MASTER

EIT spectroscopy on rubidium Rydberg states

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EIT spectroscopy on rubidium Rydberg states

Master Thesis

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Abstract

Rubidium Rydberg atoms can potentially be used as building blocks for a quantum simulator. The Rydberg atoms are created in a two-step excitation process. First, rubidium in the $5S_{1/2}$ ground state is excited to the $5P_{3/2}$ state with a probe laser beam and then the rubidium atoms are excited to either an $nD_{5/2}$, $nD_{3/2}$ or $nS_{1/2}$ Rydberg state, with a coupling laser beam. Frequency stabilization of the coupling laser requires an atomic reference signal and is essential for reproducible Rydberg experiments. To obtain such a signal, a spectroscopy setup based on electromagnetically induced transparency (EIT) is built. Here, resonance of the coupling laser beam results in an enhanced transmission of the probe laser beam through a vapor cell.

To predict the shape and magnitude of experimental EIT spectroscopy signals, a semi-analytical model based on the optical Bloch equations is used. These equations describe the interaction between an atomic three-level system with two coherent light fields and take decoherence mechanisms into account. The second stage of the model is to solve the steady state solution of the optical Bloch equations for the density matrix element which describes the coherence between ground state and the excited state. This matrix element is related to the electrical susceptibility of the vapor cell. The third stage of the model is to include a velocity distribution for the rubidium atoms to include thermal effects. The imaginary part of the electrical susceptibility is used to calculate the transmission of the probe laser beam through the vapor cell.

A setup is built to measure EIT spectroscopy signals in a rubidium vapor cell. The transmission of a resonant probe laser beam through a vapor cell is measured while the wavelength of the coupling laser is scanned. Because the expected signal to background ratio of an EIT signal is about 5%, modulation techniques are implemented to enhance the signal to background and noise ratio. Amplitude modulation (AM) with a modulation frequency of 2.5kHz of the coupling laser beam in combination with a lock-in amplifier is used to measure characteristics of the signal. With this technique experimental signals of 40D, 41D, 42S and 43S Rydberg lines are obtained. The line widths and relative intensities of the experimentally obtained signals are in good agreement with the theoretical predictions.

Drawback of the AM technique is that it can not be used to lock the coupling laser to the reference signal. A potential faster technique is to use a Pound Drever Hall scheme. Therefore frequency modulation (FM) of the probe laser beam is needed. The probe laser beam is modulated with an electro optical modulator to create sidebands at $\pm 20$MHz of the main frequency. Despite the proven presence of side bands on the probe laser beam, no experimental signal is obtained using this FM technique.

We conclude that it is possible to find atomic resonance signals as a reference for the coupling laser. The signals obtained with amplitude modulation experiments are in good agreement with the theoretical predictions. Direct locking of the coupling laser to these AM signals is however not possible and FM spectroscopy signals are not obtained. Therefore using EIT spectroscopy as a technique to lock the coupling laser to the atomic transition did not work.
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Chapter 1

Introduction

1.1 Quantum information with Rydberg atoms

Physics is about understanding how the world works. Physicists develop models and put them to the test by simulating physical processes and compare the outcome of those simulations with experiments. This way knowledge is gained about how processes in nature work. With the obtained knowledge, physical processes can be understood and predicted and systems can be designed that behave as expected. Examples can be found in everyday life. A microwave oven heats up your food, an LED emits visible light and mobile phones enable you to speak to someone on the other side of the world. Key to inventing these products and technologies is having a model that describes the physics that is involved. Most of these models need to be translated to discrete mathematics to calculate the expected outcome of experiments with a classical computer.

But a classical computer has some limitations; a lot of technologies work with principles that cannot be understood without quantum mechanics \[1, 17\]. And in general quantum mechanical problems cannot be calculated fully with a classical computer, as Richard Feynman pointed out already in 1981:

> “And I’m not happy with all the analyses that go with just the classical theory, because nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical” - Richard Feynman, 1981. \[1\]

Technologies based on quantum systems are difficult to simulate with a classical computer. Equations that describe quantum mechanical processes can look quite simple, but a numerical solution is often hard to calculate, especially if the number of degrees of freedom becomes large. For example: simulating a quantum system containing \(N\) interacting \(s = 1/2\) spins would require evaluating a vector with \(2^N\) components. For systems where \(N\) is large, it becomes impossible to take all the interactions into account. Although it is often possible to make approximations and still get valuable results, it can be useful to have the option to simulate a system, taking all the interactions between particles into account \[1, 2\].

Therefore it would be useful to have another quantum system with similar properties that can be used to simulate the original system: a quantum simulator \[1\]. Such a system can help answering questions in a range of exciting fields such as high temperature super-conductors, complex spin systems and quantum phase transitions. A quantum simulator is a quantum system with the properties that it is possible to map other quantum systems on it; it is easier to prepare and control than the system that is to be simulated and it is easy to read out \[17\]. A lattice of interacting Rydberg atoms is a candidate for such a quantum simulator. These highly excited atoms have strong interactions over distances up to micrometers and are highly tunable \[3\].

Also for certain classical problems, for example the factorization of very large numbers, quantum
CHAPTER 1. INTRODUCTION

information can be used, not in the form of a quantum simulator, but in the form of a quantum computer [2]. Quantum computers can in theory be much faster and more efficient than classical computers, but quantum computers need quantum algorithms and quantum gates. These algorithms and gates work on units of quantum information: qubits, which are the quantum mechanical equivalent of a classical bit. In contrast with a classical bit, which can be either 0 or 1, a qubit can be in a superposition of 0 and 1 and multiple qubits can be entangled [2]. One of the building blocks needed for a quantum computer is a cNOT gate. This gate can be used to entangle or disentangle qubits. Rydberg atoms can be used to build such a cNOT gate [17].

Besides applications in quantum information, Rydberg atoms are also of interests for network theorists. For example, random access wireless networks and interacting Rydberg atoms show a similarity as both a Rydberg atom as a wireless network access point can block neighbors from being ‘active’ [18].

Rydberg atoms are atoms with their valence electron in a highly excited electronic state, or in other words: atoms with a high principal quantum number \( n \) [10]. This high principal quantum number corresponds with a large orbital radius of the valence electron. Therefore a Rydberg atom can be seen as a very large atom. Many properties of Rydberg atoms scale with \( n \). A few of these properties are given in table 1.1 [3, 5, 10, 15, 8].

<table>
<thead>
<tr>
<th>Property</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital radius of valence electron</td>
<td>( n^2 )</td>
</tr>
<tr>
<td>Transition dipole moment</td>
<td>( n^{-3/2} )</td>
</tr>
<tr>
<td>Polarizability</td>
<td>( n^7 )</td>
</tr>
<tr>
<td>Van der Waals interaction strength coefficient</td>
<td>( n^{11} )</td>
</tr>
<tr>
<td>Lifetime</td>
<td>( n^3 )</td>
</tr>
<tr>
<td>Binding Energy</td>
<td>( n^{-2} )</td>
</tr>
<tr>
<td>Energy Separation between levels</td>
<td>( n^{-3} )</td>
</tr>
</tbody>
</table>

Table 1.1: Scaling properties of Rydberg atoms

In our setup Rydberg atoms will be created in a two step excitation process. Because of their property to have only one valence electron alkali atoms are suitable atoms for Rydberg experiments. In our setup rubidium is used, the rubidium is first laser cooled to a few \( \mu \)K [24, 23]. After cooling the rubidium atoms in a magneto optical trap (MOT), the atoms are first excited from the \( |5S_{1/2}, F = 3\rangle \) ground state to the \( |5P_{3/2}, F = 4\rangle \) intermediate state with a 780 nm laser beam that we will call the probe laser. This transition is also used in the MOT to cool the atoms. From this intermediate state the atoms are excited to a Rydberg state with a 480 nm laser beam that we call the coupling laser [3].

Preliminary experiments in the MOT are already performed [27]. The problem with these experiments was that the 480 nm laser was not frequency stabilized. This makes it difficult to perform experiments and makes the results hard to reproduce as the exact experimental conditions are not known. Therefore it is important to stabilize the frequency of the coupling laser, so it is resonant with the transition between the intermediate excited state and a Rydberg state. An important step in the frequency stabilization is the detection of the atomic transition as a function of the laser frequency. Therefore a spectroscopy setup is built. The spectroscopy technique that is used is called electromagnetically induced transparency (EIT). The main goal of the work performed for this thesis is to build and test an EIT spectroscopy setup.

Electromagnetically induced transparency means that electromagnetic radiation (light) changes the electromagnetic properties of a medium such that the medium becomes transparent. This name is somewhat misleading; the effect happens only for a very narrow region of the electromagnetic spectrum [9]. Therefore this technique can not be used to make people, cars or complete buildings transparent or invisible, but it can be used to make a cloud of atoms invisible for a laser that would normally be absorbed by those atoms. EIT needs two lasers: the first laser, the probe laser, couples an atomic ground state to an excited state and the second laser, the coupling laser, couples the excited state to a third state. This can be a Rydberg state, but
it can also be another ground state [9]. EIT can only happen if both the lasers are nearly resonant and if two photons, one of each laser, have the same energy as the energy difference of both transitions. Because of this property EIT spectroscopy can be used as a tool to find high Rydberg states provided that the laser that excites the rubidium from the ground state to the intermediate state is locked to the atomic resonance [6].

The goal of this master project is to build and test an EIT spectroscopy setup. This setup should provide an atomic reference signal for the coupling laser. Further the goal is to find if the setup can be used to lock the coupling laser to a Rydberg line.

1.2 This Thesis

This thesis consists of five chapters. In the first chapter (Introduction) the background of the research project is explained and the two most important concepts of the physics in this thesis are introduced: Rydberg atoms and EIT. In the second chapter we will dive into the interaction between atoms and light to understand how EIT works. We will start by understanding the similarities of rubidium Rydberg atoms and the hydrogen atom. Then we take a two level atom into account that interacts with one laser. From that basis we will add a third level and a second laser to and study the interactions. Finally we take the motion of atoms into account. This physics is put in a semi analytical model to predict the line shape and relative line strengths and line widths of EIT spectroscopy signals. In the third chapter the spectroscopy setup will be explained as well as the setup with the MOT. After this chapter we will see that EIT signals are measured and can be understood with our semi analytical model that is derived in chapter 2. In the fifth chapter we will discuss some challenges that have yet to be solved and present an outlook for finalizing the stabilization of the coupling laser and the Rydberg experiments that can be performed with a stable coupling laser. Finally we propose an outlook for the experiments of the Rydberg project.
Chapter 2

Electromagnetically induced transparency

In this chapter a semi-analytical model for electromagnetically induced transparency (EIT) is derived. The model is dedicated to EIT in a rubidium vapor cell were the rubidium \( |5S_{1/2}, F = 3 \rangle \) state is taken as a ground state, the \( |5P_{3/2}, F = 4 \rangle \) state is taken as an intermediate state and an \( S \) or \( D \) Rydberg state is taken as the third state in a three level atomic system. First the energy of a rubidium atom in a Rydberg state is calculated as function of the principal quantum number. Then, atom-light interactions are derived for a two-level atom. This leads to an understanding of the absorption of a resonant laser beam in a gas reference cell. After the two-level atom, the interaction between a three level atomic system with coherent light fields is taken into account which lead to EIT. Finally thermal effects are taken into account.

2.1 Atoms

In our experiments the element rubidium is used. Rubidium is an alkali metal: part of the atomic species in group I of the periodic system of elements. When alkali atoms are excited to Rydberg levels they obey similar scaling laws as hydrogen. Because of this, and because hydrogen is the simplest atom, it is instructive to take a closer look at the hydrogen atom. This will allow us to introduce Hartree atomic units. After the description of the hydrogen atom we will discuss what is similar and what is different for rubidium atoms.

2.1.1 The hydrogen atom

We start with the atomic structure of hydrogen. A hydrogen atom consists of two parts, a positive nucleus and a negative electron. Because the nucleus is much heavier than the electron, this system is similar to a planet orbiting a star. For the ground state of hydrogen the average distance between the electron and the proton is called the Bohr radius \( a_0 \). In atomic physics the value of the Bohr radius,

\[
a_0 = \frac{(4\pi\varepsilon_0)\hbar^2}{me^2} = 5.29177 \times 10^{-11} \text{m},
\]

is often taken as the unit of length: \( a_0 = 1 \). Besides this unit of length, other properties can also be expressed in so called Hartree atomic units or just atomic units (a.u.): the reduced Planck constant \( \hbar = 1 \) and the elementary charge \( e = 1 \). These unit values make atomic units easy to work with. In table 2.1 the atomic units that are used in this thesis are given. The fine structure constant \( \alpha \) is a dimensionless constant defined by the physical constants of elementary charge, the permittivity of vacuum, the reduced Planck constant and the speed of light \( c \):
CHAPTER 2. ELECTROMAGNETICALLY INDUCED TRANSPARENCY

Table 2.1: Atomic units versus SI units

<table>
<thead>
<tr>
<th>property</th>
<th>atomic unit</th>
<th>value in a.u.</th>
<th>value in SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance</td>
<td>Bohr radius $a_0$</td>
<td>1</td>
<td>$5.29177 \times 10^{-11}$ m</td>
</tr>
<tr>
<td>charge</td>
<td>elementary charge $e$</td>
<td>1</td>
<td>$1.602 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>permittivity</td>
<td>permittivity of the vacuum $\epsilon_0$</td>
<td>1/4\pi</td>
<td>$8.854 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>angular momentum</td>
<td>reduced Planck constant $\hbar$</td>
<td>1</td>
<td>$1.055 \times 10^{-34}$ m^2 kg/s</td>
</tr>
<tr>
<td>speed of light $c$</td>
<td>fine structure constant</td>
<td>137</td>
<td>$2.998 \times 10^8$ m/s</td>
</tr>
</tbody>
</table>

\[
\alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} \approx 1/137, \tag{2.2}
\]

it has this numerical value in all systems. The fine structure constant characterises the strength of the electromagnetic interaction between elementary charged particles.

