has identified. The drift will result in an increased estimation error. If the influence of the drift is substantial, drift correction is required.

### 4.6.3  Experiment for model estimation: PRBNS experiment

The Fast PRBNS experiment has been skipped since all information needed for the final PRBNS experiment is known. The range of linearity and the largest relevant time constant has been estimated from the stair experiment. The system bandwidth can be determined the pressure TFE given in Fig. 3.1.

**PRBNS experiment duration:** To ensure robust estimation, also of the large time constants, the experiment should last at least 10 times the largest time constant. The estimated largest time constant is 0.5 s: an experiment duration of 20 s is chosen. Half of the resulting data set is used for model estimation, the other part for model validation.

**PRBNS minimum switching time:** To obtain equal excitation of all dynamics within the system bandwidth the maximum switching frequency have to be chosen at least three times the system bandwidth. Since the bandwidth is 80 Hz the minimum switching time \( \theta \) is chosen \( \theta = 4 \text{ ms} \).

**PRBNS amplitude:** The PRBNS amplitudes are chosen such that the pressure responses stay within a range where the behaviour can be said steady state linear. When \( u_p \) and \( u_s \) are excited by PRBNS signals with amplitudes of 0.7 volt and a constant value of 0.3 volt the pressure responses have an average level of 13.50 bar and a standard deviation of 2.4 bar.

The sample frequency is chosen 4 times the maximum switching frequency, i.e. 1000 Hz.

### 4.6.4  Signal processing

**Scaling and offset correction**

The input and output signals are subtracted by their average values and the resulting signals are scaled with respect to their power contents. As a consequence, all system transfers will be equally important for the identification algorithms. See Section 4.5.7.
Filtering

To prevent that aliasing-effects distort the interesting part of the frequency spectrum below the Nyquist frequency, the outputs are filtered before sampling by an analog 6th order Butterworth filter with a cut-off frequency of 300 Hz.

Sample rate reduction

After scaling, offset correction and filtering sample rate reduction is applied. A reduction factor of 5 is used. This results in flat power spectra of the input signals up to half the reduced sample frequency.

4.6.5 Model estimation: FIR model estimation

Fitting a FIR model with 180 Markov parameters on the processed input/output data results in the impulse responses given in Fig. 4.13.1. Fig. 4.13.2 shows the model validation,

![4.13.1: FIR estimates](image)

![4.13.2: FIR model validation](image)

Figure 4.13: FIR model estimation results.

i.e. the model output and the output error signal vs. time.

4.6.6 Model estimation: Reduction of the FIR model to an MPSSM model

Determinant of the degree of the minimal polynomial

The minimal polynomial degree is found by determining the number of independent Markov parameters, i.e. the number of singular values of the block Hankel matrix significantly greater than the noise level of the singular values.

In Fig. 4.6.6 the singular values of the block Hankel matrix of the estimated FIR model are plotted in decreasing order. The estimated minimal polynomial degree is 3.
Determination of a MPSSM model from a FIR model

In the next section a MPSSM model is estimated from the input/output data. A good initial estimate of the MPSSM parameters, required to avoid local minima and to speed up the estimation, is obtained by fitting the FIR model to a 3rd order MPSSM model.

4.6.7 Model estimation: Direct estimation of MPSSM parameters

Direct estimation of the MPSSM model parameters from input/output data yield an output error model validation given in Fig. 4.6.7. The standard deviation of the output error signals is 0.193 and 0.200 for \( p_p \) and \( p_s \) respectively. These errors are substantial larger than the ratio change error. This can be caused by drift.

