MASTER

Eigenfrequency shifting due to fluid structure interaction analysis of fluid boundary conditions with DIANA

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Analysis of Fluid Boundary Conditions with DIANA 

Luuk de Bot 
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Eindhoven University of Technology 
August 1994
Eigenfrequency Shifting due to Fluid Structure Interaction

Analysis of Fluid Boundary Conditions with DIANA

Faculty of Mechanical Engineering
Eindhoven University of Technology

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Summary

The dynamic behaviour of structures can change considerably due to a partially enclosed fluid volume. Interaction is a result of the coupling between the equations of motion of both fluid and structure. A difficulty in modeling the partially enclosed fluid volume is its restriction with respect to the applied boundary conditions and the amount of fluid which must be taken into account. A simple boundary condition is ‘p=0’ which implies the absence of a pressure rise at the boundary. More advanced is the radiation boundary condition, which represents the radiation of pressure waves to the free space.

Analysis of the fluid boundaries has been done with a recently developed pilot version of the DIANA Finite Element System. The pilot version includes a non-symmetric eigenvalue subroutine to solve problems according to the fluid structure interaction theory. The finite element model consists of a rectangular plate, a rigid wall and a fluid layer in between. The model has a two dimensional character for fundamental purpose. The fluid structure model supplies more eigenfrequencies compared to a pure structural model. The additional eigenfrequencies result from the fluid and do not influence the structural dynamic behaviour. The fluid dominated eigenfrequencies are important in noise reduction problems.

Structure dominated eigenfrequencies change due to the added mass effect. Only the fluid directly between the rectangular plate and the rigid wall is relevant to calculate the frequency shift. The ‘p=0’ boundary of the ‘p=0’ condition is allowed because the difference with the radiation boundary condition is negligible with respect to the structure dominated eigenfrequencies. If the important structure dominated eigenfrequencies are smaller then the first fluid dominated eigenfrequency it is allowed to apply the ‘cheap’ symmetric eigenvalue subroutine from the present fluid structure interaction module in DIANA. The present module is, however, not suitable for acoustic analysis purposes.

Determination of the fluid dominated eigenfrequencies requires an accurate fluid model. The fluid boundary conditions have to be located at a sufficient distance from the structure which results in a larger fluid model. The sufficient distance depends on the considered problem. Both boundary conditions ‘p=0’ and ‘radiation’ supply, on this condition, good results. The radiation boundary condition is physical more realistic then ‘p=0’ because it includes acoustic damping in the model and it shows a better pressure distribution when the boundary is located close to the structure.
Symbols

Variables

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<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( c )</td>
<td>wave propagation velocity</td>
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<td>excitation force</td>
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<td>( n )</td>
<td>normal vector</td>
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<td>distance from a disturbance</td>
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<td>( u_a )</td>
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<td>matrix with interpolation functions</td>
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<td>( N_a(z) )</td>
<td>matrix with interpolation vectors</td>
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 Operators

\[
\nabla u = \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right) \quad (\text{grad})
\]

\[
\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad (\text{Laplace's operator, div grad})
\]

\[
\nabla \cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}
\]

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Subscripts

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<tr>
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</tr>
<tr>
<td>( I )</td>
<td>fluid structure interface</td>
</tr>
<tr>
<td>( P )</td>
<td>prescribed condition</td>
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<tr>
<td>( E )</td>
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<td>( \text{ext} )</td>
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Chapter 1

Introduction

This assignment has its origin in research to calculate eigenfrequencies of solar arrays. It is of major importance to have knowledge of the dynamic behaviour of solar arrays. During the launch of a satellite vibrations can be disastrous for solar arrays. It is proved by experiments that the air between solar arrays has a large influence on the eigenfrequencies. In previous research on this subject at the University of Eindhoven [3, Harrewijne] both experimental and numerical analysis have been carried out. Conclusions and recommendations of that report led to the continuation on the subject.

