MASTER

Multiple delay coherence imaging charge exchange recombination spectroscopy

Urlings, P.J.

Award date:
2015

Link to publication
Multiple Delay Coherence Imaging
Charge Exchange Recombination Spectroscopy

Peter Urlings

9th of October, 2015

A report submitted in fulfilment of the requirements of the degree of
Master of Applied Physics at the
Eindhoven University of Technology

Supervisors:
Dr. Roger Jaspers, Eindhoven University of Technology
Prof. Dr. John Howard, Australian National University
Dr. Clive Michael, Australian National University
**Abstract**

Currently, the only possible way to obtain the local ion temperature, rotational velocity, and concentration in a tokamak fusion reactor is by means of the Charge eXchange Recombination Spectroscopy technique (CXRS). In a conventional way a 1D profile can be obtained by means of spectrometry. In this report, an attempt is made to obtain 2D images of the temperature, rotation and concentration measurements obtained from a non-Gaussian spectrum, by developing a multi delay coherence imaging technique.

Coherence imaging is a fairly new approach of performing CXRS measurements using optical Fourier transform spectroscopy, capable of measuring a 2-dimensional image across a wide field-of-view. In 2010, a coherence imaging CXRS system has been successfully tested at the TEXTOR tokamak in Germany. In 2015, it was agreed that coherence imaging would be used in the ITER divertor.

Multiple delay coherence imaging is an expansion of the (single delay) coherence imaging used at TEXTOR. Single delay coherence imaging is ideal for measuring Gaussian spectra. In practice, the CXRS spectrum will be non-Gaussian, but consist of passive and bremsstrahlung components. The multiple delay systems are designed to be able to reconstruct these non-Gaussian spectra, by measuring additional interferometric data points inside a single camera frame.

In this report, the concept of multiple delay systems is tested via simulation, two multiple delay systems are designed and build, and one system is applied to the South Korean KSTAR tokamak. The simulations reveal that multiple delay systems are capable of measuring non-Gaussian spectra. Simulated plasma parameter reconstruction was achieved with accuracies up to 0.25 keV for the active temperature and 25 km/s for the active velocity, for 95% of the points in the resulting 2D image. With 0.10 keV and 10 km/s accuracy, respectively, for 68% of the points. The simulations also characterised the noise sensitivity. Unfortunately, we were unable to perform spectrum reconstruction on KSTAR, as improved calibrations are needed.
Contents

1. Introduction ........................................................................................................................................ 1
   1.1. Nuclear Fusion ................................................................................................................................. 1
   1.2. Charge Exchange Recombination Spectroscopy ........................................................................... 2
   1.3. Coherence Imaging CXRS ................................................................................................................ 3
   1.4. Multiple Delay Coherence Imaging ................................................................................................. 3
   1.5. Research Question ........................................................................................................................... 4
   1.6. Report Outline ................................................................................................................................. 4
2. CXRS and Coherence Imaging ........................................................................................................... 5
   2.1. CXRS Principle ................................................................................................................................. 5
   2.2. Coherence Imaging CXRS ................................................................................................................ 7
   2.3. Coherence Imaging Expressions and Derivations ......................................................................... 10
   2.4. Multiple Delay CXRS ..................................................................................................................... 14
   2.5. Outlook and Advantages of CXRS systems ................................................................................... 19
3. Implementation of Multiple Delay Systems ....................................................................................... 21
   3.1. Introduction: Fourier Separation system, Image Separation system ....................................... 21
   3.2. General Design Considerations ..................................................................................................... 21
   3.3. Fourier Separation Approach ......................................................................................................... 26
   3.4. Image Separation Approach .......................................................................................................... 33
4. Spectrum Reconstruction of the Multiple Delay System ..................................................................... 40
   4.1. Obtaining the contrast and the phase sets ..................................................................................... 41
   4.2. Obtaining the spectrum ................................................................................................................... 50
   4.3. The listfit method ........................................................................................................................... 51
   4.4. Alternative fitting methods ............................................................................................................. 55
   4.5. Spectrum reconstruction conclusion ............................................................................................. 56
5. Simulation of the Multiple Delay System ......................................................................................... 57
   5.1. Goals of the simulation ................................................................................................................... 57
   5.2. Methodology .................................................................................................................................. 59
   5.3. Simulation results ........................................................................................................................... 63
   5.4. Conclusions on the simulations ..................................................................................................... 84
1. Introduction

1.1. Nuclear Fusion

The world has an ever increasing desire for energy. This energy demand is met with a mixture of energy sources, the main ones being fossil fuel (oil, coal, and natural gas), nuclear fission, and renewable energy. Nuclear fusion is a new energy source, still in the development phase, that could one day complement this list. The advantages of nuclear fusion are clear:

- The fuels needed for fusion are effectively inexhaustible (unlike fossil fuels).
- The fusion reaction can't spin out of control, and doesn't produce long lasting radioactive waste (Unlike nuclear fission)
- A fusion reactor would be a centralised, controlled means of producing electricity (Unlike most renewable energy sources)

The successful development of an industrial nuclear fusion power plant would therefore be a pronounced achievement for mankind.

Nuclear fusion, as opposed to nuclear fission, is the reaction where atomic nuclei fuse together to form a compound nuclei. Where it is exothermic for heavy elements to split apart (e.g. Uranium), it is exothermic for light elements to fuse together. Inside a star such as the sun, this reaction occurs spontaneously. Without the immense pressure inside a star, the coulomb repulsion between the positively charged nuclei prevent fusion from occurring spontaneously. Fusion can be achieved on earth by forcing two elements together, for example by using a particle accelerator. However, a particle accelerator is ill-suited for the production of net energy from a fusion reaction, due to the energy losses involved. The most promising avenues for energy production from fusion, is to heat a confined plasma, and maintain a thermonuclear reaction.

For the purpose of energy production, present day research is focussed on the fusion of deuterium and tritium, as this D-T mixture is easiest to ignite. It should be noted that tritium is rare on earth, in contrast to the statement above that the fuel is inexhaustible. Tritium will need to be bred on site, in a closed loop circuit of the fusion reactor, until the advent of fusion reactors powerful enough to tackle fuel mixtures such as D-D. Only then will the fuel be truly inexhaustible, as Deuterium is the most prevalent atom in the universe after Hydrogen.

A first condition that any hot plasma machine must reach in order to produce energy is called the Lawson criterion. It states that the product of the plasma density, temperature, and energy confinement time must be equal or greater to a critical value. There are several fusion reactor designs that attempt to meet this criteria. The most promising, and most developed system is called the tokamak. A tokamak magnetically confines the D-T plasma in a torus, where it needs to be heated to 150 million degrees Celsius, at atmospheric pressure, and have a confinement time of the order of 10 seconds to meet the Lawson criterion.
The magnetically confined plasma has complex structures and instabilities. It is of importance to monitor the plasma using specialised hot plasma diagnostics to prevent confinement loss of the plasma.

Fusion energy is currently in the development phase, supported by an international collaboration consisting of thousands of scientists. At a price tag of 15 billion euro (current estimate), the ITER tokamak is the largest fusion project currently underway.

With its construction taking off in 2013 in Cadarache, France, ITER is an experimental fusion reactor, made possible by the collaboration between the European Union, the USA, China, Russia, Japan, Korea and India. ITER is designed to generate 500 MW of sustained (~20 minutes) thermal energy, therefore reaching a power amplification of 10, compared to the 50 MW of input power required to maintain the fusion reaction. ITER is therefore a big jump up from the previous fusion power record, set by the JET tokamak (Oxfordshire, UK) in 1997, of 16 MW, lasting just under a second. While ITER is roughly the same size as a commercial fusion power plant would be, it is not designed to supply electricity to the grid. Rather it is an experiment packed with analytics, to form the bridge from JET towards a future demonstration fusion power plant. ITER is expected to be operational (first plasma) in 2020, and using the D-T fusion fuel starting from 2027.

The hot plasma inside a fusion reactor is controlled by heating systems, magnetic coils, and gas input, to optimise the energy output, ensure fuel and ash circulation, and maintaining plasma confinement and stability. In return, plasma diagnostics are needed to establish the current state of the plasma so that the reaction may be steered towards those goals. CXRS is one such diagnostic providing operators and engineers with important information of the plasma state. In particular: CXRS measures the plasma temperature, the plasma rotation, and the concentration of specific plasma impurities.

1.2. Charge Exchange Recombination Spectroscopy

Charge eXchange Recombination Spectroscopy (CXRS) is a well established diagnostic on large fusion devices, and has been used for the past three decades.[2] The diagnostic will also be applied on ITER, where it has a crucial role in diagnosing burning plasmas.[3]

Several important quantities of the fusion plasma can be measured with CXRS: Flow velocity, ion temperature, and impurity concentration. Temperature data is fundamentally important for fusion, as heat is the main trigger for the fusion reaction. Additionally, CXRS can measure the temperature in various points and surfaces of the plasma, giving an insight into the stability and confinement of the plasma. The plasma rotation is another important component of fusion reactors, as it too is linked with plasma stability. There are even advanced operation schemes being investigated where the plasma rotation changes direction between the top and bottom half of the reactor. Such flow, and other flows generated by instabilities can be measured by CXRS. Lastly, CXRS can measure the density of certain impurities in the plasma. For example, this can be used to measure the concentration of helium nuclei, the ‘ash’ of the fusion reaction, as ash removal is another current topic in the fusion community.

The mechanics of CXRS are discussed in Chapter 2. In this intro, it suffices to say that CXRS is an optical diagnostic, and the above data can be extracted from the emission spectra of a fusion reactor.
The standard approach for CXRS measurements is to obtain the emission spectrum directly using a spectrometer.[4] Generally, a grating imaging spectrometer using an array of optical fibres are used to provide several data points at different plasma positions. Ion temperature and velocity profiles can be generated this way in real time. Combined with external measurements or assumptions of the plasma shape, this allow for the reconstruction the flow velocity and ion temperature across the plasma.

1.3. Coherence Imaging CXRS
The implementation of a coherence CXRS diagnostic system is relatively new.[5] In this system, Fourier transform spectroscopy is used to obtain the emission spectrum. With this method, the spectrum is not measured directly. Instead, the coherence of the light emission is measured by passing the light through an optical interferometer. The measured interferogram is the Fourier transform of the spectrum - the spectrum can then be recovered by applying an inverse Fourier transform.

These coherence imaging CXRS systems provide 2D CXRS data which allows for the direct visualisation of plasma structures. The number of generated data points is several orders of magnitude larger than with conventional CXRS. When using a megapixel camera, for example, $10^4$ data points can be obtained. Additionally, imaging systems don’t suffer from the trade off between spatial resolution and light throughput which are fundamental to the grating spectrometer arrays used for standard CXRS. One difficulty of imaging CXRS is the post processing of the data that is necessary to be able to reconstruct the spectral scene from the measured light coherence.

In 2010, such a system was developed and tested at the TEXTOR tokamak.[1] The results of that campaign showed that sufficient spatial and temporal resolution could be obtained to observe plasma structures and asymmetries. The designed range of flow speeds and temperatures was reached, and the obtained data was found to be consistent with the measurements of the established 1D CXRS system.

The TEXTOR campaign proved the principle of imaging CXRS, and the results were consistent with the standard CXRS system.[5] The imaging system featured a single delay interferometer, capable of reconstructing a simple Gaussian spectrum. It is however known that in general, the spectral band of interest contains multiple components. In particular, it seems that the spectrum consists of two Gaussians and a steady background radiation. A proposed solution to reconstruct this more complicated spectrum, is a multiple delay coherence imaging system.

1.4. Multiple Delay Coherence Imaging
A multiple delay coherence imaging system is an expansion of the single delay system which may allow for the reconstruction of a complex (non Gaussian) spectral scene. Some multiple delay systems already exist outside of fusion energy research[6], but these systems often require multiple frames to be taken of the object, and are hence limited to a static object. As fusion reactors feature fast paced dynamic processes, is it required that the spectrum can be reconstructed from a single frame.

For these reasons new multiple delay coherence imaging systems are needed, capable of resolving a complex spectrum in a single frame, which can serve as a viable fusion diagnostic.
1.5. Research Question
This report aims to answer the question of whether a multiple delay coherence imaging system can be used to unravel a complex (non Gaussian) spectrum as emitted from a fusion reactor.

The research question is answered in two steps: Firstly, realistic multiple delay systems are designed and virtualised on a computer. To test the multiple delay principle a virtual fusion plasma is generated, then send through our virtual diagnostic, and analysed and fitted as if it were real data. Provided the fit is successful, the result can then be compared to the starting, generated plasma, and characterised.

In the second step, the systems are build and tested on a real tokamak. The data from the diagnostic can then be analysed and fitted, and if the fit is successful, the results can be compared to the standard 1D CXRS data, available at the tokamak, and taken at the same time.

Prof. John Howard, Dr. Clive Michael, PhD student Alex Thorman and myself formed the research group in charge of designing, building and testing these multiple delay imaging coherence systems.

The systems were designed and build at PRL (Plasma Research Laboratory) at the ANU (Australian National University) in Canberra, Australia, and tested at KSTAR (Korean Superconducting Tokamak for Advanced Research) located at NFRI (National Fusion Research Institute) in Daejon, South Korea.

A note on my personal involvement: Most design decisions were a group effort, and it is therefore difficult to accredit specific design features to an individual for the purpose of my Master’s Thesis. However, the data analysis (calibrations, fitting methods, and the optimisation thereof) associated with the multiple delay systems was setup by myself, using input from my supervisors as starting points.

Furthermore, the literature about the Coherence Imaging systems is, due to their novelty, confined to a few concise research articles, where many formulas and derivations were left to be worked at the readers discretion. I feel it is therefore justified to devote a portion of this report to detailing the theory, operation and practical implementation of the (multiple delay) Coherence Imaging systems, at the level of a master student.

1.6. Report Outline
Chapter 2 sets the backdrop for this report. The principles of CXRS are given, together with the operation of conventional CXRS and coherence imagine CXRS. The differences and advantages of the methods are discussed. At the end of this chapter (2.4) the discussion moves towards multiple delay CXRS, and justifies its application on KSTAR. Chapter 3 describes the multiple delay coherence imaging systems which we have developed for the KSTAR tokamak. Both the physics principles responsible for the operation of the systems, and the physical implementations of the systems are given here. The data analysis and spectrum reconstruction associated with the multiple delay systems are unravelled in chapter 4. With the data analysis in place, the multiple delay systems are simulated in chapter 5. It will be shown here that the multiple delay systems as designed are theoretically capable of reconstructing a KSTAR CXRS spectrum. In chapter 6, the attempted application of the system on KSTAR is presented. A conclusion is given in chapter 7.
2. CXRS and Coherence Imaging

Charge eXchange Recombination Spectroscopy or CXRS is a mature diagnostic tool for large fusion devices.[2] The diagnostic is tuned to a specific ion in the plasma, and returns the ion temperature, flow velocity and direction, and concentration.

An injection of neutral particles is needed for the operation of the diagnostic; commonly a neutral beam is used for this purpose. It is otherwise a passive diagnostic, requiring only the tools to examine the light emitted by the charge exchange recombination reaction. In the conventional case, spectrometers are used for this purpose.

This chapter describes the CXRS principle (2.1), coherence imaging CXRS (2.2), the expressions and derivations of coherence imaging formulas (2.3), the KSTAR spectrum, model, and need for the multiple delay systems (2.4), and a summary of 1D CXRS versus 2D CXRS versus Multiple Delay 2D CXRS (2.5).

2.1. CXRS Principle

2.1.1. Source of Emission

The fundamental principle is the charge exchange collision between neutral atoms $H^0$ and ionised plasma ions $A^Z$. In a high temperature fusion device it is safe to assume that A is fully ionized. The injected neutral atoms are typically hydrogen or deuterium, as deuterium is the fuel of the fusion reactor.

$$H^0 + A^Z \rightarrow H^+ + (A^{Z-1})^*$$

The reaction leaves the ion $A^{Z-1}$ in an excited state, which will cause line radiation when the electron decays to lower energy levels. For the purpose of obtaining the ion temperature and flow velocity, the ion of focus should have an easy to measure line radiation, typically a low-Z impurity in the plasma with associated line radiation in the visible range. Alternatively, if the purpose is to establish the concentration of alpha’s, fuel or impurities, the system has to be targeted to the specific ions involved.

It is important to note that line radiation is only prevalent where there is a neutral source. A neutral beam is predominantly used for this purpose. Alternatively, CXRS can also be employed on the edge of the plasma (neutral particles coming from the wall), or by injecting neutral particles through gas puffing.[7] Without the use of tomography, the spatial resolution of a CXRS measurement is determined by the overlapping of the viewing line, and the neutral source. For a neutral beam, the spatial resolution is much more defined then for neutrals along the plasma edge.

2.1.2. Encoding of Information

The emitted line radiation is affected by the Doppler effect. In this way, the plasma conditions are encoded onto the line radiation. The Doppler effect, depending on the ion’s velocity, encodes two pieces of information: The ion’s rotational velocity around the tokamak, and the ion’s temperature.
The ion’s rotational velocity $v$ around the tokamak induces a Doppler shift, which shifts the wavelength of the emission, according to:

$$
\lambda = \frac{\lambda_0}{1 + \frac{v}{c}}
$$

Where $\lambda_0$ is the starting wavelength associated with the line radiation, $\lambda$ is the resulting wavelength and $c$ is the speed of light. The resulting wavelength $\lambda$ can be higher or lower than $\lambda_0$, depending on whether the particles are respectively moving away or towards of the point of view. The difference depends on the sign of $v$.

The ion’s temperature is determined by the thermal motion of the particle, and these motions also induce a Doppler effect. The combined effect for a distribution of these fast motions is a broadening of the spectral line. For a Maxwellian velocity distribution, the broadening is Gaussian in shape. The standard deviation $\sigma_\lambda$ of the resulting Gaussian broadened spectral line is given by:

$$
\sigma_\lambda = \lambda_0 \sqrt{\frac{E_i}{m_i c^2}}
$$

Where $E_i$ and $m_i$ are respectively the thermal energy and the mass of the ion.

In summary, measuring the spectrum of an impurity ion CXR transition can provide information on the rotational velocity and temperature of the impurity ions. In addition, the intensity of the line radiation can be used as a measure of the impurity concentration.

### 2.1.3. Conventional CXRS

To obtain an ion temperature and flow profile, a prevalent impurity ion is chosen, of which the CXR line radiation is easily measurable. Generally, this means that the line radiation has a strong intensity and minimal overlap with other sources of emission. The line radiation is then captured using a spectrometer.

In KSTAR, the CXRS system is aimed at the 529.05 nm Carbon VI (n 8-7) spectral line with a neutral heating beam as the neutral source. A Czerny–Turner spectrometer with a focal length of 1.33 m is used to capture the spectrum of this transition line.[8] Typically, this setup provides 8 CXRS data points.[9]

The data point are typically taken along a radial line through the plasma, allowing for ion temperature and velocity profiles to be generated. These are important variables for the operators, and show the strength of CXRS. If in addition the flux surfaces are known, either through simulations or external measurements, these data points can be extrapolated across the plasma, giving rise to an estimation of ion temperature and velocity profile throughout the plasma.
2.2. Coherence Imaging CXRS
This section explains key concepts of Coherence imaging CXRS.

In the first section, the concepts of coherence, Fourier Transform Spectroscopy, and single and multiple delay systems are discussed.

In the second section, a step-for-step detailed example of a single delay Coherence Imaging system based on polarisation interferometry is given, which will be the foundation for the construction of multiple delay systems in chapter 3.

The mathematics of the coherence imaging systems, including the definitions and derivations of the contrast and the phase, are saved for section 2.3.

2.2.1. FTS using single and multiple delay coherence systems
The ‘coherence’ is a property of light, referring to the degree of coherence of the electromagnetic (EM) radiation. It is a property that can be calculated through the correlation of one electric field with another (or itself). This correlation can be measured in practice through interferometry, where two EM waves are made to interfere.

Fourier Transform Spectroscopy (FTS) is a method of performing spectroscopy, by measuring the coherence of a light source.[10] The coherence is captured by interfering the light with itself, across a range of path length differences, to produce an interference pattern. At the hearth of FTS is the Wiener-Khinchin theorem, which connects the spectrum of a slight source with its interferogram, which will return in section 2.4. For now, it suffices to say that the spectrum and interferogram of a light source are directly linked as they are each other’s Fourier transformation. If a full interferogram is measured, the spectrum can be readily calculated, and vice versa, as illustrated in Fig. 1. The x-axis of the interferogram shows the path length difference of the interferometer, which is the same as the effective delay of the light waves through the coherence systems.

![Fourier Transform Diagram]

Fig. 1: An illustrative (Gaussian) spectrum and its interferogram. A section of the interferogram is marked as an example of the range of a single delay coherence imaging system
In order to reconstruct a ‘simple’ spectrum, i.e. a spectrum that can be characterised by a limited number of parameters, the optical coherence imaging systems measure a segment out of the interferogram. The single delay coherence systems scan one section of the interferogram as shown in Fig. 1, which contains enough information to reconstruct a Gaussian spectrum.

The name of the multiple delay coherence imaging system originates from the fact that multiple sections of the interferogram are measured, at different delay values. This technique should allow for a spectrum reconstruction with a greater number of parameters, such as two Gaussians, as illustrated in Fig. 2.

![Fig. 2: An illustrative (double Gaussian) spectrum and its interferogram. A section of the interferogram is marked as an example of the range of a multiple delay coherence imaging system](image)

It should be noted that the above graphs are of mock spectra. The width of the Gaussians are made 30x times larger than can be expected from real CXRS signals, in order to visualise the fringes in the interferogram. For realistic data, the fringe density in the image and the number of ‘waves delay’ would be 30x higher. Also the separation of the Gaussians is made ~100x larger, to visualise the modulation of the fringe amplitude that occurs when the spectrum is no longer a single Gaussian. The effect is still present for realistic data, but it will be more subtle. A real CXRS spectrum will be discussed in section 2.5.

The figures do illustrate that for non-Gaussian spectra, the reconstruction of the spectra will be more involved, due to the summation of the fringes, and resulting modulating intensity. The reconstruction aspects will be discussed in chapter 4.

### 2.2.2. Single delay Polarisation Interferometry

Polarisation interferometry can be used as a method to perform FTS.[6] A basic setup of a 2D imaging interferometer is discussed here as it forms the foundation for the multiple delay systems. The setup is shown in Fig. 3.
The setup uses a displacer plate, which serves several functions at once (see the appendix for details of the displacer plate). The light will be split at the displacer plate into ordinary and extraordinary rays. Both rays will be delayed by moving through the crystal, but one will be delayed more than the other. This difference will induce the necessary path difference for the interferometer, referred to in this report as the delay of the interferometer. Additionally, the displacer plate will induce a spatial shear, as the two rays are moved apart.

The displacer plate is put between two polarisers, with polarisation angle 45 degrees to that of the displacer plate. The first polariser ensures that the ordinary and extraordinary rays have equal intensity. The second polariser is needed to align the polarisation directions of the e and o rays, allowing the two rays to interfere.

The actual interference occurs at the image plane, after a focussing lens rejoins the two sheared rays. The delay induced by the displacer depends on the angle of incidence. A first lens is needed so that the rays originating from a point will be parallel when moving through the displacer, and hence have the same phase delay. Effectively, the polarisers and displacer act on the light in Fourier-space, sandwiched between two lenses.

![Fig. 3: A basic polarisation interferometry setup. The optical axis of the displacer is in plane with the image as indicated by the arrow. The polarisers 'P' are rotated 45 degrees to the plane of the image. Interference occurs as a single ray is split at the displacer, and recombined at the detector after travelling different path lengths. Different pixels at the detector will measure different delays due to the displacer plate's sensitivity to input angles, resulting in a fringe pattern at the detector.](image)

The delay, or path length difference, depends on the plate thickness, its birefringence and the angle of incidence on the plate. The path length difference is clear from the different paths of the blue and green ordinary and extraordinary rays in Fig. 3. When a camera is used in the image plane with a set focussing lens, every pixel has a fixed delay which can be calculated using the phase shift formula by Veiras (see appendix).

The position dependent delay causes fringes (regions or lines with high or low light intensity) to appear at the image plane. The fringes are caused by the gradual change of the delay. The Doppler effects change the contrast and the phase of these fringes.
2.3. Coherence Imaging Expressions and Derivations
This section explores the mathematics behind the Doppler interferometry of the single delay coherence system discussed in the previous section. An understanding of the origin of the expressions for the phase and contrast, and their relation to the physical properties of particle temperature and velocity, will be useful for expanding the application to multiple delay system. The derivations are based on [11] and [12].

