Dynamic response of high-rise building structures to blast loading

van der Meer, L.J.

Award date:
2008
Dynamic response of high-rise building structures to blast loading

L. J. van der Meer (0520033)

Graduation committee:
Dr ir M. C. M. Bakker
Prof dr ir J. G. M. Kerstens
Dr ir J. Weerheijm


April 2008
Acknowledgements

First of all I would like to thank my graduation committee for their guidance, suggestions and advice:

- Dr ir Monique Bakker, Eindhoven University of Technology
- Prof dr ir Jan Kerstens, Eindhoven University of Technology
- Dr ir Jaap Weerheijm, Delft University of Technology, TNO

Furthermore I am very thankful to David Rijlaarsdam, Mechanical Engineering student in the direction of Dynamics and Control, for his help with Matlab and the discussions we had regarding the contents of this graduation project. I would also like to express gratitude to Henk van Harn of Adams Bouwadvies, for the opportunity to gather information about the La Fenêtre building in the Hague, although the building structure turned out to be too complex to function as an example in this report.

L. J. van der Meer
Summary

In the Netherlands, the railway line from the harbor in Rotterdam to the Ruhr area in Germany is used for the transport of many hazardous substances. One of the risks involved is that of a BLEVE from a vessel containing a liquified gas such as LPG. A BLEVE is a boiling liquid expanding vapor explosion, which is a result of the catastrophic failure of the vessel, a sudden decompression and explosive evaporation of the liquified gas. One of the hazards associated with a BLEVE is a shock wave, the BLEVE blast. Buildings in the neighborhood of the railway line have a certain risk of being struck by this extreme type of loading. In densely built areas such as Tilburg, residential and office buildings are within a range of 20m of the railway line.

Dutch guidelines use empirical data from the Second World War to estimate damage of buildings up to four stories due to blast loads. For higher buildings, a single degree of freedom (SDOF) approach is proposed. Other literature also focuses on SDOF response, which means that the building is assumed to vibrate with a single shape at the first natural frequency. This graduation project was used to evaluate the continuous response of high-rise building structures to blast loading in general and BLEVE blast loading in particular.

Continuous response is an infinite sum of mode shapes vibrating at their natural frequencies. To obtain dynamic response of buildings to blast loading, a blast load and impulse distribution on a facade needs to be established first. Blast parameters are available in literature versus range and charge mass for high explosives. The relations between the various blast parameters are based on gas dynamics and also apply to BLEVEs, except for duration and impulse which are underestimated. More recent literature is available to determine BLEVE overpressure and duration versus range and liquid mass. Impulse can be derived from over-pressure and duration. Other blast parameters can be derived from overpressure. Hence, a blast load and impulse distribution on a facade can be determined.

The second requirement for the determination of dynamic response is an appropriate dynamic model of the building structure. For SDOF response, an equivalent SDOF system is obtained. For continuous response, a stability element is modeled as an equivalent continuous beam. Special attention is given to assumptions, which are done during the modeling process.

The SDOF response is determined analytically for linear-elastic material behavior, idealized blast loads and without damping. The response is divided in three regimes: impulsive, dynamic and quasi-static. The response regime is determined by the ratio of the duration of the blast load on the building and the natural period of vibration of the building. The maximum response in the different regimes is given in response diagrams, such as the pressure-impulse diagram, which gives a damage envelope for a certain degree of damage or failure criterium. For a SDOF system which represents a building, internal forces are proportional to the top displacement, which is chosen as the degree of freedom. Therefore, all failure criteria can be linked to a critical top displacement.
The continuous response is determined by analysis of a continuous Timoshenko beam, representing a building. The Timoshenko beam includes deflection due to bending, shear and rotation of the cross section. Modal contribution factors for base moment and base shear are derived, which give the contribution of a mode as a percentage of the total response. The modal response can be analyzed as a SDOF system. The total response is a sum of the modal responses multiplied by the modal contribution factors. Often a few modes are enough for sufficient accuracy. A method is given to approximate the overall maximum of the continuous response. The failure criterium is directly linked to base shear or base moment response.

Upon comparison of SDOF and continuous response, it is concluded that the SDOF system is not conservative for response in the impulsive regime, whereas the quasi-static response is similar for both models. BLEVE blast loading on high-rise buildings is likely in the impulsive regime. It can be concluded that response of high-rise buildings to loads in the impulsive regime, among which BLEVE blast, can not be analyzed with a SDOF system.

These conclusions are based on overall response. It is assumed that the building facade is able to transfer all loading to the bearing structure. The influence of failing facade elements on the blast propagation and on the load transfer to the bearing structure should be the subject of future research.
In Nederland wordt het spoor van de haven in Rotterdam naar het Ruhr-gebied in Duitsland gebruikt voor transport van gevaarlijke stoffen. Een van de mogelijke risico’s is het optreden van een BLEVE van een wagon met een vloeibaar gas zoals LPG. BLEVE staat voor kokende vloeistof, uitzettende damp explosie en is het resultaat van het volledig bezwijken van de wagon, gevolgd door een plotselinge decompressie en explosieve verdamping van het vloeibare gas. Eén van de gevolgen van een BLEVE is een schokgolf, de BLEVE blast. Gebouwen vlakbij het spoor lopen het risico om aan deze extreme belasting onderworpen te worden. In dichtbebouwde gebieden zoals Tilburg staan woon- en kantoorgebouwen binnen een afstand van 20 meter van het spoor.

Nederlandse richtlijnen maken gebruik van empirische gegevens uit de Tweede Wereldoorlog om schade ten gevolge van blast aan gebouwen tot en met vier verdiepingen te schatten. Voor hogere gebouwen wordt een single degree of freedom (SDOF) methode voorgesteld. Andere literatuur richt zich ook op SDOF respons, hetgeen betekent dat aangenomen wordt dat het gebouw trilt met één vorm en één eigenfrequentie. Dit afstudeerproject is gebruikt om de continue respons van hoogbouw op blast in het algemeen en BLEVE blast in het bijzonder te bepalen. Continue respons is een oneindige som van trillingsvormen die trillen met de bijbehorende eigenfrequenties.

Om de dynamische respons van gebouwen voor blast te verkrijgen, moet ten eerste de verdeling van belasting en impuls op de gevel worden bepaald. Blast parameters zijn beschikbaar in literatuur versus afstand en massa van de lading voor krachtige explosieven. De relaties tussen de verscheidene blast parameters zijn gebaseerd op gasdynamica en kunnen ook toegepast worden op BLEVEs, met uitzondering van duur en impuls, welke onderscheid worden. Recente literatuur is beschikbaar waarmee BLEVE overdruk en duur versus afstand en massa vloeibaar gas bepaald kunnen worden. Impuls kan van overdruk en duur afgeleid worden. Andere blast parameters kunnen van overdruk afgeleid worden. Het is dus mogelijk om een verdeling van belasting en impuls op de gevel te bepalen.

Een tweede vereiste om de dynamische respons te kunnen bepalen, is een geschikt dynamisch model van de gebouwconstructie. Voor SDOF respons werd een SDOF systeem gemodelleerd. Voor continue respons werd een stabiliteitselement gemodelleerd als een equivalent continue balk. Speciale aandacht is gegeven aan de aannames die gedurende het modelleerproces gedaan zijn.

De SDOF respons werd analytisch bepaald voor lineair-elastisch materiaalgedrag, geïdealiseerde blast belastingen en zonder demping. De respons kan verdeeld worden in drie gebieden: impuls, dynamisch en quasi-statisch. Het responsgebied wordt bepaald door de ratio van de duur van de blast belasting op het gebouw en de trillingsperiode van het gebouw. De maximum respons in de verschillende gebieden kan worden vastgelegd in responsdiagrammen, zoals het druk-impuls-diagram, waarin een schadegebied voor een bepaald schade- of bezwijkcriterium
wordt gegeven. Voor een SDOF systeem dat een gebouw voorstelt, zijn de inwendige krachten evenredig met de gekozen vrijheidsgraad, de verplaatsing aan de top. Daarom kunnen alle bezwijkcriteria worden gekoppeld aan een kritische verplaatsing van de top.

De continue respons werd bepaald aan de hand van een continue Timoshenko balk, die een gebouw voorstelt. De Timoshenko balk neemt verplaatsing mee ten gevolge van buiging, dwarskracht en rotatie van de doorsnede. Contributiefactoren voor moment en dwarskracht aan de voet van het gebouw kunnen worden afgeleid, waarmee het percentage, dat door een trillingsvorm of mode aan de totale respons wordt bijgedragen, is vastgelegd. De respons van een enkele mode is analogo aan de respons van een SDOF systeem. De totale respons is de som van de respons van de modes vermenigvuldigd met de bijbehorende contributiefactoren. Vaak zijn enkele modes genoeg voor voldoende nauwkeurigheid. Een methode om het globale maximum van de continue respons te benaderen, werd bepaald. Het bezwijkcriterium is direct gekoppeld aan dwarskracht- of momentrespons aan de voet.

Na vergelijking van SDOF en continue respons, kan geconcludeerd worden dat het SDOF systeem niet conservatief is voor respons in het impulsgebied, terwijl respons in het quasi-statische gebied voor beide modellen ongeveer gelijk is. Het is aannemelijk dat BLEVE blast op hoogbouw in het impulsgebied ligt. Daaruit kan geconcludeerd worden dat respons van hoogbouw op belasting in het impulsgebied, waaronder BLEVE blast, niet met behulp van een SDOF systeem bepaald mag worden.

Deze conclusies zijn gebaseerd op globale respons. Er werd aangenomen dat de gevel alle belasting aan de draagconstructie kan overdragen. De invloed van het bezwijken van gevelelementen op de blastvoortplanting en op de krachtsoverdracht van de gevel naar de draagconstructie moet verder onderzocht worden.
List of symbols

The list of symbols is shown below in alphabetical order, small letters first, then captives and then Greek symbols. If $a$ is a symbol, time derivatives of $a$ are denoted by $\dot{a}$ and $\ddot{a}$, while space derivatives of $a$ are denoted by $a'$ and $a''$. If $b$ is a dimensional symbol, the dimensionless equivalent is written as $\bar{b}$. If $A$ is a vector or matrix, it is displayed as $\mathbf{A}$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-first wave number for the eigenfunction of a beam, $2\pi$ times the number of cycles in the beam length</td>
<td>-</td>
</tr>
<tr>
<td>$a_c$</td>
<td>-critical wave number, see (5.10)</td>
<td>-</td>
</tr>
<tr>
<td>$a_n$</td>
<td>-first wave number $a$ of the $n^{th}$ mode</td>
<td>-</td>
</tr>
<tr>
<td>$A$</td>
<td>-area of a cross section</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_b$</td>
<td>-area of cross section of beam</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>-area cross section of column</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_d$</td>
<td>-area cross section of diagonal</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-dimensionless ratio of shear stiffness and bending stiffness, $SH^2/B = s^2/\gamma^2$</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>-angle of incidence between a blast wavefront and the surface of an object, see figure 6.1</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$b$</td>
<td>-second wave number for the eigenfunction of a beam, not physically interpretable</td>
<td>-</td>
</tr>
<tr>
<td>$b_n$</td>
<td>-second wave number $b$ of the $n^{th}$ mode if $a \leq a_c$</td>
<td>-</td>
</tr>
<tr>
<td>$\dot{b}_n$</td>
<td>-second wave number $b$ of the $n^{th}$ mode if $a &gt; a_c$</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-dimensionless ratio of rotary spring stiffness and bending stiffness, $CH/B$</td>
<td>-</td>
</tr>
<tr>
<td>$B$</td>
<td>-bending stiffness, usually denoted as $EI$, the product of Young’s modulus and the second moment of area of the beam cross section</td>
<td>Nm$^2$</td>
</tr>
<tr>
<td>$c_n$</td>
<td>-modal contribution factor in the $n^{th}$ mode</td>
<td>-</td>
</tr>
<tr>
<td>$c_{n, bm}$</td>
<td>-modal contribution factor in the $n^{th}$ mode for base moment</td>
<td>-</td>
</tr>
<tr>
<td>$c_{n, bs}$</td>
<td>-modal contribution factor in the $n^{th}$ mode for base shear</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: List of symbols

continued on next page
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>-damping constant</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$C_c$</td>
<td>-critical damping constant</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$C_1 - C_4$</td>
<td>-constants</td>
<td>-</td>
</tr>
<tr>
<td>$C_d$</td>
<td>-drag factor for blast wind (dynamic pressure) on an object</td>
<td>-</td>
</tr>
<tr>
<td>$C_{db}$</td>
<td>-drag factor for blast wind on the back of an object (suction)</td>
<td>-</td>
</tr>
<tr>
<td>$C_{df}$</td>
<td>-drag factor for blast wind on the front of an object</td>
<td>-</td>
</tr>
<tr>
<td>$C_b$</td>
<td>-rotary stiffness of the foundation of a building</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$C_r$</td>
<td>-reflection factor for a blast wave on an infinite plane, see figure 2.6</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_b$</td>
<td>-part of deflection which is due to bending</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>-part of deflection which is due to shear</td>
<td>-</td>
</tr>
<tr>
<td>$d$</td>
<td>-length of the diagonal in a trussed frame</td>
<td>m</td>
</tr>
<tr>
<td>$D_1 - D_4$</td>
<td>-constants</td>
<td>-</td>
</tr>
<tr>
<td>$D(t)$</td>
<td>-time dependent dynamic load factor (DLF), $\tilde{y}(t)/\tilde{F}_m$ =</td>
<td>-</td>
</tr>
<tr>
<td>$D_{det}(t)$</td>
<td>-dynamic load factor for idealized detonation</td>
<td>-</td>
</tr>
<tr>
<td>$D_{imp}(t)$</td>
<td>-impulsive dynamic load factor</td>
<td>-</td>
</tr>
<tr>
<td>$D_m$</td>
<td>-maximum of time dependent dynamic load factor</td>
<td>-</td>
</tr>
<tr>
<td>$D_{m:n}$</td>
<td>-approximate maximum dynamic load factor in n$^{th}$ mode</td>
<td>-</td>
</tr>
<tr>
<td>$D_{m:qs}$</td>
<td>-maximum dynamic load factor in quasi-static regime</td>
<td>-</td>
</tr>
<tr>
<td>$\delta_{nn}$</td>
<td>-Kronecker delta, equal to 1 if n=m, 0 otherwise</td>
<td>-</td>
</tr>
<tr>
<td>$E$</td>
<td>-Young’s modulus</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$E_k$</td>
<td>-kinetic energy</td>
<td>J</td>
</tr>
<tr>
<td>$E_s$</td>
<td>-strain energy</td>
<td>J</td>
</tr>
<tr>
<td>$E_w$</td>
<td>-work done</td>
<td>J</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>-correction factor for the SRSS rule applied to blast loading</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_{cbuc}$</td>
<td>-buckling factor for column of trussed frame</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_{dbuc}$</td>
<td>-buckling factor for diagonal of trussed frame</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\eta}_n(t)$</td>
<td>-time solution within the n$^{th}$ mode</td>
<td>-</td>
</tr>
<tr>
<td>$f(t)$</td>
<td>-time function</td>
<td>-</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>-space function</td>
<td>-</td>
</tr>
<tr>
<td>$f_n$</td>
<td>-$n^{th}$ natural frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_y$</td>
<td>-yield stress</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$F$</td>
<td>-force</td>
<td>N</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>-forcing function or force-time profile</td>
<td>N</td>
</tr>
<tr>
<td>$\tilde{F}(t)$</td>
<td>-dimensionless forcing function, $\tilde{F}(t)/\tilde{F}_m$</td>
<td>-</td>
</tr>
<tr>
<td>$F_{eq}$</td>
<td>-equivalent force</td>
<td>N</td>
</tr>
<tr>
<td>$F_m$</td>
<td>-maximum of forcing function</td>
<td>N</td>
</tr>
<tr>
<td>$\tilde{F}_m$</td>
<td>-dimensionless maximum of forcing function, $\tilde{F}_m/R_c$</td>
<td>-</td>
</tr>
<tr>
<td>$F_{m:mc}$</td>
<td>-quasi-static asymptote when moment is critical</td>
<td>N</td>
</tr>
<tr>
<td>$F_{m:sc}$</td>
<td>-quasi-static asymptote when shear is critical</td>
<td>N</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
<td>SI Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>$F_n$</td>
<td>-participation factor of the $n^{th}$ mode, $\int_0^1 W_m(\bar{x})P(\bar{x})d\bar{x}$</td>
<td>-</td>
</tr>
<tr>
<td>$G$</td>
<td>-shear modulus</td>
<td>N/m²</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-material property, $\gamma = \sqrt{2(1 + \nu)/k^l}$</td>
<td>-</td>
</tr>
<tr>
<td>$h$</td>
<td>-story height</td>
<td>m</td>
</tr>
<tr>
<td>$H$</td>
<td>-building height</td>
<td>m</td>
</tr>
<tr>
<td>$i$</td>
<td>-impulse of $p\Delta t$</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$i_n$</td>
<td>-impulse of the negative phase of the blast</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$i_r$</td>
<td>-reflected impulse</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$I$</td>
<td>-impulse of $F\Delta t$</td>
<td>N·s</td>
</tr>
<tr>
<td>$\bar{I}$</td>
<td>-dimensionless impulse, $I\omega_1/R_c$</td>
<td>-</td>
</tr>
<tr>
<td>$I_{eq}$</td>
<td>-equivalent impulse</td>
<td>N·s</td>
</tr>
<tr>
<td>$I_{mc}$</td>
<td>-impulsive asymptote when moment is critical</td>
<td>N·s</td>
</tr>
<tr>
<td>$I_{sc}$</td>
<td>-impulsive asymptote when shear is critical</td>
<td>N·s</td>
</tr>
<tr>
<td>$I_z$</td>
<td>-second moment of area of the cross section about the weak (z-)axis</td>
<td>m⁴</td>
</tr>
<tr>
<td>$j$</td>
<td>-imaginary unit, $j^2 = -1$</td>
<td>-</td>
</tr>
<tr>
<td>$J$</td>
<td>-impulse of force per unit length</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$J_{mc}$</td>
<td>-impulse per unit length when moment is critical</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$J_{sc}$</td>
<td>-impulse per unit length when shear is critical</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$k$</td>
<td>-integer, $k = 1, 2, \ldots$</td>
<td>-</td>
</tr>
<tr>
<td>$k'$</td>
<td>-shear shape factor, $k' \approx 1$</td>
<td>-</td>
</tr>
<tr>
<td>$K$</td>
<td>-spring stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>$K_b$</td>
<td>-spring stiffness which represents only bending deflection</td>
<td>N/m</td>
</tr>
<tr>
<td>$K_{eq}$</td>
<td>-equivalent spring stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>$K_L$</td>
<td>-load factor, equation [5, 7]</td>
<td>-</td>
</tr>
<tr>
<td>$K_{LM}$</td>
<td>-load-mass factor, $K_M/K_L$</td>
<td>-</td>
</tr>
<tr>
<td>$K_M$</td>
<td>-mass factor, equation [5, 5]</td>
<td>-</td>
</tr>
<tr>
<td>$K_R$</td>
<td>-resistance factor, equation [5, 6], but also $K_R = K_L$</td>
<td>-</td>
</tr>
<tr>
<td>$K_s$</td>
<td>-spring stiffness which represents only shear deflection</td>
<td>N/m</td>
</tr>
<tr>
<td>$\ell$</td>
<td>-lever between columns in trussed frame</td>
<td>m</td>
</tr>
<tr>
<td>$L$</td>
<td>-length of building in direction of blast wave propagation</td>
<td>m</td>
</tr>
<tr>
<td>$L()$</td>
<td>-stiffness operator, see equation [5, 23]</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>-modification factor for impulsive asymptote for inclusion of higher modes</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{qs}$</td>
<td>-modification factor for quasi-static asymptote for inclusion of higher modes</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>-distributed mass</td>
<td>kg/m</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>-dimensionless distributed mass, $mH^4\omega_1^2/B$</td>
<td>-</td>
</tr>
<tr>
<td>$M$</td>
<td>-concentrated mass</td>
<td>kg</td>
</tr>
<tr>
<td>$M()$</td>
<td>-mass operator, equation [5, 23]</td>
<td>-</td>
</tr>
<tr>
<td>$M_{eq}$</td>
<td>-equivalent mass</td>
<td>kg</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
<td>SI Units</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>$M_{\text{TNT}}$</td>
<td>-charge mass</td>
<td>kg TNT</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-moment</td>
<td>Nm</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>-dimensionless moment, $\mu H/B$</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>-base moment</td>
<td>Nm</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>-critical moment</td>
<td>Nm</td>
</tr>
<tr>
<td>$n$</td>
<td>-integer, $n = 1, 2, \ldots$</td>
<td>-</td>
</tr>
<tr>
<td>$N_{\text{cc}}$</td>
<td>-critical axial force for a column in a trussed frame</td>
<td>N</td>
</tr>
<tr>
<td>$N_{\text{cd}}$</td>
<td>-critical axial force for a diagonal in a trussed frame</td>
<td>N</td>
</tr>
<tr>
<td>$N_w$</td>
<td>-axial force in a column due to the weight of the upper stories</td>
<td>N</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-Poisson constant, $\nu = 0.3$ for steel</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>-natural circular frequency of mode $n$</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\bar{\omega}_n$</td>
<td>-dimensionless natural circular frequency, $\omega_n/\omega_1$</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_\zeta$</td>
<td>-damped natural circular frequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>$p$</td>
<td>-static overpressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_0$</td>
<td>-ambient pressure, at sea level $p_0 = 101.3$ kPa</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{\text{trans}}$</td>
<td>-translational pressure due to static overpressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_\text{drag}$</td>
<td>-translational pressure due to dynamic pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_m$</td>
<td>-maximum static overpressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{\text{m:n}}$</td>
<td>-maximum under-pressure in negative phase of blast</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_r$</td>
<td>-reflected overpressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{\text{r:st}}$</td>
<td>-reflected static overpressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{\text{trans}}$</td>
<td>-translational pressure, resultant pressure on a building or object</td>
<td>Pa</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>-distributed load</td>
<td>N/m</td>
</tr>
<tr>
<td>$\bar{P}(x)$</td>
<td>-dimensionless distributed load, $\bar{P} = PH^3/B$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{P}(x)$</td>
<td>-dimensionless distributed load divided by its maximum,</td>
<td>-</td>
</tr>
<tr>
<td>$P_l$</td>
<td>-linear distributed load</td>
<td>N/m</td>
</tr>
<tr>
<td>$P_m$</td>
<td>-maximum of $P(x)$</td>
<td>N/m</td>
</tr>
<tr>
<td>$\bar{P}_m$</td>
<td>-maximum of $\bar{P}(x)$</td>
<td>-</td>
</tr>
<tr>
<td>$P_{\text{m:mc}}$</td>
<td>-maximum distributed load when moment is critical</td>
<td>N/m</td>
</tr>
<tr>
<td>$P_{\text{m:sc}}$</td>
<td>-maximum distributed load when shear is critical</td>
<td>N/m</td>
</tr>
<tr>
<td>$P_q$</td>
<td>-quadratic distributed load</td>
<td>N/m</td>
</tr>
<tr>
<td>$P_u$</td>
<td>-uniform distributed load</td>
<td>N/m</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-angle of rotation due to bending moment</td>
<td>rad</td>
</tr>
<tr>
<td>$\tilde{\varphi}$</td>
<td>-dimensionless angle of rotation due to bending moment,</td>
<td>rad</td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>equal to $\varphi$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\psi}(x)$</td>
<td>-eigenfunction or mode shape for angle of rotation due to</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>bending moment</td>
<td></td>
</tr>
</tbody>
</table>

continued on next page
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>-dynamic pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$q_r$</td>
<td>-reflected dynamic pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$r$</td>
<td>-range from the charge center of the blast</td>
<td>m</td>
</tr>
<tr>
<td>$r_m$</td>
<td>-maximum of a response quantity</td>
<td>...</td>
</tr>
<tr>
<td>$R$</td>
<td>-resistance</td>
<td>N</td>
</tr>
<tr>
<td>$R_c$</td>
<td>-critical resistance</td>
<td>N</td>
</tr>
<tr>
<td>$R_{mc}$</td>
<td>-resistance when moment is critical</td>
<td>N</td>
</tr>
<tr>
<td>$R_{sc}$</td>
<td>-resistance when shear is critical</td>
<td>N</td>
</tr>
<tr>
<td>$s$</td>
<td>-slenderness ratio, $s = \sqrt{\alpha \gamma^2}$</td>
<td>-</td>
</tr>
<tr>
<td>$S$</td>
<td>-shear stiffness, usually denoted $GA$, the product of the shear modulus and the area of cross section</td>
<td>N</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>-dimensionless shear stiffness, equal to $SH^2/B = \alpha$</td>
<td>-</td>
</tr>
<tr>
<td>$t$</td>
<td>-time</td>
<td>s</td>
</tr>
<tr>
<td>$t_a$</td>
<td>-time from explosion to blast wavefront arrival</td>
<td>s</td>
</tr>
<tr>
<td>$t_{ax,b}$</td>
<td>-time of blast arrival at the back of a building</td>
<td>s</td>
</tr>
<tr>
<td>$t_{ax,f}$</td>
<td>-time of blast arrival at the front of a building</td>
<td>s</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td>-dimensionless time, $\omega_1 t$</td>
<td>-</td>
</tr>
<tr>
<td>$t_d$</td>
<td>-duration of the positive phase of the blast</td>
<td>s</td>
</tr>
<tr>
<td>$\bar{t}_d$</td>
<td>-dimensionless equivalent of $t_d$, $\omega_1 t_d$</td>
<td>-</td>
</tr>
<tr>
<td>$t_{df;f}$</td>
<td>-time to diffract from the front to the sides of a building</td>
<td>s</td>
</tr>
<tr>
<td>$t_{dn}$</td>
<td>-duration of the negative phase of the blast</td>
<td>s</td>
</tr>
<tr>
<td>$t_{dp}$</td>
<td>-duration of the static overpressure $p$</td>
<td>s</td>
</tr>
<tr>
<td>$t_{dq}$</td>
<td>-duration of the dynamic overpressure $q$</td>
<td>s</td>
</tr>
<tr>
<td>$t_m$</td>
<td>-time from $t_a$ to the first maximum of the response to blast loading</td>
<td>s</td>
</tr>
<tr>
<td>$t_{m;1}$</td>
<td>-time from $t_a$ to the first maximum of the response in case of multiple modes</td>
<td>s</td>
</tr>
<tr>
<td>$\bar{t}_m$</td>
<td>-dimensionless equivalent of $t_m$, $\omega_1 t_m$</td>
<td>-</td>
</tr>
<tr>
<td>$T_1$</td>
<td>-natural period of the $1^{st}$ mode</td>
<td>s</td>
</tr>
<tr>
<td>$T_n$</td>
<td>-natural period of the $n^{th}$ mode</td>
<td>s</td>
</tr>
<tr>
<td>$\bar{T}(t)$</td>
<td>-time solution</td>
<td>-</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-integration domain of $t$</td>
<td>-</td>
</tr>
<tr>
<td>$u$</td>
<td>-wavefront velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$u(t - t_d)$</td>
<td>-unit-step function which is 0 if $t &lt; t_d$ and 1 if $t \geq t_d$</td>
<td>-</td>
</tr>
<tr>
<td>$v$</td>
<td>-shear force</td>
<td>N</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>-dimensionless shear force, $vH^2/B$</td>
<td>-</td>
</tr>
<tr>
<td>$v_b$</td>
<td>-base shear</td>
<td>N</td>
</tr>
<tr>
<td>$v_c$</td>
<td>-critical shear force</td>
<td>N</td>
</tr>
<tr>
<td>$w$</td>
<td>-transverse deflection</td>
<td>m</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>-dimensionless transverse deflection, $\bar{w} = w/H$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>-vector of $\bar{w}$ and $\phi$</td>
<td>-</td>
</tr>
</tbody>
</table>