For the numerical analysis of fluid structure interaction problems the finite element package DIANA, [11] is used. A model consists of a structural part and a fluid part\(^1\). Coupling between the two parts is achieved by so called fluid structure interfaces. In a numerical analysis of fluid structure interaction systems, a major problem is the definition of boundary conditions of the fluid domain. This is necessary to restrict the fluid domain and consequently to restrict the numerical model to a reasonable number of degrees of freedom. Two boundary conditions have been analysed to restrict the fluid domain: a simple boundary condition called ‘p=0’ and the more advanced radiation boundary condition. Aim of this report is to examine the difference in numerical results and applicability of both boundary conditions.

Conclusion of the report of [3, Harrewijne] was to apply the ‘simple’ boundary condition ‘p=0’ instead of the radiation boundary condition because no difference was found in numerical results. The CPU-time for calculations of models with the ‘p=0’ boundary condition was much smaller compared to models with the radiation boundary condition. However this conclusion is not satisfying because the ‘p=0’ boundary condition is physically hard to understand. Another reason to doubt this conclusion was the rather primitive way to calculate eigenfrequencies of the numerical model with radiation boundary conditions.

In chapter two it is shown that the ‘p=0’ boundary condition only results in a simple eigenvalue problem if the fluid is assumed non-compressible. In all other cases the fluid structure interaction system results in a non-symmetric eigenvalue problem. Very recently TNO has developed a pilot version of DIANA which is capable of eigenvalue calculation of non-symmetric matrices in coupled fluid structure systems. This method arises when the radiation boundary

\(^1\)In the framework of this report a fluid should be considered as an acoustic volume, but for general purposes the term fluid is used.
condition is applied and the fluid is assumed compressible. Solution methods of coupled fluid structure systems are discussed in chapter three.

During the research on this subject experiments were carried out by [4, Kiel]. The experiments involved the influence of the air layer between two rectangular steel plates. Because of practical considerations, bolts were used to retain the distance between the plates, and to enable variation of the distance for experiments with different thickness of air layer. The experiments show a decrease in eigenfrequencies for thinner air layers.

For further experimental research [4, Kiel] recommends an experimental set-up without connections at the edges of the plates. Another recommendation is to measure eigenfrequencies of the model in a vacuum tank. Comparison between experiments in air and in vacuum enables quantification of the acoustic influence. In theory, results of vacuum experiments and results of numerical analysis without air should be similar. This enables the fitting of numerical data of the FEM-model. Model validation is then more correct to accomplish.

The main purpose of this research project is the study to the fluid boundaries. Because of computational aspects a simple numerical model is sufficient for this purpose. A two dimensional model is chosen which enables easy adaptation of the dimensions of the fluid domain. Dynamic analysis is done at several numerical models to study the influence of the boundary condition ‘p=0’ and the radiation boundary condition on the eigenfrequencies. The results of numerical analysis are discussed in chapter four. In the last chapter conclusions and recommendations will be discussed.
Chapter 2

Fluid structure interaction theory

In order to obtain fluid structure interaction in a finite element method it is necessary to describe the fluid behaviour analogous to the equations of motion for structures. The fluid domain is characterized by a common wave equation. This wave equation is discretized by means of the Galerkin method. With the appropriate boundary conditions this results in an equation of motion.

At the fluid structure interface coupling is achieved by equality between pressure in the fluid nodes and acceleration of the structure nodes. Finite element formulations for the fluid domain can be derived using pressure, displacements or displacements potential as independent variables. This chapter discusses only the formulation using pressure as independent variable to characterize the linear fluid.