4.6.8 Model estimation: Final model

By means in Eq.(4.11) and Eq.(4.17) the unscaled input-output relation in State Space form can be obtained. The Bode representation of the resulting transfer functions \( H_{kl} \) is given in Fig. 4.6.8. The transfer function \( H_{kl} \) relates valve signal \( u_k \) of pulley \( k \) \((= p, s)\) to the pulley pressure \( p_l \) of pulley \( l \) \((= p, s)\).
From this figure it can be seen that \( H_{pp}, H_{ss}, H_{ps} \) and \( H_{ps} \) have the same static gain, i.e. the pulley interaction is strong at this pressure level. The model has one pole at 0.8 Hz and a pair of underdamped at 23.3 Hz. The model transfers have three zeros, of which the smallest is minimum phase and the others non-minimum phase. Table 4.2 shows the values of the estimated zeros together with the gains and poles.

4.6.9 Influence of working points

To investigate the pressure behaviour around other ratios, experiments were done for \( i \) around 0.67 and 1.5. This is done by changing the set-point of the ratio controller. Besides an experiment was done for a higher average pressure level. By applying 0.4 volt to \( u_{pr} \), instead of 0.3, an average pressure level of 19 bar is obtained.

The static gains, the poles and the zeros of the resulting models are given in Table 4.2. From this table it can be seen that the static gains of the different transfers are different for different working ratios. This is caused by the ratio controller. To keep the CVT at a constant ratio \( i \), a pressure ratio \( \rho \equiv \frac{p_t}{p_s} \) is required. See v.d. Leest [6]. Table 4.2 shows the measured pressure ratios required to keep the CVT at a certain ratio. The ratio controller realizes the desired ratio \( i \) and, hence, the pressure ratio \( \rho \). The obtained pressure levels are being used to estimate the static gains of the pressure model. This explains the difference in static gains for different ratios. In Table 4.2 the gain ratio \( \frac{H_{pp}(\rho)+H_{ps}(\rho)}{H_{ss}(\rho)+H_{ps}(\rho)} \) is given for the different models. The gain ratios correspond quite well with the measured pressure ratios.

When the poles and zeros for different ratios are considered it can be concluded that the models are not exactly the same, but they also do not differ significantly.

The static gains of the estimated model with \( u_{pr} = 0.4 \) is larger than the model with \( u_{pr} = 0.3 \). This can be explained by steady state non-linearities discussed in Subsection
4.6.2. Besides, the gains of $H_{ps}$ and $H_{sp}$ are smaller than the gains $H_{pp}$ and $H_{ss}$, which means less pulley interaction for this pressure level. This corresponds to Fig. 4.12. Compared to the models for $u_{pr} = 0.3$, the cross-transfers $H_{ps}$ and $H_{sp}$ of the model for $u_{pr} = 0.4$ have two minimum phase zeros instead one.

### Table 4.2: Pressure model quantities for different working points. 

<table>
<thead>
<tr>
<th>$i/u_{pr}$</th>
<th>Static gains</th>
<th>$H_{sp}(0) + H_{pr}(0)$</th>
<th>$H_{ss}(0) + H_{ps}(0)$</th>
<th>$\rho$</th>
<th>Poles</th>
</tr>
</thead>
<tbody>
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#### 4.7 Discussion

The models identified in this chapter describe the input-output relation between input(s) and output(s) obtained from dedicated experiments. These experiments were done under certain operating conditions, i.e. certain rotational speeds, certain oil temperatures, etc. It is likely that the investigated dynamics are different for different operating points. For example, during experiments it is seen that the pressure behaviour depends on the oil temperature.

However, it is not realistic to identify the system dynamics for all operating points. I therefore propose to design the CVT controller based on the identified models. If the controller performance is influenced significantly by some phenomenon, such as for example the oil temperature, more research on this influence is required.
Chapter 5
Conclusions and recommendations

5.1 Conclusions

CVT control:

- When two independent controllers based simple SISO system models are used for pressure control respectively ratio change control, the ratio change controller is disturbed significantly by the pressure control actions. The pressure controller is not disturbed by the ratio change (Section 3.6.3);

- The pressure controller, designed by Ploemen, based on a pressure model which takes the pulley interaction into account does not improve the CVT control performance significantly.