In the hydrogen atom the electron and proton are bound by the Coulomb force. The allowed energetic levels are the eigenvalues of the Hamiltonian; to find them we have to solve the stationary Schrödinger equation [16]

\[
\hat{H}\Psi = E\Psi; \tag{2.3}
\]

with the wave function $\Psi$, element of the Hilbert space, $E$ the energy and the Hamiltonian $\hat{H}$ which in atomic units is given by

\[
\hat{H} = -\frac{\nabla^2}{2} - \frac{1}{r}, \tag{2.4}
\]

where the distance between the electron and the nucleus is given by the radius $r$. In the Schrödinger equation the wave function can be separated in a radial and a angular part, where the radial part is only dependent on $r$ and the angular part is dependent on the polar angle $\theta$ and the azimuthal angle $\phi$:

\[
\Psi = R(r)Y(\theta, \phi). \tag{2.5}
\]

The solutions that represent the radial and angular part of the wave function are well known for hydrogen. They can be described by using Laguerre $L_n^2(x)$ and Legendre $P_l^m(\cos\theta)$ polynomials in terms of the spherical coordinates $r, \theta, \phi$ and the quantum numbers $n, l$ and $m$. The quantum number $n$ is known as the principal quantum number and is related to the distance between the electron and the nucleus, the quantum number $l$ is related to the angular momentum and the quantum number $m$ is related to the projection of angular momentum on the z-axis. In general, only the solutions with the quantum numbers $n$ and $l$ real and integer exist. Moreover, $l$ has to be between zero and $n-1$ and $m$ must have a value between $l$ and $-l$. These solutions may seem rather complicated with the polynomials, but the hydrogen system is one of the few real systems that can be fully solved analytically:

\[
R_{nl}(r) = r^l L_{n-l-1}^{2l+1}(r)e^{r/2}, \tag{2.6}
\]

\[
Y_{lm}(\theta, \phi) = P_l^m(\cos\theta)e^{im\phi}. \tag{2.7}
\]

The eigenvalues of the Hamiltonian are the allowed energies for atomic states. These energies are only dependent on the principal quantum number $n$ and are given by:

\[
E_n = -Ry \frac{1}{n^2}, \tag{2.8}
\]

with $Ry$ the Rydberg constant defined as $Ry = \frac{1}{2}$ in atomic units, or in SI units:

\[
Ry = \frac{m_e c^2 \alpha^2}{2} = \frac{m_e c^2 \alpha^2}{2}. \tag{2.9}
\]

EIT spectroscopy on rubidium Rydberg states
2.1.2 Rubidium Rydberg atoms

Alkali atoms are the atoms in group I of the periodic system of elements. These elements are lithium (Li), sodium (Na), potassium (K), rubidium (Rb), cesium (Cs) and francium (Fr). Like hydrogen these atoms have one valence electron in their outer shell, therefore the energy levels scale similar to hydrogen. The energy levels are not completely the same, to see what is different we take a closer look to the interaction with the valence electron and the rest of the atom. The core electrons of alkali atoms fill the inner shells entirely and will remain almost unperturbed by the valence electron. Therefore it is not necessary to solve the complete system of inner electrons, the nucleus and the valence electron. The Hamiltonian for the valence electron can be written in atomic units as:

$$\hat{H}_\alpha = -\frac{1}{2} \nabla^2 + V_{\text{core}}(r) + \frac{\alpha^2}{r^3} \hat{L} \cdot \hat{S}. \quad (2.10)$$

The first term on the right hand side of the equation is the kinetic energy operator that is the same as for hydrogen. The second term is a core potential $V_{\text{core}}$ which is experienced by the valence electron due to the interaction with the nucleus and the core electrons; this core potential is spherically symmetric. The third term on the right hand side of the equation is the spin-orbit coupling. This term contains the fine structure constant $\alpha$, the distance between the core and the valence electron $r$ and the orbital $\hat{L}$ and spin $\hat{S}$ angular momentum. As hydrogen also has a nuclear spin the difference between hydrogen and alkali Rydberg atoms in the term $V_{\text{core}}(r)$. Because of the last term, containing both spin and orbital angular momentum, the valence electron has no longer both a definite orbital angular momentum state and a definite spin angular momentum state. Therefore another quantum number is needed, $j$, which is associated with the eigenvalue of the operator $\hat{J}^2$ were $\hat{J} = \hat{L} + \hat{S}$ is the total angular momentum operator. Now certain properties can be written in terms of the quantum numbers $n, l$ and $j$. For example the energies of atomic states can be written in terms of the quantum number $n, l$ and $j$:\![3]:

$$E_{nlj} = -\frac{Ry^*}{(n - \delta_{nlj})^2}, \quad (2.11)$$

where the Rydberg constant for rubidium in atomic units, $Ry^* = 0.4999673250$ is different from the Rydberg constant for hydrogen, $Ry = 0.5$, and the quantum defect $\delta_{nlj}$ can be calculated with the Rydberg-Ritz formula [3]:

$$\delta_{nlj} = \delta_{0j} + \frac{\delta_2}{(n - \delta_{0j})^2} + \frac{\delta_4}{(n - \delta_{0j})^4} + ... \quad (2.12)$$

The parameters $\delta_0, \delta_2, \delta_4, ...$ have to be determined experimentally as calculating them would involve solving the complete many body system of a rubidium atom. This term depends on the quantum number $l$ as this term is correlated with the probability that the valence electron can be found in the core of the atom. For the first values of $l$, $\delta_{0j}$ is found to be [3] $\delta_0(l = 0) = 3.13, \delta_0(l = 1) = 2.65, \delta_0(l = 2) = 1.35, \delta_0(l = 3) = 0.016$; for $l > 3$ the quantum defect is small enough to neglect, further we ignored $j$ dependence here. Further there is the hyperfine splitting, which do not occur for high lying Rydberg states but show up in the lower states. These are denoted by the quantum number $F$.

Besides the scaling of the rubidium Rydberg energies it is useful to include some general properties of rubidium. Some general properties of Rb$^{85}$ are given in table 2.2 [14].
2.2 Atom-light interactions in two-level atomic systems

The solution of the Schrödinger equation allows only energetic states that have discrete values. For an atom to undergo a transition to a higher state, energy from the environment is needed. This energy can come from collisions with other atoms or from radiation. We will now focus on controlled transitions using electromagnetic radiation, i.e. photons, as energy carriers to excite atoms to a higher state. We start by looking at an atom that can be in two states, a ground state \( |g\rangle \) and an excited state \( |e\rangle \). The energy difference between \( |e\rangle \) and \( |g\rangle \) is \( \hbar \omega_e \), with \( \omega_e \) the atomic transition frequency. A laser is used to couple the atomic states [13]. A schematic view of this system is given in figure 2.1, the hyperfine levels of both the ground state \( |g\rangle = |5S_{1/2}\rangle \) and the excited state \( |e\rangle = |5P_{3/2}\rangle \) are shown; the transition is chosen because it is a so called closed transition. Due to the selection rules (\( \Delta l = \pm 1 \) and \( \Delta F = 0, \pm 1 \)) all the atoms that are excited will fall back to their original state [13]. While this is true for the ideal case (\( F=3 \), \( F’=4 \)), in practice a repump laser is needed to excite atoms from the \( F=2 \) ground state to the \( F’ = 3 \) excited state because a small fraction of atoms are excited from the \( F=3 \) ground state to the \( F’=3 \) excited state because the \( F’=3 \) and \( F’=4 \) excited states are close together. We will not take this further into account in our theoretical models for two and three level atoms.

In our experiments this transition is used to laser cool the rubidium atoms and excite rubidium to the intermediate state from where the atoms can be excited to a Rydberg state. In this specific case the transition properties are given in table 2.3.

### 2.2.1 Optical Bloch equations

A transition between the ground state and an excited state primarily occurs due to coupling of the electric field of a (near) resonant laser with the electric dipole moment of the atom. Due to spontaneous emission an atom in that excited state will decay to the ground state and emit a photon. The photon is emitted in a random direction. This process can be observed by measuring the intensity of a laser beam that has passed through a cloud of atoms. The laser beam can be seen as a photon reservoir, when one photon from that reservoir is taken to excite an atom this will be detected as a dip in intensity of the laser. As the photons that are emitted by excited atoms travel in a random direction they are not likely to be measured by a photodiode installed to measure the power of the laser. The strength of the coupling between the laser and the atom depends on the matrix element \( d_{eg} \) given by [13]:

\[
d_{eg} = e \langle e | r | g \rangle
\]

### 2.2 Atom-light interactions in two-level atomic systems

The solution of the Schrödinger equation allows only energetic states that have discrete values. For an atom to undergo a transition to a higher state, energy from the environment is needed. This energy can come from collisions with other atoms or from radiation. We will now focus on controlled transitions using electromagnetic radiation, i.e. photons, as energy carriers to excite atoms to a higher state. We start by looking at an atom that can be in two states, a ground state \( |g\rangle \) and an excited state \( |e\rangle \). The energy difference between \( |e\rangle \) and \( |g\rangle \) is \( \hbar \omega_e \), with \( \omega_e \) the atomic transition frequency. A laser is used to couple the atomic states [13]. A schematic view of this system is given in figure 2.1, the hyperfine levels of both the ground state \( |g\rangle = |5S_{1/2}\rangle \) and the excited state \( |e\rangle = |5P_{3/2}\rangle \) are shown; the transition is chosen because it is a so called closed transition. Due to the selection rules (\( \Delta l = \pm 1 \) and \( \Delta F = 0, \pm 1 \)) all the atoms that are excited will fall back to their original state [13]. While this is true for the ideal case (\( F=3 \), \( F’=4 \)), in practice a repump laser is needed to excite atoms from the \( F=2 \) ground state to the \( F’ = 3 \) excited state because a small fraction of atoms are excited from the \( F=3 \) ground state to the \( F’=3 \) excited state because the \( F’=3 \) and \( F’=4 \) excited states are close together. We will not take this further into account in our theoretical models for two and three level atoms.

In our experiments this transition is used to laser cool the rubidium atoms and excite rubidium to the intermediate state from where the atoms can be excited to a Rydberg state. In this specific case the transition properties are given in table 2.3.

#### 2.2.1 Optical Bloch equations

A transition between the ground state and an excited state primarily occurs due to coupling of the electric field of a (near) resonant laser with the electric dipole moment of the atom. Due to spontaneous emission an atom in that excited state will decay to the ground state and emit a photon. The photon is emitted in a random direction. This process can be observed by measuring the intensity of a laser beam that has passed through a cloud of atoms. The laser beam can be seen as a photon reservoir, when one photon from that reservoir is taken to excite an atom this will be detected as a dip in intensity of the laser. As the photons that are emitted by excited atoms travel in a random direction they are not likely to be measured by a photodiode installed to measure the power of the laser. The strength of the coupling between the laser and the atom depends on the matrix element \( d_{eg} \) given by [13]:

\[
d_{eg} = e \langle e | r | g \rangle
\]
Figure 2.1: A schematic view of the Rb$^{85}$ 5$S_{1/2}$ and 5$P_{3/2}$ levels. The two states that will be considered in the description of a level atom are the hyperfine states 5$S_{1/2}$, $F = 3$ as a ground state and 5$P_{3/2}$, $F = 4$ as the excited state. This is a closed transition. The red arrow represents a laser coupling the ground state to the excited state. The energy of the photons in the laser $\hbar \omega_p$ ($\omega_p$ is the laser frequency) is in general slightly different from the difference in energy between the two atomic states $\hbar \omega_e$. This difference is called the detuning $\delta$ and is defined as $\delta = \omega_e - \omega_p$.

where we neglected the effects of polarized light.

For the transition between the rubidium ground state $5S_{1/2}, F = 3 \rightarrow 5P_{3/2}, F = 4$, the value of this transition dipole matrix element in atomic units is 1.956 [14]. For the transition between the intermediate state and an S or D Rydberg state, which will be discussed in more detail later, the transition dipole matrix element can be calculated by separating the it into a radial and spherical part [8]. For large principal quantum number the radial part is given by $d_{rad} \simeq C_1 n^{n^*/2}$ where $n^*$ is the earlier introduced reduced principal quantum number, $C_0 = 4.508 a.u.$, and $C_2 = 8.475 a.u.$ The spherical part of the transition dipole moment is given by $d_{sph} = \sqrt{1/3}$ for $l = 0$ states and $d_{sph} = \sqrt{2/5}$ for $l = 2$ states [8].

The interaction between an atom and a laser is given by the inner product of the laser field with the electric dipole moment of an atom. The electric dipole moment is given by (2.14) were the raising and lowering operators of the atomic dipole for the transition, $\hat{a}^\pm$, are defined as $\hat{a}^+ = |e\rangle \langle g|$, $\hat{a}^- = |g\rangle \langle e|$. Non resonant processes (absorption of a photon with an de-excitation of the atom or emission of a photon with the excitation of an atom) can be removed because they are unlikely to happen. This is called the rotating wave approximation. This gives the interaction Hamiltonian:

$$V_{AL} = \frac{\hbar \Omega}{2} (\hat{a}^- + \hat{a}^+) \quad (2.15)$$

were the Rabi frequency $\Omega$ is introduced. This Rabi frequency is a measure of the coupling strength between the atom and the laser and is given in terms of the electric dipole moment of the atom and the electric field strength of the laser:

$$|\Omega| = \frac{E \cdot d}{\hbar} \quad (2.16)$$

The laser frequency $\omega_p$ is in general slightly different from the atomic transition frequency $\omega_e$. This difference is called the detuning $\delta$ of the laser: $\delta = \omega_e - \omega_p$. An illustrative view of the dressed atom approach is shown in figure 2.2. In this figure the rows are the number of photons in the reservoir and the columns are the atomic states. If one photon is taken from the reservoir and the atoms is excitated to a higher state, the effective energy difference is $\hbar \delta$. For a resonant laser the diffence is zero. The atomic Hamiltonian can be written in terms of the detuning:

$$H_A = -\hbar \delta \hat{a}^+ \hat{a}^- = -\hbar \delta |e\rangle \langle e| \quad (2.17)$$

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Figure 2.2: In the dressed atom approach only the energy difference of the detuning is taken into account in the atomic part of the Hamiltonian. The columns are the atomic states, the rows are the number of photons. The inset shows that an energy difference between a photon and an atomic transition can be written as a detuning.