2.1 Wave equation

The behaviour of fluid domain is entirely defined by the wave equation for fluid structure interaction purposes. The wave equation is simply postulated here without details about assumptions and simplifications. Reference is made to the report of [12, Van Veen] and the thesis of [9, Roozen].

\[
\nabla^2 p(x, t) = \frac{1}{c_s^2} \frac{\partial^2 p(x, t)}{\partial t^2} \quad (2.1)
\]

Multiplying the wave equation by arbitrary weight functions and integration at the fluid volume \( \Omega \) provides a weak formulation.

\[
\int_\Omega w(x) \frac{1}{c_s^2} \frac{\partial^2 p(x, t)}{\partial t^2} \, d\Omega - \int_\Omega w(x) \nabla^2 p(x, t) \, d\Omega = 0 \quad (2.2)
\]

Partial integration and applying Gauss theorem provides for the second integral term in equation 2.2.

\[
\int_\Omega w(x) \nabla^2 p(x, t) \, d\Omega = \int_\Gamma w(x) \nabla p(x, t) \cdot nd \Gamma - \int_\Omega \nabla w(x) \cdot \nabla p(x, t) \, d\Omega 
\]

Substitution of equation 2.3 in the weak formulation results in :

\[
\int_\Omega w(x) \frac{1}{c_s^2} \frac{\partial^2 p(x, t)}{\partial t^2} \, d\Omega + \int_\Omega \nabla w(x) \cdot \nabla p(x, t) \, d\Omega = \int_\Gamma w(x) \nabla p(x, t) \cdot nd \Gamma \quad (2.4)
\]
2.2 Boundary conditions

The boundary integral in the right-hand side of equation 2.4 is dependent on boundary conditions corresponding to different types of boundaries of the fluid domain. Considering a fluid domain partially connected to a structure, three possible boundaries are present.

2.2.1 Interface between fluid and structure

Air is considered as a non-viscid fluid, this implies the absence of friction between fluid and structure. Displacements at the structure $u_s$ and fluid displacement $u_f$ have to be equal along a normal at the structure. This requires:

$$ u_s \cdot \mathbf{n} = u_f \cdot \mathbf{n} \quad (2.5) $$

The velocity $v$ of fluid particles at the fluid structure interface are supposed to be equal to displacements $u_s$ differentiated with respect to time parallel to a normal vector $\mathbf{n}$.

$$ \frac{\partial p(x, t)}{\partial n} = \nabla p(x, t) \cdot \mathbf{n} = -\rho \frac{\partial u_s(x, t)}{\partial t} \cdot \mathbf{n} \quad (2.6) $$

$$ \nabla p(x, t) \cdot \mathbf{n} = -\rho \frac{\partial^2 u_s(x, t)}{\partial t^2} \cdot \mathbf{n} \quad (2.7) $$

Multiplying equation 2.7 by an arbitrary weight function and integration over the interface $\Gamma_I$ provides a boundary condition for interaction among fluid and structure.

$$ \int_{\Gamma_I} w(x) \nabla p(x, t) \cdot \mathbf{n} d\Gamma = -\rho \int_{\Gamma_I} w(x) \frac{\partial^2 u_s(x, t)}{\partial t^2} \cdot \mathbf{n} d\Gamma \quad (2.8) $$

2.2.2 Prescribed condition or rigid wall

In case of a rigid wall equation 2.5 gives $u_s \cdot \mathbf{n} = 0$. Because a rigid wall does not have an acceleration, equation 2.7 results in $\nabla p(x, t) \cdot \mathbf{n} = 0$. The boundary condition is simplified as shown in next equation.

$$ \int_{\Gamma_I} w(x) \nabla p(x, t) \cdot \mathbf{n} d\Gamma = 0 \quad (2.9) $$

2.2.3 Infinite extent

Although there are several possibilities to model the infinite extent, only two of them are discussed here. The first method is the Sommerfield condition or radiation boundary condition, the second method is the 'p=0' boundary condition. An approach to model the infinite extent was proposed by Sommerfield, [10, Sandberg]. Vibrations in the fluid domain causes radiation of pressure waves to the infinite extent where they will damp down. In a correct model, the pressure proceeds to zero and reflections of waves is not allowed. The Sommerfield method is exact if the incident wave is perpendicular to the boundary. In the existence of only outgoing waves, no waves are reflected. If the boundary is located at a sufficiently large distance from structural parts and from sources of disturbance in the fluid, it is a quite accurate method. This is shown in [8, Reinink].
Sommerfield employs the wave equation for an arbitrary scalar $\Phi$.