2.3.1. The signal at the detector
An incoming wave with scalar wave component $u(t)$ is split by the interferometer in two rays with equal amplitude. They each experience a different delay, with relative delay $\tau$, before recombining at a square-law detector. The signal intensity $S$ at the detector is proportional to:

$$ S = \frac{I_0}{2} \left(1 \pm \Re[\Gamma]\right) $$

Where $I_0 = \langle uu^* \rangle$ is the spectrally integrated irradiance, or brightness, and $\Gamma$ is the complex coherence, given by:

$$ \Gamma = \frac{1}{I_0} \langle u(t)u^*(t + \tau) \rangle $$

2.3.2. Quasi-monochromatic radiation
For quasi-monochromatic radiation with frequency $\nu$, the field $u$ can be described as:

$$ u(t) = A(t)\exp \left(-i2\pi_\nu t\right) $$

Where the complex amplitude $A(t)$ varies slowly compared to the exponent. The complex coherence is then:

$$ \Gamma = \frac{1}{I_0} \int_{-\infty}^{+\infty} A(t) \exp(-i2\pi_\nu t) \left[A(t + \tau) \exp(-i2\pi_\nu (t + \tau))\right]^* dt $$

$$ = \gamma(\tau)\exp(i2\pi_\nu \tau) $$

Where $\gamma(\tau) = \frac{1}{I_0} \langle A(t)A^*(t + \tau) \rangle$ is the temporal coherence of the spectral line.

2.3.3. Phase delay
To continue with the analysis, the phase delay $\phi$ is introduced:

$$ \phi(\nu) = 2\pi_\nu \tau(\nu) $$

The dependency of the time delay $\tau(\nu)$ on the wavelength of the ray is due to optical dispersion: the refractive indexes of the birefringent plates are wavelength dependent. In particular, the time delay is given by: $\tau(\nu) = \frac{LB(\nu)}{c}$, where $L$ is the optical path length, and $B$ is the wavelength dependent birefringence.
The Doppler effect will cause the frequency to fluctuate around a central frequency $\nu_0$, allowing a Taylor expansion around that point. Using $\xi$ as the dimensionless frequency coordinate: $\xi = (\nu - \nu_0)/\nu_0$, $\tau_0 = \tau(\nu_0)$ and $\phi_0 = 2\pi\nu_0\tau_0$, it follows:

$$\phi(\xi) = \phi_0 + \xi\nu_0 \left[ 2\pi\nu_0 + 2\pi\nu \frac{\partial\tau(\nu)}{\partial\nu} \right]_{\nu_0} + O(\xi^2)$$

$$\phi(\xi) = \phi_0 + \xi\kappa\phi_0 + O(\xi^2)$$

Where $\kappa = 1 + \frac{\nu_0}{\tau_0} \frac{\partial\tau(\nu)}{\partial\nu}$ corrects for the optical dispersion. Defining the group phase delay coordinate as $\hat{\phi}_0 = \kappa\phi_0$, then for small $\xi$:

$$\phi(\xi) = \phi_0 + \xi\hat{\phi}_0$$

2.3.4. Expansion complex coherence

The Wiener-Khinchin theorem states, under the appropriate mathematical conditions, that a self-correlation (coherence) can also be written as a Fourier-transform.

Using the Wiener-Khinchin theorem, the complex coherence can be related to the spectral distribution of the irradiance, $I(\nu)$. This step effectively links the coherence as measured via interferometry, to the spectrum of the incident light:

$$\Gamma(\tau) = \frac{1}{I_0} \int_{-\infty}^{+\infty} I(\nu) \exp(i2\pi\nu\tau) \, d\nu$$

Which can be rewritten as:

$$\Gamma(\phi) = \frac{1}{I_0} \int_{-\infty}^{+\infty} I(\xi) \exp(i\phi(\xi)) \, d\xi$$

$$\Gamma(\phi) = \frac{1}{I_0} \int_{-\infty}^{+\infty} I(\xi) \exp(i\xi\hat{\phi}_0) \, d\xi \exp(i\phi_0)$$

The result is in the same form as found earlier for the quasi-monochromatic radiation:

$$\Gamma(\phi) = \gamma(\hat{\phi}_0) \exp (i\phi_0)$$

The alternative definition for the temporal coherence of the spectral line is hence given by:

$$\gamma(\phi) = \frac{1}{I_0} \int_{-\infty}^{+\infty} I(\xi) \exp(i\xi\phi) \, d\xi$$
Furthermore, if \( I(\xi) = I_0 g(\xi) \), and \( g \) a normalised spectrum, with Fourier transform \( G \) then:

\[
\gamma(\phi) = \int_{-\infty}^{+\infty} g(\xi) \exp(i\xi \phi) \, d\xi
\]

\[
\gamma(\phi) = G(\phi)
\]

### 2.3.5. The signal at the detector, expressed by a contrast and a phase

The signal at the detector can now be written as:

\[
S = \frac{I_0}{2} \left( 1 \pm \Re \left[ G(\hat{\phi}_0) \exp(i\phi_0) \right] \right)
\]

For a Gaussian spectrum, characterised by a normalised central wavelength \( \xi_D \) and RMS width \( \sigma \):

\[
g_{\xi_D,\sigma}(\xi) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left( \frac{-(\xi - \xi_D)^2}{2\sigma^2} \right)
\]

It follows that the Fourier transform is given by:

\[
G_{\xi_D,\sigma}(\phi) = \exp \left( -\frac{(\phi\sigma)^2}{2} \right) \exp(i\xi_D\phi)
\]

The signal at the detector can then be written as:

\[
S = \frac{I_0}{2} \left( 1 \pm \Re \left[ \exp \left( -\frac{(\hat{\phi}_0\sigma)^2}{2} \right) \exp \left( i(\xi_D\hat{\phi}_0 + \phi_0) \right) \right] \right)
\]

\[
S = \frac{I_0}{2} \left( 1 \pm \zeta \cos(\phi_D + \phi_0) \right)
\]

Where we have introduced the contrast \( \zeta \) and the phase offset \( \phi_D \)

\[
\zeta \equiv |\gamma| = |G(\hat{\phi}_0)|
\]

\[
\phi_D \equiv \hat{\phi}_0 \xi_D
\]

These definitions remain valid for different non-Gaussian, symmetrical spectra.

It can be seen that the contrast is nearly independent from the wavelength offset, while being highly dependent on the width of the spectrum. In the case of a Gaussian, this is seen by the factor \( \sigma \) occurring in the expression:
\[ \zeta \equiv \exp \left( -\frac{\phi_D \sigma^2}{2} \right) \]

At the same time, the phase is dependent of the frequency offset, but independent of the width of the spectrum. This decoupling is an important advantage of this imaging Doppler interferometry technique. Unfortunately, this decoupling is no longer true for asymmetrical spectra, as is the case in KSTAR. This will be further discussed in section 2.4

2.3.6. Recovering the rotational velocity and temperature

For a simple spectral shape, such as a Gaussian, it can be seen that temperature and rotational velocity take advantage of this decoupling: a wavelength shift only affects the phase \( \xi_D \) while the temperature of the particles induce the Doppler broadening, \( \sigma \) which determines the contrast \( \zeta \), for any given delay.

The component of the rotational velocity in the direction of the viewing line, \( v_D \), is proportional to the Doppler phase shift \( \phi_D \).

The temperature of the ion species \( T_i \) is related to the measured contrast, at a fixed delay offset, by:

\[ \zeta = \exp \left( -\frac{T_i}{T_C} \right) \]

Where \( T_C \) is the ‘instrument characteristic temperature’, depending on the mass of the ion species \( m_i \), and the delay according to:

\[ kT_C = \frac{2m_i c^2}{\phi_D^2} \]
2.4. Multiple Delay CXRS

The motivation for using multiple delay CXRS to measure a non-Gaussian spectrum can be found in the first section. 2.4.2 shows the active, passive, and background signals that make up the non-Gaussian KSTAR CXRS spectrum model, and the parameters by which they are described. In section 2.4.3 the interferometry formulas are extended for this KSTAR spectrum. In 2.4.4 we give examples of non-Gaussian KSTAR spectra being analysed by a single delay system, to show the need for the a multiple delay system.

2.4.1. Gaussian and Non-Gaussian spectra

As mentioned in 2.3.5, the contrast and phase of a Gaussian spectrum are very decoupled. Another way to see this, is by the knowledge that the Fourier transform of a Gaussian is another Gaussian, where the width of the resulting Gaussian is inversely proportional to the width of the spectrum. This can be seen in Fig. 1. The fringes are caused by the offset of Gaussian (529.05 nm, instead of 0 nm). This envelope of the fringes, is exactly the contrast as derived in the previous section. While the fringes are induced by the phase as derived in the previous section. It is clear that a measurement of the contrast and phase, can be easily translated into a Doppler broadening, and Doppler shift.

For a non-Gaussian spectrum, such as a double-Gaussian spectrum, as shown in Fig. 2, this decoupling is no longer the case. Each Gaussian \( i \), with relative intensity \( R_i \) will generate its own complex coherence:

\[
\Gamma_i = R_i \zeta_i \exp(i \phi_{i,0})
\]

Where \( \phi_{i,0} = \phi_i + \phi_0 \), and the contrast and phase shift \( (\zeta_i, \phi_i) \) for each component can be determent from the means outlined in section 2.3. The total coherence of the spectrum will be:

\[
\Gamma = \Gamma_1 + \Gamma_2 = R_1 \zeta_1 \exp(i \phi_{1,0}) + R_2 \zeta_1 \exp(i \phi_{2,0})
\]

Alternatively, expressed in real values, this can be written as:

\[
\zeta_m \cos \phi_{m,0} = R_1 \zeta_1 \cos \phi_{1,0} + R_2 \zeta_2 \cos \phi_{2,0}
\]

This equation marks the difficulty in recovering a non-Gaussian spectrum. A measurement of the contrast and phase \( (\zeta_m, \phi_m) \) can no longer be linked one to one to the width and displacement of a Gaussian. There is a clear degeneracy: multiple non Gaussian spectra could result in the same contrast and phase value. A measurement at a single delay is hence insufficient to reconstruct the spectrum. Using a multiple-delay system would be one solution at overcoming this problem.

2.4.2. KSTAR CXRS spectrum model

Fig. 3 shows an example spectrum of the conventional KSTAR CXRS system. Based on example spectra like these, we have decided to model the KSTAR spectrum consisting of an Active CXRS signal, a Passive emission, and a Background level.
The active CXRS signal is effectively a Gaussian defined by the temperature and velocity of the particles of interest, as from the previous section. The Passive emission consists of unwanted CXRS contributions to the signal, originating from outside the scrape off layer. It is also modelled as a Gaussian, with a passive temperature and velocity. The Background signal originates from the Bremsstrahlung of the plasma, and has a near constant value.

The spectrum hence consists of three Gaussians, with the following 7 variables:

- $I_A$ The intensity of the active signal
- $\sigma_A$ The RMS width of the active signal
- $\lambda_A$ The central wavelength of the active signal
- $I_P$ The intensity of the passive signal
- $\sigma_P$ The RMS width of the passive signal
- $\lambda_P$ The central wavelength of the passive signal
- $I_B$ The intensity of the background signal

This makes up the model of the spectrum. The RMS width and the central wavelength of the Doppler broadened and shifted Gaussians relate directly to the temperature and rotational velocity of the particles.

The total intensity of the spectrum is trivially attained by the coherence imaging systems, as the intensity count of the pixels. It is therefore easy, and more instructive, to focus on the relative intensities of the different components of the model spectrum. The rest of the report will focus on the 6 unknowns of the spectrum: The temperature, velocity, and intensity ratio of both the active and passive component. The 7th variable: the relative intensity of the background signal, is redundant when the relative intensities for the active and passive signals are known.
2.4.3. Interferometry of the KSTAR CXRS model spectrum

The coherence imaging system is fitted with a wavelength filter, which results in the measured spectrum being a multiplication between the plasma spectrum and the filter passband. The analysis performed in chapter 4 takes this full filter characteristic into account. For the purpose of the discussion here, it is sufficient to assume that the filter in a wide Gaussian: wide enough such that the shape of the active and passive emissions doesn’t change, and such that the constant background emission takes the form of the filter passband, with a known width and central wavelength.

The total spectrum is given by:

\[ I(\xi) = I_A g_A(\xi) + I_P g_P(\xi) + I_B g_B(\xi) \]

Where the Gaussians \( g_A, g_p \) and \( g_B \) are characterised by known or to-be-determined quantities as discussed above. The spectrum results in the signal at the detector:

\[ S = \frac{I_0}{2} \left( 1 \pm \Re \left[ (G_A(\phi_0) + G_P(\phi_0) + G_B(\phi_0)) \exp(i\phi_0) \right] \right) \]

\[ S = \frac{I_0}{2} (1 \pm \zeta_A \cos(\phi_A + \phi_0) + \zeta_B \cos(\phi_B + \phi_0)) \]

Where:

\[ \zeta_A \equiv \exp \left( -\frac{(\phi_0 \sigma_A)^2}{2} \right) \]

\[ \phi_A \equiv \hat{\phi}_0 \zeta_A = \hat{\phi}_0 \left( \frac{\lambda_0 - \lambda_A}{\lambda_0} \right) \]

And similar for the contrast and phase induced by the passive and background spectra. \( \lambda_0 \) is the reference wavelength, taken to be the Carbon VI spectral line of 529.05 nm.

For a single delay, the interferometry system returns the measured contrast and phase shift \( \zeta_m \) and \( \phi_m \), so that:

\[ S = \frac{I_0}{2} (1 \pm \zeta_m \cos(\phi_m + \phi_0)) \]

From a system of equations point of view, a measurement at a single delay results in 2 variables. Hence, a first requirement is that three or more delays are measured in order for the system with 6 unknowns to be solvable. A second requirement is that the system of equations should be well conditioned. This won’t always be the case here, since for example the model of the spectrum does not disallows the active and passive spectra to be identical, which will result in degenerate solutions. As the spectrum is not symmetrical, the contrast and phase are more coupled, which will worsen the condition of the system of equations. For example, the contrast from the Active signal can now influence the phase of the measured signal. In order to optimise the condition of the system, the optical delays are chosen as
to span the range of delays where the contrast fluctuates, and to minimise the chi-squared solution of fitting the contrast of example spectra.

The implications of using a multiple delay approach with this spectral model and its potential degeneracy will be seen in chapter 5.

2.4.4. Single delay system applied to the model spectrum

Let’s take the following realistic KSTAR example spectrum, derived from example spectra from the 1D CXRS system (See Fig. 4 for an example):

Active temperature: 1.2 keV  
Active velocity: 62 km/s  
Passive temperature: 1.1 keV  
Passive velocity: 40 km/s  
Ratio active signal: 45%  
Ratio passive signal: 20%

If this spectrum were to be measured with a single delay system, under the wrongful assumption that only one Gaussian is present in the spectrum, the recovered temperature and velocity would be dependable on the chosen delay, as shown in Fig. 5. If the spectrum had been a single Gaussian, the recovered temperature and velocity would be constant, for any delay.

![Single delay fit applied to KSTAR spectrum](image)

**Fig. 5**: The result of a single-Gaussian fit applied to the example KSTAR spectrum. The result is inconsistent, as expected, as the ‘measured’ temperature and velocity changes depending on the chosen delay for the interferometer.

At \( \phi_0 = 362 \) waves delay (the characteristic delay for KSTAR, as we will see in chapter 3) the example spectrum has a contrast value of \( \zeta_m = 0.49 \), and a phase of \( \phi_m = \phi_{m,0} = -0.41 \) radials. This corresponds with a temperature of 3.0 keV, and a velocity of 54.1 km/s. In this case, the reason a much higher temperature than the actual plasma temperature is recovered, is due to the broad background (Bremsstrahlung) component. For illustration, Fig. 6 shows the results for the same spectrum, with the background component removed (effectively raising the active and passive signal ratio to 70\% and 30\% respectively).
Fig. 6: The result of a single-Gaussian fit applied to the example KSTAR spectrum without the background signal. Above 100 waves of delay, the fit is nearly constant. (compare with Fig. 5)

There is a high temperature peak visible at low delays, which is caused by the filter function included in the analysis: As the active and passive signals pass through the filter, they are slightly deformed by the non constant passband. In the interferogram, this means the active and passive Gaussians are convoluted with the passband. At low delays, the wide passband will be the dominant factor in this convolution.

Outside of this filter induced peak, the reconstructed temperature and velocity appear constant. They appear so, as the active and passive component are very similar in this example, effectively forming a single averaged Gaussian, that is measured using the single delay system. The fit (at $\phi_0 = 362$ waves delay) gives a contrast value of $\zeta_m = 0.72$, and a phase of $\phi_m = \phi_{m,0} = -0.42$ radials. This corresponds with a temperature of 1.16 keV, and a velocity of 56 km/s, which indeed very similar to (the average of) the two input signals.

Lastly, it is illustrative to show that the recovered temperature and velocity does indeed differ at different delays, when the active and passive signal are dissimilar. For example, take the following the realistic ‘mismatched’ spectrum, containing a hot and fast active component, and cool and slow passive component, again omitting the background signal for clarity.

| Active temperature:        | 4.3 keV                     |
| Active velocity:           | 100 km/s                    |
| Passive temperature:       | 1.1 keV                     |
| Passive velocity:          | 10 km/s                     |
| Ratio active signal:       | 50 %                        |
| Ratio passive signal:      | 50 %                        |
The fit in Fig. 7 shows that even without a background signal, the fitted parameters depend on the delay. This is as expected from the initial discussion in section 2.2.1. The multiple delay systems rely on this delay dependency to reconstruct the non-Gaussian spectrum.

2.5. Outlook and Advantages of CXRS systems

2.5.1. Application of conventional CXRS vs Coherence Imaging CXRS

Conventional 1D CXRS is a mature diagnostic with proven reliability. It is capable of supplying the reactor operators with ion temperature and velocity profiles, in real time. It is limited to a few (~10) data points, providing a rough temperature and velocity profile. As a standard diagnostic technique, it will be used in ITER.

Coherence imaging CXRS can provide a large 2D array of data points (~10,000), which are available some time after the measurement (currently ranging from a few seconds to ~30 minutes, depending on the resolution, and the type single/multiple delay). It is hence more useful as a research tool rather than an operator tool. Coherence imaging CXRS is a novel technique that has already been proven to work, and now awaits adaptation on more fusion devices. The new research possibilities are the key value of the new concept of coherence imaging CXRS. As of May 2015, it was confirmed Coherence Imaging CXRS will also be used in ITER, to monitor the plasma flows in the divertor.[13]
The coherence imaging CXRS advantages for the fusion community are:

- An additional dimension of information, on which new analytics can be deployed that are otherwise not possible. The visualisation of the plasma flows, temperature and impurity density can be used to characterise the plasma by detecting asymmetries and plasma structures. Depending on the setup, plasma flows in the SOL and Divertor can be observed.
- Advanced plasma models often describe the chaotic flow of particles within the plasma. These models can be benchmarked to the 2D temperature and velocity data.
- ‘bad spots’, such as viewing lines which are contaminated by an intense reflection of the plasma vessel, are easily identifiable.
- Coherence imaging systems also opens the pathway for use with gated image intensifier cameras. These camera’s are capable of locking onto MHD instabilities, and can integrate the instability across several frames, allowing for the first time to visualise such instabilities. Because of their periodicity, the sawtooth oscillation would be a potential target to observe. As the sawtooth is a mayor cause for disruptions, being able to control it is an important challenge in the fusion community. Being able to observe and model MHD instabilities will benefit the ability to maintain a stable fusion plasma in future fusion power plants.

For the foreseeable future, Multiple Delay 2D CXRS with full megapixel resolution will not be available in real time. The large datasets generated by imaging CXRS prohibit real time analysis. Instead the data will become available in the order of ~15 minutes past the event.

### 2.5.2. Coherence Imaging CXRS: single delay vs multiple delay

The multiple delay systems are developed as an expansion to the single delay coherence imaging system, to attempt to reconstruct non-Gaussian spectra, fitting the active, passive and bremsstrahlung at once. Because of this additional generated information, and the laser calibrations system, the expectation is that system does not need external measurements to obtain the passive and background components.

Section 2.4 showed how a multiple delay coherence imaging system could allow to measure a non-Gaussian spectrum, and in particular a standard KSTAR CXRS spectrum.

In large (power generation grade) fusion reactors, Bremsstrahlung and passive emission provide a large contribution to the CXRS spectral range. The multiple delay systems can separate these components, whereas the single delay system would require external measurements, as shown in section 2.4.4

The multiple delay systems can effectively serve as a multiple single delay systems. Therefore they have only a few disadvantages compared to the latter: only the resolution and/or the dynamic range of the data is reduced (depending on implementation), as the camera image is shared for the multiple delays. The main difference is the added complexity of the implementation of the system, as seen in Chapter 3. Additionally, the data analysis required to combine the info of the separate single delay systems into one spectrum is a new challenge, which is covered in Chapter 4, and tested in Chapter 5.
3. Implementation of Multiple Delay Systems

3.1. Introduction: Fourier Separation system, Image Separation system

Two multiple delay systems are developed in this report: the Fourier separation system, and the image separation system. Both utilise a different principle to obtain the contrast and phase for multiple delays.

The systems are designed to be operated at the KSTAR tokamak. The image separation system consists of 4 single delay systems, each imaged on to a quadrant of the CMOS camera. The Fourier separation system generates the fringe pattern of three single delay systems superposed on each other. Both systems can measure the coherence of the light at different delays within a single frame, allowing the measurement of fast changing plasmas.

The Fourier separation approach is the main system for the remainder of the report. The image separation system could not be applied on KSTAR for the purpose of this report, due to manufacturing errors on some of the critical components (Wollastons and Quad Delay Plate). Replacement components were supplied too late for the 2013 KSTAR campaign, although it has since been tested on the 2014 KSTAR campaign (results pending).

Comparison of the two systems

The Fourier separation approach measures the additional delays at the cost of some amount of dynamic range of the camera, while the image separation approach measures the additional delays at the cost of image resolution. Theoretically, there should be little difference between the results of the two proposed methods. Due to the essential averaging involved in the data analysis (see section 5.2.2) both the contrast and dynamic range will be averaged out.

Chapter outline

The remainder of this chapter covers the physical operation and design of the multiple delay systems. Before moving on to the details of the two systems in 3.3 and 3.4, section 3.2 will cover the general design considerations posed by the multiple delay systems and the KSTAR tokamak.

3.2. General Design Considerations

There are general design considerations for applying multiple delay coherence imaging devices at KSTAR. The relevant mechanical layout at KSTAR is discussed first, followed by a discussion for the required wavelength filter, the interferometer, and the laser calibration system.

3.2.1. KSTAR I-port

We were assigned the KSTAR I-port to use for CXRS during the 2013 KSTAR campaign. The optical rail with the multiple delay setup would fit entirely into this port. A few design considerations related to KSTAR are as follows:

- The inside of the fusion reactor is visible through a vacuum window at the end of the port. A metal mirror is used up front of our optical rail to align the setup with the viewing direction.
• The size limitations dictates that the laser (used for calibrating the setup) is located outside the port, and fed through with an optical fibre.
• The camera should be located as far back as possible, to minimise the flux of neutrons, and for easy maintenance. For this purpose, two large 300 mm lenses are used in the setup, to relay the light across from the front of the port to the back of the port.

Fig. 8 shows the top down view of KSTAR and the relation between the I-port, The M-port, and the neutral beam. The M port contains the 1D CXRS system - both systems even happen to be roughly aimed at one another.

![Fig. 8: Top down view of KSTAR, showing the relation between the I-port (Housing our 2D CXRS system) the M-port (Housing the 1D CXRS) and the Neutral Beam.](image)

### 3.2.2. Wavelength filter

A wavelength filter is necessary to block out most of the light that is not part of the Carbon VI emission, although the use of a filter will affect the interferometer. The filter used in this report, and the resulting extension to the interferometer formula in section 2.4.3 will be discussed here.

The filter requires a pass band around the Carbon VI spectral line. In particular the wavelength filter used at KSTAR can be roughly approximated by a flattened Gaussian with central wavelength 529.48 nm, and FWHM of 2.85 nm. At the (laser) calibration light wavelength of 532 nm, the transmission is already reduced to the order of 1%. (Which is compensated by the high intensity of the laser), see Fig. 9.
The main side effect of the wavelength filter is to cause an artificial component to the spectrum. The constant Bremsstrahlung emission from KSTAR’s energetic plasma is observed by the interferometer through the wavelength filter, and hence, appears to have the spectrum of the passband. This side effect should be fully mitigated by incorporating the passband in the model spectrum.

Assuming a constant emission component (bremsstrahlung), the pass band shape effectively forms a spectrum, which will be picked up by the interferogram. As the pass band is much wider than the typical CXRS emission, the Fourier transform will drop off much faster, and is only visible at low delays.

Knowing the precise structure of the passband filter therefore allows for better simulation of the expected contrasts and phases, used in the listfit function (section 6.3.1) especially at low delays. As a secondary issue, the central wavelength of the pass band depends on the angle of incidence (a feature of the interference filter used). Hence, different pixels may require different corrections.

Simulation of the wavelength filter

The simulation is needed for the listfit fitting method, in section 4.3. The contrasts and phases, as found in section 2.4.3. are adjusted for the wavelength filter.

