MASTER

Signaling in real options investment games
finding optimal investment strategies using the binomial lattice model

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Signaling in Real Options Investment Games:
Finding Optimal Investment Strategies using the Binomial Lattice Model

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In this research I am going to approximate a real option investment game with asymmetric information - which has up to now only been investigated in a stochastic nature - by using the binomial lattice model. The game consists of an incumbent having the information advantage, and an entrant who lacks information about a specific parameter, the investment costs. The investment decision depends on the present value of the project, the binomial parameters, the market share, the continuous-time discount rate and of course the private information which are the investment costs. A firm will undertake an investment if he expects his payoff to be positive and his timing of investment is a result of the possible option value of waiting.

The main goal of this research is to show the influence of asymmetric information (the information value) as well as possible option values of waiting (the option value) on the behavior of the firms, and ultimately demonstrate the combination of both and their superadditive value to the payoffs of the firm having the information advantage position. The major adjustments I undertake to justify the binomial lattice model are a finite investment horizon, fixed investment opportunities on the time horizon with equal interval length, a discrete parameter that represents the type, and the determination of the updated beliefs in the simulations. Because of the incompleteness of information, this game initially is described as a Bayesian model in pure strategies, which is the model I intend to investigate. However, due to the assumptions to come up with a simulation model, I deviate from this Bayesian model so that the results are not perfect Bayesian equilibria but optimizations in an investment game.

According to the simulations, this research, with its adjustments compared to a Bayesian model, shows the outcomes and optimal strategies in four different games. It can be concluded that games with only option value have optimal strategies with higher payoffs for the incumbent, but the effects is very limited. The same holds for games with only information value. However, when a game possesses those two characteristics, they appear to be superadditive since the game becomes more dynamic and therefore the incumbent can take more advantage of his position and get a optimal strategy with a bigger payoff than the two other games. The same result is also found in a game with an extra period added. Next to that the effects of the separate parameters (present value share, volatility and initial belief) show that the results depend on more factors and that a change in every separate parameter has an effect of the pattern of the optimal strategies and the corresponding outcomes.
Summary

In August 2010, I met with mr. Reindorp for the first time, to discuss our expectations from the project and which areas might be of an interest to write a master thesis about. It was at that point already clear that I would start to participate in a second master program simultaneously with the Operations Management and Logistics master at the TUe, namely Econometrics at the Maastricht University in September 2010. The reason I choose to follow that second master was my keen interest in game theory and especially its applications to financial problems. With that knowledge, we decided in the first meetings that the subject of my internal research would be in an area where game theory would be involved. Since mr. Reindorp’s expertise is in the field of option theory and that there is still a lot to be investigated where options theory and game theory are combined, we started investigating the literature in the first months to find an interesting subject that is worth investigating.

The first part of the thesis was the literature review and mr. Reindorp proposed that I would read a set of articles that were divided in three main subjects, and we decided in the beginning that we would focus on game theory in option games. Both of us were looking for interesting articles and the planning was that in the first months I would prepare around three articles for every meeting and discuss them to find unsolved problems, limitations that could be investigated or follow-up questions I could tackle. After reading and discussing around twenty articles I started on the literature review document where I created a summary including all the relevant parts of the articles and make an overview of the possible problems or questions that could be improved or solved. At that point the research question gradually started to develop. After the period of searching through literature I found a subject that I thought was worthwhile investigating and was an ideal combination of the two theoretical areas. I tried to form a research question where I would create a model where two firms would play a game in finding the ideal timing of investment using the option value of waiting in a market where a certain value can be obtained. Since the previous literature in general focused on models that occurred in a stochastic nature, I also used the stochastic nature as the main background for my research. However, after trying to form the mathematics of the model where in a stochastic nature two firms need to make the perfect decision to get the highest payoff, I ran into a couple of problems. It appeared to be too complex to create a model where, using a Geometric Brownian Motion, the decisions of investment of two firms are analyzed using game theory, since the stochastic behavior leads to an infinite amount of future paths the stochastic parameter can take. This seemed a problem that was also explicitly explained in some recent articles as well (Watanabe, 2010). So I needed to modify the framework so that it could fit my research question in such a way that it was possible to create and simulate a model that would result in an output that could be analyzed.

To go from the stochastic model to a model with a finite amount of outcomes and therefore a more analyzable character, I had to make adjustments. In literature some articles made adjustments as well (i.e. Watanabe (2010) worked with a discrete parameter to limit the amount of possible outcomes) so I created a model based on the following adjustments that were all related to each other. First of all, the freedom to invest at every single point in time was discarded. Companies cannot decide to invest at any point in time when the investment appears most profitable, but rather only invest at certain points in time (with equal interval
length). That means they can invest once a day / week / month or even longer. The second major adjustment is a finite investment horizon. Companies can decide, for example, to invest in the project every week with a maximum of 52 weeks since after that deadline the project has lost its profitability due to new models, updates, replacements, customer preferences etc. The third adjustment is based on Watanabe (2010) where he uses a discrete parameter for the private information instead of a stochastic parameter; the amount of demand is either high or low. In my research I consider the private information to be about the project’s investment cost like in Grenadier and Malenko (2011) and I assume the costs to be a discrete parameter, either high or low and the entrant has an initial belief about the low and high costs. As previously stated, in almost all studies, at least one stochastic process is incorporated, usually as the public information in the market. However now that I discretized the investment periods and made the horizon finite, the stochastic process of the public information can be approximated by a binomial options pricing model which is now the framework for the analysis of decision made over time.

My research will be based on the gap in literature about signaling in real options investment games. Now that the adjustments are incorporated, the model that will be analyzed is shortly described as follows. I consider a market with two firms that have to make a decision about investing in a project that has a certain present value \((PV)\) and a cost to undertake the investment. Both firms are not in the market prior to the game. The \(PV\) is the public information and the investment costs are the private information (type of the incumbent). In period 1, both firms observe the \(PV\) and know the multiplications of the binomial movements in subsequent periods, so the binomial pricing tree is assumed to be public information. After period 1, the binomial movement will result in a multiplication of the first period’s \(PV\). This multiplication is either \(u\) leading to a higher \(PV\) in period 2 or \(d\) leading to a lower \(PV\) in period 2. This process continues for a predefined number of periods. Observing this binomial pricing tree of \(PVs\), the incumbent sends a message containing his investment decisions for the entire game and the entrant responds by choosing an action, which he bases on his belief about the incumbent’s type. So the actual signal is the incumbent’s investment strategy for the game, knowing the structure of the binomial tree and the entrant’s response he can induce. In a 2-period game there are three decision nodes (derived from a two-period binomial tree), one in period 1, and two in period 2. Only one of these second period nodes will be reached, but the firms don’t know which so have to make an investment decision for both prior to the game. Their strategies (messages and actions) contain investment decision for every binomial path that can occur. What will happen in this game is that the incumbent has to make the perfect decision how to send his signal to maximize his payoffs conditional on the entrant’s belief and actions. So both firms eventually choose their optimal investment timing to maximize their payoffs. They evaluate their payoffs at any point in the game according to the possible option values. These option values determine their optimal investment timing taken into account the other firm’s actions. In the end this will result in an optimal investment decision for both.

However, when simulating the model I ran into the problem of finding updated beliefs, since there is no clear defined method in doing so. Beliefs depend on external factors and are not solely bound to mathematical laws, but also on emotions and other not measurable factors. When this model so far with the adjustments was still analyzable
according to the rules of a Bayesian game, I had to find a way to come up with the type-contingent probabilities that result in the updated beliefs of the entrant about the incumbent’s message and that method violated one of the principles of a Bayesian equilibrium. Therefore, the model undergoes a last adjustment. For the model to be able to simulate these parameters as well as doing that in a consistent way, I let the type-contingent probabilities, normally obtained by the entrant by external factor that results in updated beliefs, to be in the power of the incumbent. These updated beliefs are obtained in such a way that the incumbent has the power to influence them and will lead to the incumbent’s optimal outcome. So next to choosing the message, the incumbent can influence the strategy of the entrant even further. This adjustment leads to a model that can be simulated and has a method of obtaining type-contingent probabilities.

In this research I intended to find the value of both information asymmetry due to signaling and the possibility of option values by waiting to invest, and I did that by isolating these two subjects and then combine them to find the difference in the outcomes of the firms. So in the analysis of the model, I simulate four different games to show the influence of both the major theories. One game where there is no information asymmetry or option values of waiting, another with only information asymmetry, a third with only option values and the final and fourth where both option values and information asymmetry are combined.

It is shown by the 2-period game that a competition without information asymmetry and no option value of waiting simply shows that the firms are identical in their payoffs. When the same competition faces information asymmetry about the investment costs there is still only one point on the horizon for both firms to make an investment decision on so still the incumbent has very limited power with his information advantage. When the game is symmetric in information, but has the option value of waiting and observe which two outcomes can be chosen by nature in the next period, the payoffs only slightly increase, but for both firms. Since they still have the same information, no firm can ‘outplay’ the other by sending messages so both will invest in the second period, which will increase their payoff slightly. When we combined the option value and asymmetry in information, the payoffs of the firms deviate completely from any of the three situations analyzed before. Now the incumbent has the advantage of the information and can also decide when to invest. Even though the entrant has no information advantage, also he can decide to wait with his investment and therefore impose a sort of threat to the incumbent.

It can be concluded that the combination of option value and information value are in some sense superadditive since both of them individually result in little or no increase in the payoffs. There is an interaction between the firms and both chose their strategy in a way they think will be optimal for their payoff. In addition to the main goal of the simulations and research, I also investigated the effect of the parameters separately by varying the market share they get, the volatility in the market and the initial belief the firms have about the occurrence of both costs. Increasing the spread between the PV shares in favor of the incumbent intuitively would result in a decrease of the entrant’s expected payoff but surprisingly also could increase the entrant’s expectation of his payoff, which depends on the PVs and the strategies resulting from that. This is noticeable conclusion that can be drawn from the effects of the PV share on the optimal strategies and payoffs. Increasing the volatility does lead to an increasing PV in the up-node and a decreasing PV in the down-node,
but also a decreasing $p_u$ and increasing $p_d$. The consequence of the volatility increase is that the payoffs of both will eventually approach a limit. That limit fully depends on the $PV_1$ as also do the strategies chosen by the firm. The last parameter tested is the initial belief about the costs by the entrant. Increasing the gap between the initial probabilities eventually results in a symmetric game with full knowledge.

Next to that I simulated one game with an extra period to find if the same patterns occur as in the small games and in the most general three period game, this happened. The overall conclusion is that signaling in option games has proved to add a lot of value for the firm having the information advantage and gives him a power, to some extent, to manipulate the market with his strategies. To obtain the highest advantage, there are a lot of parameters that need to be taken into account before the incumbent will choose his signal to make sure that his message is the profit-maximizing one for him conditional on the entrant's response.
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1. Introduction

Choosing the optimal timings for investment projects are important and difficult decisions to be made by a firm in any kind of competition. A lot of information has to be processed to make this decision and capture the most earnings for the firm in a strategic way. In this paper, I am going to investigate an asymmetric game with two equal sized firms, which only differ in their information about the market and the determination of their timing of investment along a real options framework. In literature this type of research has received some attention, all with different assumptions and applications. In this introduction I will explain all the relevant theory and literature in this research field and the different directions that were investigated about this subject. In the next section, my research will be explained in detail and I will point out the difference between what has been investigated so far and my research intentions. In the third section the model will be exposed and all the formulas and assumptions I used to solve the model are explained. In chapter 4, the description of the equilibrium calculations, which I will use to calculate optimal strategies, is clarified and again used in chapter 5, where the two-period game will be fully exposed into the finest detail to show how a game with the given parameters will produce optimal strategies that correspond to investment timings and payoffs. This most basic game will be displayed to show payoff calculations, optimal investment decisions, and how the assumptions about the game relate to the behavior of the firms and their resulting strategies. In the sixth chapter, the bigger games are explained and it is shown that the two-period game serves as a backbone of the framework for games of all sizes. Chapter 7 deals with programming the games in excel, how this simulation model deviates from a Bayesian game and how exactly optimal strategies are found by maximizing an objective function which is the incumbent’s payoff. Chapter 8 is devoted to the actual simulations of several games done in Excel. I handle different kinds of 2-period and 3-period games and will show results and pattern that occur in both firms’ behavior in choosing their strategies. The conclusions that are drawn according to these simulations are summed up in chapter 9 and in the final chapter I will point out the limitations of this research and directions that future research can take to increase the knowledge about this subject.

1.1 Games with Incomplete Information

The most important aspects about game theory for this research are the games with incomplete information and in particular games with signaling effects. In games with incomplete information, the players only know the structure of the game and do not know which decisions have been taken prior to their decision. The reaction has to be based on beliefs or rational thinking. Games with signaling aspects are a type of incomplete information games with information asymmetry. In this type of game, one player has private information the other players cannot observe. The player possessing this private information has a strategy set that consists of signals (or messages) that could be sent to the other player. The uninformed player then reacts on a signal by choosing a specific action. So, this sender-receiver game is a two-player Bayesian game with communication in which the Sender has private information but no choice of actions, and the Receiver has a choice of actions but no
private information (Myerson, 1991). In this game the Sender has a type which contains his private information and the Receiver has to choose an action based on his belief about the type of the Sender, so the Receiver assigns probabilities to every possible type of the Sender. Stated otherwise, the Receiver is initially uncertain, but could get an indication about his position in the game by a (strategic) signal of the Sender possessing some private information. The sender is informed about the outcome of a random variable, whereas the receiver is not. Signaling and its applications to my research are discussed in more detail in the next chapter.

1.2 Valuation Methods (NPV and Real Options)

The two valuation methods mostly used in literature for options analyses are NPV (discrete-time) and Black-Scholes (continuous-time). The NPV (net present value) is a method used to analyze the profitability of an investment or project. The NPV is the sum of present values of future cash flows minus the current costs. NPV is the core instrument for discounted cash flow (DCF) analyses. However, nowadays it is stated that the standard DCFs when not used properly often undervalue project with options and strategic interactions. Therefore Real Options are calculated by the Black-Scholes method, which is the most widely used technique in the history of finance. Real options are options that can be used to take advantage of. It is the right, but not the obligation, to obtain the gross present value of expected cash flows in the future by making an irreversible investment in an uncertain environment on or before the expire date of this option. The option value is the asymmetry between the right and obligation to invest. This looks similar to NPV but real options only have value when investment involves an irreversible cost of exercise in an uncertain environment. According to Reindorp (2009), the essence of the real options approach consists of making explicit allowance for the value of flexibility in the timing of an investment decision. So in short, real options valuation can be used when according to Weeds (2002) all of the following three conditions apply: an uncertain future; an irreversible (investment) decision and the fact that the firm holding the (investment) option has the ability to delay.

In the Black-Scholes formula, stock prices are described according to a Geometric Brownian Motion (GBM) which will be described in the following chapters (Cox et al., 1979). The deterministic drift of this stochastic process determines the characteristic shape of the sample paths. The option used in the Black-Scholes method is of a European type, it can only be exercised at the pre-defined expiration date of that option, and the stock pays no dividends before this expiration date.

1.3 Literature

All studies about option games in a duopolistic market are focused on a variant of determining the optimal timing of an investment. In literature the emphasis mostly lies on games with (some) parameters having a stochastic nature. The reason is that these processes capture more uncertainty than the games with discrete processes and are therefore a more realistic representation. According to Kong and Kwok (2006) the net present value model fails to accommodate market uncertainty, irreversibility of investment and ability to delay entry. Dixit and Pindyck (1994) state that the real option analysis is the perfect translation of a firm’s investment into a financial call option. They point out that irreversible risky projects
should use the option value of waiting. Postponing investment makes it possible for new information to arrive. By increasing the time of delay before investing and therefore receiving more information, a monopolist can predict with increasing probability which direction the market influence will take, which is the option value of waiting to invest. As soon as the first firm makes his investment decision, the market reveals its true state. The decision problem can therefore be seen as a game with incomplete information, where at time zero, nature chooses the state of the world. Through time the incompleteness of the information resolves because of the signals that arrive.

Almost all research is done in a duopolistic setting. However, the relation between the firms has several variants. In Lambrecht and Perraudin (2003) and Sparla (2002) the firms are assumed to be identical. They are equal sized and symmetric in their characteristics i.e. costs, revenues or the opportunity to capture the first mover advantage. Contrary to that are the articles that focus on the inequalities between the firms as in a leader-follower context (i.e. Watanabe (2010), Grenadier and Malenko (2011), Mason and Weeds (2009), Villani (2009)) or the related monopolist-duopolist structure (Thijssen (2001)). In some cases firms have the same size and opportunities but are asymmetric in at least one characteristic, i.e. costs (Wu and Xuan (2005), Kong and Kwok (2006), Pawlina and Kort (2001)).

The structure of the competition often entails the possibility for preemption in the market. The articles about the leader-follower relation between the firms allow for the fact that a firm can capture (economic) advantages by being the first to invest as in Mason and Weeds (2009) or at least hold the opportunity to threat to preempt the market (Lambrecht and Perraudin (2003)). In addition, other articles, i.e. Kong and Kwok (2006), Villani (2009), Thijssen et al. (2002) investigate variants of preemption and the conclusions are mixed about having a first-mover advantage or a second-mover advantage. For example, Pawlina and Kort (2001) investigate a duopolistic market with asymmetric investment costs and its impact on the optimal real option exercise strategies of both firms. They conclude that increasing uncertainty delays the investments even though there is extra value in being the first investor, so there is a first-mover advantage. In Huisman et al (2003) they describe an asymmetric game where the firms obtain imperfect information about the project’s profitability at some point in time. Investing first can lead to a higher market share, however investing as second can be beneficial because of information spillovers by the other firm’s investment. Villani (2009) created an option game where two firms face an R&D investment opportunity. He assumes the possibility of a first-mover advantage which leads to a higher market share; however the downside is that the option exercise by the leader reveals information that benefits the follower as also in Dixit and Pindyck (1994). This information revelation only appears when both firms do not decide simultaneously. In most studies the markets allow for preemption, so both firms are (equally) able to take the first-mover advantage. However in Watanabe (2010) the entrant is only allowed to invest after the incumbent so the incumbent is assumed to be the first mover in all cases.