The total Hamiltonian describing the coherent processes is the sum of the atomic Hamiltonian with the interaction potential and can be written as the matrix [13]:

\[ H = \frac{\hbar}{2} \begin{pmatrix} -2\delta & \Omega \\ \Omega & 0 \end{pmatrix}. \]  

(2.18)

To calculate the transition probability we have to derive and solve the optical Bloch equations. These equations describe the density matrix \( \rho \) as a function of time and take also spontaneous processes into account. The density matrix for a pure quantum state \( |\Psi\rangle \) is defined as [13]

\[ \rho = |\Psi\rangle \langle \Psi|. \]  

(2.19)

For a mixture of pure states the density matrices of the separate pure states can be added to form the density matrix of the complete system,

\[ \rho_{\text{atom}} = \begin{pmatrix} \rho_{gg} & \rho_{ge} \\ \rho_{eg} & \rho_{ee} \end{pmatrix}. \]  

(2.20)

In contrast to the wave function \( \Psi \) the density matrix can be used for systems where not all the information is contained, so called statistical mixtures. The off-diagonal elements, called coherences, can be calculated by solving the Schrödinger equation if and if only the density matrix is describing a pure quantum state. Due to spontaneous emission or the finite line width of the laser that interacts with the atoms, these coherences decay. These processes create loss of information about the exact quantum state; this way the mixture of pure states becomes a statistical mixture. The loss of information can be written in terms of a decoherence matrix \( L(\rho) \). The optical Bloch equations are the set of differential equations given by [12]

\[ \frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + L(\rho). \]  

(2.21)

One of the decoherence matrices is related to spontaneous emission. In the most general form this Lindblad matrix for spontaneous emission \( L_{\text{se}} \) is calculated via the formula [4, 11]:

\[ L_{\text{se}}(\rho) = \bigg( C^\dagger C \rho C^\dagger C - \frac{1}{2} (C^\dagger C \rho + \rho C^\dagger C) \bigg) \]  

(2.22)

where the operator \( C = \sqrt{\Gamma_e} |e\rangle \langle g| \), with \( \Gamma_e \) the inverse of the lifetime of the excited state \( |e\rangle \) that decays to the ground state \( |g\rangle \). For the two level atom this gives the decoherence matrix:

\[ L_{\text{se}} = \begin{pmatrix} \Gamma_e \rho_{ee} & -\frac{1}{2} \rho_{ge} \Gamma_e \\ -\frac{1}{2} \rho_{eg} \Gamma_e & -\Gamma_e \rho_{ee} \end{pmatrix}. \]  

(2.23)

Another decoherence megaism is the effect of the finite line width of the laser \( \gamma_p \), which is captured by the decoherence matrix \( L_{\text{lw}} \):
\[ \mathcal{L}_{lw} = \begin{pmatrix} 0 & -\gamma_p \rho_{eg} \\ -\gamma_p \rho_{eg} & 0 \end{pmatrix}. \] (2.24)

Now we can write down the optical Bloch equations as

\[ \frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}_{se}(\rho) + \mathcal{L}_{lw}(\rho). \] (2.25)

To find the density matrix elements, the left hand side of the equation is set to zero. This way the steady state solution of the density matrix is obtained. This steady state solution is related to macroscopic observables such as absorption and phase shift of light that passes a medium that is described by the density matrix.

The matrix element we are interested in is \( \rho_{eg} \), that contains the information about the transition from the ground state to the excited state. For this two-level atom, the steady state matrix element is given by:

\[ \rho_{eg} = \frac{\Omega_p^2}{2} \frac{\delta_p - i(\Gamma_e/2 + \gamma_p)}{\Omega_p^2/2 + (\Gamma_e/2 + \gamma_p)^2 + \delta_p^2}. \] (2.26)

### 2.2.2 Absorption

The microscopic process of an atom absorbing a photon and being excited has to be translated to a macroscopic observable that can be measured in an experiment. A laser beam that is interacting with atoms induces polarization. Let us assume to have \( N \) atoms contained in a volume \( V \) that couple to the electric field of that laser. Now the polarization can be written as the sum

\[ P = -\sum_{i=1}^{N} \frac{\langle er_i \rangle}{V} \] (2.27)

which for large \( N \) is written in terms of the density matrix element \( \rho_{eg} \), the transition dipole moment \( d_{eg} \), the frequency \( \omega_p \) of the laser and the volume \( V \) as

\[ P = -\frac{N}{V} [d_{eg} \rho_{eg} e^{-i\omega_p t} + \text{c.c.}]. \] (2.28)

The density \( N/V \) can be calculated, assuming that the rubidium atoms behave as an ideal gas, by dividing the vapor pressure \( P_{\text{vap}} \) found in [14] by the average energy per atom \( k_bT \):

\[ \frac{N}{V} = \frac{P_{\text{vap}}}{k_bT}. \] (2.29)

In the vapor cell used in the experiments about two thirds of the atoms are Rb\(^{85} \) atoms, the other atoms are Rb\(^{87} \) atoms, from the Rb\(^{85} \) atoms, only half of the atoms is in the correct ground state. The density of usable atoms is therefore a factor of three smaller: 3 \( \cdot \) \( 10^{15} \) atoms per cubic meter. Note that the amplitude of the electric field of the laser is contained in \( \rho_{eg} \). The macroscopic definition of the polarization of a medium is given in terms of the electric field strength \( E_p \), laser frequency \( \omega_p \), the permittivity of the vacuum \( \epsilon_0 \) and the susceptibility \( \chi \) of a cooled gas of rubidium-85 atoms:

\[ P = \frac{1}{2} \epsilon_0 E_p \left[ \chi e^{-i\omega_p t} + \text{c.c.} \right]. \] (2.30)

From these two expressions, 2.28 and 2.30, we can derive an expression for the electric susceptibility \( \chi \):

\[ \chi = -\frac{2N d_{eg}^2}{\epsilon_0 V \hbar \Omega_p} \rho_{eg}. \] (2.31)


Figure 2.3: The imaginary part of the susceptibility of an ultra cold gas of rubidium-85 atoms is plotted as a function of the detuning of the laser. The different colors are for different Rabi frequencies: the red curve is for $\Omega_p = 0.1\Gamma_e$; the blue curve is for $\Omega_p = 0.5\Gamma_e$; the orange curve is for $\Omega_p = \Gamma_e$ and the green curve is for $\Omega_p = 1.5\Gamma_e$. The density of the cloud of ultra cold rubidium atoms for this plot is $4\times10^{16} \text{m}^{-3}$. For this transition $\Gamma_e = 2\pi \times 6 \text{ MHz}$.

The susceptibility is in general a complex number. The imaginary part of the susceptibility $\chi_I$ is related to the absorption of light by a medium and can therefore be measured with an absorption experiment. In figure 2.3 the imaginary part of the susceptibility is plotted as a function of the detuning of the laser $\delta_p$. We will now explain how this works. Let us assume that we have a medium with length $l$, refractive index $n$ and a laser with an electric field amplitude $E_0$, wave number $k_p$ and frequency $\omega_p$ that is propagated through the medium. The electric field of the laser after passing the medium $E_l$ is given by:

$$E_l = E_0 e^{i(k_p n l - \omega_p t)},$$

(2.32)

were the refractive index $n$ can be written as:

$$n = \sqrt{1 + \chi} \approx 1 + \frac{\chi_R + i\chi_I}{2},$$

(2.33)

were $\chi_R$ and $\chi_I$ are the real and imaginary part of the susceptibility. This approximation is valid for small $|\chi|$. Both in a rubidium vapour cell as in an ultra cold cloud of rubidium atoms, the susceptibility $|\chi_I| \leq 10^{-3}$ so this approximation is valid for the experiments described in this thesis.

Now the electric field of the laser beam, after passing the medium, can be written in terms of the original amplitude multiplied by an absorption and a phase shift term:

$$E_l = E_0 e^{-\frac{\chi_I l}{2}} e^{i(\phi - \omega_p t)}.$$

(2.34)

With the phaseshift $\phi = kl(1 + \frac{1}{k^2})$. A direct measurement of the laser electric field is not possible, in stead EIT is detected by measuring the intensity of the laser using a photo diode. The intensity is a function of the refractive index, the square of the electric field and $\epsilon_0$:

$$I_P = \frac{E_0^2 c n_{air} \epsilon_0}{2}.$$

(2.35)
In this equation, $E_P$ is the electric field of the laser beam at the photo detector and $n_{\text{air}}$ is the refractive index of the ambient air.

### 2.2.3 Doppler effect in a room temperature vapor cell

So far the system of a two level atom is considered for an atom that does not move, the so called frozen gas approximation. In practice such a system does not exist. In a cloud of laser cooled atoms, we come as close as we can get to this condition. But most spectroscopy setups used for locking lasers to atomic transitions work with room temperature vapor cells. In these cells the atoms move around and hence the Doppler effect plays a role: the atom observes a different laser frequency if it has a velocity component in the direction of the laser beam. Therefore Doppler-free spectroscopy setups are often used [24]. In these setups the laser is counter propagating through the cell so Doppler effects cancel out. Later in this thesis we will describe a doubly driven three level atom spectroscopy setup with counter propagating lasers of different wavelengths. That setup is not completely Doppler-free [6]. Therefore we will explain for this simple two-level atom system how the Doppler effect changes the absorption spectrum.

The Doppler effect can be included in the system by introducing an effective detuning with a Doppler term. This effective detuning $\delta^D$, is given in terms of the velocity component in the direction of the laser beam $v_z$ and the wave number of the laser $k_z$ by:

$$\delta^D = \delta^0 + k_z v_z.$$  (2.36)

The susceptibility of the gas at room temperature, $\chi_{rt}$, can now be calculated by averaging over all velocities, solving the integral:

$$\chi_{rt} = \int_{v_z} \chi(v_z) N(v_z) dv_z,$$  (2.37)

where $N(v_z)$ is the normalized distribution of $v_z$ at $T = 300K$ given by:

$$N(v_z) = \sqrt{\frac{mR_b}{2\pi k_b T e^{-\frac{mv_z^2}{2k_b T}}}}$$  (2.38)

and $\chi(v_z)$ is the susceptibility were the effective detuning is the parameter that contains $v_z$.

Now we understand how the density matrix element $\rho_{eg}$ that is calculated, based on microscopic arguments, is related to the susceptibility of a medium and to macroscopic observations as phase shift and absorption.
2.3 A doubly-driven three-level atom

Now we will discuss a doubly-driven three-level atom. A new level is added to the atom so there are three possible states: the ground state $|\text{g}\rangle$, the intermediate state $|\text{e}\rangle$ and a high lying Rydberg state $|\text{r}\rangle$. The ground state is coupled to the intermediate state with a laser which we will call the probe laser and the Rydberg state is coupled to the intermediate level with a laser which we will call the coupling laser. A schematic view of this system is given in figure 2.4. In the previous section we have seen that photons of a resonant probe laser are absorbed by atoms in the ground state to excite them to the intermediate state. When atoms in the intermediate state decay to the ground state a photon is emitted in a random direction. Therefore this process is observed as a dip in intensity in the probe laser beam as the frequency of that laser is scanned over the atomic resonance. In the case of a doubly driven three-level atom something remarkably can happen. If the combined energy of two photons, one of each laser, is equal to the energy difference between the Rydberg state and the ground state, the probe laser is no longer absorbed. Even if the probe laser is perfectly resonant, if the coupling laser is resonant too, the atom becomes transparent for the probe laser.

This phenomenon is called electromagnetically induced transparency (EIT). EIT is a quantum effect and cannot be understood classically. When the two lasers together are resonant with the total transition the excitation pathways to excite the intermediate level interfere destructively so the intermediate level is no longer coupled to the system.

Similar to the derivation of the Hamiltonian of a driven two-level atom, we can describe a doubly-driven three-level atom using the rotating wave approximation to include only the resonant terms and the dressed atom approach to write the energy part of the Hamiltonian in terms of the laser detuning.

We will include a second set of raising and lowering operators working on the transition between the intermediate state and the Rydberg state: $\hat{b}^+ = |\text{r}\rangle \langle \text{e}|$ and $\hat{b}^- = |\text{e}\rangle \langle \text{r}|$ that have a similar function as the earlier introduced operators.

The Hamiltonian describing the coherent processes of the three-level atom interacting with the two laser beams is now given by:

$$
H = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_p & 0 \\ \Omega_p & -2\delta_p - \Omega_c & \Omega_c \\ 0 & \Omega_c & -2(\delta_p + \delta_c) \end{pmatrix}
$$

were we used a matrix notation for convenience.

To see that this system can actually become transparent for the probe laser, we will have a look at the system for the condition of two-photon resonance. There can still be a non zero detuning of the probe laser and the intermediate state, but this is compensated by a detuning of the laser coupling to the upper state $\delta_p + \delta_c = 0$. In this case we call our hamiltonian $H_{\text{res}}$ which can be written as:

$$
H_{\text{res}} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_p & 0 \\ \Omega_p & -2\delta_p - \Omega_c & \Omega_c \\ 0 & \Omega_c & -2\delta_p \end{pmatrix}.
$$

When $H_{\text{res}}$ is diagonalized we find new eigenstates, which are linear combinations of the atomic states given by [4]:

$$
|\text{+}\rangle = \sin \theta \sin \phi |\text{g}\rangle + \cos \phi |\text{e}\rangle + \cos \theta \sin \phi |\text{r}\rangle \\
|\text{−}\rangle = \sin \theta \cos \phi |\text{g}\rangle - \sin \phi |\text{e}\rangle + \cos \theta \cos \phi |\text{r}\rangle \\
|\text{D}\rangle = \cos \theta |\text{g}\rangle - \sin \theta |\text{r}\rangle
$$

where the angles $\phi$ and $\theta$ are known as the stuckelberg mixing angles. The angles are, given by [4]:

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Figure 2.4: A model of a doubly driven three level atom, the two bottom levels are discussed in the section about the two level atom. For EIT a third level with a high principal quantum number is added. This state can either be an S state or a D state. The D states are split into two different J states. EIT occurs if the total detuning \((\delta_p + \delta_c=0)\)

\[
\tan \theta = \frac{\Omega_p}{\Omega_c}, \quad \tan 2\phi = \frac{\sqrt{\Omega_p^2 + \Omega_c^2}}{\delta_p}.
\] (2.42)

Using goniometry we find that the cosine and sine of the Stuckelberg angle theta are given by:

\[
\cos \theta = \frac{1}{\sqrt{1 + \Omega_p^2/\Omega_c^2}} \quad \text{and} \quad \sin \theta = \frac{\Omega_p/\Omega_c}{\sqrt{1 + \Omega_p^2/\Omega_c^2}}.
\] (2.43)

Atoms can decay from the states \(|+\rangle\) and \(|-\rangle\) to the dark state \(|D\rangle\). When an atom is in the dark state it can no longer couple to the light field. Let the Hamiltonian for two photon resonance work on the dark state:

\[
H_{\text{res}} |D\rangle = \frac{\hbar}{2} (|e\rangle \langle g| \cos \theta |g\rangle - |e\rangle \langle r| \sin \theta |r\rangle),
\] (2.44)

and we see that there is no coupling to the light field at all:

\[
H_{\text{res}} |D\rangle = \Omega_p \frac{1}{\sqrt{1 + \Omega_p^2/\Omega_c^2}} |e\rangle - \Omega_c \frac{\Omega_p/\Omega_c}{\sqrt{1 + \Omega_p^2/\Omega_c^2}} |e\rangle \equiv 0.
\] (2.45)

This results in destructive interference of excitation pathways and is known as Electromagnetically Induced Transparency or in short: EIT. In this approach we have assumed that the energy between the atomic ground state and the Rydberg state is equal to the combined energy of the two photons needed for a transition to the Rydberg state. This leads to an understanding of why we should observe transparency but does not give information about the bandwidth and the influence of experimental parameters. Therefore we will derive a semi-analytical model which enables us to calculate the shape of the EIT signals.