If the filter transmission is denoted by $T(\xi)$, then the actual measured spectrum $I'(\xi)$, using the KSTAR model spectrum of section 2.4.3., is:

$$I'(\xi) = I(\xi)T(\xi) = T(\xi)[I_A g_A(\xi) + I_B g_B(\xi)]$$

Initially, it is instructive to assume that the transmission spectrum is presumed a Gaussian, $T(\xi) = g_f(\xi)$. To find calculate the contrast and the phase of this modified spectrum the Fourier transform of the intensity needs to be known, as in 2.4.3. Each term in the fourier Fransform is now a convolution between two Gaussians. The first term, for the active CXRS emission, is:

Fig. 9: Wavelength passband filter used in the multiple delay systems. It can be accurately approximated by using three Gaussians, as indicated in dashed lines. The error of this approximation is less than 0.5 \% throughput.
\[ G_A' = G_A \otimes G_f = FT[g_Ag_f] \]

Where \( G_f \) is the Fourier transform of the Gaussian passband. The analytical solution is:

\[
G_A' = FT\left[ \frac{1}{\sigma_A} \exp\left( -\frac{(\xi - \xi_{D,A})^2}{2\sigma_A^2} \right) \ast \frac{1}{\sigma_f} \exp\left( -\frac{(\xi - \xi_{D,f})^2}{2\sigma_f^2} \right) \right] (\phi) 
\]

\[
G_A' = \frac{1}{\left(\sigma_A^2 + \sigma_f^2\right)^{\frac{1}{2}}} \exp\left( -\frac{\sigma_A^2 \sigma_f^2 \phi^2 + \xi_{D,A}^2 + \xi_{D,f}^2 - 2\xi_{D,A} \xi_{D,f}}{2\left(\sigma_A^2 + \sigma_f^2\right)} + i\phi \left( \frac{\sigma_A^2 \xi_{D,f} + \sigma_f^2 \xi_{D,A}}{\sigma_A^2 + \sigma_f^2} \right) \right) 
\]

Using this analytical solution, and similar equations for the passive and background components, the contrast and phase can be calculated for any spectrum, adjusted for the Gaussian wavelength filter.

As a final expansion, the filter spectrum is not a Gaussian, but it can be approximated by the sum of Gaussians. It was found that just 3 Gaussians (fitted using Matlab) were sufficient for an accurate representation, as shown in Fig. 8. Therefore, the filter transmission can be described as \( T(\xi) = g_{f,1}(\xi) + g_{f,2}(\xi) + g_{f,3}(\xi) \). Therefore, the signal at the detector can be described by 9 convolutions, the formula of which is as shown above. One for each combination of the 3 spectrum components (active passive background) and the 3 Gaussians that make up the wavelength filter.

This is the method used in the listfit CP generation module, in section 4.3.1 to obtain contrasts and phases which are corrected for the wavelength filter.

### 3.2.3. Delays of the Interferometer

For the multiple delay system, the delays are chosen such that the fitting of the typical KSTAR spectral scene has a minimal chi-squared solution.

The chi-squared solution (not performed by the author) reveals that the three delays for the Fourier separation approach should be (120, 355, 835) waves respectively.

It is instructive to look at the single delay system for some example values. If such a system was going to be set up at KSTAR, a first task would be to determine the characteristic temperature and delay. The characteristic temperature of a single delay interferometer is the temperature which causes the contrast to drop with a factor of \( 1/e \). The delay of such a system is hence chosen such that the characteristic temperature of the system matches the expected ion temperature maximum.

The maximum temperature of KSTAR is 50 million degrees Kelvin, or 4.3 keV, with expected temperatures and flow speeds of 1 keV and 100 km/s.[14] If 4.3 keV is taken as the characteristic temperature, the associated delay can be calculated from the inverse of the formulas in section 2.3.6. The result is 362 waves delay, which is indeed the mid delay as revealed by the chi-squared solution.
3.2.4. **Calibration system**

Preliminary experiments indicated that the phase shift induced by the birefringent materials can drift over time. Even when insulated within a thermal cell, the phase was seen to fluctuate over the course of several hours. It was hence decided to monitor the phase shift at KSTAR using a diffused laser as image source. Before every plasma pulse, a laser reference image was to be taken.

The system additionally provides the system contrast, and the reference phase required for the data analysis. Effectively, it provides an in-situ calibration for the interferometer.

The system relies on a polarisation cube (see Fig. 10, left) to allow both the plasma light, and the calibration light to feed into the interferometer. The polarisation cube also serves as the first polariser of the interferometer. The calibration light and plasma light will have perpendicular polarisation directions. As the laser calibration is taken before the onset of plasma, there is no contamination of plasma light into the calibration shot.

The practical setup is shown in the right of Fig. 10. The calibration light will be provided by a Nd:YAG laser at 532 nm. Due to the limited space in the KSTAR port, the laser is located outside of the port, and fed through with an optical fibre. When the beam exits the fibre, it is spread out by widening lens, and reflected from a mirror onto a diffuser plate. It passes through a pre-polariser, before entering the polarisation cube.

The pre-polariser was added as the polarisation cube manufacturer noted that the extinction ratio (‘goodness of polarisation’, specifically the ratio of light with the intended polarised direction to the unintended direction exiting the cube) was lower for the light being reflected when compared to the light the passes through. In particular, the extinction ratio for the plasma light is >1000:1, whereas for the calibration light, it is ~100:1. The pre-polariser is added to increase the level of polarisation of the calibration light exiting the cube.

---

**Fig. 10:** LEFT: Function of the polarisation cube is shown. At the output, both Plasma and Calibration light will be (orthogonally) polarised and fed through to the system. RIGHT: System as implemented. Calibration light is provided by a laser beam fed through an optical cable, which is widened, diffused, and pre-polarised.
3.3. Fourier Separation Approach
The Fourier Separation generates 3 different delays, by generating 3 different overlapping fringe patterns.

Each fringe pattern is said to have its own carrier wave. The individual fringe patterns, or carriers, can be separated in the Fourier domain, generated by taking a 2D Fourier transform of the image. This separation process, which is explained further in next chapter (4.1.2), explains the name of this approach. The superposition of the fringe patterns is visualised in Fig. 11. The Fourier separation approach for use in hot plasma fusion reactors is, to our knowledge, a novel technique developed at the Plasma Research Laboratory.

The interferometer consists of a displacer plate triplet. The effect of combining the three displacers can be described by Stokes’ vectors and Mueller Calculus, which is detailed in the next section. A following section discusses the resulting superposition of fringe patterns. With the interferometer explained, the final section gives an overview of the full setup.

3.3.1. Stokes Vectors & Mueller calculus
Stokes vectors describe polarised light, while Mueller Matrixes describe optical elements. Mueller calculus is a means for calculating how the stokes vector (polarisation) is changed by a Mueller Matrix (optical element). This section will only describe parts that are relevant to understand the Fourier separation setup. A full description of Mueller calculus can be found in textbooks, such as [15].

Stokes’ vectors describe polarised light as a vector with 4 elements. These elements correspond to the amount of unpolarised light, the amount of vertically or horizontally polarised light, the amount of diagonally polarised light, and the amount of circularly polarised light. A positive sign of the latter three elements indicates horizontal (0 degrees), diagonal (45 degrees) and right handed circular polarisation. A negative sign indicates the orthogonal polarisation: Vertical (90 degrees), diagonal (135 degrees) and left handed circular polarisation. The Stokes vector for unpolarised light, with 1 unit intensity is given by:

\[
\text{source} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\]

Mueller matrixes are 4x4 in size. When they act on a Stokes vector, the result is a new Stokes vector, which represents the light output. Textbook Mueller matrixes can be found for any linear birefringent plate.

There are three Mueller matrices necessary to understand the Fourier separation approach:

Polarizer

\[
P = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]
Rotator, rotates the polarisation in clockwise direction when viewing the object, with angle $\theta$.

$$r(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Delay plate, with fast axis on the horizontal, and phase shift $\phi$.

$$D(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\phi) & \sin(\phi) \\ 0 & 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

Note that a delay plate Mueller matrix can be used to model the displacer plates in the setup, as a displacer plate is a delay plate, with an added spatial shear. The spatial shear aspect of the displacer plates does not show up in Mueller calculus.

The rotator is a tool that can be used to determine the Mueller Matrix of an optical plate, that is rotated. An optical plate rotated by $\theta$ can be modelled by rotating the polarisation by $-\theta$ degrees, applying the optical plate, and then rotating the polarisation back by $\theta$ degrees. In particular, the formulas for a polariser and displacer plate, rotated by an angle $\theta$ are given by:

$$rP(\theta) = r(\theta) \times P \times r(-\theta)$$

$$rD(\theta, \phi) = r(\theta) \times D(\phi) \times r(-\theta)$$

Where $\times$ denotes matrix multiplication. Also note that for matrix multiplications, the Stokes vector, or light input, is added on the right of the equation. The equations resemble the light path, starting on the right, and moving towards the left.

The interferometer of the Fourier separation approach consists of the following elements:

1. HWP at 22.5°
2. 6 mm Displacer 90°
3. 3 mm Displacer 0°
4. 4 mm Displacer 45°
5. Polariser 22.5°

Starting from the plasma, the light (source) is unpolarised. The calibration cube will polarise the light ($P$). The HWP will rotate the polarisation by $r(2 \times 22.5°) = r(45°)$. The Mueller calculation hence starts with: $r(45°) \times P \times source$. The resulting equation for the full setup is:

$$S = rP(22.5°) \times rD(45°, \phi_{4 mm}) \times rD(0°, \phi_{3 mm}) \times rD(90°, \phi_{6 mm}) \times r(45°) \times P \times source$$

The resulting Stokes vector can be readily calculated. Of interest is the first component of the Stokes: the total intensity:
\[ S_{\text{Intensity}} = \frac{1}{4} \left( 1 + \frac{\cos(\phi_1 + \phi_2)}{2\sqrt{2}} + \frac{\cos \phi_2}{\sqrt{2}} - \frac{\cos(\phi_2 - \phi_1)}{2\sqrt{2}} \right) \]

Where we have defined:

\[ \phi_1 \equiv \phi_{4\,mm} \]
\[ \phi_2 \equiv \phi_{6\,mm} - \phi_{3\,mm} \]

The system can be described by two fundamental delays, \( \phi_1 \) and \( \phi_2 \). The first is simply the delay induced by the third displacer. The second is a combination of the 6 mm and 3 mm displacer plates: since there is a 90° rotation between them, they effectively combine to form an asymmetrical Savart plate, which can be described by a single delay, \( \phi_2 \).

When the two effective elements (Displacer plate, and asymmetrical Savart plate) are used in together, they result in 4 possible combinations for the delays: \( \phi_1, \phi_2, \phi_1 + \phi_2 \) and \( \phi_1 - \phi_2 \). As only three delays are necessary to unfold the spectrum, it was decided to extinguish one of the possible combinations, by choosing the angle between the elements such that the weight of that carrier becomes zero. This was done in favour of the weights of the other carriers, which results in additional contrast resolution for these carriers.

The delays of the three carriers can be identified as: \( \phi_1 + \phi_2, \phi_2, \) and \( \phi_1 - \phi_2 \), with respective weights: \( 8^{-1/2}, 2^{-1/2}, 8^{-1/2} \). These delays are labelled d1, d2 and d3 respectively for the remainder of this report. The factor of \( \frac{1}{4} \) in front is the resulting intensity reduction caused by the two polarisers.
3.3.2. Source and distinction of the different delays

The previous sections showed that the three waveplates gave rise to two fundamental delays, and that from this three actual delays were obtained, using Mueller calculus. This section will offer a more physical explanation of the origin of these different delays, by tracing the rays and polarisation combinations. It will also show how these delays form individual fringe patterns, and show up as individual spots in the Fourier image, so they can be separated.

Source of the different delays

A delay is brought on by a path length difference between two rays. For a single plate, the delay is simply the path length difference between the o-ray and e-ray of the plate. For multiple plates, more combinations are possible. Each ray has multiple routes it could take, and any two such rays will induce a certain delay. We will trace the possible options in this section to find a conclusion on the possible resulting delays.

Let's first take a look at a combination of two displacer plates. Any ray moving through the two plates has 4 possible options: oo, oe, eo, ee, where the first character indicates the identity the ray took in the first plate, and so on. The path length difference is generated by the path length difference between any two rays. If the delay between the o-ray and e-ray of the displacers are denoted as $\phi_1$ and $\phi_2$ for the first and second displacer, then all possible path length differences are listed in the following table:

<table>
<thead>
<tr>
<th></th>
<th>oo</th>
<th>oe</th>
<th>eo</th>
<th>ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>oo</td>
<td>0</td>
<td>$\phi_2$</td>
<td>$\phi_1$</td>
<td>$\phi_1 + \phi_2$</td>
</tr>
<tr>
<td>oe</td>
<td>$\phi_2$</td>
<td>0</td>
<td>$\phi_1 - \phi_2$</td>
<td>$\phi_1$</td>
</tr>
<tr>
<td>eo</td>
<td>$\phi_1$</td>
<td>$\phi_1 - \phi_2$</td>
<td>0</td>
<td>$\phi_2$</td>
</tr>
<tr>
<td>ee</td>
<td>$\phi_1 + \phi_2$</td>
<td>$\phi_1$</td>
<td>$\phi_2$</td>
<td>0</td>
</tr>
</tbody>
</table>

Outside of the trivial zero delay, there are 4 delays uncovered: $\phi_1$, $\phi_2$, $\phi_1 + \phi_2$ and $\phi_1 - \phi_2$.

This is exactly the case in the Fourier separation setup. Despite the setup containing 3 displacer plates, two of them for a pair, giving rise to a single delay. Therefore, the above analysis applies to our setup. The polarisers are merely positioned in such a way that one combination has zero intensity, leaving the three delays needed for the designed Fourier separation system.

Distinction between the different delays

The combination of two arbitrary rays not only determines its delay, but also its spatial shear. The spatial shear is the spatial offset between the two input rays in a plane, which end up on the same spot on the camera’s image plane. The spatial shear determines the fringe direction, in the same manner as was seen for the single delay system in section 2.2.2.
By tracing the motions of the two interfering rays through the three displacer plates, the spatial shear can be found. $d_4$ is the simplest example: the two rays take the same path through the asymmetrical savart plate, thus omitting $\phi_2$. They only differ in the last displacer plate, where one ray follows the ordinary route ($o$) and its counterpart follows the extraordinary route ($e$) resulting in first ray having zero offset, and the second an offset in the direction of the displacer plate's optical axis, and proportional to plate thickness. The same can be done for the rays that take the same path through the last displacer, hence omitting $\phi_1$, and resulting in $d_2$. The remaining delays, $d_1$ and $d_3$ are combinations of $\phi_1$ and $\phi_2$.

These rays are visualised in Fig. 11, for all 4 delays. The starting location of two interfering rays are plotted in the top row, and the resulting the spatial shear is the imaginary line connecting the two points.

In the second row, the induced fringe pattern of the spatial shears can be seen. Note that the fringes are in the direction of the spatial shear, and their frequency is proportional to the size of the shear.

Since the fringe patterns are sinusoidal, they appear as symmetrical dots in the Fourier transform image, which is shown in the bottom row. For visibility, one dot is encircled.

On the right side, the combination of the fringe patterns is given, according to the intensities obtained from the Mueller calculus. This Fourier image matches the experimental one, as can be seen in Fig. 24.

---

**Fig. 11**: The different ray offsets, fringe patterns, and Fourier images, corresponding to the 4 different delays, as generated by the Fourier separation system’s waveplates. On the right side, the combination image is shown, using the intended weights.
Note that the zero carrier is not shown in the image, but will induce the central dot in the Fourier image. It is generated by the rays that take the same path through all three plates, and hence have a spatial shear of zero, inducing no fringe pattern.

Notice that by the choice of the angles of our displacer plates, it can be seen that the fringes corresponding to the different delays are always in different direction. And that the dots in the Fourier image are separated from each other. This provides the means of extract the fringe patterns for a specific delay. This separation process is detailed in section 4.1.2.

3.3.3. Fourier Separation Setup Overview
The optical setup is visualised in Fig. 12, and a picture of the back-end of the setup in our lab is shown in Fig. 13.

![Diagram of the Fourier separation system](image)

**Fig. 12:** Diagram of the Fourier separation system. F: Wavelength filter, HPW: Half-wave plate, D: Displacer plate, P: polariser.

![Picture of the back-end of the Fourier separation system](image)

**Fig. 13:** Picture of the back-end of the Fourier separation system. The calibration cube is visible, connected to Lens 3 and 4. The remainder of the system is made light tight, starting with the wavelength filter. The tube containing the HWP, Displacers, and Polariser is visible, which is fixed to the 5th lens and to the camera.
Setup Rundown

The first elements the plasma light encounters are the mirror, the first lens, and the two large lenses, which form the relay, moving the light from the front to the back end of the setup. Then follows the calibration system, with polarisations cube, and a following lens generates another Fourier-space for the interferometer. These elements are part of the general design consideration listed in section 3.2.

Then follows the optical wavelength filter (F), and the system is once more made light tight beyond this point. A HWP (rotated 22.5 degrees) is added to rotate the vertically or horizontally polarised light exiting the polarisation cube into diagonal polarisation required as input for the displacer triplet. As indicated in the previous section, the final polariser needs to be oriented at 22.5 degrees.

A final lens then images onto the CMOS sensor, where the rays are combined, forming the superposed fringe pattern.

Lenses

The first lens is focussed on the plasma, roughly two metres in front of it, and last lens is focussed on the interference pattern. The remaining 3 lenses are focussed at infinity. A list of the lenses used can be found in the table below. The choice of lenses were decided by the lenses available, the system requirements, and to optimise the light throughput and vignetting. For the optimisation step, etendue calculations and Zemax optical simulation software was used.

<table>
<thead>
<tr>
<th>ID</th>
<th>Focal length</th>
<th>F-number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 mm</td>
<td>1/1.2</td>
</tr>
<tr>
<td>2</td>
<td>300 mm</td>
<td>1/2.8</td>
</tr>
<tr>
<td>3</td>
<td>300 mm</td>
<td>1/2.8</td>
</tr>
<tr>
<td>4</td>
<td>85 mm</td>
<td>1/1.8</td>
</tr>
<tr>
<td>5</td>
<td>85 mm</td>
<td>1/1.8</td>
</tr>
</tbody>
</table>

Outside of the plasma and CMOS, there are 2 more image planes, neither of which are needed, but are a consequence of the first focussing lens, and the required light relay system. The calibration system is placed in the second image plane, close to the interferometer. This posed no harm, as the high finesse of the polarisation cube doesn't distort the image.
3.4. Image Separation Approach

Using the image separation approach, the multiple delays are obtained by generating 4 duplicates of the plasma image. Each of these quadrants are given a different delay (phase shift), before they are passed through an interferometer. All 4 quadrants are then observed simultaneously on a single CMOS detector. Effectively, every quadrant can be seen as a single delay system.

To generate 4 duplicate images with the same FOV of the plasma, Wollaston prisms are used. The 4 different delays are added to the quadrants using a carefully designed quad delay plate (QDP). The Wollaston prisms and QDP are vital components to the image separation system, and are discussed in the next two sections, before the system is discussed in its entirely in the third section.

3.4.1. Wollaston Prisms

Wollaston prisms are made up of two orthogonal prisms of birefringent material, cemented together at their base. They unravel an incoming beam of light into two orthogonal linearly polarized outgoing beams. The resulting rays will symmetrically exit the Wollaston prism with a certain splitting angle between them. Wollaston Prisms can be used as beam splitters, polarisers, or even interferometry, when they are designed with very small splitting angles.

The splitting angle depends on the frequency of the light, and on properties (thickness and material) of the Wollaston prism. The plane at which the split occurs is fixed by the cemented triangular prisms. The polarization directions are determined by the optical axis of the two prisms.

Fig. 14 shows a common Wollaston prism: the polarization directions are parallel and perpendicular to the plane of the splitting. The figure also shows a simple setup where a Wollaston is used to duplicate an unpolarized light source. When the light source is polarized, the two images may have different intensities. A diagonal polarizer could be added before the Wollaston to ensure equal intensities.

Fig. 14. Left: A Wollaston Prism, with common splitting and polarization of input and output rays. (Image from Wikipedia)
Right: a setup where a Wollaston Prism is used to duplicate an image of an unpolarized source.

For the image separation system, one ‘normal Wollaston’ and one ‘special Wollaston’ are needed. The normal Wollaston split into 0° and 90° polarization (the common, horizontal/vertical directions), while the special Wollaston split into −45° and 45° polarization (diagonal), when compared to the (horizontal/vertical) splitting plane. Fig. 15 displays the output polarization of the individual Wollaston prisms. Fig. 16 displays the combined effect of a polarizer (at 45 degrees), the normal, and the special Wollaston. Also shown is the resulting image when this setup is placed in the Fourier domain of an optical system.
Fig. 15: Arrows indicate the output polarisation for the Wollaston prisms. Left: the normal Wollaston prism with a horizontal splitting, giving rise to 2 images, one horizontally polarised, and one vertically polarised. Right: ‘special’ Wollaston.Exiting rays are diagonally polarised compared to the vertical splitting.

**Polarisation after the:**

<table>
<thead>
<tr>
<th>Polarizer</th>
<th>First Wollaston</th>
<th>Second Wollaston</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Polarizer" /></td>
<td><img src="image2.png" alt="First Wollaston" /></td>
<td><img src="image3.png" alt="Second Wollaston" /></td>
</tr>
</tbody>
</table>

**Resulting image:**

- ![Resulting image](image4.png)
- ![Resulting image](image5.png)
- ![Resulting image](image6.png)

Fig. 16: The exit polarization direction and resulting images after inserting a polariser and two Wollastons into the Fourier domain of an optical system, visualising the image duplication of the Wollaston prisms.

In order for the duplicate images not to overlap, they need to be confined. This is achieved by adding a rectangular mask in an image plane prior to the Wollastons.

The desired splitting angles in the image separation system depend on the CMOS size, the QDP size, and the available lenses. For the our setup, we required splitting angles of 5.92 and 5.00 degrees, for the normal and special Wollaston respectively.
3.4.2. Quad Delay Plate

The four images generated by the Wollaston Prisms will pass through an interferometer, which will cause the light to interfere with itself. One can set the delay between the interfering rays by placing a delay plate between the Wollaston and the interferometer. A single delay plate would give every quadrant an identical, fixed delay. To assign different delays to every quadrant, a Quad Delay Plate (QDP) is needed, see Fig. 17.

![Quad Delay Plate](image)

Fig. 17: The quad delay plate, a custom build optic for use in the image separation system. Each quadrant is a Field-Widened delay plate, and assigns a different delay to the image passing through.

The QDP is a custom made optic (assembled to our specification by an external company), consisting of 4 field widened delay plates. In practice, it’s made out of 9 pieces: 4 pairs of delay plates, and one low order half waveplate (HWP, with twice the length and width of the delay plates). Each pair of delay plates with identical thickness is glued to both sides of the HWP, forming one FW delay plate quadrant. The result is a QDP which is symmetrical around the HWP, with 4 different delays, and thicknesses, for every quadrant (see Fig. 17). The optical axis for the plates are: Delay plate: 0 degrees, HWP: 45 degrees, paired delay plate: 90 degrees.

As the 4 quadrants need to be imaged onto the plate, the plate necessarily needs to be in placed at an image plane. This means the QDP is sandwiched between two lenses: The first lens is required to form an image plane at the QDP after the light has passed through the Wollastons in Fourier-space. Similarly, the second lens is needed after the QDP to convert back to Fourier-space for the interferometer.

As the delay plates are in an image plane, the light will pass through it with a large range of angles. It is therefore chosen to have a field widened (FW) delay plate, as these feature a more homogeneous performance for large angles of incidence: The delay of the two plates are added up, while spatial variations are reduced (see the appendix, where this property is derived).

The varying thicknesses, combined with the refraction of BBO causes an issue when placed in air: the image can’t be focused on all 4 quadrants at once. The solution was to submerge the QDP in an optical oil, with matching refractive index. There were some challenges here: An oil tight transparent optical mount was build to precision, as good QDP alignment with the image plane was crucial, in all three dimensions. The assembly took place in a clean room environment, to avoid dust contamination on the QDP. The matching oil needed to be inserted carefully, without any bubbles forming. And finally, the
sealing o-ring needed to be chemically resistant (e.g. Viton), as we discovered when our initial generic o-ring reacted to and discoloured the matching fluid over time. These assembly issues aside, the completed QDP mount was very robust once assembled.

3.4.3. Image Separation System Setup Overview

Fig. 18 shows an example picture of the setup in our lab.

Fig. 18: The image separation system in our lab. The Wollaston prisms are in place, allowing a live feed of the 4 duplicate images on the display. The interferometer, wavelength filter and QDP are removed here.

