MASTER

Parametric design and calculation of circular and elliptical tensegrity domes

van Telgen, M.V.

Award date:
2012

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Parametric design and calculation of circular and elliptical tensegrity domes

Eindhoven University of Technology
Faculty of Architecture, Building and Planning
Master Structural Design

M.V. (Michael Vincent) van Telgen
Spanvlianderplein 73
5641 EH Eindhoven
Tel. 06-42900315
St. id. nr. 0632047

Date/version:
December 7, 2011, Final version

old versions:
November 3, 2011, version 3
October 12, 2011: version 2
September 6, 2011: version 1

Graduation committee
prof. ir. H.H. Snijder
ir. A.P.H.W. Habraken
dr. Dipl.-Ing. J. Beetz
**Foreword**

Since the goal of the thesis is to design a non-circular tensegrity dome, relevant information must be collected to realize a well-founded project. In this literature study it was set out to learn about tensegrity, both as a principle and as an architectural application. To that end, literature was studied, case-studies were made, structural analyses were performed, and models were built and tested. This resulted in a well-informed insight in the interesting world of tensegrity. A study into generative designing is also done. This is a quick way to design tensegrities, and making variations in the designs.

Important design aspects were found while writing the literature study. This includes for instance, the amount of possible variations in geometry and topology, and the consequences of designing a compression hoop which is not based on a circular shape. The found information provides the basis for the research methodology and forms a guideline for analyses of tensegrities which will be made in the future.
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1. Tensegrity

1.1. What is tensegrity?

Definition

Much has been written about the subject tensegrity. And so, there are many explanations as to what a tensegrity exactly is, as its exact definition seems to be subject to debate. It looks like it is not so easy to capture the exact meaning of what tensegrity is in a few words of even a full sentence, let alone what characteristics such a tensegrity would have.

Many engineers and artists have tried to explain what a tensegrity exactly is. Richard Buckminster Fuller, the world famous engineer, coined the name tensegrity, a contraction of tension and (structural) integrity, because there was no name for the type of structure at the time. He defined it in his book ‘Synergetics’ as: "Tensegrity describes a structural relationship principle in which structural shape is guaranteed by the finitely closed, comprehensively continuous, tensional behaviours of the system and not by the discontinuous and exclusively local compressional behaviors" [1]. For someone who has never seen a tensegrity though, this is completely incomprehensible.

In his book ‘Tensegrity Systems of the Future’ René Motro, an engineer and tensegrity researcher, acknowledges it is not easy to define tensegrity [2]. He states: “The difficulties in establishing a clear definition of tensegrity systems are well known. It seemed some years ago to be useful to give a "patent based" definition that could serve as a reference for comparison with the other known definitions”. He continues: “the following definition is established on the basis of patents, which have been registered by Fuller, Snelson and Emmerich”. The definition is:

“Tensegrity Systems are spatial reticulate systems in a state of self-stress. All their elements have a straight middle fibre and are of equivalent size. Tensioned elements have no rigidity in compression and constitute a continuous set. Compressed elements constitute a discontinuous set. Each node receives one and only one compressed element.”

Since René Motro believes this is incomplete, and prone to controversy, he provides his own definition: “a tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components”.

This is also not very informative for someone who has never seen a tensegrity. Perhaps a better description for tensegrity may be given when a more poetic approach is used. Buckminster Fuller said tensegrity might be considered as “compression elements in a sea of tension”. Although this does not describe what a tensegrity is in any practical sense. Sculptor Kenneth Snelson calls tensegrity “floating compression” [3], which could lead someone to believe some kind of liquid is involved, which of course is not the case.

Arguably, a tensegrity is well described as ‘a stable prestressed structural system comprised of only compression and tension elements in which the compression elements are discontinuous, and the tension members are continuous. Bending moments are not allowed and the system partially or completely collapses when just a single element is removed’. It is recognized that this statement may not be adequate as a scientific description, but it captures what is believed to be important about tensegrity. It is urged to everyone to form his or her own opinion about what the definition of tensegrity is. Thinking about this helps to understand tensegrity.
**What does tensegrity look like?**

Tensegrities contain two types of components. There are struts, which are components that only that bear compression forces, and ties, which only allow tensile forces to take place. These two types of components are connected to each other in such a configuration that no two separate struts ever touch each other. Not at the struts ends, where the ties are connected, nor along its length. This makes the compression system discontinuous. The ties are allowed to converge at the strut ends, but not along its length. Because of this, the tension system becomes continuous.

Struts may be made out of metals like steel or aluminum, or of wood. Pneumatic struts are also possible, advantageously combining low weight and compression strength. Ties are usually made from steel cables, but ropes and elastic bands may also be possible depending on the magnitude of the forces involved, the desired flexibility and its use.

Simply connecting struts and ties is not enough to obtain tensegrity. A certain amount of pretensioning is necessary for that. This makes the tensegrity rigid. How much pretension is applied depends on requirements for stiffness. For tensegrities which are loaded by external forces, adding pretension ensures the structural stability of the system.

It is common for tensegrities to (at least partially) collapse when just a single component is removed. The participating forces are highly dependent on the structure as a whole to be able to stay rigid. Depending on the total number of components, the effect of a single component failure is large or small.

When designing tensegrities, the designer is limited by configuration conditions. Not all configurations of struts and ties result in a tensegrity. The directions of the components are important, as are the number of components used. Force equilibrium is needed in every node; otherwise the tensegrity will not be stable.

When a tensegrity has been built, it cannot simply be reconfigured. Shortening or elongating ties is not an easy task, since all ties are interdependent. Prior to building a tensegrity, the designer should already have all the information available on component lengths and amount of pretension. That is not an easy task. The mathematics behind tensegrity is very complex.

The simplest possible tensegrity configuration is seen in Figure 1. It is a 1-dimensional tensegrity, consisting of only one strut and one cable. The system is similar to a pretensioned column. In practice, a small bending moment occurs due to the eccentricity of the connection between tension and compression element in the configuration which is drawn in the figure. It is noteworthy that if a bending moment occurs, the system will not be a true tensegrity.

A simple 2-dimensional ‘tensegrity’ is also possible (see Figure 2). It is not a tensegrity in the strictest sense, due to the crossing compression elements; a true tensegrity consists of discontinuous compression elements. Still, this is a very useful configuration for kites, when it is constructed in lightweight materials.
The simplest possible configuration for a three dimensional tensegrity is shown in Figure 3. It consists of three struts in compression, and nine ties in tensile tension. It has many names, but is commonly called the 3-strut T-prism, due to the number of struts and the type of polyhedron it resembles, the prism.

The tensegrity in Figure 4 achieves stability in point A by combining the three tensile (+) forces of the cables (in blue) so that the vector of the resulting compression (-) force (in red) is in the exact same direction of the strut. On the other side of the strut, this must also be the case for equilibrium. This seems very easy, but in reality, it is much more complicated. This is explained in detail in chapter 2.7. A fun and interesting tool for varying and playing with tensegrities can be found at [4].
1.2. Origins of tensegrity

Tensegrity has a short history, even as a concept. Sculptor Kenneth Snelson made his first tensegrity in 1948 when he was a student attending a summer session at Black Mountain College in North Carolina [5]. There he met Buckminster Fuller, who in the years to come became the inspirer for Snelson’s early work, but also an adversary by claiming the term for how such a system should be called. The definition ‘tensegrity’ was devised by Buckminster Fuller and is a contraction of the words ‘tensional’ and ‘integrity’. The term proposed by Kenneth Snelson was ‘floating compression’. This never caught on with the larger public, much to his dismay. During the summer session next year, Fuller claimed authorship over a number of ideas Snelson says he invented. This means it is not clear who actually invented the first tensegrity, but it is safe to say it was invented around that time. Snelson later created many tensegrity sculptures throughout his career as an artist. One of his tensegrities which is in the Netherlands is the ‘Needle Tower II’ (see Figure 5), and is found at the Kröller-Müller Museum. It consists of 20 layers, each containing 3 struts. Kenneth Snelson views the ideas of tensegrity as a form of art. His desire is to show the ‘essence of elementary structure’.

Buckminster Fuller realized its potential as a building principle as well. He played with the idea of spanning incredible distances using tensegrity principles. He experimented with lots of geometries for tensegrities and later created a tensegrity dome which was eventually named after him (more on this later). Buckminster Fuller called these types of tensegrities ‘aspension domes’, a derivation of ascending and suspension.
Before Kenneth Snelson and Buckminster Fuller, Karl Ioganson, who was an artist, had experimented with the idea of a ‘tensegrity prism’ (see Figure 6) in 1921, though it was not a true tensegrity because the cables are not pretensioned [6].