Dynamic response of high-rise building structures to blast loading
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>-width of the front of a building</td>
<td>m</td>
</tr>
<tr>
<td>$\tilde{W}(x)$</td>
<td>-eigenfunction or mode shape for transverse deflection</td>
<td>-</td>
</tr>
<tr>
<td>$\vec{W}_n$</td>
<td>-vector of $\tilde{W}_n$ and $\Psi_n$ of the $n^{th}$ mode</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{W}_m^T$</td>
<td>-transpose of $\vec{W}_m$</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>-axial coordinate from bottom $x = 0$ to top $x = H$ of beam</td>
<td>m</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>-dimensionless equivalent of $x$, $x/H$</td>
<td>-</td>
</tr>
<tr>
<td>$y$</td>
<td>-transverse deflection</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{y}$</td>
<td>-transverse velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\ddot{y}$</td>
<td>-transverse acceleration</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>-dimensionless transverse deflection $\bar{y} = y/y_c$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{b}_y$</td>
<td>-dimensionless transverse velocity $\bar{b}_y = \dot{y}/(\omega_1 y_c)$</td>
<td>-</td>
</tr>
<tr>
<td>$y_b$</td>
<td>-bending deflection</td>
<td>m</td>
</tr>
<tr>
<td>$y_c$</td>
<td>-critical deflection</td>
<td>m</td>
</tr>
<tr>
<td>$y_{ce}$</td>
<td>-critical elastic deflection</td>
<td>m</td>
</tr>
<tr>
<td>$y_{cp}$</td>
<td>-critical plastic deflection</td>
<td>m</td>
</tr>
<tr>
<td>$y_l$</td>
<td>-deflection due to linear load distribution</td>
<td>m</td>
</tr>
<tr>
<td>$y_m$</td>
<td>-maximum dynamic deflection</td>
<td>m</td>
</tr>
<tr>
<td>$y_{m,pt}$</td>
<td>-maximum static deflection, $F_m/K$</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{y}_m$</td>
<td>-maximum transverse velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$y_{mc}$</td>
<td>-deflection when moment is critical</td>
<td>m</td>
</tr>
<tr>
<td>$y_q$</td>
<td>-deflection due to quadratic load distribution</td>
<td>m</td>
</tr>
<tr>
<td>$y_r$</td>
<td>-deflection due to rotation of the base</td>
<td>m</td>
</tr>
<tr>
<td>$y_s$</td>
<td>-deflection due to shear</td>
<td>m</td>
</tr>
<tr>
<td>$y_{sc}$</td>
<td>-deflection when shear is critical</td>
<td>m</td>
</tr>
<tr>
<td>$y_u$</td>
<td>-deflection due to uniform load distribution</td>
<td>m</td>
</tr>
<tr>
<td>$z$</td>
<td>-scaled distance, $r/(M_{TN}^T)^{1/3}$</td>
<td>kg/m$^{1/3}$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-damping ratio, $C/C_c$</td>
<td>-</td>
</tr>
</tbody>
</table>
Contents

Acknowledgements i

Summary ii

Nederlandse samenvatting iv

List of Symbols vi

1 Introduction 1

1.1 Problem statement 2

1.2 State of the art 2

1.2.1 Guidelines 2

1.2.2 Research 2

1.3 Research outline 3

2 Explosion, blast and interaction 7

2.1 Explosive loading 7

2.2 Blast loading 8

2.2.1 Pressure-time profile 8

2.2.2 Scaled distance 9

2.2.3 Dynamic blast pressure 10

2.3 BLEVE blast loading 10

2.3.1 Blast wave from a vessel burst 11

2.3.2 Blast wave from a BLEVE 12

2.4 Blast-structure interaction 12

2.4.1 Blast waves on an infinite rigid plane 12

2.4.2 Blast waves on a building 13

2.4.3 Blast distribution 17

3 Modeling of high-rise building structures 19

3.1 Example of a realistic building structure 20

3.2 A stability element: the trussed frame 20

3.3 The equivalent beam 21

3.4 The equivalent single degree of freedom system 22

3.4.1 Load-mass factors for slender beams 25

3.4.2 Load-mass factors for simple shear beams 27

3.4.3 Load-mass factors for non-slender beams 28

Dynamic response of high-rise building structures to blast loading
### CONTENTS

3.4.4 Load-mass factors for non-slender beams with rotary spring .......................... 29
3.4.5 Comparison of models .................................................................................. 30

4 Dynamic response of a SDOF system to blast loading ......................................... 33
  4.1 Dynamic response of SDOF systems to idealized blast loading ............................. 34
    4.1.1 Natural frequency of vibration .................................................................. 35
    4.1.2 Damping ............................................................................................... 35
    4.1.3 Material model ...................................................................................... 36
    4.1.4 Forcing function .................................................................................... 36
    4.1.5 Response to idealized blast loads .............................................................. 37
  4.2 Time of maximum deflection and the dynamic load factor ................................. 39
    4.2.1 Time of maximum deflection .................................................................. 41
    4.2.2 The dynamic load factor ........................................................................ 42
    4.2.3 Limits of the dynamic load factor ............................................................... 43
  4.3 The pressure-impulse diagram .......................................................................... 45
    4.3.1 Pressure-impulse diagrams in literature .................................................... 46

5 Dynamic response of a continuous beam to blast loading ..................................... 49
  5.1 The differential equation of motion of a transversely vibrating Timoshenko beam . 50
    5.1.1 Dimensionless variables ........................................................................ 50
    5.1.2 Equation of motion of a Timoshenko beam .............................................. 50
  5.2 Determination of mode shapes and natural frequencies of a beam ....................... 51
  5.3 The mode superposition method: forced response of a beam ............................ 52
    5.3.1 Orthogonality and normalization of eigenfunctions .................................. 53
    5.3.2 Mode superposition method .................................................................. 54
    5.3.3 Modal contribution factors ..................................................................... 56
  5.4 Approximation of response including higher modes ........................................... 59
    5.4.1 Modal combination rules ....................................................................... 59
    5.4.2 Dynamic load factor including higher modes .......................................... 60
    5.4.3 Estimates of maximum response .............................................................. 66
  5.5 Pressure-impulse diagram including higher modes ............................................. 66

6 Results for example building structure .................................................................. 69
  6.1 BLEVE blast loading on example building ......................................................... 69
    6.1.1 Blast distribution ................................................................................... 70
  6.2 Modeling of example building structure ............................................................. 71
    6.2.1 From trussed frame to equivalent beam .................................................... 72
    6.2.2 From equivalent beam to equivalent SDOF system ................................. 72
  6.3 Single degree of freedom response ................................................................... 73
    6.3.1 Pressure-impulse diagram for SDOF response ......................................... 75
  6.4 Response of a continuous beam ....................................................................... 76
    6.4.1 Pressure-impulse diagram including higher modes .................................... 77
  6.5 Comparison of results ...................................................................................... 78
    6.5.1 Comparison of response ........................................................................ 78
    6.5.2 Comparison of $p-i$ diagram .................................................................. 78
    6.5.3 Modification factors for inclusion of higher modes ................................... 78

L. J. van der Meer
Chapter 1

Introduction

This report is about the dynamic response of high-rise building structures to blast loading in general and BLEVE blast loading in particular. In this chapter, a topical problem is stated concerning BLEVE blast loading on high-rise building structures. Subsequently the state of the art is described, including rules, guidelines and ongoing research. Then an outline of the research underlying this report is presented.

In chapter 2 the effects of an explosion and in particular of a blast on a building structure are described. The chapter does not go into detail about the causes of an explosion nor about the physical and chemical circumstances. The blast wave produced by an explosion is studied in terms of pressure in the free field. Finally the influence of obstructions such as buildings on the blast wave pressures is illustrated.

Chapter 3 is dedicated to the modeling of a realistic, but simple building structure in steps of decreasing complexity. After the building structure has been presented, it is reduced to a single stability element, a trussed frame. The stability element is further reduced to an equivalent beam. The final model is a single-degree-of-freedom (SDOF) spring mass system. Every step of modeling is described in detail and the assumptions necessary for each step are given.

Succeeding chapter 3 in which the SDOF system describing the building structure has been established, chapter 4 focusses on the dynamic response of the SDOF system. The difference between static and dynamic response is explained and concepts of dynamic load factor (DLF) and pressure-impulse (p-i-)diagram are introduced.

The response of a SDOF system is only useful if the assumptions made during the steps of modeling are acceptable. An important lack of the SDOF system is the fact that higher modes of vibration are ignored. Therefore, chapter 5 moves up one step in the modeling process by determining the response of a continuous beam with infinite degrees of freedom. First the type of beam is determined, then the equation of motion is given and the mode shapes and natural frequencies are obtained. The forced response of the continuous beam is determined with the mode superposition method. Special attention is given to modal contribution factors for base shear and moment, which give the percentage of participation of a certain mode. The maxima of the time dependent base shear and moment, which are approximated using modal combination rules and assumptions regarding the dynamic load factor of the higher modes of vibration. The pressure-impulse diagram is constructed using these approximation including higher modes of vibration.

In chapter 6 the response of the example building structure, which is modeled in chapter
as a continuous beam and a SDOF system, is determined with and without higher modes of vibration using the methods described in chapters 5 and 4 respectively. The p-i diagrams of both models are compared and the location in the diagram of a BLEVE blast according to both Dutch guidelines and more recent research is given in order to assess damage of the structure.

Conclusions and recommendations are given in chapter 7 and 8 respectively.

1.1 Problem statement

In the Netherlands, residential and office buildings near the railway line from the Rotterdam harbor to Germany, risk being struck by a BLEVE. A BLEVE is a type of explosion that can occur when a train wagon containing a liquified gas is damaged. It is described in more detail in section 2.3. A case study including risk assessment was done on this particular problem by Sjoerd Mannaerts, another graduation student from Eindhoven University of Technology. The graduation project of Sjoerd Mannaerts (2008) focussed on risk assessment and blast wave propagation. This graduation project focusses on the dynamic response of high-rise building structures to explosive loading. Different types of loading from an explosion are described in chapter 2. First the state of the art in the Netherlands, Europe and worldwide is reviewed and an outline of the research project is presented, including limitations.

1.2 State of the art

Guidelines, rules and ongoing research in the field of explosion effects on building structures in the Netherlands, in Europe and worldwide, are reviewed in this section.

1.2.1 Guidelines

In the Netherlands, the Ministry of Housing, Spatial Planning and the Environment, has published a series of documents about toxic and hazardous substances. One part of this series is about explosion effects on structures [22]. It describes idealized blast loads, blast-structure interaction, the response of single-degree-of-freedom (SDOF) systems to blast loading, the dynamic load factor, pressure-impulse diagrams, the translation of a structure to a SDOF dynamic model, the strength of glass windows, debris and the use of empirical data based on explosive events during the Second World War. Empirical data is only valid for building structures up to four stories. For building structures of more than four stories, schematization as a SDOF system is suggested. The explosion itself is described in Chapter 5 of [23].

The Dutch guideline [22] is partly based on [3], an American manual about *Structures to resist the effect of accidental explosions* by the US Army, Navy and Air Force. The manual is almost 1800 pages thick and contains information about blast, fragment and shock loads, principles of dynamic analysis, reinforced concrete design and structural steel design with respect to explosive loading.

1.2.2 Research

Explosion resistance was part of a workshop in Prague, March 2007, by the *European Cooperation in the Field of Scientific and Technical Research*, titled *Urban Habitat Constructions*
under Catastrophic Events [24]. In one of the papers [18] collected in this reference, current research in the area of impact and explosion engineering is summarized and categorized:

- Whole building and building element response and robustness in the face of blast and impact loading. Robustness is defined as the resistance of the structure to progressive collapse due to local damage;
- Response of structures to blast loads from both high explosive events and gas explosions using experimental and numerical methods;
- Response of buildings to underground explosions and the utility of seismic design methodologies to produce blast and impact resistant structures;
- Response of structures and structural elements to impact from missiles and vehicles;
- Development of existing expertise in general 'dynamic loading' towards impact and explosion studies.

In another paper [11] from the same reference, future research that is required in the area of impact and explosion engineering is described. The paper is based mainly on terrorism, but could be applied to explosions from other causes as well. Existing design manuals, for example [3], are listed and commented upon. According to the article, future research should include among others:

- Protection methodology and risk assessment.
- Load and environment definition.
- Material behavior.
- Computational capabilities.
- Behavior and effects of building enclosure.
- Building and structural behavior.
- Combined knowledge of explosion, earthquake and wind engineering.

The author also states that 'the traditional concept of pressure-impulse diagrams should be re-evaluated'(p. 287 of [11]), without explaining why. However, a recent article [12] by the same author about the numerical determination of the pressure-impulse (p-i) diagram makes clear that he thinks of p-i diagrams as a useful tool for damage assessment of structural components and wants to overcome the limitations of the analytical determination of these diagrams.

1.3 Research outline

Objectives of the research project are:

- Verify the assumption that high-rise buildings can be analyzed for blast loading as a SDOF system.
• Apply the concept of pressure-impulse diagram to high-rise building structures.

• Model a high-rise building with different levels of complexity and predict response to blast loading.

• Include higher modes in the p-i diagram.

• Compare results.

A framework is presented in figure 1.1. The research projects focuses on the modeling (from actual structure to (discrete) FE model to continuous model to SDOF model) and the pseudo-analytical part. It is called pseudo-analytical because some parts of the analysis such as the determination of the mode shapes and natural frequencies of the continuous model can only be done with help of numerical software, depending on beam type and boundary conditions. Also, if p-i diagrams for a complex material model and load-time history are needed, numerical methods are necessary. The structural variables material model and damping are between brackets, because the material model is linear-elastic throughout this investigation and damping is ignored (conservative). The part concerning coupled numerical response is shown in dashed lines, because it is not within the scope of this research project. An example of this approach can be found in [15].
Figure 1.1: Framework of graduation project.

- Actual blast
  - literature
  - (experiments)
  - CFD

- Actual structure
  - experiments

- Actual response
  - Coupled response

- FE structural model
  - -2D
  - -3D

- Continuous model
  - mode superposition analysis (analytical)

- SDOF model
  - -p – i – diagram

- Blast variable
  - -idealised blast load

- Structural variables
  - -frequency ratio
  - -slenderness ratio
  - -(material model)
  - -(damping)

- Numerical MDOF response
  - compare

- Continuous \( \infty \text{DOF} \) response
  - \(-p – i – diagram\)

- Coupled FE model of blast and structure
  - coupled transient dynamic analysis

- Continuous

- SDOF response

- Complex

- Simple

- Assumptions
Chapter 2

Explosive loading, blast loading and blast-structure interaction

In this chapter, explosive loading is described. In section 2.1 different types of loading on a building structure, caused by an explosion are described. The blast wave, one of these types of loading, is described in more detail in section 2.2. It is the only load effect considered in the rest of the report. Special attention is given to blast from a BLEVE in section 2.3. In section 2.4 the influence of a building structure as a rigid object on the blast wave is illustrated and the blast loading on the building structure is given in terms of pressure and impulse.

2.1 Explosive loading

Explosion effects on building structures can be divided into primary and secondary effects. The primary effects include:

1. Airblast: the blast wave causes a pressure increase of the air surrounding a building structure and also a blast wind. More attention to this phenomena will be given in section 2.2.

2. Direct groundshock: an explosive which is buried completely or partly below the ground surface, will cause a groundshock. This is a horizontal (and vertical, depending on the location of the explosion with regard to the structural foundation) vibration of the ground, similar to an earthquake but with a different frequency.

3. Heat: a part of the explosive energy is converted to heat. Building materials are weakened at increased temperature. Heat can cause fire if the temperature is high enough.

4. Primary fragments: fragments from the explosive source which are thrown into the air at high velocity (for example wall fragments of an exploded gas tank). Fragments can hit people or buildings near the explosion. They are not a direct threat to the bearing structure of the building, which is usually covered by a facade. However, they may destroy windows and glass facades and cause victims among inhabitants and passers-by.

An overview of the explosion effects on buildings, summarized by the author of this report, is given in figure 2.1. Secondary explosion effects, such as secondary fragments and blast-induced groundshock are not considered.
2.2 Blast loading

An elaborate description of explosions and blast waves is given in reference [19] of which some parts that are relevant to this report are explained below.

During an explosion an oxidation reaction occurs that is called combustion. When explosive materials decompose at a rate below the speed of sound (subsonic), the combustion process is called deflagration. Gas and dust explosions are of this type. Under specific conditions a deflagration to detonation transition can occur. Detonation is the other form of reaction which produces a high intensity shock wave. The reaction rate is 4-25 times faster than the speed of sound (supersonic). An explosion of TNT is an example of a detonation. The two types of explosions have significantly different pressure-time profiles and will therefore be treated separately in this report.

2.2.1 Pressure-time profile

The meaning of a few important blast parameters can be seen in figure 2.2. In this figure, $t_a$ is the arrival time of the blast, $t_d$ is the positive (overpressure) phase duration of the blast,
2.2 Blast loading

$t_{d,n}$ is the negative (under-pressure i.e. negative overpressure) phase duration of the blast, $p_0$ is the ambient pressure, $p_m$ is the peak static overpressure, $p_{n,m}$ is the maximum value of under-pressure, $i$ is the impulse of the positive phase of the pressure-time curve and $i_n$ is the impulse of the negative phase of the pressure-time curve. The pressure-time profile in the figure is that of a detonation. The deflagration pressure-time profile is different as can be seen in figure 2.3. The deflagration pressure-time profile will transform to a detonation profile if the peak-static overpressure exceeds the value of approximately 3kPa ($p_m > 3$ kPa).

\[ t_{d,n} \]
\[ p_0 \]
\[ p_m \]
\[ p_{n,m} \]
\[ t_d \]
\[ t_{d,n} \]

Figure 2.2: Blast wave pressure time profile, taken from [19].

\[ p(t) \]
\[ p_m \]
\[ i \]
\[ t \]

(a) Detonation

\[ p(t) \]
\[ p_m \]
\[ t \]

(b) Deflagration

Figure 2.3: Detonation and deflagration pressure-time history.

2.2.2 Scaled distance

An important parameter for determination of air-blast pressure and impulse is the scaled distance $z$, which is dependent of the distance $r$ from the charge center in meters and the charge mass $M_{TNT}$ expressed in kilograms of TNT:

\[ z = \frac{r}{(M_{TNT})^{1/3}} \] (2.1)

Other blast parameters can conveniently be plotted against the scaled distance. Such graphs can be found in reference [3]. In figure 2.4 the peak static overpressure $p$, impulse $i$, time of

L. J. van der Meer
Figure 2.4: Blast parameters for TNT equivalent explosions \[3\]

blast arrival \(t_a\) and positive phase duration \(t_d\) are shown depending on the scaled distance \(z\).

### 2.2.3 Dynamic blast pressure

Apart from a static overpressure \(p\), there is also a dynamic pressure \(q\) (i.e. blast wind) associated with a blast wave. This dynamic pressure is higher than the static overpressure for small scaled distance and lower than the static overpressure for large scaled distance. The positive phase duration of the dynamic pressure \(t_{dq}\) and static overpressure \(t_{dp}\) is also different, but in this report it will be assumed that both durations are equal. In figure 2.5 the static overpressure \(p\) and the dynamic pressure \(q\) are displayed versus the scaled distance \(z\).

### 2.3 BLEVE blast loading

BLEVE is an abbreviation for *boiling liquid expanding vapor explosion*. According to \[5\], a BLEVE is the explosive release of expanding vapor and boiling liquid when a container holding a pressure-liquified gas fails catastrophically. Catastrophic failure means that the container is fully opened to release its contents nearly instantaneously. This total loss of containment can take place for a number of reasons including flawed materials, fatigue, corrosion, poor manufacture, thermal stresses, pressure stresses and reduction in material strength due to high wall temperatures. Other definitions state that the liquid temperature must be above the atmospheric *superheat limit*, but BLEVEs were also observed for liquid temperatures beneath this limit. However if the material is a pressure-liquified gas, its temperature at atmospheric pressure must be above the superheat limit if explosive boiling is to occur. According to \[1\] a BLEVE gives rise to the following:

- Splashing of some of the liquid to form short-lived pools, which would be on fire if the liquid is flammable.
- Blast wave.
2.3 BLEVE blast loading

- Flying fragments ('missiles').
- Fire or toxic gas release. If the pressure-liquified vapor is flammable, as is often the case, the BLEVE leads to a fireball. When the material undergoing BLEVE is toxic, adverse impacts include toxic gas dispersion.

A list of BLEVE accidents is also available in reference [1]. The accidents concern for example road and rail transport of pressure-liquified gasses (PLGs) or storage of PLGs at petrol stations or near factories/plants. The worst BLEVE accident occurred in 1984 in Mexico City and was responsible for 650 deaths and over 6400 injured.

2.3.1 Blast wave from a vessel burst

In this report, only the blast wave produced by the BLEVE is considered. It should be noted that the other effects can do significant damage to buildings and people near to or in them.

In the Netherlands the methods for calculating blast effects from vessel bursts in general are divided in two categories (paragraph 7.3.2 of [23]):

- Methods of solving the differential equations of fluid mechanics by which the shock wave can be described. Analytical solving is only possible when a lot of simplifications are made. Numerical solving can be done by means of finite-difference methods or Eulerian-Lagrangian codes. However, these methods require a lot of knowledge and give results for specific situations.

- Generalized methods based on thermodynamic terms and the available energy for blast generation. Two methods developed in this category are described.

  1. The first method is based on high explosives, for which the blast effects are known. Unfortunately, blast effects from vessel bursts include lower initial overpressures, a slower decay of the overpressure with distance, longer positive phase durations, much larger negative phases and strong secondary shocks. This method gives

L. J. van der Meer
reasonable results only at far range, which means larger than 10-20 times the vessel diameter.

2. The second method is Baker’s method, based on research by W. E. Baker in 1977, which is comparable to the previous method at far range. At close range experimentally verified results for ideal gases are used. The method includes a correction method for the influence of a nearby surface and the shape of the vessel.

2.3.2 Blast wave from a BLEVE

For a BLEVE, which is a special case of a vessel burst, Baker’s method includes a modification for vessels with flashing liquids. In [23] this last method is selected and described in paragraph 7.5.2 and in the example on page 7.63. The method might be non-conservative because it results in too short positive phase durations.

More recently, in [21], a new method to calculate the blast effects from BLEVE was introduced. By using acoustic blast modeling, it was found that the blast effects depend strongly upon the exact release and evaporation rate of the liquified gas. If it is assumed that the pressure vessel nearly instantaneously disintegrates, then the release rate approximates infinity and the evaporation rate of the superheated liquid is fully determined by the rate at which the developing vapor can expand by moving the mass of vapor and the surrounding air. This is called expansion-controlled evaporation. The assumption of nearly instantaneous disintegration is conservative. In [21], BLEVE blast charts for propane in half space are shown, which are determined by gas dynamic modeling using the assumption of expansion-controlled evaporation. Compared to Baker’s method, presented in [23], the blast parameters are less conservative. It should be noted that the modification factors for a nearby surface and the shape of the vessel are not included in the blast parameters of [21]. This means that the parameters are valid for a hemispherical blast originating at the surface. When the modification factors are applied to Van den Berg’s method, the overpressures are similar to those of Baker’s method for far range and the positive phase durations (and consequently impulses) are larger.

In this report both a BLEVE following [23] and [21] will be used in example calculations.

2.4 Blast-structure interaction

Blast parameters are given in literature for TNT blasts in free space (free-air burst) or half space (surface burst). In case of a surface burst, parameters for free-air bursts should be multiplied by a reflection factor of 1.8 [19]. Theoretically this factor should equal 2, but some energy is dissipated in the deformation of the surface. A reflection factor of 1.8 gives good agreement with experimental results.

2.4.1 Blast waves on an infinite rigid plane

If a blast wave with a certain (time dependent) static overpressure $p(t)$, dynamic pressure $q(t)$ and impulse $i(t)$ encounters an infinite, rigid plane, it is reflected. Because the incident wave and the reflected wave coincide, the pressure on the rigid plane is higher than the pressure of the incident wave and is denoted $p_r(t)$, reflected overpressure. The reflected impulse associated with the reflected overpressure is denoted $i_r(t)$. 

Dynamic response of high-rise building structures to blast loading
The reflected overpressure and impulse are dependent on the angle of incidence $\alpha_i$ of the blast wave, which is the angle between the blast wavefront and the target surface. The reflection coefficient $C_r$ is defined as the ratio of the reflected overpressure and the incident overpressure (overpressure if the wave were not obstructed, sum of static overpressure $p(t)$ and dynamic pressure $q(t)$). If the angle of incidence is $90^\circ$, the blast wave travels alongside the plane and the overpressure is equal to the static overpressure, which is also referred to in literature as side-on overpressure. The dynamic pressure $q(t)$ works in this case only in the direction parallel to the plane and is therefore (almost) not obstructed. The friction between the moving air and the rigid plane is negligible. For all $\alpha_i$ between $0 - 90^\circ$, the reflected pressure $p_r(t)$ is dependent on the static overpressure $p(t)$ and the dynamic pressure $q(t)$. The reflection coefficient $C_r$ is shown versus the angle of incidence $\alpha_i$ in figure 2.6 for a detonation. As the figure points out, the reflection coefficient is dependent on the static overpressure $p(t)$. The reflection coefficient also depends on the type of explosion, detonation or deflagration. For a deflagration, figures are available in [3] and [22].

Starting at an angle of incidence $\alpha_i$ of approximately $40^\circ$, depending on the static overpressure, the reflection coefficient $C_r$ increases and has a local maximum which is sometimes higher than the reflection coefficient at $\alpha_i = 0^\circ$. This is due to Mach reflection, which occurs when the reflected wave catches up and fuses with the incident wave at some point above the reflecting surface to produce a third wavefront called the Mach stem [19]. According to [22], these local peak values are the result of theoretical derivations that could not be verified by experiments. Therefore it is suggested that these peak values are flattened for simple calculations.

### 2.4.2 Blast waves on a building

If a blast wave encounters a building, the building is loaded by a pressure, which is a summation of two parts: the first part is due to the static overpressure and the second part is due to the dynamic pressure or blast wind. These pressures are shown in figure 2.7. Both static overpressure and dynamic pressure are a function of time, but also of the unobstructed distance to the charge center. Time and distance are coupled by the velocity at which the blast wave is propagating, although this velocity is not constant.

Due to the coupling of time and distance, the part of translational pressure $p_{trans}(t)$ which is due to the static overpressure $p(t)$ depends on the size of the building (or other object) compared to the positive phase duration of the blast $t_d$. The translation pressure is defined as the nett pressure due to blast loading.

If the building is very small in one direction (width or height), the overpressure on the front and the back of the building is approximately equal and hence the translational pressure is approximately zero (figure 2.8a overpressure).

If the building is larger (width or height) compared to the blast duration and the angle of incidence is $0^\circ$, the front of the building is loaded with the reflected static overpressure $p_{r,sl}(t)$. The sides are loaded with the static overpressure. However, this pressure works in opposite direction on both sides and does not result in translational pressure. The pressure in the blast wave traveling along the sides equals the static overpressure (dynamic pressure is considered further on). This pressure is lower than the reflected static overpressure, which causes large local pressure differences. These pressure differences cause the blast wave to diffract around the building. When the diffraction from the front to the sides of the building is completed, the static pressure on the front is decreased from the reflected static overpressure to the static
Figure 2.6: Reflection coefficient vs angle of incidence for a detonation (figure 3-3 from TM5-855, predecessor of [3])

Overpressure. Similarly, pressure differences cause the blast wave to diffract from the sides to the back of the building, once the blast wave front passes by the back of the building. Translational pressure resulting from diffraction is significant only when the time to diffract around the building is approximately equal to or larger than the positive phase duration of the blast. For this type of building and blast, the building is called a diffraction target (figure 2.8b, overpressure).