Fig. 19 shows the diagram of the optics involved in the image separation system. At the bottom, a bar indicates where image planes are formed, and what regions are Fourier-space i.e. where the different rays originating from an image point are parallel. The contents of the diagram are discussed below.

Fig. 19: Diagram of the image separation system. F: Wavelength filter, HPW: Half-wave plate, W: Wollaston, D: Displacer plate, P: polariser.

Setup Rundown
Similar to the Fourier separation system, starting at the plasma the first elements the light encounters are the mirror, the first lens, the polarisation cube, and the two large lenses, forming the light relay from the front to the back end of the setup. The only deviation with the Fourier separation system is that the calibration system is moved to the front end, to make place for the mask in the back end. These elements are part of the general design consideration listed in section 3.2.

The mask is placed in the image plane, to prevent the Wollaston duplicated images from overlapping. In the next Fourier-space, the optical wavelength filter (F) is added. Beyond this point, the system is made light tight. A HWP is added for polarisation reasons (see below) followed by the Wollaston prisms.

The QDP and container is sandwiched between two lenses, in order to form another image plane.

The interferometer consists of FW Savart plate to add a spatial shear (fringes) and a polariser to align the polarisations of the rays to be combined.

A final lens images onto the CMOS sensor, where the rays are combined, forming the fringe pattern.

Removing the mask, Wollaston prisms, and QDP from the presented system, reveals the fundamental single delay system.

**Lenses**

The first lens is focussed on the plasma, roughly two metres in front of it, and last lens is focussed on the interference pattern. The remaining 5 lenses are focussed at infinity. This causes the light to consistently change between an image plane and Fourier-space. Outside of the plasma and CMOS, there are 3 more image planes. The first image plane is not necessary, but could not be avoided. The mask and QDP require the other two image planes. Due to space restrictions at the front, the polarisation cube could not be placed in front of the first lens, and was instead placed in the first image plane. As with the Fourier separation system, placing the polarisation cube in an image plane poses no harm, as the high finesse of the polarisation cube doesn’t distort the image.

A list of the lenses used can be found in the table below. The choice of lenses were decided by the lenses available, the system requirements, and to optimise the light throughput and vignetting. For the optimisation step, etendue calculations and Zemax optical simulation software was used. Fig. 20 shows the Zemax model of the setup.

<table>
<thead>
<tr>
<th>ID</th>
<th>Focal length</th>
<th>F-number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 mm</td>
<td>1/1.2</td>
</tr>
<tr>
<td>2</td>
<td>300 mm</td>
<td>1/2.8</td>
</tr>
<tr>
<td>3</td>
<td>300 mm</td>
<td>1/2.8</td>
</tr>
<tr>
<td>4</td>
<td>180 mm</td>
<td>1/2.8</td>
</tr>
<tr>
<td>5</td>
<td>85 mm</td>
<td>1/1.8</td>
</tr>
<tr>
<td>6</td>
<td>55 mm</td>
<td>1/1.2</td>
</tr>
<tr>
<td>7</td>
<td>50 mm</td>
<td>1/1.2</td>
</tr>
</tbody>
</table>
Fig. 20: Model of the image separation multiple delay system, in Zemax optical software. This image is to scale, and showcases the distinction between the front end and back end, separated by the light relay. The Wollaston prisms simulate a single splitting here, resulting in only half of the CMOS detector receiving light in the image. The back end shows 3 image planes: at the mask, QDP, and CMOS.

**Interferometer**

The interferometer is formed with FW Savart plate, build up from its individual parts: two identical displacer plates, rotated by 180 degrees, with a HWP in between (see appendix). The FW Savart plate is chosen for the system as it induces fringes that are very straight.[Reference] This allows for 1D Fourier transforms to be performed on the rows of pixels perpendicular to the fringes.

The thickness of the displacer plates, together with the final lens, determine the fringe density on the CMOS. The aim is get about 1 fringe per 10 pixels, as to maximising the fringes while retaining the ability to resolve them accurately.

**Polarisation rundown**

The light originating from the plasma or calibration system is generally unpolarised. The polarisation cube polarises the light: horizontally for the plasma light, and vertically for the calibration light.

Fig. 16 shows that the Wollaston pair requires diagonally polarised input light, in order for the quadrants to have equal intensity. This mismatch is present as the Wollastons were not originally designed to be used with the polarisation cube. A HWP is added before the Wollaston prisms in order to rotate the polarisation by 45 degrees. To induce this rotation, the HWP must be positioned with a 22.5 degrees rotation, relative to the vertical or horizontal axis. The appendix has more information on these plates, and other birefringent plates.

The FW delay plates all have a horizontal/vertical polarization direction. This means they will unravel the diagonal polarization coming from the final Wollaston (Fig. 16) into a fast ray and a slow ray, thereby setting the specified delay between the two rays.
The effective FW Savart plate in the interferometer is also oriented to have a horizontal/vertical polarization direction. This way, the Savart plate does not unravel the polarized light, but only adds a spatial shear to the slow and fast ray.

The final polariser is rotated by 45 degrees. As such, it forces the horizontally and vertically polarised fast and slow rays to align with a diagonal polarisation. This allows the rays to interfere at the CMOS detector.
4. Spectrum Reconstruction of the Multiple Delay System

The devices outlined in Chapter 3 produce images containing fringe patterns. This chapter details how the localised plasma spectra can be recovered from these images. There are two main steps: First, the interferometric contrast and the phase is recovered from the image. There will be one contrast and phase value per pixel and per delay. The second step, is to fit a spectrum to the contrast and phase for every pixel or pixel group. The spectrum then reveals the Doppler broadening and shift which can be readily converted into a plasma temperature and velocity. The blue arrows in Fig. 21 and Fig. 22 visualise these two steps, respectively for a single delay system, and a multiple delay system. The images show the added difficulty of the multiple delay system, that is required to recover a more complicated spectrum. The first step produces more contrast and phase sets for the multiple delay setup, through separating the delays. The second step requires a fitting program, explained in this chapter, to uncover the more complex spectra, whereas this step consists of a single equation in the case of the single delay setup.

![Diagram](image)

**Fig. 21:** The flow of data from raw image to plasma parameters, for the single delay case. The contrast and phase are extracted and calibrated at the first arrow. The second arrow is a one-to-one calculation.

![Diagram](image)

**Fig. 22:** The flow of data from raw image to plasma parameters, for the multiple delay case. The contrasts and phases are extracted and calibrated at the first arrow. At the second arrow, the plasma parameters are non-trivially fitted to the measured contrast and phase.
4.1. Obtaining the contrast and the phase sets
This section covers how the contrasts and phases are extracted from a raw camera image. There are a few steps to this process:

First the raw image is cleaned up (4.1.1), and then the different carriers belonging to the multiple delays are separated (4.1.2). This procedure produces the initial contrasts and phases. These will then have to be calibrated (4.1.3 & 4.1.4). Lastly, our setup and calibration is prone to a k-shift, detailed in (4.1.5).

4.1.1. Preparing the raw image data
Neutrons from the reactor will strike the CMOS, generating regions of pixels with high count values. They are very common and unpredictable, see the left of Fig. 23 for an example. During the KSTAR run, every frame contained hundreds of these spikes. As these spikes generally disappear in the following frame, it is possible to remove them using a short-windowed adaptive time median filter. This filter will replace the pixel value with the median of its value, taken over three consecutive frames. Additionally, the filter is adaptive, meaning the correction is only applied when the current pixel value is out of the norm (outside of a specified number of standard deviations around the average pixel values over which the median is taken) as to not distort the data, or temporal resolution, by taking the median of all points.

Additionally, some pixels of the CMOS are dead or pulsating. While these are persistent in time, they tend to be localised to individual pixels. It is hence possible to remove them using a small-windowed adaptive median filter. This is a normal median filter, over the data points surrounding the pixel in question, within a single frame. The same adaptive approach as described above is used here.

The result is that large spatial domain errors due to the neutrons can be filtered as they are short-lived, while long lived errors can be filtered as they tend to be small in space. A section of a typical image before and after filtering is shown in Fig. 23.

![Fig. 23: Left: 540x320 section of the 2560x2160 raw image, before any filtering. Right: the same section, with both types of filtering in place. Notice the reduction of neutron induce noise (white specks).](image-url)
4.1.2. Separating the carriers
The multiple delay systems will produce images that are a composition of multiple fringe patterns, where each fringe pattern is carried by a delay group. We’ll call these fringe patterns the carriers. Each carrier effectively contains the data for a single interferometer, and can be analysed just as a single delay coherence imaging system.

For the Fourier separation approach, all the carriers are superposed in the same image, as shown in the previous chapter. To separate the carriers, a 2D Fourier transform is applied to the frame, which reveals the different carriers as dots, with two point symmetrical dots per carrier. The bandwidth filter is then formed by setting the Fourier transform to zero except in the disk around the carrier dot of interest. The disks are chosen such that they do not overlap each other, in order to truly separate the different carriers. (One improvement added later on is a hanning window for the phase, see 6.3.3). As only one dot is selected of each pair belonging to a certain delay, it is ensured that the following inverse Fourier transformation is complex, with readily attainable contrast and phase.

Fig. 24 shows the (cropped) Fourier transform image of a measured frame on the left, and the filtered carriers on the right. The measured frame is from a KSTAR laser calibration.

In the right image, the 3 carriers belonging to the 3 delays of the interferometer are labelled d1, d2, d3. Also visible are the 0\textsuperscript{th} delay DC component, needed for calibration, and the 4\textsuperscript{th} delay, at a low intensity, as it is not cancelled out perfectly. The 4\textsuperscript{th} delay is not used in further analysis.

The left image shows the 4 carriers (d1-d4), and their mirror images, around the centre, d0. In addition, there are a few low intensity higher order impurities, which are automatically excluded by the bandwidth filters.
Fig. 24: Left: The Fourier transform of an image captured during a KSTAR plasma shot. Comparing with the expected carriers (Fig. 11), reveals that low intensity higher order dots (such as d4) are present. Right: The bandwidth filters applied in the Fourier domain. All bandwidth filters are shown here simultaneously for convenience. During the extraction, only one carrier (circle) is made transparent at a time.

For the image separated system, the carriers can be extracted more easily. The image is split up into 4 quadrants, and each quadrant is effectively a separate single-delay system, and can be analysed as such. Since the fingers are equidistant and parallel with the horizontal due to FW Savart plate, it is possible to reconstruct the contrast and the phase column by column. This is done by taking the Fourier Transform for every column of pixels perpendicular to the fringes, apply a bandwidth filter, and inverse Fourier transform. The bandwidth filter is typically a step function at the Nyquist frequency, to remove the mirror image. This again ensures the inverse Fourier transformation is complex, with readily attainable contrast and phase (carriers). The values will need to be calibrated as covered in the next sections.
4.1.3. Contrast Calibration

This section is split up into two parts. The first part explains how the contrast can be obtained from a (point of the) interferogram. This is what the contrast would be if the setup was ideal. In reality, there will be a maximum system contrast which is less than unity, caused by the non exact focussing of the last lens, optical aberrations, CMOS pixel crosstalk, and other potential real world limitations. This is corrected for by using the laser calibration shot, in the second part.

Obtaining the contrast:

In Section 3.3.1, the resulting stokes vector was found for the interferometer. This showed that the total intensity at the detector, for a normalised input was:

\[ S_{\text{intensity}} = \frac{1}{4} \left( 1 + \frac{\cos(\phi_1 + \phi_2)}{\sqrt{2}} + \frac{\cos \phi_2}{\sqrt{2}} - \frac{\cos(\phi_2 - \phi_1)}{2\sqrt{2}} \right) \]

This is the ideal case, without any added Doppler shift or broadening. Adding the phase shift, the loss of contrast, and changing the normalised intensity to the intensity measured at the detector \( I_0 \), the equation becomes:

\[ S_{\text{intensity}} = I_0 + c_1 \frac{I_0}{2\sqrt{2}} \cos(p_1) + c_2 \frac{I_0}{\sqrt{2}} \cos p_2 - c_3 \frac{I_0}{2\sqrt{2}} \cos p_3 \]

These 4 terms are the different components, \( d_0, d_1, d_2, d_3 \) as measured in Fig. 24.

Consider \( d_1 \). The measured complex interferometry point is:

\[ \text{Carrier 1} = \pm c_1 \frac{I_0}{4\sqrt{2}} \exp(i \ p_1) \]

The extra division by two occurs as 1 of the two complement carriers (dots in Fig. 24) is passed through. Which of the two is taken also determines the sign of the measured point. \( C_1 \) can then be obtained by taking the modulus of Carrier 1, and a factor:

\[ c_1 = \frac{4\sqrt{2}}{I_0} |\text{Carrier 1}| \]

The total intensity can be obtained by measuring carrier 0. Effectively,

\[ c_1 = 4\sqrt{2} \frac{|\text{Carrier 1}|}{|\text{Carrier 0}|} \]

And similar for the other carriers. The factors up front are respectively \( 4\sqrt{2}, 2\sqrt{2}, 4\sqrt{2} \)

Laser calibration contrast:

In reality a contrast of 1 can’t be achieved, due to real world components of the setup. There will instead be an intrinsic maximum contrast. The corrected contrast is then:
\[
c_i_{\text{corrected}} = \frac{c_{i,\text{plasma}}}{c_{i,\text{maximum}}} = \frac{c_{i,\text{plasma}}}{c_{i,\text{laser}}}
\]

The intrinsic maximum contrast can be found by measuring the contrast (using the techniques above) of the laser calibration shot. Using the knowledge that the laser features a non-broadened, narrow spectral line, it’s corrected contrast has to be unity. Therefore, the actual measured contrast is equal to the intrinsic maximum contrast.

4.1.4. Phase Calibration

The phases associated with the fringe pattern are uncovered by separating the carriers. But to obtain the Doppler induced phase shift, a zero point is needed: the instrumental fringe pattern at the central wavelength of 259.05 nm. This fringe pattern can’t be measured directly, as there is no convenient commercially available cold spectral source at this wavelength. The first step of the solution is to simulate the fringe pattern using a model of the setup. The second step is to correct this model for the real system, using the laser calibration shots. The combined model and laser calibration shot correction result in an accurate phase zero point, which can then be used in both the fitting approach and matrix inversion approach to extract the CXRS data.

The spatial heterodyne system consists of three displacer plates. Their phase shifts are given by the Veiras formulas (see appendix). For the three plates, the phase shifts are:

\[
\Delta \phi_{\text{Displacer, 6 mm}}(\alpha, \delta + \frac{\pi}{2}), \quad \Delta \phi_{\text{Displacer, 3 mm}}(\alpha, \delta), \quad \Delta \phi_{\text{Displacer, 4 mm}}(\alpha, \delta + \frac{\pi}{4})
\]

As the neighbouring 6 mm and the 3 mm displacers are rotated 90 degrees, their phase shifts readily subtract from each other: The pair effectively forms an asymmetrical Savart plate.

The next step is to generate two 2-dimensional maps \( \phi_1 \) and \( \phi_2 \), containing the phase shifts caused by the displacers, for every pixel of the CMOS. The angles of incidence \( \alpha \) and \( \delta \) can be calculated for every pixel.

\[
\phi_1, \, \text{Sim, 532 nm} = \Delta \phi_{\text{Displacer, 4 mm}}(\alpha, \delta + \frac{\pi}{4})
\]

\[
\phi_2, \, \text{Sim, 532 nm} = \Delta \phi_{\text{Displacer, 6 mm}}(\alpha, \delta + \frac{\pi}{2}) - \Delta \phi_{\text{Displacer, 3 mm}}(\alpha, \delta)
\]

These are the simulated phase shifts for the interferometer, for light with a wavelength of 532 nm.

Generating an image with these phase shifts (via the Mueller formula for the system, with added vignetting based on a laser calibration shot) results in an image with very similar appearance to the experimental data. The shape and frequency of the fringes largely match, however, the phase generally does not.

The discrepancy is caused by the non-ideal setup. Temperature fluctuations, small misalignments, non exact focal length of the final lens, etc. all contribute to small changes in the phase shift. For the purpose of fitting the contrast and the phase (the latter in particular), it is vital to correct the simulation.
For this purpose, the laser calibration shots, prior to the plasma pulse can be used. For both the laser calibration shot and the simulated image, the Fourier transform is taken, and the carriers are separated:

\[ z_{\text{car}_i} = \text{Fourier}^{-1}[CF_i(\text{Fourier}[\text{image}])] \]

\( z_{\text{car}_i} \) denotes the \( i \)th carrier, \( CF_i \) is the filter function, isolating the \( i \)th carrier in Fourier-space, \( \text{image} \) can be the simulated image, or an experimental image.

To obtain the phase difference, the carriers are divided, and the phase is recovered.

\[ \Delta \phi_{\text{car}_i} = \text{Phase}\left[ \frac{z_{\text{laser car}_i}}{z_{\text{sim car}_i}} \right] \]

\( \Delta \phi_{\text{car}_i} \) obtained this way is limited to a \( 2\pi \) range, and will contain phase jumps (if it didn’t, the simulation agreement would be so good that a correction wouldn’t have been needed!) As the simulation is fairly close to the measured data, the phase jumps will be limited, and can be unravelled. This does imply an uncertainty (\( \#\text{phase jumps} \times 2\pi \)) on the total phase, which results in the phenomena of a k-shift, as detailed in the next section.

Using the relations between the carrier phases and the phase shifts, the corrected phase shifts can be uncovered:

\[ \phi_{1, \text{ Corr}, \ 532 \text{ nm}} = \phi_{1, \text{ Sim}, \ 532 \text{ nm}} - \frac{\Delta \phi_{\text{car}_3} - \Delta \phi_{\text{car}_1}}{2} \]

\[ \phi_{2, \text{ Corr}, \ 532 \text{ nm}} = \phi_{2, \text{ Sim}, \ 532 \text{ nm}} - \Delta \phi_{\text{car}_2} \]

The phase shift for any wavelength \( \lambda \) (in particular, the zero point of 529.05 nm can then be calculated using:

\[ \phi_{1, \text{ Corr}, \ \lambda} = \frac{\lambda}{532 \text{ nm}} \phi_{1, \text{ Corr}, \ 532 \text{ nm}} \]

\[ \phi_{2, \text{ Corr}, \ \lambda} = \frac{\lambda}{532 \text{ nm}} \phi_{2, \text{ Corr}, \ 532 \text{ nm}} \]

If an image is generated using these corrected phase shifts with the same procedure, the result is an image with excellent contrast and phase agreement to the experimental data.

4.1.5. k-shift
The k-shift is an unknown parameter in the setup of the multiple delay systems, which is caused by the sensitive waveplates in the interferometer.

The phase calibration detailed in 4.1.4 corrects for any plate misalignments or construction tolerances, but leaves the possibility for a k-shift induced phase shift.
The three delays for every pixel in the image vary between 10-950 waves. With typical values of (120,355,835) for the 3 delays. The k-shift effectively adds or subtracts a number of waves from this, which could affect the results of the measurement and data analysis.

### 4.1.5.1. k-shift description

The fitting methods, including the listfit method, require the interferometric delays as an input, together with the contrasts and phases. The delays determine the point in the interferogram, and directly affect the expected contrast and phase values. Therefore, they need to be known with great precision for any successful fit.

The delays are calibrated for practical experiments. The calibration, described in 4.1.4, involves using a laser beam to generate a fringe pattern. The phases of this fringe pattern are then used to calibrate the delays. If a fringe pattern would be simulated using these corrected delays, it would line up exactly with the experimental laser beam generated fringe pattern.

However, the shortcoming of this calibration is that the phases, and hence delays, can be off by a number of wavelengths. The phases can only be measured using $p_{meas} = p_{true} + k2\pi$. Typically, the number of full radials two angles are separated by, is indicated by letter k. This integer k is the basis of the k-shift.

When a k-shift is present, i.e. when the corrected delays are off by $k2\pi$ with nonzero k, the fringe patterns simulated at the laser frequency will still be correct, as any additional full wave will not influence the fringe pattern. The k-shift forms an issue when the delays are calculated for the CXRS wavelength. The k-shift then causes an offset in the delay:

$$delay_{529.05\, nm,\, true} = (delay_{532\, nm,\, calibrated + k2\pi}\times\frac{529.05\, nm}{532\, nm}) + k2\pi \times 0.9945$$

This offset in delay is large enough to affect the fitting results, even for $k = 1$, as shown in 5.3.4.

If for example $k = 4$, than the delay will have an offset of 0.018 radials. This is already of the order of the carrier extraction, which has an accuracy of about 0.02 radials. This is usually enough to ruin any fitting procedure. In case of the listfit method, a k-shift of 4 would affects the CP deviation, adding ~0.4-1.0 to the original value, usually leaving the result untrustworthy.

While there are 3 delays for the Fourier separation approach, there are only 2 ground path-length differences (See section 3.3) this means that there are two k-shifts variables needed to describe the physical variation of the waveplates. The k-shift is therefore denoted as 2-dimensional variable $(k_i, k_j)$ in this report.

### 4.1.5.2. Solutions to the k-shift

Once the k-shift is known for a setup, it shouldn’t change, barging physical and thermal shocks to the setup. So it only needs to be determined once.
Option 1: As the k-shift can only take on a finite number of values, it is possible to brute-force the results. By attempting different values for k when fitting the data, the correct k should stand out. Using the listfit method, the correct k would provide the best fit, and have the lowest cp deviation value.

Option 2: Additional calibrations, such as using a secondary laser beam, could remove the k-shift ambiguity. Pending on the stability of the wave plates during installation at KSTAR, the second laser might have to be installed in-situ.

Option 3: Under the assumption that the plasma velocity is low during the start up of a KSTAR shot, the plasma is effectively a light source of 529.05 nm. The fringe pattern generated by this light can be used as a calibration for the delays, removing the k-shift ambiguity.

In this report, option 1 was used to resolve the k-shift.

4.1.5.3. Brute-force solution to the k-shift

The k-shift offset can be found by brute forcing the data. It is argued above that even the smallest k-shift, such as (0,1), affects the results, and causes an elevated CP deviation value. The brute force solution to finding the k-shift is therefore straightforward: try to fit the KSTAR data using all possible k-shifts, and find the k-shift that results in the lowest CP deviation value.

This lowest CP deviation value should be easy to find: The tests in section 5.3.4 reveal that k-shift offsets greater than (0,3) or (2,2) will cause the fit to not resolve, and have CP deviation values greater than 1. Therefore, the majority of the results while performing a brute force k-shift on KSTAR data will result in high CP deviation values. Only a small cluster in the k-shift 2D parameter space will result in a fit with CP deviations less than 1.

The k-shift brute force range:

As said in the intro, the k-shift is induced by misalignments and plate tolerances. The expected wave delays of around (120,355,835) will be offset by a number of waves, relating to the k-shift.

During assembly of the systems, it was obvious that alignment of the delicate waveplates and lenses can only be done within a 10-20 wave tolerance.

The delay plates that induce the delays and cause the fringe patterns have higher order components that result lightly curved fringes. The fringe pattern, with its curved shape, provides an additional marker for the delay of the interferometer. At high offsets, greater than about 50 waves, the fringe pattern becomes visibly distorted compared a zero-offset pattern. There is no need for the brute force method to extend beyond this range, as such distortion would have been picked up through other means.

Taking these considerations into account, together with plate tolerances, a conservative, large range for the k-shift was chosen: [-80..80], for both $k_i$ and $k_j$. This value is roughly 10% of the maximum delay waves.
Computation time and optimisation of the brute force method:

Without optimisation, each possible k-shift is evaluated using the standard listfit procedure. There are 161 values for both $k_i$ and $k_j$, resulting in 25921 combinations. Assuming 5 plasma variables, this calculation would take 10 minutes * $161^2 \approx 180$ days.

A first optimisation, is to reduce the fitting time per k-shift. The default fitting procedure in this report generates an output of 24 x 20 points, convenient for visualisation of the data (see section 5.2.2). It is unnecessary however to generate 480 points to find the k-shift. The resolution is further reduced by a quarter in both dimensions, resulting in 6 x 5 or 30 points. This is still sufficient to establish the typical cp-deviation value for the fit, and rule out any odd points.

The second optimisation is that the k-shift search can be performed in two stages. In the first stage, the area/cluster of k-shifts is found wherein lies the correct k-shift. In this first stage, the k-shift does not have to be incremented by one. The results of section 5.3.4 show that the 1-sigma test is still passed when a k-shift offset of (2,1) is applied. Therefore, the k-shift can be incremented by (5,3), which will ensure the k-shift will be found with a +/- (2,1) tolerance. The brute force grid is effectively made less dense, as the target is a group of points, rather than a single point. This stage hence takes only $33 \times 55 = 1815$ evaluations.

Once the area is found a second stage brute force, where the k-shift will be incremented by one, will determine the exact k-shift. As this range is only $([-2..+2], [-1..+1])$, this second stage only involves 15 evaluations.