2.3.1 Semi analytical model of EIT in an ultracold gas

To calculate the shape of the EIT signals as a function of the Rabi frequencies a semi analytical model is created and solved with Mathematica. In this model the optical Bloch equations
are calculated and the steady state solution is determined. It is assumed that the atoms do not move: the frozen gas approximation. Further we assume that the lasers overlap perfectly. The model is based on the optical Bloch equations and is derived in a similar way as for the driven two level atom. The density matrix is now given by:

\[
\rho_{\text{atom}} = \begin{pmatrix}
\rho_{gg} & \rho_{ge} & \rho_{gr} \\
\rho_{eg} & \rho_{ee} & \rho_{er} \\
\rho_{rg} & \rho_{re} & \rho_{rr}
\end{pmatrix}.
\]  

(2.46)

Written componentwise:

\[
\rho_{gg} = \frac{i}{2} \Omega_p (\rho_{ge} - \rho_{eg}) + \Gamma_c \rho_{cc};
\]

\[
\rho_{ee} = \frac{i}{2} \Omega_p (\rho_{ge} - \rho_{eg}) + \Omega_c (\rho_{er} - \rho_{re}) - \Gamma_c \rho_{cc} + \Gamma_r \rho_{rr};
\]

\[
\rho_{rr} = \frac{i}{2} \Omega_c (\rho_{re} - \rho_{er}) - \Gamma_r \rho_{rr};
\]

\[
\rho_{eg} = \frac{i}{2} (\delta_{p} \rho_{eg} + \Omega_p (\rho_{ce} - \rho_{eg}) - \Omega_c (\rho_{pg} - \rho_{eg}) - \Gamma_c \rho_{ce} - \gamma_{pg} \rho_{eg});
\]

\[
\rho_{er} = \frac{i}{2} (2 \Omega_c \rho_{re} + \delta_{p} \rho_{rg} + \Omega_c (\rho_{re} - \rho_{ce}) - \Gamma_c \rho_{re} + \Gamma_r \rho_{er} - \gamma_{c} \rho_{re});
\]

\[
\rho_{gr} = \frac{i}{2} (\Omega_p \rho_{re} - \Omega_c \rho_{ce} + 2 (\delta_{c} + \delta_{p}) \rho_{pg}) - \frac{i}{2} \Gamma_r \rho_{rg} - \gamma_{res} \rho_{rg};
\]

\[
\rho_{ij} = \rho_{ji}^*.
\]

We are interested in the steady state solution of the optical block equations. To obtain this solution we set the left hand side of equation 2.49 to zero. To overcome the problem that we now have to solve a problem of the form \( A \ddot{x} = 0 \) we replace one one of the equations by the condition that the trface of the density matrix must be equal to one: \( \rho_{gg} + \rho_{ee} + \rho_{rr} = 1 \). From the matrix element \( \rho_{eg} \) the susceptibility of the doubly-driven three-level atom for the probe laser can be calculated in the same way as for the driven two level atom.

\(^{1}\)In practice we can take \( \Gamma_r = 0 \), because this value is much smaller than the other parameters in the OBE. Taking \( \Gamma_r = 0 \) saves computation time and gives the same curves.

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2.3.2 Results of the ultra cold EIT model

Now it is time to look at some of the results of this model. We will investigate the role of three different experimental parameters: the strength of the probe and coupling laser and the two-photon line width line width. The first step is to look at the role of the laser intensities. We will examine the influence of the Rabi frequency of the probe laser and coupling laser, again compared to Γe which we take as the characteristic frequency in our model. As we discuss here the results of EIT in an ultra cold system, we will take the density of atoms to be \(4 \cdot 10^{16}\) atoms per cubic meter, a typical density of atoms in our magneto optical trap [22]. The result of this model is shown in figure 2.6. We observe that a larger Rabi frequency of the probe laser broadens the absorption and a larger Rabi frequency of the coupling laser broadens the EIT feature. For the laser linewidth we assume that \(\gamma_p = \gamma_c\) and that \(\gamma_{res} = \gamma_p + \gamma_c\), which means that the probe- and coupling laser are uncorrelated. Because we are mainly interested in the EIT effect we will look at the imaginary part of the susceptibility. The different parameter are scaled to the line width of the transition \(5S_{1/2} \rightarrow 5P_{3/2} : \Gamma_e\). In figure 2.5 the imaginary part of the susceptibility is plotted as a function of the detuning of both the probe and coupling laser. The detuning in the figures 2.5, 2.6 and 2.7 is for both lasers plotted from -20 to 20 Mrad/s. Let us discuss what we see in this figure in detail. The parameters in this plot are: \(\Omega_c = 2\Gamma_e; \Omega_p = \Gamma_e\) and the line width of both lasers is \(1\% \) of \(\Gamma_e\). The first feature that we observe is that \(\chi_I\) becomes zero along the line \(\delta_p + \delta_c = 0\), this is the EIT effect as explained earlier. The eigenstates of the Hamiltonian give in this case rise to destructive interference of excitation pathways for atoms in the dark state. The macroscopic effect is that the imaginary part of the susceptibility becomes zero. Another feature that we observe is that for a sufficient detuning of the probe laser (for example \(\delta_p = 10\) Mrad/s) there is a peak of maximum absorption next to the EIT feature. This peak occurs at the side of the EIT feature were \((\delta_p + \delta_c) > 0\), but not for \((\delta_p + \delta_c) < 0\). Further this figure is symmetric along the line \((\delta_p + \delta_c) = 0\). In the figures 2.6 and 2.7 similar plots are shown for different parameters. The same features can be seen as discussed here, but the width and dept of the EIT feature depend on these parameters.

We see in figure 2.7 that for large line widths the EIT effect becomes more shallow. In our setup we can assume that both laser line widths are below one Mrad/s, hence we do not expect that this feature plays a role in our experiment.

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Figure 2.5: The imaginary part of the susceptibility (colors) is plotted as a function of the detuning of the coupling laser (horizontal axis) and probe laser (vertical axis). The parameters are: $\Omega_p = \Gamma_e$; $\Omega_c = 3\Gamma_e$ and $\gamma_p = \gamma_c = \Gamma_e/100$. The detunings of both lasers are plotted from -30Mrad/s to 30Mrad/s. The imaginary part of the susceptibility is zero along the line $(\delta_p + \delta_c = 0)$, where one of the eigenstates of the Hamiltonian becomes a dark state, atoms in the dark state are no longer coupled to the light field this is called EIT.

2.3.3 Semi analytical model of EIT in a vapor cell

In our spectroscopy experiment a rubidium vapor cell is used. The rubidium vapor in this cell is far from ultra cold: the cell is at room temperature. This means that the rubidium atoms in the cell are moving around and Doppler effects can become important. Due to the velocity component of the atoms in the z-direction, along the path of the lasers, the atom observes a Doppler shifted frequency of both lasers. We can incorporate this in our model by changing the detuning of the lasers into the effective detuning, similar to the case of the Doppler effects of a two level atom coupled with one laser as we did with equation 2.36 An atom moving towards the laser observes a larger effective detuning and an atom moving in the direction of the laser beam observes a lower effective detuning, the effective detunings are given in equation 2.51.

To minimize the Doppler effect the lasers are counter-propagating [6]. A schematic view of the Doppler effect in this atomic system is shown in figure 2.8 [7, 4]. The effective detunings become:

$$\delta_p^D = \delta_p - k_pv_z \quad \delta_c^D = \delta_c + k_cv_z.$$  \hspace{1cm} (2.51)

Now we can substitute the effective detuning of the probe and coupling laser 2.51 in the steady state solution of the optical Bloch equations that are given in equation 2.50.

We will consider a system were the probe laser is resonant for atoms that not move, so $\delta_p = 0$. For different $v_z$ the imaginary part of the susceptibility is plotted as a function of the detuning of the coupling laser. If $\chi_I = 0$ the atom is subject to EIT.

One of the observed features is the shift of the resonance. Because of the Doppler mismatch between the probe and coupling laser, an atom with a larger $v_z$ needs a larger detuning of the coupling laser to become resonant. An other feature is the susceptibility for a far detuned coupling laser. The value of $\chi_I$ is lower for atoms with a larger $v_z$, also this is easy to understand.

The value of $\chi_I$ in absence of EIT is determined by the effective detuning of the probe laser.

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Figure 2.6: The imaginary part of the susceptibility (colors) are plotted as function of the detuning of the coupling laser (horizontal axis) and probe laser (vertical axis). The horizontal rows have the same Rabi frequency of the probe laser, the vertical columns have the same Rabi frequency of the coupling laser. Along the horizontal axis the detuning of the coupling laser is plotted from $-20 \text{ Mrad/s} < \delta_c < 20 \text{ Mrad/s}$ and along the vertical axis the detuning of the probe laser is plotted with the same scale. The density of Rb$^{85}$ atoms in the correct ground state in this plot is $8.2 \times 10^{14} \text{ m}^{-3}$.
Figure 2.7: The imaginary part of the susceptibility (colors) is plotted as function of the detuning of the coupling laser (horizontal axes) and probe laser (vertical axes). It is assumed that the probe laser line width is the same as the coupling laser line width, $\gamma_P = \gamma_C$, and that the two photon line width is given by: $\gamma_{res} = \gamma_P + \gamma_C$. The Rabi frequencies are set to $\Omega_P = 0.1\Gamma_e$ and $\Omega_C = \Gamma_e$. Along the horizontal axes the detuning of the coupling laser is plot from $-20MHz < \delta_C < 20MHz$ and along the vertical axes the detuning of the probe laser is plotted with the same scale. The density of $\text{Rb}^{85}$ atoms in the correct ground state in this plot is $8.2 \times 10^{14} m^{-3}$. 
Figure 2.8: Left: Atoms (gray dots) in a vapor cell move around. The velocity component in the direction of the probe laser is defined as the positive $v_z$ component. The red arrow represents the probe laser, the blue arrow represents the coupling laser. Right: The shift in detuning can be interpreted as the shift in energy of atomic resonances. For atoms with a non-zero $v_z$ component the energy of the top level is shifted more than the energy of the intermediate level.

Figure 2.9: The imaginary part of the susceptibility for atoms that have a velocity component along the optical path is shown as function of the detuning of the coupling laser. The dashed line is the susceptibility for atoms with $v_z = 0$. The atoms that move, are moving in the direction of the coupling laser.
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For a large $v_z$ the effective detuning of the probe laser is large. The probe laser is resonant for atoms with $v_z = 0$. The last feature we notice in the line shape is the peak in susceptibility that becomes sharp for atoms with a larger $v_z$, this is similar to the ultra cold EIT with a detuned probe laser. We believe that this feature emerges due to the ac Stark effect. Atomic levels can be shifted if a strong enough light field is present. It is known that the ac Stark shift exists in a doubly driven three level atom [20].

The next step is to consider an ensemble of atoms [7, 5], the procedure is similar to the case of a driven two level atom: by integrating over the velocity the susceptibility of an atomic vapor can be calculated. Note that $v_z$ in the detuning is a variable in the matrix element $\rho_{eg}$ and hence we can write the susceptibility as a function of $v_z$. For the susceptibility of the atomic vapor in the cell, we add up all the contributions of the individual atoms. The atomic density in the vapor cell is $3.2 \cdot 10^{15}$ atoms per cubic meter, see section 2.2.2. We assume a Maxwell-Boltzmann distribution for the velocity components, so that the normalized distribution of $v_z$ is given in terms of the Rubidium mass $m_{Rb}$, the Boltzmann constant $k_B$ and the temperature $T$ by:

$$N(v_z) = \sqrt{\frac{m_{Rb}}{2\pi k_B T}} e^{-\left(\frac{m_{Rb} v_z^2}{2 k_B T}\right)} \tag{2.52}$$

Now the susceptibility can be calculated as:

$$\chi_{\text{vapor}} = \int_{-\infty}^{\infty} \chi(v_z) N(v_z) dv_z \tag{2.53}$$

To see how the exact shape of the susceptibility arises it is illustrative to look at the susceptibilities of atoms with different velocities along the optical path. In figure 2.10, $\chi_I$ is plotted for different velocity classes, from $v_z = -23.5 m/s$ to $v_z = 23.5 m/s$. Also $\chi_I$ for a vapor cell is plotted (red, dashed line), note that this is not on the same scale as the signals for the velocity classes, but it serves as a guide for the eye to see how the signal is formed. The essential difference between an ensemble of atoms that does not move and a thermal ensemble is that the imaginary part of the susceptibility does not completely go to zero and that there are two peaks in the imaginary part of the susceptibility next to the resonance.
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Figure 2.10: $\chi_I$ is plotted as function of $\delta_C$ (gray lines), $\chi_I$ for $v_z = 0$ and $v_z = 10.5 m/s$ are plotted in blue and purple to show the difference in line shape. As a guide for the eye $\chi_I$ for an atomic vapor is also plotted (dashed, red line). The red dotted line is not plotted on the same scale as the other lines as the collective effect is about two orders of magnitude smaller than the one-atom effect.