The bicycle wheel principle, an invention by Sir George Cayley, is also a tensegrity system. This idea was already around in the 1800’s, so arguably this is the first tensegrity. When the axle is loaded, the forces are redistributed through the spokes to the rim, and from the rim to the ground. Under compression loads, the spokes will buckle, and therefore the spokes are pretensioned, so no compression forces can occur.

1.3. Tensegrity domes

Spodek Arena

One of the first tensegrities in architecture is the roof of the Spodek arena in Katowice, Poland, which was designed by Wacław Zalewski. The building is shaped like a flying saucer, because this shape resulted from the desire to accommodate both a sports venue as well as a music arena [7]. Work on this arena began in 1964, and it was finished in 1972 [8]. 120 spans were realized from the edge to the center. These cables are positioned both at the top and bottom layer (see Figure 9), as are the ring cables. Diagonals span between the struts. The outer ring is in tension unlike normal tensegrity domes, which have outer rings in compression. This tension is caused by the support structure itself. Due to eccentric gravity loading, the ‘columns’ want to rotate at the support and fall outward. This is prevented by the outer ring and due to the direction of the forces this results in tension in the ring. The steel structure weighs 300 tons. At the center of the dome a taller dome was placed (see Figure 10).
Design by Richard Buckminster Fuller

The original design for an aspension dome with a closed ring system was patented by Buckminster Fuller in 1964 (see Figure 11). [9]. Aspension is derived from ascending and suspension, referring to the tension rings which are consecutively placed higher in the structure as seen from outward to inward.

The Georgia Dome in Atlanta, United States is built with this type of structure. This dome will be treated later in the case studies.
Engineer and architect David H. Geiger designed multiple tensegrity domes for arenas hosting major sporting events, including the Florida Suncoast Dome in St. Petersburg, Florida, in 1990 (see Figure 13), as well as the Gymnastics (see Figure 14) and Fencing Arenas for the Olympics in Seoul, Korea, in 1988.

All these designs include ridge cables spanning from the outer compression ring to the inner tension ring, but only in the top layer. The ridge cables are connected to the hoop cables at the bottom layer by compression posts and diagonal cables. Geiger used membrane roof claddings to keep roof weight as low as possible.

In Figure 12, a more detailed sketch of the Geiger tensegrity dome is displayed. Geiger designed the much simpler looking Cabledome tensegrity based on the example of Fuller’s tensegrity dome [10].

Figure 11: Buckminster Fuller tensegrity dome viewed in 3D and from the top

Designs by David Geiger

Figure 12: David Geiger tensegrity dome viewed in 3D and from the top
Figure 13: Florida Suncoast Dome

Figure 14: The Gymnastics dome roof for the Korean Olympics
**Bicycle wheel type roofs**

Some bicycle wheel type roofs are built. For example, the United States Pavilion was built in 1958, for the world expo in Brussels, Belgium and the New York State Pavilion was built in 1964, in New York, the United States. The weight of the roof causes a compression force in the outer ring. The spokes are in pretensioned and a center tension ring hangs in the middle. Bicycle wheel type roofs are much like tensegrity domes, but lack struts and diagonals.

Another bicycle wheel roof was built in Zaragoza Spain, in 1988. It is a bullring which has a partially retractable PVC coated polyester fabric roof.

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**Figure 15:** U.S. Pavilion  
**Figure 16:** New York State Pavilion  
**Figure 17:** Zaragoza bullring
1.4. Other tensegrity structures

Kurilpa Bridge

The Kurilpa Bridge in Brisbane, Australia, illustrates another purpose of tensegrity in architecture (see Figure 19). It was designed by Cox Rayner Architects and Arup Engineers and was completed in 2009 [11]. The longest span this bridge contains is 120 meters. It allows only pedestrians and cyclists to cross the Brisbane River.

The bridge is the first multi-mast cable-stay bridge, based on the principles of tensegrity [12]. The tubes are the compression elements and the cables are in tension. These are connected to the deck.

Space exploration

In space exploration, where wind loading is not present and rectangular spaces are not required, tensegrity offers solutions due to its low structural weight and the ability to deploy from a smaller volume. This is useful when many devices have to be brought into space using a rocket, since loading space is always limited.
1.5. **The construction sequence of a tensegrity dome**

It is important to note that contrary to usual building methods, the tensegrity dome structure only assumes its final form when it is fully completed. Very large deformations must be possible in the tensegrity. This has consequences for detailing and section sizes. Supports and nodes must be able to rotate freely, to prevent unforeseen stresses in sections. Section sizes may need to be larger during erection, and calculations must be made to confirm this. There are variations in tensegrity dome erection sequence [13], but most agree on the following basic steps (see Figure 20 and Figure 21).

![Figure 20: Erection sequence for the Olympic arenas in Seoul, Korea](image)

![Figure 21: Erection sequence for the Georgia Dome in Atlanta, United States](image)
The generalized implementation method is as follows:

**Step 1:**
Firstly, the compression hoop must be installed. Alternatively, anchor points may be used if an open tensegrity is chosen. This could be done when the support reactions are not balanced.

![Figure 22: Step 1; Compression hoop](image)

**Step 2:**
The top cables are connected to the compression hoop at one side, and to the top of the inner tension hoop. The top cables are slack at this point, and follow a line known as the catenary.

![Figure 23: Step 2; Top cables added and center upper tension hoop](image)

**Steps 3 to 5:**
Struts may be hung on the top cables one at a time, or all at the same time, depending on preferences. The tension hoop then is placed, followed by the diagonals. As more diagonals are installed, the forces in the cables increase.

![Figure 24: Step 3; First struts added](image)

![Figure 25: Step 4; First tension hoop added](image)
**Steps 6 to 8**

This process is repeated for each ring.

**Step 9:**

During the last phase the tensegrity dome assumes its final form.

### 1.6. Scale models of tensegrities

There are many possible tensegrities, varying in shape, number of struts, number of cables and size. In addition, each different tensegrity responds in its own way when subjected to deformations. Local or global collapse may occur when cutting away one or more elements. It is very insightful to see this in practice, and so a few tensegrities were built. The models
which were created are the '3-strut T-prism', a '6-strut tensegrity', and a '30-strut icosahedron tensegrity'.

All of these tensegrities have been made out of wooden rods and elastic band. It is convenient to use rubber bands, because they can simulate a pretensioning force by scaling the needed length of the rubber band. The rods and bands are linked together by drilling a hole in the rod and screwing in a small screw with a washer. Example pictures were used to build the tensegrities, and AutoCAD models were made to know what lengths should be used for the struts and cables.

Understanding these basic tensegrities helps understanding more complex tensegrities like tensegrity domes. Therefore, some of them are investigated. The basic form is described, as well as in which ways the tensegrity is symmetrical, how many elements there are, and how the model interacts when subjected to deformation.

3-strut T-prism

As the name implies, the 3-strut T-prism has three struts. All elements are 3-fold rotational symmetrical. This particular tensegrity has wooden struts which are 200mm long, with a diameter of 9mm. It has nine cables made from elastic bands, each measuring around 133mm long. Small discrepancies in cable length are explained by the differences in elasticity per band. Each rod end is attached to three cables. The angles made by the struts and cables are about 45°. It is a very flexible tensegrity, even withstanding deformations until it is completely flat. When any one of the cables is cut, the tensegrity completely collapses. If one cable is made shorter, the others stretch a bit to maintain equilibrium in the system.
6-strut tensegrity

Made with only six 200mm long struts, this tensegrity shows exceptional amounts of symmetry. It is based on the icosahedron and plane symmetrical along all its x, y and z-axis. This is particularly noticeable when viewing two parallel struts in plain. It is also 6-fold rotational symmetrical. This is easily seen when looking at the plain of one of the cable triangles. 24 identical cables, each measuring 125mm are connected to the struts. All strut-ends are connected to four cables, two of which at an angle of 35° and two at 66°. The tensegrity deforms, but remains in equilibrium when any cable is made longer or shorter. When the offset between one set of parallel struts is reduced by exerting external forces on it, the two other parallel struts will also reduce their offset to compensate for the change in equilibrium conditions. Partial collapse occurs when a single cable is cut. When one strut-end is freed of its cables, there is still equilibrium possible, without any struts touching each other.