Next to Huisman (2003) and Villani (2009) other studies also assume incomplete information i.e. information arriving over time or information revelation. In Thijssen et al. (2001), the focus is on the arriving information in time when investing under vanishing uncertainty. Over time, the firm gets stochastically signals on the probability of the project. Dixit and Pindyck (1994) point out that irreversible risky projects should use the option value
of waiting. Postponing investment makes it possible for new information to arrive. They state that through time the incompleteness of the information resolves because of the signals that arrive. However, next to the assumptions of information arriving in the market over time, studies like Watanabe (2010) and Grenadier and Malenko (2011) describe markets with information asymmetry and firms having private information. Lambrecht and Perraudin (2003) describe a real options model with incomplete information in a market with two identical firms and both firms have private information about their own investment cost.

The asymmetry in information is different to an asymmetry in a specific parameter i.e. cost, revenues, size or any other clear measurable characteristic of a firm. Having an asymmetry between firms about a parameter does not automatically lead to private information as in Huisman et al (2003) which investigates an asymmetric game where the firms obtain imperfect information at some point in time, but there is no private information about this asymmetry. Also in Kong and Kwok (2006) the firms are asymmetric, but no firm has an information advantage so there is no private information.

The information asymmetry aspect is the most relevant for my research. In Watanabe (2010) and Grenadier and Malenko (2011) the focus of their research is the most related to the direction I would like to investigate. These two articles are both very recent and also relatively unique since they incorporate signaling in option games. The fact that these articles are very recent proves the interest in this area nowadays. I consider Watanabe (2010) to be the most useful article based on its framework of competition and asymmetry in information about a single discrete parameter. He assumes a duopolistic market with an incumbent having an information advantage about the demand which is either low or high. The revenue flows are stochastic and common knowledge. Both firms are in competition for the optimal timing of investment, but the entrant cannot invest before the incumbent. Early investment by the incumbent reveals information to the entrant, so the entrant would follow this investment faster, which reduces the incumbent’s profits. Grenadier and Malenko (2011) also use signaling in real options games. The option exercise by a firm is the signal of private information to the other firms. These other firms have a belief about these signals which has an influence on the decision-maker’s utility. The signaling incentives determine the timing of the option exercise. Signaling could either erode the option value of waiting, or enhance it, which depends on the direction of beliefs about the signals.

The types of equilibria that are analyzed in literature can be divided in two main groups: equilibria in non-Bayesian games, where a strategy profile contains strategies that are the optimal response to the other players’ strategies in the profile; and in Bayesian games, where the players maximize their expected payoffs because they have a belief about the other players’ strategies. Most of the articles focus on the non-Bayesian games and investigate the following three types of equilibria: the preemptive, sequential and simultaneous equilibrium (i.e. Wu and Xuan (2005); Bouis et al. (2006); Pawlina and Kort (2001), Kong and Kwok (2006)). Others investigate only one or two of those equilibria i.e. Thijssen et al. (2002), Sparla (2002) who both put the emphasis on preemptive equilibria, and Mason and Weeds (2009) on sequential or simultaneous equilibria. In a preemptive equilibrium one of the firms invests earlier than the other(s), and in a simultaneous equilibrium both firms delay their investment considerably (Huisman et al. (2003)). Because of the information incompleteness in Lambrecht and Perraudin (2003), they analyze the perfect Bayesian Nash equilibria. The
actions have to be taken sequentially instead of simultaneously because after the signal is arrived the second firm makes a decision. Grenadier and Malenko (2011) focus on the Bayes-Nash separating equilibria, in which the decision-maker reveals their type through the option exercise strategy.

It appears that in all studies, equilibria can be described by formulas combined with some threshold values, but the actual investment equilibria and its corresponding payoffs cannot be calculated since the stochastic nature results in an enormous complexity of the model with infinite amount of investment opportunities on the time horizon. This means that an infinite amount of option values for every firm need to be calculated which is impossible. The general tendency of the most related articles to my subject of signaling in option games is in a duopolistic market with a certain kind of asymmetry. The parameters usually are stochastic, and this stochastic nature is the core of the model in almost every study because it is realistic and highly applicable. However, if a stochastic process and a GBM in particular, is approximated by a binomial pricing method (which is in fact the discrete approximation of the GBM) the calculations can lead to much clearer answers, equilibrium strategies can be approximated by finding optimal strategies and conclusion which could be generalized to the stochastic models. I will show this in my research.

2. The Research

2.1 The basic model

My research will be based on the gap in literature about signaling in real options investment games. Since only two firms are able to enter the market, this game is in a duopoly market (a market with the possibility of two large buyers of a specific product). It will have some parallels with Watanabe (2010) where real options are incorporated in signaling games for strategic investment. In this specific research, an incumbent (the sender) and an entrant (the receiver) in the market compete and try to optimize their investment timing. The incumbent is assumed to be the company that is in the market the longest and will be in the information advantage position, because of his accumulated knowledge of the market so far. Therefore the incumbent is assumed to be the first to invest. The entrant only observes the public information and knows the incumbent has valuable information and will use this as strategically as possible in his favor. The incumbent does this by sending signals that contain information about his type (which is his private information about the market). These signals are the incumbent’s timing of investment or better yet, his investment decisions at any point on the investment timing horizon. The entrant, who receives a signal imposes a belief (probability) on it being of a certain type and responds with an action, to maximize his outcome based on these signals and his corresponding beliefs. Therefore the incumbent can ‘lie’ about his type by timing the investment strategically, leaving the entrant puzzled with the signal.

The public information, which both firms can observe, is the present value ($PV$) of a project. This $PV$ displays the total value that can be obtained in the entire market by investing
in the project. This means that in case of a buyer’s monopoly, only the incumbent obtains a \( PV \) which is equal to the market’s entire \( PV \). If the entrant decided to join and also invest in the project, both firms share the \( PV \) and the competition model will be a duopoly. Since it is assumed that both firms only differ in information about the market and are equal sized, they split the \( PV \) equally and get a fifty percent share of the market’s \( PV \) when they invest in the same period. Until the moment the entrant decides to join the market, the incumbent acts as a monopolist.

The private information contains the amount of the project’s investment costs. The incumbent knows the actual project’s investment costs (determined by nature) which are either high or low and based on that he will determine his messages that will be sent to the entrant. The entrant knows the costs are either high or low and knows the amount of these costs also, but doesn’t know the actual cost for this investment. The entrant, since he observes prior decisions, knows he lacks information and is ‘behind’ in the market and therefore will attach a belief to the possibility that the costs are either high or low for any signal he receives.

Considering the research objective, I am going to explain in this chapter what I want to investigate and what the framework and methods will be to accomplish this. The actual model outline will be described in the next chapter. The main area of interest will be on option games with information asymmetry. In literature, there is very little specific research that investigates signaling incorporated in real options games. This area of research seems very useful, yet no article accomplished to combine these aspects of real options and signaling games with equilibrium strategies and payoffs as a result because of its complexity. Investment decisions often depend on the information that is available on the market and to act on that as soon as possible. In reality there is always an information asymmetry in the market about some aspects of information, if not, every firm in a market will react perfectly similar and there won’t be any difference between them. The purpose of my research is to investigate a model that approaches a realistic setting in multiple ways and adds knowledge to the literature in this field so far. I will do this by making some major adjustments compared to the literature previously described.

The studies using stochastic processes for at least one parameter in the model are not able to calculate clear equilibrium strategies and payoffs. They only describe a way to solve the model or the calculations needed to find the equilibriums assuming some threshold values, but because of their highly uncertain nature and complexity the actual calculations could not be made. The reason for this is the infinite amount of combinations of variables since there are an infinite amount of investment timing possibilities on the time line of a stochastic parameter. My contribution to this problem is a couple of adjustments to the models analyzed in the previous chapter. First of all, the freedom to invest at every single point in time will be discarded. Companies cannot decide to invest whenever the \( PV \) reaches a certain level that seems most profitable to them. In this paper, companies can only invest at certain points in time (with equal interval length). That means they can invest once a day / week / month or even longer. The second major adjustment is a finite investment horizon. Companies can decide, for example, to invest in the project every week with a maximum of 52 weeks since after that deadline the project has lost its profitability due to new models, updates, replacements, customer preferences etc. These two restrictions of discretized
decision points and a finite horizon solve for the complexity previous models have to find valuable solutions to equilibrium calculations (Watanabe (2010) explains explicitly in his paper, that his setting is too complex and equilibria cannot be found) or other useful numerical outcomes. Also, these adjustments make the analysis of the model simpler and more evident to find an optimum of strategies for the firms.

2.2 From real scenario to my final model

My research objective is derived from a complicated real-life situation which happens in most investment decision in a market: how does a firm decide when to invest to maximize his own long-term payoff given the other firm’s information and investment decisions? In realistic investment decision scenarios, the PV of the project is a stochastic process following a geometric Brownian motion. The same pattern occurs in the investment costs that also follows a GBM (unless price negations resulted in a fixed price) since price fluctuations, as for example in stock prices, are usually stochastic. In that case the private information would have the same complex fluctuations as the public information over time. As pointed out before, the signal sent by the incumbent is simply his decision to invest or wait at every point in the game. His investment decision can reveal information about his type, because waiting can for example mean the incumbent is waiting for a higher PV in the future since the costs are too high at this point. On the other hand, waiting can also mean the incumbent wants the PV to decrease in the next period so that he can be the monopolist since it is unprofitable for the entrant to join the market with the low PV then. The entrant can respond to the signal by also making an investment decision resulting in the payoffs for both. However, as in line with Watanabe (2010), I assumed in the previous chapter that the incumbent, being in the market the longest, has obtained more knowledge about this particular market, the entrant can only decide to invest after the incumbent has invested, because he has to anticipate the signal of the incumbent first and then choose his response based on what he believes the signal tells about the actual investment costs. When looking at the framework of the real situations, we can conclude that it is too complex to model these multiple stochastic processes, analyze the resulting investment behavior and come up with relevant solutions, optimizations and conclusions. In this description both the public and private information are stochastic and investment decisions can be made on any time on a specific time horizon which means there are an infinite amount of points in time the firms can decide to invest in. It simply cannot be simulated to result in an actual equilibrium of investment timings and their payoffs. This complexity needs adjustments and simplifications to make it possible to analyze such investment decisions and come up with optimal strategies and relevant numerical outcomes.

The first and second adjustments both are already explained and incorporated to make the model more applicable and able to optimize.

The third adjustment is based on Watanabe (2010) where he uses a discrete parameter for the private information instead of a stochastic process; the amount of demand is either high or low. In my research I consider the private information to be about the project’s investment cost like in Grenadier and Malenko (2011). Since I assume the costs to be a discrete parameter, it is like in Watanabe (2010) either high or low and the entrant has an initial belief about the low and high costs and assigns a probability to them; p and 1-p, resp.
As previously stated, in almost all studies, at least one stochastic process is incorporated, usually as the public information in the market. However now that I discretized the investment periods and made the horizon finite, the stochastic process of the public information can be approximated by a binomial options pricing model. I will explain this binomial options pricing model (or binomial lattice model) in full detail in the next chapter and in Appendix A the justification of using this binomial options pricing model as an approximation for the GBM is outlined.

Finally, the beliefs of the entrant about the costs of investment depend on the message. The entrant has prior beliefs about the costs being low or high. After receiving the message, he updates his beliefs using Bayes’ rule resulting in posterior beliefs upon which he bases his action, because these posterior beliefs directly determine the expectation the entrant has about the investment costs.

2.3 The final model

With these adjustments and simplifications that improved the analyzability and understandability of the game, my research objective is to investigate the effects of information asymmetry on investment strategies and payoffs in a duopoly market and the consequential signaling on the investment decisions by the firms using a binomial lattice model which results in the possibility of option values. This results in the following framework for my research.

I consider a market with two firms that have to make a decision about investing in a project that has a certain \( PV \) and a cost to undertake the investment. Both firms are not in the market prior to the game. The \( PV \) is the public information and the investment costs are the private information (type of the incumbent). In period 1, both firms observe the \( PV \) and know the multiplications of the binomial movements in subsequent periods, so the binomial pricing tree is assumed to be public information. After period 1, the binomial movement will result in a multiplication of the first period’s \( PV \). This multiplication is either \( u \) leading to a higher \( PV \) in period 2 or \( d \) leading to a lower \( PV \) in period 2. This process continues for a predefined number of periods (see next chapter). Observing this binomial pricing tree of \( PVs \), the incumbent sends a message containing his investment decisions for the entire game and the entrant responds by choosing an action, which he bases on his belief about the incumbent’s type. So the actual signal is the incumbent’s investment strategy for the game, knowing the structure of the binomial tree and the entrant’s response he can induce. In a 2-period game there are three decision nodes (derived from a two-period binomial tree), one in period 1, and two in period 2. Only one of these second period nodes will be reached, but the firms don’t know which so have to make an investment decision for both prior to the game. Their strategies (messages and actions) contain investment decision for every binomial path that can occur. What will happen in this game is that the incumbent has to make the perfect decision how to send his signal to maximize his payoffs conditional on the entrant’s belief and actions. So both firms eventually choose their optimal investment timing to maximize their payoffs. They evaluate their payoffs at any point in the game according to the possible option values. These option values determine their optimal investment timing taken into account the other firm’s actions. In the end this will result in an optimal investment decision for both.
3. The Model

3.1 Basic Model Description
In this research I consider two firms in an asymmetric competition, an incumbent and an entrant. The incumbent has the information advantage. Both firms have the opportunity to invest immediately when possible, but also have an option to wait and invest sometime in the future. The incumbent and entrant are denoted by firm $F = INC$ and $F = E$, resp. In the game, the firms have the opportunity to make an investment in a single project to capture a share of the project’s present value ($PV$). This investment can be made at most only once by each firm in a game. Both firms have common knowledge about this project’s $PV$, which represents all the earnings that can be obtained in the finite lifetime of an investment.

As discussed in the previous chapter, in most realistic investment scenarios, the $PV$ of the project is a stochastic process following a geometric Brownian motion (GBM). A geometric Brownian motion is a continuous-time stochastic process in which the underlying stochastic factor has a logarithm that follows the pattern of a Brownian motion. The GBM is used for the Black-Scholes Option Pricing (Cox et al., 1979) to price stocks and options. Here, $Y_t$ is assumed to be the underlying stochastic aggregate economic factor affecting the $PV$. The continuous-time discount rate is denoted by $r$, $\mu \in [0, r)$ is the drift parameter, $\sigma > 0$ is the instantaneous standard deviation or volatility parameter and $dW$ is the increment of a standard Wiener process which is normally distributed, $dW_t \sim N(0, dt)$:

$$dY_t = \mu Y_t dt + \sigma Y_t dW_t$$

The parameters $\mu$, $\sigma$ and $r$ are common knowledge (also in my binomial lattice model) and assumed constant over time (Mason and Weeds, 2009). The incumbent’s private information contains the actual cost of investing in the project. In reality this investment cost also follows a GBM (unless price negations resulted in a fixed price) since price fluctuations are usually stochastic. In that case the private information would have the same stochastic structure as the $PV$. In the model, I consider the project’s investment cost, $C^\theta$, to be either low or high, so $C^\theta \in \{C^L, C^H\}$. The actual investment cost determined by nature is assumed to be constant throughout the game. The investment cost is equal for the two firms and they also both know the actual value for $\theta = L$ and $\theta = H$. So in general, the entrant has an information deficit about which cost has been determined by nature prior to the game. The prior probability of drawing $\theta = L$ or $\theta = H$ is denoted by the $p$ and $1-p$, which is also the prior belief about the investment cost and is common knowledge.

3.2 Binomial Approximation of the GBM
In the previous chapter I described the meaning of a real option and its applicability for the calculation of option values. Also the reasoning behind the approximation of a GBM by a binomial lattice is explained (see appendix A), but figure 1 shows its actual format. The start of the game, which is the same for the GBM, is a single point from where the stochastic parameter starts its path of fluctuations. I call this first node: period 1, because this is the first
period a firm can make an investment decision from the set $d_t \in \{I, N\}$ which are the decisions Invest ($I$) and Not Invest ($N$). In the next period, the $PV$ will take a new value which is a multiple of the first period, either after an up movement or a down movement in the binomial tree with a certain probability. At this new node, the same pattern repeats itself: an investment decision is made and the parameter follows a binomial up or down movement. This binomial lattice continues this process infinitely when approximating a GBM. However, I imposed a restriction that the investment horizon is finite so the number of periods is also finite. In the last period, independent of which node, the firm(s) making a decision can only invest then or never again, so in a last-period node there is no option value of waiting, only an investment payoff (if positive) or a zero payoff of not investing. The nodes in the penultimate period use these last-period payoffs to calculate the option value of waiting one period. If this option value is positive, so waiting is more profitable than investing immediately, the firm will not invest and go to the next period to either of the two subsequent nodes and make an investment decision at that node. This process continues via this backward reasoning until the first period. So at every node in the game, the firm can calculate option values of waiting for every possible subsequent period.