2.3.4 Results of the semi-analytical model of EIT in a vapor cell

For calculating the imaginary part of the susceptibility in a vapor cell, we numerically integrate equation 2.53. In figure 2.11 the imaginary part of the susceptibility is plotted as a function of the detuning of the coupling laser. The detuning is plotted from $-50 \text{ Mrad/s}$ to $50 \text{ Mrad/s}$. We used four different settings of the Rabi frequency of the coupling laser and three different setting of the Rabi frequency of the probe laser, again scaled to $\Gamma_e$. We observe that for a smaller $\Omega_P, \chi_I$ becomes larger and that for a larger $\Omega_C$ the EIT feature becomes larger as well as the peaks for maximum absorption related to the AC stark shift. The line width of the EIT signal scales with $\Omega_C$. For a larger Rabi frequency of the coupling laser the line width of the EIT signal becomes larger.

For a thermal ensemble of atoms the difference between the minimum and maximum value of $\chi_I$ is, depending on the Rabi frequencies, between $5 \times 10^{-9}$ and $2.5 \times 10^{-8}$. In the case of ultra cold atoms the difference between the maximum and minimum value of $\chi_I$ is between $3 \times 10^{-7}$ and $1 \times 10^{-6}$. Further for ultracold atoms there is actually EIT, the value of $\chi_I$ becomes zero if both lasers together are resonant. For thermal atoms $\chi_I$ never reaches zero, therefore a thermal gas never becomes completely transparent for the probe laser.

The next step is to calculate the expected intensity $I$ after passing the vapor cell. This will be calculated with the formula:

$$I = \left(\frac{\hbar \Omega_p}{d_{eg}}\right)^2 \frac{c n_{air} \epsilon_0}{2} e^{-k_p L \chi_I}, \quad (2.54)$$

with $L$ the length of the vapor cell, note that frequency dependence is in the term $\chi_I$. The factor $(\hbar \Omega_p/d_{eg})$ is equal to the amplitude of the electric field $E_0$. We will discuss four regimes of probe laser Rabi frequencies, $\Omega_p = 0.1 \Gamma_e$, $\Omega_p = 0.2 \Gamma_e$, $\Omega_p = 0.5 \Gamma_e$ and $\Omega_p = 1 \Gamma_e$, these Rabi frequen-
Figure 2.11: The imaginary part of the susceptibility is plotted as function of the detuning of the coupling laser for a resonant probe laser. These results of the numerical integration of equation 2.53 for different rabi frequencies of the lasers are obtained for a temperature of 295K and density of $8.2 \times 10^{14} m^{-3}$. The colors represent different Rabi frequencies of the probe laser, the red curves are for $\Omega_p = 0.5 \Gamma_e$, the blue curves for $\Omega_p = 1 \Gamma_e$, the orange curves for $\Omega_p = 1.5 \Gamma_e$ and the green curves for $\Omega_p = 2 \Gamma_e$. The figures represent for different Rabi frequencies of the coupling laser. Top left: $\Omega_c = 0.5 \Gamma_e$, top right: $\Omega_c = \Gamma_e$, bottom left: $\Omega_c = 2 \Gamma_e$ and bottom right: $\Omega_c = 5 \Gamma_e$. 

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Figure 2.12: The expected laser intensity for the probe laser after passing the vapor cell as a function of coupling laser detuning is plotted for different values of the probe and coupling laser Rabi frequencies. The intensities are given in W/m$^2$. The atom density is $8.2 \times 10^{14}$ m$^{-3}$ and the temperature is 295K. The colors represent different Rabi frequencies of the coupling laser. Red: $\Omega_c = 5\Gamma_e$, Blue: $\Omega_c = 2\Gamma_e$, green: $\Omega_c = \Gamma_e$. The four figures are each for a specific Rabi frequency of the probe laser. Top left: $\Omega_p = 2\Gamma_e$, top right: $\Omega_p = 1.5\Gamma_e$, bottom left: $\Omega_p = 1\Gamma_e$ and bottom right: $\Omega_p = 0.5\Gamma_e$.

These expected intensities are plotted in figure 2.12. We observe that the largest amplitude of the signal is obtained by a high $\Omega_p$. Despite the result that for a given $\Omega_c$ a small $\Omega_p$ gives the maximum signal for the imaginary part of the susceptibility, the expected photo diode signal is higher if a large $\Omega_p$ is chosen. Because a high $\Omega_p$ means a high intensity of the probe laser, the expected photodiode signals are also larger. But the expected maximum signal to noise ratio is 1% for all probe laser intensities. This means that in order to see the signal a modulation technique should be used to get the signal out of the background. The technique that is used to measure the signals in the experimental setup is to chop the coupling laser with an optical chopper use a lock in amplifier to look at the difference between the photodiode signals for the case were the coupling beam is on and were the coupling beam is off. This technique is further discussed in the next chapter.

2.4 Theoretical versus experimental parameters

In this chapter we used the Rabi frequency $\Omega$ to describe the laser strength, in most plots the strength is given in the ratio between a Rabi frequency and the transition linewidth of the bottom transition $\Gamma_e$. This way the graphs are as universal as possible. The drawback of this ap-
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Figure 2.13: The ratio between the Rabi frequency and $\Gamma_e$ is plotted as a function of laser power for a laser beam with a radius of 0.13 mm (probe laser beam) and 0.2 mm (coupling laser beam). For the probe laser (a) the power is given in $\mu$W, for the coupling laser (b) the power is given in mW. The purple line in the graph gives the result for a transition to the 41D Rydberg state, the blue line in (b) is the result for a transition to the 41S state.

Approach is that it is not possible to directly measure the Rabi frequency in the lab. In an experimental environment most of the time the power of a laser is measured. Therefore two plots are presented in figure 2.13 were the ratio between the Rabi frequency and $\Gamma_e$ is plotted as function of the laser power, where we assume a uniform laser profile with a radius of 0.13 mm for the probe laser beam and 0.2 mm for the coupling laser beam. The Rabi frequency as a function of the power $P$ and radius $r$ of a laserbeam and the transition dipole moment $d$ is given by:

$$\Omega(P, r, d) = \frac{d}{\hbar} \sqrt{\frac{2 P}{\epsilon_0 c \pi r^2}}. \quad (2.55)$$

In the theory the linewidth of the signal is given in radial per second. In the experiments only the wavelength of the laser can be measured. The frequency of a laser in terms of the speed of light $c$ and the wavelength $\lambda$ is

$$\omega = \frac{2\pi c}{\lambda}. \quad (2.56)$$

Finally we express the frequency change $\Delta\omega$ in terms of the change in wavelength $\Delta\lambda$ for $\Delta\lambda \ll \lambda$:

$$\Delta\omega = -2\pi c \frac{\Delta\lambda}{\lambda^2}. \quad (2.57)$$

These equations can be used to determine the distance between peaks in a spectrum or the linewidth of peaks in terms of the radial frequency in stead of wavelength. This enables direct comparison of spectral features that emerge at different wavelengths.
Chapter 3

Experimental setup

3.1 EIT spectroscopy

3.1.1 Optical lay out of the spectroscopy setup

For the EIT spectroscopy two laser beams are needed: a probe laser beam and a coupling laser beam. The probe laser is resonant with the rubidium $|5S_{1/2}, F = 3⟩ \rightarrow |5P_{3/2}, F = 4⟩$ transition and the coupling laser is (nearly) resonant with the transition between the intermediate and a Rydberg state, remember figure 2.4. A schematic view of the optical lay out of the setup is given in figure 3.1.

The probe laser beam is coupled out of a fiber and can be frequency modulated with an electro optical modulator (EOM: Thorlabs EO-PM-NR-C1) to create side bands. This laser is used to excite rubidium atoms from the ground state to the intermediate state 2.2.2.

The coupling laser beam is split using a half wave plate and a polarizing beam splitter cube. This gives the option to use a branch of the coupling laser in another setup. To ensure that the coupling laser beam completely overlaps the probe laser beam, the beam is expanded using a telescope. It is important that the coupling laser beam completely overlaps the probe laser beam as EIT will only happen in the part of the probe beam that has overlap with the coupling beam. The amplitude of the coupling laser beam can be modulated using an optical chopper wheel.

The probe laser beam and coupling laser beam are mixed and demixed using dichroic mirrors. The laser beams are counterpropagated through a rubidium cell. The power of the probe light after passing the vapor cell can be measured using a photodetector (Thorlabs DET210 or Thorlabs PDA 36-A/M). A band-pass filter of $780 \pm 10\text{nm}$ is placed in front of the photodetector to ensure that only the light of the probe laser beam is measured.

The power of the probe laser beam can be changed using neutral density (ND) filters and the power of the blue light can be controlled with either the half-wave plate and polarizing beam splitter cube or with ND filters. The laser systems providing these laser beams are discussed in more detail in the sections 3.2.1 and 3.2.2. It is important that the coupling laser beam diameter is larger than the probe laser beam diameter. Only the part of the probe laser beam that is covered by the coupling laser beam is subject to EIT. Therefore the signal to background ratio would decrease if a part of the probe laser beam is not covered by the coupling laser beam. Therefore alignment is also of major importance.
Figure 3.1: A schematic overview of the experimental setup for EIT spectroscopy. The Toptica TA-SHG laser system (1) provides (250-300) mW of 480 nm light as coupling laser. The probe laser is coupled out of a fiber (2) and provides 80 $\mu$W of 780 nm light as probe laser. The power of the coupling laser in the EIT setup can be regulated with the combination of half wave plate (3) and polarizing beam splitter cube (4). The mirrors (5) are E02 or E03 coated broadband mirrors. The probe laser beam profile is enlarged with a telescope consisting of a negative lens with a focal distance of -50 mm (6) and a positive lens with a focal distance of 150 mm (8). The coupling beam can be amplitude modulated with a chopper wheel (8) to enable the use of a lock in amplifier to increase the signal to noise ratio. The probe and coupling beam are mixed and separated using dichroic mirrors (9) and are counter propagated through a rubidium gas reference cell (10). The probe power is measured with a photo detector (11) that is placed behind a narrow band pass filter (12). The wavelength of the fundamental laser light of the Toptica TA-SHG system can be measured with a wave meter. A part of the fundamental laser light is coupled into a fiber (13) for this purpose. For noise reduction by frequency modulation sidebands are needed on the probe laser beam. To frequency modulate the probe laser beam an electro optical modulator (EOM) is installed (14).


3.2 Laser system

The laser beams in the EIT spectroscopy are part of the laser system of the ultra cold plasma (UCP) setup. Three laser laser systems will be discussed. The first system provides the probe laser beam for the EIT spectroscopy and the cooling laser beam for the magneto optical trap (MOT), this part of the setup will be discussed later in section 3.4. The second system provides repump laser beams for the MOT and the third system provides the coupling laser beam in the EIT spectroscopy and the Rydberg excitation laser beams for experiments with ultra cold rubidium Rydberg atoms.

3.2.1 EIT probe laser and MOT lasers

A Toptica DLX110 780 nm diode laser is used for the Rb$^{85}$ $|5S_{1/2}, F = 3⟩ → |5P_{3/2}, F = 4⟩$ transition. The laser is locked to this atomic transition using frequency modulation (FM) spectroscopy [24]. This laser is used for cooling of rubidium atoms in the MOT, excitation of ground state atoms in the MOT to the intermediate state and as probe laser in the EIT spectroscopy. With acousto optical modulators (AOMs) it is possible to give the excitation and trapping branches of the laser each an individually adjustable detuning. For efficient laser cooling a detuning is required [13], while the excitation laser beam and probe laser beam needs to be resonant with the atomic transition.

In figure 3.2, a schematic overview of the MOT laser system is given, the red lines are the laser beams from the Toptica DLX110 780 nm diode laser described above. A part of the light in the frequency offset lock part of the setup is split off and coupled in a fiber to be used as a probe laser beam in the EIT spectroscopy setup. The waist of the probe laser beam as it is coupled out of the fiber is 250 $\mu m$. The maximum power of the probe laser on the EIT table is about 85 $\mu W$.

A Toptica DL100 laser is used to provide repump laser beams. Remember figure 2.1, atoms can accidentally end up in the $5P_{3/2}, F = 3$ state because the frequency difference between this state and the $5P_{3/2}, F = 4$ state is only 121 MHz [23]. Those atoms can fall back to the $5S_{1/2}, F = 3$ or $5S_{1/2}, F = 2$ state. Atoms that end up in the $5S_{1/2}, F=2$ state cannot be excited with the excitation laser. Therefore the repump laser is needed. This laser pumps the atoms from the $5S_{1/2}, F = 2$ to the $5P_{3/2}, F = 3$ state. The repump laser is frequency locked using a frequency offset lock where the beat signal of the repump and excitation laser is measured and locked to the difference in atomic frequencies: 2915 MHz [24]. The repump laser beams are also drawn in figure 3.2, the blue lines are repump laser beams. Simulations have shown that a weak repump laser is sufficient [26].

3.2.2 EIT coupling laser

A Toptica TA-SHG 110 frequency doubled high power laser system is used as a Rydberg excitation laser between the $5P_{3/2}, F = 4$ state and a Rydberg state: $nS_{1/2}, nD_{3/2}$ or $nD_{5/2}$, and as the EIT coupling laser. This laser system has a 960 nm continuous wave diode laser, a tapered amplifier and a doubling cavity. The output wavelength of the laser system is 480 nm. We will take closer look to this laser system [28]. A photo of the inside of the inside of this system is shown in figure 3.3. The coupling laser is slightly elliptical: with a horizontal waist of 90 $\mu m$ and a vertical waist of 160$\mu m$, the waist of the coupling laser increased with a three times magnifying telescope (a negative lens with focal length $f = -50mm$ and a positive lens with $f = 150mm$ placed 100 mm apart). This increases the horizontal waist of the coupling laser to 270$\mu m$ and the vertical waist to 480$\mu m$.