If the positive phase duration of the blast is much smaller than the time to diffract around the building, the building is loaded sequentially. A sequentially loaded building feels a pressure from the blast wave either at the front, at the sides or at the back, but not at the same time (figure 2.8c, overpressure).

Summarizing the effects of static overpressure on a building, three essentially different types can be distinguished:

- No translational pressure (long positive phase duration compared to time of diffraction).
- Translational pressure due to difference between overpressure on front and back (positive phase duration approximately equal to or larger than the time of diffraction).
- Translational pressure due to local overpressure only (small positive phase duration compared to time of diffraction).

The effects of static overpressure determine the translational pressure on a building together with effects of the dynamic pressure or blast wind. Whereas the static overpressure is...
caused by an increased density of the air, the dynamic pressure is the result of the movement of air away from the blast source. Similar to ordinary wind, the blast wind causes a pressure on the front of a building and a negative pressure (suction) at the back of a building. Both pressures are translational in the same direction. Because of their translational nature, these pressure are called drag pressures. The drag pressure on a building or object is equal to the dynamic pressure multiplied by a drag factor $C_d$. Drag factors can be found in [22], table 4-1, for loads on the front side of an object. The drag coefficient for suction at the back of the object is smaller and negative ($C_d = -0.3$ for a boxed-shape object and $\alpha = 0^\circ$).

The dynamic pressure on the front is reflected similar to the static overpressure. In literature, the reflected overpressure $p_r(t)$ includes both the reflected static overpressure $p_{r, st}(t)$ and the reflected dynamic pressure $q_r(t)$. This is rather confusing and leads to mistakes such as in figure 3.27 of [19], where the drag force is superimposed on the force from the reflected overpressure. If a building is small (height or width), the pushing and sucking drag pressures on front and back are applied at approximately the same time. Because of the small dimensions of the front face, no reflection occurs (figure 2.8a, dynamic pressure).

For the other types of buildings, the behavior is similar as for static overpressure (figure 2.8b-2.8c, dynamic pressure), except for the direction of the pressure on the back of the building, which is opposite of the direction of static overpressure. The reason for this similarity is found in the similarity of the pressure-time history of static overpressure and dynamic pressure.

Summarizing the effects of both static overpressure and dynamic pressure, the target (building with respect to blast) can be divided into three categories:

- Drag target: translational pressure from dynamic pressure, crushing from static overpressure (long positive phase duration compared to time of diffraction). See figure 2.8a.

- Diffraction target: translational pressure due to both dynamic pressure and static overpressure (positive phase duration approximately equal to or larger than time of diffraction). See figure 2.8b.

- Sequentially loaded target: translational pressure due to both dynamic pressure and static overpressure, but only local (positive phase duration much smaller than time of diffraction). See figure 2.8c.
Other objects near the blast source as well as inequalities in the surface all influence the blast pressure-time history. It can be concluded that a lot of information is needed about the explosive and its location, the geometry of the target and its surroundings to be able to quantify the pressure-time history. In complex built environments, a lot of reflections can occur, leading to a complex pressure-time history \[20\]. It is impossible to calculate all reflections and possible re-reflections under different angles of incidence as well as the influence of the geometry of the target and surrounding structures on the reflected overpressure analytically. In this case it is necessary to do experiments or to use CFD, Computational Fluid Dynamics, to simulate the blast. In uncoupled CFD buildings are modeled as rigid objects. In coupled CFD the influence of movement and failure of building elements on the blast wave propagation is also considered. An example can be found in \[15\]. However, a lot of input is required by the user as well as expertise in finite element modeling.

Dynamic response of high-rise building structures to blast loading

---

### Figure 2.8: Target categories for blast loading on a building.

<table>
<thead>
<tr>
<th>Target</th>
<th>Translational pressure</th>
<th>Idealized pulse shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overpressure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dyn. pressure</td>
<td></td>
</tr>
</tbody>
</table>

(a) Drag target

![Drag target diagram]

(b) Diffraction target

![Diffraction target diagram]

(c) Sequentially loaded target

![Sequentially loaded target diagram]
2.4 Blast-structure interaction

2.4.3 Blast distribution

Using the knowledge from the previous sections and the methods from literature to obtain quantitative information about the translational pressure and impulse on a building, the distribution of pressure and impulse over the facade of the building has not been taken into account. For blasts at large range, the distribution is approximately uniform. At shorter ranges, this assumption is not valid, since the blast propagation from a surface burst is hemispherical. At close range, the range (and scaled distance) to different heights on the facade is significantly unequal. For example, if the building is 65m high and the range from the surface burst to the base of the building is 20m, the range from the surface burst to the top is 68m. This difference in range (and scaled distance) does not only result in different pressure and impulse, but also in a different arrival time $t_a$ of the blast and a different positive phase duration $t_d$. So, by calculating the range $r$ and scaled distance $z$ from the blast source to every point on the facade and taking into account the angle of incidence $\alpha_i$, blast parameters can be determined using figures 2.9 and 2.6. Figure 2.9 is similar to figure 2.4, but including the reflected pressure $p_r$ and impulse $i_r$. Also the wavefront speed $u$ is shown which can be used in calculations of diffraction.

![Shock wave parameters for hemispherical TNT surface bursts at sea level](image)

Figure 2.9: Shock wave parameters for hemispherical TNT surface bursts at sea level (figure 3-7 from TM5-855, predecessor of [3])

Using Baker’s method and figures 2.6 and 2.9, BLEVE blast parameters at a range of 20m and 50m were calculated, for a propane BLEVE of 108 m$^3$ and 70 volume % liquid flashing from 329K. These blast parameters were used to get a general idea of the blast parameters’ distribution over the facade of the building. Reflected pressure $p_r$ and impulse $i_r$ are shown in figure 2.10a and time of blast arrival $t_a$ and time of blast departure $t_a + t_d$ in figure 2.10b. From the first figure it can be seen that the distribution is not at all uniform at $r = 20m$ and approximately uniform at $r = 50m$. From the latter figure it can be concluded that the building is loaded sequentially at both ranges.
Figure 2.10: Distribution of blast parameters over a facade at ranges of 20m and 50m, based on [3].
Chapter 3

Modeling of high-rise building structures subjected to dynamic loading

In this chapter, an example building with a steel trussed frame structure is presented. The building is used as an example building in chapter 9 of [10], in which it is subjected to preliminary design calculations for static and wind loading. The beam, column and diagonal dimensions from these calculations are assumed to be correct. In section 3.1 the example building is described. After certain assumptions, the building structure is reduced to a single stability element, a trussed frame, for 2D calculations in section 3.2. Section 3.3 further reduces the trussed frame to an equivalent beam including the assumptions necessary for this simplification. Finally, in section 3.4, the beam is reduced to a single-degree-of-freedom (SDOF) system, which is a concentrated mass on a spring loaded by a time varying concentrated force. An overview of this chapter is shown in figure 3.1, in which $M$ is a concentrated mass, $B$ is the bending stiffness, $S$ is the shear stiffness, $F_{eq}$ is an equivalent concentrated force, $M_{eq}$ is an equivalent concentrated mass and $K_{eq}$ is an equivalent stiffness.

Figure 3.1: Overview of the building section and subsequent models in this chapter.
3.1 Example of a realistic building structure

The building plan is shown in figure 3.2. The building dimensions are \( L \times W \times H = 21.6\text{m} \times 50.4\text{m} \times 64.8\text{m} \). The columns are placed on a grid of 7.2m\( \times \)7.2m and the story height \( h = 3.6\text{m} \). Columns are HD400\( \times \)744, diagonals are HF RHS300\( \times \)12.5 and beams are HEB360. The stability is assured by 6 stability elements in the considered direction (the weak axis) and 4 in the other direction. The stability elements are K-shaped trussed frames. The rotary stiffness of the foundation of a single trussed frame is \( C_\phi = 5.44 \cdot 10^7 \text{kNm} \). The steel quality is S355. The mass carried by a single stability element is \( M = 2.06 \cdot 10^6 \text{kg} \).

![modeled part](image)

Figure 3.2: Building plan.

3.2 A stability element: the trussed frame

The following assumptions are made to convert the 3D building to a 2D trussed frame:

- The load distribution over the width \( W \) of the building is symmetric, so that there is no torsion.
- The floors are infinitely stiff in their plane.
- The total load is divided equally over the 6 trussed frames.

The distributed reflected overpressure \( p_r(x,t) \) on the building is converted to a number of concentrated forces \( F(x,t) \) applied at floor levels. Since the total load is divided equally over 6 trussed frames, the load \( F(x,t) \) applied at a floor level is approximately equal to

\[
F(x,t) = \frac{p_r(x,t)Wh}{6}
\]

in which \( W \) is the building width and \( h \) the story height. This expression is exact if the load distribution is linear. Otherwise it is an approximation.

One story of the trussed frame is shown in figure 3.3.

The trussed frame model was built in the finite element (FE) program ANSYS 11.0 using link elements with an axial stiffness (LINK1) and connected by hinges. The masses (MASS21) are lumped to the connections and the link elements are weightless. The rotary stiffness \( C_\phi \) of...
the foundation (if taken into account) is converted to two translational springs (COMBIN14) with stiffness $K$ between the columns of the trussed frame and the constraints. The columns carrying the facade and the floors between these columns and the trussed frame are not modeled (see figure 3.2), except for the part of the mass that is carried by the trussed frame. The trussed frame was modeled in ANSYS 11.0 to obtain the static deflection under wind loading as well as the natural frequencies of the trussed frame, which is necessary to compare it to other models. The ANSYS 11.0 input file can be found in appendix B.

### 3.3 The equivalent beam

To reduce the trussed frame to an equivalent beam, the following assumptions are made:

- All connections are hinges.
- The bending stiffness $B$ is completely determined by the axial stiffness $EA_c$ of the columns (figure 3.4a):
  \[ B = \frac{\ell^2}{2} EA_c \]  
  (3.2)
- The shear stiffness $S$ is completely determined by the axial stiffness $EA_d$ of the diagonals (figure 3.4b):
  \[ S = \frac{\ell^2 h}{2d^3} EA_d \]  
  (3.3)
- The beams do not contribute to the shear stiffness, because they are assumed to be infinitely stiff in their plane.
- The bending stiffness, shear stiffness and mass are constant over the height of the beam. The fact that columns in actual buildings have smaller sections on upper stories is neglected.

In the FE program ANSYS 11.0 the model was built with beam elements (BEAM3) and lumped masses (MASS21). The beam elements were configured to include shear deflection. The ANSYS 11.0 input file can be found in appendix B. The trussed frame and the equivalent beam are compared by means of static analysis under wind loading and modal analysis in...
Figure 3.4: Bending and shear in a trussed frame.

Table 3.1: The static analysis results in a maximum displacement of the top. The modal analysis gives the natural frequencies and mode shapes. Vertical degrees of freedom are constrained. The first four natural frequencies are compared. Uniform loading is assumed and base rotation is ignored. The error percentage is defined as

\[ \text{EB \ - \ TF} \ \cdot \ 100\% \]

in which EB is the value for the equivalent beam and TF the value for the trussed frame. The static deflection is overestimated (+9.5%) by the equivalent beam model and the first natural frequency is underestimated (−4.4%). This indicates that the equivalent beam model is less stiff than the trussed frame, which is conservative. The higher natural frequencies are slightly overestimated. This inconsistency might be explained by the reduced accuracy of the finite element solution for higher modes.

3.4 The equivalent single degree of freedom system

In this section, the equivalent beam is reduced to a single-degree-of-freedom (SDOF) system, which is the most basic dynamic system that allows easy response calculations. A degree of freedom is a single translation or rotation of a concentrated mass. The beam has infinite degrees of freedom if it is continuous (distributed mass) or it has the sum of degrees of
freedom of all lumped masses. If only the transverse deflection is considered, then the number of lumped masses is equal to the number of degrees of freedom. A beam (and a structure in general) has as many natural frequencies and mode shapes as it has degrees of freedom. A mode shape or mode of vibration, is a particular shape that the structure adopts during vibration at a specific natural frequency. The SDOF system vibrates only at the first natural frequency with a single shape, ideally the first mode shape. The following assumptions are made for the reduction of the equivalent beam to the equivalent SDOF system:

- The dynamic response is determined completely by the first frequency and a single response mode, ideally the first mode shape.

- The SDOF system is energy equivalent to the beam in this single response mode.

The second assumption is used to calculate the equivalent mass $M_{eq}$ and stiffness $K_{eq}$ of the SDOF system as well as the equivalent force $F_{eq}$ on the system. The lumped mass of the SDOF system can be made equivalent to the distributed mass of the beam, by assuming that both have the same kinetic energy. The spring stiffness of the SDOF system can be made equivalent to the bending stiffness, shear stiffness and the rotary stiffness of the foundation of the beam, by assuming that both have the same strain energy. The concentrated force on the SDOF system can be made equivalent to the distributed force on the beam, by assuming that the work done by both forces is the same. The energy equivalence results in conversion factors for equivalent mass, stiffness and force in the SDOF system. For simplicity, it is assumed that the beam is continuous, i.e. it has a uniform distributed mass, a distributed load and a constant bending and shear stiffness. In a real building, more mass is concentrated in the floors than between the floors, so a real building is a continuous system with added mass at the floor levels. To determine the conversion factors for mass, stiffness and force, consider the continuous beam and equivalent SDOF system as shown in figure 3.5, in which $H$ is the building height, $P(x)$ is the distributed load, $y_m$ is the maximum deflection at the top, $\dot{y}_m$ is the maximum velocity at the top (first time derivative of $y_m$), $y(x)$ is the shape function for transverse deflection, $B$ is the bending stiffness, $S$ is the shear stiffness, $C_\phi$ is rotary stiffness of the foundation, $m$ is the distributed mass, $F_{eq}$ is the equivalent force, $M_{eq}$ is the equivalent mass and $K_{eq}$ is the equivalent stiffness.

The conversion factors can be found in literature [4] as the mass factor $K_M$, resistance factor $K_R$ and load factor $K_L$. The mass factor is obtained from the kinetic energy equation, the resistance factor is obtained from the strain energy equation and the load factor is obtained from the work done equation. The conversion factors are defined as follows:

\begin{align}
K_M &= \frac{E_{k,\text{beam}}}{E_{k,\text{sdof}}} = \frac{m}{\frac{1}{2} \int_{x=0}^{x=H} \dot{y}(x)^2 dx} \frac{\frac{1}{2} \dot{y}(H)^2}{\frac{1}{2} \dot{y}(H)^2} \\
K_R &= \frac{E_{s,\text{beam}}}{E_{s,\text{sdof}}} = \frac{\frac{1}{2} \int_{x=0}^{x=H} [v(x)]^2 dx + \frac{1}{2R} \int_{x=0}^{x=H} [\mu(x)]^2 dx + \frac{C_\phi}{2} \left(\frac{y(H)}{H}\right)^2}{\frac{1}{2} Ky(H)^2} \\
K_L &= \frac{E_{w,\text{beam}}}{E_{w,\text{sdof}}} = \frac{\int_{x=0}^{x=H} y(x)P(x) dx}{y(H) \int_{x=0}^{x=H} P(x) dx}
\end{align}

L. J. van der Meer
In these equations, $x$ is the axial coordinate of the beam, which is $H$ at the top and 0 at the base (the $x$–coordinate can also be taken from top to base as long as this is done consequently). $E_k$, $E_s$ and $E_w$ are the kinetic energy, strain energy and work done respectively.

Fortunately, it is not necessary to calculate $K_R$. It can be proven that $K_R = K_L$. In order to prove this, a deflected shape function $y(x)$ must be defined. This shape can be either the first mode shape of the continuous beam or the deflected shape of the continuous beam under the load $P(x)$. In the first case, the natural frequency of the beam and equivalent SDOF system will match and the static deflection will be approximated. In the latter case, the static deflection of both systems will match and the first natural frequency will be approximated. Determining mode shapes for continuous beams is quite complicated, as can be seen in section 5.2. Therefore, in this section the static deflected shape will be used as shape function. Before making the SDOF system energy equivalent to the continuous beam, we reduce the continuous beam to a non-equivalent SDOF system. This system has the following properties:

$$
F = \int_0^H P(x) dx = P_m H \\
M = \frac{m H}{y(H)} \\
K = \frac{F}{y(H)} \\
y(H) = \frac{F}{K} \tag{3.8}
$$

In equation (3.8), $y(H)$ is the static deflection at the top due to the load $P(x)$. The equivalent system should have the same static deflection $y(H)$ at the top as the non-equivalent system, because both are defined to have the same static deflection at the top as the continuous beam. This results in:

$$
y(H) = \frac{F_{eq}}{K_{eq}} = \frac{K_L F}{K_R K} = \frac{F}{K} \quad \rightarrow \quad K_L = K_R \tag{3.9}
$$

Figure 3.5: Continuous beam vs equivalent SDOF system.
It can be concluded that the energy equivalence does not influence the static deflection. However, it does influence the natural circular frequency $\omega_1$:

$$\omega_1 = \sqrt{\frac{K_{eq}}{K_{M}}} = \sqrt{\frac{K_{L}K_{M}}{K_{LM}M}} = \sqrt{\frac{K}{K_{LM}M}}$$

The load-mass factor $K_{LM}$ is the only necessary factor to transform a continuous structure into an equivalent SDOF system. The load-mass factor is found in literature, such as table 5.1-5.6 in [4], table 10.2-10.3 in [19] or table 3.12-3.13 in [3]. The number of cases for which these factors can be found in literature are unfortunately limited. A continuous beam that is equivalent to a trussed frame is generally a non-slender beam, which means that the shear deflection cannot be neglected. The load-mass factors in literature are meant for structural elements that, instead of entire structures, are generally slender. So load-mass factors for trussed-frame equivalent SDOF systems can not be found in literature and have to be calculated. This is done in this section for three load cases: a uniform distributed load, a linear distributed load and a quadratic distributed load. If the rotary stiffness of the foundation is taken into account as well, an extra term is added to the static deflection, which also influences the load-mass factor. In the next subsections, the load-mass factor will be calculated for an Euler-Bernouilli (slender) beam, a simple shear beam (non-slender without bending), a Timoshenko beam (non-slender) and finally a Timoshenko beam with a rotary spring at the base.

### 3.4.1 Load-mass factors for slender beams

Slender beams have negligible shear deflection. The deflected shape of a slender beam can be found by using the Euler-Bernouilli theory. By integrating the load function $P(x)$ four times, the shear force, moment, angle of rotation and deflection are obtained. The boundary conditions are used to determine the integration constants. In chapter 2 it was concluded that the load distribution $P(x)$ is more or less uniform for long-range surface explosions (range much larger than the height of the building), while for short-range surface explosions it is more concentrated at the base. The actual load distribution is not a simple function and therefore three simple load functions are used, which approximate short- and long-range load distributions.

$$P_u(x) = \frac{P_m}{x}$$  \hspace{1cm} (uniform distributed load, long-range)  \hspace{1cm} (3.11)

$$P_l(x) = \frac{2P_m}{H}(-x + H)$$  \hspace{1cm} (linear distributed load, short-range)  \hspace{1cm} (3.12)

$$P_q(x) = \frac{3P_m}{H^2}(x^2 - 2Hx + H^2)$$  \hspace{1cm} (quadratic distributed load, short-range)  \hspace{1cm} (3.13)

The load functions $P(x)$ are chosen so that they cause equal shear force at the base (the load functions have the same area), see figure 3.6.
With the \( x \)-coordinate from base to top, the following boundary conditions have to be satisfied:

\[
\begin{align*}
y(0) &= 0 \quad (3.14) \\
\frac{dy}{dx}(0) &= 0 \quad (3.15) \\
B \frac{d^2y}{dx^2}(H) &= 0 \quad (3.16) \\
B \frac{d^3y}{dx^3}(H) &= 0 \quad (3.17)
\end{align*}
\]

Respectively, deflection at the base is 0, angle of rotation at the base is 0, moment at the top is 0 and shear force at the top is 0. \( B \) is the bending stiffness \( EI_z \) which can be ignored in the boundary condition since \( EI_z \neq 0 \). Using these boundary conditions with the load functions, the shape functions are obtained:

\[
\begin{align*}
y_{bu}(x) &= \frac{P_m}{B} \left[ \frac{x^4}{24} - \frac{Hx^3}{6} + \frac{H^2x^2}{4} \right] \\
y_{bl}(x) &= \frac{P_m}{BH} \left[ \frac{x^5}{60} + \frac{Hx^4}{12} - \frac{H^2x^3}{6} + \frac{H^3x^2}{6} \right] \\
y_{bq}(x) &= \frac{P_m}{BH^2} \left[ \frac{x^6}{120} - \frac{Hx^5}{20} + \frac{H^2x^4}{8} - \frac{H^3x^3}{6} + \frac{H^4x^2}{8} \right] \quad (3.18)
\end{align*}
\]

These are the deflected shapes for uniform load distribution \( y_u \), linear load distribution \( y_l \) and quadratic load distribution \( y_q \). The functions are plotted in figure 3.7. These shape functions can be used to calculate the mass factor (3.5) and load factor (3.7).

In (3.19), \( \dot{y} \) is the variable instead of \( y \), which is the time derivative of \( y \). However, if the response of both the continuous beam and the equivalent SDOF system are written as a product of a spatial function and a time function, which is the same for both systems, then the ratio of the spatial functions will remain the same if the time function is differentiated:

\[
\begin{align*}
\text{Beam:} & \quad \begin{cases} f(x)f(t) & \dot{f}(x) = f(x)\dot{f}(t) \end{cases} & \dot{f}(x) &= f(x) \\
\text{SDOF:} & \quad \begin{cases} f(H)f(t) & \dot{f}(H) = f(H)\dot{f}(t) \end{cases} & \frac{\dot{f}(x)}{f(H)} &= \frac{f(x)}{\dot{f}(H)} \quad (3.19)
\end{align*}
\]
So the time functions or derivatives of time functions of both systems will cancel each other when the load or mass factor is calculated.

In Table 3.2 the load-mass factors and the natural frequencies $\omega_1$ for the three load functions are shown. The exact value for the natural frequency $\omega_1 = 3.516\sqrt{\frac{B}{mH^2}}$ [17].

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Load distribution} & K_M & K_L & K_{LM} & \omega_1 \cdot \sqrt{\frac{mH^2}{H}} \\
\hline
\text{Uniform} & \frac{104}{405} \approx 0.26 & \frac{2}{5} = 0.40 & \frac{52}{81} \approx 0.64 & 3.53 \\
\text{Linear} & \frac{125}{362} \approx 0.27 & \frac{5}{27} \approx 0.24 & \frac{25}{27} \approx 1.14 & 3.63 \\
\text{Quadratic} & \frac{51}{182} \approx 0.28 & \frac{1}{5} \approx 0.17 & \frac{153}{91} \approx 1.68 & 3.78 \\
\hline
\end{array}
\]

Table 3.2: Load-mass factors for slender beams

The equivalent SDOF systems are optimized for static deflection, the natural frequencies are approximated. The deflected shape under uniform loading gives the best approximation of the natural frequency, because this shape resembles most the first mode shape. Mode shapes and natural frequencies are determined by free vibration, so they are not influenced by the load.

### 3.4.2 Load-mass factors for simple shear beams

In this subsection, only shear deflection is considered and all other contributions (bending, base rotation) to deflection are ignored. Using the same method as in the previous subsection,
the load-mass factors are determined for the following shape functions:

\[
\begin{align*}
ys:u(x) &= \frac{P_m}{S} \left[ -\frac{x^2}{2} + Hx \right] \\
ys:l(x) &= \frac{P_m}{SH} \left[ \frac{x^3}{3} - Hx^2 + H^2x \right] \\
ys:q(x) &= \frac{P_m}{SH^2} \left[ -\frac{x^4}{4} + Hx^3 - \frac{3H^2x^2}{2} + H^3x \right]
\end{align*}
\] (3.20)

The shape functions for shear deflection are plotted in figure 3.8.

![Shear deflection for three load functions.](image)

Figure 3.8: Shear deflection for three load functions.

The load-mass factors are shown in table 3.3.

<table>
<thead>
<tr>
<th>Load distribution</th>
<th>(K_M)</th>
<th>(K_L)</th>
<th>(K_{LM})</th>
<th>(\omega_1 \cdot \sqrt{\frac{mH^2}{S}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>(\frac{8}{15})</td>
<td>(\frac{2}{3})</td>
<td>(\frac{4}{5})</td>
<td>0.80</td>
</tr>
<tr>
<td>Linear</td>
<td>(\frac{9}{14})</td>
<td>(\frac{3}{5})</td>
<td>(\frac{15}{14})</td>
<td>1.07</td>
</tr>
<tr>
<td>Quadratic</td>
<td>(\frac{32}{25})</td>
<td>(\frac{1}{7})</td>
<td>(\frac{55}{36})</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Table 3.3: Load-mass factors for simple shear beams

3.4.3 Load-mass factors for non-slender beams

So far, either bending or shear stiffness was used to determine the load-mass factor. However, neglecting shear stiffness is only allowed for slender beams while the simple shear beam is a purely theoretical model. Therefore, load-mass factors for non-slender beams are considered in this section, in which both bending and shear deflection are incorporated. In the previous subsections, the load-mass factors depended only on the load distribution and were independent of material and geometrical properties. Load-mass factors for non-slender beams...
are dependent on the slenderness ratio \( s \) (geometrical property) and the material property \( \gamma \) (both defined in [9]):

\[
s = \sqrt{\frac{H^2A}{I_z}} \quad \gamma = \sqrt{\frac{E}{G}} = \sqrt{\frac{2(1+\nu)}{k'}}
\]  

(3.21)

The slenderness ratio \( s \) depends on the height \( H \) of the beam (not the height of the cross section!), the area of the cross section and the second moment of area of the cross section. The material property \( \gamma \) depends on \( \nu \), which is the Poisson ratio and \( k' \), which is the shear shape factor. For steel, \( \nu = 0.3 \) and in general, \( k' \approx 1 \), so \( \gamma^2 \approx 2.6 \). If we introduce a dimensionless parameter \( \alpha \), defined as

\[
\alpha = \frac{SH^2}{B} = \frac{s^2}{\gamma^2},
\]  

(3.22)

the load-mass factors can be written as a function of this single parameter. The shape functions used are simply the sum of equations (3.18) and (3.20). The load-mass factor functions are displayed in table 3.4. For \( \alpha \to 0 \) the load-mass factors are similar to those of the simple shear beam, while for \( \alpha \to \infty \) the load-mass factors approach those of the slender beam. The load-mass factors are plotted versus \( \alpha \) in figure 3.9.