For example, a firm that finds itself in node 4.2 can calculate the option value of investing in period 5, 6, 7 (etc.) until the last period using the option values and utilities of investing in node 5.2, 5.3, 6.2, 6.3, 6.4, (etc.) but not node 5.1, 6.1, 5.4, 6.5 since these are not subsequent to 4.2 and therefore have no influence on the option value calculations of node 4.2. As a result for a game using this binomial lattice, calculations for the option values of investing of every period being in any node can be made. To accomplish this, the nodes in a certain period need to be averaged according to their probabilities of occurrence (see next section) since before the start of the game the firms don’t know the path that is going to be taken by nature and therefore base their decision on the expected payoff in a period instead of the fixed payoff at a node. So the nodes in period 3 all three determine the option value of waiting to invest until period 3 being in period 1. These option values are used to determine the best and most profitable investment decision for both firms.
In the thesis, I will explain a two-period game in full detail and show every calculation made in the simplest game to obtain strategies and payoffs for both firms. Since this simplest game consists of two periods with one node in the first and two nodes in the second period, this represents the essentials of every ‘subgame’ consisting of three nodes: one followed by its two subsequent nodes. Every node is the basis of a subgame in a bigger game.

3.3 Method of Analysis

There are two ways a game can be analyzed: one where a strategy determines the period of investment (for detailed information about this method see Appendix B) independent of the binomial path that is taken; and the other where a strategy explicitly makes a decision for every possible binomial path, so an investment decision at every separate node in the game.

The major difference is that in the first method an investment is decided to be made in a period (independent of its nodes), whereas in the second method an investment is done in a certain node along a binomial path, so the binomial movements are explicitly incorporated. In the first method, all the period’s nodes contribute to that period’s average payoff which determines the investment period. However, in the second method, the payoffs at every node are analyzed separately to result in the optimal investment. The following extensive-form game (figure 2) represents the situation where at every node in a 2-period game a decision can be made for every type and binomial movement and this figure is used to show the model for every separate decision within a strategy.
I will investigate my model according to the decision-per-node method. In this analysis, every strategy needs to be defined explicitly for all the binomial movements that can occur in the game. The 2-period game can take on two different paths and the firms need to pick a strategy that is optimal for both paths. The strategies now incorporate all the decisions that can be taken for every possible binomial path so that in any case they invest rationally. In this case a strategy has the form:

\[
\{d_1, \{d_{2,1}, d_{2,2}\}\}
\]
Where in every node a binary decision $d_{i,j}^t \in \{I,N\}$ is made, where $i$ is the period and $j$ is the node ranked from top to bottom$^1$. There is one decision in the first period and two in the second period (and obviously three in a third period). In the second period there are four possible combinations of decisions, two for each possible path, namely invest or not invest. Since the message is sent after observing the cost, $\{d_1, \{d_{2,1}, d_{2,2}\}\}$ is either the left part or the right part of figure 2. This also holds for the action of the entrant even though he might not make a decision at all in the first period. This results in the following set of messages and actions for the incumbent and entrant:

$$M = \{(I, \{\cdot, \cdot\}), \{N, \{I, I\}\}, \{N, \{I, N\}\}, \{N, \{N, I\}\}, \{N, \{N, N\}\} \}$$

$$A = \{(I, \{\cdot, \cdot\}), \{\cdot, \{I, I\}\}, \{\cdot, \{I, N\}\}, \{\cdot, \{N, I\}\}, \{\cdot, \{N, N\}\}, \{N, \{I, I\}\}, \{N, \{I, N\}\}, \{N, \{N, I\}\}, \{N, \{N, N\}\}, \{\cdot, \cdot\} \}$$

In both methods of analyzing a game, the cost is chosen by nature, followed by a message and finally an action. In the first method this resulted in the inability of a firm to redo the analysis when finding itself at a node with a non-optimal payoff that can be increased by waiting at least one more period. The second type incorporated this ability by sending a message consisting of decisions at every node so for every possible binomial path. Then the firms can decide beforehand which nodes are profitable to invest in. This is the most realistic representation since in reality it is irrational not to increase your payoff when possible. My research will therefore be based on the decision-per-node analysis and in the next paragraph the transformation of the separate decisions into strategies is explained in detail. The difference between the two methods also becomes clear in chapter 4.

A summary of a Bayesian model, which is the starting point of my research will be the following:

1. $F = INC, E$ is the set of players in the game
2. $M$ is the set of strategies for player $I$
3. $A$ is the set of strategies for player $E$
4. $\Theta$ is the set of types for player $I$, player $E$ has no types
5. $u^F: \Theta \times (M \times A) \rightarrow R$  
6. $p(\theta)$ is prior probability for the types being ‘low’ or ‘high’, $p$ and $1-p$, resp.
7. I analyze the game in pure strategies

During the analysis of the games I will deviate from this Bayesian model, mainly because of nr. 6, but I will elaborate on that in detail in chapter 7.

### 3.4 Explanation of the extensive-form games

In this paragraph I will use the decision-per-node representation because of its dynamic character and rational representation. I will show how a game with decisions at every

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$^1$ When referring to node 1 in period 1, $i.j$ is simply node $I$ instead of node $1.1$

$^2$ Even though this utility is an expectation as will be shown in later in this chapter, I use the term payoff instead of expected payoff when no confusing can occur.
separate node is transformed into a game that analyzes a message or action as a bundle of these separate decisions. A firm makes at every node a decision $d_{i,j}^k \in \{I, N\}$. In a 2-period game, both firms can make at most three decisions which are bundled into a strategy of the form $\{d_1, \{d_{2,1}, d_{2,2}\}\}$. What the order of decision making is in the game is displayed in figure 2.

In period 1, the incumbent, knowing the costs, can make a decision to either invest or wait for the next period. If he invests, his influence on the game is over. Then the entrant can make a decision in the same node since the incumbent is done playing, however he doesn’t know if he is on the ‘low’ or ‘high’ part of the figure because of the information asymmetry. The entrant can now choose also to invest, so they will both invest in node 1, or wait and see what the binomial movement will do. It can go either up or down and after this movement, the entrant can make his last investment decision again, invest or not. Whenever the incumbent decided not to invest in the first node, he will wait for the binomial movement and then make his second-period decision in either the up or down node. The decision for both nodes is incorporated explicitly in the strategy. If he invests, the entrant can only have a decision in that particular node, but if the incumbent didn’t invest in either period, the entrant can never invest leaving both a zero payoff.

Figure 2 shows how this game is played according to a binomial model. At every node, the firms make a decision to invest or wait if they expect the payoff in a subsequent period to be higher (which is the option value concept, discussed later in this chapter). However, a message or action cannot contain just a decision at a node, but should be a bundle of all decisions the firm will make if he was to decide what to do at every node that could be reached in a game. So in this 2-period game there are three nodes, which means that both firms can make at most three decisions. The reason why it is not always three is that by investing in the first node, the incumbent loses his ability to invest in either second node, he already made his investment. The same holds for the entrant, which also can lose his first-period decision if the incumbent decided not to invest in node 1.

A strategy, which can be both a message and action now contains all decisions that could be made (the ones that cannot be made are represented with a dot). The incumbent has five combinations of separate decisions, the entrant ten (five with the first-node decision and five without). This message-action extensive-form game containing all possible decision bundles is displayed in figure 3:
In this representation, in line with the previous one, it is shown that whatever the incumbent sends as a signal, the entrant does not know in which part of the game he actually is, represented by the information sets. The incumbent chooses one of his five messages and the entrant will respond with one of his ten actions. However, the entrant will not choose from the same set of actions for every message he receives, since some actions as a response to a message are irrational and lead to a negative payoff (i.e. $a = \{N, \{N, I\}\}$ as a response to $m = \{N, \{I, N\}\}$ is irrational).

The extensive-form games of the decision at every node (figure 2) and the combination of all strategies (figure 3) are directly connected with the representation in figure 4.

This figure shows in the most basic form the decisions they can make: invest or not in period 1 and after a binomial movement make a decision at both nodes in period 2 of which one will actually be reached. As stated before, some decisions cannot be made and are represented by a dot. A strategy profile gives an outcome of the game and this strategy profile consist of two ‘figures’ as in figure 4 combined. The strategy profiles are shown in figure 5, where all the strategies are a set of decisions as the extensive-form game of figure 3, which is used for the determination of the optimal strategies and the resulting payoffs. Figure 5 shows the exact relation between figure 2 and 3:
It is clear how I represent messages and actions as a bundle of the separate decision of the form \( d_{i,j}^E \in \{I, N\} \) as in figure 2. The decisions \( d_{i,j}^E \) that are green are the ones that can actually be made by the firms; the ones that are red will never be reached and are represented by a dot in the strategy. Let’s look at the strategy profile on the left, where the incumbent plays \( m_1 = \{I, \{i, \cdot \}\} \) and the entrant responds with \( a_7 = \{N, \{I, N\}\} \). What will happen can be seen in figure 2: assume low costs, then the incumbent invests and the entrant can decide in the same node and in this case he chooses to wait. The chance then determines the binomial path and therefore the following node, if that is the up-node, the entrant will invest and the payoffs will be \((x_1, y_1)\) in figure 2 for the incumbent and entrant, resp. If the down node is reached the entrant won’t invest and the payoffs will be \((x_2, y_2)\). The payoffs \((a_1, b_1)\) in figure 5 are simply the addition of the two payoff outcomes of figure 2 times their binomial probability (chapter 3.7 shows the calculations of the binomial probabilities):

\[
(a_1, b_1) = p_a(x_1, y_1) + p_d(x_2, y_2)
\]

The three extensive-form games show the connection between separate decisions and strategies that contain these decisions. In this research I will obtain equilibria and calculate payoffs according to figure 3, but it will be clear that the calculations for the possible payoffs at every node are an input for that.

### 3.5 PV share

As stated before, the PV of the investment is assumed to capture all the earnings that will be obtained in the project’s finite lifetime. Since we then know the amount of the whole PV of a project, the firms can calculate their share according to their investment timing. Since I
assumed that, except for the information asymmetry, the firms are perfectly equal, both get the same share of the PV when they decide to invest in the same period. However, when they don’t invest (almost) simultaneously, the division of the PV will be unequal. The share they get depends on the time between their (possible) investments. Obviously, the longer the time between investments, the bigger the incumbent’s share $\phi^{INC}$ and the smaller the entrant’s $1 - \phi^{INC} = \phi^{E}$ since the incumbent is in the market significantly longer. The number of periods between the two investments is denoted by $z$ and $\phi_D$ is the difference between their shares per period in between investments, so the total difference between their shares is $z\phi_D$. So if we assume a game with three periods in which the incumbent invests in period 1 and the entrant in period 3 with $\phi_D = 0.2$, then $\phi^{INC} = 0.7$ and $\phi^{E} = 0.3$. If the entrant does not invest in a game where the incumbent did, then $\phi^{INC} = 1$. This shows that the PV shares of the firms are a function of the message and action that determine the investment timings and therefore $z$, so the PV share when investing in node $i,j$ by firm $F$ and predefined $\phi_D$ is given by the function $\phi^{ij}_{F}(m, a)$.

This is also assumed in Thijssen et al. (2003) and Villani (2009) where it is stated that investing sooner leads to a higher market share. The decrease in market share for the entrant can be linear or exponential. Both are realistic and applicable depending on the kind of investment and the structure of the market. The notations I used describe the linear decrease in PV share.

3.6 The signaling aspect

The basics of the signaling aspect in option games are already described in chapter 1. However, the signaling aspect I will use in my research is not equal to the pure definition of signaling games. The calculations for the optimal payoffs (chapter 4) are done according to the same basic requirements, namely that an optimal strategy means that deviating cannot improve the payoff. Even though the effect of the message on an entrant’s decision is different (footnote 4), and the incumbent has the power to influence the beliefs, the entrant still needs to update his belief according to the type-contingent probabilities and message he receives.

---

3 Assume $\phi_D = 0.2$, $m = \{N, \{I, I\}\}$ and $a = \{\emptyset, \{I, N\}\}$, then $\phi^{I+}_1 = 0.5$, $\phi^{E}_1 = 0.5$, $\phi^{INC}_2 = 1.0$ and $\phi^{E}_2 = 0$

4 The difference between signaling in games and pure signaling games lies in the use of the Bayes’ posterior beliefs in the overall utility calculations and the fact that I only assume pure strategies and no mixed strategies. In the signaling games, the calculations are as follows: A strategy for the incumbent prescribes a probability distribution $P_1[\cdot | \theta]$ over the messages $m$ for each type $\theta$. A strategy for player 2 prescribes a probability distribution $P_2[\cdot | m]$ over actions $a$ for each message $m$. The expected payoff for player 1 before the game with type $\theta$ and strategy $P_1[\cdot | \theta]$ when player 2 plays $P_2[\cdot | m]$ is:

$$u_1(P_1, P_2) = \sum_m \sum_a P_1[m | \theta] P_2[a | m] u_1(m, a, \theta)$$

And the expected payoff for player 2 conditional on $m$ when he uses strategy $P_2[\cdot | m]$ and posterior belief $\mu(\cdot | m)$ can be computed as follows:

$$u_2(m, P_2, q) = \sum_{\theta} \sum_a q(\theta | m) P_2[a | m] u_2(m, a, \theta)$$

The $P_1[\cdot | \theta]$ are the signals used for the entrant’s Bayes’ updated beliefs. The incumbent knows all $P_2[\cdot | m]$ which are the reactions of the entrant for incumbent’s every strategy. In my analysis of signaling, the updated beliefs are not used to multiply with the utility expected to obtain: $u^e(m, a, \theta)$, but rather are a used to calculate the expectation of a specific parameter within the utility calculation as will be shown in chapter 3.7.
In this section I will explain the signaling calculations with the mathematical notations. Signaling in games is based on the fact that information is not equally distributed among the players. As already described in the previous section, the investment cost is determined by $C^\theta \in \{C_L, C_H\}$, so the incumbent has a type containing the information about the investment cost $\theta \in \Theta = \{L, H\}$. The incumbent sends a signal which is a message from a set $m \in M$ that contains (some) information about his type. After observing $m$, the entrant updates his beliefs about $\theta$ according to Bayes’ Rule and bases his choice of actions $a \in A$ on the posterior distribution $q(\theta|m)$ over $\theta$.

In the game that is played, the incumbent has a probability of investing in a particular node according to his message conditional on his type: $P[m|\theta]$. The entrant assigns these probabilities to every message for $\theta$, which I call the type-contingent probabilities, and updates his belief about the investment cost according to them. The prior beliefs for low and high cost are $p$ and $1-p$, resp. The beliefs are updated according to Bayes’ rule:

$$q(m) = P[L|m] = \frac{pP[m|L]}{pP[m|L] + (1-p)P[m|H]}$$

Given these updated beliefs the incumbent can induce his best strategy. The type-contingent probabilities $P[\cdot|\theta]$ that the entrant attaches to the investment timings are common knowledge, so the incumbent can chose his message in such a way that he can try to make the entrant belief the costs to be of a certain value which results in an specific action that is optimal to the incumbent. Stated differently, the incumbent knows the Bayes’ updating calculations of the entrant and will pick a message that evokes a response that will maximize his own payoff (the method of obtaining type-contingent probabilities is explained in full detail in chapter 7 and 8). The formula for Bayes’ updated results in the posterior belief that the message contains type $L$:

$$q_m = q(L|m) = \frac{pP[m|L]}{pP[m|L] + (1-p)P[m|H]}$$

Where the sum of the type-contingent probabilities for a type have to be equal to one:

$$\sum_{m \in M} P[m|\theta = L] = 1 \quad \text{and} \quad \sum_{m \in M} P[m|\theta = H] = 1$$

In this model $P[m_1|\theta]$, where $m_1 = \{I, \{\cdot,\}\}$, means the probability the entrant assigns to the incumbent investing in node $I$ when the type is assumed $\theta$, with $i$ being the period and $j$ being the node in the period ranking from top to bottom. In essence in this game the incumbent can only influence the payoffs with his message-sending strategies and will do that in a way that, knowing the updated beliefs, he will obtain an optimal payoff. For the simulation model, which starts from chapter 7, we can see that next to choosing the messages, the incumbent can also choose the type-contingent probabilities since he has the most

---

5 Node 3.2 correspond to the middle node in period 3; the node after an up and down (or vice versa) move.
influence in the game. Therefore the type-contingent probabilities can also be viewed as part of a strategy.

Contrary to the signaling games, the signaling in my research has slightly other effects. In the signaling games, it could occur that mixed strategies are played. I consider only pure strategies in my research; in the optimal outcome the optimal (most profitable) investment consists of a single message. At the start of the game, after the cost has been given, the firms choose a strategy that includes all the separate investment decision based on how they induce the other firm will behave if acting rationally in every node. This means that the incumbent chooses his strategy in such a way so that he influences the entrant to choose an action which is optimal for the incumbent. The incumbent knows the costs, while the entrant has an expectation over the costs based on the updated beliefs over the messages. So the fact that the payoff has an expectation has nothing to do with the mixed strategies of the firms but with the uncertain binomial movements in the game. In case of a game with incomplete information and a setting where the updated beliefs are determined by consistent external factors, using the concept of PBE would be satisfied. However, now that I am determining the type-contingent probabilities in a different way, the concept of perfect Bayesian equilibrium will be replaced by the determination of optimal strategies. Even though I cannot use the PBE concept, the method of obtaining payoffs can still be done using the same equilibrium payoff calculations of chapter 4. The formulas are all based on the same concept of optimal strategies, only the updated beliefs are not in line with the theory from this point.