960 nm fundamental laser and tapered amplifier

The fundamental laser is a Toptica DL100 960 nm diode laser. The frequency of this laser can be tuned. Rough tuning of the wavelength is done by adjusting the angle of a grating inside the
Figure 3.2: A schematic overview of the laser system for the magneto optical trap. The trapping laser (red lines) is used to cool and trap rubidium atoms and as an excitation laser for ultracold Rydberg experiments. A part of the resonant light is split off with a glass plate (A1) and coupled in a fiber (A2) to be used as a probe laser beam in the EIT spectroscopy setup. A 80MHz shifted laser beam (B) is used to test if side bands are created on the probe beam in the EIT spectroscopy setup. This 80MHz shifted light is mixed with the resonant electro optic modulated light to create a beatnote for the fundamental frequency and the sidebands. The repump laser beams are the blue lines. Figure adapted from [24].
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Figure 3.3: A photograph of the coupling laser system without protective covers is shown. The system consists of three parts: a DL100 diode laser (green box); a tapered amplifier TA-100 (yellow box) and a second harmonic generator SHG 110 (orange box). The red lines indicate the 960 nm light, the blue line indicates the 480 nm light. The wavelength of the fundamental diode laser (A) can be roughly tuned by adjusting the angle of an external grating with screw (B). An optical isolator (C) prevents feedback that would damage the diode. A part of the fundamental light is coupled out (D) for monitoring the wavelength. Most of the fundamental light is amplified with the tapered amplifier (E). About 800 mW of 960 nm light is transmitted through another optical isolator. Two mode matching lenses (F) and (G) are used to focus the light in the cavity. The light is coupled into the cavity with the coupling mirrors (H). The cavity is a ring resonator (I-J-L-N): with mirror (I), connected to a piezo element to scan the cavity length; mirror (J) that has a photodiode on the back that monitors the intensity of the light in the cavity; (K) is the doubling crystal (KNbO$_3$) that is temperature controlled; the mirror (K) is a dichroic mirror that transmits blue light to the exit of the laser (M) and the mirror (N) transmits a small part of the red light that is monitored by a fast photodiode that is connected with the Pound Drever Hall detector (PDD 110). The electronics and electrical connections of the doubling cavity (P) and the fundamental laser and amplifier (O) are separated.

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Figure 3.4: The output power of the Second Harmonic Generator is given as a function of crystal temperature. The input light in the SHG has a wavelength of 960.329 nm and a power of 807 mW. This wavelength is used to address the 41D Rydberg states.

DL100 (figure 3.3, screw (B)). Fine tuning of the wavelength can be done by either using a piezo element that controls the distance between the external grating and the diode or by modulating the current through the diode. The piezo is controlled by a scan controller (Toptica SC110). After the fundamental laser an optical isolator is placed. This optical isolator suppresses reflected light by 60 dB to avoid feedback in the diode laser. After the optical isolator the laser power is amplified by a tapered amplifier. Most of the 960 nm light is coupled into the Tapered Amplifier, but a small part of it is split off. The split-off light is coupled into a fiber so that the wavelength and power can be measured with a wavelength meter (Lambdameter LM-007). The tapered amplifier amplifies the power of the 960 nm light, the maximum measured output power of the amplifier is about 850 mW. After the tapered amplifier another isolator is placed to prevent feedback in the amplifier.

The coupling laser system has to produce 480 nm light. Therefore the frequency of the fundamental light is doubled. The amplified light is coupled into an external ring resonator to increase the power. A frequency doubling crystal is placed inside this resonator to create the second harmonic light. The resonator has a bowtie shape and is also called a bowtie cavity. The coupling of the fundamental light in the cavity can be optimized by adjusting the mode matching lens and using the adjustable incoupling mirrors.

Doubling cavity

The doubling cavity is a ring resonator defined by four mirrors in bowtie configuration with a crystal that doubles the frequency of the light. The first mirror partially transmits the fundamental 960 nm light, this mirror is used to couple the fundamental light in the cavity. The second mirror is mounted on a piezo element to enable adjustment of the cavity length. Behind the third mirror a photodiode is placed to measure the intensity of the fundamental light in the cavity. Between the third and fourth mirror a potassium niobate (KNbO$_3$) crystal is installed to generate second harmonic light, this light has a frequency equal to twice the frequency of the fundamental light. The fourth mirror is a dichroic mirror that reflects the fundamental wavelength and transmits the frequency doubled light. The efficiency of the doubling crystal is strongly dependent on the crystal temperature as is shown in figure 3.4. The optimal temperature for a fundamental wavelength of 961.329 nm (needed to address the 41D Rydberg states) is found to be 39.6 degrees Celsius.
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Figure 3.5: Two images of the coupling laser beam profile are shown. Left is the correct zero order Gaussian mode, right a higher order cavity mode. The zero order mode of the doubling cavity should be locked to the fundamental laser.

The doubling cavity needs to be locked to the fundamental laser. The current through the laser diode is modulated with 20MHz to create sidebands on the fundamental laser beam, a piezo on one of the cavity mirrors can be used to scan the cavity length. A Pound Drever Hall locking scheme is used to lock the cavity to the master laser diode [21].

Optimizing the blue laser power output

In daily use, the output power of the blue laser system is about 250 mW. For perfect alignment of the doubling cavity and optimized temperature of the doubling crystal a maximum power of 306 mW is measured. The output power is dependent on a large number of settings such as the crystal temperature, the cavity mode, the cavity alignment and the modulation debt of the master laser. If the output power is down to less than 50 mW it proved useful to follow a checklist in a specific order.

The first step is to check if the right cavity mode is locked to the master laser. In general several modes will fit in the cavity [25], and only the zero order Gaussian mode can ensure a maximum power output. For this check, a camera can be placed inside the beam line of the laser. The beam profile of the laser should in the ideal case be a zero order Gaussian ellipse. Two camera images are shown in figure 3.5: the left image shows the correct zero order mode; the right image shows a higher order mode.

If the wrong cavity mode is locked to the master laser, the cavity has to be scanned and locked to another peak in the Fabry-Perot spectrum, then the beam profile has to be checked again to confirm the right mode is found. If the peak of the correct zero order mode is not the highest peak of the Fabry-Perot spectrum, the coupling mirror can be used to maximize those peaks in the spectrum. For this procedure it is best to scan the cavity over a few spectral ranges. For fine-tuning the zero order Gaussian mode of the cavity should be locked to the fundamental laser and the incoupling can be optimized by measuring the power of the 480 nm light.

If the right order mode of the cavity is locked to the fundamental laser but the output power is not high enough, it is useful to change the crystal temperature by a few tenths of a degree. As is shown in figure 3.4 the correct temperature of the doubling crystal is of major importance to maximize the output power.

If the crystal temperature is optimized and there is still a lack of power, it might be useful to check the output power of the tapered amplifier, which should be about 800 mW and the incoupling of that light into the cavity as explained earlier.

The last step in maximizing the power is to change the modulation dept of the 20MHz modulation of the master laser. The maximum amount of blue power is achieved for a small modu-
lation depth, however if the modulation depth is too small, the lock of the cavity to the fundamental laser may get unstable. This is a trade-off: the modulation depth should be as low as possible as it also creates sidebands to the blue light that might interfere with the experiments.

### 3.2.3 The wave meter

To monitor the wavelength of the lasers that are used in the experiments, a commercial wave meter, a LambdaMeter LM-007, is used. This wave meter employs four interferometers. The output value of the wave meter is given with a precision of $10^{-5}$ nm. Absolute calibration of the wave meter with such a precision is rather difficult as the absolute given value of the wavelength depends on the individual intensities of the interferometers. This depends on the incoupling of the light in the wave meter. The light is coupled into the wave meter with a home-made fiber coupler to provide more stability than when the laser is coupled in via free space.

### 3.3 Measuring spectroscopy signals

#### 3.3.1 Tuning the laser wavelength

**Computer control**

All the experiments of the UCP setup can be controlled with a computer. For the EIT measurements, two programs are written. One for scanning the wavelength of the blue laser: BlueLaserScan, and one for monitoring the intensity of the red laser: GetPDVoltage. Both programs can be controlled by the scan controller software and read out by the experiment monitor software. A schematic view of the software to hardware communication is shown in figure 3.6.

Both programs work via a National Instruments USB 6259 DAQ. The program BlueLaserScan divides an external reference voltage in 16 bits. Via the scan controller software, a voltage can be set on an analog output. This voltage is used to modulate the frequency of the fundamental diode laser by either changing the distance between the piezo and the laser diode with the SC110 module or by changing the current through the laser diode. With increasing current, the wavelength is increased, the current steps are small enough to prevent any measurable effect on the output power. Because the resolution of the wavelength meter is only $10^{-5}$ nm, the wavelength is interpolated to plot a more smooth graph. The scanning program also reads a value from the wave meter. The program GetPDVoltage reads the intensity. This can be the output voltage of a photodetector, but it can also be an output of electronics that enhances the signal to background ratio. In our experiment, the output of a lock-in amplifier is read by the program GetPDVoltage. The lock-in amplifier is used to increase the signal to background ratio. There is a built-in function to average a number of measurements and calculate the variance.

The result of a measurement is a folder that contains a number of text files. These files contain the settings of the experiment and comments written down by the experimenter. Other files contain settings of experimental steps and values acquired from the experiment.

**Modulation voltage on SC110**

If the wavelength of the coupling laser is scanned via the piezo, a modulation voltage is set on the analog input of the laser electronics which is connected with the scan controller SC110. A scan resolution of $2 \cdot 10^{-2}$ nm/V is obtained. The drawback of this scanning method is that the wavelength is not increased with every step in the modulation voltage. The wavelength as function of modulation voltage is more like锯齿形。Therefore it is not possible to interpolate the wavelength in a correct way so that the wavelength resolution is limited by the resolution of the wavelength meter. The behavior is shown in figure 3.7.
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Figure 3.6: A schematic overview of the computer control over the experiment. Left in the picture the experimental hardware is shown. The current control of the laser (DCC 110), the output of the lock-in amplifier (Lock-in output) and an external reference voltage source. In the center the National Instruments 16 bit DAQ is shown that allows communication between the PC and the experimental hardware. The software is shown completely left. All programs are connected to the scan controller. In this program a scan script can be written that stepwise increases the modulation input voltage on the DCC110 or SC110 via the program BlueLaserScan. This enables scanning of the wavelength of the coupling laser (red arrows). The dotted box around Scan script indicates that this is part of the program Scan controller. Each step in the measurement the output value of the Lock In amplifier is stored by the Experiment Monitor program via the program GetPDVoltage (blue arrows). Also the output value of the wave meter (purple arrow) and the value of the external reference source are stored (green arrow), both values via the program BlueLaserScan. The data is stored in a folder that contains files with the experimental details and for each measurement step a file with the scan controller data and experiment monitor data.

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Modulation voltage on DCC110

If the wavelength is scanned by changing the current through the master laser diode, by modulating the output of the current control DCC110, a scan resolution of $4.6 \times 10^{-3}$ nm/V is obtained. The wavelength increases directly proportional with the modulation voltage, as is shown in figure 3.8. A possible drawback of this method is that the output power of the fundamental diode laser is dependent on the current through the diode; therefore scanning the current over a large range is not advisable as for most experiments the output laser power should be constant. For the used scan ranges in the experiments the power of the 480 nm light did not change more than 1 mW, which is the resolution of the used power meter. Further this method ensures that for each step in modulation voltage the wavelength is increased. This enables us to interpolate wavelength so that the resolution of the wavelength meter is no longer the limiting factor.

3.3.2 Amplitude modulation

The expected EIT signals are small. Therefore it is important to improve the signal to noise ratio. One way of doing this is to use a lock in amplifier to amplify a modulated signal. The signal is amplitude modulated using an optical chopper in the coupling laser beam. A lock in amplifier has two inputs, the signal input and the reference input. A signal with a certain frequency is fed into the reference input. A signal (with noise) is fed into the signal input. The Lock In amplifier (Princeton Applied Research: model 5101) only amplifies the signal that has the same frequency as the reference signal. In our case we measure the voltage of a photodiode that is monitoring the power of the probe light that has passed the vapor cell. The signal from the photodiode is expected high if the coupling laser is on, when the absorption of the probe light vanishes due to EIT. The signal is expected to be low when the coupling laser is off, or when the coupling laser is on but off resonance. A reference output for the chopper is available to for the reference input of the lock in amplifier.

To perform quantum information experiments with Rydberg atoms, a frequency stabilized coupling laser is needed. Similar to the group in Durham [6] we aim to use EIT spectroscopy to lock the coupling laser to the desired Rydberg state with frequency modulation locking setup. Using an optical chopper in the coupling beam in combination with a lock in amplifier is a way to obtain a weak spectroscopy signal, but this method a some drawbacks. First of all the method with the lock-in amplifier is inherently slow. Further the coupling laser beam needs to be chopped, which might be a problem in other experiments. Using only a branch of this beam is not an op-
CHAPTER 3. EXPERIMENTAL SETUP

Figure 3.8: Left: the wavelength of the fundamental laser is plotted as function of the modulation voltage on the DCC110 module, for increasing modulation voltage the wavelength increases. Right: zoom; it is clear that for each step in the voltage the wavelength is increased. The wavelength meter has limited resolution hence the discrete steps in the wavelength. In this case it is possible to interpolate the wavelength to obtain a higher resolution than the resolution of the wavelength meter.

3.3.3 Frequency modulation

To use FM spectroscopy the probe laser beam is frequency modulated with an electro-optical modulator (EOM), this creates sidebands on the probe laser. The EOM is driven with a 20 MHz signal. A 20MHz signal is provided by the PDD module, this signal is used as the internal oscillator of the PDD module. However this 20MHz signal is too weak to drive the EOM. To avoid the use of too many amplifiers a resonance circuit is used. First the 20MHz is amplified by a Mini Circuits amplifier and then a buffer amplifier is used to reduce the impedance of this part of the setup, $R_s$, from 50Ω to about 1Ω. Then the signal is further amplified with a resonance circuit.

In this circuit the EOM is one capacitor with a capacitance of 12.2 pF, a coaxial cable acts as a second capacitance parallel to the EOM to tune the resonance frequency of the circuit. A coil box provides the inductance. The resonance frequency $\omega_r$ of the resonant circuit is given in terms of the capacitance $C$ and inductance $L$ by:

$$\omega_r = \frac{1}{\sqrt{LC}}. \quad (3.1)$$

Because the EOM should be driven with the 20MHz originating from the PDD module the circuit should be tuned to this resonance frequency. Therefore tunable capacitance in the form of a coax cable ($C_{coax} = 100pF/m$) is added to enable tuning the circuit with the cable length as tuning of the inductance is unpractical. A schematic view of the circuit is given in figure 3.9.