CVT identification:

- When $i$ is chosen as the system output, instead of $\frac{di}{dt}$, the differentiating operation which very susceptible to high frequency noise and, hence, the required strong low-pass filtering can be cancelled (Section 4.2);

- The pulley values can be steered such that the CVT ratio change is completely decoupled from the pulley chamber pressures (Subsection 4.3);

- Decoupling of the driving pulley pressure from the CVT ratio change requires a pressure model that also describes the pulley interaction (Subsection 4.3);

- Using linearizing functions the CVT ratio change behaviour can be linearized (Subsection 4.5.2 and 4.5.3);

- The ratio change behaviour can described accurately by low order linear models (Subsection 4.5.12);

- The pressure behaviour shows some drift and is slightly steady state non-linear (Subsection 4.6.2);

- Despite drift, the relevant MIMO pressure behaviour can be described adequately by low order linear models (Subsection 4.6.7).
5.2 Recommendations

- Since ratio $i$ can be determined more accurate and with a higher bandwidth than $\frac{di}{dt}$, it is advantageous to chose $i$ instead of $\frac{di}{dt}$ as the quantity to control with;

- The disturbance of the change of ratio by pressure control actions can be eliminated completely when the CVT controllers are designed using the CVT steering proposed in Subsection 4.3;

- The obtained ratio change and pressure models can be used for CVT controller design;

- If a more accurate pressure model is required, I propose to identify the model for a drift corrected and a steady state linearized pressure behaviour. The drift can be corrected using a feedback-loop and a steady state linear behaviour can be obtained by applying linearizing functions.
Appendix A

Ratio change control

In this section the rate of ratio change control law of Spijker is derived in a somewhat different way. An adaptive control strategy is used to compensate for non-modelled system dynamics. Furthermore, in order to reduce the noise of the measured rate of ratio change, a Kalman filter is employed.

The control law is based on the following starting-points:

- The pulley sheave velocity model for the ratio controlled pulley is given by Eq. (3.9):

  \[ \dot{v}_1 = -a_q v_1 + \frac{a_q}{A} Q_1 \]
  \[ \text{Substitution of Eq. (2.1) results in:} \]
  \[ \dot{v}_1 = -a_q v_1 + \frac{a_q}{A} c_q V_q^* u_1 \]

  Here, \( v_1 \) is measured, \( V_q^* \) and \( A \) are parameters that are assumed be known exactly, \( a_q \) and \( c_q \) are parameters which value have to be estimated.

- The desired behaviour of \( v_1 \) is specified by means of a reference model. This model is chosen such that \( v_r \) converges to the desired value \( v_{1,\text{set}} \) of \( v_1 \):

  \[ \dot{v}_r = -a_{qr} v_r + a_{qr} v_{1,\text{set}} \quad a_{pr} > 0 \]

  where \( a_{qr} = 40\pi \) is chosen to realize the desired closed-loop bandwidth of 20 Hz. Furthermore, \( v_{1,\text{set}} \) is obtained from desired rate of ratio change \( \frac{d}{dt} V_q^* \) via Eq. (2.2) and Eq. (2.7).

- We wish that the output error defined as \( e_q = v_r - v_1 \), converges to zero according to the following relation:

  \[ \dot{e}_q = -a_{qr} e_q. \]