3.7 Value functions

The most important parameter which will in the end determine the investment decisions and corresponding strategies is of course the profitability of the investment, or simply the payoff of an investment decision. The payoff of an investment at a specific node is a function of the PV, the PV share and the (for the entrant, expected) costs. The firms will invest if they expect to obtain a profit. The entrant’s decision whether to join the incumbent in the market or not depends on his expectation of the project’s profitability, whereas the incumbent has full knowledge about his possible payoffs. The firms will only invest in node \( i.j \) if they expect a positive payoff.

However, the calculations needed to determine at which nodes to invest, are done according to a more complex formula. The decision not to invest in the current period because the expected payoff in a future period is higher (which means a positive option value, explained later) needs more input than just the PV share, PV and (expected) costs. A future period consists of multiple nodes which all occur with a certain probability. The probability that node \( i.j \) is reached is denoted by \( p_{i,j} \). The probabilities that the binomial movement goes up or down, resp. are given by the following formulas (Appendix A):

\[
p_u = \frac{(r-d)}{(u-d)} \quad \text{and} \quad p_d = 1 - p_u
\]

So being in period 1 then \( p_{2,1} = p_u \) and \( p_{2,3} = p_u^2 \) etc. The expected payoff of investment in a future period consists of the separate investment decisions at the nodes that could occur in
that period, so the payoffs of all nodes that can occur in that period need to be calculated separately and combined with the probability of occurrence determine the expected payoff of a future investment. Since a message and action contain a decision at every node to be of the set \{I, N\} the representation of the formulas for the payoff can be described more compact. Only the nodes which contain decision \(d_{i,j} = \{I\}\) are used in the calculations since all the other nodes, when reached, result in a zero payoff.

The value of the entrant is a function of the entrant’s belief for the investment costs \(\theta\) and both firms’ strategies:

\[
\begin{align*}
\mathcal{E}(m(\theta), a(m(\theta)), q_m) &= \sum_{\{(i,j) | (d_{i,j} = \{I\}) = a\}} p_{i,j}(PV_{i,j} \varphi_{i,j}^{\mathcal{E}}(m, a) - \mathbb{E}[C_{m}^\theta]) \\
\end{align*}
\]

Where the payoff for an investment at a specific node is \(u_{i,j}^E = PV_{i,j} \varphi_{i,j}^{\mathcal{E}}(m, a) - \mathbb{E}[C_{m}^\theta]\). The expected payoff for the entrant is a summation over all the nodes \(i,j\) for which the corresponding decision \(d_{i,j}\) in the action \(a\) is an investment conditional on their probability of occurrence. The expected costs are determined by the posterior beliefs about both types:

\[
\mathbb{E}[C^\theta] = \sum_{\theta} q(\theta | m) C^\theta
\]

The entrant can observe the \(PV\) but can only have an expectation of the costs, which he bases his investment decisions on. Even though the cost is chosen by nature and will not change during the game, the entrant has a different expected cost for every message, because the bundle of investment decisions all have a different meaning. For example, the message consisting of an investment only in node 2.1 could indicate that the costs are higher than the message consisting of investments in both nodes 2.1 and 2.2, since the entrant could perceive that investing only in node 2.1 means that 2.2 is not profitable enough. The incumbent knows that the entrant makes these calculations about the costs so he will send his messages strategically to take advantage of the Bayes’ updated beliefs \(q_m\).

The incumbent has a trivial belief about the investment costs, so he is certain about his possible payoffs conditional on the entrant’s actions.

\[
\begin{align*}
\mathcal{E}(m(\theta), a(m(\theta)), q_m) &= \sum_{\{(i,j) | (d_{i,j} = \{I\}) = m\}} p_{i,j}(PV_{i,j} \varphi_{i,j}^{\mathcal{E}}(m, a) - C^\theta) \\
\end{align*}
\]

Where the payoff for an investment at a specific node is \(u_{i,j}^{INC} = PV_{i,j} \varphi_{i,j}^{INC}(m, a) - C^\theta\).

3.8 Net Present Value and Option Value

To calculate the option values of waiting, the firm needs to know the expected payoff of investing in a future period, which I will refer to as the current value or net present value (\(NPV\)) of a future investment. When calculating the \(NPV\) of the next period, being in a certain
node, only the two subsequent nodes can be used for the calculation (see figure 6). In this figure it is shown that being in node 2.1 and calculating the NPV of investing in period 3, only node 3.1 and 3.2 can be used.

![Figure 6: binomial tree with relevant nodes to calculate NPVs](image)

The NPV is simply the PV minus the (expected) cost of an investment in a future period, discounted for the amount of periods between the current node and the future investment opportunity. Using the payoffs of the firms at every necessary node, \( u_{i,j}^E \) and \( u_{i,j}^{INC} \), results in the analysis of the NPVs. According to Cox et al. (1979) the method of calculating the NPV for the next period in the future is done via the following formula, with:

\[
NPV_{(i,j)\rightarrow(i+1)}^F = \frac{p_u(u_{i+1,j}^E) + p_d(u_{i+1,j+1}^E)}{r}
\]

The notation \( NPV_{(i,j)\rightarrow(i+1)}^F \) means calculating the NPV of period \( i+1 \) being in period \( i \) in node \( j \). The payoffs of the two subsequent nodes are needed, and \( u_{i+1,j+1}^E \) refers to the payoff investment at a specific node as described above: \( u_{i,j}^{INC} = PV_{i,j}^{INC}(m,a) - C^\theta \) and \( u_{i,j}^E = PV_{i,j}^E(m,a) - \mathbb{E}[C_m^\theta] \). The NPV of an option in a 2-Period game can be calculated via the following formula:

\[
NPV_{1\rightarrow2}^F = \frac{p_u u_{2,1}^E + p_d u_{2,2}^E}{r}
\]

These calculations are very similar to the payoff formulas for the firms, only now the probabilities of occurrence of a node are not simply \( p_{i,j} \) calculated from the start of the game but the probability of reaching a node, being in another. So for calculating \( NPV_{3,1\rightarrow4}^F \), \( p_{4,1} \) is not \( p_u^3 \) but just \( p_u \).

Having all the payoffs and NPVs, it is possible to calculate the option value of waiting for both firms. The option value is a future’s NPV minus the value of immediate investment in the project. I added to this definition the assumption that the OV is non-negative since a negative OV means investing in the current period and hence is trivial. This results in the following formula:

\[
OV_{(i,j)\rightarrow(i+1)}^F = (NPV_{(i,j)\rightarrow(i+1)}^F - u_{i,j})^+
\]
In chapter 5 the OVs of a two-period game are calculated and in chapter 8 I show how OVs are obtained in the simulations, the reason for appearance and the OVs in combination with the value of the information advantage.

4. Determining Optimal Strategies

The equilibrium concept that is relevant for games with incomplete information is the Perfect Bayesian equilibrium. A perfect Bayesian equilibrium is a strategy profile \( \sigma^* = (m^*, a^*) \) and posterior beliefs \( q(\theta|m) \) such that three requirement are ensured. It was already pointed out that the concept of PBE cannot be used, however the calculations for finding optimal strategies can still be done using the following requirements:

First it has to be guaranteed that \( m^* \) is the optimal strategy of the ‘subgames’ of each type \( \theta \) of the incumbent, given the action. Second, this also holds for the entrant that \( a^* \) is the optimal strategy of the ‘subgames’ of the entrant, given the belief. And third, posterior (updated) beliefs have to be obtained. For the analysis of the signaling effect (and not the pure signaling game, Gibbons (1992)), I use the following solution concept. The assessment:

\[
\{(m(L), m(H)), a(\cdot), q(\cdot)\}
\]

consists of three components: \( m(L) \) and \( m(H) \) are the incumbent’s messages for private information \( L \) and \( H \), resp.; \( a(m) \) is the entrant’s action for the observed incumbent’s message about his investment decisions; and \( q(\theta|m) \) is the entrant’s belief about the observed incumbent’s message assuming a specific type. The optimal solution is the assessment:

\[
\{(m^*(L), m^*(H)), a^*(\cdot), q^*(\cdot)\}
\]

satisfying the following conditions:

First, an optimum in a signaling game is pair of strategies \( m^*(\theta) \) and \( a^*(m) \) and a belief \( q(\theta|m) \) satisfying the following requirements:

\[
\sum_{\theta \in \Theta} q(\theta|m) = 1
\]

Second, given the incumbent’s (Sender’s) message and the entrant’s (Receiver’s) belief, the entrant’s unique optimal action is characterized as follows: For each \( m \in M \), the entrant’s action \( a^*(m) \) must maximize his utility, given the belief \( q(L|m) = q_m \) about which types could have sent \( m \). That is \( a^*(m) \) solves:

\[
\forall m, \quad a^* \in \arg\max_{a \in A} u^E(m(\theta), a(m), q_m)
\]

\[
u^E(m(\theta), a^*(m(\theta)), q_m) = \max_{a \in A} u^E(m(\theta), a(m), q_m)
\]
The same requirement also applies to the incumbent, but he has complete information (and therefore a trivial belief) so the incumbent’s strategy must be optimal given the entrant’s strategy. For each \( \theta \in \Theta \), the incumbent’s message \( m^*(\theta) \) must maximize his own utility, given the entrant’s strategy \( a^*(m^*(\theta)) \).

Therefore, third, \( m^*(\theta) \) is the optimal message of the incumbent for \( \theta = L, H \). That is, \( m^*(\theta) \) solves:

\[
\forall \theta, \quad m^* \in \arg \max_{m \in M} u^{INC}(m(\theta), a^*(m), q_m)
\]

\[
u^{INC}(m^*(\theta), a^*(m^*(\theta)), q_m) = \max_{m \in M} u^{INC}(m(\theta), a^*(m), q_m)
\]

Finally, \( q_m \) is the entrant’s updated belief for his optimal strategy for the low cost when receiving message \( m \). This message \( m \) contains a set of investment decisions which the entrant can perceive as information about the type of the incumbent and every message can result in a different \( q_m \):

\[q_m = q(L|m) = \frac{pP[m|L]}{pP[m|L] + (1 - p)P[m|H]}\]

\[q(H|m) = 1 - q(L|m)\]

An assessment is said to be a (weak) perfect Bayesian equilibrium in pure strategies (PBEP) if the previous requirements are satisfied. I am going to investigate the optimal pure strategies, because I assume only one node is the optimal to invest in and the long-term utility is at maximum when this optimal investment decision is made.

When analyzing the PBE in my 2-period model, its corresponding strategic-form game will be figure 7:

![Figure 7: strategic-form 2-period game](image)

There are five possible messages and two types, so in theory, the incumbent can have 25 sets of \((m(L), m(H))\). The entrant has five information sets, one after each message, and therefore has to choose five actions and can choose from maximum ten at each information set, so in theory the entrant has \(10^5\) sets of \((a(m_1), a(m_2), a(m_3), a(m_4), a(m_5))\). The number of strategy combinations \(((u^{INC}(H), u^{INC}(L)), u^E)\) in the strategic-form game will then be 2,500,000. The number of actual possible combinations that can occur will be less but still an extreme amount in the strategic-form game and therefore I cannot show this full...
strategic-form game. The optimum however can be found in chapter 8 since the requirements are satisfied and the maximizations were able to be programmed and simulated.

5. Two-Period Game

5.1 General Description

In this chapter I describe the 2-period game in detail, including all figures, assumptions and formulas I used. The 2-period game consists of three nodes in a binomial model and therefore three decision points in total; one in period 1 from where the game always starts (this node will be in the path every time the game is played) and two decisions in the second period of which only one will appear in a random path. At every node a separate analysis is made given the project’s PV at the game (figure 8).

Figure 8: Binomial Representation of a 2-Period Game with all essential formulas for the firms

At three points in the game, of which two will actually occur, a payoff determines the profitability of an investment undertaken at that point. These three payoffs are the core of the option calculations. It is described previously that in this game consisting of a binomial lattice the analysis starts in the last period and then continues until the first via backward reasoning. The reason is that it is a finite game and therefore the last period’s nodes have no option value of waiting. So these last period’s nodes are the only nodes that only have a direct exercise value. By knowing the last period’s payoffs, the option values for investing in that period, being in a penultimate period can be derived.

All payoffs are a function of the message-action-belief tuple \((m(\theta), a, q_m)\) resulting in the formulas of chapter 3.7:

\[
u^{NC}(m(\theta), a, q_m) = \sum_{[(i,j)](d_{ij}=(l))\in m} p_{i,j}(PV_{i,j}\varphi_{i,j}^{NC}(m,a) - C^\theta) \\
u^{E}(m(\theta), a, q_m) = \sum_{[(i,j)](d_{ij}=(l))\in a} p_{i,j}(PV_{i,j}\varphi_{i,j}^{E}(m,a) - E[c^\theta])
\]
In this 2-period there are five type-contingent probabilities for $L$ as well as for $H$, resulting in all the posterior beliefs $q_m$ obtained by Bayes’ Rule:

$$q_{m_1} = \frac{pP[m_1|L]}{pP[m_1|L] + (1-p)P[m_1|H]}$$

$$q_{m_2} = \frac{pP[m_2|L]}{pP[m_2|L] + (1-p)P[m_2|H]}$$

$$q_{m_3} = \frac{pP[m_3|L]}{pP[m_3|L] + (1-p)P[m_3|H]}$$

$$q_{m_4} = \frac{pP[m_4|L]}{pP[m_4|L] + (1-p)P[m_4|H]}$$

$$q_{m_5} = \frac{pP[m_5|L]}{pP[m_5|L] + (1-p)P[m_5|H]}$$

These five posterior beliefs can all have a different value and therefore every message can result in a different expectation about the costs for the entrant.

In addition to the payoffs at the three nodes, figure 8 also shows the net present value ($NPV$) and option value ($OV$) which are the most important calculations to determine the actual investment timings. The meaning of these formulas is already described in chapter 3, but it is clear that a positive $OV$ means that investing in the second period is in the long-term more profitable than investing in period $I$, so the firm will wait in the first period when the $OV$ is positive.

$$NPV_{1\rightarrow2}^F = \frac{p_u u_{2,1}^F + p_d u_{2,2}^F}{r}$$

$$OV_{1\rightarrow2}^F = (NPV_{1\rightarrow2}^F - u_1^F)^+$$

5.2 Numerical example

Now we have all the important formulas I will show how these formulas lead to investment equilibria for both firms and especially that the incumbent sends his message strategically to make the entrant believe what is optimal for the incumbent. This numerical example shows the behavior of an asymmetric competition between equal sized firms.

Let’s assume that $PV_1 = 50$ and in node 2.1 a low cost duopoly and high cost monopoly are profitable, $u = e^{\sigma \sqrt{t/n}} = 1.5$ and $d = e^{-\sigma \sqrt{t/n}} = 0.667$ and $r = 1.1$ ($\sigma = 0.2$ and $\frac{t}{n} = 4$) so $p_u = 0.52$ and $p_d = 0.48$ resulting in:
Furthermore, the costs are $C^\theta \in \{C^L, C^H\} = \{20, 40\}$ and assume nature chooses $\theta = L$. The initial beliefs for the costs are $P[\theta = L] = P[\theta = H] = 0.5$. The messages and actions have the form of $\{d_1, \{d_{2,1}, d_{2,2}\}\}$ and $\varphi_\Delta = 0.2$ so the PV is shared 60/40 when there is one period between investments. For every node they make a separate investment decision so that for any path that nature determines they can stick to their strategy. If the incumbent for example chooses to send $m = \{N, \{I, N\}\}$ that means that he will invest in node 2.1 if nature determines that the binomial movement goes up and won’t invest if the binomial movement goes down.