The computation time after the optimisations is therefore $10 \text{ min} * \frac{1}{16} * 1830 \approx 19$ hours. Since the k-shift only needs to be found once, this is an acceptable outcome.
4.2. Obtaining the spectrum

4.2.1. Fitting procedures

Every pixel in an multiple-delay image will produce a set of contrasts and delays. The contrast and phase will depend not only on the plasma conditions, but also delays - the path length difference for the interferometer - which will be different for every pixel. This means that every pixel will need to be fitted individually.

The fitting module will require not only the set of contrasts and phases, but also the associated delays as input parameters.

Not every pixel needs to be fitted. Due to the optical dispersion of the system, and the Fourier separation approach, there will be a point spread function of at least 10 pixels, averaging the information contained in the image across many pixels. As outlined in section 5.2.2, the resolution will be averaged and reduced, resulting in a 20 x 24 grid. This resolution is large enough to provide a visual representation of the plasmas in this report, and to provide a sufficient number of points to perform analysis on.

4.2.2. Fitting methods

The set of measured contrasts and phases for a pixel needs to be mapped to a spectrum. The problem presents itself as a non-linear transformation, with 6 inputs and 6 outputs. This analysis is preformed in IDL®.

Several attempts were made at designing a method capable of fitting a spectrum to a set of contrasts and phases at different delays. Initial attempts utilised least squares techniques, including “MPFIT”, an IDL fitting routine, based on a dampened-least-squares method, and “CONSTRAINED_MIN”, another IDL fitting routine which uses a generalized Reduced Gradient method. Due to the 6 dimensions of the system, and the objective minimisation function featuring many local minima, none of the attempts using these build in fitting methods were very successful.

A “listfit” method was devised, that solves the problem of fitting functions not converging, or getting trapped in a local minima, whilst simultaneously being easy to work with. The downside of the listfit method, is that in its simplicity, it requires a large computation time, as each pixel is effectively brute-forced into a solution. The next section (4.3) is dedicated to detailing the listfit method.

Alternatively to the listfit method, there exist more advanced statistical methods to recover the plasma parameters, such as Bayesian data analysis and Function Parametrization. A discussion of these three different approaches will follow in section 4.4.
4.3. The listfit method

The idea behind the listfit method is that a large number of possible plasma states (spectrum parameters) are generated, its contrast and phase sets calculated for every spectrum, and recorded into a large list, called CPlist. The measured data is then compared to the list, and the best match is returned as the fit to the data.

The method that generates CPlist, and the method that finds the point in CPlist with the lowest deviation to the measurement, are described in more detail below.

4.3.1. CPlist generation module

The CPlist is a list of a certain length, with every entry containing 6 real data points: the contrast and phase for all 3 delays. It is hence digitally stored as a length x 6 matrix. The purpose of the generation module’s is to fill this list with as many plasma configurations as possible, while keeping the computation time and memory usage within reason.

In the simplest case, the CPlist generation module consists of 6 nested for-loops, (one for-loop for every plasma parameter) and the evaluation function which returns the contrasts and phases of each spectrum.

The evaluation function depends on the interferometer that is used. In general, this will be the interferometer that takes into account the full passband of the wavelength filter, described in Section 3.2.2.

Computational resources:

As described in section 2.4, there is an Active, Passive, and Background component to the spectrum, which can be described using 6 plasma variables. If each variable is allowed to take on 10 different values, then CPlist will have a length of \(10^8\) to cover all possible plasma configurations:

Here, for every pixel, a million sets of plasma states are defined, and the resulting contrasts and phases calculated. They are then stored in a \(10^6 \times 6\) floating point matrix, consuming 24 MB of ram. If every variable was allowed to take on 20 different values, the computation time would be 64x longer, and the ram usage for storage alone would increase to 1.5 GB. This example shows the high computational resources that the listfit method requires, and the importance of selecting the correct range, parameters, and number of values each parameter is allowed to take on.

The time consumption of each calculation depends on the complexity of the virtual interferometer, and how efficiently it is programmed into the generation module. Where appropriate, the wavelength filter could be approximated by a Gaussian, or omitted altogether. Or alternatively, the Background bremsstrahlung could be omitted in some cases, to simplify and speed up the calculation.

For the purpose of this report, the CPlist generation module has been rebuild from scratch a total of 8 times over the course of a year, with each rebuild providing either greater functionality, different interferometers, or speed optimisations.
The efficiency of the final version of the CPlist generation module can be demonstrated by using the default, full interferometer as described in section 3.2.2, and a default distribution of parameter steps (See section 4.3.2). It then takes 1.73 seconds to generate a list of 3.2 million elements using a single CPU core, clocked at 2.53 GHz. This means it takes 540 ns to generate every element in the list, with each contrast and phase value consuming 90 ns of CPU time, or only 225 CPU clock cycles. Without the added complexity of the wavelength filter, and simply assuming a Gaussian background signal, the calculation time is reduced to only 40 clock cycles. In these few clock cycles, the CPU processes exponents of complex numbers, arc-tans, and moduli, in addition to multiplications and summations. This indicates that there is little left to be optimised in the IDL code.

There are other means of speeding up the calculation, such as updating the code to work with multiple CPU cores, or enabling GPU processing, but these were not pursued in this report.

**Spectrum generation grid:**

The CPlist generation module calculates the contrasts and phases of all spectrums in a 6 dimensional grid, in the 6 dimensional space of all possible spectrum parameters.

The maxima and minima for every parameter are fixed: the temperature and velocity have a minimum of 0, and a maximum set by what can be expected at KSTAR. The relative intensities are also assigned a limited range, to reduce the number of unnecessary calculations. (e.g. if the Active intensity is 0, then there is no point in trying to fit it) They are shown in the table below.

The maxima and minima set the boundary for the grid. The next question is the number of steps that are generated for every parameter. The upper boundary is set by the maximal size of CPlist, to limit the computational resources required, as indicated above. The step sizes can’t be made too small either, as per following example:

Whilst the main focus of the fit is on the active temperature and velocity, the other parameters can’t be neglected. If, for example, only two steps are generated for the passive ratio (say 25%, and 75%) where the actual plasma has a value far removed from these values (say 50%), then the difference in contrast and phase for the measured data, and those of the simulated plasma will always be very large, regardless of the fact that 30 steps might have been used for the temperature and velocity.

This example shows that whilst the step size might be small, say 5 km/h for the active velocity, this is no measurement of accuracy, as the accuracy of the fit depends on the successful fitting of all parameters simultaneously.

The step sizes have a profound effect of the success rate and accuracy of the listfit method. Tweaking the step sizes typically meant the difference between ~70% and ~90% success rate for the listfit method. Whilst attempts have been made to derive an analytically optimal step size distribution, based on the derivations and sensitivities between the spectrum parameters and contrasts and phases, no clear model could be established. Instead, The number of steps have eventually been determined
through trial and error, to result in the highest success rate. (This is about ~90 % in ideal simulations, as will be seen in chapter 5). The results are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Active</th>
<th>Passive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.3</td>
<td>100</td>
</tr>
<tr>
<td>Steps</td>
<td>31</td>
<td>18</td>
</tr>
</tbody>
</table>

The values in the table determine the grid for the listfit method. The number of steps used for the passive velocity is left as a variable. If set to 1, the problem will be reduced to 5 parameters, under the assumption that the passive velocity is fixed. This will feature in chapter 5.

One possible improvement to this grid system, would be to select a random spectrum inside every 6D-hyperrectangle, rather than using fixed, linearly spaced spectra. As this may increase the success rate of spectra who’s parameters fall just in between those of simulated spectra.

4.3.2. CPlist deviation module
The CPlist deviation module compares the measured data point to those in the CPlist, and returns the position in the list that matches best. The it also returns the deviation value, a measure for how well the match is.

Omitting speed optimisations for now, the process that the CPlist deviation module goes through is as follows: the measured data point is subtracted from all the points in CPlist. Then, the phase is rewrapped, and the absolute value is taken of all contrast and phase differentials. Without the rewrapping step, a comparison between a measured phase of 3.1 radians, and a simulated phase of -3.1 radians would be seen as -6.2, instead of the actual difference of $\frac{6.2}{2\pi}$ = 0.083.

Each contrast and phase ‘column’ of the CPlist matrix is then multiplied by a weight, as is described further down. The weighted deviations are then summer together, to form a single ‘deviation value’ for every spectrum in CPlist.

The deviation value describes how well the measurement point matches a simulated spectrum. It is designed, via the weights, that the deviation should be << 1. A value greater than 1 implies that the deviation between the measurement and simulation is generally greater than the measurement error, and thus it can’t be a good fit. Typical deviation values for simulations, as shall be seen in chapter 5, are 0.00 to 0.30.

The spectrum in CPlist with the lowest deviation value is forwarded as the ‘fit’ of the measured data point.
While it is clear that a large deviation marks a bad fit, and a low deviation marks a likely good fit, the deviation value can't be easily interpreted as an error bar on the fitted velocity and temperature, due to it having to pass the non-linear Fourier transform that links the contrast and phase with the temperature and velocity. Rather, it is a handy means of eliminating obvious bad fits.

Weights:

There are 6 weights, one every contrast and phase of the three different delays. The 6 weights are fixed for all calculations in this report. They are build up from an importance value, and a noise value. The weight is the importance divided by the noise.

The importance value of a parameter is essentially another weight, indicating in percentages how important the contrast or phase value is for a successful fit, compared to the other contrast and phase values. The derivatives of the contrasts and phases, in function of the plasma parameters provides a good initial indicator as to which contrasts or phases are the most sensitive to chance, and hence should be assigned a greater importance value. Based on simulations (such as those in chapter 5) it was found that slightly higher success rates could be attained by tweaking these initial values. In reality, the optimal values are likely dependent on many aspects, such as the position of the pixel, the most likely plasma states, and the precision of the setup. The values that were settled on for this report are listed in the table below.

The noise value is the maximum software induced noise level that could occur. It is a measure of precision too. If the contrast or phase difference is larger than this noise value, then that is a good indicator that the simulated spectrum and the measurement do not match.

The noise here stems from the Fourier techniques used to extract the contrast and the phase (Section 4.1). The Fourier methods to separate the carriers typically produce a small oscillation in the data, even when the data is otherwise simulated without error. The values listed in the table below are obtained by generating a multiple-delay image, performing the separation of the carriers, and observing the difference between input and output contrasts and phases.

<table>
<thead>
<tr>
<th>Importance (%)</th>
<th>Contrast</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>delay 1</td>
<td>delay 2</td>
</tr>
<tr>
<td>Importance (%)</td>
<td>7</td>
<td>32</td>
</tr>
<tr>
<td>Noise</td>
<td>0.015</td>
<td>0.01</td>
</tr>
<tr>
<td>Weight</td>
<td>470</td>
<td>3200</td>
</tr>
</tbody>
</table>

In this table, delay 1 stands for the highest delay, and delay 3 for the lowest. A few observations: The phase noise is higher than the contrast noise. This is intuitively sensible, as the phase domain is larger than the contrast domain ($2\pi$ vs 1.0). The importance of the middle delay is by far the highest. This is also sensible, as for 3 well chosen delays, the middle delay should observe the largest phase and contrast fluctuations. Fluctuations will be greatest.
Computational optimisation

The main optimisation (outside of general efficient coding practices) incorporated in the CPlist deviation module is to start with only a single column the nx6 CPlist. The results with a deviation value (difference x weight) greater than a maximum deviation can be omitted from further calculations. A safe value for the maximum deviation would be 1, or any other value that removes a large percentage (e.g.: 90 %, for fitting typical KSTAR spectra) of the potential spectra. Time is then saved by avoiding calculating the contrast and phase differences, phase jumps, and absolute values, for spectra already guaranteed to have a high deviation value.

The phase column of delay 2 is selected for this purpose, as it features a high weight value, and a wide range of potential values.

With this optimisation, the CPlist deviation module takes just 0.25 s to return the best match, out of 3.2 million candidates, on a 2.53 GHz CPU.

4.4. Alternative fitting methods

As chapter 5 will show, the listfit method was successful at fitting simulated multiple delay data. There are however alternative methods, that are perhaps more suitable to problems of this type. In particular, the statistical techniques Bayesian Data Analysis (BDA) and Function Parametrization (FP) seem suitable. Both of these methods can be used when a forward model of the system into the diverse measurements exist (in our case, the interferometer), but when the inverse function is difficult to attain (in our case, recovering the spectrum). Both methods have been successfully applied in a variety of fusion diagnostics.

BDA compares the measurement results with a set of forward calculated system states. If these system states are selected on a gird, then this is similar to the listfit method. The comparison itself is more advanced then the listfit method: where the listfit method minimises the difference between the measured data and a list of forward calculated data, the BDA takes the accuracy of the measurement into account, and applies statistical methods to determine the system state which has the highest likelihood to correspond to the measured data. The BDA is a general method that requires little prior assumptions on the data and features excellent error propagation handling. But like listfit, the error minimisation/likelihood maximisation is a time consuming process.

The FP method is generally used within the fusion community where real-time data analysis of complex measurements are required. Prior to the measurements, a database consisting of a large set of possible system states and their forward calculation is generated. The known mapping from system state to measurement is then inversed using a statistical analysis on this database, resulting in a low-order polynomial fit. Hence, FP is suited for problems where mapping of the system states to the measurements can be approximated well by a low order polynomial fit. This might be a caveat for the spectrum reconstruction, as the contrast and phase do not necessarily behave in this way. In particular, the occurrence of phase jumps in the measurement data would immediately rule out an FP approach. However, phase jumps can normally be avoided by the design of the system carrier extraction. (A quick test using our system, reveals that phase jumps can be avoided as long as the rotational velocities are
below 250 km/s, which is indeed the case for KSTAR.) The statistical analysis of the database also provides the option to calculate basic of error propagation.

In terms of computation speed, the FP method has a clear advantage. The inversion procedure is slow, comparable to listfit and BDA, but once the inversion is known, it can be applied fast. Since there are different pixel groups, with different delays, each group will need their own inversion, implying it will initially take FP just as long to analyse an image as the other methods, but unlike the other methods, the calculated inversion can be stored, and readily applied to other images. The storage space requirements will depend on the order of the FP, and the number of pixel groups it’s calculated for. Even for high estimates of both, storing this data is much more resource friendly than the computational requirements of listfit and BDA.

As a last remark, the listfit and BDA methods have means of incorporating prior knowledge into the analysis. A specific parameter region of interest, or an external measurement of, e.g., the passive signal, could be incorporated into the listfit or BDA analysis. For FP, such prior knowledge can’t be incorporated without recalculating the inversion, which would negate the main advantage of using FP.

In summary, the expected error analysis (propagation of errors), computation time, and general fit accuracy for the three methods are listed in the table:

<table>
<thead>
<tr>
<th></th>
<th>Listfit</th>
<th>BDA</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error analysis</td>
<td>none</td>
<td>advanced</td>
<td>good</td>
</tr>
<tr>
<td>Computation time</td>
<td>slow</td>
<td>slow</td>
<td>fast</td>
</tr>
<tr>
<td>Fit accuracy</td>
<td>good</td>
<td>best</td>
<td>good</td>
</tr>
</tbody>
</table>

Due to time constraints, BDA and FP were eventually not pursued in this report. However, the table shows that both BDA and FP are expected to have advantages over the listfit method, and any follow up study is therefore advised to chose one or the other by comparing the benefits of speed and accuracy. The listfit method, being conceptually the simplest, was chosen as the starting point for this report, as it could be build from scratch with relative ease, and help to clarify the problem. While it was insightful to create the listfit method, it likely would have been possible to implement BDA or FP in a similar time span.

4.5. **Spectrum reconstruction conclusion**

The listfit method will be used to reconstruct the spectra in this report. By default, it’s range encompasses all possible KSTAR plasma states, as defined by the model in 2.4.2. When the passive velocity is set to zero, as will often be the case in the simulations, it takes about 2 seconds for the listfit method to generate 3.2 million possible spectra, and fit the best one to a point.

When the passive velocity is not fixed, the number of possible spectra generated and calculation time are multiplied by the number of values the passive velocity is allowed to take on.

The listfit method is optimised to for the active CXRS temperature and velocity. This means that generally, listfit will have a higher accuracy for these two parameters.
5. Simulation of the Multiple Delay System

5.1. Goals of the simulation

The goal of the simulation is to test the data analysis methods, and in particular the spectrum reconstruction method (listfit) unique to the multiple delay systems, before moving on to real data from a tokamak.

The simulation should be designed in such a way that the data analysis and fitting methods remain the same for both the simulation and real tokamak measurements. Specifically, the simulation should generate a raw image, calculated from an arbitrary plasma, similar to the raw image that would be taken by a multiple delay system at a tokamak.

The success and accuracy of the fit is determined using a 1 & 2 sigma test, which reports the absolute error range on the fitted plasma parameters required, for a minimum of 68% (1 sigma) or 95 % (2 sigma) of the fitted points. It is further described in section 5.2.3.

An often made assumption in CXRS is that the passive velocity component on fusion reactors is small. It is therefore often justified to turn the passive velocity into a constant, rather than a variable of the fit. Most simulations in this report are set to fit 5 components, keeping the passive velocity constant at 0. This has also the benefit of reducing computation time. The validity of setting the passive velocity to 0 is checked in the third test, where it is once more incorporated, and all 6 variables are fitted.

Test series 1: Basic listfit tests

In section 5.3.1, we will establish the basic functionality of the simulation and listfit method. To do this, a uniform plasma will be generated, and it will be fitted by a 5 parameter spectrum using listfit. This test will demonstrate the ability of the listfit method to recover the spectrum across the field of view, with different delays for every pixel.

The test will be performed at first by the direct fit method, which leaves out the additional steps of generating an image, and re-separating the carriers (as will be described in 5.2) before moving on to the default image fit method. This will determine the effect that the Fourier separation method will have on the fit.

To test the listfit method for a variety of spectra, the above mentioned tests will be repeated for a non-uniform plasma. In particular, the active temperature and velocity will be ramped from zero to maximum along the horizontal and vertical axis correspondingly, while keeping the other parameters constant.

Test series 2: Noise response

The listfit fitting methods has the contrast and phase measurements as input. In this series of tests, noise is added to the contrast and phase, so that the stability and noise response of the listfit method can be established.
There are three tests: first, noise is added to the contrast and phase after they have been averaged and the resolution is reduced. This test effectively simulates systematic errors on the CP measurement, as the error persists after averaging. Second, noise is added to the individual contrasts and phases. This tests reveals the sensitivity of the fit on each separate contrast or phase. The third and final test adds noise to the contrasts and phase before averaging, to simulate non-systematic noise.

These tests will define allowable measurement tolerances for the contrast and phase when the fitting method is applied to real tokamak data.

**Test series 3: 6 Variables**

As stated in the intro, most simulations in this report fit 5 components, while keeping the passive velocity fixed at 0 km/s. In this section, the fit is expanded to include the passive velocity as well, fitting all 6 variables.

Two tests are performed, with each a fixed plasma image. The first simulated plasma sweeps the active temperature and passive velocity throughout the image. It will be analysed using different configurations of the listfit method, including the default configuration, which assumes zero passive velocity. The second plasma sweeps both active and passive velocities, and all 6 components will be fitted.

The first test will determine and discuss the accuracy of the fit when 6 parameters are fitted compared to 5, while the second test provides insight into the interaction between the passive and active components.

**Test series 4: k-shift.**

As indicated in section 4.1.5, the k-shift is an unknown in the setup of the multiple-delay systems. It is a 2-dimensional variable \((k_i, k_j)\) that describes the phase offset in the waveplates. It only needs to be found once: the k-shift should remain constant provided the setup remains unchanged. Further plasma shots at KSTAR would only require the laser calibration shot, and the previously found k-shift to calibrate the delays and phases.

By simulating an artificial k-shift, one can establishing what range of k-shifts would be acceptable, via the sigma tests. The resulting interval of acceptable k-shifts can be used to speed up the brute force method of finding the k-shift on practical KSTAR data (See section 4.1.5).

The additional feature of these tests is to demonstrate the effects of the k-shift, and how it affects the resulting fit.
5.2. Methodology

5.2.1. Simulation & Image generation

The first step is to define a plasma. We’re not concerned with a realistic tokamak geometry at this point, so the shape of the plasma is arbitrary. It is convenient to fill the entire image with a plasma, and ramp two plasma parameters in the horizontal and vertical direction. This way, the accuracy of the fit across a range of plasma parameters can be determined.

The resulting multiple delay measurement image is then calculated using a forward model of the measurement setup. The starting point is the virtualisation of the multiple-delay system. The resulting set of contrasts and phases can be combined into an image by applying the result from the Mueller Calculus in section 3.3.1:

\[
S_{\text{intensity}} = \frac{1}{4} \left( 1 - c_1 \frac{\cos p_1}{2\sqrt{2}} + c_2 \frac{\cos p_2}{\sqrt{2}} + c_3 \frac{\cos p_3}{2\sqrt{2}} \right)
\]

Where \( c_i \) and \( p_i \) are the contrasts and phases of a pixel, and \( S_{\text{intensity}} \) is the calculated pixel intensity.

The forward calculation and the listfit method also require the delays of the pixels as input. These delays are generated using a model of the system, as in section 3.3.

Image fit and direct fit

What has been described above is the default image fit method. The direct fit method removes the intermediate step of generating an image. Instead the forward calculated contrasts and phases are fed directly into the listfit method. The results obtained using the direct fit (shown in section 5.2) are hence free of any noise induced by the image generation and carrier separation methods.

Forward model expansions

Additional details of the system could be taken into account in the forward model, such as camera noise, optical aberration of the lenses, or focus mismatch. They are not included in our analysis, for the following reasons: The aberration of the lenses and the focus mismatch affect the maximum attainable resolution. This will have no effect on the results in this chapter, since the resolution is already limited by the computation time. Only 24 x 20 pixels are analysed, and the spacing between these pixels is far greater than the point spread function induced by the aberration and focus. The camera noise can also be ignored, as the bulk of it would be filtered out by the data preparation in section 4.1.1, and any remaining noise can be interpreted as general measurement noise, which will be investigated in 5.3.

Default simulation spectrum

The go-to spectrum, while performing the simulations is the default KSTAR spectrum, as discussed in section 2.4.4. We’ll reiterate this default plasma spectrum, together with the listfit parameter range, as mentioned in section 4.3.1:
Active Passive Ratios

<table>
<thead>
<tr>
<th></th>
<th>Active</th>
<th>Passive</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temp.</td>
<td>Velocity</td>
<td>Temp.</td>
</tr>
<tr>
<td>Minimum</td>
<td>0 keV</td>
<td>0 km/s</td>
<td>0 keV</td>
</tr>
<tr>
<td>Default</td>
<td>1.2 keV</td>
<td>62 km/s</td>
<td>1.1 keV</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.3 keV</td>
<td>100 km/s</td>
<td>4.3 keV</td>
</tr>
</tbody>
</table>

In most simulation, 2 parameters will be ramped from minimum to maximum, while keeping the other parameters constant at the values set in this table.

### 5.2.2. Resolution reduction throughout the simulation

The resolution is reduced throughout these simulations. Initially, the generated images are 2560 x 2160 pixels, to match the real camera resolution. This resolution is maintained throughout the carrier separation, resulting in a 2560 x 2160 x 6 array, containing the contrasts and phases for every pixel.

Then, for several reasons, the resolution is reduced to 24 x 20 points:

First, it is reasonable to assume a point spread function of at least 10 x 10 pixels, due to the known point spread function of the optics, and the Fourier separation method. A resolution larger than 256 x 216 would therefore contain no additional information.

Computation time is the next limiting factor. As section 4.5 concludes, it takes about 2 seconds to fit a point assuming the default case, without any prior knowledge, and the passive velocity omitted. A resolution of 256 x 216 using the default listfit method would therefore take 31 hours. A resolution of 24 x 20 takes only 16 minutes.

The resolution accuracy of the multiple delay systems is generally an important aspect for its practical use, being one of the prime advantages of coherence imagine. In this report however, the focus is the fundamental operation of the multiple delay systems. A 24 x 20 resolution is sufficient to determine a visual representation of the accuracy of the fitting method, at a sensible computation time. For the KSTAR data in particular, a 24 x 20 resolution is sufficient: Enough points to determine whether a fit converged or not, while limiting the computation time, so that different test and settings can be preformed quickly.

The reduction is applied as follows. 100 x 100 groups of pixels are averaged into a single point. The first group covers the pixels starting from pixel (80,80) to (180,180), and the last group starts at pixel (2380,1980) and ends at (2480,2080). Resulting in 24 x 20 points. The data from the 80 pixels wide border is not used, conveniently omitting any edge effects induced by the Fourier separation.

Due to phase jumps, the phases can’t be averaged directly. Converting to the phase offset, remembering that the phases are the sum of the delay, and the phase offset, will resolve this. The phase jumps will only be removed if the phase offset is sufficiently small. For the multiple delay system in this report, this is indeed the case, provided the plasma velocities are below 250 km/h.
5.2.3. Sigma test - determining the accuracy of the fit.
Each fitted frame will return 24 x 20 spectra with 6 parameters per spectrum. The resulting 2880 parameters have to be formatted in a way so that it’s easy to conclude whether a fit was successful or not, and what the typical accuracy is.