  From Eq. (A.2) and Eq. (A.3) it is easily seen that \( \dot{e} \) can be written as:

  \[ \dot{e}_q = \dot{v}_r - \dot{v}_1 = -a_{qr} e_q + [(a_q - a_{qr}) v_1 + a_{qr} v_{1,\text{set}} - a_q c_q V_q^* u_1]. \]
The input $u_1$ is chosen as:

$$u_1 = \frac{A}{V_q^*} \left[ \frac{1}{\hat{c}_q} v_1 + \frac{a_{qr}}{\hat{a}_q \hat{c}_q} (v_{1,\text{set}} - v_1) \right] \quad \text{(A.6)}$$

where $\hat{a}_q$ and $\hat{c}_q$ are the available estimates of $a_q$ and $c_q$, respectively. It is assumed that $V_q^*(\Delta p_1)$ can be calculated exactly by substitution of the measured $p_1$ and the constant $p_{\text{line}}$. Then the error equation (A.5) becomes:

$$\dot{e}_q = -a_{qr} e_q + \vartheta_1 v_1 + \vartheta_2 (v_{1,\text{set}} - v_1) \quad \text{(A.7)}$$

where $\vartheta_1$ and $\vartheta_2$ are given by:

$$\vartheta_1 = a_q \left( 1 - \frac{c_q}{\hat{c}_q} \right) ; \quad \vartheta_2 = a_{qr} \left( 1 - \frac{a_q c_q}{\hat{a}_q \hat{c}_q} \right) \quad \text{(A.8)}$$

To arrive at an adaptation law for the parameters $a_q$ and $c_q$, the following candidate Lyapunov function $V$ is chosen:

$$V = \frac{1}{2} e_q^2 + \frac{1}{2} \vartheta_1^2 + \frac{1}{2} \vartheta_2^2 ; \quad e, \zeta > 0, \quad \text{(A.9)}$$

which represents an accumulation of the sheave velocity output error and the parameter error. Differentiation of this relation with respect to time yields, after substitution of Eq.(A.7):

$$\dot{V} = -a_{qr} e_q^2 + \vartheta_1 [\epsilon \dot{\vartheta}_1 + e_q v_1] + \vartheta_2 [\zeta \dot{\vartheta}_2 + c_q (v_{1,\text{set}} - v_1)] \quad \text{(A.10)}$$

and therefore $\dot{V} < 0$ for each $e_q \not= 0$ if $\dot{\vartheta}_1$ and $\dot{\vartheta}_2$ satisfy:

$$\dot{\vartheta}_1 = -\frac{1}{\epsilon} e_q v_1 , \quad \dot{\vartheta}_2 = -\frac{1}{\zeta} c_q (v_{1,\text{set}} - v_1) \quad \text{(A.11)}$$

If the model parameters $a_q$ and $c_q$ are constant or are slowly varying this results in the following adaptation law for the controller parameters:

$$\dot{\hat{e}}_q = \frac{1}{\epsilon} \hat{e}_q \hat{c}_q \epsilon v_1 \quad \text{and} \quad \dot{\hat{a}}_q = -\hat{a}_q \hat{c}_q \left[ \frac{1}{\zeta} a_{qr} (v_{1,\text{set}} - v_1) - \frac{1}{\epsilon} v_1 \right] e_q \quad \text{(A.12)}$$

Since the values of the parameters $a_q$ and $c_q$ are not known, they have to be estimated. The error initiated by this parameter uncertainty only affects the adaptation speed when is assumed that $a_q$ and $c_q$ are constant. Therefore, $a_q$, $c_q$, $\epsilon$ and $\zeta$ are replaced by $\epsilon_b$ and $\zeta_b$, respectively:

$$\dot{\hat{e}}_q = -\frac{1}{\epsilon_b} \hat{e}_q^2 e_q v_1 \quad \text{and} \quad \dot{\hat{a}}_q = -\hat{a}_q \hat{c}_q \left[ \frac{1}{\zeta_b} a_{qr} (v_{1,\text{set}} - v_1) - \frac{1}{\epsilon_b} v_1 \right] e_q \quad \text{(A.13)}$$