Every message can have an individual set of type-contingent, for example, $P[m_1|L]$ is the probability the entrant assigns that the incumbent will invest in node 1 given that the costs are low. These probabilities are public information in this situation since the incumbent can influence them so both firms can observe these probabilities. The ten type-contingent probabilities are obtained by the entrant as a result of the combination of all parameters but the most important input are the PVs and the amount of the costs, therefore the type-contingent probabilities change when the PVs change. When the PVs all increase with a certain percentage then the type-contingent probabilities also change since it could be that new nodes become profitable for investment (keep in mind that $\sum_{m \in M} P[m|\theta] = 1, \forall \theta$): The entrant can attach the following probabilities to the investments of the incumbent for every message:

$$
\begin{align*}
P[m_1|L] &= 0.3 & P[m_1|H] &= 0.15 \\
P[m_2|L] &= 0.55 & P[m_2|H] &= 0 \\
P[m_3|L] &= 0 & P[m_3|H] &= 0.8 \\
P[m_4|L] &= 0.1 & P[m_4|H] &= 0 \\
P[m_5|L] &= 0.05 & P[m_5|H] &= 0.05
\end{align*}
$$

Why these probabilities? - The first probability represents the probability that the incumbent will invest in node 1 given that the costs are low. The incumbent can invest there but the entrant can follow and both can make a little profit. The entrant would not invest here if he expects the costs to be high, or in general, if his expectation is higher than 25. The entrant assigns all ten probabilities and the incumbent knows them since they are public information (these probabilities are manually chosen in a way that looks like a possible situation but by no means the actual unique probabilities, which depend on the entrant and how he interpreted all parameters and expects the incumbent to behave). The entrant now sees that when he receives message $m_1$ it could mean high or low but the probabilities of occurrence need to be updated according to Bayes’ Rule since he cannot induce with a reasonable certainty what the type is:
which represents the entrant’s posterior belief about an investment of the incumbent meaning that the costs are low, and \( p(H|m_1) = 0.333 \). With these updated beliefs, the entrant calculates his expectation about the costs in the following way:

\[
\mathbb{E}[C^\theta_m] = \sum_{\theta} q(\theta|m) C^\theta
\]

\( \mathbb{E}[C^\theta_{m_1}] = 26.67 \). This results in the following table:

| \( P[X|L] \) | \( P[X|H] \) | \( q \) | \( \text{E}[\text{cost}] \) |
|----------------|----------------|--------|-------------------|
| m1 0.3 | 0.15 | 0.666667 | 26.66667 |
| m2 0.55 | 0 | 1 | 20 |
| m3 0 | 0.8 | 0 | 40 |
| m4 0.1 | 0 | 1 | 20 |
| m5 0.05 | 0.05 | 0.5 | 30 |

*Figure 10: posterior beliefs about low costs and resulting expected costs*

Based on his expectation of the costs in this message, the entrant will make his investment decision by maximizing his expected payoff so he chooses action \( a_7 = \{N, \{I, N\}\} \) as a response to \( m_1 \), so \( a_7(m_1) \). The other four responses are \( a_3(m_2) \), \( a_5(m_3) \), \( a_5(m_3) \) and \( a_{10}(m_5) \). It appears that (assuming equal shares of the PV when investing in the same node) the investment of the entrant will result in his expected payoff:

\[
u^E(m_1(\theta), a_7(m_1), q_{m_1}) = \sum_{\{(i,j)|(d_{i,j}=i)\in a\}} p_{i,j}(PV_{i,j}q_{i,j}^E(m, \alpha) - \mathbb{E}[C^\theta_m]) \]

Which is equal to \((0.52 * (75 * 0.4 - 26.67))/1.1 = 1.58\). It is profitable for the entrant to choose action \( a_7 \) as a response to \( m_1 \). The incumbent induces all the responses of the entrant and can therefore also induce his expected outcomes for every message. He observes that \( q_{m_3} = 0 \) which means the entrant expects the costs to be 40 when receiving that message. The entrant thinks that \( P[m_3|L] = 0 \) since it will be irrational that the incumbent wouldn’t invest in node 2.2 when the costs are low; he could be the monopolist in node 2.2 in any case. The incumbent observes this and will therefore send \( m_3 \) (in the next game this \( P[m_3|L] \) will then be updated and gets a positive number). The optimal response will be \( a_5(m_3) \) leaving the incumbent a monopolist where in fact a duopoly would be profitable for both.

\[
u^{INC}(m_3(L), a_5(m_3), q_{m_3}) = \sum_{\{(i,j)|(d_{i,j}=i)\in m\}} p_{i,j}(PV_{i,j}q_{i,j}^{INC}(m, \alpha) - C^\theta) \]

\[= 26\]
They could easily share 75 and both make a profit. In case of symmetric information, he would have sent $m_2 = \{N, \{I, I\}\}$ where he will share the market in node 2.1 and be the monopolist in node 2.2. This message-sending by the incumbent shows the ‘fooling’ behavior of this particular game. The entrant in the end observes the type and comes to the conclusion that he is ‘fooled’ so he will adjust his type-contingent probabilities, which again the incumbent will use as strategically as possible. The optimal set of strategies of this example according to the solution assessment is (however, for the simulations, I need to do it separately for high and low costs):

$$\left\{ (m_3(L), m_3(H)); a_5(m_1), a_3(m_2), a_5(m_3), a_5(m_4), a_{10}(m_2); q^*(m_1) = 0,667, q^*(m_2) = 1, q^*(m_3) = 0, q^*(m_4) = 1, q^*(m_5) = 0,5 \right\}$$

And the actual payoffs that will be obtained for both firms in the low cost and high cost scenario are:

$$u^F \left( m_3(L), a_5(m_3), q_{m_3} \right) = (26 ; 0)$$
$$u^F \left( m_3(H), a_5(m_3), q_{m_3} \right) = (16,55 ; 0)$$

The fact that $m_3$ is the optimal message, it means that investing in the second period appears more profitable than investing in the first period which is message $m_1$. This is also shown by the positive option value:

$$OV_{1\rightarrow 2}^F = (NPV_{1\rightarrow 2}^F - u_1^F)^+ = 26 - 19,6 = 6,4$$

This example showed how to get from messages and actions to a solution assessment and in the end to the payoffs for both firms. To find the influence of both information asymmetry and real options, the payoff of the incumbent will be the focus point since he will be the firm with the information advantage and the ability to move first.

The incumbent knows these ten type-contingent probabilities and therefore choses his best message according to that. With the given ten type-contingent beliefs the incumbent will choose $m_3 = \{N, \{I, N\}\}$ to make the entrant belief costs are high enough to make an investment for the entrant unprofitable and $m_2 = \{N, \{I, I\}\}$ would imply low costs. A message results in certain updated beliefs and therefore the incumbent will send his optimal message, which will result in his optimal payoff thanks to the updated belief.

The ten type-contingent probabilities cannot be given clear and straightforward values for every separate situation in a simulation. The values could depend on exogenous information the entrant has about the incumbent or how the entrant interprets the combination of the parameters, so there is no theory or method that results automatically in these probabilities. Therefore I will generate these type-contingent probabilities as if they can be fully influenced by the incumbent. I will explain in chapter 7 and 8 how this is done in the simulation and why this method deviates from the Bayes’ equilibrium concept.
6. M-Period Games

In this chapter I show that the 2-period game is the most important part of the skeleton of every bigger game. Since the 2-period game is fully described and is the framework of all bigger games, I will point out how games are extended and how equilibria are found. When extending to a 3-Period game (figure 11), what actually happens is that the binomial tree of the 2-Period games are merged into one binomial tree of the 3-Period game, where the decisions in the nodes of the 2-Period games have to be analyzed separately and together make up for the set of decisions of the bigger games which results in investment equilibria.

![Figure 11: Binomial representation of the 3-Period game containing all essential formulas](image)

When extending the games to an M-Period game, basically the binomial tree of the game consists of a chain of 2-Period binomial trees that all have decisions in every node that leads to an investment payoff, an NPV for the next period based on the two subsequent nodes and possible option values. Every 2-period binomial tree is a part of a 3-Period binomial tree which again is a part of a 4-Period binomial tree etc.

![Figure 12: every node is part of a 2-Period binomial tree](image)

Essentially every node (except the ones in the last period) are similar to figure 13 where every subsequent node is used for NPV and OV calculations for every subsequent period. So basically, a GBM is an infinite repetition of figure 13.
Even though the games get more nodes and therefore more decision points when a period is added, the extensive form game that is analyzed remains figure 3. Still one message and one action will be chosen, however the set of decisions that a strategy contains is in a 3-Period game represented as \( \{d_1, \{d_{2,1}, d_{2,2}\}, \{d_{3,1}, d_{3,2}, d_{3,3}\}\} \).

In appendix C extra detailed information about the 3-period game is explained along with a numerical example using the same reasoning as chapter 5. All these calculations are done in the simulations of chapter 8.

### 7. Programming in Excel

The simulations for the games are programmed and run in Excel. As explained before, there are two ways of analyzing the games, decision-per-period and decision-per-node. I only simulated the decision-per-node games because of the more dynamic character. I start by creating the 2-period game which is then used to model the 3-Period game. The sequence of programming for this 2-Period game is done as follows.

First I set up all the base parameters needed for the model: the amount of both costs, the actual cost (private information), the \( PV_1 \), binomial multiplication \( u \) and the initial belief of low costs by the entrant. These parameters will return the probabilities of the up and down movement, \( p_u \) and \( p_d \), and the \( PVs \) after period 1. However, from this chapter, the model that I will investigate will be adjusted compared to the described model to be able to simulate the games. The two-period game already changes from a real life situation to a model that results in optimal strategies as described in chapter 2. From now on the model will be modified even further, which I point out with the following comparison between the initially intended model, namely the Bayesian model, and the simulated model.

#### 7.1 Bayesian model (fig 3)

**Step 1:** All public information is given (\( PV, u, d, etc. \) as input for Step 7)

**Step 2:** nature chooses the cost: \( H/L \)
**Step 3:** Incumbent observes the Cost and chooses a message \( m \in M \) by anticipating what the entrant might choose as a rational response to the message, so the incumbent maximizes his payoff in a Bayesian game:

\[
\max_m u'(m, a(m), q_m)
\]

s.t. \( m \in m_1, ..., m_5 \) and \( a \in a_1, ..., a_{10} \) for \( \forall \theta \)

**Step 4:** Entrant observes message and understands the content as being all investment decisions at every node.

**Step 5:** Entrant updates its belief about the message using Bayes’ rule and has rational outcomes for the type-contingent probabilities based on external factors (such as incumbent’s past behavior, seasonal factors etc.)

\[
q(m) = P[L|m] = \frac{pP[m|L]}{pP[m|L] + (1 - p)P[m|H]}
\]

**Step 6:** Entrant chooses action from \( a \in A \) which will maximize his payoff given message \( m \). So he uses the maximization function

\[
\max_a u^E(m, a(m), q)
\]

**Step 7:** The message-action combination results in a Bayes equilibrium with the final payoffs for both companies (bottom of fig. 3)

In the above described model, there is only one component that cannot be simulated using clear defined methods or theories: the type-contingent probabilities. For the model to be able to simulate these parameters as well as doing that in a relevant way, I let these type-contingent probabilities to be in the hands of the incumbent. These updated beliefs are obtained in such a way that the incumbent has the power to influence them and will lead to the incumbent’s optimal outcome. In the numerical example in chapter 5 I manually obtained the ten type-contingent probabilities as an example how the model calculates optimal strategies and outcomes, but when doing analysis for multiple examples and show patterns when i.e. the \( PV \) increases proportionally, it is impossible to keep obtaining these probabilities manually. Another reason why to include the type-contingent messages in the power of the incumbent is that there is no defined formula or method to come up with them automatically, since they depend on the entrant’s knowledge, experience etc. However, the biggest influence on the updated beliefs are of course the incumbent decisions, so by given him the power in the excel model, there is a method obtaining consistent (but by no means consistently realistic) type-contingent messages quickly. Because of this method, the results are consistent and a pattern occur whenever a parameter changes which I will show in chapter 8. Even though my model has adjusted assumptions compared to a Bayesian model, my goal is still to approach the outcomes of a possible Bayesian model; however I have no reference
material to make that claim. The modification of the type-contingent probabilities in the simulation model results in the following sequence of steps.

7.2 Simulation Model

**Step 1:** All public information is given \((PV, u, d, \text{ etc. as input for Step 7})\)

**Step 2:** Nature chooses the cost: \(H/L\)

**Step 3a:** Incumbent observes the cost and chooses a message from the set \(m \in M\) by anticipating what the entrant might choose as a rational response to the message.

**Step 3b:** Incumbent chooses the ten type contingent probabilities \(P[\cdot | \theta]\) by inducing what the entrant might choose as a rational response to the message. So for every message / type-contingent probability combination the incumbent anticipates what the entrant will pick when he thinks rationally and the combination that results in the highest payoff for the incumbent will then be chosen. This results in the maximization function:

\[
\max_{m, x_1^t, \ldots, x_5^t, x_1^h, \ldots, x_5^h} u^t(m, a(m), q_m)
\]

s.t. \(m \in m_1, \ldots, m_5\) and \(a \in a_1, \ldots, a_{10}\)

There are ten \(P[m|\theta]\) which I will describe as \(x_1^t, \ldots, x_5^t, x_1^h, \ldots, x_5^h\). So the maximization function which is in the power of the incumbent does not only include maximization over all his messages, but also over the type contingent probabilities. Even though there are ten variables subject to change, there are twelve constraints in this model. All ten type-contingent probabilities should be smaller or equal to one (as well as positive) and also the addition of the type-contingent messages on every possible path has to be equal to one, 

\[
\sum_{m \in M} P[m|\theta] = 1, \forall \theta.
\]

A disadvantage of this method is that the type-contingent probabilities can be different in case of \(H\) or \(L\) since the incumbent can first observe the cost and then choose his type-contingent probabilities.

**Step 4:** Entrant observes message and understands the content as being all investment decisions at every node

**Step 5:** Entrant updates its belief about the message using Bayes’ rule

\[
q(\cdot) = P[L|\cdot] = \frac{pP[\cdot | L]}{pP[\cdot | L] + (1 - p)P[\cdot | H]}
\]

**Step 6:** Entrant chooses action from \(a \in A\) which will maximize his payoff. So he uses the maximization function
\[
\max_a u^E(m, a(m), q)
\]

**Step 7:**

The message-action combination results in an optimization of the final payoffs using the following formulas:

\[
u^E(m(\theta), a(m(\theta)), q_m) = \sum_{(i,j) \mid (d_{ij} = l) \in a} p_{i,j} \left( PV_{i,j} q^{E}_{i,j}(m, a) - \mathbb{E}[C^0_m] \right)
\]

\[
u^{INC}(m(\theta), a(m(\theta)), q_m) = \sum_{(i,j) \mid (d_{ij} = l) \in em} p_{i,j} \left( PV_{i,j} q^{INC}_{i,j}(m, a) - C^0 \right)
\]

So the maximization of the incumbent and entrant are the following:

\[
\max_{m,x^L_1, \ldots, x^L_5, x^H_1, \ldots, x^H_5} \sum_{(i,j) \mid (d_{ij} = l) \in em} p_{i,j} \left( PV_{i,j} q^{E}_{i,j}(m, a) - C^0 \right)
\]

\[
\max_a \sum_{(i,j) \mid (d_{ij} = l) \in a} p_{i,j} \left( PV_{i,j} q^{E}_{i,j}(m, a) - \mathbb{E}[C^0_m] \right)
\]

Where the expected investment costs as a direct input to the investment decision is given by:

\[
\mathbb{E}[C^0_m] = \left( \frac{px^L_m}{px^L_m + (1-p)x^H_m} \right) C^L + \left( 1 - \frac{px^L_m}{px^L_m + (1-p)x^H_m} \right) C^H
\]

Since the incumbent has the power to influence the type-contingent probabilities directly, the assumptions for the Bayesian equilibrium do not hold (see fig. 3b and 3c). Therefore the concept of the simulation changes from finding a Bayes equilibrium into obtaining an optimal combination of strategies. From this point on, I will refer to the optimal outcome as the optimal strategy tuple which comes down to a “belief intimidation tuple” (since the incumbent dictates the type-contingent probabilities) whereby deviation from an optimal strategy cannot lead to better payoffs.
To summarize, the maximization by the incumbent is done by obtaining type-contingent probabilities in such a way that the incumbent finds it the optimal combination for him. Even though this is not what happens in reality, it at least gives consistent type-contingent probabilities and outcomes.

In the most extreme situation, the entrant believes at a node in a low cost game to have (very) high costs. For example the costs are 20 but the entrant has an expectation about them to be (close to) 40 as in the previous example. In reality this seems a rare case since whenever a node is profitable with high costs; it should also be profitable with low costs for the incumbent you think. However, it can actually happen when the entrant expects that the incumbent will not invest in a node with low costs but only with high costs. That is because investing with low costs can result in a duopoly resulting in a payoff that is lower than investing there at a high costs in a monopoly. For example, $PV_{i,j} = 70$. When low costs, $C^\theta \in \{20,40\}$, than the duopoly payoff is $u^l_{i,j} = 0.5 \times 70 - C^l = 15$ whereas with high costs the monopoly payoff is $u^H_{i,j} = 70 - C^H = 30$. Therefore the entrant can believe that the incumbent will only invest in that particular node when costs are high, when in fact they are low. The incumbent, knowing this reasoning of the entrant can use this and with low costs invest in the node $i,j$ anyway, fooling the entrant in making him believe the costs are high and then obtain the highest possible payoff in that node, namely $u^l_{i,j} = 70 - C^l = 50$. In chapter 8.1 is explain with an example why this method of obtaining type-contingent probabilities leads to the results I would like to find and that this method does not harm the intentions of my research.

Finally, I built payoff matrices that obtain all possible outcomes for every message-action combination. One of these combinations will be the optimal strategy tuple such that no unilateral deviation can lead to a better payoff. As a result of this process the simulation obtains the end result table that contains for both the asymmetric as well as the symmetric (full information) games the payoffs for both firms, the strategies they choose, the resulting optimal strategies, the option values and information value. The difference between the incumbents payoff in the symmetric and asymmetric game is referred to as the Information

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See appendix D for these payoff matrices and how they obtain the optimal messages and actions for the firms.
Value (IV). When the optimal payoff in asymmetric game is higher than in the symmetric game, the IV has a clear and quantitative value that is a result of the information advantage.

The optimization of the incumbent’s payoff is done with the Excel Solver. The objective cell is the incumbent’s payoff, obviously. The values that are subject to change are the ten (in a 2-period game) type-contingent messages $P[m|\theta]$. These ten values are obtained such that the incumbent finds them optimal for an investment decision. So by sending a message, the incumbent influences the expectation of the costs at every node by the entrant, because of the Bayes’ updated beliefs.

The same way of simulating is done in the 3-Period game, however this model becomes, by the addition of just one period, already extremely complex. For example, the number of constraints in the 3-Period Game is doubled to 38. The 3-Period game can be simulated in the same way as in the 2-Period by creating matrices for both the symmetric as well as the asymmetric case. However, in the 2-Period game, there are 5 messages and 10 actions for the incumbent and entrant, resp. In the 3-Period game this number increases severely to 18 messages and 44 actions. Therefore, the binomial mode of the 3-Period game, which essentially is three 2-Period binomial models combined, is the most complex game that can be simulated and optimized by Excel, bigger games could not be handled but also do not add much to the findings of the 2-period and 3-period games.

8. Simulations and Results

This chapter is devoted to the actual simulations of different games. I will test the effects of several important parameters, i.e. the PVs, costs, the PV shares, volatility etc., on the results of the outcomes and find their impact on the optimal strategies. It is important that the results found in these 2-period games can be generalized to bigger games and eventually could be able to approximate a GBM. These results can subsequently be checked by analyzing a 3-period game (which has the exact same algorithm as the 2-period game in chapter 7, only more messages and actions). Games consisting of more than three periods are too complex to model in Excel. Not only the values separately, but also the combination of them strongly influence the outcome of a game, so the results should be analyzed carefully taking into account all the parameters.