Figure 32: Left: A 6-Strut tensegrity. Middle: Symmetry visible in two directions. Right: 6-fold rotational symmetry

Figure 33: Left: equilibrium with one cable made shorter. Right: When one strut-end is disconnected, the model’s shape is left somewhat intact
**30-strut icosahedron tensegrity**

This 30-strut tensegrity uses 90 identical cables to maintain equilibrium. For building purposes, this tensegrity has been built using only 30 cables, and subdividing, but not cutting, each cable into three equal lengths. The rods are 120mm long. The cables are only somewhat longer and measure 130mm. The model could be described as two sets of shells, one made by cables on the outside providing pretension, and one made by the struts on the inside. Both cables and struts show the same pattern, namely 12 pentagons and 30 triangles. Due to way these are arranged, this tensegrity is 5-fold of 3 fold rotational symmetrical, depending on the location of the rotation axis. Shortening a single cable has little effect on the form of the tensegrity. Separation of a cable also has little effect on the tensegrity, as the effect is absorbed by the other cables.

![Figure 34: Left: 30 strut tensegrity. Right: 5-fold rotational symmetry visible](image1)

![Figure 35: Left: deformation when force is applied to the top. Right: cutting a single cable](image2)
1.7. *Scale models of tensegrity domes*

**Geiger tensegrity dome**

The Geiger tensegrity dome is a very simple looking tensegrity. One may see some resemblance to a spider web when viewed from the top. This particular Geiger dome has six top cables measuring 250mm in length. The top cables are divided into three equal 83mm long segments, to which the struts are attached. The struts have lengths 45mm for the outermost tensile hoop, 35mm for the middle tensile hoop, and 30mm for the innermost tensile hoop. The struts are connected to the three tensile hoops on the lower side of the tensegrity. Note that above the innermost tensile hoop, there is also a tensile hoop at the top. Diagonals are attached between each two hoops from the top cable to the intersection with the strut and tensile hoop. The outermost hoop is a compression hoop, in this model an old bicycle rim is used. The span is 520mm.

![Figure 36: Top left: a Geiger tensegrity dome. Top right: the dome from the top. Bottom right: the dome from the side](image)

More top cables may be added if desired. This will affect the stresses in the tensegrity, but will not change its fundamental behaviour. The number of top cables influences the number of sides the polygon has, which is always the same as the number of top cables. The number of top cables also determines the number of mirror symmetries the dome has. One can be found at the section from midpoint between two top cables to the center, and one at the line section where the top cable is. In addition, the dome is also rotational symmetrical.

When examined, the pretensions in cables seem to differ predominantly. This is easily noticeable when pressing down on a rubber cable. Pretension in the tension hoops is lower in the hoops at the center. Likewise, a lower pretensioning force is found at the innermost top cable parts, when compared to the top cable section connecting the compression hoop to the first strut. In addition this is also the case for the diagonals; they are more tensioned at the outside and less when nearer to the inside. This is all anticipated when comparing the dome to real life examples.
The Geiger dome deforms easily. This is due to the high elasticity of the rubber bands and the low amount of present pretension.

Cutting any cable will instantly force the dome to adapt and find a new equilibrium state. Cutting cables near the compression ring has a more profound effect than near the center. This is of course also a consequence of the pretensioning in the cables.

*Buckminster Fuller tensegrity dome*

Though the Buckminster Fuller tensegrity dome looks difficult to comprehend, it is actually very simple. Like the Geiger dome it consists of top cables, but these measure 300mm due to the curvature (see Figure 38). In this model, there are 9 top cables in the blue direction, and 9 in the red direction. Each top cable has three diagonals, connected from top to the bottom of the struts when seen from the outer hoop towards the center. There are four tension hoops, three at the lower side, and one at the center of the upper side. Red and blue top cables share struts at their intersections. From outside to inside these measure 55, 45 and 30mm. Equal to the Geiger dome, the span is 520mm.

Many symmetry planes may be found as well as rotational symmetry planes. This is of course determined by the number of top cables, as with the Geiger dome. The designer is free to determine the number of top cables, but for practical reasons 18 top cables were used in this model. Building time, costs, loads, span length and available resources will determine how many should be used.
By building the Fuller dome it was learned that it was much more difficult to construct compared to the Geiger dome. When completed, the struts were in tension and were not vertical in the model but slant. The amount of needed pretension seems to be much higher, especially in the outermost tension hoop, diagonals and the outermost parts of the top cables. For both the Geiger and Fuller domes a predetermined amount of strain is used in all of the rubber bands. For the Geiger dome this was 20%, and for the Fuller dome this was 30%. The choice to increase the strain was made because the cables in the Geiger dome were relatively slack, and this model was to be more firm. Seemingly, there is more difference between the most and least pretensioned cables in the Fuller dome. This might be explained by the number of top cables, and the need for higher amounts of pretension needed in the outermost cables, or maybe both. This should be examined in finite element calculations of the domes. It was decided to add rigid strings, to get the model into the desired shape. These are placed at the outermost parts of the top cables, the outermost tension hoop and the middle tension hoop. This released strain on those parts and increased strain on the innermost parts of the top cables, the innermost tension hoops and the diagonals. All of the struts became compressed.
Because of the newly added rigidity by the strings, the model is not easily deformed. The effects of deformation cannot be examined due to this property. It is assumed that similar to the other dome, the Fuller dome finds new states of equilibrium when subjected to deformations. Cutting cables leads to large deformations, probably because of the pretension. This also leads the dome to find new equilibrium states. It is noted that some compression elements may be found under tension when this happens. This is possible because of the large amount of redundant force which may be still applied to the model before structural failure of the different parts.

Figure 40: Many geometric shaped are found in the Fuller tensegrity dome

Conclusions

Building the tensegrity models became a particularly well informing exercise. Some interesting features became apparent, and should reoccur in all designs in the future.

One of these features is symmetry. It seems that any type of tensegrity can be made in a symmetrical way, but it is not necessary. This is also the case for tensegrities which are not discussed in this paper. A possible reason for the appearance of symmetry is the factorization of forces. If mirror symmetry is the case, the component forces will be equal and opposite of each other, and perfectly aligned with the compressive elements. In cases where elements are not of equal length, surplus forces are passed along to other elements so equilibrium will still be achieved, but then there is no symmetry. This is easily seen in the 6-strut tensegrity of Figure 41. Suppose one of the blue cables were not aligned as in the figure. The tensegrity would have to deform to find a new equilibrium of forces, perhaps so much that the object would become ‘unusable’. Large tensile or compressive forces may also
occur, which may be undesirable or even unpredictable. A more violent example may be the case when one of the cables snaps.

Numbers of components also seem to be important when considering failure due to breaking cables. In general, the fewer components, the more devastating a cable failure becomes. For the 3-strut T prism, one snapped cable leads to total system collapse, and in the 30-strut icosahedron tensegrity this incites small deformations. For the domes, a snapped cable also leads to small deformations, but formerly compressive struts may need to act as tensile elements.

The needed amount of pretension may be unpredictable, including the locations were more and less pretension has to be applied. This is primarily the case for the dome models, where more pretension is necessary near the rim of the dome. The needed pretension in the 3, 6 and 30 strut tensegrities are unknown, but that could easily be found in FEM calculations. Pretension can be induced as strain on the cables, but other methods may be applied as well. The stress-strain ratio and the amount of pretension in the cables determine the stiffness of the overall structure. The ratio is of course called Young’s modulus. For linear elastic behavior, Hooke’s law of elasticity is applicable. Pretension should be higher in cases where larger spans are needed and when fewer elements are available.
2. Structural Analysis of tensegrities

2.1. The shape of a tensegrity dome

An untrained eye might say that tensegrities have strange, inefficient shapes, because cables and struts seem to be placed in somewhat random locations. It may not be so obvious why this is necessary, because surely simpler looking structures are also possible?

A similar looking structure is easily found in structural engineering, the truss. This structure, like the tensegrity, may accomplish great spans, for instance in the case of bridges. But there are striking differences between trusses and tensegrities. Trusses are not pretensioned and they are somewhat laterally supported due to its available moment of inertia.

By definition, a tensegrity is a self-stable structure which is always pretensioned. Compression and tensions forces are allowed, but no bending moments are possible. Also, the tension in the system is continuous, and the compression in the system is discontinuous. These conditions define the shape of the structure. Bearing these conditions in mind, it is easy to derive the shape of a tensegrity dome.

Figure 42: Structural problem to be solved

Consider this very simple and common engineering problem: a span with a central point load. Normally, the shape is considered satisfactory when bending moments are possible in a beam. However, bending moments are not allowed in tensegrities, and thus the span will not be a beam but a bar. Also, no normal force is possible in this situation; all forces are perpendicular to the beam. No solution can be found to this problem.