When this law is compared with the law of Spijker [1]:

$$\dot{\hat{e}}_q = \frac{1}{\zeta} e_q^2 v_{1,\text{set}} \quad \text{and} \quad \dot{\hat{a}}_q = 0, \quad \text{(A.14)}$$

it can be seen that Spijker does not adapt $\hat{a}_q$ on-line and that $\hat{e}_q$ is adapted as function of $v_{1,\text{set}}$. This is the consequence of the way Spijker derived the law. Because of the unreliable
determination of \( v_1 \), he designed the law such that the terms \( \theta_1 v_1 \) and \( -\theta_2 v_1 \), of error Eq. \((A.7)\), are not used. The error equation than becomes:
\[
\dot{e}_q = -a_{qr} e_q + \theta_2 v_{1,\text{set}} 
\]
\((A.15)\)

The Lyapunov function Spijker used is:
\[
V = \frac{1}{2} e_q^2 + \frac{1}{2} \zeta \dot{q}_2^2; \quad \zeta > 0, 
\]
\((A.16)\)

With these functions the law given in Eq.\((A.14)\) has been obtained. It has to be noticed that \( v_1 \) is still needed in this law for the determination of \( e_q \), since \( e_q = v_r - v_1 \).

Since the purpose of the research is to test Ploemen’s interactive pressure controller the ratio change controller has not been changed, i.e. Spijker’s law has been used during experiments.

A pulley sheave velocity signal \( v_1 \) is used in the feedback law and in the adaptation law to form the error signal \( e_v \). Since this signal can not be measured in a direct manner, it is derived from the CVT ratio change by using Eq. \((2.7)\) and Eq. \((2.2)\). The rate of ratio change is obtained by differentiating the measured CVT ratio. In order to reduce the remaining noise, a Kalman filter is used. This estimator, designed independently of the control law, is based on the flow model given by Eq. \((3.9)\). The Kalman filter uses the measured signal to form the error signal \( e_v \) which is used to obtain estimates of \( v_1 \):

\[
\dot{v}_1 = -a_q \dot{v}_1 + \frac{\dot{a}_q}{A} Q_1 - l_q e_v, \quad e_v = v_1 - \dot{v}_1 
\]
\((A.17)\)

\[\text{Figure A.1: Sheave velocity control diagram}\]
The flow $Q_1$, the input of the model, is determined by the estimated $\hat{v}_1$ and $v_{1,\text{set}}$, according to $Q_1 = A \left( \frac{\partial z}{\partial y_0} \hat{v}_1 + \frac{z_r}{\alpha_q} v_{1,\text{set}} \right)$. More information on the design of this Kalman filter is given in Spijker [1]. Fig. A gives the block scheme of the ratio change controller, including the Kalman filter.
Appendix B

Ratio and ratio change determination

B.1 Angular velocity measurement

To measure the angular velocity of the CVT shafts a slotted disc mounted on the shaft is used in conjunction with inductive pick-ups. Measuring the time interval $\Delta t$ between two passing slots yields a sampling period which is inversely proportional to the rotational frequency of the disc. It is easily seen that the angular velocity $\omega_i$ of shaft $i$ is given by

$$\omega_k = \frac{2\pi}{n_{\text{disc},k} \Delta t_k} \quad k = p, s$$

(B.1)

where $n_{\text{disc},k}$ denote the number of slots on disc $k$. In the test-rig a slotted disc with 20 slots is used on the primary pulley while the disc on the secondary pulley contains 4 slots.

The method employed here yields the average angular velocity for the time interval between two passing slots. The Bode plot of the transfer function of the continuous speed signal to the discrete speed signal obtained by this method is given in Fig. B.1.1. From this Figure it can be concluded that the method employed here yields accurate values for low frequencies and that the high frequency components are suppressed. But since only frequencies up till 20 Hz are of interest, the averaging operation hardly influences the speed measurement.