$$\nabla^2 \Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} \quad (2.10)$$

Consider a proceeding plane wave as a result of disturbances at the origin $r = 0$. A general solution of the differential equation is, with arbitrary scalars $\Phi_1$ and $\Phi_2$:

$$\Phi = \Phi_1(r - ct) + \Phi_2(r + ct) \quad (2.11)$$

No waves are admitted, so $\Phi_2 = 0$. Partial differentiation of equation 2.11 leads to:

$$\frac{\partial \Phi}{\partial r} = \Phi_1$$
$$\frac{\partial \Phi}{\partial t} = -c\Phi_1 \quad (2.12)$$

Eliminating the scalar $\Phi_1$ results in:

$$\frac{\partial \Phi}{\partial r} = \frac{-1}{c} \frac{\partial \Phi}{\partial t} \quad (2.13)$$

For a wave perpendicular to the boundary the following equation is written:

$$\frac{\partial \Phi}{\partial t} = -c (\nabla \Phi \cdot \mathbf{n}) \quad (2.14)$$

By replacing the arbitrary scalar $\Phi$ with the fluid pressure $p(x,t)$ and substituting the fluid wave propagation velocity in equation 2.14:

$$\frac{\partial p(x,t)}{\partial t} = -c_a (\nabla p(x,t) \cdot \mathbf{n}) \quad (2.15)$$

Equation 2.15 is multiplied by an arbitrary weight function and integrated at the boundary.

$$\int_{\Gamma_E} w(x) \nabla p(x,t) \cdot \mathbf{n} d\Gamma = -\frac{1}{c_a} \int_{\Gamma_E} w(x) \frac{\partial p(x,t)}{\partial t} d\Gamma \quad (2.16)$$

Boundary condition ‘$p=0$’ is physically difficult to understand. The effect of radiation waves is simply ignored, which results in a fluid model where damping is absent. It is presumed that the pressure at the boundary is zero in relation to an arbitrary reference pressure. The pressure at the nodes of the fluid model is based on pressure variations, not on absolute pressure.

2.2.4 Weak formulation with boundary conditions

Finally the boundary integrals in the previous section and the weak formulation, equation 2.4 are combined. This provides an equation that describes the behaviour of the fluid part of the fluid structure problem.

$$\int_{\Omega} w(x) \frac{1}{c_a^2} \frac{\partial^2 p(x,t)}{\partial t^2} d\Omega + \int_{\Omega} \nabla w(x) \cdot \nabla p(x,t) d\Omega =$$
$$-\rho_0 \int_{\Gamma_f} w(x) \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{n} d\Gamma - \frac{1}{c_a} \int_{\Gamma_E} w(x) \frac{\partial p(x,t)}{\partial t} d\Gamma \quad (2.17)$$
2.3 Equations of motion

It is necessary to have equations of motion for both the fluid and structure. Equation 2.17 will be written in discretized form in section 2.3.1 for finite element application describing a fluid domain. In section 2.3.2 the familiar equations of motion are provided for the structure. Combining the equations of motion results in coupled equations, section 2.3.3.

2.3.1 Fluid

Pressure $p(x,t)$ is approximated by a matrix comprising interpolation functions $N_a(x)$ and pressure at the fluid nodes $P_a(t)$ according to:

$$p(x,t) = N_a^T(x)P_a(t) \quad (2.18)$$

The following is valid for the structure displacement vector $u_s$ applying the matrix comprising interpolation vectors $N_s(x)$ and displacements $U_s(t)$ at the structure nodes.

$$u_s = N_s^T(x)U_s(t) \quad (2.19)$$

According to Galerkin's method, the matrix including interpolation functions $N_a(x)$ is used as weight function $w(x)$. Substitution in equation 2.17 results in:

$$\frac{1}{c_s^2} \int_\Omega N_a(x)N_a^T(x)\,d\Omega \hat{P}_a + \int_\Omega \nabla N_a(x) \cdot \nabla N_a^T(x)\,d\Omega P_a =$$

$$-\rho \int_{\Gamma_f} N_a(x)N_s^T(x)\,n_d\,d\Gamma \bar{U}_s - \frac{1}{c_s} \int_{\Gamma_B} N_a(x)N_s^T(x)\,d\Gamma \hat{P}_a \quad (2.20)$$