Figure 43: Step 1

To enable the possibility of a normal force, the structure must change shape. When this is done, the load $F$ is decomposed into component forces. This will lead to a rather large reaction force $R_H$ (depending on the angle of the bar), a reaction force $R_V$ and tension in the bar (cable). Sufficient deformation has to be possible for the reaction forces to become acceptable.
In this configuration, only vertical reaction forces are needed to carry load $F$. However, lengthy compression bars are subject to considerable buckling problems.

Increasing the angles between bars and cables improves the level of forces in the system. Adding a vertical compression bar (strut) stiffens the structure, causing smaller deformations. However, this is not a tensegrity, since the compression bars are continuous and the ‘horizontal’ compression bars still have major buckling problems.

The obvious solution to steps 2 and 3 is to remove the ‘horizontal’ compression bar. This results in an unstable system, since the compression bar may freely rotate around node ‘S’.

Adding a pretension force to the system in step 3 easily solves the problem in step 4. For a planar structure, this complies with all requirements for being a tensegrity.
Figure 48: Step 6

Dividing the system into more sections would be clever, since it shortens the buckling length of the struts. Unfortunately, this raises another problem, namely the unhindered rotations around nodes ‘S’. Thus, the structure as proposed in step 6 will collapse due to rigidity problems.

Figure 49: Step 7

The problem in step 6 is easily solved. Since the displacements at nodes ‘S’ are equal and opposite of each other, adding a cable between these two nodes results in a stable system. This proposed solution works for any number of divisions along the top.

Two problems still exits though, and one is that this structure is planar. This implies nodal restraining in the direction perpendicular to the plane of the structure. Since in real life structures are always spatial, a solution is required for the stability of a tensegrity. Also, the desired shape of the tensegrity is a dome, and not a narrow beam. The solution to this problem is presented in step 8. The second problem is that the system is not self-stable. This means the system is only coherent due to the supports, which disallow any displacements. The tensegrity should do this itself.

Figure 50: Step 8. Tension forces in blue, compression forces in red.

Figure 50:

Simply adding the same section as in Figure 49, but in the perpendicular direction, results in a stable tensegrity dome. Of course, the designer is free to add as much of these in other directions (not drawn). This significantly improves the spread of the loading on the supports,
and thus decreases axial forces in the struts. But it also worsens the detailing problem in the top and bottom node of the central struts, where many cables come together.

Figure 51:
By creating a tension hoops in the center of the structure and at the underside of the outer struts, the tensile forces can still be distributed through the system and allows the design to have simpler node designs. This is very useful, especially when many top cables and struts are present. Albeit that in this figure the tension hoop is actually a square. The point load $F$ cannot be placed at the center anymore, since there are no struts at that position. They are hence redistributed to the center struts.

Figure 52:
By finally adding a compression hoop, the structural system becomes a closed tensegrity. It is then fully self-stable.

Of course many other geometries and topologies are possible. This derivation is arbitrary, and only serves to illustrate why and how the elements are placed. It would be very
interesting to know if other topologies and/or geometries would provide us with better results.

2.2. Hand calculations of planar tensegrities

It is easy to show the distribution of forces in a planar tensegrity dome using a Cremona diagram. The Cremona diagram is a useful graphical method for determining forces in structures (but not displacements). However, tensegrities are actually better analyzed using a non-linear analysis, because tensegrities must deform somewhat in order to compensate for newly introduced forces. This is always done using FEM software, since it would be too much work when done by hand. Due to the initial shape of the tensegrity dome, these deformations should be small compared to the system as a whole, and therefore a linear analysis may provide accurate results as well. Both Peter van den Heuvel and Mariëlle Rutten have included Cremona diagrams of planar tensegrities in their graduation work, to compare hand calculations as well.

Tensegrity domes must always have its struts under compression, and all cables under tension. This guarantees the system is discontinues in compression and continuous in tension.

Failure of the system may occur due to several reasons. High deformations can take place when the stiffness of the structure (depending on the elasticity and cross-section area) is too low, or when the applied pretension is not high enough. Instability is possible when struts buckle or when cables are slack due to a lack of pretension. Collapse of the structure is possible when material strengths for cables and struts are too low and one or more of these elements break.

Consider the planar tensegrity in Figure 53. Suppose we would have already built this tensegrity and we cannot change its geometry, but are able to freely apply pretension force and external forces. At first, only an arbitrary pretension force $F_p$ of 15 kN is applied, and no vertical loads are allowed ($F$ and $F_v$ are both 0 kN). Self-weight is also neglected to better illustrate how Cremona diagram works. The Cremona diagram is then drawn as in Figure 54.

D: Diagonal cable with $\beta$ for all diagonals  
T: Top cable with $\alpha_1 > \alpha_2 > \alpha_3$  
H: Hoop cable  
S: Strut

![Figure 53: Planar tensegrity](image)
We can see all cables are in tension and all struts are in compression. Adding pretension force scales the diagram.

When we add some vertical loads \( F \) of 1,2kN we can draw the Cremona diagram in Figure 55. Notice the direction in which \( T_3 \) is drawn has reversed. This means that part of the top cable is not under tension anymore. Also \( H_4 \) is now loaded in compression. Since cables can only be subjected to tensile forces, the system will deform heavily prior to this Cremona diagram ever taking place. This may lead to collapse of the system.

The solution is presented in Figure 56. By adding enough pretension force \( F_p \) until all cables are in tension again solves the structural problem. In this case \( F_p \) had to be raised to 20 kN in order for the model to become a tensegrity again. Note that especially sections \( D_1, H_1, S_1 \) and \( T_1 \) must be able to cope with the increase of force.
2.3. Comparison between hand and FEM calculations for planar tensegrities

In the previous chapter it was stated that hand calculations should yield the same results as finite element method calculations, because only small deformations take place in the planar tensegrity. This paragraph investigates if this is truly the case. In the FEM calculation, both a linear and a non-linear analysis are performed, so that we may compare the different results.

Suppose we import the planar tensegrity from Figure 53 into GSA. Since we did not need to know how long all the elements were in the hand calculation, we should first consider whether or not it is needed for the FEM calculation. We saw before that when the angles of the geometry remain the same, only the pretension force $F_p$ and the external load $F$ had effects on the Cremona diagrams. Increasing $F$ and $F_p$ equally resulted in scaling of the figures. So if the forces and angles are kept the same, the span length should not matter for the FEM calculation also. Since we must define a span length in order to import the model in GSA, 15m span length is chosen. Also, $F_p = 20 \text{ kN}$, $F = 1,2 \text{ kN}$ and $F_v = 3,6 \text{ kN}$.

Section forces are determined by measuring the lengths of the lines in the Cremona diagrams for the hand calculation. This is easily done in AutoCAD, in which the figures were drawn, using the ‘dist’ (distance) command. GSA provides this data by viewing the ‘Beam and spring forces and moments’ output. For the linear analysis, ‘beam’ elements are used, since rigid connections are needed for the analysis. For the non-linear analysis ‘bar’ elements are chosen. Using beam elements causes bending moments in the system, but these are negligible due to their sizes.

The results of the hand and FEM calculations are collected in Table 1.
As can be seen in the table, all results are exactly the same. Therefore we can also state that the geometry of the system has not changed significantly due to the applied loads. It appears that the modeled system is very stiff, and the difference between linear and non-linear behavior is hence negligible.

What about displacements? GSA provides us with information regarding those results as well. The hand calculation unfortunately, does not. The nodal displacements found in z-direction (the same direction as force F) are collected in Table 2.

<table>
<thead>
<tr>
<th>Maximum nodal displacement (z-direction) (mm)</th>
<th>Hand calculation</th>
<th>Linear FEM calculation</th>
<th>Non-linear FEM calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No result</td>
<td>-0,65</td>
<td>-0,65</td>
<td></td>
</tr>
</tbody>
</table>

The maximum nodal displacement is the same for the linear and the non-linear analysis. This suggests the results are coherent, but they may not be exact, since the deformations highly depend on the used cross section of the members.

The displacements are also really small, compared to the span of the tensegrity. This is easily explained by the height/span ratio. The height was specifically chosen to keep a clear demonstration for the hand calculation, but it clearly results in small deformations.
Other methods may be considered to further compare these results, such as applying pretension by nodal displacements or temperature changes in members. In theory, these methods should yield the same results, but this is yet to be verified.

2.4. **Comparison between linear and non-linear FEM calculations for 3D tensegrities**

The previous calculations apply to planar tensegrities. For 3D tensegrities, the model has a total of four top cables, with the same geometric properties as in Figure 53. See Figure 57.