The main objective of the simulations, however, is to find the most important aspects of my research: the option value, the information advantage value, their interaction and the resulting equilibria and payoffs. In the beginning I explained that my research is focused on signaling in option games which means that these two aspects determine the evolution of the game and the strategies played. Both these aspects have a quantitative value which I will show with the simulations in this chapter. In general, four different games can be compared: a game with only $OV$; a game with only $IV$; a game with both $OV$ and $IV$; and a game without any of these values, the base case. Obviously, the games containing both the $OV$ and $IV$ are the most interesting, however by checking the base games with only an $OV$ or $IV$, the relation between the two can be explained (for the given parameters). A game without $OV$ means that the firms can only invest in period 1 or not at all; there is no option value in waiting. Or
stated differently, he cannot observe the $OV$ and will invest in the first period whenever the payoff is positive and renounce from investment otherwise. A game without the $IV$ simply is a symmetric game, so both firms have the same knowledge about the market. For every type of game I determine the optimal strategy tuple and compare their outcomes.

First of all I will shortly explain an example that clearly shows what happens in a game when a parameter, and in this scenario the $PV$, changes. The resulting graph in figure 14 clarifies what can be found in simulations and justifies the method I use for obtained type-contingent probabilities as explained in Chapter 5 and 7. Throughout this chapter the payoffs for the entrant are an expectation in the sense that he expects to obtain this payoff due to his expectation of the costs.

8.1 Expectation and Explanation

I already explained how I will come up with type-contingent probabilities in previous chapters, but now I will work out one situation manually to show the method I use to find type-contingent probabilities and how they can be used to conclude a pattern. The parameters for this example and the ones in the next paragraph are all set equal: the costs are $C^\theta = \{20, 40\}$, the actual costs for that simulation being chosen by nature either $C^L = 20$ or $C^H = 40$ which is the incumbent’s private information so that both cases are separately analyzed (that is because these two situation could lead to different type-contingent probabilities), the up-multiplication $u = 1,5$, $r = 1,1$, the difference between $PV$ shares is $\varphi_\Delta = 0,2$, so the shares of $PV$ are divided 60/40 when one period in between investments for the incumbent and entrant, resp., and the initial belief for the costs to be either low or high is 0,5.

To explain what I expect that the simulations will show, I analyze this model assuming that it has a pooling equilibrium for every value of the $PV$s. A PBE in pure strategies is said to be a pooling equilibrium if $m^*(L) = m^*(H)$. This implies that $q^*(m^*(L)) = q^*(m^*(H)) = p$ (prior belief). In this case the incumbent’s message does not transfer information about the project’s investment costs and the entrant predicts low costs with the prior probability $p$ in the equilibrium behavior of the incumbent $m^*(L) = m^*(H)$. So in a pooling equilibrium the actions of the entrant in equilibrium are $a^*(m^*(L)) = a^*(m^*(H))$. In that case the entrant always, independent of the message, expects the costs to be the average of the low and high costs since the prior belief for the costs to be either low or high is 0,5. The expected costs the entrant has for every message is now $E[C_m] = 30$, since his updated beliefs about low costs are now always 0,5. Basically he gains no further information by the incumbent’s message and decides to use this average cost as his threshold in every situation to invest or not. What I expect to happen in this situation is exactly what can be concluded from figure 14 where I depict the firms’ payoffs for high and low costs against the $PV$ in period 1 ($x$-axis stands for $PV_1$ and $y$-axis for payoff in this chapter).
The payoff lines for the incumbent both show three drops followed by a proportional increase whereas the expected payoff for the entrant seems to increase faster than usual at the same $PV$ values. These patterns can be explained fairly simple. The incumbent obviously is monopolist in the relative low $PV$ area because it is not profitable enough for the entrant to follow the investment. However, at some point the entrant observes that the highest $PV$ in the game, $PV_{2.1}$, reaches a certain level so that investment becomes profitable enough for him to invest. Since I assumed a 60/40 market share and $u = 1.5$, then this point is reached when $PV_{2.1}$ exceeds 75 ($75 \times 0.4 - 30 = 0$), so when $PV_1 > 50$. From that point the entrant will change his strategy into one where investment is only done in node 2.1: action $a_7$ (assuming the incumbent invests in period 1). The strategies stay constant until the second highest $PV$ value, $PV_1$, also reached a certain value. When $PV_1$ exceeds 60, node 2.1 also becomes profitable as an investment for the entrant. However, at which $PV_1$ value does the entrant exactly changes his strategy? That is when investing in the first period becomes more profitable than investing according to his current action $a_7$. That is, he deviates from $a_7$ to $a_1$ when:

$$\varphi^E_1(m,a) \times PV_1 - 30 > p_u (\varphi^E_{2.1}(m,a) \times PV_{2.1} - 30)/r$$

Since $PV_{2.1} = 1.5PV_1$, $\varphi^E_1(m,a) = 0.5$ and $\varphi^E_{2.1}(m,a) = 0.4$, the entrant changes his strategy when:

$$PV_1 > \left( -\frac{30p_u}{r} + 30 \right) \div \left( 0.5 - \left( \frac{0.6p_u}{r} \right) \right)$$

And $r = 1.1$ and $p_u = 0.52$, so when $PV_1 > 73.1$. Since the drops in the figure are supposed to be vertical, the drop occurs exactly at the calculated $PV_1$, between $PV_1 = 70$ and $PV_1 = 80$. At that exact point the entrant deviates to $a_1$, but the incumbent anticipates on that and

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7 Actually, the drops in this and all further graphs should be exactly vertical. The reason is that at some threshold value for the PV the competition becomes profitable so that the entrant can join and make a profitable investment. The reason the drops in these graphs are not vertical in this chapter is that it is very time consuming to run a simulation for every .1 increase in PV, so I tested increases in steps of 10.
sends message \( m_2 = \{N, \{I, I\}\} \) and the entrant, in turn, immediately responds with \( a_3 = \{; \{I, N\}\} \). This manual example shows that the firms at any point are maximizing their payoff and always choose the optimal strategy given the other firm’s strategy (also derived from the simulation).

The last drop occurs when even the lowest node reaches a value that results in a profitable investment for the entrant. It is simple to see that the entrant, still playing \( a_3 \) (only invest in node 2.1) will play \( a_2 \) (investing in node 2.1 and 2.2) when \( PV_{2.2} \) exceeds 30/0.5 = 60, again (they share the market 50/50 because the incumbent plays message \( m_2 \) which is investing in the second period). This equals at \( PV_1 \) equal to 90 and also this point in the figure is clearly depicted.

This example of the pooling situation shows that the three drops simply correspond to the fact that the incumbent loses his monopoly position one node at a time as a result of an increasing \( PV \), until they will always share the market and the payoff will increase proportionally from there to infinity. For the given parameters, when:

- \( PV_1 \leq 50 \): the incumbent is monopolist in the entire game; at every node
- \( 50 < PV_1 \leq 73,1 \): the incumbent is monopolist in nodes 1 and 2.2
- \( 73,1 < PV_1 \leq 90 \): the incumbent only has a monopolist position in node 2.2
- \( PV_1 > 90 \): both firms will share the market in every node as in a duopoly

Even though this pooling example is very straightforward and probably not very realistic to happen in a lot of situations, this pattern of drops and increases is now shown and will also occur in the situations where every message has a different meaning for high and low costs, but it could sometimes be less clear to recognize them.

8.2 Effects of OV and IV

The games I test in this paragraph only vary in their ability to have either or both \( IV \) and \( OV \). The parameters for the four analyses are all set equal as in the previous paragraph: the costs are \( C^H = \{20,40\} \), with the actual costs chosen by nature being \( C^L = 20 \) to compare payoffs (could also be done with high costs) for all different types of games, \( u = 1,5, r = 1,1 \), the share of \( PV \) is 60/40 for the incumbent and entrant, resp., and the initial belief for the costs to be either low or high is 0,5.

Because the costs and \( PV \) in period 1 as absolute values are not very significant, I use, when no confusion can occur, the \( PVC \) (-ratio) which represents the ratio of the \( PV \) in period 1 to the low costs. That way, the results can be analyzed and compared in a more relevant way. To make the graphs similar and comparable, for the remaining chapter, the \( x \)-axis represents the \( PVC \) and the \( y \)-axis the payoff.

8.2.1 Base case

In the base case, the game has no \( OV \) or \( IV \). It is a symmetric game, both firms have the same information, and it is not possible to look into the future and calculate the option value of waiting for an investment. This results in a very basic graph of both firms’ payoffs:
The payoffs increase proportional to the increase of the $PV$ except for very low $PVC$s since only there a monopoly is the most profitable competition for both. The firms are equal in every aspect except that the incumbent can invest first, so has a first-mover advantage. Therefore the firms have equal payoffs in every $PVC$ that has payoffs high enough for a duopoly competition.

8.2.2 Base Case + IV

Now information asymmetry is added to the previous game, making signaling possible. There is still no $OV$ and investment can therefore only be done in period $i$ or not at all since they lack the ability to look into the future and forecast the profitability of later investments. The $IV$ has limited power in case of low costs and no power in case of high costs. When costs are low the incumbent can try to send his message to imitate high costs and be monopolist until $PV_1 = 80\ (PVC = 4)$, since he can choose the type-contingent probabilities and therefore make the entrant belief high costs (this is the most extreme case and by no means true, however it shows how the incumbent can use his power in this one-period game). Above this amount, independent of the messages, every $PV_1$ guarantees that the entrant can make a profitable investment independent of his expectation of the costs. In the symmetric case with low costs the incumbent will be the monopolist until $PV_1 = 40$ because a higher $PV_1$ allows for the entrant to enter and make a profit for the same reasons as before. The entrant observes that the costs are low and will follow. In figure 16 it is obvious that the incumbent’s payoff does not increase compared to the Base Case for the middle and high PVCs. Only at relatively low PVCs the incumbent can fool the entrant about the costs and be a monopolist. At higher ratios the payoffs of the incumbent do not deviate from the base case. The entrant’s expected payoffs however appear to be lower at every point since he is informed that the costs are high when in fact they are low. It actually does not matter at that point anymore what the incumbent tries to make the entrant believe, he will follow anyway and therefore

This is the highest PV that does not 100% guarantee a profitable investment of the entrant, in case he expects the costs to be high. I explained in chapter 3 and 5 that these are just the most extreme and highly unlikely scenarios, but are still in line with patterns that occur in the strategies the firms choose. This is also the reason why in the graph the difference of the payoffs lines is exactly 20, which means the entrant expects the costs to be 40 and the incumbent knows they are 20. Therefore, in reality, the payoff lines should be closer together and the incumbent’s drop should occur sooner than at $PVC = 4$. The entrant could not have a rational belief that the costs are that high. In this case it should be more around 30 and in the games with also OV it could be almost 40 at some nodes, certainly not all. In order to succeed in those results, a highly applicable method to assign the most realistic values to the type-contingent probabilities is needed. In this research, for the comparability with the other models, I hold on to this method.
from PVC = 4 the payoff lines could also be exactly equal, since it does not influence the investment decisions anymore.

![Figure 16: payoffs vs. PVC-ratio in base case + IV](image)

In general, a game which consists of information asymmetry but no OV has limited power if the costs are low and even no power if the costs are high.

### 8.2.3 Base Case + OV

Again, the Base Case (one-period) is assumed but now only OV is incorporated in the model. This means that the game is symmetric again and there is no possibility of signaling. However, now the firms can look into the future one period and observe that the PV can evolve into a high amount or low amount, with a certain probability, thereby influencing the investment decision in period 1.

![Figure 17: payoffs in base case + OV for low costs](image)

In the graph, for low costs it can be seen that only for low PVCs, the OV has a very limited power. The reason is that for those values, there are nodes that only allow for a monopoly to be profitable and both firms have the same information so only the first-mover advantage occurs now in favor of the incumbent. When PVC exceeds 3 than at every node a duopoly is profitable for both firms and their payoffs will be equal and are all 1,8181 higher than the payoffs in the base case. This extra amount comes from the addition of the extra period that gives an OV equal to 1,8181 because the game has converged to an duopoly and only the PVs change proportionally and therefore the OV quickly converged to a constant, since they will both wait until period 2 to invest in this low cost scenario.
When looking at the high-cost variant in figure 18 the incumbent’s payoff has the noticeable shape with three clear drops now.

![Figure 18: payoffs in base case + OV for high costs](image)

These drops also occur in the low cost graph but are hard to notice since the payoff lines become equal very quickly, so the drops occur very close to each other, and a simulation is run for every PV increase of 10 which could discard a drop in between these intervals. The reason for these drops lies in the amount of nodes in the game. Because there are three nodes with different PVs, one node at a time becomes profitable for the entrant when the PV increases, so the incumbent loses his monopoly position one node at a time, corresponding to the drops in his payoff until both are in a duopoly in the whole game.

Like the previous game, the Base Case with only OV also has very limited power in scenarios where the costs are low and mediocre power when the costs are high. However, in the final and most important game described in the next paragraph, it can be seen that this OV in combination with an IV results in much more power for the incumbent than either value alone, which is shown in the next section.

### 8.2.4 IV and OV: Complete Game

The games with both the OV and IV incorporated are the most important games and the purpose of this research. The combination of the OV and IV result in a graph that shows a payoff line for the incumbent that has three clear drops followed by an increase and a payoff line for the entrant that has steeper increases simultaneous with these drops.

![Figure 19: payoffs in complete game for low costs example](image)

The spread between the payoffs of the firms is wide until a high PVC from where both firms will always be in a duopoly, since they will both invest in the game, which does not have to
be in the same period. It is obvious that the addition of both the \( IV \) and \( OV \) instead of just either one leads to totally different paths of the payoff graphs, when the parameters are kept constant throughout all simulations. This graph and the following paragraph prove the power of the combination of both information asymmetry and the option value of waiting.

8.2.5 Overview of the four games
When putting the incumbent’s payoffs together for all four games in one graph for low costs (because the incumbent’s payoff is the objective function), the conclusion is fairly simple. Only in a very rare case does the game with only \( IV \) has the highest payoff for the incumbent, but this can be explained simply by the fact that no \( OV \) is present and the entrant cannot wait to reach a the up-node in period 2 with a higher \( PV \) that allows for a duopoly to be profitable. The incumbent is monopolist in the \( IV \) game for \( PVC \) slightly higher than in the complete game, since in the second period there already appears a node that allows for a duopoly for \( PVC \)s between 3 and 4,5.

![Figure 20: incumbent’s payoffs in all four games compared for low costs](image)

So the beauty of this graph lies in the combination of three lines that are straight from \( PVC = 4,5 \) results in a payoff line with drops and increases that are not found in the other three lines. This shows that the combination of the \( IV \) and \( OV \) is in some sense ‘superadditive’. I will explain in the next section how these drops occur out of the combination of straight lines.

8.3 Effects of other parameters
Next to the analysis of the four different appearances a game can take, specific parameters also have a significant effect on the firms’ strategies and payoffs. I will show the effects on
the equilibrium payoffs when the spread between the PV shares increases, the volatility of the option increases and the initial belief about the low cost changes.

8.3.1 PV share

The effect of the PV share of the firms is very interesting. The increase of the PV share of the incumbent does not necessarily mean a decrease of the entrant’s payoff. In fact, in some cases, the expected payoff of the entrant even increases as a result of the decrease of his PV share and the cause of that lies in the specific strategy the incumbent choses to make the entrant have a belief about the costs. Below are two very simple graphs for the incumbent and entrant’s payoffs in three PV₁ scenarios.

![Figure 21a and 21b: For three PV₁ scenarios, the effect of the PV share on inc and ent payoffs, resp. for low costs](image)

In figure 21a it can be observed that on the left part the payoff of the incumbent increases if his PV share increases, simultaneously for the entrant the payoff along with his PV share decrease. This all seems obvious. However, for PV₁ = 90 and PV₁ = 110 the payoff of the entrant increase at a certain PV share where you would expect them to fall even further. The reason lies in the updated beliefs of the entrant. For example, the PV₁ = 90 case, when the PV share is 70/30, instead of trying to make the entrant believe the costs are high, i.e. 40, the incumbent chooses his signals in such a way that the entrant believes the costs are very low, around 25 to motivate an investment. This way the entrant will invest in node 2.1 believing he obtains 6.31 which is more than his expected profit when he would invest in period 1 (half the PV minus 40 which is 5). In fact the entrant would obtain much more in reality when investing in period 1, so he is fooled to invest a period later than the incumbent to optimize the incumbent’s payoff since he gets a much bigger PV share. Therefore, when the PV share changes to 80/20 the entrant expects to obtain more instead of less, because the PV share for him is now in any case too small to invest in node 2.1 when the incumbent invests in period 1 and from now both will invest in period 2, and therefore they will share the market equally 50/50 at all times. The same reasoning can also be observed for the PV₁ = 110 situation, but here the ‘fooling’ already happens at the PV shares 60/40.

This example shows that instead of making the entrant belief the costs to be higher than reality to demotivate investment resulting in a monopoly position for the incumbent, also the opposite occurs. The entrant is now let to believe the costs are low in a certain node, to motivate him to invest there, where in fact the entrant would have obtained more when investing in another node, but now the incumbent fully uses his advantage of having the much bigger PV share and therefore tries to make the entrant invest as late as possible. These
simulations therefore prove that the strategic character of the message-sending by the incumbent goes both ways. It can fool the entrant by trying to make him believe the costs are higher than then reality, or in other cases the exact opposite.