There is no clear-cut way of achieving this goal. To determine if such a fit is good or bad, we’ve used a test that we’ll name the sigma test. The reasoning behind this test and its procedure is outlined here:

First note that after a fit, the error between the fitted spectrum parameters and the original data can be different for the different parameters. E.g.: the error of the velocity of a fit could be high, while the error on the temperature is low, for the same fit. It therefore makes sense to keep the different parameters separate.

Focussing on a single parameter then, the error between fitted and original will fluctuate throughout the image. The nature of the multiple delay fitting problem (with possible degeneracy) implies that there will always be points that do not fit well. The result is satisfactory, when a large enough number of points have a low enough error. One way of determining this is to have a predetermined number of points that you require to be fitted, and then observe the absolute range of errors one would have to allow in order to reach that number. That range then becomes the measure for accuracy.

For the predetermined number of points, we’ve taken the values of 68% or 95% in this report, corresponding to the requirement that a statistical 1-sigma or 2-sigma amount of points fit with a certain accuracy. The 1-sigma check is suitable for the visualisation of the data: with more than 68 % of the pixels fitting correctly, an operator could easily use this data to interpret the plasma behaviour. The 2-sigma check is one of prestige. It implies that out of the 480 points typically fitted per frame in this report, less than 25 failed. This is essentially the highest accuracy test that can reasonably be performed in this setup. A 3-sigma test would not be statistically useful, as it would hinge on only 2 points out of the 480.

To determine if the 1-sigma or 2-sigma test is passed, the measure of accuracy is compared with a maximum allowable error. The maximum allowable absolute errors, are set in this report as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>25</td>
<td>1.1</td>
<td>25</td>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>

These values are chosen by taking 1/4 of the range of the listfit parameters. In a sense the maximum allowable errors can be set arbitrarily, depending on the needs of the analysis.

Typically, low rotational velocity and temperatures are expected around the plasma edge, which is the main contributor to the passive emission. In contrast to this, is the KSTAR 1D CXRS system, which suggests that the passive emission can be as hot and fast as the active emission. The parameters expected values, and maximum allowable errors in this report are based on this KSTAR 1D CXRS data and the simulations are run with this high passive velocity and temperature.
Should the KSTAR plasma have a colder, slower passive component, the simulations would have a smaller range, which can affect the result. However the high temperature and velocity simulations preformed here would still provide a good baseline.

Note that for a successful/passed, 1-sigma test, the following equation holds:

\[
\text{The error values of 68\% of the points} < \text{1-sigma test absolute error value} < \text{Maximum allowable errors}
\]

The sigma test, by definition, reveals an absolute error value that is larger than the error values for 68\% of the calculated points. For a successful, passed, 1-sigma test, this absolute error value is still smaller than the maximum allowable error. For example, in test 1, section 5.3.1, the active temperature and velocity absolute error value are around 100 eV and 10 km/s.

Since we are interested in the visualisation of the plasma, we require a high fraction of the points (68\%) to fit, but the precision is allowed to be somewhat large, as even with an absolute error of 1/4 of the range of the parameters, the general plasma trend will still be visible in an image. Therefore the values in the table above are chosen to be rather broad.

If the accuracy of the parameter of a fit (expressed in the absolute error value required to fit 68\% of the points) is smaller than the maximum allowable error, the fit passes the 1-sigma test, the fit is said to be acceptable for that parameter, and the box is coloured green. Ideally, the accuracy is much smaller than the maximum allowable error, marking a good fit. If the accuracy is higher, the fit is said to have failed for that parameter.
5.3. Simulation results

5.3.1. Basic listfit tests

In the basic listfit tests, the passive velocity is set to zero, reducing the fit to 5 unknown spectrum parameters per pixel. The first listfit tests will be performed on an uniform plasma, which will be fitted using the direct fit method and the image fit method. After that, the method will be tried on non-uniform plasma. Again, using both the direct fit method, and the image fit method.

The spectrum to be fitted is the default KSTAR spectrum, see the section 5.2.1 under Methodology.

5.3.1.1. Uniform plasma using direct fit method

This simulation provides an starting point for the discussion of many aspects of the fitting program. The results from this test are shown in Fig. 25. The CP deviation values returned by the listfit method (defined in 4.3.2) were small, all ranging between 0.07 and 0.09.

First off, the simulated results are completely uniform: The same value was fitted for each parameter across the image, resulting in 5 homogeneous images. This is a non trivial result: while the same spectrum (with the same interferogram) is fitted everywhere, each point in the image is associated with a different set of delays. The delays fluctuate heavily throughout the image, as they are the cause for the fringe patterns. Hence, the measured contrasts and the phases are going to be different for every point. The images, though homogeneous, are included here as a reference, as other results won’t be quite this uniform. The result of Fig. 25 implies that the fitting program performs consistently for a wide range of delays.

In this particular case, all points and parameters have the same absolute errors. This means the 1 sigma and 2 sigma tests both have the same result:
Both the 1 sigma and 2 test passes for all parameters. The active temperature was recovered with good agreement with the original data, resulting in a value of 1.22 keV, where the input was 1.15 keV, over the range of 4.3 keV.

As the table shows, the passive temperature is recovered with an absolute error of 0.3 keV. In actuality, a value of 1.4 keV was recovered for a 1.1 keV input.

The recovered active velocity was 47 km/s, as opposed to the input value of 62 km/s.

This is a 15 km/s error, on a total fitting range of 100 km/s. This error is higher than both the 1 and 2 sigma absolute errors in section 5.3.1.3. A deeper investigation offers an explanation for this, which satisfies both the results posted here, and additional simulations, such as those in 5.3.1.3:

The fitting program generates 18 possible values for the active velocity. Specifically, the values in km/s are:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.08</td>
<td>15</td>
<td>0.3</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

The input value of 62 km/s happens to be one of the unfortunate values that are far removed from the fitting points, with a distance of 3 km/s from its nearest neighbour. Data points that are this far removed from one of the brute-force points will have a harder time fitting. It is hard to predict whether these values will never fit, or whether they still fit a reasonable amount of time. To answer this question it makes more sense to look at a situation where the input varies across a range. This is covered in section 5.3.2. In this instance, the data that was fitted was only 2.5 ‘steps’, or 15 km/s away from the true value. This absolute error is for many application still an acceptable outcome.

The recovered ratios for the active and passive signal are: 60 % and 5 %, where the input was 45 % and 20 %. While the recovered values differ from the original values, note that the total has remained constant, at 65 %, which implies that the remaining intensity component, the bremsstrahlung at 35 %,
has fitted correctly. The discrepancy of the ratios and the is likely due to the fit compensating for the active velocity.

The fact that the fit has forwarded these values, implies that the deviation of the contrast and phase between this fitted point \( T_A = 47 \text{ km/s}, R_A = 60 \%, R_B = 5 \% \) and the original point \( T_A = 62 \text{ km/s}, R_A = 45 \%, R_B = 20 \% \) is smaller than any other points, that may intuitively seem closer to the input value, such as for example the point \( T_A = 65 \text{ km/s}, R_A = 47 \%, R_B = 18 \% \)

A potential solution to this issue would be to add an additional fitting round. After the initial point is found in the neighbourhood of the true solution, the fit could run again, on a reduced parameter space, to find a better solution. This idea of the two-step fitting is explored in section 6.3.2. The conclusion is that for practical reasons, it is not efficient to perform.

5.3.1.2. Uniform plasma using image fit method
The same test with an uniform plasma is now repeated using the image fit method. A zoomed in segment of the generated image is shown in Fig. 26, and the results of the fit are shown in Fig. 27. The results for the 1 & 2 sigma tests were exactly the same as for the direct fit method, and can be seen in the previous section. The CP deviations are again very low. The values were in the range of 0.04 - 0.19, which is somewhat larger than the direct fit approach.

---

Fig. 26: Example of a simulated image, generated using the KSTAR default uniform plasma values, zoomed in by a factor of 10 for clarity.
As a first observation, the results are comparable the direct fit results. The sigma tests reveal the same absolute errors, and for the majority of points, the recovered values are indeed identical.

Some points have fitted to a different spectrum, which can be seen by the non uniform features in Fig. 27. It was observed that the absolute errors of the parameters has remained constant throughout the time, despite those points that have fitted to a different spectrum. For example, where the original active velocity was 62 km/s, the majority of the pixels have been fitted to 47 km/s, while the remaining part fitted to 76 km/s. The error remained surprisingly similar at around 15 km/s. The same holds for the other parameters. This means that the direct fit and the image fit resulted in the same outcome for the sigma tests.

It is observed that in this test, that the added steps of using the simulated contrasts and phases to generating an image, followed separating the carriers again from the image, has little ill effects on the outcome of the fit. The perturbations induced by these extra steps merely cause some pixels to fit to a slightly different spectra.

5.3.1.3. Non-uniform plasma using direct fit method
The non uniform plasma consists of an active temperature and velocity ramps. The 1 & 2 sigma tests for the fit of this plasma are:
The table shows that parameters of focus, the active velocity and the phase, are correctly fitted with an absolute error less than 0.07 keV, and 7 km/s for 68% of the points, or less than 0.19 keV, and 19 keV for 95% of the points.

A numerical data analysis shows that all parameters are fitted well, and that there is a logical distribution: Parameters have a good chance of fitting correctly with a low error, and even if they don’t, they are likely not far removed from the original data. This is a logical result, but non-trivial for the non-Gaussian Fourier transformation spectrum fit. This is shown as most parameters manage to pass the 2-sigma test, with the exception of the passive ratio.

Observe that the absolute errors for the 1-sigma test achieved here are better than those in the uniform plasma test. This supports the finding in 5.3.1.1 regarding plasma values that are just in between the listfit simulated spectra. By ramping the velocity and temperatures, the majority of points in the image will not be as far removed from the simulated spectra, and more points will achieve a higher accuracy. Even the active velocity has resolved better than in the uniform image case, with error values typically 7 km/s.

Fig. 28 shows the original plasma data, the fitted data, as well as the difference. The difference image column is added with a reduced scale for clarity.
Fig. 28: The 5 plasma parameters for a simulated plasma with ramping active temperature and velocity. The scale of the difference column has been reduced (1/4) for clarity. See text for the interpretation.
As the 1-sigma test passed, the visualisation of the fit should be sufficiently clear to observe the core plasma features, such as the active temperature and velocity ramp, and the constant value for the other parameters. As Fig. 28 shows, this is indeed the case.

The difference images, in particular the active velocity and the passive ratio differences, seem to indicate that the fitting error is highest in the centre of the image, while the edges are fitted with greater accuracy. One explanation for this is the degeneracy between the active and passive signal: The passive temperature and velocity is fixed, while the active is ramped. When they become similar, degeneracy can occur, resulting in a reduced accuracy. There is however too little data present here to perform a useful statistical analysis.

The CP Deviation plot in Fig. 29 shows no particular features, indicating that the listfit method is functional across the whole range of active temperature and velocity. The range of attained CP deviations was 0.03 - 0.37.

Fig. 29: A plot of the CP deviation value belonging to the direct fit of the non-uniform plasma. As the CP deviation values are smaller than 1, this fit is said to have resolved.

5.3.1.4. Non-uniform plasma using image fit method
The same test in 5.3.1.3 is repeated using the image fit method. The sigma tests can be found in the previous section, 5.3.1.3, together with the 1 & 2 sigma results of the direct fit method, for easy comparison. The fringe patterns are displayed in Fig. 30.
Although the active CXRS parameters are ramped from zero to maximum, the visual difference in the fringe patterns is rather subtle. In the bottom left corner, the temperature and velocity are close to zero. Resulting in maximum contrast, and default fringes. In the top left, the plasma velocity is close to maximum. The main effect of this, is a phase shift of the of the individual carriers. Without a reference, it is visually hard to distinguish. The bottom right has high temperature, of which the main effect is to reduce the contrast of the fringes. This is the most visible effect, as the fringes appears to be more washed out. The top right corner will be a combination of the two previous effects.

The absolute errors of the image fit data is contained in the sigma test table in 5.2.1.3. It is observed that the same parameters pass the tests as before. As expected, the absolute errors are equal or slightly higher than with the direct fit method.

The recovered spectra using the image fit method were very similar to the direct fit method, so instead of showing the full results we’ll display just an illustrating example: the error of the active velocity of both fitting methods, is shown in Fig. 31.
As could be anticipated from the sigma tests, the two methods image fit and direct fit produce very similar results, as illustrated by Fig. 31. The same conclusion is reached here as for the uniform plasma test in 5.3.1.2: the carrier separation method works, and only results in a light increase in absolute errors. The majority of the errors are caused by listfit, and the degeneracy of the multiple-delay system, and not by the carrier separation.

In this image fit test featuring a non-uniform plasma, the active CXRS temperature and velocity are fitted with an absolute error less than 0.08 keV and 7 km/s for 68% of all points, and with an absolute error less than 0.25 keV and 21 km/s for 95% of the points.

Fig. 31: Comparison of the similarity between the active velocity difference using the direct and image fit methods.
5.3.2. Noise response

In this section the accuracy of the fit will be determined when noise is added to the measured contrasts and phases. As described in 5.2.2, the resolution is reduced by averaging the contrasts and phases. If the noise is added after the averaging, the noise can be seen as systematic noise, persisting throughout averaging.

The noise can be added to all 6 contrasts and phase variables simultaneously, or to each individual variable. The former allows us to assess the accuracy with a realistic distribution of measurement errors, while the later is an experiment to see how each cp variable affects different plasma parameters. Both tests are preformed.

In the third test the noise is added to the cp variables before the averaging takes place.

The noise is expressed in percentages of the range for the cp variables. The ranges are 0 to 1 for the contrasts, and 0 to 2π for the phases. The noise effected variable is then calculated by adding or subtracting a random number within this range. For example, 3 % noise on the ‘phase2’ variable implies \( \text{phase2}' = \text{phase2} + 0.03 \times \text{rand}(0..2\pi) \).

Due to the random nature of the noise, there is some fluctuation in repeat experiments of these tests, in particular, the generated images would look different. The results remain consistent however, as the sigma tests utilise all 480 points.

The test setup is otherwise identical to 5.3.1.4: The default hot KSTAR plasma is used, with a ramping active temperature and velocity, fitted using the image fit method.

5.3.2.1. Systematic noise on contrasts and phases simultaneously

The test is run with a systematic cp noise of 1%, 3% and 5%. The results of the 1-sigma and 2-sigma tests are given here.

<table>
<thead>
<tr>
<th>1 σ test</th>
<th>( T_A ) [keV]</th>
<th>( v_A ) [km/s]</th>
<th>( T_P ) [keV]</th>
<th>( R_A ) [%]</th>
<th>( R_P ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.23</td>
<td>12</td>
<td>0.6</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>3%</td>
<td>0.67</td>
<td>39</td>
<td>1.1</td>
<td>9</td>
<td>41</td>
</tr>
<tr>
<td>5%</td>
<td>1.4</td>
<td>52</td>
<td>1.1</td>
<td>6</td>
<td>41</td>
</tr>
</tbody>
</table>

The 1-sigma test reveals that the accuracy of the fit drops of very quickly when adding systematic noise: For 1 % noise the sigma test still passes for all variables, but at 3 % three out of five variables already fail the test. At 5% both the active temperature and velocity fail the test.

The only parameter still fitting correctly in the three tests is the ratio of the active signal. But without all other parameters fitting, this is likely to be a fluke: The listfit method is not as optimised to measure the active ratio compared to the temperature and velocity.
The only variable to pass the 2-sigma test is the active temperature, at 1% noise. The 2-sigma test reveals the large leap in absolute errors between 1% and 3% noise. The active temperature error grows by a factor greater than 10, and the active velocity triples in absolute error.

Fig. 32 shows the CP deviation value, together with the fitted results for the active temperature and velocity, for the cases of 1%, 3% and 5% added noise.

![Image](image.png)

Fig. 32: The CP Deviation values, fitted active temperatures and fitted active velocities are shown, for the fit of a plasma with a temperature and velocity ramp, with noise of 1%, 3%, and 5% added noise to all cp parameters. See text for interpretation.

The cp deviation images show that the confidence of the fit reduces dramatically as the systematic noise goes up. At 3% noise, about 50% of the points have a cp deviation value of 1 or greater, these points show up as white in the image. At 5% noise, the image is nearly all while, with about 90% of the points
having a cp deviation value of 1 or greater. On the other hand, at 1 % noise the cp deviation is never 1 or
greater, indicated by all points in the image being grey to black, and marking a much more healthy fit.

The fitted temperature and velocity images show the parameters returned by the fit. In a successful fit,
we would expect to see the input temperature and velocity ramp in this image. At 1 % cp noise, this
ramping temperature and velocity is clearly visible. At 3 and 5% noise, the images contains many more
black spots. This happens as cp deviation is so high that the fit returns zero for the temperature and/or
velocity. The gradual degradation of the result is visible going from 1% to 3% to 5%.

The conclusion here is that the listfit method does not cope well with systematic noise, failing the 1-
sigma test for most parameters, including the active velocity, at just 3% added noise.

5.3.2.2. Noise on individual contrasts and phases

3 % noise is applied to the 6 cp measurements individually (c1,c2,c3 for the three contrasts, and
p1,p2,p3, for the phases). The 1-sigma tests reveals:

<table>
<thead>
<tr>
<th>1 σ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>0.09</td>
<td>9</td>
<td>0.5</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>c2</td>
<td>0.17</td>
<td>12</td>
<td>0.6</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>c3</td>
<td>0.18</td>
<td>10</td>
<td>0.5</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>p1</td>
<td>0.24</td>
<td>14</td>
<td>0.5</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>p2</td>
<td>0.30</td>
<td>24</td>
<td>1.0</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>p3</td>
<td>0.11</td>
<td>13</td>
<td>0.3</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

All tests passed, except for the ratio of the passive signal, for the p2 noise test. In fact, the p2 test has
the highest absolute errors for all plasma variables for all 6 tests by a clear margin, and is therefore the
most sensitive.

The trivial, yet subtle result here is that adding noise to any cp measurement, causes an increase of the
absolute error of all parameters. Unlike single-delay CXRS, where the contrast corresponds directly to
the temperature, and the phase to the velocity, all parameters are coupled here.

This relation between contrast and temperature, and phase and velocity, does however still show up in
the 2-sigma test:

<table>
<thead>
<tr>
<th>2 σ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>0.4</td>
<td>22</td>
<td>1.1</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>c2</td>
<td>0.4</td>
<td>24</td>
<td>2.7</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>c3</td>
<td>0.6</td>
<td>24</td>
<td>2.7</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>p1</td>
<td>0.9</td>
<td>27</td>
<td>1.3</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>p2</td>
<td>1.0</td>
<td>76</td>
<td>1.3</td>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td>p3</td>
<td>1.7</td>
<td>62</td>
<td>1.1</td>
<td>17</td>
<td>41</td>
</tr>
</tbody>
</table>

The active velocity failed when 3% noise was added to any of the phases, while passed when the noise
was added to any contrasts. Additionally, the passive temperature error, while it failed all tests, was much higher for c2 and c3 than it was for any phase. Interestingly however, the phase errors seem to affect the active temperature much more than the contrast errors.

The CP deviation values are shown in Fig. 33:

![Figure 33: CP Deviation values for the fit of the plasma with a temperature and velocity ramp, with 3% noise added to each individual CP measurement, resulting in 6 different fits. See text for interpretation.](image)

The deviation values are lowest for contrast 1 and phase 1. The deviation values are visibly higher for phase 2 and phase 3, with about \( \sim 15\% \) of the points having a cp deviation value \( \geq 1 \), whereas the 4 other tests the cp deviations were \( < 1 \). For phase 2, this agrees with the 1 and 2-sigma test tables above, as it was shown to be the most sensitive.

Phase 3 shows a higher CP deviation, in addition to high absolute errors for the 2-sigma test. However, it also featured generally low absolute errors in the phase 1 test.

The different output parameters have similar trends, as an example error between original and fitted active velocity is shown in Fig. 34.
This image, and together with those of the other parameters, show that the error of the phase 2 is indeed highest, whilst the error of phase 3 is moderate, comparable to the other contrasts and phase.

From these tests, and in particular when looking at phase 3, it seems that passing the 1-sigma test is a better sign for a good fit, then the CP deviation value is. This is sensible, as a good fit is marked by a low absolute error on the majority of the pixels, which is indeed the goal of the 1-sigma test. The 1-sigma test indicated that phase 2 was the most sensitive parameter, and Fig. 34 indeed shows that the fitting errors were highest for phase 2. This is an affirmation that the 1-sigma test is a good means to determine the accuracy of the fit.

The CP deviation value seems to correspond to the 2-sigma test: whenever the absolute error values in the 2-sigma test where high or low, the CP deviation resembled this. However, there is not enough data generated in this test to employ any meaningful statistics.

The conclusion of these tests with noise applied to individual contrasts and phases, is that the relationship between the accuracy of the input (the measured contrasts and phases) is difficult to relate to the accuracy of the output (the fitted spectrum). The input and output parameters of the listfit function are very coupled, which is to be expected of a multiple-delay system, so that all input errors influence all output parameters.
The 1 and 2 sigma tests reveal no extreme results. While the phase 2 measurement is slightly more important/noise sensitive than the other measurements, there isn’t one input parameter that does not affect the result, nor is there one that overpowers the others. This result implies that the listfit function and the weights included therein are well calibrated.

5.3.2.3. Noise on all contrasts and phases simultaneously with averaging

Non-systematic noise can be simulated in the usual way of adding noise to the contrasts and phases of the full image (2560 x 2160) before averaging takes place. The expectation is that the contrasts and phases will be averaged appropriately, and the noise will be suppressed. The result for 10% of non-systematic noise together with the reference of 0 noise is shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg noise</td>
<td>0.09</td>
<td>7</td>
<td>0.3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Avg noise</td>
<td>0.33</td>
<td>19</td>
<td>0.9</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>reference</td>
<td>0.08</td>
<td>7</td>
<td>0.3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>reference</td>
<td>0.25</td>
<td>21</td>
<td>0.9</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>

Observe that the non-systematic noise is indeed suppressed, the obtained absolute errors are nearly identical, and the pass/fail results of the sigma tests are the same.

As discussed in 5.2.2, the averaging takes place over $100 \times 100 = 10,000$ pixels in this simulation. In practical multiple-delay implementations, the averaging would likely occur over fewer pixels to maintain a high resolution. Because of the optical dispersion and the carrier separation/extraction method, there will always be some amount of averaging, and thus noise reduction, involved. The conclusion is hence that non-systematic noise does not cause much concern for the multiple-delay systems. Systematic noise is the main concern.
5.3.3. 6 Variables
In these tests, all 6 parameters are fitted: active temperature, active velocity, active ratio, passive temperature, passive velocity and passive ratio. In the other simulations, only 5 parameters were fitted, as the passive velocity was set constant. Correspondingly, the to be fitted plasmas are also generated using a nonzero passive velocity in these tests.

In the first test, the active temperature and passive velocity are ramped up through the image. The fit is performed using a variable number of steps for the passive velocity in listfit. This test can be used to compare the accuracy of the 6 parameter test versus the previous 5 parameter test, and determine the number of steps required for the fit.

In the second test the active and passive velocities are ramped up across the image. The focus of this test is on the interaction between both velocities, i.e. how does the fit behave when the velocities are equal to each other.

5.3.3.1. Active temperature and passive velocity sweep
This test is similar to the Active temperature and Active velocity test in 5.3.1.4, with the exception that it's the passive velocity being ramped up in this case. Despite the passive velocity being the dynamic variable, the first test to be preformed is the typical 5 parameter fit, as used in the previous simulations. This test uses the erroneous assumption that \( v_p = 0 \). Next, all 6 parameters are fitted. The number of steps of the passive velocity that the listfit function generates (See 4.3.1 for the background info) is varied from 3 to 5 to 10. For the 10 steps test, the steps for the other parameters were actually, due to computer ram restrictions. The results for the 1 & 2 sigma tests, with added passive velocity, are shown in the table:

<table>
<thead>
<tr>
<th>( v_p = 0 )</th>
<th>( T_A ) [keV]</th>
<th>( v_A ) [km/s]</th>
<th>( T_p ) [keV]</th>
<th>( v_p ) [km/s]</th>
<th>( R_A ) [%]</th>
<th>( R_p ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( v_p ) steps</td>
<td>0.30</td>
<td>14</td>
<td>0.9</td>
<td>66</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>5 ( v_p ) steps</td>
<td>0.40</td>
<td>14</td>
<td>1.0</td>
<td>35</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>10 ( v_p ) steps</td>
<td>0.35</td>
<td>14</td>
<td>0.9</td>
<td>22</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( v_p = 0 )</th>
<th>( T_A ) [keV]</th>
<th>( v_A ) [km/s]</th>
<th>( T_p ) [keV]</th>
<th>( v_p ) [km/s]</th>
<th>( R_A ) [%]</th>
<th>( R_p ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( v_p ) steps</td>
<td>1.0</td>
<td>32</td>
<td>2.7</td>
<td>94</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>5 ( v_p ) steps</td>
<td>1.6</td>
<td>39</td>
<td>2.7</td>
<td>80</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>10 ( v_p ) steps</td>
<td>1.7</td>
<td>39</td>
<td>2.7</td>
<td>69</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

For the \( v_p = 0 \) test, all parameters pass the 1-sigma test, except for the passive velocity which wasn’t fitted. The unfitted passive velocity does increase the absolute errors, which can be seen by comparing this sigma test with the one in 5.3.1.3. The \( v_p = 0 \) result shows that the use of a 5 parameter fit,
excluding the passive velocity, is acceptable in terms of the 1-sigma test, despite there being a 
fluctuating passive velocity in the plasma.