![Figure 57: Model for the 3D-tensegrity test](image)

Again, for the linear analysis, ‘beam’ elements are used, since rigid connections are needed for the analysis. For the non-linear analysis ‘bar’ elements are chosen. The results are displayed in Table 3.
Table 3: Comparison between 3D FEM calculations

<table>
<thead>
<tr>
<th>Element type</th>
<th>Number:</th>
<th>Linear FEM calculation</th>
<th>Non-linear FEM calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Section force (kN)</td>
<td>Section force (kN)</td>
</tr>
<tr>
<td>Top cable</td>
<td>T1</td>
<td>8,51</td>
<td>8,52</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>2,16</td>
<td>2,17</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>0,05</td>
<td>0,05</td>
</tr>
<tr>
<td>Diagonals</td>
<td>D1</td>
<td>15,18</td>
<td>15,20</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>6,21</td>
<td>6,20</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>2,26</td>
<td>2,26</td>
</tr>
<tr>
<td>Struts</td>
<td>S1</td>
<td>-8,15</td>
<td>-8,17</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>-3,33</td>
<td>-3,33</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>-1,22</td>
<td>-1,22</td>
</tr>
<tr>
<td>Tension hoops</td>
<td>H1</td>
<td>9,06</td>
<td>9,06</td>
</tr>
<tr>
<td></td>
<td>H2</td>
<td>3,70</td>
<td>3,70</td>
</tr>
<tr>
<td></td>
<td>H3</td>
<td>1,35</td>
<td>1,35</td>
</tr>
<tr>
<td></td>
<td>H4</td>
<td>0,03</td>
<td>0,04</td>
</tr>
</tbody>
</table>

The differences between linear and non-linear analysis are minimal. The small discrepancies can be explained by the difference in used element type.

The difference between maximum nodal displacements is also minimal. This may also be due to the difference in chosen element type. See Table 4.

Table 4: Comparison between nodal displacements for 3D FEM calculations

<table>
<thead>
<tr>
<th>Element type</th>
<th>Linear FEM calculation</th>
<th>Non-linear FEM calculation</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum nodal displacement (z-direction) (mm)</td>
<td>-0,53</td>
<td>-0,59</td>
<td>-10,2%</td>
</tr>
</tbody>
</table>

2.5. Comparison between 2D and 3D FEM calculations

A comparison between the planar tensegrity and the 3D tensegrity can also be made. It is quite apparent not much changes in the top cables, diagonals and struts when we compare values from Table 1 and Table 3. Something else does change quite a bit. The forces in the tension hoop are lowered between the 2D and 3D analysis. This is fully explained by the change in direction of the tension hoop cable. See Table 5.
<table>
<thead>
<tr>
<th>Element type:</th>
<th>Number:</th>
<th>2D Non-Linear FEM calculation</th>
<th>3D Non-linear FEM calculation</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension hoops</td>
<td>H1</td>
<td>12,81</td>
<td>9,06</td>
<td>-29,3%</td>
</tr>
<tr>
<td></td>
<td>H2</td>
<td>5,23</td>
<td>3,70</td>
<td>-29,3%</td>
</tr>
<tr>
<td></td>
<td>H3</td>
<td>1,91</td>
<td>1,35</td>
<td>-29,3%</td>
</tr>
<tr>
<td></td>
<td>H4</td>
<td>0,05</td>
<td>0,04</td>
<td>-20%</td>
</tr>
</tbody>
</table>

Table 5: Comparison between nodal displacements for 3D FEM calculations

2.6. Conclusion for the comparisons

Based on the data as found in the previous paragraphs, the hand calculations, linear and non-linear calculations provide the same results for element forces. For node displacements a decisive conclusion cannot be made since these are too small to compare. When large scale comparisons are going to be made in the thesis, it will be necessary to further investigate this.

Hand calculations are easy and provide good data for pretensioning forces in planar tensegrities. Unfortunately, no displacements can be calculated using this method, and hence it is not suitable for large scale comparisons between different tensegrities. The major disadvantage for the linear calculations is the use of beam elements. While the results seem to be accurate, the elements do not reflect the components properties. Cables do not have any bending stiffness whatsoever. Non-linear analysis takes more time, but takes the need for the tensegrity to deform somewhat to bear external loads into account. It also provides good results for displacements.

Ultimately, using non-linear analysis provides the best results and is the best choice for tensegrity analysis.

2.7. Investigation of a 3-strut T-prism tensegrity

The different aspects of tensegrity domes have been treated in the previous paragraphs. It may also be useful to examine the 3-strut T-prism. The shape, the forces in the tensegrity and a FEM calculation are studied.

Shape

It is interesting to analyze even the most simple of tensegrities, the 3-strut T-prism. The shape for instance, is associated by equations [14]. The relation between the struts ‘s’, the length of the ties at the bottom ‘a’, the ties at the top ‘b’ and the ties between the bottom and the top ‘c’ (see Figure 58) is given by:

\[ s^2 = \frac{ab}{\sin 60} + c^2 \]  

[1]
See Figure 59. Some extra lines are drawn. ‘x’ extends in direction ‘a’, so that a rectangular triangle can be made with ‘y’. ‘z’ is the height of the tensegrity. ‘r’ is the radius of the circle. The relationship between the element lengths can be captured in:

\[ s^2 = (a+x)^2 + y^2 + z^2 \]
\[ c^2 = x^2 + y^2 + z^2 \]

Combining these equations gives:

\[ c^2 = s^2 - a^2 - 2ax \]

The length of ‘x’ is always set by:

\[ x = r - \frac{1}{2}a \]

The length of the radius ‘r’ is governed by:

\[ r = \frac{1}{2}b \sin(60°) \]

Combining the last three equations brings forth equation 1. The element length of the strut is thus bound by the lengths of the ties. This is useful for designing such tensegrities. It stands to reason such relations exist for tensegrity domes as well, which is convenient for tensegrity dome design. Suppose we want to alter the design of the 3-strut T-prism tensegrity in Figure 60. Here the length of ties ‘a’ is decreased.
We can only do so by changing lengths ‘s’ or ‘c’ if ‘b’ is kept the same. However, we may create any configuration as long as the values for the element lengths are positive and non-zero integers.

**Forces**

The forces in the tensegrity are interrelated, but not always identical in components ‘a’, ‘b’, ‘c’ and ‘s’. For static equilibrium, the forces exerted by the ties must be counteracted by the struts. The scale of these forces is unknown, but the directions of the forces are. There is also a relationship between element length and amount of force in the element. So if element lengths have to be scaled, the forces in the elements must be scaled just as much.

See Figure 61. Elements ‘a’, ‘b’, ‘c’ and ‘s’ have different lengths and section forces. If we consider the static equilibrium for the strut-end at the top left of the picture and place the blue tensile lines in succession, Figure 62 emerges. It is evident that no static equilibrium is possible in this configuration. Since components ‘a’ are interdependent, only two scaling operations are possible, scaling ‘a’ and ‘c’. Scaling ‘c’ does not result in equilibrium, but scaling ‘a’ does. This results in Figure 63.
This accounts for the half the equilibrium for this strut element, since it is also dependent on the forces in 'b' on the other side of the strut. So, the tensile forces for the bottom right side of the strut are now drawn in succession in Figure 64. Forces 'b' must now be scaled, resulting in 'b*' for Figure 65. Static equilibrium is now achieved for this strut. This process may be repeated for the other struts, and results in the same scaling of lines 'a' and 'b'.

Now the forces in the elements may be calculated. The element lengths of the tensegrity are:
\[ l_a = 1020\text{mm} \]
\[ l_b = 2040\text{mm} \]
\[ l_c = 2072\text{mm} \]
\[ l_s = 2588\text{mm} \]

Lengths \( l_a^* \) and \( l_b^* \) are (measured in AutoCAD):
\[ l_a^* = 1178\text{mm} \]
\[ l_b^* = 589\text{mm} \]

Note that ‘\( l_a^* \)’ and ‘\( l_b^* \)’ are not actual element lengths, but reflect the ratio for the occurring forces with respect to the unchanged element lengths for ‘\( s \)’ and ‘\( c \)’.

Suppose the compression load \( N_s \) in ‘\( s \)’=–100 kN (an arbitrary load), then:
\[ N_a = \frac{l_a^* \times 100}{l_s} = \frac{1178 \times 100}{2588} = 45.5\text{kN} \]
\[ N_b = \frac{l_b^* \times 100}{l_s} = \frac{589 \times 100}{2588} = 22.8\text{kN} \]
\[ N_c = \frac{l_c^* \times 100}{l_s} = \frac{2072 \times 100}{2588} = 80.1\text{kN} \]

The loads are positive, since they act in the opposite direction of the force in the strut.
This is an easy method for the calculation of the occurring forces, provided an AutoCAD model, of any other 3D drawing application is available.