For a fixed resonance frequency $\omega_r$, coil box inductance $L$ and EOM capacitance $C_1$, a second capacitance parallel to the EOM should be chosen with the value $C_2$. This value can be calculated with the formula:

$$C_2 = \frac{1}{\omega_r^2L} - C_1. \quad (3.2)$$

The amplitude of the 20MHz signal at the EOM is $Q$ times the amplitude of the signal at the source, where $Q$ is the Q-factor of the circuit in terms of the source impedance $R_s$, inductance $L$
CHAPTER 3. EXPERIMENTAL SETUP

Figure 3.9: A resonant circuit is used to amplify the 20MHz of the PDD module before it is given to the EOM. The EOM act as one of the capacitors in the circuit. A buffer amplifier reduces the source impedance with a factor 50 to increase the Q-factor of the circuit. A coil box acts as an inductor. The system can be tuned to resonance with a coax cable acting as a parallel capacitance to tune the resonance frequency to 20MHz.

and capacitance $C$ are given by:

$$Q = \frac{1}{R_s} \sqrt{\frac{L}{C}}. \quad (3.3)$$

To test if the EOM actually produces sidebands at 20MHz, the probe laser beam is beatened with a branch of the cooling laser (laser beam B in figure 3.2). In figure 3.10 a beat signal of the probe laser and cooling laser is shown. The amplitude of the 20MHz is high enough to create even second order sidebands. Therefore an attenuator can be placed between the 20MHz source and buffer amplifier. In figure 3.10 four beat signals are shown with different attenuators. The Pound Drever Hall module (PDD 110) can transform an EIT signal of a probe laser with sidebands and a regular coupling laser beam in an error signal. Effectively the PDD takes the derivative of the EIT signal. Therefore it is no longer needed to lock on the top of signal. The PID controller can lock the laser to the slope of an error signal. If the intensity of the probe laser is not stable this would cause problems when the laser would be locked to top, because the maximum value of the signal would be variable. If the derivative is taken one can always lock to point were the error signal goes through zero.

The PDD module is characterized by feeding the output of the 20MHz internal oscillator to the input. The 20MHz signal is attenuated to create different effective input signals, the signal strength is expressed in dBm. The result is shown in figure 3.11.

3.3.4 Estimates of the signals

Estimate of the AM EIT spectroscopy signal

To estimate the amplitude of the EIT signal when amplitude modulation is used we will perform a ‘back on the envelope’ calculation. We assume to be in the regime of chapter 2, were the numerical integrations show that we can expect an signal to background ratio of 5%. We will use that the power of the probe laser is 80 $\mu$W at the detector. Because the coupling laser is chopped, the lock-in amplifier will amplify the difference between the probe laser with EIT and without EIT. The expected difference in power is $\Delta P = 4\mu W$ The photodetector has a detector efficiency of 0.5 A/W, so the expected photocurrent is 2 $\mu$A. The input impedance of the lock-in amplifier is set to 50k$\Omega$, which gives a voltage of 0.1V. Depending on the settings of the lock-in the magnitude of the output signal can be calculated.

Estimate of the FM EIT spectroscopy signal

Now we will perform a ‘back on the envelope’ calculation to find the expected amplitude of the error signal in the case FM spectroscopy is used. We assume to be in the regime discussed in
CHAPTER 3. EXPERIMENTAL SETUP

Figure 3.10: The intensity of a beat signal between the probe laser and a 80 MHz shifted branch of the probe laser is plotted as function of the frequency. An EOM is used to create sidebands on the probe laser. The 20MHz output of the PDD module is amplified and fed to the EOM via a resonant circuit. With attenuators the amplitude of the 20MHz signal that goes into the EOM can be modified.

Figure 3.11: The output voltage of the PDD110 module is shown as a function of the effective strength of a 20MHz input signal. The PDD 110 module of Toptica is characterized by feeding the internal oscillator signal of 20MHz back to the input of the PDD. The 20MHz signal is attenuated with mini circuits attenuator to produce low intensity 20MHz signals. For effective input signals between -90 dBm and -30dBm the PDD behaves linearly.
chapter 2, were the numerical integrations show an expected signal to background ratio of 0.3%, as in figure 2.12. For the calculation we assume to have a total probe laser power of $P_{\text{probe}} = 80 \mu\text{W}$, with the EOM set to create sidebands in the order of 10% of the total power. So the power of the main peak is assumed to be $P_0 = 64 \mu\text{W}$ and the power in each first order side band is assumed to be $P_s = 8 \mu\text{W}$. The maximum power of the beat signal is given by

$$P_{B_{\text{max}}} = \sqrt{P_0P_s}.$$  \hfill (3.4)

We expect that 0.3% of this number is the power of the EIT beat signal:

$$P_{B_{\text{EIT}}} = 0.07 \mu\text{W}.$$ \hfill (3.5)

The detector efficiency is 0.45 A/W [29] so the expected photodiode current $I_{B_{\text{EIT}}} = 31.5 \mu\text{A}$. The input impedance of the electronics is 50Ω, which gives an expected beat voltage amplitude of $V_{B_{\text{EIT}}} = 1.7\mu\text{V}$. The expected power in the electronic signal is then

$$P_{E_{\text{EIT}}} = 35\mu\text{A} \cdot 1.7\mu\text{V} = 59\text{fW},$$ \hfill (3.6)

which is $10 \log(59 \cdot 10^{-9}) = -132\text{dBm}$. To increase this number three mini circuits amplifiers are used to amplify the signal with 66dB to get an expected input signal of the PDD of -66dBm. The expected maximum output of the PDD, the amplitude of the error signal is expected to be in the order of 10mV.

### 3.4 Ultra cold plasma setup

Finally we will discuss the ultra cold plasma (UCP) setup [19]. For this thesis the UCP is only used as a reference to provide absolute calibration of the wavelength meter. But later in the project, the UCP will be used to perform experiments with ultra cold Rydberg atoms. Prove of principle experiments with Rydberg atoms in the UCP setup have already been performed, where the rubidium Rydberg blockade radius is measured [27].

The name UCP setup refers to the combination of a magneto optical trap inside an accelerator that is used to trap and cool rubidium atoms to a few $\mu\text{K}$ and than ionizing them. The charged particles can be accelerated from the MOT to a detector. For a short period of time, the atoms are ionized but still in the same region. This can be described as an ultra cold plasma as the ion and electron temperatures are very low. The minimum temperature of electrons in the UCP setup is found to be $14 \pm 3 \text{ K}$ [22]. The minimum temperature for rubidium ions is in the order of several $\mu\text{K}$ so the name ultra cold plasma is justified. A schematic view of the vacuum chamber, that contains the MOT coils and the accelerator is given in figure 3.12.

The UCP setup is not new: earlier the setup is used to create ultra cold coherent electron bunches [23, 22], and it can also be used as a source for ultra cold ions [24]. With an accelerator the electrons and ions are separated and one of the two species is accelerated from the MOT into the beam line [23, 24, 22]. The polarity of the accelerator can be switched to chose between the negative electrons or the positively charged ions as particles that enter the beam line.
Figure 3.12: Schematic view of the UCP vacuum chamber. The accelerator geometry includes access holes for the lasers. The excitation laser (red) is focused in the MOT and can be spatially controlled with a spatial light modulator. The blue laser is the ionization or Rydberg laser. This laser can be used to either photo-ionize the excited atoms in the MOT or excite them to a Rydberg level. After excitation to the Rydberg level the Rydberg atoms can be ionized by switching on the accelerator voltage. The ions are accelerated to a detector. This figure is adapted from [24]
Chapter 4

Experimental Results

4.1 Ultracold Rydberg excitation

Before studying EIT, it is necessary to find the wavelength of the coupling laser that excites rubidium from the intermediate state to a Rydberg state. The wavelength meter cannot be calibrated absolutely up to the precision of $10^{-5}$ nm with an off-the-shelf reference source, as there is no reference source with this precision available.

The wavelength meter is calibrated by finding the wavelength needed to excite atoms that are in the $5P_{3/2}$ state to a Rydberg state, remember figure 2.4. The theoretical value of the wavelength can be compared with the value given by the wavelength meter. The ultra cold plasma setup (UCP) is used to create and detect Rydberg atoms from a cloud of laser-cooled rubidium atoms [27, 3].

The theoretical wavelength needed for the coupling laser can be calculated if the energy of a Rydberg state $E_{nlj}$ is known. The energy of a Rydberg state can be calculated with equation 2.11, further the energy of the intermediate state $E_{5P_{3/2}}$ and the ionization energy $E_{ion}$ are needed, which are respectively given in table 2.3 and table 2.2. The wavelength of the coupling laser as function of the energy of the Rydberg state is given by:

$$\lambda_c = \frac{2\pi \hbar c}{E_{nlj} + E_{ion} - E_{5P_{3/2}}}.$$  \hspace{1cm} (4.1)

In the experiments with Rydberg atoms, ultra cold rubidium atoms in the MOT can be excited to either S or D state Rydberg atoms. This is a two step excitation process, remember figure 2.4. The laser beam used in the first step is both spatially and frequency wise controllable. The exact frequency can be adjusted with an AOM, see section 3.2. To image the Rydberg atoms, they can be ionized by switching on an accelerator electric field. The rubidium ions are then accelerated to a detector. This ion signal is used to detect the wavelength at which the coupling laser excites atoms to the Rydberg level.

In the experiments the 780 nm cooling beam is also used as the excitation beam and is continuously on. The coupling laser is switched on for 150 $\mu$s, 50 $\mu$s after the coupling laser is switched off, the electric field is switched on to ionize the present Rydberg atoms and accelerate the rubidium ions to the detector. Using this experiment, six Rydberg lines were found around $n = 40$. The result is shown in table 4.1. The expected wavelength of the infrared light $\lambda_{IR_{theory}}$ is compared with actual value given by the wave meter $\lambda_{IR_{exp}}$. Also the wavelength of the blue light, $\lambda_{blue} = \frac{1}{2} \lambda_{IR_{exp}}$ is given. We can calculate that the difference between the expected wavelength and the value of the wave meter is $(0.0024 \pm 0.0002)$ nm. The line width of the ion signal is found to be less than $2.5 \cdot 10^{-5}$ nm. In the domain of the blue light this is about 200 Mrad/s. Within ten digits on the Wavelength meter, an ion signal was found. This line width is much larger than expected as the line width of the transition to a Rydberg level, $\Gamma_r$ is small. However, it helps us to find the Rydberg lines within such a precision that we can perform EIT.
CHAPTER 4. EXPERIMENTAL RESULTS

<table>
<thead>
<tr>
<th>Rydberg state</th>
<th>$\lambda_{IR_{\text{theory}}}$ (nm)</th>
<th>$\lambda_{IR_{\text{exp}}}$ (nm)</th>
<th>$\Delta \lambda$ (nm)</th>
<th>$\lambda_{\text{blue}}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40D$_{5/2}$</td>
<td>961.49595</td>
<td>961.4987 ± 0.00005</td>
<td>0.0025</td>
<td>480.7494</td>
</tr>
<tr>
<td>40D$_{3/2}$</td>
<td>961.49623</td>
<td>961.4984 ± 0.00005</td>
<td>0.0025</td>
<td>480.7492</td>
</tr>
<tr>
<td>42S$_{1/2}$</td>
<td>961.45854</td>
<td>961.4610 ± 0.00005</td>
<td>0.0025</td>
<td>480.7305</td>
</tr>
<tr>
<td>41D$_{5/2}$</td>
<td>961.32718</td>
<td>961.3297 ± 0.00005</td>
<td>0.0022</td>
<td>480.6649</td>
</tr>
<tr>
<td>41D$_{3/2}$</td>
<td>961.32691</td>
<td>961.3294 ± 0.00005</td>
<td>0.0028</td>
<td>480.6647</td>
</tr>
<tr>
<td>43S$_{1/2}$</td>
<td>961.29226</td>
<td>961.2947 ± 0.00005</td>
<td>0.0024</td>
<td>480.6474</td>
</tr>
</tbody>
</table>

spectroscopy experiments.
It should be noted that one needs to take care when performing these measurements in the MOT. The voltage over the accelerator electrodes, needed to ionize the Rydberg atoms, is in general not zero between two pulses. Only for a specific combination of the ionization voltage and the experimental repetition rate the voltage over the accelerator plates is zero at time of Rydberg excitation. If the voltage is not zero at that time, there is an electric field and due to the Stark effect [3] ‘false’ lines can be found. This can also help us explain the large line width found in the UCP setup; if the electric field strength varies over several repetitions o the Rydberg excitation, the energy levels of the Rydberg states shift, which results in a large line width.

4.2 EIT spectroscopy results

In the EIT spectroscopy setup the same Rydberg lines are measured. All the transitions that are found with the UCP setup are also found with the spectroscopy setup. The four spectral lines are plotted in figure 4.1. For this measurement the probe laser is resonant with the transition between the rubidium ground state $5S_{1/2}$, $F = 3$ and the intermediate state $5P_{3/2}$, $F = 4$. The coupling laser is scanned by modulating the current through the fundamental laser diode. The power of the probe laser is 18 mW and the power of the coupling laser is 230 mW. The wavelength of the fundamental laser is measured and divided by two to obtain the wavelength of the coupling laser. The coupling laser beam is modulated with a chopper to enable the use of a lock-in amplifier to enhance the signal to noise ratio. The spectrum with the lines listed in table 4.1, measured in the EIT spectroscopy setup is shown in figure 4.1.

In table 4.1 we see that we expect the D line to be a doublet of lines, one for quantum number $J = 5/2$ and one for quantum number $J = 3/2$. If we zoom in on the spectrum and look at each separate line, we also see that the D lines are doublets. We zoom in on the lines from figure 4.1 and express the intensity (lock-in output) as a function of the detuning of the coupling laser beam. The frequency axis is obtained as follows; a linear fit of the wavelength step per measurement step (see figure 3.8) is performed to correct for the limited resolution of the wavelength meter. The frequency step in radials per second is calculated using equation 2.57.

The expected difference in coupling laser beam wavelength between excitation to the 41D$_{5/2}$ and 41D$_{3/2}$ Rydberg states that follows from equation 4.1 is $1.32 \cdot 10^{-4}$ nm, which corresponds with a frequency detuning of -1076 Mrad/s. For the 40D line, the difference between the different $J$ states is $1.43 \cdot 10^{-4}$ nm, which corresponds with a frequency detuning of -1165 Mrad/s. The expected frequency difference between the 40D$_{3/2}$ and 40D$_{5/2}$ lines as well as the expected frequency difference of between the 41D$_{3/2}$ and 41D$_{5/2}$ lines are agreement with the experiment as can be seen in figure 4.2.
Figure 4.1: The output of the lock-in amplifier as function of the coupling laser wavelength. Four lines are clearly visible: the light blue line is the $43S_{1/2}$ Rydberg line; the red line is the $42S_{1/2}$ Rydberg line, the green line is the $41D$ Rydberg line and the dark blue line is the $40D$ Rydberg line.