<table>
<thead>
<tr>
<th>Load distribution</th>
<th>( K_M )</th>
<th>( K_L )</th>
<th>( K_{LM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>8(3024 + 999( \alpha ) + 91( \alpha^2 )) ( \frac{2835(4 + \alpha)^2}{60 + 15\alpha} )</td>
<td>40 + 6( \alpha ) ( \frac{4(3024 + 999\alpha + 91\alpha^2)}{189(80 + 32\alpha + 3\alpha^2)} )</td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>5(4455 + 1078( \alpha ) + 75( \alpha^2 )) ( \frac{1386(5 + \alpha)^2}{21(5 + \alpha)} )</td>
<td>63 + 5( \alpha ) ( \frac{5(4455 + 1078\alpha + 75\alpha^2)}{66(5 + \alpha)(63 + 5\alpha)} )</td>
<td></td>
</tr>
<tr>
<td>Quadratic</td>
<td>256256 + 49296( \alpha ) + 2805( \alpha^2 )) ( \frac{10010(6 + \alpha)^2}{42(6 + \alpha)} )</td>
<td>144 + 7( \alpha ) ( \frac{3(256256 + 49296\alpha + 2805\alpha^2)}{715(6 + \alpha)(144 + 7\alpha)} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Load-mass factors for non-slender beams

### 3.4.4 Load-mass factors for non-slender beams with rotary spring

To include rotation of the foundation, a rotary spring with stiffness \( C_\phi \) can be added at the base of the beam model. If this is a linear elastic rotary spring, then the deflection is proportional to the moment at the base of the beam. The shape functions are simple:

\[
y_{r,u}(x) = \frac{P_mH^2}{2C_\phi}x
\]

\[
y_{r,l}(x) = \frac{P_mH^2}{3C_\phi}x
\]

\[
y_{r,q}(x) = \frac{P_mH^2}{4C_\phi}x
\]  

(3.23)

The total deflection is the sum of (3.18), (3.20) and (3.23):

\[
y(x) = y_0(x) + y_s(x) + y_r(x)
\]  

(3.24)
Similar to the dimensionless parameter $\alpha$ for shear deflection, a dimensionless parameter $\beta$ for deflection due to base rotation is introduced:

$$\beta = \frac{C_\phi H}{B}$$  \hspace{1cm} (3.25)

The load-mass factors are given in Table 3.5.

<table>
<thead>
<tr>
<th>Load distribution</th>
<th>$K_{LM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>$\frac{4(3024\beta^2 + 27\alpha\beta(175 + 37\beta) + 7\alpha^2(270 + 117\beta + 13\beta^2))}{189(80\beta^2 + 4\alpha\beta(35 + 8\beta) + 3\alpha^2(20 + 9\beta + \beta^2))}$</td>
</tr>
<tr>
<td>Linear</td>
<td>$\frac{5(4455\beta^2 + 77\alpha\beta(81 + 14\beta) + 15\alpha^2(154 + 55\beta + 5\beta^2))}{66(315\beta^2 + 2\alpha\beta(245 + 44\beta) + 5\alpha^2(35 + 12\beta + \beta^2))}$</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$\frac{3(256256\beta^2 + 624\alpha\beta(539 + 79\beta) + 165\alpha^2(728 + 221\beta + 17\beta^2))}{715(864\beta^2 + 6\alpha\beta(245 + 31\beta) + 7\alpha^2(54 + 15\beta + \beta^2))}$</td>
</tr>
</tbody>
</table>

Table 3.5: Load-mass factors for non-slender beams including base rotation

To plot the load-mass factors versus $\alpha$ and $\beta$, a three dimensional plot would be needed. In Figure 3.9 the load-mass factor is given versus $\beta$ for $\alpha \rightarrow 0$ (simple shear beam) and $\alpha \rightarrow \infty$ (slender beam). Since both the parameter for shear deflection $\alpha$ and the parameter for deflection due to base rotation $\beta$ are related to the bending stiffness $B$, load-mass factors for simple shear beams with base rotation can not be obtained by letting $\alpha = 0$. If $\alpha = 0$, $\beta$ also disappears from the load-mass factor, which reduces to the value for simple shear beams. For $\alpha = 17.09$, the value for the example building of Section 3.1 and a linear distributed load, the influence of $\beta$ is negligible.

### 3.4.5 Comparison of models

In Table 3.6 the maximum static deflection $y_{\text{max}}$ under wind loading and the first natural frequency are compared for the trussed frame (TF), equivalent beam (EB) and SDOF system.
3.4 The equivalent single degree of freedom system

Figure 3.10: Load-mass factor vs $\beta$ for limits of $\alpha$.

Table 3.6: Comparison of static deflection under wind loading and natural frequency for different models.

<table>
<thead>
<tr>
<th>Deflected shape</th>
<th>$y_{m, st}[\text{mm}]$</th>
<th>$f_1[\text{Hz}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{b,u}$</td>
<td>77.50</td>
<td>0.386</td>
</tr>
<tr>
<td>$y_{s,u}$</td>
<td>63.90</td>
<td>0.474</td>
</tr>
<tr>
<td>$y_{r,u}$</td>
<td>40.43</td>
<td>0.663</td>
</tr>
<tr>
<td>$y_{b,u}$ + $y_{s,u}$ + $y_{r,u}$</td>
<td>94.70</td>
<td>0.360</td>
</tr>
<tr>
<td>$y_{b,u}$ + $y_{s,u}$</td>
<td>81.10</td>
<td>0.445</td>
</tr>
<tr>
<td>$y_{s,u}$</td>
<td>59.50</td>
<td>0.650</td>
</tr>
<tr>
<td>$y_{r,u}$</td>
<td>36.20</td>
<td>0.630</td>
</tr>
</tbody>
</table>

TF=trussed frame, EB=equivalent beam, lump=lumped mass, dis=distributed mass, SDOF=SDOF system

for different deflected shapes. Uniform distributed loading is assumed for the shape function. The trussed frame should be compared to the equivalent beam with lumped mass. The SDOF system should be compared to the equivalent beam with distributed mass. The static deflection is slightly overestimated by the equivalent beam and the SDOF system compared to the trussed frame. The static deflection of the equivalent beam and the SDOF system are approximately equal. This makes sense, since the static deflected shape was assumed for the SDOF system. The natural frequency of the trussed frame is slightly underestimated by the equivalent beam. The difference between the equivalent beam with lumped and distributed mass, can be explained by the fact that the mass of each story is lumped to the floor above that story. Therefore the mass distribution over the height is slightly different. The natural frequency is overestimated by the SDOF system with respect to the equivalent beam with distributed mass. This is due to the assumption that the beam vibrates with the static deflected shape. Because the natural frequencies of the trussed frame are underestimated by the equivalent beam and overestimated by the SDOF system, the trussed frame and the SDOF system match quite well. For bending and shear deflection, without base rotation, the error percentage is +1.3%. The error percentage is defined as

$$\frac{\text{SDOF} - \text{TF}}{\text{TF}} \cdot 100\%$$

in which SDOF is the SDOF value and TF is the trussed frame value.

L. J. van der Meer
Chapter 4

Dynamic response of a single degree of freedom system to blast loading

In chapter 2 blast loading in general has been described as well as blast loading on building structures in particular. An example building structure has been modeled in chapter 3. In this chapter, the focus is on the response of the single-degree-of-freedom (SDOF) system to blast loading. This SDOF system was described in section 3.3. The response of this system to idealized blast loading is described in section 4.1.

In section 4.2 the dynamic load factor (DLF) is explained, which describes the ratio of the maximum dynamic response and the static response if the maximum dynamic load were applied as a static load. The DLF is dependent on the ratio of $t_d$ and $T_1$, the positive phase duration of the blast and the natural period of vibration of the structure respectively. The diagram of DLF versus the $t_d/T_1$-ratio contains three regimes, namely an impulsive regime, a quasi-static regime and a dynamic transition region in between.

The pressure-impulse ($p-i$-) diagram, described in section 4.3, is a modification of the $DLF-t_d/T_1$-diagram, which divides all possible combinations of pressure and impulse of the blast loading into two categories: either the blast exceeds the chosen damage criterium or it does not. The $p-i$-diagram is a powerful assessment tool, which is generally used to estimate the maximum blast load capacity of structural elements, such as beams, slabs, plates, shells etcetera. However, the method can also be applied to building structures as a whole, if only the overall response is considered. The overall response determines the internal forces in the elements. The response of building structure elements is only useful if the element is directly exposed to the blast load. Yet structural stability elements, such as cores, stability walls or (trussed) frames, are often not even in the neighborhood of the building facade. Therefore, they should be assessed for overall response. Element response can be used to assess damage of facade elements or structural elements close to the facade. If any of these elements fail, this failure should be taken into account when assessing the overall response. In literature, the resistance of the building to progressive collapse due to failing structural elements is called robustness. However, this is not within the scope of this research.

In this chapter, response of a SDOF system is considered, which is a simplification of an example building structure. In chapter 5 the example building structure is modeled as a continuous beam with infinite-degrees-of-freedom (IDOF).
4.1 Dynamic response of single degree of freedom systems to idealized blast loading

Most structural engineers are used to the fact that a structure has to satisfy static equilibrium. For example, if the end of a spring with stiffness $K$ is moved over a distance $y$ (figure 4.1a), the other end of the spring should be held in place with a reaction force (or resistance) in the opposite direction of the displacement of $R = K \cdot y$. This is actually a simplification of Newton’s second law of motion, which can be used to obtain the equation of motion. This equation will be given for a basic dynamic model, a spring-mass-damper system, as shown in figure 4.1b. A damper dissipates energy, causing the response of the system to gradually fade out when no external forces are present, see subsection 4.1.2. For a mass $M$ on a spring with stiffness $K$ and damper with damping coefficient $C$, to which a force $F(t)$ is applied, the equation of motion is:

$$M \ddot{y}(t) + C \dot{y}(t) + Ky(t) = F(t) \quad (4.1)$$

This is a differential equation, not only involving the displacement $y(t)$, but also the first and second order time derivatives of the displacement, being the velocity and the acceleration of the displacement. When a structure and the dynamic forces acting on it satisfy its equation(s) of motion, the structure is said to be in dynamic equilibrium. The structural response depends on $M$, $K$, $C$, $F(t)$ and its initial conditions $y(0)$, $\dot{y}(0)$ and $\ddot{y}(0)$. A system in which only the translation (or rotation) of one concentrated mass is considered, is called a single degree of freedom system (SDOF system).

Before the response of the SDOF system can be explained, some properties of the system have to be described first. These properties are the natural frequency $\omega_1$, the damping ratio $\zeta$ which is related to the damping constant $C$, the material model which is related to the stiffness $K$ and the forcing function $F(t)$ of the blast.

This section is not intended as a comprehensive introduction to structural dynamics. If the reader is not familiar with structural dynamics, chapter 1-8 of [17] are advised reading material for understanding the structural dynamics used in this report.
4.1 Dynamic response of SDOF systems to idealized blast loading

4.1.1 Natural frequency of vibration

The SDOF system as shown in figure 4.1b has a natural frequency of vibration, equal to:

\[ \omega_1 = \sqrt{\frac{K}{M}} \text{ [rad/s]} \]  \hspace{1cm} (4.2)

With some sort of damping with damping constant \( C \), the damped ‘frequency’ of vibration is:

\[ \omega_{1;\zeta} = \sqrt{\frac{K}{M} - \left(\frac{C}{2M}\right)^2} \text{ [rad/s]} \]  \hspace{1cm} (4.3)

These frequencies are circular frequencies in rad/s. The natural frequency \( f_1 = \omega_1/(2\pi) \). The natural period \( T_1 = 1/f_1 = 2\pi/\omega_1 \).

4.1.2 Damping

Without damping, the resulting vibrations of an applied force that is no longer present would go on forever. Damping is the dissipation of energy of the system, which causes the vibrations to gradually fade-out if the force is no longer present. The energy which is lost is converted to heat or sound. Different types of damping can be present in a system simultaneously:

- **Viscous damping** is the damping caused by resistance of a fluid medium such as for example air, gas or water.

- **Dry friction damping** is the damping caused by friction between surfaces.

- **Material damping** is the damping caused by cyclic plastic deformation of materials.

When a building vibrates, viscous damping is caused by the resistance of the surrounding air and material damping occurs when the elastic limits of structural elements are exceeded. Dry friction damping can occur during earthquakes between the building foundation and the soil. It is difficult to estimate the damping ratio of a vibrating building and it is conservative to ignore damping. However, ignoring damping is not always allowed. In general, for structures loaded with periodic forces, damping can not be ignored, because damping has a significant influence on the response to periodic forces near resonance. Without damping, response at resonance is infinite.

For a SDOF system, the critical damping coefficient is \( C_c = 2M\omega_1 \), which is the damping coefficient at which so much energy is dissipated in damping, that the mass does not oscillate at all but only returns to its equilibrium position. The damping ratio is defined as:

\[ \zeta = \frac{C}{C_c} \]  \hspace{1cm} (4.4)

Therefore, the natural circular frequency of a damped system can be written as:

\[ \omega_{1;\zeta} = \sqrt{1 - \zeta^2} \omega_1 \]  \hspace{1cm} (4.5)

For small amounts of damping the damped and undamped frequency are almost equal, so that the use of the undamped frequency is permitted.
In this chapter, damping is neglected, because the maximum response to a blast load, is always the first maximum of the response after the maximum of the blast load. The effect of damping increases with the number of oscillations. The first maximum of response is usually within the first oscillation, where the effect of damping is still small. Moreover, damping always decreases the maximum response, so neglecting damping is always conservative.

4.1.3 Material model

In differential equation 4.1 all terms are linear functions of $y$, $\dot{y}$ or $\ddot{y}$. If one or more of the terms are non-linear functions of $y$ or its derivatives, the vibration is called non-linear. Examples of nonlinearity are a pendulum swinging with large amplitudes (geometric nonlinearity), a nonlinear spring (material nonlinearity), nonlinear damping or a variable mass system. Material nonlinearity is likely to occur, because structural materials such as steel and concrete behave elastic-plastic. Because buildings are generally not designed for blast loads, stresses above the yield limit might occur. In this report, only the material steel is considered, which behaves linear-elastic and has a critical resistance related to the yield stress. Therefore the influence of ductility on the response is not investigated. Ductility is the ratio of the critical plastic deformation $y_{c,p}$ and the critical elastic deformation $y_{c,e}$, see figure 4.2. Ignoring the influence of ductility in combination with a critical resistance related to the yield stress is conservative, because it ensures that the structure is loaded in the elastic range even though there is always some additional plastic capacity.

\[ R(y) = K y_c \]

$y_{c,e}$ $y_{c,p}$

Figure 4.2: Linear-elastic (continuous line) vs elastic-plastic (dashed line) material model.

4.1.4 Forcing function

How the response of a SDOF system is determined, depends on the forcing function $F(t)$. When the forcing function is an harmonic function, the response will also be an harmonic function and is relatively easy to determine. In case of a periodic non-harmonic function, the function can be written as a sum of harmonic functions using Fourier series expansion. When the function is non-periodic, other methods are available to determine the response, see subsection 4.1.5. To determine the response analytically, blast pressure-time profiles such as in figure 2.3 have to be given analytically. Since the actual blast pressure-time variation is...
not predictable, blast pressure-time profiles are often idealized. These idealized pressure-time profiles are given in figure 4.3 for a detonation (figure 4.3a) and a deflagration (figure 4.3b).

![Idealized blast pressure-time profiles](image)

Figure 4.3: Idealized blast pressure-time profiles.

### 4.1.5 Response to idealized blast loads

In this section, the response of the SDOF system is determined for the following conditions:

- Damping is neglected. The equation of motion reduces to:
  \[ M\ddot{y}(t) + Ky(t) = F(t) \] (4.6)

- The spring is linear-elastic.

- The forcing function is an idealized blast load, from either figure 4.3a or 4.3b.

- The system is at rest before the forcing function is applied.

To determine the response of a SDOF system to a non-periodic forcing function, the convolution integral or Duhamel integral is often used. If the system is at rest before an impulse \( I = F\Delta t \) is applied to it, the response is:

\[ y(t) = \frac{I}{M\omega_1} \sin\omega_1 t \] (4.7)

When a non-periodic forcing function is divided into impulses \( F(\tau)d\tau \), the response to the sum of impulses is:

\[ y(t) = \frac{1}{M\omega_1} \int_0^t F(\tau) \sin\omega_1 (t - \tau) d\tau \] (4.8)

This equation is called the convolution integral. It applies only if the system is at rest before the forcing function is applied. Since this is one of the conditions listed in this subsection, it is automatically satisfied.
It is convenient to write response in dimensionless form. Therefore, the following expressions are defined (as in reference [13]):

\[
\bar{F}_m = \frac{F_m}{R_c}, \quad \bar{t}_d = \omega_1 t_d, \quad \bar{I} = \bar{F}_m \int_0^{\bar{t}_d} \bar{F}(\bar{t}) \, d\bar{t}, \\
\bar{y} = \frac{y}{y_c}, \quad \bar{\dot{y}} = \frac{\dot{y}}{\omega_1 y_c}, \quad \bar{\dot{t}} = \omega_1 t, \quad \bar{F} = \frac{F}{F_m}
\]

(4.9)

In which \(\bar{F}_m\) is the non-dimensional maximum force, \(F_m\) is the maximum force, \(R_c\) is the critical resistance, \(\bar{t}_d\) is similar to the ratio \(t_d/T_1\) but in radians, \(\omega_1\) is the natural circular frequency of the structure, \(\bar{I}\) is the non-dimensional total impulse, \(\bar{y}\) is the response as a fraction of the critical response and \(\bar{F}\) is the non-dimensional force. The parameters are explained graphically in figure 4.4.

![Figure 4.4: Parameters for response in dimensionless form.](image)

**Response to an idealized detonation**

Using the dimensionless parameters defined in subsection 4.1.5 the force function for an idealized detonation (figure 4.3a) is:

\[
\bar{F}(\bar{t}, \bar{t}_d) = \left(1 - \frac{\bar{t}}{\bar{t}_d}\right) \left[u(\bar{t}) - u(\bar{t} - \bar{t}_d)\right]
\]

(4.10)

where \(u\) is a unit-step function. The response to this force function using the Convolution integral is:

\[
\bar{y}(\bar{t}, \bar{t}_d) = \bar{F}_m \int_0^{\bar{t}} \bar{F}(\tau) \sin(\bar{t} - \tau) d\tau = \bar{F}_m \left\{ \begin{array}{ll}
-\frac{\bar{t}}{\bar{t}_d} + 1 - \cos \frac{\bar{t}}{\bar{t}_d} + \sin \frac{\bar{t}}{\bar{t}_d} & \text{if } \bar{t} \leq \bar{t}_d \\
-\cos \bar{t} + \frac{\sin \bar{t}}{\bar{t}_d} - \frac{\sin(\bar{t} - \bar{t}_d)}{\bar{t}_d} & \text{if } \bar{t} > \bar{t}_d
\end{array} \right.
\]

(4.11)

Response for different \(\bar{t}_d\) is given in figure 4.5. For completeness the response is also given if damping is present (\(\zeta = 0.10\)). The dimensionless blast duration \(\bar{t}_d\) is equal to the ratio \(t_d/T_1\) multiplied by 2\(\pi\). If this ratio becomes greater, more oscillations occur during the presence of the forcing function. It is also observed that for a damping percentage of 10\%, the time of maximum response is barely influenced and the maximum response itself is slightly lower. This agrees with the assumption that ignoring damping is conservative.
4.2 Time of maximum deflection and the dynamic load factor

As mentioned before, only the first maximum of the response that occurs after the maximum load is of interest, when the resistance to blast loading is to be determined. In figures 4.5 and 4.6 it can be concluded that a detonation causes higher response than deflagration for large $\bar{t}_d$. Another important conclusion is the fact that the maximum response always occurs after the maximum force. For a detonation this is trivial but it is also true for a deflagration. For small values of $\bar{t}_d$ it does not seem to matter significantly whether the blast is a detonation or deflagration.

**Response to an idealized deflagration**

Using the dimensionless parameters defined in subsection 4.1.5, the force function for an idealized deflagration (figure 4.3b) is:

\[
\bar{F} (\bar{t}, \bar{t}_d) = \frac{2\bar{t}}{\bar{t}_d} \left[ u(\bar{t}) - u(\bar{t} - \frac{\bar{t}_d}{2}) \right] + 2 \left( 1 - \frac{\bar{t}}{\bar{t}_d} \right) \left[ u(\bar{t} - \frac{\bar{t}_d}{2}) - u(\bar{t} - \bar{t}_d) \right]
\]

(4.12)

To which the response is:

\[
\bar{y}(\bar{t}, \bar{t}_d) = \frac{2\bar{t}}{\bar{t}_d} - \frac{2\sin \bar{t}}{\bar{t}_d} + u \left( \bar{t} - \frac{\bar{t}_d}{2} \right) \left[ -\frac{4\bar{t}}{\bar{t}_d} + 2 + \frac{4\sin \left( \bar{t} - \frac{\bar{t}_d}{2} \right)}{\bar{t}_d} \right] + u(\bar{t} - \bar{t}_d) \left[ \frac{2\bar{t}}{\bar{t}_d} - 2 - \frac{2\sin(\bar{t} - \bar{t}_d)}{\bar{t}_d} \right]
\]

(4.13)

Again $u$ is a unit-step function. Response for different $\bar{t}_d$ is given in figure 4.6. For completeness the response is also given if damping is present ($\zeta = 0.10$). From comparison of figures 4.5 and 4.6 it can be concluded that a detonation causes higher response than deflagration for large $\bar{t}_d$. Another important conclusion is the fact that the maximum response always occurs after the maximum force. For a detonation this is trivial but it is also true for a deflagration. For small values of $\bar{t}_d$ it does not seem to matter significantly whether the blast is a detonation or deflagration.
4.6 it can be observed that the maximum response is dependent on the parameter \( \bar{t}_d \) which is equivalent to the \( t_d/T_1 \) -ratio. A convenient way to represent response of SDOF systems to blast loading, is in a plot of the maximum response versus the \( t_d/T_1 \) -ratio. However, the maximum response also depends on \( F_m \), the maximum of the forcing function. Therefore the dynamic load factor (DLF) is defined as the ratio of the maximum dynamic deflection \( y_m \) and the static deflection if the maximum dynamic force were applied statically, \( y_{st;m} = F_m/K \).

The following procedure is required to obtain the DLF-plot:

- Solve the equilibrium equation:

\[
M \ddot{y}(t) + Ky(t) = F(t) \quad (4.14)
\]

In which \( M \) is the mass, \( K \) is the stiffness and \( F(t) \) the force or pressure function. If the equation can be solved analytically, the solution is the response \( y(t) \) of the structure, which is the deflection in the direction of the applied force function. The response can be written in dimensionless form \( \bar{y}(\bar{t}) \) as well (see equations (4.11) and (4.13)).

- Solve velocity \( \dot{y}(\bar{t}) = 0 \) to find the time \( \bar{t}_m \) at which maximum deflection occurs.

- Determine maximum deflection \( \bar{y}(\bar{t}_m, \bar{t}_d) \). The dynamic load factor \( \bar{D}_m \) is:

\[
\bar{D}_m = \frac{\bar{y}(\bar{t}_m, \bar{t}_d)}{F_m} = \frac{y_m}{y_c} \cdot \frac{R_c}{F_m} = \frac{y_m}{y_c} \cdot \frac{K y_c}{K y_{m;st}} = \frac{y_m}{y_{m;st}} \quad (4.15)
\]

The first step was already achieved for idealized detonation and deflagration loads. The second step, the determination of the time of maximum deflection is described next.

Dynamic response of high-rise building structures to blast loading
4.2 Time of maximum deflection and the dynamic load factor

4.2.1 Time of maximum deflection

The time of maximum deflection is found by equating the first time derivative of the response to zero and solving for $\bar{t}$. The first solution for $\bar{t}$ that occurs after the maximum of the blast load is the time of maximum deflection $\bar{t}_m$. Because the response is different for $\bar{t} \leq \bar{t}_d$ and $\bar{t} > \bar{t}_d$, there are also two values of $\bar{t}_m$, namely one for $\bar{t}_m \leq \bar{t}_d$ and one for $\bar{t}_m > \bar{t}_d$. The first solution is valid when the maximum response occurs during the blast and the latter is valid when the maximum response occurs after the blast load has ended.

Time of maximum deflection for an idealized detonation

The time of maximum deflection for an idealized detonation is obtained by differentiating (4.11) to $\bar{t}$, equating the result to zero and solving for $\bar{t}_m$. The result is:

$$\bar{t}_m = \begin{cases} \arccos \left[ \frac{(\cos \bar{t}_d - 1)(\bar{t}_d - \sin \bar{t}_d)}{\sqrt{(\cos \bar{t}_d - 1)^2 + 2 + (\bar{t}_d)^2 - 2 \cos \bar{t}_d - 2 \bar{t}_d - 2 \bar{t}_d \sin \bar{t}_d}} \right] & \text{if } \bar{t}_m > \bar{t}_d \\ 2 \arctan \bar{t}_d & \text{if } \bar{t}_m \leq \bar{t}_d \end{cases}$$

(4.16)

Maximum deflection occurs either during or after the blast. The shifting point is at $\bar{t}_m = \bar{t}_d = 2.33112$.

Time of maximum deflection for an idealized deflagration

The time of maximum deflection for an idealized deflagration is obtained by differentiating (4.13) to $\bar{t}$, equating the result to zero and solving for $\bar{t}_m$. The result is:

$$\bar{t}_m = \begin{cases} 2 \arctan \frac{1 + \tan \left( \frac{\bar{t}_d}{4} \right)}{1 - \tan \left( \frac{\bar{t}_d}{4} \right)} & \text{if } \bar{t}_m \geq \bar{t}_d \\ 2 \arctan \left[ \frac{-2 \tan \left( \frac{\bar{t}_d}{4} \right) - \sqrt{2} \sqrt{\tan \left( \frac{\bar{t}_d}{4} \right)^2 + \tan \left( \frac{\bar{t}_d}{4} \right)^4}}{-1 + \tan \left( \frac{\bar{t}_d}{4} \right)^2} \right] + k2\pi & \text{if } \bar{t}_m < \bar{t}_d \end{cases}$$

(4.17)

in which $k$ is an integer. The maximum of the blast is at $\bar{t}_d/2$, therefore the maximum response always occurs after or at this time. The shifting point is at $\bar{t}_m = \bar{t}_d = \pi$.

In figure 4.7 the time of maximum deflection is shown versus the duration of the blast. For convenience, the dimensionless parameters have been converted to ratios of dimensional parameters. Figure 4.7a shows the ratio of $t_m$ and $T_1$ versus the ratio of $t_d$ and $T_1$. In figure 4.7b, $t_m/T_1$ is changed to $t_m/t_d$ because this ratio shows immediately whether the maximum response occurs during ($t_m/t_d < 1$) or after ($t_m/t_d > 1$) the blast.

L. J. van der Meer
4.2.2 The dynamic load factor

The dynamic load factor is obtained by calculating $\bar{t}_m$ and substituting it for $\bar{t}$ in $\ddot{y}(\bar{t}, \bar{t}_d)$. For a detonation this means substituting (4.16) in (4.11) and for a deflagration (4.17) should be substituted in (4.13). The resulting expressions could be given analytically, but would be awkward. A better method is to calculate $\bar{t}_m$ and subsequently $\bar{D}_m$ for a range of $\bar{t}_d$. In figure 4.8, $\bar{D}_m$ is given versus $t_d/T_1$ for a detonation and a deflagration. The figure agrees with similar figures in literature, for example [4].

Figure 4.8: Dynamic load factor vs $t_d/T_1$ for idealized blast loads.
4.2 Time of maximum deflection and the dynamic load factor

4.2.3 Limits of the dynamic load factor

To interpret the $D_m - t_d/T_1$-diagram, it is instructive to look at the limits of $D_m$ when $t_d/T_1 \to 0$ and $t_d/T_1 \to \infty$. In the first case, the duration of the blast is very short compared to the natural period of the SDOF system. An impulse has exactly this property, so to evaluate this limit, $D_m$ is determined for impulse response. Impulse response is displayed in (4.7). Written in dimensionless variables:

$$\bar{y}(\bar{t}) = \bar{I} \sin \bar{t} \quad \rightarrow \quad \bar{D}(\bar{t}) = \frac{\bar{y}(\bar{t}_m)}{\bar{F}_m} = \frac{\bar{t}_d}{2} \sin \bar{t} \quad \rightarrow \quad \bar{D}_m = \frac{\bar{t}_d}{2} = \frac{\pi \bar{t}_d}{T_1} \quad (4.18)$$

Apparently the diagram contains an impulsive asymptote for which $\bar{D}_m$ can be determined with $\pi \bar{t}_d/T_1$. Notice that $\bar{D}_m$ for the impulsive asymptote is independent of the pressure-time profile and only depends on the ratio $t_d/T_1$.