**FEM calculation**

Of course, the element forces may also be determined in a FEM calculation. The same tensegrity as in Figure 61 is therefore imported into Oasys GSA. One node is restrained in x, y, z, xx, yy and zz directions to prevent errors due to stability. Bar elements are chosen and the struts are loaded, so that there is a 100kN normal force in them. A non-linear analysis is then performed to find the tension forces in the ties. The results are:
\[ N_a = 45.5\text{kN} \]
\[ N_b = 22.7\text{kN} \]
\[ N_c = 80.0\text{kN} \]

Which are the same results as the forces found by the hand calculation. See also Figure 66.
Figure 66: GSA results for element normal forces
3. Case studies

Many studies have been conducted on the subject tensegrity. It was useful to explore graduation projects and real-life examples, because much can be learned from other people’s experience. Though none of the found projects aim to compare many tensegrity domes or to design multiple non-circular tensegrity domes, similarities and general findings are definitely applicable in my own project. Consequently, a one real-life example and two graduation projects are studied.

3.1. Graduation project ir. Peter van den Heuvel

Peter van den Heuvel set out to design a new ice skating stadium in the city of Heerenveen [15]. He considered the current facility to be outdated, and found that it should be replaced with a state of the art design, adhering to current standards in comfort and facilities. This stadium must have the roof to be able to regulate atmospheric conditions needed for ice-skating.

![Figure 67: Structural design of the ice-skating stadium](image)

This roof became a tensegrity structure with a pneumatic membrane cladding system, and an interesting 8-shape resembling ground plan (see Figure 68). It is a low-weight roof system, which is still able to span the distance and allow light to enter the building. He made many models for the roof geometry, all of which were analyzed for usability, spatiality and esthetics. Of course, the structural system also had to be safe.

The geometry is based on the Geiger dome principle. All top cables are perpendicular to either the compression hoop (in the circular section) or perpendicular to the tension hoops (in the curved section), and do not deflect. Due to the inward deflections of the long side, the ‘compression hoop’ becomes a tension arch in those two places. This leads to unusual forces at the inflection point. Normally, a tensegrity is a closed system of forces, but here an extra compression element (not seen in Figure 68) is needed between the deflections points to counter the inward forces at the base of the tension arch.

Peter van den Heuvel first researched the different variables in a planar tensegrity structure. He states that the following variables are important to the structural design:

1. The number of tension hoops
2. The total height of the structure
3. The shape of the top cable
4. The height above the support
5. The length of the compression bars
6. The distance between the tension hoops

Most of these variables are linked together. It is not possible to increase the length of the compression bars without adjusting the total height of the system. Also, the angles of the top cable and diagonals will alter when varying the bar length. This is important to know, because it complicates the comparisons of different topologies. There may also be more variables possible, so it will be a good exercise to analyze these variables myself. The interaction of the variables decides whether a design is efficient or not.

![Figure 68: Geometry of the tensegrity structure](image)

He then evaluates the system on three aspects. These are the amount of pretension needed (in kN), the amount of steel needed (in kg) and the deformations in the structure (in m). From his analyses he concludes the relative importance for the height above the support, the concave shape of the top cable, the length of the compression bars and the distance between the tension hoops. Those properties in particular seem to decide the amount of pretension needed in the structure.

To simulate a pretension for the planar models he displaces nodal supports in GSA. This has the advantage that the structure is able to find its optimal amount of pretension in the cables itself. To find the needed amount of pretension, the node of a support is displaced until all of the cables are in tension. The displacing is then stopped, so the structure can be analyzed. For instance: a span of 120m needed its support node to be displaced by 0,2m for all the cables to be in tension. All of the pretensions were thus found by experimenting. Further increasing the pretension makes the system stiffer, and less prone to deflecting. However, Peter states that it is wiser to adjust the geometry of the system when smaller deflections are desirable.
For designing the 3D tensegrities, Peter used temperature changes in the compression bars to simulate pretension. This was done because displacing nodes is difficult in 3D structures, as each node needs to be displaced exactly in line to the cables. Changing the temperature in the bars linearly increases the length of the bars. This results in pretension in the tensegrity. Thankfully, this does not change the topology of the system by much, though some variables, such as the angles made by the top cable and diagonals may differ. The geometry of the deformed system should of course be used as the final design of the tensegrity. Other methods, such as adjusting the lengths of top cables, diagonals and tension hoops may also be used.

For his comparison, Peter included load cases for pretension, air pressure and both pre-stress and air pressure. It should be noted that self-weight of the structure is neglected when the preliminary designs are made. Self-weight only constitutes a small part of the loading on the system. Also, the needed cross-sections of the tensegrity are usually unknown beforehand, so it is difficult to directly apply the right section sizes. In the final stages of the design process, self-weight may be included in the calculations to determine the impact on the loads of the system. Large forces (and thus sections) suggest inefficient behavior, and make the tensegrity relatively heavy. Then, the geometry must be changed to improve performance.

Obviously, costs are important, and this is determined by material prices and personnel costs. When considering all the things that contribute to an optimal design, construction period should thus be considered too. It is not easy to calculate the exact costs for each node, since material and personnel prices vary a lot over time. Large numbers of nodes can safely be considered as more expensive, even if detailing is the same for each node.

On the subject of symmetry, Peter states that he considers it to make the designing of tensegrities a lot more manageable. However, when designing complex shaped tensegrities it may be necessary to differ from symmetry. Mutual differences between parts may also be needed.

### 3.2. Graduation project ir. Mariëlle Rutten

Would we be able to improve design freedom in tensegrity domes when we replace the struts with pneumatic systems? This was the main question for Mariëlle Rutten in her graduation work [16]. Her goal was to design such a tensegrity cable dome. She researched many different topologies and geometries. These may also be useful in my research. Other conclusions may also proof useful. Ultimately, it was found that inflating pneumatic struts is a good and easy way to apply pretension on tensegrities.

To find the type of tensegrity most suited for her purpose, Mariëlle decides to compare tensegrities with the following specifications:

- Straight, convex and concave top cables
- 2 tension hoops, and 3 tension hoops
- 6, 8 and 12 top cables in total
- Geiger and Fuller topologies

She then designed the tensegrity cable domes to have a span of 24 meters and compared results found by calculations made in Oasys GSA.
For the Geiger topology, it became clear that it was better to have a concave top cable shape. This is analogous to Peter’s findings. Also, the fewer the number of tension hoops and the lower the numbers of top cables, the better it was for the amount of steel necessary. The numbers of top cables were the more important factor. Deformations however, were higher compared to the Fuller topology, and large deformations were found when applying a single point load.

The Fuller topology has different preferences. Here, a concave top cable was also most efficient, but more top cables were preferable for the optimal amount of steel. The number of tension hoops needed to be low. Deformations were smaller, compared to the Geiger dome. A point load leads to smaller deformations, so the stiffness is somewhat higher.

For both Geiger and Fuller domes, only applying pretension loads led to concave top cable shapes as the more efficient variant when comparing deformations. Point loads need concave top cables for minimal deformation. Distributed loads were best transferred by tensegrities with convex top cables.

Mariëlle states that the number of connections is an important deciding factor for comparing tensegrities. This is due to build costs, both determined by building time and material use.

When building a GSA model, it is important that some foreknowledge is applied. Models should always be built out of ‘bar’ elements, so that tension and compression is possible in all elements. Nodes should be hinged, so no bending moments are allowed. Mariëlle simulated pretensions by displacing nodes, in contrast to Peter, who uses temperature change in the struts. This is due to the simple and predictable geometry of the compared models.

Finally, the compression ring is an important feature of the tensegrity. One should decide whether or not it must be included in the design optimization. Connections between the compression hoop and top cable may also be simulated as pins. Reaction forces on the pins can be used to design the ring structure. An open structure (i.e. without a compression ring) is easier to design, but means that the global system is not yet stable by itself. A check needs to be performed on the bearing construction to determine if the construction is stiff, stable and strong enough. A closed system is stable by itself, but may be harder to design if the system is non-circular.

The number of top cables and diagonals connecting to the compression hoop is important to consider. More connections reduce the buckling length of the compression hoop. According to Euler’s formula for buckling, the quadratic length of the member highly determines the available buckling strength. On the other hand, the applied forces have to be decomposed into component forces and the magnitude of these component forces is dependent on the angles made by the geometry of the structure.