The 3, 5 and 10 steps tests show that the absolute error of the passive velocity decreases as the number 
of steps increase. At 10 steps, the absolute error is small enough to pass the 1-sigma test.

In general the accuracy of fitting one parameter influences the accuracy of the others, since all 
parameters are coupled in the multiple delay system. This does not appear to be the case here: the 
absolute error values for the other parameters remain nearly constant. This is an interesting result, but 
without additional tests it can’t be confirmed here.

The CP deviation values belonging to these tests are:

<table>
<thead>
<tr>
<th>k-shift</th>
<th>Median CP deviation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_p = 0$</td>
<td>0.19</td>
</tr>
<tr>
<td>3 $v_p$ steps</td>
<td>0.13</td>
</tr>
<tr>
<td>5 $v_p$ steps</td>
<td>0.11</td>
</tr>
<tr>
<td>10 $v_p$ steps</td>
<td>0.11</td>
</tr>
</tbody>
</table>

These reveals a decreasing CP deviation value for increased number of $v_p$ steps. This is a sensible result: 
The fit becomes more accurate, resulting in a lower CP deviation, when there are more generated fitting 
steps in listfit.

5.3.3.2. Active and passive velocity sweep
In this test the active and passive velocity are ramped up across the image. The active velocity in the 
vertical direction, and the passive in the horizontal. To observe the effects induced by the fluctuating 
velocities, the active temperature was doubled in this test to 2.4 keV, therefore avoiding the similarity 
between the two default KSTAR temperatures (1.2 keV active, 1.1 keV passive) which may results in a 
sub optimal fit for this test. The sigma tests are:

<table>
<thead>
<tr>
<th>1 $\sigma$ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$v_P$ [km/s]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 $v_p$ steps</td>
<td>0.4</td>
<td>16</td>
<td>0.5</td>
<td>8</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 $\sigma$ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$v_P$ [km/s]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 $v_p$ steps</td>
<td>1.0</td>
<td>43</td>
<td>0.7</td>
<td>29</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

The 1-sigma tests reveals that most parameters pass the sigma test, with the exception of the ratio’s. 
The absolute error value for the passive velocity is significantly lower than those in section 5.3.3.1. By 
sweeping both velocities, the same effect is achieved as was observed between 5.3.1.1 and 5.3.1.3: the 
fluctuation in velocity is beneficial for the fit, due to the decreased distance between the measured 
values and the simulated values. The 2-sigma test reveals that neither velocity parameter pass the 95% 
sigma test.

The difference between the fitted plasma and the original is shown in Fig. 35.
The images display a feature around the diagonal from the bottom left to top right, where the two velocities are approaching each other. When they are equal, the active and passive Doppler Gaussians in the spectrum overlap. The features in Fig. 35 show that while the fit is indeed visibly affected by this overlapping, it is still functional: the fit doesn’t break along the diagonal.

It is unclear whether the absolute errors of the velocities increase or decrease around the diagonal in these images. It is however clear that they increase for the temperatures. The regions of the active and passive temperature with the lowest absolute errors are the top left and bottom right corners, where the velocity difference is greatest, and hence the separation between the two Doppler Gaussians is greatest as well.
5.3.4. k-shift

The k-shift is simulated by fitting an image with a different set of delays than those that were used to generate it. For the image, The default hot KSTAR plasma is used, with a ramping active temperature and velocity, fitted using the image fit method, as in 5.3.1.4, with an added artificial k-shift in listfit.

The k-shift consist of $k_i$ and $k_j$, the shifts of the two ground path length differences for the Fourier-separation approach (see section 3.3). The total k-shift is denoted as $(k_i, k_j)$.

The first tests will look at $(0,0)$ and $(1,0)$ separately, and the third test will combine the two. For the range, there is no need to go beyond $k = 3$, as the CP deviation value is already expected to be unusable at $k = 4$.

Additional points outside of this range (such as (4,0)) were calculated to confirm consistency. As expected, they did not resolve (median CP deviation value > 1, and failed sigma tests) and were therefore not included below.

5.3.4.1. k-shift parameter 1

The results for the 1 & 2 sigma tests are shown in the table.

<table>
<thead>
<tr>
<th>k-shift</th>
<th>1 σ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td></td>
<td>0.08</td>
<td>7</td>
<td>0.3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(1,0)</td>
<td></td>
<td>0.14</td>
<td>11</td>
<td>0.3</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>(2,0)</td>
<td></td>
<td>0.23</td>
<td>13</td>
<td>0.6</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>(3,0)</td>
<td></td>
<td>0.37</td>
<td>19</td>
<td>1.0</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k-shift</th>
<th>2 σ test</th>
<th>$T_A$ [keV]</th>
<th>$v_A$ [km/s]</th>
<th>$T_P$ [keV]</th>
<th>$R_A$ [%]</th>
<th>$R_P$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td></td>
<td>0.19</td>
<td>19</td>
<td>0.6</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>(1,0)</td>
<td></td>
<td>0.78</td>
<td>25</td>
<td>1.3</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>(2,0)</td>
<td></td>
<td>0.89</td>
<td>32</td>
<td>1.3</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(3,0)</td>
<td></td>
<td>3.6</td>
<td>57</td>
<td>2.2</td>
<td>20</td>
<td>41</td>
</tr>
</tbody>
</table>

The absolute errors in the 1 sigma test steadily increase with increasing $k_i$. $k_i = 2$ is the highest value for which all plasma variables pass the 1 sigma test. The 2 sigma test reveals that even at $k_i = 1$ the majority of plasma variables do not pass the test.

The median CP deviation values that belong to these fits are:

<table>
<thead>
<tr>
<th>k-shift</th>
<th>Median CP deviation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.13</td>
</tr>
<tr>
<td>(1,0)</td>
<td>0.35</td>
</tr>
<tr>
<td>(2,0)</td>
<td>0.60</td>
</tr>
<tr>
<td>(3,0)</td>
<td>0.86</td>
</tr>
</tbody>
</table>
As expected from the discussion in 4.1.5, the CP deviation are very sensitive to k-shift. The CP deviation values are a weighted sum of the difference between the measured contrast and phase and the listfit generated ones. Under the assumption that the induced errors (k-shift) are small, the listfit method will still find the correct plasma spectrum to fit (as indicated by the 1 sigma table). If in addition, the contrasts errors are low, then it follows that the CP deviation value is proportion to the phase errors, which is in turn proportion to the k-shift. This linear relationship Is exactly what is found for the median CP deviation values.

5.3.4.2. k-shift parameter 2
The results for the 1 & 2 sigma tests are shown in the table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.08</td>
<td>7</td>
<td>0.3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(0,1)</td>
<td>0.14</td>
<td>9</td>
<td>0.6</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>(0,2)</td>
<td>0.23</td>
<td>13</td>
<td>0.7</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>(0,3)</td>
<td>0.9</td>
<td>67</td>
<td>1.1</td>
<td>6</td>
<td>41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.19</td>
<td>19</td>
<td>0.6</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>(0,1)</td>
<td>0.53</td>
<td>23</td>
<td>1.3</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>(0,2)</td>
<td>0.65</td>
<td>36</td>
<td>1.7</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>(0,3)</td>
<td>3.3</td>
<td>94</td>
<td>1.2</td>
<td>20</td>
<td>41</td>
</tr>
</tbody>
</table>

It is again found that the absolute errors in the sigma tests increase when the k-shift is increased. The second k-shift variable appears to have a greater impact compared to the first. $k_j = 1$ is the highest value for which all plasma variables pass the 1 sigma test. Although a few more plasma variables pass the 2 sigma test, the absolute error range is comparable to the 1<sup>st</sup> k-shift parameter.

The median CP deviation values that belong to these fits are:

<table>
<thead>
<tr>
<th>k-shift</th>
<th>Median CP deviation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.13</td>
</tr>
<tr>
<td>(0,1)</td>
<td>0.41</td>
</tr>
<tr>
<td>(0,2)</td>
<td>0.73</td>
</tr>
<tr>
<td>(0,3)</td>
<td>1.00+</td>
</tr>
</tbody>
</table>

The CP deviation values confirm that the fit is more sensitive to the 2<sup>nd</sup> k-shift parameter. For the k-shift (0,3), the fit even has a CP deviation greater than 1, which generally implies the fit has failed to resolve.
5.3.4.3. k-shift both parameters

The combinations between the k-shift parameters are also calculated. Only the results ([1,2],[1,2]) are shown here, as the other fit results (such as (3,1)) did not resolve (median CP deviation value > 1, and failed sigma tests).

<table>
<thead>
<tr>
<th>1 σ test</th>
<th>( T_A ) [keV]</th>
<th>( \nu_A ) [km/s]</th>
<th>( T_P ) [keV]</th>
<th>( R_A ) [%]</th>
<th>( R_P ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.08</td>
<td>7</td>
<td>0.3</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(1,1)</td>
<td>0.18</td>
<td>14</td>
<td>0.7</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>(2,2)</td>
<td>1.8</td>
<td>67</td>
<td>1.1</td>
<td>6</td>
<td>41</td>
</tr>
<tr>
<td>(1,2)</td>
<td>0.43</td>
<td>39</td>
<td>1.1</td>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>(2,1)</td>
<td>0.24</td>
<td>20</td>
<td>1.0</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 σ test</th>
<th>( T_A ) [keV]</th>
<th>( \nu_A ) [km/s]</th>
<th>( T_P ) [keV]</th>
<th>( R_A ) [%]</th>
<th>( R_P ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0</td>
<td>0.19</td>
<td>19</td>
<td>0.6</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>1-1</td>
<td>0.69</td>
<td>36</td>
<td>1.8</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>2-2</td>
<td>3.6</td>
<td>94</td>
<td>1.1</td>
<td>8</td>
<td>41</td>
</tr>
<tr>
<td>1-2</td>
<td>2.7</td>
<td>85</td>
<td>1.8</td>
<td>19</td>
<td>41</td>
</tr>
<tr>
<td>2-1</td>
<td>1.9</td>
<td>53</td>
<td>2.2</td>
<td>20</td>
<td>41</td>
</tr>
</tbody>
</table>

In general, the combination of the two k-shift parameters worsens the condition of the fit, as both shifts cause an offset of the delays. While (2,0) fitted well, and (0,2) fitted reasonably, the combination (2,2) does not resolve at all, with 5 plasma variables failing the 1 sigma test.

It is found that k-shifts (1,1) and (2,1) both pass the 1 sigma test for all parameters. The CP deviation values are:

<table>
<thead>
<tr>
<th>k-shift</th>
<th>Median CP deviation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.13</td>
</tr>
<tr>
<td>(1,1)</td>
<td>0.65</td>
</tr>
<tr>
<td>(2,2)</td>
<td>1.00+</td>
</tr>
<tr>
<td>(1,2)</td>
<td>0.96</td>
</tr>
<tr>
<td>(2,1)</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The k-shift (2,1) is the largest k-shift that still passes the 1-sigma test, and has a CP deviation value less than 1. This makes it the optimum k-shift for the brute force technique to find the k-shift of the setup on KSTAR (See section 4.1.5)
5.4. Conclusions on the simulations

Test series 1: Basic listfit tests

The homogeneous fitting results for a uniform plasma in section 5.3.1.1 and 5.3.1.2 indicate the fit works, since it gives reasonable results across the field of view, and for all combinations of interferometric delays in the multiple delay system.

A comparison between the direct fit and image fit methods in 5.3.1 reveal that the image fit method produces nearly identical results. The difference for the 1 sigma test is insignificantly small, and the 2 sigma test reveals only some minor increased errors. This result implies that the procedure for the carrier separation (Section 4.1), essential for analysis of experimental data, can be performed with hardly any increase in the noise of the measurement.

For a simulated plasma with a fluctuating active temperature and phase component, the listfit procedure using the image fit method is successful at passing the 1-sigma test, and aside from one parameter, also the 2-sigma test.

The 1-sigma test reveals that for 68% of the pixels, the original plasma parameters can be recovered with an absolute error less than 0.1 keV for the active temperature, and less than 10 km/s for the active velocity component.

The 2-sigma test reveals that for 95% of the pixels, the original plasma parameters can be recovered with an absolute error less than 0.25 keV for the active temperature, and less than 25 km/s for the active velocity component.

Test series 2: Noise response

It is found that systematic noise can quickly reduce the accuracy of the fit, and cause it to not resolve. When systematic noise is applied to the contrasts and phases simultaneously of 0.06 peak to peak for the contrast and 0.06 radials for the phase (+/-3%), the fit will no longer resolve.

This result implies that it is very important to avoid any systematical errors on experimental (KSTAR) data.

When systematic noise of the same intensity is added to an individual contrast or phase, the fit still resolves, and the sigma test is still passed. It is only when multiple CP measurements experience systematic noise, that the accuracy of the fit deteriorates.

The tests also reveals that the result of the fit is most sensitive to the second phase component, associated with the mid-range delay. Noise added to this component results in higher absolute error values in the sigma tests, compared to the other component.

A last test confirms that non-systematic noise, such as noise of 10% of the contrast and phase range spanning just a few pixels, can be averaged out without any measurable effect on the results of the fit.
Test series 3: 6 Variables

These tests showcased the difference in accuracy of the fit between the 6 variable fit and 5 variable fit (with and without passive velocity). When the simulated plasma had a non zero passive velocity component, the absolute error values were increased for the active temperature and velocity: from ~0.1 keV and ~8 km/s to 0.4 keV and 16 km/s respectively. Despite this increase, the 1-sigma tests were passed, indicating that it is possible to accurately fit 6 variables.

A logical result was obtained: the absolute error values for the passive velocity decreased as the number of steps in listfit increased.

It was found that while the fit showed some features/artefacts when the two (active and passive) velocities were similar, they were minor, and did not 'break' the fit, nor significantly influence its accuracy. The images reveals that the highest accuracy for the two temperatures, was obtained when the two velocities were furthest apart. A logical result, stemming from the separation or overlapping of the Doppler Gaussians.

Test series 4: k-shift:

The k-shift tests confirm the issue of the k-shift: just a few waves offset at the calibration wavelength of 532 nm, causes a significant phase offset at the CXRS wavelength of 529.05 nm, which results in a reduced accuracy of the fit, and an increased CP deviation value. An offset of 4 waves for the first parameter, or 3 for the second parameter, was sufficient to cause the 1-sigma tests to fail, and CP deviation value to be greater than 1.

It was found that the second k-shift parameter was more sensitive than the first. An offset of (2,1) for the 2 k-shift parameters was the highest available offset that still passes the 1-sigma test and CP deviation checks. The size of this allowable offset is used to optimise the brute force determination of the k-shift on experimental KSTAR data.
6. Multiple Delay System on KSTAR

6.1. Introduction
The Fourier separation system was shipped to Daejeon, South-Korea, and applied to and tested on the KSTAR tokamak. The setup and related peripherals were as designed in Chapter 3, and analysed as described in Chapter 4.

Unfortunately, no meaningful CXRS temperature or velocity data could be obtained from these KSTAR measurements. This chapter therefore covers the trouble shooting performed to find the cause of, and solutions to this issue.

In this chapter, the rundown of the data analysis is given for a KSTAR measurement. (6.2) Some attempts that were made to resolve the fit and increase the accuracy by improving fixed assumptions, reducing errors, and by testing the data and setup for inconsistencies were described. (6.3) And some solutions to prevent these potential calibration flaws in the future are proposed. (6.4)
6.2. Results from KSTAR

6.2.1. Rundown: obtaining contrast and phase
The analysis starts when a picture is taken. Fig. 36 shows a raw camera image. The image was acquired during a high temperature KSTAR pulse, using a single heating neutral beam.

![Fig. 36: Raw image taken during KSTAR operation. KSTAR pulse 9240. On the left is the zoomed in image, showing the fringe pattern, and neutron induced noise. On the right is the full image, showing the LCFS and several features and reflections from the other side of the reactor.]

The last closed flux surface is visible on the right side, in particular in the top corner, where the intensity drops abruptly. The LCFS is not to be confused with vignetting, which can be easily verified as the LCFS moves during plasma startup. In the background, some reflections are visible: Circular and rectangular features of internal tokamak equipment. The fringe pattern and neutron bombardment specks are visible in the zoomed image.

Section 4.1 describes how the images are processed further. Black pixels, hot pixels, and neutron bombardment spots are cleaned up, according to section 4.1.1. (to perform the time median neutron noise filtering, at least 3 frames are needed). Then the carriers are separated, according to section 4.1.2. For illustration, Fig. 37 shows 2 example data sets obtained after separating the carriers. On the left is the extracted intensity of the zero carrier, which is used in the calibration of the other carrier's contrasts. On the right is the extracted phase from the first carrier, visualising the fringe pattern belonging to that carrier.
Fig. 37: Example image of Fourier-separated components. Left: Intensity of the zero carrier, to be used in the contrast calibration. Right: Phase extracted from the first carrier, 4x zoomed in.

Contrast 1
Contrast 2
Contrast 3
Phase 1
Phase 2
Phase 3

Fig. 38: Contrasts and phases extracted from KSTAR pulse 9240. These images are interpreted in section 6.2.2. The delays are 835, 358, and 120 waves delays respectively. The ranges for the contrasts are 0.0 to 1.0 (black to white), the ranges of the phases are limited to [-1.5; 2.0], [-2.5; 0.0], [-2.0; -0.5] respectively.
The contrasts and phases are then calibrated, according to 4.1.3 and 4.1.4. The contrasts are calibrated against the (frame specific) zero carrier’s intensity, and (instrument specific) maximum contrast, as obtained by the laser calibration frame. The phases are calibrated by subtracting a reference fringe pattern, revealing smooth phase differences. The reference fringe pattern is based on the model of the interferometer, and the laser calibration frame. The results are shown in Fig. 38, and discussed in the following section.

6.2.2. Contrast and phase data discussion

Contrast

To interpret the contrast, it is useful to keep in mind the interferometry of a single-Gaussian (SG). In the case of a SG, the temperature of the plasma corresponds to the width of the spectrum. The interferometry signal will also be a Gaussian, of which the contrast width is reversely proportional to the spectrum width.

The SG approximation thus implies that higher delays, the contrast goes down. This general trend is visible in Fig. 38, where the images get lighter left to right, as the delays go down. This also explains the first contrast, having the highest delay, being nearly zero everywhere during this hot plasma pulse. Contrast 1 also reveals the plasma is colder (not black in the image) at the plasma edge, which is indeed expected to be more cool.

While the SG approximation is useful to show trends (contrast typically decreases as delays go up) and for extreme cases (contrast zero at high delays), it is not analytically accurate when multiple Gaussians make up the spectrum, as shown in 2.2.1 and 2.4.4. This is also found here: a SG analysis of the three different contrasts, reveal three different temperatures for every point. This is the inconsistency that the (listfit) spectrum reconstruction method is intended to resolve.

Contrast 3 reveals some unphysical results: 7% of the points have a contrast higher than 1. (4% have a contrast between 1.00 and 1.05, 2% between 1.05 and 1.10, and 1% higher than 1.10). This is indicative of a noisy calibration.

Phase

Comparing the three phase images, phase 1 stands out as being very dynamic. As contrast 1 approaches zero, the phase of the signal is ill defined, resulting in a phase extraction consisting of mostly noise. The phase is only smooth and clear in the top right corner, there where the contrast is nonzero. The tokamak’s reflections are visible in the phase 2 and phase 3, which are more clear.

Note that the calibration step turns the phase images like the one shown on the right of Fig. 37, into those shown in Fig. 38. The fact that tokamak features can be observed indicate that the reference delays and fringes are functioning as expected: accurate, aside from a potential k-shift.

The signs of the phases are troubling however. A single Gaussian simulation of the experiment, with the assumption of the plasma rotating towards the camera at 100 km/s, and using the same analysis and Fourier separation techniques, reveals that the phase offset should be [1.70, -0.75, 0.25] for the centre.
To illustrate the results, Fig. 39: Illustration of the two example points in the KSTAR frame.

6.2.3. Spectrum fitting debugging
As stated earlier, any fitting attempts on the data have been unsuccessful. The CP deviation values of all fitted points were always large, greater than 1. To uncover the source of the error, the listfit program was adjusted to fit the contrast and the phase separately. This allows us to observe whether the contrasts or phases, and their respective calibration methods, may be to blame.

6.2.3.1. Separate contrast and phase fit
By separating the fitting of the contrast and the phase, the fitting function can no longer be expected to return physically accurate data (only three unknowns are used to fit 6 parameters). However, the CP deviation value can still be observed. By only fitting the contrast or the phase, the CP deviation value will report on the goodness of the fit for the contrast and phase separately.

In practice, this separation is performed by running the listfit method, while setting the weights for the phases or contrasts to zero in the CPlist deviation module (Section 4.3.2). The remaining weights are unaltered.

Observe that the CP deviation value for the total fit will be greater than or equal to the sum of the CP deviation values for the separate contrast and phase fits. Therefore, the cause of the high CP deviation may be identified. For example, if the contrast fit returns a high CP deviation, then the fit will fail regardless of the values for the phase. Therefore, the problem has been pinpointed to the contrast with certainty (although additional problems outside of the contrast may exist). Also note that the full fit may return a high CP deviation value, even when the separate contrast and phase fits return low deviation values, as the optimum contrast values do not need to coincide with the optimal phase values.

6.2.3.2. Spectrum fitting debugging results
Neither of the separate contrast or phase fits resolved. That is to say, out of the 30 points fitted for every k-shift combination, never more than 6 points at a time had CP deviation values lower than 1. A discussion of the CP deviation value is given with its introduction in Section 4.3.2. An example of how the CP deviation value corresponds with the accuracy of the data in terms of noise can be seen in Fig. 32.

To illustrate the results, the contrast of two example points are described here, as shown in Fig. 39:

Example point 1
This is the central pixel in the frame. The measured values for the three contrasts were 0.05, 0.2, 0.5. When the fit was restricted to fitting only the contrasts, the resulting lowest CP deviation was 1.01. This is without adding the CP deviation value of the phases, and it is therefore too high. It is worth noting that if the fit is restricted even further, to the individual contrasts, each contrasts fits.

In different words, the measured contrasts are within the range of the contrasts generated by the list fit method. However, while plasmas can be found that match individual contrasts, no plasma generated by the listfit method satisfies all three contrasts at once. The best solution, the plasma whose contrasts are the most similar to the measured ones, is too far separated from the measurement, as indicated by the deviation value.

Example point 2

This point is in the top right corner of the image, closer to the plasma edge, located 75% along the diagonal. The measured values for the three contrasts were 0.08, 0.5, 0.68.

The result here is more obvious than in example 1, the fit clearly failed due to contrast 3: The measured value was 0.68, while the generated listfit values range between 0.77 and 1.00. As the generated listfit values should cover all potential plasma states, including high and low intensity for the active, passive and background signal, the contrast 3 measurement must corresponds to an unphysical result. The measured contrast 3 being lower than expected is a general result, which applies for a multitude of pixels

6.2.4. Conclusion KSTAR data

The raw data is converted to contrast and phase data, and the listfit method is then tried on this data. It is found that the fit does not resolve. There are several potential causes: The listfit method doesn't work, the data is too noisy, there are wrong assumptions, or the calibration or setup is faulty.

Chapter 5 proves that the listfit method does work. In terms of noise, section 6.2.1 shows that the contrast and phase can be obtained and generally produces smooth images. Non-systematic noise is therefore not an important aspect of why the fit doesn't resolve. The cause must therefore be a systematic error: a fault in an assumption, the calibration, or the setup.

A wrong assumption may be, for example, the shape of the spectrum. If the shape is different than the suggested model in section 2.4.2, the listfit function might not produce comparable spectra. A faulty calibration might be caused by a different instrumental contrast for the plasma light and the laser light. A faulty setup may be caused by alignment or waveplate orientation errors.

These potential errors are amplified by the nature of the multiple-delay fitting, and the sensitivity to systematic errors (as shown in 5.3.2). It is therefore important to try and reduce any systematic errors, induced by the analysis, setup, or the calibration.

Section 6.3 covers some of the attempts that were made to increase the accuracy of the fit by improving assumptions and analysis methods, and by testing the data and setup for inconsistencies.
6.3. Attempts to resolve the data

6.3.1. Wavelength filter simulation improvements
The wavelength filter, as described in 3.2.2, was initially approximated by a single Gaussian, before the filter characteristic was more accurately modelled by three Gaussians.