The other limit, where $t_d/T_1 \to \infty$, the detonation blast load becomes a step force whereas the deflagration blast load becomes a linear rising load. The response $\bar{D}(\bar{t})$ to a step force can be determined with, for example, the convolution integral:

$$\bar{F}(\bar{t}) = 1 \quad \rightarrow \quad \bar{D}(\bar{t}) = \int_0^{\bar{t}} \sin(\bar{t} - \bar{\tau})d\bar{\tau} = 1 - \cos \bar{t} \quad \rightarrow \quad \bar{D}_m = 2 \quad (4.19)$$

Thus the maximum dynamic load factor for an idealized detonation is equal to 2. Moreover, the maximum dynamic load factor for all decaying blast loads is equal to 2. However, some blast loads such as the idealized deflagration have an initial rise and reach their maximum after some time. The maximum force of the idealized deflagration is reached at $\bar{t}_d/2$.

$$\bar{F}(\bar{t}) = 2 \frac{\bar{t}}{\bar{t}_d} \quad \rightarrow \quad \bar{D}(\bar{t}) = \int_0^{\bar{t}} 2 \frac{\bar{t}}{\bar{t}_d} \sin(\bar{t} - \bar{\tau})d\bar{\tau} = \frac{2\bar{t} - 2 \sin \bar{t}}{\bar{t}_d} \quad (4.20)$$

At first sight this expression does not seem to give the maximum dynamic load factor for an idealized deflagration. Yet, when $t_d/T_1 \to \infty$, $t_m = t_d/2$, because the natural period $T_1$ becomes infinitely small compared to $t_d$. Since the response has a maximum in every oscillation, it also has a maximum in the oscillation where $t = t_d/2$. Another requirement for the time of maximum deflection is $\bar{t}_m = k2\pi$ with $k = 1, 2, \ldots$, because otherwise $\bar{D}(\bar{t})$ does not have a maximum.

$$\bar{t}_m = k2\pi = \frac{\bar{t}_d}{2} \quad \rightarrow \quad \bar{D}_m = \frac{2\bar{t}_m - 2 \sin \bar{t}_m}{\bar{t}_d} = \frac{\bar{t}_d}{\bar{t}_d} - 2 \sin(k2\pi) = 1 - 0 = 1 \quad (4.21)$$

It can be concluded that the $\bar{D}_m - t_d/T_1$-diagram of an idealized detonation and deflagration have horizontal asymptotes at $\bar{D}_m = 2$ and 1 respectively. These asymptotes are referred to in literature as quasi-static asymptotes.

The limits of $t_m$ follow simply from the expressions $\bar{D}(\bar{t})$ in equation (4.18) and (4.19). The impulse response is a sine function. A sine function (with phase=0) has its first maximum at $T_1/4$. So:

$$t_d/T_1 \to 0 \quad t_m/T_1 \to 1/4 \quad (4.22)$$
The impulsive limit is independent of the forcing function. The quasi-static asymptotes do depend on the forcing function. For a detonation, the response to a constant force equals a (1-cosine) function. This function has its first maximum at \( T_1/2 \). So:

\[
t_d/T_1 \to \infty \quad t_m/T_1 \to 1/2
\]  

(4.23)

For a deflagration, the time of maximum response is equal to the time of maximum load.

\[
t_m/T = \frac{1}{2}t_d/T
\]  

(4.24)

The time of maximum response can also be written as the ratio of time of maximum response and the blast duration. In this case:

\[
t_d/T_1 \to \infty \quad \begin{array}{l}
t_m/t_d \to \infty \quad \text{(detonation and deflagration)} \\
t_m/t_d \to 0 \quad \text{(detonation)} \\
t_m/t_d \to 0.5 \quad \text{(deflagration)}
\end{array}
\]  

(4.25)

**Limits of DLF by conservation of energy**

As an alternative, the asymptotes can be determined with the law of conservation of energy. An impulse causes an initial velocity \( \dot{y}(0) \), which is equal to:

\[
I = M\dot{y}(0) \quad \rightarrow \quad \dot{y}(0) = \frac{I}{M}
\]  

(4.26)

Thus the kinetic energy induced by the impulse is equal to:

\[
E_k = \frac{1}{2}M\dot{y}(0)^2 = \frac{I^2}{2M}
\]  

(4.27)

The critical resistance \( R_c = Ky_c \). The strain energy required to deflect \( y_c \), is:

\[
E_s = \frac{1}{2}Ky_c^2
\]  

(4.28)

If the impulse is to cause critical deflection, the strain energy should be equalled to the kinetic energy induced by the impulse, from which follows:

\[
y_c = \frac{I}{\sqrt{KM}} \quad I = \frac{Fmnt_d}{2} \quad y_m = y_c \quad \bar{D}_m = \frac{ym}{yst;\bar{m}} = \frac{IK}{\sqrt{KM}F_m} = \frac{\omega_1 t_d}{2} = \frac{\pi t_d}{T_1}
\]  

(4.29)

In the last step \( \sqrt{K/M} = \omega_1 \) was substituted. The asymptote is in this case independent of the forcing function, because the impulse for both detonation and deflagration is equal to \( F_m t_d/2 \).

The quasi-static asymptotes are dependent on the forcing function. For a detonation \( t_m/t_d \to 0 \) if \( t_d/T \to \infty \). This is shown graphically in figure 4.9(a). The work done by the quasi-static force \( F_m \) is the area underneath the \( F(y) \) graph in figure 4.9(b). The strain energy for the spring to reach critical deflection is equal to the area underneath the \( R_c - y_c \) graph in the same figure. So if the work done \( E_w \) and the strain energy \( E_s \) are equated, the dynamic load factor is obtained:

\[
E_w = F_my_m \quad E_s = \frac{1}{2}Ky_c^2 \quad y_c = y_m \quad y_m = \frac{2F_m}{K} = 2yst;\bar{m} \quad \bar{D}_m = 2
\]  

(4.30)

The same can be done for a deflagration (see figure 4.10):

\[
E_w = \frac{F_my_m}{2} \quad E_s = \frac{1}{2}Ky_c^2 \quad y_c = y_m \quad y_m = \frac{F_m}{K} = yst;\bar{m} \quad \bar{D}_m = 1
\]  

(4.31)

The conservation of energy is a very convenient principle for determination of the asymptotes.
4.3 The pressure-impulse diagram

First of all, pressure-impulse diagram is a misleading name, because it is used in literature for force-impulse diagrams as well. In this report, this tradition is followed, but the axes labels always make clear what diagram it is. $p$ is pressure (force per square meter), $F$ is force, $P$ is force per meter. Similarly, $i$ is the impulse from pressure, $I$ is the impulse from force and $J$ is the impulse from force per meter. Because SDOF systems are loaded by a force, the constructed diagrams are $F−I$–diagrams.

The pressure-impulse diagram is a modification of the $\bar{D}_m−\bar{t}_d/T_1$–diagram. Instead of $\bar{D}_m$ and $\bar{t}_d$, the $x$– and $y$–axis are labeled with $\bar{F}_m$ and $\bar{I}$. This results in a modification of the asymptotes. The impulsive asymptote is modified as follows:

$$
\bar{D}_m = \frac{\bar{t}_d}{2} \quad \bar{F}_m = \frac{1}{\bar{D}_m} = \frac{2}{\bar{t}_d} \quad \bar{I} = \frac{\bar{F}_m\bar{t}_d}{2} = 1
$$

(4.32)

So instead of linear, the impulsive asymptote is now vertical. The quasi-static asymptotes are simply the reciprocal values of the asymptotes in the DLF-diagram.

$$
t_d/T_1 \to \infty \quad \bar{I} = \frac{\pi \bar{F}_m t_d}{T_1} \to \infty \quad \bar{D}_m \to 2 \quad \bar{F}_m \to \frac{1}{2} \quad (\text{detonation})
$$

(4.33)

$$
t_d/T_1 \to \infty \quad \bar{I} = \frac{\pi \bar{F}_m t_d}{T_1} \to \infty \quad \bar{D}_m \to 1 \quad \bar{F}_m \to 1 \quad (\text{deflagration})
$$
The pressure-impulse diagram is shown in figure 4.12. It was determined in Excel. Alternatively, $p-i$-diagrams can be determined in Matlab, see appendix A. It agrees with similar diagrams in literature, for example 19.

The pressure-impulse diagram can be divided in three distinct regimes:

- The impulsive regime: the response is determined by the impulse. The shape of the blast load pressure-time profile does not have any influence on the response. Transient analysis is not necessary.

- The quasi-static regime: the response is determined by the load, which is applied as a step force (detonation) or as a linear rising force (deflagration). The shape of the blast-load pressure-time profile does not have influence on the response, except for the difference between decaying and linear rising forces. Transient analysis is not necessary.

- The dynamic regime: the response is determined by load, impulse and shape of the blast load pressure-time profile. The response can not be approximated by one of the asymptotes. The time of maximum response $t_m$ and the duration of the blast $t_d$ are approximately equal. Transient analysis is necessary.

A schematic pressure-impulse diagram is shown in figure 4.11, where the regimes and the related force and resistance functions are shown. Only for the relatively small dynamic regime, dynamic response calculations are necessary. The diagram is constructed by using the damage criterium $\bar{y} = y_m/y_c = 1$. If the damage criterium is intensified, the asymptotes in the diagram move towards the axes. When the damage criterium is loosened, the asymptotes in the diagram move away from the axes. The combinations of force and impulse, which are exactly on the line in the diagram, satisfy the damage criterium exactly. Combinations below or left of the diagram do not cause damage whereas combinations to the right or above the diagram exceed the damage criterium. The damage criterium or critical resistance $R_c$ can be formulated for any type of failure. Often different types of failure can occur in a structure. In that case, the failure that occurs at the lowest resistance governs the critical resistance. Or alternatively, $p-i$-diagrams for several failure types can be plotted together.

### 4.3.1 Pressure-impulse diagrams in literature

The pressure-impulse diagrams in this report were created by using the methods and dimensionless variables of 13 and 14, which describe the construction of pressure-impulse diagrams for elastic and elastic-plastic material behavior respectively for three different pulse loading shapes: rectangular, linear decaying (triangular) and exponential decaying. One of the early investigators of the pressure-impulse diagram describe several aspects of the diagram in 2, such as pulse loading shape sensitivity and spring characteristics. An important feature is also described: the possibility to display diagrams for different elements or different types of failure in one figure. Also, pressure-impulse diagrams for rigid-plastic beams and plates and dynamic buckling of cylindrical shells are discussed. Pressure-impulse diagrams for elastic-plastic-hardening and softening SDOF models subjected to blast loading are described in 8. Pressure-impulse method for combined failure modes of rigid-plastic beams is the subject of 16, where both shear and bending failure are considered. The problem with pressure-impulse diagrams is that they can not be derived analytically for complex blast loads, non-linear material models and including damping, especially when several of these
difficulties are combined. In all these cases, the pressure-impulse diagrams can be determined numerically, as demonstrated in [12].

Summarizing, the following possibilities exist for pressure-impulse diagrams of SDOF systems, as found in literature:

- The influence of the blast loading pulse shape on the \( p - i \)-diagram can be determined analytically or numerically.
- The influence of the material model on the \( p - i \)-diagram can be determined analytically or numerically.
- The influence of damping on the \( p - i \)-diagram can be determined analytically or numerically.
- Any structure or structural element can be converted to an equivalent SDOF system by the method described in 3.4. This implies that the load-time profile has to be similar at every location of the structure or structural elements, with the amplitude as the only variable (in other words: the spatial variation of the load is allowed, but all loads share the same time variation).

The most important conclusions concerning pressure-impulse diagrams in literature are the following:

- As far as knowledge of the author of this report goes, \( p - i \)-diagrams in literature are based on equivalent SDOF systems, implying that the response is determined by a single response mode of the structure or structural elements.
- Structures or structural elements have in fact infinite degrees of freedom (IDOF). Therefore, it is not evident that the first mode determines the response. The contributions
of modes to the response depend on the load distribution, the load-time profile, the mode shapes of the structure and the type of response considered. For IDOF systems, displacement and internal forces are not proportional as for SDOF systems. For example, the contributions of higher modes to shear force and moment are higher than contributions to displacement.

So, additional research in the area of \( p-i \)-diagrams should include:

- The influence of higher modes of vibration on the \( p-i \)-diagram. The result should be compared to the SDOF \( p-i \)-diagram for different failure criteria. This aspect is further investigated in chapter 5.

Figure 4.12: \( F_m - \bar{I} \)-diagram for idealized blast loads
Chapter 5

Dynamic response of a continuous beam to blast loading

In chapter 4 it was concluded that the influence of higher modes of vibration is not included in the pressure-impulse-diagram, because the diagram is generally set up for a failure criterium (or a degree of damage) of an equivalent SDOF system. To investigate the influence of higher modes of vibration on the response and equivalently, the $p-i$-diagram, this chapter focuses on the response of a continuous beam with infinite degrees of freedom, as shown in figure 3.5a but without the rotary spring at the base. It is assumed that the beam has an equally distributed mass, bending stiffness and shear stiffness. As stated before in section 3.4 the equivalent beam deduced from a trussed frame is a non-slender beam, which requires shear deflection to be included. In [9], dynamics of vibration are explained for four types of beams, of which two include shear deflection. The Timoshenko beam includes both shear deformation and rotary inertia, which is the effect of rotation of the cross section, in addition to deformation due to bending. Therefore, it is the best available model for non-slender beams.

In this chapter, the differential equation of motion of the Timoshenko beam according to [9] is described in section 5.1. Subsequently, the natural frequencies and eigenfunctions (mode shapes) of the Timoshenko beam are derived in section 5.2. In section 5.3 orthogonality and normalization of eigenfunctions, the mode superposition method and modal contribution factors for base shear and moment are discussed. The mode superposition method uses the superposition principle for linear systems to write the response as a sum of modal responses. The modal contribution factors for base shear and moment are derived from similar factors for earthquake engineering [7]. In section 5.4 the maximum of the resulting response is approximated. Finally, in section 5.5 the dynamic load factors of the higher modes are approximated for impulsive and quasi-static response, to obtain the asymptotes of the $p-i$-diagram for base shear and moment. The result of this chapter is a $p-i$-diagram including the influence of higher modes of vibration, which can be compared to the SDOF $p-i$-diagrams of section 4.3.
5.1 The differential equation of motion of a transversely vibrating Timoshenko beam

5.1.1 Dimensionless variables

The equations of motion of a Timoshenko beam are derived in [9]. The dimensionless variables used in this source will be used here also, but with some adaptations to improve the coherence with the other parameters used in this report. Axial coordinate \(x\), transverse deflection \(w\), rotation due to bending moment \(\varphi\), time \(t\) and natural frequency \(\omega_n\) are made dimensionless as follows:

\[
\bar{x} = \frac{x}{H}, \quad \bar{w} = \frac{w}{H}, \quad \bar{\varphi} = \varphi, \quad \bar{t} = \omega_1 t, \quad \bar{\omega}_n = \frac{\omega_n}{\omega_1}
\]

(5.1)

The distributed mass \(m\), force \(P\) and shear stiffness \(S\) are made dimensionless by using \(B\), \(H\), \(\omega_1\) and the parameter \(\alpha\) from (3.22):

\[
\bar{m} = m \frac{H^4 \omega_1}{B}, \quad \bar{P} = P \frac{H^3}{B}, \quad \bar{S} = \alpha = \frac{s^2}{\gamma^2} = S \frac{k'H^2}{B}
\]

(5.2)

The moment \(\mu\) and shear force \(v\) are, as a result, dimensionless:

\[
\bar{\mu} = \mu \frac{H}{B}, \quad \bar{v} = v \frac{H^2}{B}
\]

(5.3)

The time dependent part of the force \(\bar{F}\) is equal to \(\bar{F}\) in figure 4.4c. In other words, no amplitude is included in the time dependent part of the force.

5.1.2 Equation of motion of a Timoshenko beam

\[
\bar{m} \frac{\partial^2 \bar{w}(\bar{x}, \bar{t})}{\partial \bar{t}^2} - \alpha \left( \frac{\partial^2 \bar{w}(\bar{x}, \bar{t})}{\partial \bar{x}^2} - \frac{\partial \bar{\varphi}(\bar{x}, \bar{t})}{\partial \bar{x}} \right) = \bar{P}(\bar{x}, \bar{t})
\]

(5.4)

\[
\bar{m} \frac{\partial^2 \bar{\varphi}(\bar{x}, \bar{t})}{\partial \bar{t}^2} - \frac{\partial^2 \bar{\varphi}(\bar{x}, \bar{t})}{\partial \bar{x}^2} - \alpha \left( \frac{\partial \bar{w}(\bar{x}, \bar{t})}{\partial \bar{x}} - \bar{\varphi}(\bar{x}, \bar{t}) \right) = 0
\]

(5.5)

In these differential equations, the dimensionless parameters from subsection 5.1.1 are used and \(s\) is the slenderness ratio (and not the shear stiffness which is denoted with capital \(S\)).

It can be shown that \(\bar{w}(\bar{x}, \bar{t})\) and \(\bar{\varphi}(\bar{x}, \bar{t})\) are of the same form. They can be synchronized in time by assuming that they share the same time solution \(\bar{T}(\bar{t})\):

\[
\begin{bmatrix}
\bar{w}(\bar{x}, \bar{t}) \\
\bar{\varphi}(\bar{x}, \bar{t})
\end{bmatrix} = \bar{T}(\bar{t}) \begin{bmatrix}
\bar{W}(\bar{x}) \\
\Psi(\bar{x})
\end{bmatrix}
\]

(5.6)

The differential equation can be decoupled into a spatial equation and a time equation. The time equation is:

\[
\ddot{\bar{T}}(\bar{t}) + \bar{\omega}^2 \bar{T}(\bar{t}) = 0
\]

(5.7)
5.2 Determination of mode shapes and natural frequencies of a beam

The spatial equation is given by

\[
0 = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{W}'(\bar{x}) \\ \dot{\Psi}'(\bar{x}) \end{bmatrix} + \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} W''(\bar{x}) \\ \Psi''(\bar{x}) \end{bmatrix}
\]

\[
+ \begin{bmatrix} \bar{m}\omega^2 & 0 \\ 0 & \frac{\bar{m}\omega^2}{s^2} - \alpha \end{bmatrix} \begin{bmatrix} \dot{W}'(\bar{x}) \\ \dot{\Psi}(\bar{x}) \end{bmatrix}
\] (5.8)

where the forcing function is set to zero.

5.2 Determination of mode shapes and natural frequencies of a beam

For the clamped-free Timoshenko beam (i.e. cantilevered), the frequency equation is given in [9] by:

\[
(a^2 - b^2) \sin a \sinh b - ab \frac{(a^4 + a^4\gamma^4 + 4\gamma^2a^2b^2 + b^4\gamma^4 + b^4)}{(b^2 + \gamma^2a^2)(a^2 + \gamma^2b^2)} \cos a \cosh b - 2ab = 0 \] (5.9)

In which \(a\) is the wave number, which is \(2\pi\) times the number of cycles of the eigenfunction (mode shape) in the beam length, \(b\) does not have a physical meaning but is a parameter in the eigenfunction, \(\gamma\) is given in (3.21).

If \(a < a_c\), then the frequency equation is exactly as above. When \(a > a_c\), the frequency equation is obtained by replacing \(b\) by \(j\tilde{b}\) and \(j^2 = -1\). The critical wave number \(a_c\) is given by:

\[
a_c = s\sqrt{\frac{1}{\gamma^2}} + 1 \quad \text{with slenderness ratio} \quad s = \sqrt{\alpha\gamma^2} \] (5.10)

There are two eigenfunctions, one for \(a < a_c\) and one for \(a > a_c\). If \(a < a_c\), the vector of eigenfunctions is given by:

\[
\begin{bmatrix} \bar{W}(\bar{x}) \\ \Psi(\bar{x}) \end{bmatrix} = \begin{bmatrix} C_1 \\ D_1 \end{bmatrix} \sin a\bar{x} + \begin{bmatrix} C_2 \\ D_2 \end{bmatrix} \cos a\bar{x} + \begin{bmatrix} C_3 \\ D_3 \end{bmatrix} \sinh b\bar{x} + \begin{bmatrix} C_4 \\ D_4 \end{bmatrix} \cosh b\bar{x} \] (5.11)

There are only four unknowns, because \(D_n\) and \(C_n\) are interrelated by \(a\), \(b\) and \(\gamma\).

\[
D_1 = -\frac{a^2 + \gamma^2b^2}{(1 + \gamma^2)a} C_2 
D_2 = \frac{a^2 + \gamma^2b^2}{(1 + \gamma^2)a} C_1
D_3 = \frac{b^2 + \gamma^2a^2}{(1 + \gamma^2)b} C_4 
D_4 = \frac{b^2 + \gamma^2a^2}{(1 + \gamma^2)b} C_3 \] (5.12)

If \(a > a_c\) the eigenfunction becomes

\[
\begin{bmatrix} \bar{W}(\bar{x}) \\ \Psi(\bar{x}) \end{bmatrix} = \begin{bmatrix} \tilde{C}_1 \\ \tilde{D}_1 \end{bmatrix} \sin a\bar{x} + \begin{bmatrix} \tilde{C}_2 \\ \tilde{D}_2 \end{bmatrix} \cos a\bar{x} + \begin{bmatrix} \tilde{C}_3 \\ \tilde{D}_3 \end{bmatrix} \sin b\bar{x} + \begin{bmatrix} \tilde{C}_4 \\ \tilde{D}_4 \end{bmatrix} \cos b\bar{x} \] (5.13)

L. J. van der Meer
and the coefficients $\tilde{C}_n$ and $\tilde{D}_n$ are related as in equation (5.12) but with $b$ replaced by $j\tilde{b}$ (or $b^2$ replaced by $-\tilde{b}^2$).

The boundary conditions for the clamped-free beam are given by

$$W(0) = 0 \quad \text{(dimensionless deflection at clamped end)} \quad (5.14)$$
$$\Psi(0) = 0 \quad \text{(rotation due to moment at clamped end)} \quad (5.15)$$
$$\frac{\partial \Psi}{\partial \bar{x}}(1) = 0 \quad \text{(dimensionless moment at free end)} \quad (5.16)$$
$$\alpha \left( \frac{\partial W}{\partial \bar{x}}(1) - \Psi(1) \right) = 0 \quad \text{(dimensionless shear at free end)} \quad (5.17)$$

The eigenfunctions (5.11) and (5.13) can be expressed in only one unknown by using (5.12) and (5.14-5.17). This leads to

$$\Psi(\bar{x}) = D_1 \left( \sin a\bar{x} + \frac{a(b^2 + \gamma^2a^2)}{b(a^2 + \gamma^2b^2)} \sinh b\bar{x} + \frac{a \cos a + \frac{a(b^2 + \gamma^2a^2)}{a^2 + \gamma^2b^2} \cosh b}{a \sin a + b \sinh b} (\cos a\bar{x} - \cosh b\bar{x}) \right)$$

$$\tilde{W}(\bar{x}) = D_1 \left( \left[ \frac{(1 + \gamma^2)a}{a^2 + \gamma^2b^2} \sin a\bar{x} - \frac{(1 + \gamma^2)b}{b^2 + \gamma^2a^2} \sinh b\bar{x} \right] \frac{a \cos a + \frac{a(b^2 + \gamma^2a^2)}{a^2 + \gamma^2b^2} \cosh b}{a \sin a + b \sinh b} + \frac{(1 + \gamma^2)a}{a^2 + \gamma^2b^2} (\cosh b\bar{x} - \cos a\bar{x}) \right)$$

If $a > a_c$, these functions can be used but $b$ should be replaced by $j\tilde{b}$.

The eigenfunctions are functions of $\gamma$ (material property) and $s$ (geometry parameter), because $a$ and $b$ are functions of $\gamma$ and $s$. For $\gamma^2 = 2.6$ and $s = 6.66$ the wave numbers $a$ and $b$ were determined numerically with Matlab. The Matlab input is given in appendix A. The critical wave number $a_c = 7.841$ for these values of $\gamma$ and $s$. The wave numbers are displayed in table 5.1. When the wave numbers are known, the natural frequencies can be computed as follows:

$$\omega_n = \sqrt{\frac{a_n^2 - b_n^2}{1 + \gamma^2}} \sqrt{\frac{Bs^2}{mH^4}} \quad \text{and for} \quad a > a_c, \quad b_n = j\tilde{b}_n \quad (5.20)$$

This equation is given in dimensional parameters.

The eigenfunctions are known except the constants $D_1$ which determine the relative amplitudes of the eigenfunctions with respect to each other. The eigenfunctions can be orthonormalized, which is described in the next section.

### 5.3 The mode superposition method: forced response of a beam

In this section, the response is written as an infinite sum of modal responses. The mode shapes have been determined in subsection 5.1.2 except for the constant $D_1$. This constant is
5.3 The mode superposition method: forced response of a beam

<table>
<thead>
<tr>
<th>Mode</th>
<th>$a_n$</th>
<th>$b_n$ or $\tilde{b}_n$</th>
<th>$\omega_n$ [rad/s]</th>
<th>$f_n$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.84075</td>
<td>1.62824</td>
<td>2.897305</td>
<td>0.460547</td>
</tr>
<tr>
<td>2</td>
<td>4.14219</td>
<td>2.52714</td>
<td>11.06107</td>
<td>1.760424</td>
</tr>
<tr>
<td>3</td>
<td>7.07301</td>
<td>1.63825</td>
<td>23.18962</td>
<td>3.690743</td>
</tr>
<tr>
<td>4</td>
<td>8.69907</td>
<td>1.94629</td>
<td>30.04294</td>
<td>4.781482</td>
</tr>
<tr>
<td>5</td>
<td>10.83199</td>
<td>4.07270</td>
<td>39.00174</td>
<td>6.207319</td>
</tr>
<tr>
<td>6</td>
<td>11.67560</td>
<td>4.78576</td>
<td>42.52713</td>
<td>6.768403</td>
</tr>
<tr>
<td>7</td>
<td>14.27275</td>
<td>6.81980</td>
<td>53.31200</td>
<td>8.484868</td>
</tr>
<tr>
<td>8</td>
<td>15.40061</td>
<td>7.65717</td>
<td>57.96558</td>
<td>9.225509</td>
</tr>
<tr>
<td>9</td>
<td>17.58377</td>
<td>9.22948</td>
<td>66.92929</td>
<td>10.65213</td>
</tr>
<tr>
<td>10</td>
<td>19.49123</td>
<td>10.56530</td>
<td>74.72047</td>
<td>11.89213</td>
</tr>
</tbody>
</table>

Table 5.1: Wave number and frequencies for a Timoshenko beam with $\gamma^2 = 2.6$ and $s = 6.66$.

different for each mode and has to be determined to obtain the amplitudes of the modes with respect to the other modes. This can be done by normalization to the mass of the system. For the superposition principle to work properly, it is also necessary that all mode shapes are orthogonal with respect to each other, i.e. non of the mode shapes is allowed to be a linear combinations of the other mode shapes. In subsection 5.3.1 the ortho-normality condition is given, which, if satisfied, guarantees both the orthogonality and normalization to mass of the mode shapes.

Once the mode shapes are normalized and orthogonality is satisfied, they can be used in a mode superposition method, which is explained in subsection 5.3.2. When the response is obtained through this method, the response is an infinite sum of modal responses. It is impossible and luckily also unnecessary to include infinite modes.

In subsection 5.3.3 modal contribution factors for base shear and moment are defined, which give the contribution of a certain mode in percent. The contribution factors are specialized for base shear and moment, because these are usually critical in a laterally loaded cantilevered beam. Once it is known that a few lower modes contribute for example 95% to the base shear or moment, then the other modes can be neglected. The total response may be corrected with a factor to account for the missing 5% or a static correction method can be used if the response of the first mode is quasi-static (paragraph 12.12 of [7]).