### 3.3. Georgia Dome cable roof

The previous case-studies were theoretical tensegrity domes, and are thus not actually built. The Georgia Dome however, was built in 1992 for sporting events and concerts. It has a non-circular oval shaped plan, almost resembling a rectangle (see Figure 69). It was designed by
Matthys ley and is based on the Fuller tensegrity dome topology, having curved top cables in two directions.

In the top layer, 52 top cables span from the 26 supports to the center truss. In the bottom layer three tension hoops are present. Top and bottom are connected by diagonals and struts. The dome covers an area of 234m by 186m.

The highest forces occur in the most curved sections of the tensegrity dome. This is to be expected, as the angles available for the component forces are steepest in those areas. Interestingly, because the curvature in those areas is constant, the reaction forces are also almost equal. This may be a particularly useful property when designing compression rings for non-circular tensegrities in the later parts of the graduation project.

In the computer model, cable pretension was introduced by temperature change in the members. The needed cable pretension is then found by iteratively determining, applying more temperature change in members which needed it. This process repeated until all cables were under tension. It was chosen to use only 30% of the cable capacity for pretensioning. Further pretensioning was not necessary for adhering to the maximum deformation demands.

Studying details for the Georgia Dome reveals that the compression hoop is made in reinforced concrete, cast in situ (see Figure 71). Reinforced concrete is a good choice for compression ring design, since high compression forces are not a problem for reinforced concrete, but for the Georgia dome this was specifically done for quality control and speed of construction. A disadvantage is the weight of the ring, which could be lower when made in steel. The compression hoop is a hollow rectangular section, measuring about 7.9 by 1.5 meters. The width was determined by the variations of the anchoring positions of the cables. The compression ring shape is based on two different circle radii, and rests on slide bearings to avoid undue stresses on the supporting columns.
At each anchor point, two 100mm thick top cables (see Figure 73) connect the compression hoop’s inner side to the top of the first 18.3 meters high strut. The diagonal cables are connected from the underside of the compression hoop to the underside of the strut (see Figure 74). The cables are connected to steel plates, which are set into the concrete (see Figure 72).
All steel cables were prefabricated and cut to the right length, prior to installation, to avoid delays and to ensure quality control. The cables were also pretensioned twice beforehand, to avoid creep problems. To connect the cables to the post, steel plates are welded onto the steel circular tube-shaped section of the post.

The top cables, diagonals and center truss were assembled on the ground. These parts were then simultaneously jacked from the concrete compression hoop to the right height. The struts and tension hoops were installed at a height of about 75 meters. The first tension hoop to be completed was the outermost, and by jacking further, the pretension was obtained (see Figure 75). Finally, the center truss was jacked into place by using temporary support cables and by jacking from the underside of the truss.
The cladding of the roof consists of low weight Teflon-coated glass-fiber reinforced fabric. It is translucent, so that some daylight may enter the stadium through the roof panels. Because the panels vary in shape, computer aided design was used.

![Figure 76: The installation of the fabric roof cladding](image)

### 3.4. Conclusions

Based on the previous case studies, a few general conclusions can be made. They may seem obvious, but they are important to consider well when large scale comparisons between different tensegrity geometry and topology are going to be made in the later parts of the thesis.

The number of variables within tensegrity geometry and topology that can be defined is important. An example of one such variable is the lengths of the struts, and another is the total number of top cables. For the thesis, all possible variables should be determined and boundaries must be set to limit the amount in which these variables may be varied. Important design aspects should be taken into account when working with Oasys GSA. These include how to simulate a pretensioning force in the tensegrity and what type of elements should be used.

Planar tensegrity structures could be analyzed prior to analyzing 3D tensegrities. This may reveal some information which is now still unknown and it is interesting to see if there is a relationship between planar and 3D tensegrity designs.

Experimenting with a lot of different geometries for the tensegrities must be done. Since geometry has a profound influence on the structural working of the design, it is interesting to see what bounds there are for amount of occurring forces in members. Perhaps some kind of optimum could be found for a tensegrity design.
Another decision is which properties to compare. The maximum amount of pretension in the tensegrity is interesting to know, as are deflections and numbers of elements and nodes. Some of these properties could be influences by design choices. Deflections are dependent on section areas, since it abides by Hooke’s law of elasticity.
4. Non-circular tensegrity domes

4.1.  Difficulties with non-circular tensegrity domes

It would be great if structural designers were able to design tensegrity domes without a circular based compression hoop. To some extent, this is already possible. So long as the compression hoop is curved in the right direction, a tensegrity dome is well achievable. See Figure 77 for some examples. Bending moments in the compression hoop may occur though, as different loadings from the cables are applied onto it.

![Figure 77: Different shapes for possible tensegrity domes](image)

The curvature may be small or large, but it is always oriented in the same direction. It may not be as efficient as a circle though, since the forces tend to increase near the most curved parts, see Figure 78. This reference on the structural behavior of the Georgia Dome states that “much of the load on the roof gravitates toward the four corners as can be seen by the ridge and diagonal cable forces”. This is interesting to know, since adding more ‘corners’ should distribute those forces more evenly on the tensegrity dome. Also, “the shape of the moment diagram for the compression ring confirms the fact that the four corners of the roof structure attract much of the roof load. Obviously, in alternate configurations, as the oval shape tends toward a circle, moments would disappear as the circle becomes the funicular for the loads”. This answers the question whether bending moments occur, but the quantity of these forces is still unknown. This poses a problem and should be investigated.

What happens when the curvature is flattened or curved in the opposite direction (see Figure 79)? Could this be a potential tensegrity? Consider for a second a water balloon (see Figure 80). Since a water balloon at rest does not change shape or deform, it is in a state of equilibrium. That is, the applied atmospheric pressure and the stretched surface of the water balloon together form a surface load on the water inside. This water is compressed a little, but wants to return to its original volume. This results in an equal and opposing force on the surface of the water balloon. Thus, the water balloon is in equilibrium. Now, consider some force being applied on a point of the water balloon by pressing a finger into the surface. This is similar to the bend in the tensegrity dome as proposed in Figure 79. The direction of the curvature under the finger is opposite to the rest of the balloon. The material of the water balloon is incapable of resisting water pressures itself, because neither bending moments nor compression forces are possible in the material. So when the applied pressure of the finger is removed, the surface will deform into its original shape.
Figure 78: Forces in the corners of the Georgia Dome

Figure 79: Curvature in opposing direction

Figure 80: Water balloon
The analogy with a tensegrity is that the problem is reversed. The inside of the water balloon is a compressed volume (compressed water), trying to expand outward. The inside of a tensegrity dome is a tensioned system (pretensioned cables), which strains the outside to stay in the applied shape. For both systems, an indentation is not achievable without an outside influence, or without a ‘surface’ which is able to withstand bending moments. So for a dome to be a ‘true’ tensegrity system, it can arguably only be of circular shape.

This leads to the discussion whether or not the term ‘non-circular tensegrity’ could be coined at all, since bending moments and outside influences are forbidden in a true tensegrity. However, the goal of this master’s thesis is not to investigate whether a true non-circular tensegrity is possible, but to investigate whether a tensegrity based non-circular dome is possible within acceptable limits for occurring bending moments or restraint forces. It should offer practical insights and perhaps some solutions, so we may be able to build such a system at one point.

4.2. Possible compression hoop solutions to achieve a non-circular tensegrity dome

To understand the problem better, we can first simplify it. Suppose the inside elements of the cables would always exert a tensile force on the compression hoop as in Figure 81. This is the hatched area in gray and is deemed to be always equal for illustration purposes. The compression hoop must be able to resist this force. The compression hoop is drawn as the thick black line and is able to withstand bending moments due to non-circularity, but not due to the indentation. Now, an indentation is imposed in Figure 82.

Which solutions are possible is discussed next.
Allowing bending moments in the compression hoop

Suppose no changes would be made to the boundary conditions of the supports. The compression hoop would then have to be very stiff to enable such a shape, in other words; the section has to have sufficient bending stiffness (see Figure 83). This may be an aesthetically pleasing solution, since the compression hoop does not have to be aided in any way. The hoop may become very wide though.

Node restraints at inflection points

By restraining the nodes as proposed in Figure 84, the compression hoop does not have to resist the bending moment due to the indentation. Reaction forces would ensure the rigidity of the tensegrity. The nodes can be placed at the point of inflection.

Adding compression elements

This solution is seen in the graduation work of ir. Peter van den Heuvel. The compression element is placed so that the plan is still somewhat circular in shape, although the compression element is straight. The piece of the compression hoop at the indentation then becomes a tension arch. Forces from the arch and the compression hoop are resisted by the compression element. Buckling problems in the compression element can become very problematic due to the long length. See Figure 85.