8.3.2 Volatility

The up and down movement, as explained in chapter 3, depend for the most part on the volatility $\sigma$: $u = e^{\sigma \sqrt{t/n}}$ and $d = e^{-\sigma \sqrt{t/n}}$. The parameters $t$ and $n$ determine the length of the interval between two periods. Increasing the volatility results in an increase of the up-multiplication and a decrease of the down-multiplication; hence an increasing $PV_{2.1}$ and decreasing $PV_{2.2}$. At low $PV_1$, the increasing volatility leads to increasing $OV$s and payoffs for the incumbent, and at high $PV_1$ it has no $OV$. Since $p_d = (r - d)/(u - d)$ these probabilities also change by changing the volatility. Testing the effects of the volatility alone does not prove anything significant. Only in combination with different $PV_1$ values a pattern can be seen.

After the simulations which resulted in figure 22 it can be concluded that the influence of the volatility is not equal for every $PV_1$. When the $PV_1$ is very small (smaller or equal than the costs), then an increasing volatility (let’s say until $\sigma = 2$)\(^9\) results in a higher $PV_{2.1}$; therefore

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\(^9\) The reason I tested these simulations until $\sigma = 2$ is that for further increasing volatility the payoff line approaches an asymptotic value. I will prove this mathematically and show it in a graph as well.

When $\sigma \to \infty$, then $u \to \infty$ and $d \to 0$. Resulting in $p_d = (r - d)/(u - d) \to 0$ and $p_d \to 1$. It is obvious that for high volatility investing in node 2.2 is never profitable. So only node 1 and 2.1 can be used for investment. $PV_1$ never changes, only $PV_{2.2}$. What is the total expected profit that can be obtained in node 2.1 by the entrant when volatility increases?

$$u^E = p_u (q^F * u * PV_1 - E[C_{2.1}^F]) + 1 - d/r = u * \left( q^F * PV_1 - E[C_{2.1}^F]/u \right) = u - \frac{ud}{u-d} \left( q^F * PV_1 \right)$$

When $\sigma \to \infty$, therefore $u \to \infty$ then $\frac{ud}{u-d} \to \frac{u-1/r}{u-d} \to \frac{u-1/r}{\infty} \equiv 1$, and $E[C_{2.1}^F]/u \equiv 0$.

For $\sigma \to \infty$, $u^E = q^F * PV_1$ (and $u^I = q^I * PV_1$ when not investing in node 1). In case $PV_1 = 60$ and $q^F = 0.4$ then $u^E = 24$. The incumbent obtained $60 * 1.0 - 20 = 40$ ($I,0$ is his PV share because $p_d \to 1$) in node 1 since the PV doesn’t change there, so therefore the equilibrium payoff will converge to $u^E = \{40,24\}$ when the incumbent observes that costs are low. The same analysis can also be done for the high cost variant.
an increasing $OV$ and subsequently a higher payoff for the incumbent. When $PV_1$ gets higher then the games loses his $OV$ because with increasing the volatility, the $PV_{2,2}$ quickly becomes unprofitable, making the expected payoff of the second period lower than the certain payoff in period 1, thereby deleting the $OV$ which is represented by the payoff lines in figure 22 that do not increase at all. Investment will at all times be done in period 1 in these scenarios.

### 8.3.3 Initial Belief

The initial belief of the entrant about the investment costs being low was assumed to be equal to the probability that the costs are high. These initial beliefs are a direct input for the calculations of the posterior beliefs that are updated in a Bayesian way, which results in the expected cost for the entrant. So by changing the initial beliefs about the costs, and keeping the rest of the parameters constant, the posterior beliefs and therefore the expected costs change also. However, when the initial belief changes, the messages also change in such a way that the incumbent still tries to maximize his payoff what is within his power. So when the initial beliefs change slightly to, for example 60/40 or 40/60 instead of 50/50, for low and high costs, this does not influence the optimal strategies and its payoffs significantly, because the entrant still has a very high uncertainty about the type. Of course, when the initial beliefs change in the direction of 90/10 or 99/1, it is virtually impossible to fool the entrant about the cost anymore and the game tends to go to a symmetric one, where both firms have the same information about the market and $E[C^\theta_m] \rightarrow C^\theta$.

### 8.4 3-Period game

A 3-Period game consists of the 2-period binomial trees as mentioned in chapter 6. A message for a 3-period game has the same form as the 2-period game but with an extra dimension, the interaction between the 2-period games. In the first node a decision will be made to either invest or not. Then in both second period nodes, a message can be sent as if being in a 2-period game, so a message or action will, for example, have the form (appendix B):

$$\{N, \{I, I\}, \{N, \{I, N\}\}\}$$

And contain the 2-period messages: $\{N, \{I, I\}\}$ and $\{N, \{I, N\}\}$ as well as the first-period decision $\{N\}$. The 3-period game can be very useful to check the results and their
generalizability of the 2-period games. The same algorithms is used for every game independent of the amount of periods. The game still has the form of figure 3, however the strategies consist of more decisions. The formulas that calculate the payoffs therefore will also be the same since it is a summation over every node that is a (possible) investment.

This graph proves that the patterns and results of the two-period game are consistent. This game has six nodes and in line with the expectations, this graph shows six drops followed by a proportional increase, representing being a monopolist in all six nodes until being no monopolist at all and sharing the PV at every investment. It seems odd that in the middle three drops occur relatively close to each other. The reason is that node 1, 2.1 and 3.2 all reach a profitable PV for the entrant within a small range, which is also logical since $PV_1 = PV_{3.2}$. Their only difference is the PV share these nodes could obtain for both. In the end of the graph again two drops occur relatively close to each other which are the last two nodes that become profitable in a duopoly, node 2.2 and of course at last 3.3.

This 3-period game can be used to show a game that is much more complex than the 2-period game and therefore a step closer to a realistic representation and approximation of a GBM. The downside of simulating this 3-period game is that every simulation, so for every separate set of parameters, it takes between 30 and 60 second to solve and to imitate the analysis done in the previous paragraphs for the 2-period game would take extremely long and would therefore be a challenge for further research where also even bigger games should be created and analyzed.


9. Conclusions

In this paper, I explained the whole process of building basic models and add information asymmetry to it in the first place followed by the possibility of option values. This research has been done a couple of times in the recent past, however no paper managed to come up with a method of finding optimal strategies in these so called option games with signaling aspects. Therefore I tried to handle this problem by investigating the most important building blocks that create the GBMs used in option calculations, by approximating it by a binomial lattice model. This game consisting of only two periods with just two possible outcomes in the second-period is analyzed in detail. The results of the simulations show two aspects of this model. First of all the effect of information asymmetry, option value of waiting and their interaction; and second the effects of the parameters on the outcomes of the investment equilibria.

It is shown by the 2-period game that a competition without information asymmetry and no option value of waiting simply shows that the firms are identical in their payoffs. There is no difference between their knowledge, only the incumbent has a first-mover advantage, and they will, from a certain PVC-ratio on share the market equally. When the same competition faces information asymmetry about the investment costs, the incumbent has the information advantage and will send a message containing his strategy or timing of investment to the entrant. The entrant will then update his beliefs according to Bayes’ Rule about the costs and bases his investment decision on his expectation about the costs and therefore his expectation about the profit of an investment. In general, when this information asymmetry occurs in the game, the incumbent has the power to, in some cases (even in this one-period example), fool the entrant about the actual costs of investment. However, since there is still no option value of waiting in the game, there is still only one point on the horizon for both firms to make an investment decision on so still the incumbent has very limited power with his information advantage. When the game is symmetric in information, but has the option value of waiting and observe which two outcomes can be chosen by nature in the next period, the payoffs slightly increase, but for both firms. Since they still have the same information, no firm can ‘outplay’ the other by sending messages so both will invest in the second period, which will increase their payoff slightly. Therefore a game containing more periods but without information asymmetry still has little extra to obtain by the firms. When we combined the option value and asymmetry in information, the payoffs of the firms deviate completely from any of the three situations analyzed before. Now the incumbent has the advantage of the information and can also decide when to invest. Even though the entrant has no information advantage, also he can decide to wait with his investment and therefore impose a sort of threat to the incumbent.

The combination of both aspects investigated in this research show that the game gets a totally new dimension which leads to a more dynamic character. It can be concluded that the combination of option value and information value are in some sense superadditive since both of them individually result in little or no increase in the payoffs. There is an interaction between the firms and both chose their strategy in a way they think will be optimal for their payoff. The simulations show that the decisions are consistent throughout all simulations and
profit-maximizing for both firms even though the entrant has no certainty about the investment costs. The results of the simulations show that signaling in real option investment games can result in an optimal strategy tuple and that, by increasing the size of the games, the stochastic character can be approximated. Even though I am using an unusual approach of finding type-contingent probabilities, the results showed at least a clear pattern in how payoffs could behave in an option game with signaling aspects.

In addition to the main goal of the simulations and research, I also investigated the effect of the parameters separately by varying the market share they get, the volatility in the market and the initial belief the firms have about the occurrence of both costs. The results of these simulations show in the first place that varying only one parameter is not enough to draw conclusions about the effects of the parameter on the optimal strategies. Increasing the spread between the $PV$ shares in favor of the incumbent intuitively would result in a decrease of the entrant’s expected payoff but surprisingly also could increase the entrant’s expectation of his payoff, which depends on the $PV$ in period 1. This is the most noticeable conclusion that can be drawn from the effects of the $PV$ share on the optimal strategies and payoffs, since intuitively we would expect the incumbent to pretend his type to be high cost to demotivate investment of the entrant and be the monopolist, but in these remarkable cases, pretending that the costs are low, he can lure the entrant into investing in later periods in case the $PV$ shares are very unequal, thereby obtaining the biggest market share and as a result a higher payoff.

The effects of the volatility on the optimal strategy tuple are also not unilateral. Increasing the volatility does lead to an increasing $PV$ in the up-node and a decreasing $PV$ in the down-node, but also a decreasing $p_u$ and increasing $p_d$. The consequence of the volatility increase is that the payoffs of both will eventually approach a limit. That limit fully depends on the $PV_1$ as also do the strategies chosen by the firm. When $PV_1$ is very low, smaller or equal than low cost, than an increasing volatility results in an increasing $PV_{2,1}$ hence an increasing payoff for the incumbent and from a certain value of the volatility, also an increasing payoff for the entrant. For higher values of $PV_1$ this effect does not occur since it is already profitable for one or both firms to invest in period 1 and the incumbent will therefore always invest there to avoid the very high insecurity about the future $PV$s. The entrant will do that also unless it is not profitable for him and then he will invest in $PV_{2,1}$ where the higher the $PV$ value, the lower the probability of occurrence (as explained in Appendix A).

The last parameter tested is the initial belief about the costs by the entrant. The bigger the gap between the initial probabilities of low and high costs, the more certain the entrant can say he knows what the costs will be. However, he still needs to update his beliefs so the incumbent can still, by sending the ideal message, influence the updated belief of the entrant. This power, however, diminishes when the difference between the initial probabilities increases and as a result the faster the entrant will follow an investment by the incumbent when profitable, since eventually increases the difference between the probabilities will result in a symmetric game with full knowledge.

The overall conclusion is that signaling in option games has proved to add a lot of value for the firm having the information advantage and gives him a power, to some extent, to manipulate the market with his strategies. To obtain the highest advantage, there are a lot of parameters that need to be taken into account before the incumbent will choose his signal
to make sure that his message is the profit-maximizing one for him conditional on the entrant’s response.

10. Limitations and Further Research

In this research I was able to investigate signaling in option games of the simplest form. By setting up the parameters and changing them slightly, clear patterns and results occurred in the behavior of both firms. This method shows the approximation of the GBM in the smallest form which is also shown by analyzing a 3-period game. When this process of adding periods can be continued, the bigger games could show the behavior of a stochastic model and it can be proven that a GBM is modeled where signaling is incorporated in option games. However, when modeling and simulating these 2-period and 3-period games, I noticed that by just adding an extra period, the game’s complexity increases severely. The time it takes for the Excel Solver to optimize a 3-period game increases on average tenfold compared to the 2-period game. And even in the simplest form, the Excel Solver is unable to optimize a 4-period game. Therefore, to increase the number of periods in the game and make the games increasingly realistic, other software should be used that is both more powerful and able to handle a lot of input variables and constraints. The method of programming can be done in such a way that adding a period to the game can be done simply by adding an amount of 2-period games so the result is a big game, instead of the models in Excel that need to be built from scratch for every addition of a period. Instead of writing the programming language in every cell for every specific calculation, the models can be written out in the syntax so that periods can be added more easily in the formulas.

As was mentioned several times before, the only aspect that could be a weak presumption in this research is the fact that I cannot simulate the model to give Bayesian equilibrium outcomes, since I violate that concept due to the fact that the incumbent can influence the type-contingent probabilities directly. However, at this moment there is no clearly defined method that results in the determination of the type-contingent probabilities. The values that the entrant attaches to the probabilities of investment at every separate node are dependent on specific information the entrant has, which could be exogenously given. The incumbent could have behaved constantly in a certain fashion in the past, the entrant could have his own private information or external factors having an effect on both firms could all directly influence the type-contingent probabilities. Since there is no straightforward method to determine these values, the results in this research could still be improved. For future research it would be useful to come up with a theory or method to obtain the type-contingent probabilities in a way that they are (more) in line with the theory of finding Bayesian equilibria.

Another addition to this research is investigating markets other than duopolies. I assumed a market with one incumbent and one entrant, but as usually occurs in a market more firms are triggered to enter a market when it proves to be profitable. Look for example at one of the most recently evolved markets, namely the tablet market. Apple was the first to bring the tablets to the regular customer and now a handful of firms are creating their own
tablet and compete with Apple for a market share. However, Apple still behaves as the incumbent since they, until now, most of the time come up first with new additions and features for the tablet market. Therefore it can be useful to analyze oligopoly games with i.e. one incumbent and two entrants. The entrants can then decide to enter the market sequentially or simultaneously depending on their own information about the market.

In this research I assumed the private information to contain the project’s investment costs (as in Grenadier and Malenko (2011)). This seemed to me a useful assumption since these costs can directly be used for the calculations of the payoffs of the firms and in the end the objective of the firms is to maximize their profit. The value of the private information would be more difficult to calculate if the private information consisted of a more qualitative parameters, since it would have a less obvious and direct influence on the calculations. However, it might be useful to investigate games with asymmetric information about parameters other than costs, i.e. demand (Watanabe (2010)), lifetime of investment or any other main aspect of the market that influences the decisions taken by the firms prior to the game. Even though some of the parameters as private information are much harder to attach an information value to, it is highly realistic that firms act on private information that consists of more qualitative data instead of quantitative.

To check if the results of these small and abstract games approach scientific models, it might be useful to find markets with a limited amount of firms that compete for a market share and behave on their information advantage. The data of these competitions could for example show that firms in the information advantage position are obtaining more profits than their equal sized followers, hence using their information advantage strategically (which could contain information about some costs, demand, competitor’s goals or overall economic future). In that case it can be concluded that the abstract games are a good representation of the stochastic processing in real competitions.
References


Slikker, M. (2010). Game Theory with applications to supply chain management. *Lecture paper*


Watanabe, T. (2010). Real options and Signaling in Strategic Investment Games. *Draft, Ver. 1.7*

Weeds, H. (2002) Real Options and Game Theory: When should Real Options Valuation be applied? *Preliminary draft*

Wu and Xuan (2005). Optimal Timing of Firms’ R&D Investment Under Asymmetry Duopoly: A Real Options and Game-Theoretic Approach. *Algorithmic Applications in Management*
Appendix A: Binomial Approximation of a GBM

Since a Brownian Motion (BM) can take on negative values, using it directly for modeling stock prices is questionable. There are other reasons too why a BM is not appropriate for modeling stock prices. Instead, I use a non-negative variation of BM called geometric Brownian motion, $S(t)$, which is defined $S(t) = S_0 e^{X(t)}$, where $X(t) = \sigma B(t) + \mu t$ is BM with drift and $S(0) = S_0 > 0$ is the initial value (Sigman, 2006).

The binomial lattice model (BLM) is in fact an approximation to the geometric Brownian motion. For a BLM, $S_n = S_0 Y_1 Y_2 \cdots Y_n$, $n \geq 0$ where the $Y_i$ are independent and identically r.v.s distributed as $P(Y = u) = p(up)$, $P(Y = d) = 1 - p = p(down)$. Besides the initial value $S_0 (= PV_1)$, the parameters $0 < d < r < u$, and $0 < p < 1$ completely determine this model. The objective here is to estimate what these parameters should be in order for this BLM to nicely approximate GBM over a given time interval $(0, t]$.

In a GBM: $S(t) = S_0 L_1 L_2 \cdots L_n$ Where the ratios $L_i \equiv \frac{S(t_i)}{S(t_{i-1})}$, for $0 = t_0 < t_1 < \cdots t_n = t$, $1 \leq i \leq n$, are independent lognormal r.v.s. which reflects the fact that it is the percentage of changes of the stock price that are independent, not the actual changes $S(t_i) - S(t_{i-1})$

For any fixed $t$ we can re-write $S(t)$ as a similar i.i.d product to the $S(t)$ of the GBM, by dividing the interval $(0, t]$ into $n$ equally sized subintervals $(0, t/n)$, $(t/n, 2t/n)$, $\ldots$, $(n-1)t/n, t]$, defining $t_i = it/n$, $0 \leq i \leq n$ and defining $L_i = S(t_i)/S(t_{i-1})$. Each $\ln(L_i)$ has a normal distribution with mean $\mu t/n$ and variance $\sigma^2 t/n$. Thus we can approximate GBM over the fixed time interval $(0, t]$ by the BLM if we approximate the lognormal $L_i$ by the simple $Y_i$. To do so the mean and variance have to be matched so as to produce appropriate values for $u$, $d$ and $p$:

Find $u$, $d$ and $p$ such that $E(Y) = E(L)$ and $Var(Y) = Var(L)$. This is equivalent to matching the first two moments; $E(Y) = E(L)$ and $E(Y^2) = E(L^2)$.