5.3.1 Orthogonality and normalization of eigenfunctions

The eigenfunctions or mode shapes can be ortho-normalized using:

$$
\int_0^1 \left( \tilde{W}_n(\bar{x})\bar{W}_m(\bar{x}) + \frac{1}{s^2} \Psi_n(\bar{x})\Psi_m(\bar{x}) \right) d\bar{x} = \delta_{nm}
$$

(5.21)

where $\delta_{nm}$ is the Kronecker delta, which is 1 for $n = m$ and 0 otherwise.

This is the orthogonality condition when $n \neq m$. Equations (5.18) and (5.19) satisfy this condition. When $n = m$ the eigenfunctions are normalized according to (5.21), which is solved for the unknown constant $D_1$ in (5.18) and (5.19).
After ortho-normalization the eigenfunctions can be plotted, as done in figure 5.1 and used in a modal superposition analysis.

5.3.2 Mode superposition method

The spatial equation (5.8) can be written as

$$L(\bar{\omega}_n^2 M(\bar{W}_n)) = \bar{\omega}_n^2 M(\bar{W}_n)$$

(5.22)

in which $L()$ is the stiffness operator, $M()$ the mass operator and $\bar{W}_n$ the vector of eigenfunctions, $\bar{W}_n$ and $\Psi_n$. This is useful since it shortens notations and also the operators are clearly linked to stiffness and mass respectively.

$$L() = \begin{bmatrix}
\alpha \frac{d^2}{d\bar{x}^2} & -\alpha \frac{d}{d\bar{x}} \\
\alpha \frac{d}{d\bar{x}} & \frac{d^2}{d\bar{x}^2} - \alpha
\end{bmatrix}$$

and

$$M() = \begin{bmatrix}
\bar{m} & 0 \\
0 & \bar{m} \frac{\bar{s}}{s^2}
\end{bmatrix}$$

(5.23)

The displacement vector $\bar{w}(\bar{x}, \bar{t})$ can be expressed as an infinite sum of the vector of eigenfunctions $\bar{W}_n(\bar{x})$.

$$\begin{bmatrix}
\bar{w}(\bar{x}, \bar{t}) \\
\varphi(\bar{x}, \bar{t})
\end{bmatrix} = \sum_{n=1}^{\infty} \bar{\eta}_n(\bar{t}) \begin{bmatrix}
\bar{W}_n(\bar{x}) \\
\Psi_n(\bar{x})
\end{bmatrix}$$

(5.24)

Similarly, $\bar{P}(\bar{x}, \bar{t})$ can be expressed as an infinite sum of the vector $M(\bar{W}_n)$

$$\begin{bmatrix}
\bar{P}(\bar{x}, \bar{t}) \\
0
\end{bmatrix} = \sum_{n=1}^{\infty} \bar{F}_n(\bar{t}) M \left( \begin{bmatrix}
\bar{W}_n(\bar{x}) \\
\Psi_n(\bar{x})
\end{bmatrix} \right)$$

(5.25)

The $n^{th}$ mode force $\bar{F}_n(\bar{t})$ can be found by multiplying (5.25) by $\bar{W}_m^T(\bar{x})$ and integrating over the domain ($0 \leq \bar{x} \leq 1$):

$$\int_0^1 \bar{W}_m^T(\bar{x}) \bar{P}(\bar{x}, \bar{t}) d\bar{x} = \sum_{n=1}^{\infty} \bar{F}_n(\bar{t}) \int_0^1 \bar{W}_m^T(\bar{x}) M(\bar{W}_n(\bar{x})) d\bar{x}$$

(5.26)

The expression

$$\int_0^1 \bar{W}_m^T(\bar{x}) M(\bar{W}_n(\bar{x})) d\bar{x} = \delta_{nm}$$

(5.27)

is the ortho-normality condition, written slightly different then in (5.21). Therefore

$$\bar{F}_m(\bar{t}) = \int_0^1 \bar{W}_m^T(\bar{x}) \bar{P}(\bar{x}, \bar{t}) d\bar{x} = \bar{F}(\bar{t}) \int_0^1 W_m(\bar{x}) \bar{P}(\bar{x}) d\bar{x}$$

(5.28)

$$\bar{F}_m = \int_0^1 W_m(\bar{x}) \bar{P}(\bar{x}) d\bar{x}$$

(5.29)

Now $\bar{F}_m$ is a participation factor for a mode and $\bar{F}(\bar{t})$ contains only time variation of the force with amplitude 1.
5.3 The mode superposition method: forced response of a beam

Figure 5.1: *Eigenfunctions of a Timoshenko beam with $\gamma^2 = 2.6$ and $s = 6.66$.***
The equations of motion \((5.4)\) can be rewritten using \((5.24)\) and \((5.25)\) as
\[
\sum_{n=1}^{\infty} (\ddot{\eta}_n(t) M(\bar{W}_n(\bar{x})) + \eta_n(t) L(\bar{W}_n(\bar{x}))) = \sum_{n=1}^{\infty} \bar{F}_n M(\bar{W}_n(\bar{x})) \bar{F}(\bar{t})
\]
\[(5.30)\]

Using \((5.22)\) this can be simplified to
\[
\sum_{n=1}^{\infty} (\ddot{\eta}_n(t) + \bar{\omega}_n^2 \eta_n(t)) M(\bar{W}_n(\bar{x})) = \sum_{n=1}^{\infty} \bar{F}_n M(\bar{W}_n(\bar{x})) \bar{F}(\bar{t})
\]
\[(5.31)\]

which after multiplying by \(\bar{W}_m^T(\bar{x})\), integrating over the domain \((0 \leq \bar{x} \leq 1)\) and using ortho-normality becomes
\[
\ddot{\eta}_m(\bar{t}) + \bar{\omega}_m^2 \eta_m(\bar{t}) = \bar{F}_m(\bar{t}) \quad \text{for every } m = 1, 2, \ldots \infty
\]
\[(5.32)\]

Equation \((5.32)\) is equivalent to the equation of motion of a SDOF system and the same solutions for the dynamic load factor can be used within every mode. The complete solution is the infinite sum of these modes.
\[
\begin{bmatrix}
\bar{w}(\bar{x}, \bar{t}) \\
\varphi(\bar{x}, \bar{t})
\end{bmatrix} = \sum_{n=1}^{\infty} \bar{\eta}_n(\bar{t}) \begin{bmatrix}
\bar{W}_n(\bar{x}) \\
\Psi_n(\bar{x})
\end{bmatrix} = \sum_{n=1}^{\infty} \frac{\bar{F}_n}{\bar{\omega}_n^2} \bar{D}_n(\bar{t}) \begin{bmatrix}
\bar{W}_n(\bar{x}) \\
\Psi_n(\bar{x})
\end{bmatrix}
\]
\[(5.33)\]

where \(\bar{D}_n(\bar{t})\) is the dynamic load factor (DLF) obtained by applying \(\bar{F}(\bar{t})\) to a SDOF system.

Dimensionless moment \(\bar{\mu}\) and shear force \(\bar{v}\) can be derived from \((5.33)\) directly.
\[
\bar{\mu}(\bar{x}, \bar{t}) = \frac{\partial \varphi(\bar{x}, \bar{t})}{\partial \bar{x}} = \sum_{n=1}^{\infty} \frac{\bar{F}_n}{\bar{\omega}_n^2} \bar{D}_n(\bar{t}) \frac{d\Psi_n(\bar{x})}{d\bar{x}}
\]
\[(5.34)\]
\[
\bar{v}(\bar{x}, \bar{t}) = \alpha \left( \frac{\partial \bar{w}(\bar{x}, \bar{t})}{\partial \bar{x}} - \varphi(\bar{x}, \bar{t}) \right) = \sum_{n=1}^{\infty} \frac{\bar{F}_n}{\bar{\omega}_n^2} \bar{D}_n(\bar{t}) \alpha \left( \frac{d\bar{W}_n(\bar{x})}{d\bar{x}} - \Psi_n(\bar{x}, \bar{t}) \right)
\]
\[(5.35)\]

For the dimensional quantities \(w(x, t), \mu(x, t)\) and \(v(x, t)\) the dimensionless quantities should be multiplied by \(H, B/H\) and \(B/H^2\) respectively and the dynamic load factor \(\bar{D}(\bar{t})\) for the force \(\bar{F}(\bar{t})\) should be used with \(t\) instead of \(\bar{t}\) as the variable.

5.3.3 Modal contribution factors

In [7] in paragraph 16.6, an alternative method to calculate base shear and moment is given. This method can also be used to derive modal contribution factors for base shear and moment. Modal contribution factors give the ratio of the quantity in the \(n^{th}\) mode to the sum of all modes, without having to calculate all modes first. The method in the book is specialized for base acceleration, but the same principle can be used for an arbitrary applied force as well.

Equation \((5.22)\) is repeated here:
\[
L(\bar{W}_n(\bar{x})) = \bar{\omega}_n^2 M(\bar{W}_n(\bar{x}))
\]
\[(5.36)\]
This is the spatial equation, describing dynamic equilibrium. The static equilibrium is given by:

$$L(\ddot{w}(\bar{x})) = \bar{P}(\bar{x})$$  \hfill (5.37)

Using

$$\ddot{w}_n(\bar{x}, \bar{t}) = \frac{\bar{F}_n}{\bar{\omega}_n^2} \dddot{W}_n(\bar{x}) \bar{D}_n(\bar{t})$$  \hfill (5.38)

The force in the $n^{th}$ mode can be written as

$$\bar{P}_n(\bar{x}, \bar{t}) = \frac{\bar{F}_n}{\bar{\omega}_n^2} L(\ddot{W}_n(\bar{x})) \bar{D}_n(\bar{t}) = \bar{F}_n \bar{m} \ddot{W}_n(\bar{x})$$  \hfill (5.39)

If the static equilibrium (5.37) is written as

$$\bar{P}_n(\bar{x}) = \bar{F}_n \bar{m} \bar{\omega}_n^2 \bar{W}_n(\bar{x}) = \bar{F}_n \bar{m} \bar{w}(\bar{x})$$  \hfill (5.40)

Combining with (5.39) this leads to

$$\bar{v}_b = \int_0^1 \bar{P}(\bar{x})d\bar{x} = \sum_{n=1}^{\infty} \bar{F}_n \bar{m} \int_0^1 \ddot{W}_n(\bar{x})d\bar{x} = \bar{v}_b(\bar{t}) = \bar{v}_b \bar{D}_n(\bar{t})$$  \hfill (5.42)

which gives an alternative equation for base shear and defines the modal contribution factor for base shear as

$$c_{bs,n} = \frac{\bar{F}_n \bar{m} \int_0^1 \dddot{W}_n(\bar{x})d\bar{x}}{\int_0^1 \bar{P}(\bar{x})d\bar{x}}$$  \hfill (5.43)

These contribution factors over all modes equal 1, so $c_{bs,n} \cdot 100\%$ gives the percentage which is contributed by a specific mode. Also the expression for base shear (see (5.35) with $\bar{x} = 0$) is simplified to (5.42), because it is not necessary to calculate derivatives of mode shapes anymore.

For base moment, the same procedure can be followed using the second part of the equation of motion (5.4)

$$\alpha \left( \frac{d^2 \dddot{W}_n(\bar{x})}{d\bar{x}^2} + \Psi_n(\bar{x}) \right) + \frac{d^2 \ddot{\Psi}_n(\bar{x})}{d\bar{x}^2} = \bar{\omega}_n^2 \bar{m}^2 \bar{\Psi}_n(\bar{x})$$  \hfill (5.44)

This can be rewritten, using the first part of the equation of motion (5.4), to obtain

$$\frac{d^2 \ddot{\Psi}_n(\bar{x})}{d\bar{x}^2} = \bar{\omega}_n^2 \bar{m}^2 \ddot{\Psi}_n(\bar{x}) - \int_0^1 \bar{\omega}_n^2 \bar{m} \ddot{W}_n(\bar{x})d\bar{x}$$  \hfill (5.45)

and therefore

$$\frac{d\ddot{\Psi}_n(\bar{x})}{d\bar{x}} = \int_0^1 \frac{\bar{\omega}_n^2 \bar{m}}{S^2} \dddot{\Psi}_n(\bar{x}) - \int_0^1 \frac{\bar{\omega}_n^2 \bar{m}}{S^2} \ddot{W}_n(\bar{x})d\bar{x}$$  \hfill (5.46)
Remembering that \( \bar{\mu}_n(\bar{x}, \bar{t}) = \bar{F}_n \bar{\omega}^2_n \bar{D}_n(\bar{t}) \) \( \bar{\Psi}_n(\bar{x}) \bar{d} \bar{x} \) \( \) (5.47) and substituting (5.46), the alternative expression for base moment is obtained:

\[
\bar{\mu}_b; n = \bar{F}_n \bar{m} \left( \frac{1}{\bar{\tau}^2} \int_0^1 \bar{\Psi}_n(\bar{x}) \bar{d} \bar{x} + \int_0^1 \bar{x} \bar{W}_n(\bar{x}) \bar{d} \bar{x} \right) \bar{\mu}_b; n(\bar{t}) = \bar{\mu}_b; n \bar{D}_n(\bar{t}) \tag{5.48}
\]

Therefore the contribution factor for base moment is defined as

\[
c_{bm; n} = \frac{\bar{F}_n \bar{m} \left( \frac{1}{\bar{\tau}^2} \int_0^1 \bar{\Psi}_n(\bar{x}) \bar{d} \bar{x} + \int_0^1 \bar{x} \bar{W}_n(\bar{x}) \bar{d} \bar{x} \right)}{\int_0^1 \bar{x} \bar{P}(\bar{x}) \bar{d} \bar{x}} \tag{5.49}
\]

In table 5.2 the modal contribution factors are compared for base shear and moment, for three different load distributions (similar to the load distributions used in section 3.4):

- Uniform distribution: \( \bar{P}(\bar{x}) = \bar{P}_m \)
- Linear distribution: \( \bar{P}(\bar{x}) = 2 \bar{P}_m(1 - \bar{x}) \)
- Quadratic distribution: \( \bar{P}(\bar{x}) = 3 \bar{P}_m(\bar{x} - 1)^2 \)

In these expressions the amplitudes are chosen so that \( \int_0^1 \bar{P}(\bar{x}) \bar{d} \bar{x} = \bar{P}_m \). This is not necessary for evaluation of the contribution factors but it gives the load distributions the same area when they are plotted, see figure 5.2. In table 5.2 the bold contribution factors represent a total contribution of 95% for base moment and 90% for base shear. It can be seen that higher modes contribute more to base shear than to base moment. Also, when the load distribution is not uniform but more concentrated at the base, as is likely for a blast load from a short-range explosion, the higher modes have more influence.

<table>
<thead>
<tr>
<th>Load distribution</th>
<th>Uniform</th>
<th>Linear</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Shear [%]</td>
<td>Moment [%]</td>
<td>Shear [%]</td>
</tr>
<tr>
<td>1</td>
<td>61.48</td>
<td>92.49</td>
<td>35.93</td>
</tr>
<tr>
<td>2</td>
<td>23.35</td>
<td>6.78</td>
<td>34.96</td>
</tr>
<tr>
<td>3</td>
<td>5.93</td>
<td>0.98</td>
<td>12.36</td>
</tr>
<tr>
<td>4</td>
<td>1.68</td>
<td>-0.40</td>
<td>0.92</td>
</tr>
<tr>
<td>5</td>
<td>1.43</td>
<td>0.41</td>
<td>3.77</td>
</tr>
<tr>
<td>6</td>
<td>0.78</td>
<td>-0.33</td>
<td>1.49</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>0.18</td>
<td>1.80</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
<td>-0.14</td>
<td>0.46</td>
</tr>
<tr>
<td>9</td>
<td>0.60</td>
<td>0.12</td>
<td>1.31</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
<td>-0.13</td>
<td>0.46</td>
</tr>
<tr>
<td>Sum</td>
<td>96.72</td>
<td>99.96</td>
<td>93.46</td>
</tr>
<tr>
<td>Sum</td>
<td>90.76</td>
<td>99.27</td>
<td>90.31</td>
</tr>
</tbody>
</table>

Table 5.2: Modal contribution factors in % for base shear and moment.
5.4 Approximation of response including higher modes

From subsection 5.3.3, the contributions of each mode to the total dynamic response are known and the transient response can be plotted. However, the construction of a $p-i$-diagram requires the maximum response to be determined. Unfortunately, the response is a sum of harmonic functions, of which the global maximum is difficult to determine. For earthquake engineering, several modal combination rules have been established [7]. These modal combination rules use the modal maxima to estimate the total maximum of response and are described in subsection 5.4.1.

Another problem arises here, because the $n$-th modal maximum, determined by the dynamic load factor for mode $n$, is influenced by the response of the mode with the highest contribution, usually the first mode. For example, if the maximum response of the first mode occurs just after $t_d$, then all higher modes have their first maximum before $t_d$, which determines the dynamic load factor. However, if the first mode has the highest contribution, then the overall maximum response is likely to occur near the maximum of the first mode. The maxima of the higher modes in this region are lower than their first maxima. So the regime of the response of the first mode (impulsive, dynamic or quasi-static) influences the dynamic load factors of the higher modes. This is elaborated in subsection 5.4.2.

Finally, the modal maxima and the modal combination rules are used to estimate the maximum response in subsection 5.4.3.

5.4.1 Modal combination rules

Modal combination rules are described in paragraph 13.7.2 of [7] of which only two are discussed here. They will be used to approximate maximum base moment and base shear including influence of the higher modes. The problem with determining the maximum dynamic response of a continuous or MDOF system, is that the total response is a combination of dynamic load factors $D_n(t)$ with different contribution factors.

The absolute sum (ABSSUM) modal combination rule is defined as the absolute sum of the maximum response of all modes. Since the $D_n(t)$ vary sinusoidally with different frequencies, the actual maximum is always lower than the maximum from the absolute sum rule. If the
maximum of a response quantity is denoted as \( r_{m;\text{abssum}} \), the rule can be written as:

\[
r_{m;\text{abssum}} \simeq \sum_{n=1}^{N} |r_{m;n}|
\]  

(5.50)

Where \( n = 1..N \) are the modes that have a significant contribution to the response quantity according to the modal contribution factors. This rule is (very) conservative and therefore not popular in structural design applications.

The square-root-of-sum-of-squares rule (SRSS) gives a less conservative approximation of the maximum of a response quantity. If the maximum of a response quantity is denoted as \( r_{m;\text{srss}} \), the rule can be written as

\[
r_{m;\text{srss}} \simeq \sqrt{\sum_{n=1}^{N} r_{m;n}^2}
\]  

(5.51)

Where \( n = 1..N \) are the modes that have a significant contribution to the response quantity according to the modal contribution factors. The SRSS rule is based on random vibration theory and specialized for earthquake engineering, hence it is less accurate for blast loading.

5.4.2 Dynamic load factor including higher modes

The dynamic load factor for higher modes of vibration is approximated for the different regimes in the p-i diagram, assuming an idealized detonation as in figure 4.3a. For an idealized deflagration the method is similar. The method is divided in a conservative approach and a less conservative approach.

Time dependent vs maximum dynamic load factor

The time dependent dynamic load factor is denoted as \( \bar{D}_n(t) \) and defined as the time dependent dynamic response of mode \( n \) divided by the static response if the maximum dynamic load were applied statically. The maximum dynamic load factor \( \bar{D}_{m;n} \) is defined as the first maximum of the dynamic response of mode \( n \) divided by the static response if the maximum dynamic load were applied statically.

Conservative approach

The conservative approach is to assume that the dynamic load factor of all modes is determined by either the quasi-static or the impulsive asymptote. The dynamic regime in between the asymptotes is ignored. The maximum dynamic load factor is \( \bar{D}_{m;n} = 2 \) (idealized detonation) on the quasi-static asymptote and \( \bar{D}_{m;n} = \omega_n t_d/2 \). This is shown in figure 5.3. For higher modes, \( t_d/T_n \) becomes larger, as is indicated in this figure.

The maximum response in the quasi-static regime equals the response to a step force, which is equal to the value 2, independent of the natural frequency. If the load is a step force for the lowest natural frequency, it will also be a step force for the higher natural frequencies. Therefore, the maximum dynamic load factor \( \bar{D}_{m;n} \) is equal to 2 for all modes. In figure 5.4 the time-dependent response of 3 modes is shown. The maximum of each mode is equal to the modal contribution factor \( c_{b;n} \) multiplied by \( \bar{D}_{m;n} \). The modal contribution factors for base shear, uniform distributed loading, from table 5.2 are used, normalized to 100%.

Dynamic response of high-rise building structures to blast loading
5.4 Approximation of response including higher modes

Figure 5.3: Dynamic load factor and asymptotes for an idealized detonation.

Figure 5.4: Quasi-static base shear response of 3 modes to step force (uniform distribution).
Chapter 5: Dynamic response of a continuous beam to blast loading

The maximum response in the impulsive regime on the other hand, is determined by the natural frequency, because the dynamic load factor is determined by the impulsive asymptote, $D_{m,n} = \omega_n t_d/2 = \pi t_d/T_n$. This is only the case when all modes with a significant contribution are in the impulsive regime. This means that all significant modes should have a ratio of $t_d/T_n < 0.64$, see figure 5.3. The time dependent response of 3 modes is shown in figure 5.5 for an idealized detonation with $t_d = 0.1 \text{s}$. The natural frequencies are taken from table 5.1. The $t_d/T_n$ ratios are 0.046, 0.176 and 0.369 respectively, which are all in the impulsive regime (although the third mode is only just within the impulsive regime).

![Figure 5.5: Impulsive base shear response of 3 modes to idealized detonation (uniform distribution).](image)

**Less conservative approach**

The less conservative approach described here, applies only when the first mode has the largest contribution to the response. It uses the $t_d/T_n$-values of the higher modes to predict the maximum dynamic load factors $D_{m,n}$ of the higher modes including the dynamic regime. These maximum dynamic load factors depend on the maximum dynamic load factor of the first mode $D_{m,1}$. There are two possibilities:

- The maximum of the first mode occurs during forced vibration ($t_{m;1} < t_d$). This is response in the dynamic regime close to the quasi-static regime.

- The maximum of the first mode occurs during free vibration ($t_{m;1} > t_d$). This is response in the dynamic regime close to the impulsive regime.

Response in the dynamic regime, close to the quasi-static regime, is shown for 3 modes in figure 5.6b for $t_d = 2 \text{s}$. The location of the modes is shown in the $D_m - t_d/T_n$-plot of figure 5.6a. If it is assumed that the first mode has the highest contribution factor, as is the case in figure 5.6a, its maximum response governs the total response. The higher modes reach their first maximum earlier due to their shorter periods of vibration, but these maxima to not contribute to the maximum total response. Therefore, they are indicated in figure 5.6b and figure 5.6a as ‘false’ maxima.
5.4 Approximation of response including higher modes

Figure 5.6: Quasi-static/dynamic base shear response of 3 modes to idealized detonation (uniform distribution).
In figure 5.6c the dynamic load factors $D_n(t)$ for 3 modes are shown without their contribution factors. The time dependent dynamic load factor $D_n(t)$ is approximately equal to $\bar{F}(t) + 1$. Since the modal maxima of the first and higher modes occur at approximately the same time, $D_{m:n} \approx D_{m:1}$. It is conservative to use the 'false' maxima, which are simply the $D_{m:n}$ for the $t_d/T_n$ values of the modes involved (indicated in the figure with a triangle).

Response in the dynamic regime, close to the impulsive regime, is characterized by one or more modes in the impulsive regime and a number of higher modes in the dynamic regime. The MDOF or continuous impulsive response is a limit case, because there are always higher modes outside the impulsive regime. These modes have often small contribution factors, but high dynamic load factors, if they are determined by the impulsive asymptote. On the impulsive asymptote $D_{m:n} = \omega_n t_d/2$, hence the dynamic load factor is determined by $\omega_n$ which increases with mode number. If all higher modes are approximated by this dynamic load factor, almost all higher modes would have a significant contribution. However, modes with high frequencies fall outside the impulsive regime. In figure 5.7b the response in the dynamic regime near the impulsive regime is shown for 3 modes, $t_d = 0.5$ s. In this case, the first mode response is impulsive, while the second and third mode attain their maxima during the presence of the force. However, these maxima during forced vibration are 'false' maxima, since they do not contribute to the total maximum. Instead the maxima of free vibration should be determined. This can be shown by calculating the dynamic load factor for free vibration (substitute $t_m$ for $t_d$ in $D(t)$ in (4.11)), which is compared to forced vibration in figure 5.7a. For an idealized detonation, $D_n \approx 1.25$ is a conservative approximation for higher modes outside the impulsive regime. For more accurate dynamic load factors, $t_d/T_n$ values should be calculated and accompanying $D_{m:n}$ should be determined with figure 5.7a.

**Summary**

The conservative and less conservative approach for determination of maximum dynamic load factors of the higher modes are summarized in table 5.3 for an idealized detonation. For an idealized deflagration, figure 5.8 should be used.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Conservative</th>
<th>Less conservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of maximum response of first mode $t_{m:1}$</td>
<td>Asymptotes (limits)</td>
<td>Including dynamic regime</td>
</tr>
<tr>
<td>$t_{m:1} &gt; t_d$ (maximum during free vibration)</td>
<td>All modes impulsive ($t_d \rightarrow 0$) $D_{m:n} \approx \omega_n t_d/2$</td>
<td>First mode(s) impulsive, higher modes dynamic $D_{m:n} \approx \min\left(\frac{\omega_n t_d}{2}, 1.25\right)$</td>
</tr>
<tr>
<td>$t_{m:1} &lt; t_d$ (maximum during forced vibration)</td>
<td>Response to step force ($t_d \rightarrow \infty$) $D_{m:n} \approx 2$</td>
<td>All modes dynamic $D_{m:n} \approx \bar{D}_{m:1}$</td>
</tr>
</tbody>
</table>

Table 5.3: Summary: maximum dynamic load factors for higher modes (idealized detonation).
5.4 Approximation of response including higher modes

Figure 5.7: Impulsive/dynamic base shear response of 3 modes to idealized detonation.
Idealized deflagration

For an idealized deflagration the maximum dynamic load factors for the first and higher modes can be determined with figure 5.8. There is no obvious approximation for higher modes if the first mode is impulsive, so the values of $\bar{D}_{n,m}$ should be read from the free vibration part of the figure for the values of $t_d/T_n$ involved.

[Figure 5.8: Dynamic load factor for forced and free vibration for idealized deflagration.]

5.4.3 Estimates of maximum response

Using the approximations for the dynamic load factors of higher modes, the SRSS rule for modal combination and the expressions for base shear (5.42) and base moment (5.48), the approximated base shear and moment can be written as:

$$\bar{v}_{b,m} = \bar{m} \sqrt{\sum_{n=1}^{N} \left( \bar{F}_n \bar{D}_{n,m} \int_0^1 \bar{W}_n(x) dx \right)^2}$$

(5.52)

$$\bar{\mu}_{b,m} = \bar{m} \sqrt{\sum_{n=1}^{N} \left( \bar{F}_n \bar{D}_{n,m} \frac{1}{8} \left[ \int_0^1 \bar{\Psi}_n(x) dx + \int_0^1 \bar{\Psi}_n(x) dx \right] \right)^2}$$

(5.53)

in which $\bar{D}_{n,m}$ are the approximations for the dynamic load factors as described in subsection 5.4.2.