B.2 CVT ratio determination

The ratio $i$ is calculated using $\omega_s = i\omega_p$ and Eq. (B.1),

$$i = \frac{n_{\text{disc},p} \Delta t_p}{n_{\text{disc},s} \Delta t_s}$$

(B.2)

The position inaccuracy of the slots causes oscillations in the speed signal proportional to the shaft speeds. These oscillations are removed using a low pass Chebychev filter with a 20 dB stop band and a cut off frequency equal to the angular velocity of the primary pulley shaft. During identification the phase shift introduced by this filter is eliminated by filtering the signal a second time with the corresponding anti-causal filter.
B.3 CVT ratio change determination

The change of ratio is obtained by taking the time derivative of the CVT ratio by taking the difference of two successive samples of $i_i$ divided by the sample time of the primary pulley:

$$\frac{di_i}{dt} = \frac{i_i - i_{i-1}}{\Delta t_p,i}$$  \hspace{1cm} (B.3)

The Bode plots of this differentiation of $i$ is given in Fig. B.1.2.

![Bode plots](image)

Figure B.1: Bode plots of transfer functions as a result of discretization and the differentiation.

![Bode plots](image)

Figure B.1.2: Bode plots of the transfer function of the differentiation operation.

It is shown that this operation is very susceptible to high frequency noise. In order to remove this noise a low pass filter is used. Differentiation, together with low pass filtering, results in a substantial time delay.
Appendix C

PRBNS-signals

A Pseudo Random Binary Noise Sequence (PRBNS) is a periodic, deterministic signal with desirable autocorrelation characteristics. Fig.C.1 shows an example of a PRBNS-signal. PRBNS-signals can be generated by a shift-register given in Fig. C.2. The $n$ shift-register elements contain a binary number. The value of the first element is transformed to the test signal through $1 \rightarrow +a$ and $0 \rightarrow -a$. Every $\theta$ seconds the register shifts one element to the right. By an exclusive-or-function the next value of the first element is determined by the previous values of the last and the third last register element. PRBNS signals have the following properties:

- Sequence period $T$ is determined by $T = N\theta = (2^n - 1)\theta;$
The autocorrelation function, if determined over an integer number of sequence periods of sequence periods, has no stochastic element (uncertainty) in it. The autocorrelation function is shown in Fig. C.3. By choosing $N$ and $\theta$ a Dirac function can be well approximated;

The power density is given by the following equation and plotted in Fig. C.4:

$$\Phi(\omega) = a^2 \theta \left( \frac{\sin(\omega\theta)}{\omega \theta} \right)^2 \sum_{k=-\infty}^{\infty} \delta \left( \omega - k \frac{2\pi}{T} \right)$$  \hspace{1cm} (C.1)

Figure C.3: The autocorrelation function of a PRBNS-signal

Figure C.4: Power density function of a PRBS.
Appendix D

Correlation analyses

Correlation analysis is a tool for judging the time relation between two signals or of the signal with itself. It can be used for determination of process time delays, and for getting a rough estimate of process time constants. The correlation $\Phi_{uy}(\tau)$ of two discrete time signals $u(k)$ and $y(k)$ is defined by:

$$\Phi_{uy}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} [u(k) - \mu_u][y(k + \tau) - \mu_y]$$  \hspace{1cm} (D.1)

The relation between the input signal $u(k)$ and the output signal $y(k)$ of a linear system with impulse response $h(k)$ is given by the following convolution sum:

$$y(k) = \sum_{p=0}^{\infty} h(p)u(k - p)$$  \hspace{1cm} (D.2)

When this is put in the definition of $\Phi_{uy}(\tau)$ the following result is obtained:

$$\Phi_{uy}(\tau) = \sum_{k=0}^{\infty} h(k)\Phi_{uu}(\tau - k)$$  \hspace{1cm} (D.3)

In the case of a 'white noise' input signal, $\Phi_{uu}(\tau) = \delta(\tau)$, and therefore:

$$\Phi_{uy}(\tau) = h(\tau)$$  \hspace{1cm} (D.4)

So the conclusion can be drawn that the cross-correlation of a 'white noise' system input and the system output corresponds to the impulse response of the system.
Bibliography