Noting that $E(Y) = pu + (1 - p)d$ and $E(Y^2) = pu^2 + (1 - p)d^2$, and $E(L) = e^{\mu t/n + \sigma^2 t/2n}$ (from computing moments for GBM), we must solve the following two equations for $u$, $d$ and $p$:

\[
pu + (1 - p)d = e^{\mu t/n + \sigma^2 t/2n}
\]
\[
pu^2 + (1 - p)d^2 = e^{2\mu t/n + 2\sigma^2 t/2n}
\]

Since we have only two equations, there is no unique solution; we have one degree of freedom in the sense that we can apriori force one variable to take on a certain value ($p = 0.5$ for example), and then solve for the other two. The most common relationship to force is $ud = 1$, which says that $u = 1/d$, and has the effect of making the stock price in the BLM have the nice property that an up followed by a down (or vice versa) leaves the price alone: $udS_0 = duS_0 = S_0$. This is assumed. Then, letting $\omega = \mu + \sigma^2/2$, we can re-write the equations as:
\[
\begin{align*}
u d &= 1 \\
p u + (1-p)d &= e^{\omega(t/n)} \\
p u^2 + (1-p)d^2 &= e^{(2\omega+\sigma^2)(t/n)}
\end{align*}
\]

The middle of the three formulas leads to the following: \(p = \frac{e^{\omega(t/n)} - d}{u - d}\), then using this formula for \(p\) together with \(ud = 1\) to plug into the third formula allows us to solve for \(u\) (and hence \(d\))

\[
u = \frac{1}{2} \left( e^{-\omega \left( \frac{t}{n} \right)} + e^{(\omega+\sigma^2) \left( \frac{t}{n} \right)} \right) + \frac{1}{2} \sqrt{\left( e^{-\omega \left( \frac{t}{n} \right)} + e^{(\omega+\sigma^2) \left( \frac{t}{n} \right)} \right)^2 - 4}
\]

When \(n\) is large, so that \(t/n\) is small (say zero), the solution can be approximated by the more simple

\[
u = e^{\sigma \sqrt{t/n}} \quad d = e^{-\sigma \sqrt{t/n}}
\]

(in the sense that the ratio of the two formulas for \(u\) tends to one as \(n \to \infty\)). This is nice because this formula does not depend upon knowing the true value of \(\mu\); only \(\sigma\). Thus when using the BLM to price an option, we only need to estimate \(\sigma\) for the stock in question (via looking at past data) in order to get the necessary parameters (recall that the risk-neutral probability, \(p^* = (r - d)/(u - d)\)).
Appendix B: Decision-per-Period game

This method of analyzing a game is by making investment decision purely based on periods. Independent of which nodes in a period are reached in the binomial path, the firm makes his investment decision based on a period’s expected payoff. The timing of an investment is made according to the expected payoff calculated by the possible payoffs of all the period’s nodes. So which node he will find himself in does not influence his decision prior to the game. Therefore, the strategies (messages and actions), in a two-period game, have the form \( \{d_1, d_2\} \). In both periods a binary decision \( d_i \in \{I,N\} \) is made.

Whether one or two firms make a decision in a period depends on the incumbent’s strategy. Considering a two-period game, the incumbent has three different messages it can send. Every message \( m \in M \) contains two decisions, one for every period in the game. The entrant has the same strategies, as the incumbent, \( a \in A \), however every strategy can appear in two variants, one in which the entrant has the ability to make a decision in the first period since the incumbent made an investment in the first period; and one where the entrant has no decision ability in the first period, since the incumbent decided to not invest in the first period and wait until the second period. These inabilities to make a decision are all denoted by a dot. This leads to the following sets of messages and actions for the two firms:

\[
M = \{\{I,\}, \{N, I\}, \{N, N\}\}
\]

\[
A = \{\{I,\}, \{\cdot, I\}, \{N, I\}, \{\cdot, N\}, \{N, N\}, \{\cdot, \}\}\}
\]

As a response to a message, the entrant can choose any action. However, it is irrational to respond with \( a = \{I,\cdot\} \) to the message \( m = \{N, I\} \), because the entrant knows he cannot invest before the incumbent. So even though every combination of messages and actions can be chosen, some combinations are irrational and will result automatically in a zero payoff for the entrant\(^{10}\).

The set \( M \) has three strategies for the incumbent: invest in the first period, which eliminates the second period decision since a firm can only invest once; not invest in the first but in the second period; and finally no investment at all. As a result, the set of all strategy-pairs \( \sigma = (m, a) \) is:

\[
\sigma \in \{\{I,\}, \{I,\}\}, \ldots, \{\{N, N\}, \{\cdot, \}\}\}
\]

It looks like \( a = \{\cdot, I\} \) and \( a = \{N, I\} \) are the same strategy for the entrant since both have the same investment timing, namely in period 2. However in the first strategy, the entrant never had the opportunity to make an investment decision in period 1. The second strategy means that the entrant had the opportunity in period 1, but decided to wait another

\(^{10}\) \( m_1 = \{I,\}, a \in \{a_1 = \{I,\}, a_3 = \{N, I\}, a_5 = \{N, N\}\} \)

\( m_2 = \{N, I\}, a \in \{a_2 = \{I,\}, a_4 = \{\cdot, N\}\} \)

\( m_3 = \{N, N\}, a_6 = \{\cdot\} \)
period. Hence, the investment timings for the entrant are the same in both strategies; however the payoffs are distinct.

The downside of analyzing a game like this decision-per-period notation is that, even though the two second-period nodes determine the second period’s payoff, when finally arriving in this period, the investment will be made independent of which node the firm finds itself in. This is somewhat conflicting with the real options aspect I want to investigate, since that will advise in most situations to again wait if the expected payoff in a future period is higher (if the game consists of more than two periods). Consider the example in figure 3 which simply determines all the possible payoffs that could be obtained by a firm by investing in that node:

Assuming almost equal probabilities of going up or down, it is obvious that investing in period 2 is the best strategy since the expected payoff is highest there, namely 15. However, when the firm decides to wait in period 1 because of the option value and finds itself in the second period down node, then following his strategy is not optimal because he will obtain 0. The firm can improve his expected payoff by waiting another period, so again he has an option value of waiting, but only in period 2’s down node, which will give him an expected payoff of 2.5 in the third period. Therefore the decision-per-period notation fails to incorporate the rational investment decisions at every node that gives the game more flexibility of choosing optimal strategies, making it more realistic. Therefore the decision-per-node method is the superior method.

---

11 This type of binomial utility tree can actually occur. The reason why after two up-moves the utility drops, is the fact that only in this last highest node in period 3 it became profitable for the entrant to join the market and take away a chunk of the incumbent’s monopoly share. So in period 1 and 2 the incumbent would be the monopolist and only in the highest PV node, the firms will be in a duopoly.
Appendix C: 3-Period Game and Example

The 3-period game is actually a combination of 2-period games, as displayed in the figure below. In period 2, two subgames will be played which are equal to the 2-period games. However, to get there, already a 2-period game had to be played.

![Diagram of 3-period game]

This 3-period game should actually be displayed then as in the following figure where every subgame in the second period has the same messages and action as in the 2-period game. Then a message or action has the form:

\[ \left\{ d_1, \{d_{2,1}, d_{3,2}\}, \{d_{2,2}, \{d_{3,2}, d_{3,3}\}\} \right\}. \]
However, when we want to model a message and action as one set containing a decision at every node that results in one strategic-form table obtaining all possible payoff combinations, the decisions are ranked per period:

\[
\{ d_1, \{ d_{2,1}, d_{2,2} \}, \{ d_{3,1}, d_{3,2}, d_{3,3} \} \}.
\]

Because in the previous notation node 3.2 occurs twice, once for the up-subgame and once for the down-subgame.

**Numerical example**

The same formulas apply in the 3-period game as in the 2-period game:

\[
\begin{align*}
\sum_{\{(i,j) | (d_{ij} = (l)) \in m \}} p_{i,j}(PV_{i,j}^{l}(m, a) - C^\theta) \\
\sum_{\{(i,j) | (d_{ij} = (l)) \in a \}} p_{i,j}(PV_{i,j}^{E}(m, a) - \mathbb{E}[C^\theta])
\end{align*}
\]

Where:

\[
\begin{align*}
u_{l,j}^l &= PV_{i,j}^{l}(m, a) - C^\theta, \\
u_{l,j}^{E} &= PV_{i,j}^{E}(m, a) - \mathbb{E}[C^\theta], \quad \text{and} \quad \mathbb{E}[C^\theta] = \sum_{\theta} q(\theta | m) C^\theta
\end{align*}
\]

Assume the following binomial tree:

with \( u = 1.5, r = 1.1 \), the costs are \( C^\theta \in \{ C^L, C^H \} = \{20, 40\} \). The initial beliefs for the costs are \( P[\theta = L] = P[\theta = H] = 0,5 \). The messages and actions have the form of
\( \{d_1, \{d_{2,1}, d_{2,2}\}, \{d_{3,1}, d_{3,2}, d_{3,3}\}\} \) and \( \varphi_A = 0.2 \) so the PV is shared 60/40 when there is one period between investments and 70/30 when there are two periods between investments.

Assume the following example with the relative simplistic but reasonable type-contingent probabilities:

| messages | node 1 | node 2u | node 2d | node 3uu | node 3ud | node 3dd | P[X|L] | P[X|H] | q | E[cost] |
|---------|--------|--------|--------|--------|--------|--------|-------|-------|---|---------|
| 1       | I      | ·      | ·      | ·      | ·      | ·      | 0.4   | 0     | 1 | 20      |
| 2       | N      | I      | I      | ·      | ·      | ·      | 0.1   | 0     | 1 | 20      |
| 3       | N      | I      | N      | -      | I      | I      | 0     | 0     | 0.5 | 30      |
| 4       | N      | I      | N      | -      | I      | N      | 0     | 0.2   | 0 | 40      |
| 5       | N      | I      | N      | -      | N      | I      | 0     | 0     | 0.5 | 30      |
| 6       | N      | I      | N      | -      | N      | N      | 0     | 0.2   | 0 | 40      |
| 7       | N      | N      | I      | I      | I      | ·      | 0.5   | 0     | 1 | 20      |
| 8       | N      | N      | I      | I      | N      | ·      | 0     | 0     | 0.5 | 30      |
| 9       | N      | N      | I      | N      | I      | ·      | 0     | 0     | 0.5 | 30      |
| 10      | N      | N      | N      | I      | N      | N      | 0     | 0     | 0.5 | 30      |
| 11      | N      | N      | N      | I      | I      | ·      | 0     | 0     | 0.5 | 30      |
| 12      | N      | N      | N      | I      | I      | N      | 0     | 0.3   | 0 | 40      |
| 13      | N      | N      | N      | I      | N      | I      | 0     | 0     | 0.5 | 30      |
| 14      | N      | N      | N      | I      | N      | N      | 0     | 0.3   | 0 | 40      |
| 15      | N      | N      | N      | I      | N      | N      | 0     | 0     | 0.5 | 30      |
| 16      | N      | N      | N      | I      | N      | N      | 0     | 0     | 0.5 | 30      |
| 17      | N      | N      | N      | N      | I      | I      | 0     | 0     | 0.5 | 30      |
| 18      | N      | N      | N      | N      | N      | N      | 0     | 0     | 0.5 | 30      |

And message \( m_2 = \{N, \{I, I\}, \{\cdot, \cdot\}\} \) and action \( a_{12} = \{\cdot, \cdot, \{I, I, N\}\} \), then:

\[
 u'(m_2(L), a_{12}(m_2(L)), q_{m_2}) = \frac{p_{2,1}(PV_{2,1}\varphi_{2,1}^l(m, a) - C^g)}{r} + \frac{p_{2,2}(PV_{2,1}\varphi_{2,1}^l(m, a) - C^g)}{r^2}
\]

\[
 u' = \frac{0.52(60 \times 0.6 - 20)}{1,1} + \frac{0.48(26,67 \times 0.792 - 20)}{1,1^2} = 8.01
\]

Remarks:
- \( \varphi_{2,2}^l(m, a) = 0.6 \times p_u + 1.0 \times p_d = 0.792 \) because the incumbent will either share the market 60/40 if node 3.2 is reached or be a monopolist if node 3.3 is reached. Therefore the PV share is a direct function of the strategies

Entrant payoff calculation:

\[
 u^E \left( m_2(L), a_{12}(m_2(L)), q_{m_2} \right) = \sum_{(i,j) \in a} p_{i,j}(PV_{i,j}\varphi_{i,j}^E(m, a) - E[C_{i,j}])
\]

The Bayes’ updated beliefs and the resulting expected costs at the nodes are all given in the table:
\[ u^E = \frac{p_{3.1}(PV_{3.1} \varphi_{3.1}^E(m, a) - \mathbb{E}[c_m^g])}{r} + \frac{p_{3.2}(PV_{3.2} \varphi_{3.2}^E(m, a) - \mathbb{E}[c_m^g])}{r^2} \]

\[ u^E = \frac{p_u^2(90 \times 0.4 - 20)}{1,1} + \frac{2p_u p_d(40 \times 0.4 - 20)}{1,1^2} = 2.82 \]

**Remarks:**

- \( p_{3.2} = 2p_u p_d \). In case the message was \( m_2 = \{N, \{I, N\}, \{I, N, N\}\} \) then \( p_{3.2} = p_u p_d \) instead of \( 2p_u p_d \) because node 3.2 can only be reached as an investment node via node 2.1 and not via 2.2. The incumbent will not invest in 2.2 and 3.2 making it only possible for the entrant to invest in node 3.2 after node 2.1, so only when the first movement goes up and the second goes down.

For \( \sigma = (\{N, \{I, N\}, \{I, I\}\}, \{\{N, I\}, \{I, N, I\}\}) \) the payoffs are 8.01 and 2.82 resp. Both firms can deviate and improve their payoff to reach a PBE of the form:

\[ \{(m^*(L), m^*(H)); a^*(m_1), ..., a^*(m_{18}); q_{m_1}^*, ..., q_{m_{18}}^*\} \]

This 3-period game with all the remarks show that it is much more complex than the 2-period game, because some investments in the action of the entrant are unattainable, probabilities of reaching a node are not as straightforward as in the 2-period game and the shares of the \( PV \) are a clear function of the message and action which was less visible in the 2-period game.
Appendix D: Excel Model in Detail

To find the optimal message and action, all possible outcomes of a strategy-pair have to be obtained. I obtained payoff matrices in figure D1 that show for every message the optimal actions the entrant can choose, given the cost chosen by nature. So the simulation is done separately for both costs.

First, all the optimal actions as a response for every message have to be found, which is shown in figure D2. In the bottom table all actions are returned that maximize the entrant’s payoff for every message. It can occur that two actions have the same payoff for the entrant, as shown in the matrices in D2, as a response to message \( m_3 \), two actions are optimal, \( a_2 \) and \( a_3 \). When two actions have the same payoff for the entrant, the one with the highest payoff for the incumbent is chosen. Figure D2 shows for every message the optimal response: \( a_1^*(m_3) \) etc. and the payoffs are the corresponding payoffs for the incumbent for every possible optimal action the entrant can choose. In the end, the incumbent observes that \( m_2 \) is for him the optimal message given his type, since the response of the entrant will give him for that message the highest possible payoff, 37,32.

So the optimal strategy tuple can be found by this model.
### Appendix E: Messages and Actions

#### 2-Period game

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<th>messages</th>
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Appendix F: List of Parameters and Formulas

\( F: \) Set of players (firms)
\( I: \) player 1, the incumbent
\( E: \) player 2, the entrant
\( M: \) set of messages the incumbent can send
\( m: \) the message actually sent
\( A: \) set of actions the entrant can choose
\( a: \) the action chosen
\( d^F_{i,j}: \) investment decision by firm \( F \) in node \( i,j \)
\( l: \) from the set of decisions: investment (also incumbent when no confusion can occur)
\( n: \) from the set of decisions: no investment
\( \Theta: \) set of types nature can assign to the game about the investment costs
\( L: \) amount of low costs
\( H: \) amount of high costs
\( \theta: \) actual type: \( \theta \in \Theta = \{L, H\} \)
\( C: \) notation for investment costs.
\( u: \) binomial up-multiplication and utility function (as the context requires)
\( d: \) binomial down-multiplication: \( d = 1/u \)
\( r: \) continuous-time discount rate
\( p_u: \) probability of an up-movement: \( p_u = (r - d)/(u - d) \)
\( p_d: \) probability of a down-movement: \( p_d = 1 - p_u \)
\( p: \) initial belief that nature draws low costs: \( P[\theta = L] \)
\( i,j: \) node in period \( i, \) ranked \( j \) from top to bottom
\( p_{i,j}: \) probability that game reaches node \( i,j \)
\( q_{i,j}: \) Bayes’ updated belief about low costs at node \( i,j \)
\( PV_{i,j}: \) Present value at node \( i,j \)
\( \varphi^F_{i,j}: \) \( PV \) share of firm \( F \) for investment in node \( i,j \)
\( \sigma: \) equilibrium strategy pair, also the volatility of the binomial lattice model (when no confusion can occur)
\( \varphi_d: \) difference of the \( PV \) shares of both firms per period difference
\( z: \) number of periods between investment