5.5 Pressure-impulse diagram including higher modes

Only the quasi-static and impulsive criteria from subsection 5.4.2 are needed for the asymptotes of the $p-i$-diagram. Denoting the pressure and impulse for the diagram as $P_m$ and $J = P_m t_d / 2$, (5.52) and (5.53) can be rewritten for the impulsive and quasi-static regimes, using $\bar{D}_{n,m}$ for the quasi-static or impulsive regime. If we denote:

$$\tilde{P}(\bar{x}) = \frac{\tilde{P}(\bar{x})}{P_m} \rightarrow \tilde{P}(\bar{x}) = \tilde{P}(\bar{x}) \tilde{P}_m = \tilde{P}(\bar{x}) \frac{P_m H^3}{B}$$

(5.54)
then

\[
v_{b;m} = \bar{v}_{b;m} \frac{B}{H^2} = P_m H \int_0^1 \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\bar{D}_{n;m} c_{bs;n})^2}
\tag{5.55}
\]

The quasi-static asymptote is obtained by substituting \( \bar{D}_{n;m} = 2 \) and solving \( v_{b;m} = v_c \) for \( P_{m;vc} \):

\[
P_{m;vc} = \frac{v_c}{2H \int_0^1 \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} c_{bs;n}^2}}
\tag{5.56}
\]

The similar procedure can be followed for base moment, which results in:

\[
\mu_{b;m} = \bar{\mu}_{b;m} \frac{B}{H} = P_m H^2 \int_0^1 \hat{x} \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\bar{D}_{n;m} c_{bm;n})^2} = \mu_c
\]

and

\[
P_{m;mc} = \frac{\mu_c}{2H^2 \int_0^1 \hat{x} \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} c_{bm;n}^2}}
\tag{5.58}
\]

For the impulsive asymptotes \( \bar{D}_{n;m} = \omega_n t_d / 2 \) should be substituted. Notice that

\[
\bar{D}_{n;m} P_m = \frac{\omega_n t_d P_m}{2} = \omega_n J \quad \text{with} \quad J = \frac{I}{H}
\tag{5.59}
\]

\[
v_{b;m} = \bar{v}_{b;m} \frac{B}{H^2} = H J \int_0^1 \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bs;n})^2} = v_c
\tag{5.60}
\]

\[
J_{vc} = \frac{v_c}{H \int_0^1 \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bs;n})^2}}
\tag{5.61}
\]

\[
\mu_{b;m} = H^2 J \int_0^1 \hat{x} \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bm;n})^2} = \mu_c
\tag{5.62}
\]

\[
J_{mc} = \frac{\mu_c}{H^2 \int_0^1 \hat{x} \hat{P}(\bar{x}) d\bar{x} \sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bm;n})^2}}
\tag{5.63}
\]

A very important conclusion regarding the impulsive response, is that the influence of the higher modes is not only increased because of the higher contribution factors but also because of the natural frequency, which increases with the mode number. Therefore, for impulsive response, the modal contribution factors should be adjusted to account for the influence of the natural frequency. The results of these adjustments are shown in table 5.4 and compared to the quasi-static contribution factors. Both contribution factors are normalized so that the sum over 10 modes is 100%.
### Table 5.4: Modal contribution factors for 10 modes, quasi-static vs impulsive.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Shear[%]</th>
<th>Moment[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quasi-static</td>
<td>Impulsive</td>
</tr>
<tr>
<td>1</td>
<td>63.56</td>
<td>21.23</td>
</tr>
<tr>
<td>3</td>
<td>6.13</td>
<td>16.41</td>
</tr>
<tr>
<td>4</td>
<td>1.74</td>
<td>6.02</td>
</tr>
<tr>
<td>5</td>
<td>1.48</td>
<td>6.65</td>
</tr>
<tr>
<td>6</td>
<td>0.81</td>
<td>3.96</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>6.17</td>
</tr>
<tr>
<td>8</td>
<td>0.26</td>
<td>1.73</td>
</tr>
<tr>
<td>9</td>
<td>0.62</td>
<td>4.79</td>
</tr>
<tr>
<td>10</td>
<td>0.26</td>
<td>2.23</td>
</tr>
</tbody>
</table>

(a) Uniform load distribution

<table>
<thead>
<tr>
<th>Mode</th>
<th>Shear[%]</th>
<th>Moment[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quasi-static</td>
<td>Impulsive</td>
</tr>
<tr>
<td>1</td>
<td>38.44</td>
<td>8.25</td>
</tr>
<tr>
<td>2</td>
<td>37.41</td>
<td>30.69</td>
</tr>
<tr>
<td>3</td>
<td>13.22</td>
<td>22.75</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>2.19</td>
</tr>
<tr>
<td>5</td>
<td>4.03</td>
<td>11.67</td>
</tr>
<tr>
<td>6</td>
<td>1.59</td>
<td>5.03</td>
</tr>
<tr>
<td>7</td>
<td>1.93</td>
<td>7.62</td>
</tr>
<tr>
<td>8</td>
<td>0.49</td>
<td>2.12</td>
</tr>
<tr>
<td>9</td>
<td>1.40</td>
<td>6.96</td>
</tr>
<tr>
<td>10</td>
<td>0.49</td>
<td>2.73</td>
</tr>
</tbody>
</table>

(b) Linear load distribution

<table>
<thead>
<tr>
<th>Mode</th>
<th>Shear[%]</th>
<th>Moment[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quasi-static</td>
<td>Impulsive</td>
</tr>
<tr>
<td>1</td>
<td>26.83</td>
<td>4.55</td>
</tr>
<tr>
<td>2</td>
<td>38.14</td>
<td>24.71</td>
</tr>
<tr>
<td>3</td>
<td>18.44</td>
<td>25.04</td>
</tr>
<tr>
<td>4</td>
<td>2.10</td>
<td>3.69</td>
</tr>
<tr>
<td>5</td>
<td>5.91</td>
<td>13.50</td>
</tr>
<tr>
<td>6</td>
<td>1.69</td>
<td>4.20</td>
</tr>
<tr>
<td>7</td>
<td>3.39</td>
<td>10.59</td>
</tr>
<tr>
<td>8</td>
<td>0.69</td>
<td>2.33</td>
</tr>
<tr>
<td>9</td>
<td>2.05</td>
<td>8.04</td>
</tr>
<tr>
<td>10</td>
<td>0.77</td>
<td>3.35</td>
</tr>
</tbody>
</table>

(c) Quadratic load distribution

Dynamic response of high-rise building structures to blast loading
Chapter 6

Results for example building structure

In this final chapter, the response of the example building structure from chapter 3 to a BLEVE blast according to Baker’s method [23] as well as Van den Berg [21] is determined. The chapter has the same structure as the report itself. Firstly, in section 6.1 the blast load pressure and impulse distribution on the building are determined according to chapter 2. Secondly, the building structure is modeled as an equivalent beam and an equivalent SDOF system in section 6.2 as described in chapter 3. Subsequently, the SDOF response as described in chapter 4 is elaborated in section 6.3. The response of the continuous equivalent beam is given in section 6.4 by the methods of chapter 5. Finally, the results of the SDOF and continuous response are compared in section 6.5.

6.1 BLEVE blast loading on example building

Consider a BLEVE blast at the surface from a cylindrical vessel of 108 m$^3$ containing propane, 70 volume % liquid, flashing from 329 K at ranges of 20, 35 and 50m from the center of the base of the building. The method described in section 2.3, called Baker’s method, is used to determine the free field overpressure and impulse for a surface burst. Alternatively, overpressure and positive phase duration are estimated by using the BLEVE blast charts for propane in half space, presented in [21], which shall be referred to as Van den Berg’s method.

To determine positive phase duration for Baker’s method or the impulse for Van den Berg’s method, a blast pressure-time profile is needed. From literature, such as [21] and [6], it is concluded that a BLEVE blast pressure-time profile corresponds more to an idealized detonation than to an idealized deflagration. In other words, the blast wave from a BLEVE is similar to a shock wave and not to a pressure wave which is caused by ordinary gas explosions.

In Baker’s method the overpressure and impulse are adjusted for the cylindrical shape of the vessel (instead of hemispherical) and the elevation of the vessel above the surface. These modifications increase the overpressure and impulse. Therefore, the same modifications are also applied to Van den Berg’s overpressure and impulse. The two methods are compared in table 6.1. It is known that Baker’s method is less accurate for close range, which can be observed. Furthermore, assuming an idealized blast (detonation pressure-time profile) leads to short positive phase durations for Baker’s method and high impulses for Van den Berg’s method.
Table 6.1: Free field overpressure, impulse and positive phase duration for a BLEVE using two methods.

<table>
<thead>
<tr>
<th>r[m]</th>
<th>( \rho ) [kPa]</th>
<th>( \bar{r} ) [kPa·ms]</th>
<th>( t_d ) [ms]</th>
<th>( \rho ) [kPa]</th>
<th>( \bar{r} ) [kPa·ms]</th>
<th>( t_d ) [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>648</td>
<td>1723</td>
<td>5.3</td>
<td>320</td>
<td>6720</td>
<td>42</td>
</tr>
<tr>
<td>35</td>
<td>130</td>
<td>495</td>
<td>7.6</td>
<td>130</td>
<td>3120</td>
<td>48</td>
</tr>
<tr>
<td>50</td>
<td>59</td>
<td>280</td>
<td>9.5</td>
<td>54</td>
<td>1350</td>
<td>50</td>
</tr>
</tbody>
</table>

Baker’s method
Van den Berg’s method

6.1.1 Blast distribution

Blast parameters versus scaled distance are available in figures of [3], which are figures 2.6 and 2.9 in this report. The overpressure can be used to derive other parameters such as the reflection coefficient and the wavefront velocity versus scaled distance. The wavefront velocity \( u \) is needed to determine the time of diffraction of the blast wave around the building to find out whether it is a drag target, a diffraction target or a sequentially loaded target. The overpressure and positive phase duration should be determined with Van den Berg’s method [21] and the impulse should be determined by assuming a pressure-time profile. Impulse and positive phase duration from the figures in [3] should not be used.

Firstly, figure 2.9 is used to determine the amount of TNT (kg) that would produce the same overpressure. The results are shown in table 6.2.

<table>
<thead>
<tr>
<th>r[m]</th>
<th>( \rho ) [kg TNT]</th>
<th>Van den Berg’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2950</td>
<td>1170</td>
</tr>
<tr>
<td>35</td>
<td>1875</td>
<td>1875</td>
</tr>
<tr>
<td>50</td>
<td>1650</td>
<td>1400</td>
</tr>
</tbody>
</table>

Table 6.2: Charge mass of a BLEVE at different ranges.

Secondly, the equivalent amount of TNT can be used to estimate the wavefront velocity \( u \) and approximate the time of diffraction around the front of the building by:

\[ t_{d,d,f} \approx \frac{3}{u} \min \left( \frac{W}{2}, H \right) \]

This approximation is found in literature, for example [22] and [19]. Values of the time of diffraction around the front between 87-182ms were found. This is longer than the positive phase duration \( t_d \) which is between 5.3-50ms at the ranges considered. Therefore it is concluded that the building is a sequentially loaded target for the BLEVE and ranges considered. The pressure-time profile can be approximated by figure 2.8c.

Thirdly, the overpressure in kg TNT can be used to determine \( \rho_r \) from figure 2.9 directly. However, then the effect of the angle of incidence on the reflected overpressure is not taken into account. Therefore, the values of \( p \) from table 6.1 are used to determine the reflection coefficient \( C_r \). Then \( C_r \) is multiplied by \( p \) to obtain \( \rho_r \). This is done for every location on the front of the building for the appropriate values of \( r \) and \( \alpha_i \), see figure 6.1. This is done numerically with the Excel program and the average overpressure on the facade is calculated, multiplied by the facade area and divided by the number of stability elements to obtain \( F_m \), the average force on one stability element. The average impulse \( I \), is obtained by using the

Dynamic response of high-rise building structures to blast loading
$t_d$ value at the given range and the load-time profile. These are shown in Table 6.3. Some important assumptions have to be made to obtain these values:

- The pressure-time profile is that of an idealized detonation as shown in Figure 2.8c.
- The time of blast arrival $t_a$ and the positive phase duration $t_d$ are set equal at every location on the building facade. This is probably conservative since more load is assumed at one time instant than is actually present. However, the dynamic response to a sequential load which arrives first at the base and later at the top of the building (see Figure 2.10b) should be evaluated numerically to confirm this assumption.

![Figure 6.1: Range $r$ and angle of incidence $\alpha_i$ for a blast on a facade](image)

<table>
<thead>
<tr>
<th>$r$ [m]</th>
<th>$F_m$ [MN]</th>
<th>$I$ [MN·ms]</th>
<th>$t_d$ [ms]</th>
<th>$F_m$ [MN]</th>
<th>$I$ [MN·ms]</th>
<th>$t_d$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>198</td>
<td>525</td>
<td>5.3</td>
<td>89</td>
<td>1869</td>
<td>42</td>
</tr>
<tr>
<td>35</td>
<td>76</td>
<td>289</td>
<td>7.6</td>
<td>76</td>
<td>1824</td>
<td>48</td>
</tr>
<tr>
<td>50</td>
<td>47</td>
<td>223</td>
<td>9.5</td>
<td>42</td>
<td>1050</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 6.3: Average force and impulse from a BLEVE on a facade, transferred to a single stability element.

### 6.2 Modeling of Example Building Structure

In Section 6.1 the force and impulse on the example building were determined. In this section two steps of modeling are demonstrated, following Chapter 3. The first step, from building structure to trussed frame, is skipped, because the force and impulse are already divided over the stability elements. The mass is simply calculated for one stability element. The columns behind the facade carry some mass as well, but this mass does not affect the mass-stiffness ratio of the stability element. Hence the mass on the stability element is only the mass that is actually carried by the element (see Figure 6.2).
6.2.1 From trussed frame to equivalent beam

The properties of the trussed frame are shown in table 6.4. The bending stiffness $B$ is

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H [m]$</td>
<td>64.8</td>
</tr>
<tr>
<td>$a [m]$</td>
<td>7.2</td>
</tr>
<tr>
<td>$h [m]$</td>
<td>3.6</td>
</tr>
<tr>
<td>$d [m]$</td>
<td>5.09</td>
</tr>
<tr>
<td>$E [N/m^2]$</td>
<td>$2.1 \cdot 10^{11}$</td>
</tr>
<tr>
<td>$A_c [m^2]$</td>
<td>0.09481</td>
</tr>
<tr>
<td>$A_d [m^2]$</td>
<td>0.01414</td>
</tr>
<tr>
<td>$m [kg/m]$</td>
<td>31778</td>
</tr>
</tbody>
</table>

Table 6.4: Trussed frame properties.

calculated according to equation (3.2):

$$B = \frac{a^2}{2} E A_c = \frac{(7.2)^2}{2} \cdot 2.1 \cdot 10^{11} \cdot 0.9481 = 5.16 \cdot 10^{11} N m^2$$  \hspace{1cm} (6.2)

The shear stiffness $S$ is calculated according to equation (3.3):

$$S = \frac{a^2 h}{2 d^3} E A_d = \frac{(7.2)^2 \cdot 3.6}{2 \cdot (5.09)^3} \cdot 2.1 \cdot 10^{11} \cdot 0.01414 = 2.10 \cdot 10^9 N$$  \hspace{1cm} (6.3)

The load $P(x)$ on the beam is a function of $P_m = F_m/H$. One of the three load distributions from figure 3.6 can be chosen for further calculations. The actual load distribution as shown in figure 2.10a is too complex to obtain analytical results. The linear and quadratic load distributions model the fact that the blast load is more concentrated at the base (at time of blast arrival). The linear distribution is chosen in the example.

The equivalent beam is modeled without the rotary spring at the base, because the mode shapes and natural frequencies of a Timoshenko beam with a rotary spring at the base are unknown. The equivalent SDOF response can be obtained using the load-mass factors from subsection 3.4.4, but they can not be compared to the equivalent beam response.

6.2.2 From equivalent beam to equivalent SDOF system

First the mass $M$, stiffness $K$ and force $F$ for the non-equivalent SDOF system are calculated using (3.8). The load is assumed to be linear distributed as (3.12):

$$F = \int_0^H P(x) dx = P_m H = F_m$$  \hspace{1cm} (see table 6.3)

$$M = mH = 31778 \cdot 64.8 = 2.06 \cdot 10^6 kg$$

$$K = \frac{F}{y(H)} = \frac{P_m H^4}{15B} + \frac{P_m H^2}{3S} = \frac{1}{H^3} \frac{H^3}{15B} + \frac{H}{3S} = 2.20 \cdot 10^7 N/m$$  \hspace{1cm} (6.4)

Now the load-mass factor $K_{LM}$ can be obtained from table 3.4 with the parameter $\alpha$ equal to:

$$\alpha = \frac{s^2}{\gamma^2} = \frac{SH^2}{B^2} = 17.09$$  \hspace{1cm} (6.5)

Dynamic response of high-rise building structures to blast loading
6.3 Single degree of freedom response

So:

\[
K_{LM} = \frac{5(4455 + 1078\alpha + 75\alpha^2)}{66(5 + \alpha)(63 + 5\alpha)} = 1.035
\] (6.6)

The natural frequency is equal to:

\[
\omega_1 = \frac{K}{K_{LM} M} = \sqrt{\frac{2.20 \cdot 10^7}{1.035 \cdot 2.06 \cdot 10^6}} = 3.21 \text{rad/s} \quad f_1 = \frac{\omega_1}{2\pi} = 0.51 \text{Hz} \] (6.7)

It was already concluded in section 3.4 that the natural frequency is approximated less accurately by the linear and quadratic load distribution compared to the uniform distribution. In the case of uniform distributed loading, \( f_1 = 0.48 \text{Hz} \).

6.3 Single degree of freedom response

First an idea of the response regime can be acquired by calculating the \( t_d/T_1 \) ratio. This is done for the highest positive phase duration, \( t_d = 50 \text{ms} \):

\[
\frac{t_d}{T_1} = \frac{\omega_1 t_d}{2\pi} = \frac{3.21 \cdot 0.050}{2\pi} = 0.0255
\] (6.8)

This indicates that \( t_d \) is approximately 1/39th of the natural period \( T_1 \), which suggests impulsive response. The dynamic load factor \( \bar{D}(t) \) for the impulsive regime is obtained from \( y(t) \) (4.7) and dividing by \( y_{st,m} = F_m/K \) (written in dimensional form):

\[
\bar{D}_{imp}(t) = \frac{y(t)}{y_{st,m}} = \frac{I_{eq} K_{eq}}{M_{eq} \omega_1 F_{m,eq}} \sin \omega_1 t = \frac{\omega_1 t_d}{2} \sin \omega_1 t
\] (6.9)

The substitutions \( I_{eq} = F_{m,eq} t_d/2 \) and \( K_{eq}/M_{eq} = \omega_1^2 \) were made. Now consider the detonation response from (4.11) in dimensional form:

\[
\bar{D}_{det}(t) = \begin{cases} 
-\frac{t}{t_d} + 1 \cos \omega_1 t + \frac{\sin \omega_1 t}{\omega_1 t_d} & \text{if } t \leq t_d \\
-\cos \omega_1 t + \frac{\sin \omega_1 t}{\omega_1 t_d} - \frac{\sin \omega_1 (t - t_d)}{\omega_1 t_d} & \text{if } t > t_d
\end{cases}
\] (6.10)

The dynamic load factors \( \bar{D}_{imp}(t) \) and \( \bar{D}_{det}(t) \) are compared for the values \( \omega_1 = 3.21 \text{ rad/s} \) and \( t_d = 50 \text{ ms} \) in figure 6.2. The response is barely affected by the pressure-time profile of the blast load and can therefore be accurately approximated by the impulsive response. The maximum dynamic response can be approximated for each impulse distribution with:

\[
y(t) = \frac{I_{eq}}{M_{eq} \omega_1} = \frac{I}{K_{LM} M \omega_1}
\] (6.11)

The results are shown in table 6.5. Obviously, Van den Berg’s method predicts higher response due to higher impulses compared to Baker’s method. The response in the impulsive regime is not influenced by the maximum dynamic load \( F_m \).

To generate a \( p – i – \) diagram for the SDOF system it is necessary to establish a critical response \( y_c \). This response depends on the capacity of the columns and diagonals in the
trussed frame. The critical capacity of the columns of the stability element on the ground floor depends on the area of the cross section $A_c$, the yield stress $f_y$ and a buckling factor $\eta_{c,b,c}$. A part of the capacity is used to carry the vertical weight of the upper stories $N_w = 9238kN$, which should be subtracted from the original capacity:

$$N_{cc} = A_c f_y \eta_{c,b,c} - N_w = 0.09481 \cdot 355 \cdot 10^6 \cdot 0.85 - 9238 \cdot 10^3 = 19.4 \cdot 10^6 N = 19.4MN$$ (6.12)

A buckling factor of 0.85 is assumed. The steel quality is S355. The moment capacity of the beam can be determined from the column capacity:

$$\mu_c = N_{c,c} a = 19.4 \cdot 7.2 = 139.7MNm$$ (6.13)

The next step, before obtaining the critical deflection from the critical moment, is to define the part of deflection $\chi_b$ which is contributed by bending:

$$y = y_b + y_s \quad \chi_b = \frac{y_b}{y_b + y_s} = \frac{P_m H^4}{15B} \frac{1}{1 + \frac{5B}{5S_H}} = \frac{1}{1 + \frac{5}{\alpha}} = 0.774$$ (6.14)

So 77.4% of the deflection is due to bending and 22.6% is due to shear. The critical deflection when moment is critical $y_{mc}$ is equal to:

$$y_{mc} = \frac{y_{mc}}{\chi_b} = \frac{P_{mc} H^4}{\chi_b 15B} \quad \mu_c = \frac{P_{mc} H^2}{3} \quad \Rightarrow \quad y_{mc} = \frac{\mu_c H^2}{\chi_b 5B} = 0.294m$$ (6.15)
The similar procedure can be followed for critical deflection when shear is critical \( y_{sc} \). The diagonals have no initial load. It is assumed that wind loading which might be present during the blast loading, does not have a significant influence. In design practice it is safer to assume an average static wind load to be present during the blast loading. For now this is ignored.

\[
N_{dc} = A_d f_g \eta_{dc;bc} = 0.01414 \cdot 355 \cdot 10^6 \cdot 0.85 = 4.27 \text{MN}
\]  

(6.16)

The critical shear force in the diagonal at ground level can be obtained from \( N_{dc};c \):

\[
v_c = \frac{N_{dc}a}{d} = \frac{4.27 \cdot 10^6 \cdot 7.2}{5.09} = 6.04 \text{MN}
\]  

(6.17)

If \( \chi_s = 0.226 \) is the part of the deflection due to shear, the critical shear deflection \( y_{sc} \) can be obtained:

\[
y_{sc} = \frac{y_{sc}}{\chi_s} = \frac{P_{mc}H^2}{\chi_s3S} \quad v_c = P_mH \quad \rightarrow \quad y_{sc} = \frac{v_cH}{\chi_s3S} = 0.275 \text{m}
\]  

(6.18)

The critical shear deflection is lower than the critical bending deflection, therefore shear deflection is normative. Using Van den Berg’s method, it appears that 20m is the critical range for the example building with regard to failure due to overall response. This is without any safety factors, so in design practice, the diagonals should be over-dimensioned to gain more shear capacity.

6.3.1 Pressure-impulse diagram for SDOF response

To make the pressure-impulse diagram useful as a design tool, it can be specialized for a building. The \( p-i \)-diagram in figure 4.12 has asymptotes \( \bar{F}_m = 1/2 \) and \( \bar{I} = 1 \), which can be transformed to dimensional asymptotes:

\[
\bar{F}_m = \frac{F_m}{R_c} = \frac{1}{2} \quad \rightarrow \quad F_m = \frac{R_c}{2}
\]  

(6.19)

\[
\bar{I} = \frac{I_d}{2} = \frac{F_m\omega_1t_d}{R_c} = \frac{I\omega_1}{R_c} = 1 \quad \rightarrow \quad I = \frac{R_c}{\omega_1}
\]  

(6.20)

To obtain the asymptotes, \( R_{mc} = K y_{mc} \) and \( R_{sc} = K y_{sc} \) can be substituted.

\[
R_{mc} = \frac{N_{cc}aH^2}{5B} \quad \frac{K}{\chi_b} \quad R_{sc} = \frac{N_{dc}aH}{3dS} \quad \frac{K}{\chi_s}
\]  

(6.21)

The fractions \( K/\chi_b \) and \( K/\chi_s \) can be simplified because

\[
\frac{K}{\chi_b} = \frac{P_mH}{y_s+y_b} = \frac{P_mH}{y_b} = \frac{15B}{H^3} = K_b
\]  

(6.22)

and similarly

\[
\frac{K}{\chi_s} = \frac{3S}{H} = K_s
\]  

(6.23)

Substituting in \( R_{mc} \) and \( R_{sc} \) leads to:

\[
R_{mc} = \frac{3N_{cc}a}{H} = 6.47 \text{MN} \quad \quad R_{sc} = \frac{N_{dc}a}{d} = 6.04 \text{MN}
\]  

(6.24)
The asymptotes when moment is critical become:
\[ F_{m;mc} = \frac{R_{mc}}{2} = 3N_{cc}a = 3.23MN \quad I_{mc} = \frac{R_{mc}}{\omega_1} = 3N_{cc}a = 2015MN \text{ ms} \] (6.25)

The asymptotes when shear is critical become:
\[ F_{m;sc} = \frac{R_{sc}}{2} = \frac{N_{de}a}{2d} = 3.02MN \quad I_{sc} = \frac{R_{sc}}{\omega_1} = \frac{N_{de}a}{\omega_1d} = 1882MN \text{ ms} \] (6.26)

The impulse diagonal for shear failure is normative. The impulses from the BLEVE blast using Van den Berg’s method are just left of the asymptote, which indicates that the example building can just withstand the BLEVE blast if only overall response is considered and the higher modes are neglected. The results are presented graphically in section 6.5.

### 6.4 Response of a continuous beam

To include the influence of the higher modes in the response calculations, the mode superposition method is used, following chapter 5. The mode shapes and natural frequencies of a Timoshenko beam are dependent on \( \gamma \) and \( s \). \( \gamma \) is a material parameter, equal to the square root of the ratio of the Young’s and shear modulus.

\[
\gamma = \sqrt{\frac{E}{k'}} = \sqrt{\frac{2(1-\nu)}{k'}} \quad \text{with} \quad \nu = 0.3 \quad \text{and} \quad k' = 1 \quad \rightarrow \quad \gamma = \sqrt{2.6} = 1.61245 \quad (6.27)
\]

The slenderness ratio \( s \) is the reciprocal of the radius of gyration:

\[
s = \sqrt{\frac{AH^2}{I_z}} = \sqrt{\alpha \gamma^2} = \sqrt{17.09 \cdot 2.6} \approx 6.66 \quad (6.28)
\]

Coincidentally, the natural frequencies for these values are found in table 5.1, the mode shapes are plotted in figure 5.1 and the modal contributions are tabulated in table 5.4b, corrected to 100%.

Before the response is calculated, it is again instructive to calculate the \( t_d/T_n \)-ratios. In section 6.3 it was already seen that the first mode response is impulsive. In table 5.4b the contribution factors for impulsive response are also shown. The moment response is dominated by the first mode and the first three modes give 95% of the total response. The shear response is dominated by the second mode and the first nine modes are needed for a 95% contribution. In table 6.6 the \( t_d/T_n \)-ratios and the approximate \( \bar{D}_{m;n} \)-ratios are given for the various modes, using \( t_d = 42 \text{ ms} \) from Van den Berg’s method at \( r = 20 \text{ m} \). The first 7 modes are in the impulsive regime, because \( t_d/T_n < 0.371 \). The 8-10th modes can be read from figure 5.7a. \( \bar{D}_{m;n} \) can be approximated by \( \omega_n t_d/2 \) with a maximum of 1.25.