It was found that the single Gaussian approximation resulted in contrast and phase offsets, with the greatest offset for the smallest delays, as expected. For the smallest delay, the contrast offset was up to 0.08, or 8% noise, and the phase up to 0.2, or 3% noise. Since these errors are systematic, it would definitely have affected the accuracy of the fit.

A simulation was performed like those in Chapter 5, where example KSTAR-like spectra were measured using the real bandwidth filter, but analysed using the single Gaussian wavelength filter. The result was a 25% increase in absolute errors, according to the sigma tests. The test confirms the importance of correctly modelling the wavelength filter, and reducing systematic errors.

6.3.2. Two-step fitting approach
As the list fit method generates a finite number of potential spectra, there will always be data points that don’t have an exact match, but rather fall in between certain spectra. The CP deviation value between the data point and these nearby spectra are sometimes of the same order as some other spectra that is further removed from the original. Section 5.2.1 shows a mild version of this effect, where an active velocity of 47.1 km/s was fitted, instead of a velocity closer to the original of 62.3 km/s.

One idea to improve these situations was to use a two-step approach. Instead of applying the list fit method and returning a single spectrum with the lowest CP deviation, a function was made that returned all possible spectra with a sufficiently low CP deviation. The spectra would generally have some sort of common theme, e.g. a low active temperature. The parameter ranges for these spectra would then determine a reduced domain as input for a secondary list fit method. As this second step generates the same amount of candidate spectra across a smaller domain, it has a higher chance of the data point having a close match.

The results forwarded by the two-step approach eventually turned out to be very similar to the regular approach. The main issue seemed to be that some CP data points inherently have a degeneracy as to what spectrum they belong to, such that even after a secondary step, they still end up with a non perfect fit. The added complexity of the two-step approach, combined with the doubling of computation time, made it unfavourable compared to the regular list fit method. A list fit method with an optimised range and step size proved to be more efficient than the two-step approach.

6.3.3. Bandwidth filters
Section 4.1.2 describes that the carriers are separated by applying a bandwidth filter in the Fourier image. The size and shape of the bandwidth filter has been optimised, by comparing the input and output contrasts and phases, and minimising the error. The results showed that the best performance could be obtained by using different filters for the phase and for the contrast: a step function for the contrast, and a hanning window for the phase resulted in the smallest errors. The optimal size for the
bandwidth filters was the maximum, without overlapping. In our case, it was a circle with a radius of 50 pixels, as shown in Fig. 24. The resulting errors, or Fourier separation noise, are used in the calibration of the listfit method in section 4.3.2. An example of their typical values is also shown there.

6.3.4. Interferometer integrity checks
Several tests have been performed to rule out potential errors in the setup of the Fourier separation system at KSTAR. In particular, the optical plates, displacers and polarisers, might have moved during transportation.

No evidence of any such plate misalignment has been found. The best example of the system integrity are the fringe patterns for both the laser calibrations shots, as well as the plasma operations, which match the expected patterns.

6.3.5. Displacer plate rotation
If a displacer plate is rotated by 180 degrees, the measurements will be subtly different due to higher order terms (see appendix). This might constitute a small systematic error, which should be omitted.

Simulations were run to discover if any or multiple plates might have been rotated. The shape of the dots in the Fourier image (for an example Fig. 24) are sensitive to these rotations, and were used to guide the results. The conclusion was that no individual plates were rotated. It was found however, that it was difficult to distinguish whether or not the entire interferometer (all the plates) were rotated 180 degrees together.

The KSTAR data analysis was repeated for this rotated interferometer, but similar, not resolving fits were obtained.
6.4. Proposed solutions

6.4.1. Calibrating the multiple delay system on example spectra

It may sound like an obvious solution: if the multiple delay system can be tested on a well defined example spectrum in a lab, it can be easily calibrated and characterised. When it is then applied to a fusion reactor at a later date, its functionality and accuracy can be practically guaranteed.

The problem is that there currently are no easy means of producing a well defined example spectrum. Other than fusion reactors, there are no devices capable of generating the hot (Doppler Broadened) spectra, in an intrinsic manner. Alternatively, the spectrum can be build up from a range of component, the sum of which is an approximated CXRS signal. However, this would requires a very elaborate setup, rivalling the complexity of the multiple delay system itself.

This section covered what the solution would be in the ideal world. Realistically, practical limitation imply that a fusion reactor is still the best method to generate the spectrum, and to calibrate the system. This solution is covered next, in 6.4.2.

6.4.2. Calibrating the multiple delay system on a fusion reactor

Expanding on 6.4.1, the radiation emitted from a fusion reactor can be used to calibrate the multiple delay system, provided the spectrum is known and well defined.

For example, a 1D CXRS system could be used to measure the spectrum accurately, which could in turn be used to calibrate the corresponding pixels of the coherence imaging system, and in turn the entire FOV of the coherence imaging systems. The 1D CXRS system at KSTAR can in principle be used for this purpose.

The 1D and 2D data will require the same point of view to accurately compare them. With a different point of view, plasma models or tomography techniques may be used to compare the two datasets, although this will reduce accuracy of the calibration.

In the 2013 KSTAR Campaign, the point of view was different for the 1D CXRS and the multiple delay system (as shown in Fig. 8), requiring the need for tomography. This was not which was not attempted in this report. It is advised that the multiple delay system is located near to the 1D CXRS system for future campaigns utilising this calibration method.
6.4.3. Measuring cold plasmas
In the initial seconds of a plasma pulse, the plasma is still largely cold and stationary, but some CXRS emission might already be present. Measuring this cold spectrum can be very useful, and result in a string of benefits:

1) The fringe pattern will be that of the base spectral line (529.05 nm for carbon IV).
2) Having this reference fringe pattern would omit the need for phase AND contrast calibration at the laser frequency, and would avoid the k-shift altogether.
3) In addition, the k-shift issue is avoided altogether, increasing the computation efficiency.
4) This is beneficial in terms of troubleshooting any issues, as the fit could then be restricted to only the phase, as described in 6.2.3.1.

The catch is that it is impossible to know for certain how cold and/or motionless the plasma is during the early plasma start-up, without a 1D CXRS measurement. If the reference image is not of a motionless plasma, the obtained reference phase, and following hot plasma measurements, will have a phase offset resulting in an accuracy reduction for the fit (as found in 5.3.2), or won’t fit at all.

One method of reassuring that the measured plasma is indeed consistently cold during start-up, without requiring 1D CXRS, is to compare and average the cold fringe pattern of several plasma pulses. It would be highly indicative of a cold and motionless plasma if the pattern remain constant over various plasma pulses. Due to the drifting phase offset caused by the optical plates, the fringe pattern may actually be different for every pulse. The laser calibration system provides resolution here, as it can be used to offset the phase drift of the cold plasma measurements, so that they can be averaged correctly.

This method, while involving a few caveats, is promising. It could potentially provide resolution to the 2013 KSTAR multiple delay CXRS campaign, although not many cold plasma pulses are recorded.
7. Conclusion

Intro

2-Dimensional coherence imaging Charge eXchange Recombination Spectroscopy (2D CXRS) is a novel technique for measuring CXRS on hot plasma fusion reactors, on numerous ($\sim 10^4$) points across a large field of view.

In this report, we attempted to expand the 2D CXRS systems, by measuring multiple points in the interferogram at different delays in a single frame, as opposed to the single point measurement more commonly used for 2D CXRS. The additional information contained in the different delays should allow for non-Gaussian spectra to be measured. The non-Gaussian model spectrum is based on the data of the standard CXRS system at KSTAR, and consists of active and passive CXRS signals, as well as background Bremsstrahlung.

The multiple delay principle, the design and construction of the physical devices, the calibration methods and the data analysis are all worked out in this report.

Multiple Delay 2D CXRS simulations

Simulations of the multiple delay 2D CXRS systems are preformed to test the fundamental principle: a simulated plasma featuring the non-Gaussian model spectrum was virtually measured, the different delays separated, the interferometric contrasts and phases obtained, and the spectrum fitted using a custom build listfit method. The listfit function is optimised to extract the temperature and the rotational velocity of the active CXRS signal. Although it wasn’t the focus of this report, impurity density can also be measured using these systems. The results (obtained in Ch. 5) show that the multiple delays indeed allow for the reconstruction of the non-Gaussian model spectrum.

It was found that for typical plasma parameters, and for 95% of the pixels in the frame, the original input plasma parameters can be recovered with an absolute error less than 0.25 keV for the active temperature, and less than 25 km/s for the active velocity component.

Furthermore, 68% of the pixels could be fitted with an absolute error less than 0.1 keV for the active temperature, and less than 10 km/s for the active velocity component.

It was found that due to the nature of fitting problem, any systematic errors could quickly reduce the accuracy of the fit, and even cause the fit to not resolve with systematic noise larger than 3%.

The devices, and application on KSTAR

Two coherence imaging systems were developed, each capable of measuring multiple delays in the spectrum’s interferogram. Both systems use a different method to obtain the different delays. The Fourier separation system has a higher resolution, while the Image separation system has a higher dynamic range. The Fourier separation system was build in the lab, and tested on the KSTAR tokamak. The Image separation system was also trailed in the lab, but due to a shortage of components, was not tested at KSTAR.
Unfortunately, we were unable to turn the KSTAR measurements into meaningful CXRS data, as no fit was able to resolve. The interferometric contrast and phase for the multiple delays could be obtained, and various attempts were made to reduce the systematic noise in an attempt to improve the fitting process, but none of the spectrum fits resolved.

One difficulty of CXRS systems is that they can’t easily be tested outside of a fusion reactor environment, as this is the only device capable of producing the hot conditions necessary to generate broad CXRS emissions. Therefore, there is limited testing that can be performed in the lab, and most of the tests and calibrations have to take place at a tokamak, where neutron bombardment, limited operation time, and complex plasma flows make for a non-ideal testing environment. It is suspected that either a faulty calibration, or an error in the assembly of the system is the cause for the non-resolving fit. Potential solutions to overcome these problems in future operations are given (Chapter 6.4)

The design of the multiple delay coherence imaging systems, and their proven-via-simulation capability to unravel non-Gaussian spectra, and the solutions given to the problems uncovered in this report provide a good framework for a future multiple delay coherence imaging campaign, despite, the currently unresolved KSTAR data.
8. Appendix: Birefringent Optical Plates

8.1. Introduction Birefringent Optical Plates
This section details the birefringent plates that are used in this report. Birefringent plates are optical components designed to have certain properties. They are constructed from a slice of birefringent material or formed by a compound of such slices.

Birefringent materials are defined by having anisotropic refraction. In most common materials, including the ones discussed in this report, the anisotropy is uniaxial. This means that the material can be described as having an optical axis with two refractive indexes: The ordinary refractive index \( n_o \), and the extraordinary refractive index \( n_e \). The birefringence \( B \) of a material is defined by:

\[
\Delta n = n_e - n_o.
\]

Just as for non-birefringent optics, empirical Sellmeier equations exist to model the refractive indexes of birefringent materials. Sellmeier equations relate the refraction index with the wavelength of a ray, using material dependent Sellmeier constants. Two such equations are needed for birefringent materials, to model both the \( n_o \) and \( n_e \). Extensions of the Sellmeier equation often exist to include temperature (or pressure) effects on the refractive index.

If unpolarized light enters the birefringent material it will be unraveled into two orthogonal linear polarization states. One component will have its polarization perpendicular to the optical axis, and undergo standard (‘ordinary’) refraction, with refraction index \( n_o \). The other (‘extraordinary’) component will refract at a different angle, depending on the orientation of the optical axis, the angle of incidence, and the birefringence.

For a birefringent plate, with two parallel surfaces, the input and output rays are always parallel to each other. The use of the optical plates are defined by the polarization states, displacement, and phase difference of the exiting rays. Based on these properties, plates can be grouped into classes. A few of these classes are:

- Half wave plate: Changes the direction of polarization
- Delay plate: generates a phase shift
- Displacer plate: generates a displacement and a phase shift
- Savart plate: generates a displacement

The listed description is the primary function of the plate, but certainly not the only one. E.g. a Savart plate will also have a small phase shift, that is dependent on the direction of the input ray.

The construction of these different types are detailed below. Crucial in understanding the differences of the plates, is the phase shift formula stated in the next section.

8.2. Phase Shift Formula
The phase difference between the two orthogonally polarized output rays can be determined by a formula derived by Veiras et al. published in 2010.[16] The formula describes the phase difference \( \Delta \phi \)
for any ray with wavelength $\lambda_v$, described by $\alpha$ and $\delta$, respectively the angle with the surface normal and the angle between the optical axis and the ray, projected onto the surface. The formula applied to an uniaxial parallel plate with thickness $L$, with $\theta$ the angle between the optical axis and the surface plane. $n_i$ is the refractive index of the external media. Typically for air, $n_i \approx 1$.

\[
\Delta \phi = \frac{2\pi L}{\lambda_v} \left( (n_o^2 - n_i^2 \sin^2 \alpha)^{\frac{1}{2}} + \frac{n_i(n_o^2 - n_e^2) \sin \theta \cos \theta \cos \delta \sin \alpha}{n_e^2 \sin^2 \theta + n_o^2 \cos^2 \theta} \right.
\]
\[
+ \frac{-n_o(n_e^2 \sin^2 \theta + n_o^2 \cos^2 \theta) - [n_e^2 - (n_e^2 - n_o^2) \cos^2 \theta \sin^2 \delta]n_i^2 \sin^2 \alpha}{n_e^2 \sin^2 \theta + n_o^2 \cos^2 \theta})^{\frac{1}{2}} \right)
\]

This formula will be applied on the birefringent plates discussed below.

8.3. Waveplates and Delay Plates

Waveplates, or retarders have an the optical axis is in the plane of the surface. An unpolarized ray of light will be unraveled along a fast axis and a slow axis. Whether the ordinary ray, perpendicular to the optical axis, or the extraordinary ray, in plane with the optical axis constitutes for the fast or slow axis, depends on the sign of the birefringence $B$. Outside of second order effects (see [16]), there are no significant spatial displacements. In first order, a waveplate only induces a phase shift.

Knowing that $\theta = 0^\circ$, the phase shift formula becomes:

\[
\Delta \phi_{\text{delay}} = \frac{2\pi L}{\lambda_v} \left( (n_o^2 - n_i^2 \sin^2 \alpha)^{\frac{1}{2}} - \frac{n_e^2 n_o^2 - [n_e^2 - (n_e^2 - n_o^2) \sin^2 \delta]n_i^2 \sin^2 \alpha}{n_o} \right)
\]

For a ray coming in perpendicular to the surface, $\alpha = 0^\circ$, the phase difference between the two component rays becomes:

\[
\Delta \phi_{\text{Delay,\perp}} = \frac{2\pi L}{\lambda_v} (n_o - n_e)
\]

Which matches the intuitive idea of the phase difference between two rays moving in two different materials with $n_o$ and $n_e$ as respective refractive indexes.

The phase shift of a waveplate determines the function the plate:

- if the phase shift measures in whole $2\pi$ radians, it is called a delay plate. The only prominent effect is the delay between the two rays.
- If the phase shift is not an integer multiplication of $2\pi$, the EM fields of the resulting rays will combine in a way that alters the polarisation state.

In the case where the phase shift is $\pi$, the plate is called a Half Wave Plate (HWP). When linearly polarised light passes a HWP, it has the effect of rotating the polarisation direction with $2\beta$ degrees, where $\beta$ is the angle between the polarisation direction and the fast axis ($\beta$ is equal to $\delta$, in case the fast
axis is parallel with the optical axis, that is to say the extraordinary ray is the fast axis, or \( n_e < n_o \), or \( B < 0 \). Otherwise \( \beta = \delta \pm 90^\circ \). If the phase shift is \( \frac{\pi}{2} \), the plate is called a quarter wave plate, and has the ability to turn linearly polarised light into circular polarised light, and vice versa.

A zero-order HWP has a phase difference of only \( \pi \), as opposed to \( \pi + k2\pi \) with \( k > 0 \). These are generally preferred, due to their increased temperature and wavelength stability. A single crystal zero-order plate would be unpractically thin. They are hence constructed by combining two waveplates with a 90 degree rotation, such that the effective phase shift is the difference between the two plates.

### 8.4. Field Widened Delay Plates

The term Field Widened (FW) refers to the fact that optical plate is less dependent on \( \delta \) when rays have a nonzero angle of incidence, \( \alpha \). This means the plate induces less aberrations for systems with a large FOV, or otherwise large angles of incidence. It’s achieved by combining plates in such a way that some angle dependent effect cancel each other. One mathematically clean example of this is the FW Savart Plate, which follows later, but the trick can also be applied to delay plates.

The setup of a FW delay plate is a compound of a standard delay plate, a HWP at 45°, and an identical delay plate, but rotated 90°. The o-ray of the first plate will be rotated 90° by the HWP, and hence it will be the o-ray of the second delay plate as well. The total delays hence add up, while some secondary non-trivial aberrations are reduced. This can be seen by the phase shift formula:

\[
\Delta \phi_{FW delay} = \Delta \phi_{delay}(\alpha, \delta) + \Delta \phi_{delay}(\alpha, \delta + 90^\circ)
\]

\[
\Delta \phi_{FW delay} = \frac{2\pi L}{\lambda_v} \left( 2(n_o^2 - n_t^2 \sin^2 \alpha) - \frac{\{n_e^2n_o^2 - [n_e^2 - (n_e^2 - n_o^2) \sin^2 \delta]n_t^2 \sin^2 \alpha\}^2}{n_o} \right.
\]

\[
\left. - \frac{\{n_e^2n_o^2 - [n_e^2 - (n_e^2 - n_o^2) \sin^2(\delta + 90^\circ)]n_t^2 \sin^2 \alpha\}^2}{n_o} \right)
\]

\( L \) stands for the thickness of each plate. This can be written as:

\[
\Delta \phi_{FW delay} = \frac{2\pi L}{\lambda_v} \left( 2n_o \sqrt{1 - b} - n_e \left( \sqrt{1 - [1 - a \sin^2 \delta]b} + \sqrt{1 - [1 - a \cos^2 \delta]b} \right) \right)
\]

Where \( a = (1 - \frac{n_o^2}{n_e^2}) \), \( b = \frac{n_t^2}{n_o^2} \sin^2 \alpha \). Assuming small angles, a Taylor expansion can be done for \( b \ll 1 \). The result is:

\[
\Delta \phi_{FW delay} = \frac{2\pi L}{\lambda_v} \left( (2 + b)(n_o - n_e) - \frac{1}{2} n_e ab \right) + O(b^2)
\]

Which shows that, in first order, \( \delta \) dependence has disappeared.

For a ray coming in normal to the surface, \( \alpha = 0^\circ \), the phase difference between the two component rays becomes the sum of the two delay plates:
\[
\Delta \phi_{FW \text{ delay} \perp} = \frac{4\pi L}{\lambda_v} (n_o - n_e)
\]

### 8.5. Displacer Plates

A displacer is a birefringent plate which has the optical axis at an angle (\(\theta\)) between 0° and 90° to the surface plane (at 0° it would be a standard waveplate plate). The two output rays will have a different path length through the plate (delay of the rays), and an spatial offset (displacement of the rays), see Fig. 40. All the displacers discussed in this report have its optical axis at 45° to the surface, as it maximizes the displacement. The displacement and path length difference are proportional to the thickness of the plate.

![Displacer Plate](image)

*Fig. 40: A displacer plate. The output rays are displaced, and have a different path length. The optical axis shown in the plate is at 45 degrees to the surface normal, resulting a maximal displacement.*

For a displacer plate \(\theta = 45^\circ\) or sin \(\theta = \cos \theta = \frac{1}{\sqrt{2}}\), the phase shift formula becomes:

\[
\Delta \phi_{\text{Displacer}} = \frac{2\pi L}{\lambda_v} \left( (n_o^2 - n_i^2 \sin^2 \alpha)^{\frac{1}{2}} + \frac{n_i (n_o^2 - n_e^2) \cos \delta \sin \alpha}{n_e^2 + n_o^2} \right.
\]

\[
+ \left. -n_o \left[ 2n_e^2(n_e^2 + n_o^2) - 2(2n_e^2 - (n_e^2 - n_o^2) \sin^2 \delta) n_i^2 \sin^2 \alpha \right]^{\frac{1}{2}} \right)
\]

For a ray along the surface normal, \(\alpha = 0\), the phase shift is non-zero:

\[
\Delta \phi_{\text{Displacer}, \perp} = \frac{2\pi L}{\lambda_v} \left( n_o - n_o n_e \sqrt{\frac{2}{n_e^2 + n_o^2}} \right)
\]

A displacer hence induces both a phase shift and a displacement.

### 8.6. Savart Plates

A Savart plate is a compound of two identical displacer plates, with one rotated 90 degrees (Fig. 41). In this setup the delays (phase shifts) cancel out for rays normal to the surface, but a displacement remains. If \(d\) is the displacement of the displacer, then the Savart will have a displacement of \(d \sqrt{2}\).
Fig. 41: Front and Top view of a Savart plate, consisting of two displacers, rotated by 90 degrees. The output rays are displaced, but the phase shift is zero for the surface normal ray.

The phase shift formula becomes:

\[
\Delta \phi_{\text{Savart}}(\alpha, \delta) = \Delta \phi_{\text{Displacer}}(\alpha, \delta) - \Delta \phi_{\text{Displacer}}(\alpha, \delta + 90^\circ)
\]

The minus sign is due to the fact that the o-ray from the first displacer becomes the e-ray of the second displacer, and vice versa. This gives:

\[
\Delta \phi_{\text{Savart}} = \frac{2\pi L}{\lambda_v} \left( \frac{n_i(n_o^2 - n_e^2) \sin \alpha}{n_e^2 + n_o^2} (\cos \delta - \sin \delta) + \frac{-n_o[2n_e^2(n_e^2 + n_o^2) - 2[2n_e^2 - (n_e^2 - n_o^2) \sin^2 \delta]n_i^2 \sin^2 \alpha]^1}{n_e^2 + n_o^2} \right)
\]

For a ray along the surface normal, \( \alpha = 0 \), the phase shift disappears: \( \Delta \phi_{\text{Savart,\perp}} = 0 \).

8.7. Field Widened Savart Plates

As was stated for the FW delay plate, the term Field Widened refers to the fact that optical plate is less dependent on \( \delta \), when rays have a nonzero angle of incidence, \( \alpha \), indicating that the plate induces less aberrations for systems with a large FOV. A FW Savart hence has the same primary functions as a Savart plate (A displacement, with 0 phase shift along the surface normal) with typically better performance.

A FW Savart is constructed as such: A displacer, a HWP at 45 degrees, and an identical displacer at 180 degrees. Compared to the Savart plate, the second displacer is rotated an extra 90 degrees, but in return the HWP ensures that the rays are both once ordinary in one displacer, and extraordinary in the other, ensuring the phase shift across the surface normal is still zero, as visible in Fig. 42.
Fig. 42: A FW Savart plate, consisting of a HWP (45 degree rotation to the plane of the image), sandwiched between two identical displacers, rotated 180 degrees. As with a Savart, the output rays are displaced, but the phase shift is zero for the surface normal ray. The HWP rotates the polarisation direction by 90 degrees.

The phase shift formula for the FW Savart is:

$$\Delta \phi_{FW \text{ Savart}}(\alpha, \delta) = \Delta \phi_{\text{Displacer}}(\alpha, \delta) - \Delta \phi_{\text{Displacer}}(\alpha, \delta + 180^\circ)$$

Again, the minus sign is due to the fact that the o-ray from the first displacer becomes the e-ray of the second displacer, and vice versa. This results in the remarkably concise formula:

$$\Delta \phi_{FW \text{ Savart}} = \frac{2\pi L}{\lambda_v} \left( \frac{2n_l(n_o^2 - n_e^2)\cos \delta \sin \alpha}{n_e^2 + n_o^2} \right)$$

The phase shift disappears ($\Delta \phi_{FW \text{ Savart}, \perp} = 0$) for a ray parallel to the surface normal ($\alpha = 0$) as expected.

The beauty of this equation is even more apparent when written in the Cartesian coordinates $(x, y)$ of the image plane. It is easiest to first convert $(\delta, \alpha)$ into polar coordinates $(r, \theta)$. It follows that on the image plane, $r = \sin \alpha$, and $\theta = \delta$. In Cartesian coordinates then, $x = r \cos(\theta)$, and $y = r \sin(\theta)$. The equation is hence:

$$\Delta \phi_{FW \text{ Savart}} = \frac{2\pi L}{\lambda_v} \left( \frac{2n_l(n_o^2 - n_e^2)}{n_e^2 + n_o^2} \right) x$$

The phase shift is linearly dependent of $x$, and independent of $y$! This results in fringes that are both straight, and equally spaced. These plates are hence convenient to use for interferometry.
9. References