In figure 4.2 extra side bands can be seen at $\pm 126$ Mrad/s, these sidebands are due to the 20MHz\textsuperscript{1} modulation of the master laser to enable the locking of the doubling cavity to this laser. This results in 20MHz sidebands on both the fundamental 960 nm light as the 480 nm light. In the doubling crystal the frequency of the light is doubled by ‘adding’ two photons. The probability of adding a photon from the unshifted light with a photon from the 20MHz shifted light is much larger than adding two photons of the 20MHz shifted light. Therefore the sidebands on the 480 nm light are also found at 20MHz and not at 40MHz what one naively could expect.

\footnote{20MHz corresponds with $2\pi \times 20MHz = 125.7\text{Mrad/s}$.}
CHAPTER 4. EXPERIMENTAL RESULTS

4.3 EIT line characteristics

4.3.1 Dependence on the probe laser Rabi frequency

We will discuss several characteristics of the EIT spectroscopy signal. One of the most interesting features for signals obtained with a weak probe laser is the presence of Doppler wings. We will examine the features in the 41D signal. We use a D-line for this because the transition dipole moment for the transition between the intermediate and the Rydberg state $d_{re}$ is larger for a D-line than for an S-line. This results in a large Rabi frequency for the coupling laser at a given laser intensity, see figure 2.13.

For the case of a strong probe laser beam, $P_p = 84 \mu W$ which gives a Rabi frequency of $\Omega_p = 5.2 \Gamma_e$, we see in figure 4.3 that the FWHM of the experimentally obtained spectroscopy signal is $(80 \pm 5)$ Mrad/s. Both in the theoretical prediction as in the experimental signal no Doppler wings can be observed. This is due to the larger Rabi frequency of the probe laser beam in this case.

For the case of a weak probe laser beam, $P_p = 6 \mu W$ which gives a Rabi frequency of $\Omega_p = 0.9 \Gamma_e$, we see in figure 4.3 that the full width at half maximum (FWHM) of the spectroscopy peak is $(45 \pm 5)$ Mrad/s. Further we observe that both the experimental signal as the theoretical prediction have Doppler wings related to the velocity classes discussed in section 2.3.3. Because both signals are obtained with the same electronics, we expect that the ratio between the intensities found in the experiment can be explained with the theory. For that, the theoretical curve has to created in such a way that it resembles the experiment. Because a lock-in amplifier is used, not the absolute value of the intensity is measured, but the absolute difference between the intensity of the probe laser beam suspect to EIT and the intensity of the probe laser beam without the blue laser on. The signal of the weak probe laser beam is normalized and the same scaling factor is used for the strong probe laser beam, this way the magnitude of both signals can be compared.

For a strong probe laser Rabi frequency $\Omega_{p_sp} = 5.2 \Gamma_e$, a weak probe laser beam Rabi frequency $\Omega_{p_wp} = 0.9 \Gamma_e$ and a coupling laser beam Rabi frequency of $\Omega_c = 5.2 \Gamma_e$ the theoretical prediction of the line widths of both signals as well as the prediction for the relative magnitude of both signals agree with the experiments, as is shown in figure 4.3. However, the measured Doppler
CHAPTER 4. EXPERIMENTAL RESULTS

Figure 4.3: Two experimental 41D EIT signals (blue dots) are compared with the theory (red curves). In both figures the intensity is plotted as function of the detuning of the coupling laser beam in Mrad/s. The magnitude of the signal obtained with a weak probe laser beam, the right figure, is normalized. The magnitude of the signal obtained with a strong probe laser beam is scaled to the intensity of the signal obtained with the weak laser as discussed in section 4.3. The coupling laser Rabi frequency for both theoretical lines is given by $\Omega_c = 5.2 \Gamma_e$. probe laser rabi frequency is given by $\Omega_p = 5.2 \Gamma_e$ for the strong probe laser measurements and $\Omega_p = 0.9 \Gamma_e$ for the weak probe laser. The data points are in good agreement with the theoretical curves.

4.3.2 Dependence of the coupling laser Rabi frequency

In section 2.3.4 we discussed the influence of the Rabi frequency of the coupling laser. It is expected that a weak Rabi frequency of the coupling laser beam results in a signal with a smaller FWHM. To test this prediction a 42S $\frac{1}{2}$ line is compared with a 41D $\frac{5}{2}$ line. The transition dipole moment of the transition 5P $\frac{3}{2} \rightarrow 42S \frac{1}{2}$ is $9.1 \times 10^{-32}$ Cm and the transition dipole moment of the transition 5P $\frac{3}{2} \rightarrow 41D \frac{5}{2}$ is $4.5 \times 10^{-31}$ Cm, these transition dipole moments follow from equation 2.2.1 [8]. This results in a factor 5 difference between the Rabi frequencies of both transitions for the same intensity of the coupling laser beam, which follows from equation 2.16. In figure 4.4 the result is shown. The parameters for the theoretical curves in this figure are as follows: $\Omega_p = 2.2 \Gamma_e$, $\Omega_{c41D} = 6.5 \Gamma_e$ and $\Omega_{c42S} = 1.3 \Gamma_e$.

In figure 4.4 the line width (FWHM) of the 42S $\frac{1}{2}$ experimental signal is 34 Mrad/s whereas the FWHM of the 41D $\frac{5}{2}$ experimental signal is 76 Mrad/s. The line widths that follow from the theory are 29 Mrad/s for the 42S $\frac{1}{2}$ line and 67 Mrad/s for the 41D $\frac{5}{2}$ line. For the both the theoretical prediction as the experimental signals, the FWHM of the signal increases with an increase in Rabi frequency.

wings are broader and more shallow than predicted.

EIT spectroscopy on rubidium Rydberg states
Figure 4.4: Two experimentally obtained EIT signals (blue dots) with different Rabi frequencies for the coupling laser beam are compared with theoretical curves. The amplitude of both signals are scaled with respect to the $41D_{5/2}$ line. The left figure shows a spectroscopy signal of the $42S_{1/2}$ line, the theoretical curve in this figure is plotted for $\Omega_p = 2.2\Gamma_e$ and $\Omega_c = 1.3\Gamma_e$. The right figure shows a spectroscopy signal of the $41D_{5/2}$ line, the theoretical curve is plotted of $\Omega_p = 2.2\Gamma_e$ and $\Omega_c = 6.5\Gamma_e$. 

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Chapter 5

Conclusions and Outlook

5.1 Electromagnetically induced transparency

In this thesis electromagnetically induced transparency (EIT) in a rubidium vapor cell is investigated. A probe laser beam that excites rubidium atoms from the ground state to an excited state is normally absorbed. If a second laser is added that couples the excited state to a Rydberg state, the absorption of the probe laser beam vanishes if both lasers together are resonant with the energy difference between the Rydberg state and the ground state. The goal of this master project was to build an EIT spectroscopy setup and find if it is possible to use this setup to stabilize the frequency of the coupling laser. Therefore a theoretical model based on the optical Bloch equations is created to predict the amplitude and line width of experimental signals. Further an experimental setup is build and experiments are performed to find EIT spectroscopy signals. In this experimental setup signal modulation techniques are used to make small signals visible.

5.2 Theoretical model

A semi-analytical model is used to predict the experimental EIT spectroscopy signals. The model is based on the optical Bloch equations for a three-level atomic system interacting with two (nearly) resonant laser beams. The first step in the model is to define the master equation, which defines the optical Bloch equations. Then the steady state solution of the density matrix element $\rho_{eg}$ is found using Mathematica. With this matrix element the electric susceptibility of the system can be calculated. In this stage the model can predict EIT signals obtained in a ultra cold rubidium gas, these theoretical predictions are given in section 2.3.2.

The next step is adding a velocity distribution which represents the temperature of the rubidium gas in the vapor cell ($T_{\text{gas}} = 295K$). The last step is calculating the transmission of a resonant probe laser beam that couples the $5S_{1/2}$ rubidium ground state to the $5P_{3/2}$ through a rubidium vapor cell. This results in broader signals that can show Doppler wings, areas where there is actually more absorption of the probe laser 2.3.4. The Doppler wings become visible if the Rabi frequency of the coupling laser beam is higher than the Rabi frequency of the probe laser beam.

From this model it follows that the experimental conditions for EIT should be as follows; the Rabi frequencies of both lasers should be in the order of the transition line width of the bottom atomic transition. The Rabi frequency of the coupling laser beam determines the line width of the obtained reference signal; for a large line width a large Rabi frequency is needed, for a small line width a small Rabi frequency is needed. The magnitude of the signal is determined by the initial intensity of the probe laser beam.
5.3 Finding Rydberg lines

Rydberg lines have a small line width $\Gamma_r$, therefore it is important to know what wavelength should be used to couple between the intermediate state and a Rydberg state. The wavelength can be calculated using equation 4.1, and can be measured using a wavelength meter (section 3.2.3). Absolute calibration of this wavelength meter is performed with the UCP setup. In this setup rubidium atoms are laser cooled, before a two step excitation is used to excite the rubidium atoms to a Rydberg state. The Rydberg atoms are ionized by applying a pulsed electric field. The ions are measured with an MCP detector. Because there is no background signal in this setup, the signal is easy to find. The Rydberg states: $40D_{5/2}$, $40D_{3/2}$, $41D_{5/2}$, $41D_{3/2}$, $42S_{1/2}$ and $43S_{1/2}$ are detected using this technique. The experimental value of the wavelength is shifted $(0.0013 \pm 0.0002)\text{nm}$ compared with the calculated wavelength for all the Rydberg lines that are found in the UCP setup. With the Rydberg lines known with this precision it is possible to find them using the EIT spectroscopy setup.

5.4 EIT spectroscopy setup

A spectroscopy setup is built and tested. This setup contains a rubidium vapor cell in which EIT can occur. A resonant probe laser beam is available from the UCP setup. The coupling laser beam is provided by a high power frequency doubled diode laser system. The intensity of the probe laser beam, after passing the vapor cell, is measured as function of the wavelength of the coupling laser. Proper alignment of the coupling laser system is essential. In this system the frequency of a $960\text{nm}$ laser beam is doubled using a doubling crystal. To increase the intensity of the $960\text{nm}$ light, this light is confined in a bow-tie shaped cavity in which the crystal is installed. The power of the frequency-doubled light can decrease by two orders of magnitude if the laser is not coupled to the zeroth order Gaussian mode, or if the temperature of the doubling cavity is off by only $0.1\text{K}$. The scanning of the coupling laser is automated. Modulation of the current through the fundamental laser diode proved the most accurate way of scanning the wavelength (section 3.3.1). This control is implemented within the existing software architecture of the UCP setup (section 3.3.1).

5.4.1 Amplitude modulation

To increase the signal to noise rate, amplitude modulation (AM) of the coupling laser beam is provided by an optical chopper. A lock-in amplifier is used to amplify the difference in output signal of a photodetector between the probe laser beam subject to EIT and without the coupling on. Using these AM techniques the Rydberg lines: $40D_{5/2}$, $40D_{3/2}$, $41D_{5/2}$, $41D_{3/2}$, $42S_{1/2}$ and $43S_{1/2}$ are found in the spectroscopy setup at the same wavelength as they are found in the UCP setup. The experimental signals are compared to the theoretical predicted signals. For close agreement, the Rabi frequencies of both lasers are used as a scaling parameters. This is justified because of the experimental difficulties of determining the exact Rabi frequencies of the laser beams at the areas in vapor cell where EIT happens. The alignment is not exactly known and the power distribution of the laser profiles is Gaussian, where in the theoretical model the intensity (and with that the Rabi frequency) of the laser beam profile is assumed to be uniform. The line width an relative magnitudes of the signals can be explained with the theoretical model; however the absolute magnitude of the signal is much lower than predicted in section 3.3.4. A drawback of this AM technique is the time it takes to perform a single measurement, for weak signals it can take up to fifteen minutes to perform a single scan of the wavelength. Also due to the modulation of the coupling laser beam, this beam can not be used in Rydberg experiments while the EIT setup is at work. This makes it not possible to use this AM technique to directly lock the coupling laser to the EIT signal. Using an other branch
of the coupling beam is not an option as to much power of the coupling laser is then lost in the spectroscopy setup.

5.4.2 Frequency modulation

A frequency modulation (FM) technique can in theory be used to obtain a Pound Drever Hall signal that can be used to lock the coupling laser. The advantage of using a Pound Drever Hall signal is that the signal has a flank that passes through zero at resonance. Further, because the coupling laser beam is not modulated, this beam is available for Rydberg experiments after passing the EIT spectroscopy. For this technique an electro optical modulator (EOM) is installed that produces sideband on the probe laser beam. The EOM is driven by an amplified 20MHz signal. A Pound Drever Hall technique can be used to create an error signal, as this signal passes through zero at the resonance frequency, this technique can potentially be used to lock the coupling laser to the resonance frequency. For this technique a resonance circuit is build and tested to drive the EOM. Using a beatnote signal it was possible to prove that sidebands are created on the probe laser. Despite the fact that we have sidebands, no spectroscopy signal is observed using this technique. It is not clear why no signal is obtained.

5.5 Outlook

An alternative technique to lock the coupling to a Rydberg transition is to make use of an ultra stable cavity, like the cavity made by Stable Laser Systems that is bought for this research project. This technique has as advantage that it can reduce the line width of the coupling laser to a few kHz. The EIT signals obtained with the AM modulation technique can be used as a reference. Because the ultra stable cavity is temperature controlled the drift should be less than a MHz per week. An other advantage is that the lock of the coupling laser does not rely on the size of the spectroscopy signal. That is an advantage, especially for high Rydberg states, as the transition dipole moment scales with \( n^{-3/2} \) which reduces the Rabi frequency for the coupling laser beam. Using AM spectroscopy the integration time of the lock-in amplifier can be increased to find the resulting small signals.

Further it should be noted that we found a problem in the UCP setup. If a pulsed electric field is used to ionize Rydberg atoms, the field in the MOT is in general not zero at the time of Rydberg excitation. Due to the Stark effect, one can not be sure which Rydberg state is addressed [3]. The field in the MOT at the time of Rydberg excitations depends on the ionization voltage, the repetition rate of the experiment and the delay between Rydberg excitation and ionization of Rydberg atoms. A short term solution is to use a fast high voltage probe to find the 'sweet spot' where the electric field in the MOT is zero at the time of Rydberg excitation. A long term solution should be found if quantum information experiments are performed with the current setup as control over the exact Rydberg state is in that case very important.
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