Now the full response using \( \bar{D}_n(t) \) and the approximate maximum response using \( \bar{D}_{n;m} \) can be compared. Equation 5.55 and 5.57 are used to obtain the approximate maxima of shear and moment respectively. If \( \bar{D}_{n;m} \) is replaced by \( \bar{D}_n(t) \) and the SRSS rule is replaced by an ordinary sum, then the dimensional expressions for base shear and moment with respect to time can be obtained:

\[
v_{b;m}(t) = F_m \sum_{n=1}^{\infty} [c_{b;n} D_n(t)] \quad (6.29)
\]
6.4 Response of a continuous beam

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega_n$</th>
<th>$f_n$</th>
<th>$T_n$</th>
<th>$t_d/T_n$</th>
<th>$D_{m,n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.89</td>
<td>0.46</td>
<td>2.17</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>11.06</td>
<td>1.76</td>
<td>0.57</td>
<td>0.07</td>
<td>0.23</td>
</tr>
<tr>
<td>3</td>
<td>23.19</td>
<td>3.69</td>
<td>0.27</td>
<td>0.16</td>
<td>0.49</td>
</tr>
<tr>
<td>4</td>
<td>30.04</td>
<td>4.78</td>
<td>0.21</td>
<td>0.20</td>
<td>0.63</td>
</tr>
<tr>
<td>5</td>
<td>39.00</td>
<td>6.21</td>
<td>0.16</td>
<td>0.26</td>
<td>0.82</td>
</tr>
<tr>
<td>6</td>
<td>42.53</td>
<td>6.77</td>
<td>0.15</td>
<td>0.28</td>
<td>0.89</td>
</tr>
<tr>
<td>7</td>
<td>53.31</td>
<td>8.48</td>
<td>0.12</td>
<td>0.36</td>
<td>1.12</td>
</tr>
<tr>
<td>8</td>
<td>57.97</td>
<td>9.23</td>
<td>0.11</td>
<td>0.39</td>
<td>1.20</td>
</tr>
<tr>
<td>9</td>
<td>66.93</td>
<td>10.65</td>
<td>0.09</td>
<td>0.45</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>74.72</td>
<td>11.89</td>
<td>0.08</td>
<td>0.50</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Table 6.6: $t_d/T_n$ ratio and approximate DLF for 10 modes.

$$
\mu_{bm}(t) = \frac{F_m H}{3} \sum_{n=1}^{\infty} [c_{bm;n} D_n(t)]
$$

The results are shown in figure 6.3 and compared to two approximation rules, namely the absolute sum rule (ABSsum) and the square-root-of-sum-of-squares rule (SRSS). The SRSS rule is developed for earthquake engineering and based on random vibration theory, as mentioned before. It seems that the SRSS rule is non-conservative for blast loading. The ABSSUM rule is too conservative. In this particular case, the first maximum can be accurately approximated by 1.5 times the SRSS rule. The first maximum is defined as the highest maximum in the first half of the first period of the first mode. Maxima that occur later are assumed to be reduced by damping, because the higher modes are affected more by damping than the first mode.

6.4.1 Pressure-impulse diagram including higher modes

The asymptotes of the pressure-impulse diagram including higher modes are obtained by following section 5.5. First the asymptotes when moment is normative:

$$
F_{m;mc} = \frac{1}{1.5} \cdot \frac{3\mu_c}{2H \sqrt{\sum_{n=1}^{\infty} (c_{bm;n})^2}} = \frac{1}{1.5} \cdot \frac{3 \cdot 139.7 \cdot 10^6}{2 \cdot 64.8 \cdot 0.825835} = 2.61 \text{MN}
$$

$$
I_{mc} = \frac{1}{1.5} \cdot \frac{3\mu_c}{H \sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bm;n})^2}} = \frac{1}{1.5} \cdot \frac{3 \cdot 139.7 \cdot 10^6}{64.8 \cdot 3.11413} = 1385 \text{MNms}
$$

The asymptotes when shear is normative are:

$$
F_{m;sc} = \frac{1}{1.5} \cdot \frac{v_c}{\sqrt{\sum_{n=1}^{\infty} (c_{bs;n})^2}} = \frac{1}{1.5} \cdot \frac{6.04 \cdot 10^6}{2 \cdot 0.518518} = 3.88 \text{MN}
$$
\[ I_{sc} = \frac{1}{1.5} \cdot \frac{v_c}{\sqrt{\sum_{n=1}^{\infty} (\omega_n c_{bs,n})^2}} = \frac{1}{1.5} \cdot \frac{6.04 \cdot 10^6}{5.36292} = 751 \text{MNms} \quad (6.34) \]

6.5 Comparison of results

For an example BLEVE blast loading on an example building the response and pressure-impulse diagram were obtained, using both a SDOF and a MDOF approach. In this section, the top displacement, base moment and base shear response are compared. Subsequently, the asymptotes of the pressure-impulse diagrams of SDOF and MDOF response are given, including the locations in the diagram of the BLEVE blasts at different ranges according to Baker’s and Van den Berg’s method. Finally, modification factors for the asymptotes of the SDOF \( p - i \)-diagram are given to include the influence of higher modes on the response.

6.5.1 Comparison of response

The influence of the higher modes is least significant in the top displacement, see figure 6.4a. More influence of higher modes is visible in the base moment, see figure 6.4b, and the base shear has the highest contribution from the higher modes, see figure 6.4c.

6.5.2 Comparison of \( p - i \)-diagram

The \( p - i \)-diagram for the example BLEVE blast on the example building is shown in figure 6.5. The SDOF \( p - i \)-diagram is non-conservative for impulsive response. The quasi-static asymptotes are approximately equal for both failure criteria and SDOF and MDOF. It can be concluded that the response to BLEVE blast loading is impulsive, because the impulsive asymptotes are normative for determination of failure. The value of peak force from the BLEVE blast does not play a role in this. Shear is the likely type of failure, which can be explained by the higher contribution factors of higher modes, compared to moment. Of course this is dependent on the dimensions of the columns and the diagonals in the trussed frame, which determine the moment and shear capacity. However, the columns and diagonals were already dimensioned for wind loading, so their relative dimensions are realistic.

6.5.3 Modification factors for inclusion of higher modes

Pressure-impulse asymptotes for SDOF systems can be obtained easily. To include the influence of higher modes, a modification factor for the asymptote would be useful, if it were independent of the failure criterium. The asymptotes of the SDOF diagram are found in equations (6.19)-(6.20):

\[ F_{m:sdof} = \frac{R_c}{D_{m:qs}} \quad I_{sdof} = \frac{R_c}{\omega_1} \quad (6.35) \]

in which \( D_{m:qs} \) is the quasi-static dynamic load factor, equal to 2 for an idealized detonation and equal to 1 for an idealized deflagration.

Dynamic response of high-rise building structures to blast loading
The asymptotes of the MDOF diagram are given in equations (6.31)-(6.34). Denoting the correction factor 1.5 as $\varepsilon$, substituting $R_c$ for $\mu_c$ and $v_c$ and generalizing the contribution factor of the $n^{th}$ mode to $c_n$, the asymptotes are obtained independent of the failure type:

$$F_{m;mdof} = \frac{R_c}{\varepsilon D_{m;qs}} \sqrt{\sum_{n=1}^{\infty} (c_n)^2}$$

$$I_{mdof} = \frac{R_c}{\varepsilon} \sqrt{\sum_{n=1}^{\infty} (\omega_n c_n)^2}$$

(6.36)

The modification factor $\lambda$ of the asymptotes is defined as:

$$\lambda_{qs} = \frac{F_{m;mdof}}{F_{m;sdof}} = \frac{1}{\varepsilon \sqrt{\sum_{n=1}^{\infty} (c_n)^2}}$$

$$\lambda_i = \frac{I_{mdof}}{I_{sdof}} = \frac{1}{\varepsilon \sqrt{\sum_{n=1}^{\infty} (\bar{\omega}_n c_n)^2}}$$

with $\bar{\omega}_n = \frac{\omega_n}{\omega_1}$ (6.37)

So the modification factor $\lambda_{qs}$ for the quasi-static asymptote is the reciprocal of the SRSS rule applied to the contribution factors $c_n$ and multiplied by a correction factor $\varepsilon$. The impulsive asymptote depends also on the ratio of $\omega_n$ and $\omega_1$ for every $n$. The definition of $\lambda_i$ makes clear how important the higher modes are, because $\bar{\omega}_n$ keeps increasing with increasing $n$. So $c_n$ should relatively decrease more than $\bar{\omega}_n$ increases to decrease the influence of the higher modes. This is only true for impulsive behavior of all modes, which is a limit case.

Unless the load is a purely theoretical impulse, there are always high modes that have a dynamic load factor $\bar{D}_{m;n} < \omega_n t_d/2$, which reduces the influence of these modes, because $\omega_n$ disappears from the modal contribution. $\bar{D}_{m;n} \approx 1.25$ for modes with $\omega_n t_d > 2.33112$ for an idealized detonation. For an idealized deflagration, $\bar{D}_{m;n}$ for modes with $\omega_n t_d > \pi$ should be determined with figure 5.8 for the $t_d/T_n$ value of each mode.

L. J. van der Meer
Figure 6.3: Base shear and moment for 10 modes of continuous Timoshenko beam.
Figure 6.4: Response of example building to example BLEVE blast.
Figure 6.5: *Pressure-impulse diagram for example BLEVE blast on example building.*
Chapter 7

Conclusions

• Conclusions regarding BLEVE blast:
  – BLEVE blast parameters according to Baker’s method result in too short positive phase durations, which is not conservative.
  – BLEVE blast parameters according to Van den Berg’s method are preferred to those according to Baker’s method, but modification factors for elevation and tank shape are not included.

• Conclusions regarding modeling:
  – A trussed frame can be modeled as an equivalent beam, if it is assumed that the columns determine the bending stiffness and the diagonals determine the shear stiffness. The approximation is more accurate if the trussed frame has more stories.
  – A continuous beam can be modeled as a single degree of freedom (SDOF) system, if it is assumed that only the first mode contributes to the response. The equivalent SDOF system should be energy equivalent to the beam in the first mode.
  – Load-mass factors in literature apply only to slender beams. Trussed frame equivalent beams are generally non-slender.
  – Load-mass factors can be determined for non-slender beams with or without a rotary spring at the base using energy equivalence.
  – Load-mass factors for a static deflected shape do not always result in an accurate first natural frequency. The first mode of vibration is better approximated by the static deflected shape for a uniform distributed load than for a linear or quadratic distributed load.

• Conclusions regarding SDOF response:
  – Impulsive response is independent of the blast pressure-time profile.
  – The limit of quasi-static response to all decaying blast loads (starting with the maximum load) is determined by the response to a step function with a dynamic load factor equal to 2.
  – The limit of quasi-static response to all rising blast loads (which reach a maximum after some time) is determined by the static response with a dynamic load factor equal to 1.

L. J. van der Meer
– Full transient response analysis is only necessary if the response is not in the impulsive nor in the quasi-static regime.
– Pressure-impulse diagrams can be constructed numerically or analytically for any SDOF system including damping, non-linear material behavior and complicated pressure-time profiles.

• Conclusions regarding response of a continuous Timoshenko beam:
  – The trussed frame equivalent continuous beam can be modeled by a Timoshenko beam, which includes shear deformation and rotary inertia.
  – The mode shapes and natural frequencies of a Timoshenko beam can be determined numerically.
  – The mode shapes of a Timoshenko beam depend on the slenderness ratio (geometrical property) and on the ratio of the Young’s modulus and the shear modulus (material property).
  – The force and the response can be written as a sum of modal forces and responses, which share the same time solution.
  – Modal contribution factors for base shear and base moment can be defined, which sum up to 1. Base shear contribution factors are always positive. Base moment contribution factors can be positive or negative.
  – For uniform, linear and quadratic load distribution, only two or three modes are needed to have a minimum of 95% contribution to base moment.
  – For 95% contribution to base shear, more modes are needed than for base moment. Least modes are needed for the uniform load distribution, more modes are needed for the linear load distribution and most modes for the quadratic load distribution.
  – The dynamic load factor for higher modes is dependent on the response regime of the mode with the highest contribution factor, usually the first mode.
  – The maximum response including higher modes can be approximated by applying the SRSS rule to the peak values of modal response and multiplying the result by a factor. A conservative value for this factor has not been established.
  – Impulsive and quasi-static response are obtained by assuming that all modes are in the impulsive and quasi-static regimes respectively.
  – The contribution factors for quasi-static response cannot be used for impulsive response, because the impulsive response depends on the product of natural frequency and contribution factor, resulting in more influence of the higher modes.

• Conclusions regarding example:
  – The quasi-static response is similar for the SDOF system and the continuous beam.
  – The impulsive response is underestimated by the SDOF system. The modification factor for the impulsive asymptote depends on the product of natural frequency and contribution factor of all modes in the impulsive regime.
  – The impulsive response is a limit case, assuming all modes are in the impulsive regime. Depending on the $t_d/T_1$-ratio, only a number of higher modes are in the impulsive regime. Therefore the impulsive asymptote is conservative compared to the actual $p-i$-diagram.

Dynamic response of high-rise building structures to blast loading
– If $t_d = 100$ ms and $f_1 = 1$ Hz are taken as the most unfavorable combination of blast and structure, only the first two modes are in the impulsive regime. Nevertheless, the impulsive asymptote is reduced to 45% of the SDOF value. If $t_d = 50$ ms and $f_1 = 0.5$ Hz, six modes are in the impulsive regime. In this case, the impulsive asymptote is modified to approximately 35% of the SDOF value. If $t_d/T_1 \approx 0.3$, the SDOF and continuous response are approximately equal. Since $t_d/T_1 \leq 0.1$, BLEVE blast loading on high-rise building structures is in the impulsive regime.

– A SDOF system is non-conservative for BLEVE blast loading.
Chapter 8

Recommendations

• Recommendations regarding BLEVE blast loading:
  – It should be investigated whether the correction factors in Baker’s method for elevation and shape of the tank agree with experiments of BLEVEs.

• Recommendations regarding response to blast loading:
  – The influence of damping on the response of higher modes should be investigated. Modal damping can be added in the mode superposition method. Modal damping ratios influence the dynamic load factors of the modes as well as the natural frequencies. However, for small damping, the damped natural frequencies can be approximated by the undamped natural frequencies. The real challenge is to estimate the modal damping ratios. It is expected that higher mode vibrations are damped sooner, because they have more oscillations in the same time period compared to the lower modes. Depending on how soon the maximum overall response is reach, damping has a certain influence on the response. If the maximum is reached soon, damping has little influence. If the maximum is reached relatively late, damping might have a considerable influence. Damping decreases the influence of higher modes, so ignoring it might be over-conservative.
  – The influence of higher modes should be investigated in combination with non-linear material behavior. This is quite difficult, because the mode superposition method cannot be used for non-linear material behavior. The non-linear material behavior occurs only locally, in a plastic hinge, so the question is how the differential equation of motion is affected by this.
  – The influence of higher modes should be investigated in combination with other pressure-time profiles. An important aspect is the inclusion of a large negative phase followed by a second shock, because these are associated with BLEVE blast.
  – A better modal combination rule should be found for response to blast loading, which can replace the SRSS rule.
  – A reliable method to determine the maximum of a sum of several harmonic functions is needed.
  – The actual shape of a pressure-impulse diagram in the dynamic regime including higher modes should be determined numerically, to compare it with the asymptotes. The impulsive asymptote might be too conservative.

L. J. van der Meer
– The influence of failing building elements on the overall response should be investigated. In this report, it was assumed that all load on the facade is transferred to the bearing structure.

– The response of a high-rise building structure should be determined numerically for a sequential load, which arrives first at the base at the front and later at the top and sides. In this report, it was assumed that the blast arrived at the same time at every location on the front facade. Both responses should be compared.

• Recommendations regarding modeling:

  – Mode shapes and natural frequencies should be calculated for sandwich beams, which represent most types of stability elements. In addition it should be possible to include varying cross-section, bending stiffness, shear stiffness and mass.

  – Mode shapes and natural frequencies of continuous beams with base rotation should be determined.

  – Modal contribution factors for different beam types, slenderness ratios, material parameters and load distributions should be tabulated.
Appendix A

Matlab input files

A.1 Pressure-impulse diagram (SDOF)

The determination of the $p-i$ diagram consists of two parts:

- Definition of a forcing function.
- The main file where the diagram is determined.

The main file is shown below:

```matlab
clear all
close all
clc

TD = [];
TM = [];
P = [];
IMPULSE = [];
YP = [];

tau_step = 5e-3;
tau1_step = 5e-3;

step_td = 1e-4;
count = 0;

TD = 0.2:0.2:50;
for td = TD;
    t2 = 0:td.*step_td:td;
    Load2 = force(t2,td);

    [value_max index_max] = max(Load2);
    t2m = t2(index_max);

    CONV = [];
    TAU = [];
    stop = 0;
    tau = 0;
    while stop == 0;
        tau1 = 0:tau1_step:tau;
        Conv = sum(force(tau1,td).*sin(tau-tau1).*tau1_step);
        TAU = [TAU, tau];
        CONV = [CONV, Conv];
        tau = tau + tau_step;
        if tau > t2m & tau > tau_step;
            stop = 1;
        end
    end

    TM = [TM, t2m];
    IMPULSE = [IMPULSE, Load2];
end
```

L. J. van der Meer
if CONV(end) < CONV(end-1);
    stop =1;
end
end

plot(TAU,CONV,'k-')
hold on
xlabel('t')
ylabel('y')
grid on
drawnow

[value1_max index1_max] = max(CONV);
tm = TAU(index1_max);

TM = [TM, tm];
p = 1./value1_max;

IntF = sum(Load2.*td.*step_td);
Impulse = IntF.*p;
P = [P, p];
IMPULSE = [IMPULSE, Impulse];
count = count + 1
end

figure
plot(IMPULSE,P,'k-o')
axis([0 max(IMPULSE) 0 max(P)])
grid on
xlabel('i')
ylabel('p')

figure
plot(TD,1./P,'k-o')
grid on
xlabel('utd')
ylabel('ymaxp')

figure
plot(TD,TM./TD,'k-o')
grid on
xlabel('td')
ylabel('tmtd')

TD_def = TD;
TM_def = TM;
P_def = P;
IMPULSE_def = IMPULSE;
YP_def = YP;

% TD_det = TD;
% TM_det = TM;
% P_det = P;
% IMPULSE_det = IMPULSE;
% YP_det = YP;

save def.mat TD_def TM_def P_def IMPULSE_def YP_def
% save det.mat TD_det TM_det P_det IMPULSE_det YP_det

The forcing function for an idealized deflagration is:

function [out] = force(t,td);
if min(t) < 0

Dynamic response of high-rise building structures to blast loading
The determination of the wave numbers consists of two parts:

- Definition of a frequency equation (5.9).
- The main file where the wave numbers are determined.

The main file is shown below:

```matlab
clear all
close all
clc
global s g
RECALC = 1;
S = 6.6635
G = 1.61245
N = 10;
if RECALC == 1;
    ac = s*sqrt(1/g^2+1);
x0 = 1;
[a_n,fval,exitflag] = fzero(@Timoshenko_function,x0);
a(1) = a_n; x = a_n;
b(1) = 1/sqrt(2*G^2) .* sqrt(-x.^2*(1+G^4)+...
      (1+G^2)*( -s^2 + sqrt(x.^4.*(-1+G^2)^2+2*x.^2.*(1+G^2)*s^2+s^4) ));
    if x >= ac
        b(1) = 1/sqrt(2*G^2) .* sqrt(x.^2*(1+G^4)-...
          (1+G^2)*( -s^2 + sqrt(x.^4.*(-1+G^2)^2+2*x.^2.*(1+G^2)*s^2+s^4) ));
```

L. J. van der Meer
end

for n = 2:N
    k = 1;
    while abs(a_n - a(n-1)) < 0.01 | a_n < a(n-1)
        [a_n,fval,exitflag] = fzero(@Timoshenko_function,x0);
        x0 = a_n + k*0.001;
        k = k + 1;
    end
    a(n) = a_n; x = a_n; % Save a_n
    b(n) = 1/sqrt(2*g^2) .* sqrt(-x.^2*(1+g^4)+...
        (1+g^2)*(-s^2 + sqrt(x.^4.*(-1+g^2)^2+2*x.^2.*(1+g^2)*s^2*s^4) )); % Save b_n if an < a_c
    if x >= ac
        b(n) = 1/sqrt(2*g^2) .* sqrt(x.^2*(1+g^4)-...
            (1+g^2)*(-s^2 + sqrt(x.^4.*(-1+g^2)^2+2*x.^2.*(1+g^2)*s^2*s^4) )); % Save b_n tilde if an >= a_c
    end
end

A(m,:) = a
B(m,:) = b
end

save Timoshenko
else
    load Timoshenko
end

And the Timoshenko frequency equation is:

function [out] = Timoshenko_function(x)

global s g
ac = s*sqrt(1/g^2+1);

y = 1/sqrt(2*g^2) .* sqrt(-x.^2*(1+g^4)+...
    (1+g^2)*(-s^2 + sqrt(x.^4.*(-1+g^2)^2+2*x.^2.*(1+g^2)*s^2*s^4) ));
out_tmp = (x.^2-y.^2).*sin(x).*sinh(y)-...
             ( x.*y.*(x.^4+4*g^4+4*g^4+2.*x.^2.*y.^2+y.^4.*g^4.*4y.^4).*...%.
            (y.^2+g^2.*x.^2)*(x.^2+g^2*y.^2)) ).*cos(x).*cosh(y)-2*x.*y;

if x >= ac
    y = 1/sqrt(2*g^2) .* sqrt(x.^2*(1+g^4)-....%
        (1+g^2)*(-s^2 + sqrt(x.^4.*(-1+g^2)^2+2*x.^2.*(1+g^2)*s^2*s^4) ));
    out_tmp = (x.^2+y.^2).*sin(x).*sin(y)-...
             ( x.*y.*(x.^4-4*g^4-4*g^4+2.*x.^2.*y.^2+y.^4.*g^4.*4y.^4).*...%.
            (-y.^2+g^2.*x.^2)*(x.^2+g^2*y.^2) ).*cos(x).*cos(y)-2*x.*y;
end

out = out_tmp;

Dynamic response of high-rise building structures to blast loading
Appendix B

Ansys input files

The Ansys input files consist of three models:

- A trussed frame model with lumped mass.
- An equivalent beam model with lumped mass.
- An equivalent beam model with distributed mass.

Two types of analysis are done:

- Static analysis for wind loading.
- Modal analysis to determine natural frequencies and mode shapes.

First the trussed frame model with lumped mass:

/ PREP7

ET,1,LINK1 !link elements, tension/compression, no bending
ET,2,MASS21,,4 !mass elements
ET,3,CUMBIN14,0,2, !spring elements, for rotation of foundation
R,1,0.09481, !column, HD 400x744
R,2,0.01414, !diagonal, HF RHS 300x300x12.5
R,3,100 !beam, ‘infinitely’ stiff
R,4,114400.8 !mass
R,5,2.099E09 !translational spring stiffness

MP,EX,1,2.1E11 !steel Young’s modulus
MP,PRXY,1,0.30 !steel Poisson ratio

N,1,−3.6,0 !nodes left side
N,2,−3.6,3.6
N,3,−3.6,7.2
N,4,−3.6,10.8
N,5,−3.6,14.4
N,6,−3.6,18
N,7,−3.6,21.6
N,8,−3.6,25.2
N,9,−3.6,28.8
N,10,−3.6,32.4
N,11,−3.6,36
N,12,−3.6,39.6
N,13,−3.6,43.2
N,14,−3.6,46.8

L. J. van der Meer
N,15,-3.6,50.4
N,16,-3.6,54
N,17,-3.6,57.6
N,18,-3.6,61.2
N,19,-3.6,64.8

N,20,3.6,0 !nodes right side
N,21,3.6,3.6
N,22,3.6,7.2
N,23,3.6,10.8
N,24,3.6,14.4
N,25,3.6,18
N,26,3.6,21.6
N,27,3.6,25.2
N,28,3.6,28.8
N,29,3.6,32.4
N,30,3.6,36
N,31,3.6,39.6
N,32,3.6,43.2
N,33,3.6,46.8
N,34,3.6,50.4
N,35,3.6,54
N,36,3.6,57.6
N,37,3.6,61.2
N,38,3.6,64.8

N,39,0,3.6 !nodes middle
N,40,0,7.2
N,41,0,10.8
N,42,0,14.4
N,43,0,18
N,44,0,21.6
N,45,0,25.2
N,46,0,28.8
N,47,0,32.4
N,48,0,36
N,49,0,39.6
N,50,0,43.2
N,51,0,46.8
N,52,0,50.4
N,53,0,54
N,54,0,57.6
N,55,0,61.2
N,56,0,64.8

!N,57,-3.6,0 !nodes for translational springs
!N,58,3.6,0

REAL,1 !column
TYPE,1 !link element

E,1,2
E,2,3
E,3,4
E,4,5
E,5,6
E,6,7
E,7,8
E,8,9
E,9,10
E,10,11
E,11,12
E,12,13
E,13,14
E,14,15
E,15,16

Dynamic response of high-rise building structures to blast loading
| REAL,3 | !beam             |
| TYPE,1 | !link element    |
| E,1,20 |
| E,2,39 |
| E,3,40 |
| E,4,22 |
| E,41,23|
| E,5,42 |
| E,42,24|
| E,6,43 |
| E,43,25|
| E,7,44 |
| E,44,26|
| E,8,45 |
| E,45,27|
| E,9,46 |
| E,46,28|
| E,10,47|
| E,47,29|
| E,11,48|
| E,48,30|
| E,12,49|
| E,49,31|
| E,13,50|
| E,50,32|
| E,14,51|
| E,51,33|
| E,15,52|
| E,52,34|
| E,16,53|
| E,53,35|
| E,17,54|
| E,54,36|
| E,18,55|
| E,55,37|
| E,19,56|
| E,56,38|

| REAL,2 | !diagonal        |
| TYPE,1 | !link element    |

L. J. van der Meer
Dynamic response of high-rise building structures to blast loading
The input file for the equivalent beam with lumped mass:

/PREP7

ET,1,BEAM3  !beam element
ET,2,MASS21,,,4  !2D mass element without rotary inertia
ET,3,COMB14,0,6  !rotational spring

R,1,0.02599,2.457,7.2,1  !area, area moment of inertia, height, include shear deflection
R,2,114400.8  !mass of mass element
R,3,5.440608E10  !rotational spring stiffness

MP,EX,1,2.1E11  !steel Young's modulus
MP,GXY,1,8.08E10  !steel Shear modulus

N,1,0,0  !nodes
N,2,0,3.6
N,3,0,7.2
N,4,0,10.8
N,5,0,14.4
Dynamic response of high-rise building structures to blast loading
!D,20,UX !extra constraints for rotational spring
!D,20,UY
!D,20,ROTY
D,1,UX !constraints
D,1,UY
D,1,ROTY
D,2,UY !vertical constraints for mass
D,3,UY
D,4,UY
D,5,UY
D,6,UY
D,7,UY
D,8,UY
D,9,UY
D,10,UY
D,11,UY
D,12,UY
D,13,UY
D,14,UY
D,15,UY
D,16,UY
D,17,UY
D,18,UY
D,19,UY

The forces are left out because they are similar to the previous input file. If the mass is
distributed, the lumped mass elements are left out and the beam element constants are
replaced by:

R,1,0.02599,2.457,7.2,1,,31778

Static analysis is done by adding the following strings to the input files:

/SOL
ANTYPE,0
SOLVE
FINISH

Modal analysis is done by adding the following string:

/SOL
ANTYPE,2
MODOPT,LANB,18,0,,,OFF
MXPAND,18, , ,0
SOLVE
FINISH
References


Dynamic response of high-rise building structures to blast loading