4.3. Consequences for the cable geometry and topology

We have considered the consequences for the compression hoop, but not yet for the tensioned cable system. It is important to note that the same limitations for curvature direction apply, and the comparison to the water balloon in Figure 80 holds true. The problem is only reversed, in other words, compression is now tension.

Suppose we were able to build the tensegrity dome in Figure 86, and all tension hoops are in tension. The blue line represents the normal circumstances in which the tensegrity dome
operates. This means the forces (drawn in blue) from the tension hoop are in the ‘usual’ direction, and we can apply the well known structure in Figure 87 (left). At the red line, the direction of the forces is ‘wrong’, because the forces (drawn in red) point in the wrong direction. This means the structure behaves in a different fashion (Figure 87 right). Struts are loaded with a tension force. The diagonal cables and the top cables are now loaded with a compression force, which is of course impossible. Applying this situation will result in instability of the structural system.

Figure 86: Difference in tension hoop cable force component direction. Blue: normal direction, red: direction at indentation

Figure 87: Left (blue): cables in tension, struts in compression. Right (red) cables in compression, struts in tension

Unfortunately, no simple solutions exist; other that keeping the tension hoop curved in the right direction. This means the whole tensegrity works in the same fashion as in Figure 87 (left). The problem of wrongly curved tension hoops should be solved or avoided in the thesis.
5. Investigation into generative designing

5.1. Introduction

In the motivation it is stated that the design process is determined by choices. Each choice narrows down the design freedom of an object, until a state is reached where the product has the desired properties, and will not be altered anymore. It is evident that in order to provide solutions to problems, this road is usually taken. However, many roads remain unexplored. That is to say, other designs, some of which may prove to perform much better, may never be considered.

The problem is of course the amount of time given to designers in which they can explore all the options. A simple solution to this problem would be to provide more time, but for obvious reasons, this is not the best solution. A better solution is to be able to do more in the same amount of time.

Design processes are easily adapted to current state of the art software, but the designers themselves first have to spend time learning to work in new ways. This is an obstacle for them, since the initial investment time could be high, and no certainties are given as to whether the new way works better.

So now the requirements are:

1. Do more in the same amount of time
2. Master the new design technique in a very short time span

A new type of designing is called ‘generative designing’, though no one knows exactly what generative designing is. My experience is that it is a convenient way to set up designs based on control parameters to quickly investigate their variances. For someone else this will be a way to explore great new architectural designs. This adds to the versatility of this method, and the potential seems limitless. To narrow the topic down, only the implications for the structural designers will be discussed, but it is stressed that the topic is not limited to engineering applications.

Generative designing is possible due to the large and readily available computing power of modern personal computers. It is thus obvious why this is a relatively new designing technique. The design process is explained in Figure 88 [17]. All designs start with an idea, similar to traditional design processes. However, generative designing is based on algorithms which are interpreted by software. These algorithms are a mathematical representation of the idea. The designer builds his models using the basic principles of his idea, until a working model is obtained. He is then free to add variables, so the model may be changed to his liking. The mathematical model is interpreted by the computer, and an output is then given. This output is judged by the designer, who can decide whether to accept the output as it is, or to change the rules or parameters. This process is usually repeated countless times, until the designer is satisfied with the end result. Models are always easily adapted, and the designer is free to add as much detail and variables as he wants. All this adds to increasingly complex and precise models, which are still very adaptable in late stages of the design.
5.2. *Rhino and Grasshopper*

There are a few programs available, but in this graduation work Rhino and Grasshopper are used to build parametric models. Rhino and Grasshopper are both very popular in the architectural world, and among students [18]. These products are user oriented, have active communities, are easy to use, and allow external parties to build their own plug-ins. This all adds to the usability of the products.

**Rhino** is a NURBS-based 3-D modeling tool [19]. Simply said, it is a modeling application. It is commonly used because it is easy to pick up and it has many possibilities for modeling. But, most importantly, it allows plug-ins to be used. Grasshopper is one such plug-in. It uses Rhino’s graphic interface to display the output it generates from the parametric model. Grasshopper uses a separate graphic interface to display the parametric model. This makes it very easy to use and understand. Also, the designer doesn’t need to have foreknowledge about scripting, though it is of course possible to design your own scripts. These qualities make it very suitable for architectural engineers to use.

Grasshopper’s interface is easily explained. Users are free to place components (from here on referred to as <components>) into the workspace, also referred to as the canvas. Components may be params or numbers, they may create curves or 3D shapes, or modify values through functions, amongst many other things. Components can be linked together using a kind of cable, which Grasshopper draws for the designer in the interface. By doing
this, the user creates series of things that he wants to happen. For instance, the user may tell Grasshopper to draw two points using a point component, and then draw a line from point to point. There is no limit to the size of the Grasshopper parametric model, apart from the designer’s creativity and computing power.

5.3. Examples

There are many examples possible, and it would be unfeasible to discuss all of them. Instead, two examples are given so the reader may understand the basic principles.

Example 1

Suppose we want a cube to be scaled in a Grasshopper model, and compare it to a cube with the original size. Note that the left and right images are taken from the same parametric model, and only the slider is set on 2.2 for the right image. In this very simple model we can see how the right cube is scaled in Figure 89. To do this, a <number slider> component is used to create a scaling number, which is fed into a <scale> component. The cables indicate the way in which all the components are interconnected. Here, the right cube is scaled by the <scale> component, because they are interconnected. The left cube is not connected, and is therefore not scaled, but left intact. Also, some components are shaded in a different gray scale. This indicates which components are actually drawn in the graphic interface. The cube right (<box> component) is darker, and is thus not drawn. It is important to know Grasshopper does not actually alter shapes from the original form. Rather, Grasshopper copies what is fed into its components, and then redraws them using the newly acquired information. Consequently, data is never lost, which is very useful for evaluating the model and this also allows data to branch into different components.
Example 2

This distributing is visible in Figure 90. A parametric model for a wheel and spokes is created. The <slider> for the Scale factor for the pipes is connected to both Spoke diameter and the Tire diameter (both <pipe> components), but for the Tire diameter the scale factor is first multiplied by two. So it does not matter what scaling factor is chosen, the diameter of the tire is always 2 times larger. Another slider divides the circle into 10 equal segments using the <divide curve> component. It then draws <line>s from the center <point> to all those divided points. It is noteworthy that the number of spokes is independent of the diameter for both spokes and tire.

Figure 90: Parametric model of a wheel with spokes
5.4. **Advantages and disadvantages of generative designing**

Based on acquired experience with Grasshopper [20], it can already be said this method of drawing models for tensegrity structures is very useful. As with any technique, there are also some disadvantages. It is however safe to say that parametric designing will play a major role in the future for architectural engineers, though it will probably never fully replace customary procedures. Rather, it should augment the creative process and allow designers to explore more concepts than ever before.

The strength of parametric designing lies in general with the ease of use. For instance, a graphic output is always available to the designer, even when he has just started defining the rules to which the model must adhere. Unwanted results are easily spotted and mended. If the graphic output is not easily judged, it is usually possible to display the needed information in other ways. This ‘What you see is what you get’ way of working makes it easy to master parametric designing.

Another benefit of parametric designing is the inherent design freedom the model provides, even in late stages of the design. The designer is sometimes asked to modify his model in late stages, at which time, it may prove to be a large burden. This is not the case in parametric designing, since additions and changes are easily made.

Also, a great amount of information is generated with the models. It should be possible to connect parametric designing to Building Information Modeling (BIM). This makes it easy to provide information about elements and connections at an early stage. Other disciplines may start giving their opinions about how the building should work in an earlier stage. This could save a lot of money, since late adaptations to the design become unnecessary.

The major disadvantage will be the need to learn how Grasshopper works. This may take from days up to weeks to master, depending on the effort invested. Also, it will take even an experienced designer some time to work out a Grasshopper model before he may be satisfied with the result. This process will always be slower than sketching on tracing paper. However, the end result is directly presentable. Changes are easily made in Grasshopper, unlike sketches.

The designer must also know beforehand how he wants the model to behave. For structural engineers this means devising structural principals like braced framing, or how many columns should be used. That means a structural designer cannot start his parametric design until these choices are made. Fortunately, this also shortens the time spent on the parametric model, since not all structural features have to be possible.

Especially for structural engineers, a fast link to FEM software should come in handy when designing a building. Having information about how structures are loaded in an early stage allows the engineer to design better. Surprises are then omitted.

Concluding, parametric designing is a good way to quickly design (parts of) buildings, since a high degree of freedom is always available to the designer. There are limitations, since the software is still in its infancy.
**Bibliography**