Equation 2.20, is in fact, the equation of motion for the fluid domain. Written down in matrix form this results in:

$$M_a \ddot{P}_a + C_a \dot{P}_a + K_a P_a = F_a \quad (2.21)$$

According to equation 2.20, the matrices defining mass, damping and stiffness of the fluid part are extracted. Note that both the mass matrix and damping matrix are dependent on the fluid wave velocity. The damping matrix arises from the radiation boundary condition of Sommerfield.

$$M_a = \frac{1}{c_s^2} \int_\Omega N_a(x)N_a^T(x)\,d\Omega \quad (2.22)$$

$$C_a = \frac{1}{c_s} \int_{\Gamma_B} N_a(x)N_a^T(x)\,d\Gamma \quad (2.23)$$

$$K_a = \int_\Omega \nabla N_a(x) \cdot \nabla N_a^T(x)\,d\Omega \quad (2.24)$$

Column $F_a$ in the right-hand side of equation 2.21 consists of forces applied by the structure to the fluid. So, this column represents coupling between fluid and structure. By equation 2.8 this leads to:

$$F_a = -\rho \int_{\Gamma_f} N_a(x)N_s^T(x)\,n_d\,d\Gamma \bar{U}_s \quad (2.25)$$
The matrix $A_{aa}$ is defined for coupling between fluid and structure:

$$A_{aa} = \int_{\Gamma} \mathbf{N}_a(\mathbf{x}) \mathbf{n} N_a^T(\mathbf{x}) d\Gamma$$

Equation 2.25 can now be written as:

$$F_a = -\rho^a A_{aa}^T \tilde{U}_s$$

### 2.3.2 Structure

This section discusses the structural part of fluid structure interaction. Only attention is paid to a general matrix equation of motion. The linear elastic structure is characterized by a mass matrix, damping matrix and stiffness matrix with kinematic degrees of freedom at the nodes $U_s$.

$$M_{ss} \ddot{U}_s + C_{ss} \dot{U}_s + K_{ss} U_s = F_s + F_{ext}$$

The right-hand side of equation 2.28 is subdivided in a column containing external forces and forces at the interface. The latter are forces applied by the fluid to the structure. The boundary conditions at the interface (equation 2.8) result in:

$$F_s = \int_{\Gamma} \mathbf{N}_a(\mathbf{x}) \mathbf{n} p(\mathbf{x}, t) d\Gamma = \int_{\Gamma} \mathbf{N}_a(\mathbf{x}) \mathbf{n} N_a^T(\mathbf{x}) d\Gamma \mathbf{P}_a$$

Using equation 2.26 this results in:

$$F_s = A_{sa} \mathbf{P}_a$$

### 2.3.3 Coupled equations

Combining the results from section 2.3.1 and 2.3.2, a coupled set of non-symmetric matrix equations is obtained. The effect of coupling is provided by the presence of matrix $A_{aa}$ in the overall mass and stiffness matrix, equation 2.31.

$$
\begin{bmatrix}
M_{aa} & 0 \\
\rho^a A_{aa}^T & M_{aa}
\end{bmatrix}
\begin{bmatrix}
\ddot{U}_s \\
\ddot{P}_a
\end{bmatrix}
+ 
\begin{bmatrix}
C_{aa} & 0 \\
0 & C_{aa}
\end{bmatrix}
\begin{bmatrix}
\dot{U}_s \\
\dot{P}_a
\end{bmatrix}
+ 
\begin{bmatrix}
K_{aa} & -A_{aa} \\
0 & K_{aa}
\end{bmatrix}
\begin{bmatrix}
U_a \\
P_a
\end{bmatrix}
= 
\begin{bmatrix}
F_{ext} \\
0
\end{bmatrix}
$$

The coupling phenomenon considers only those structural degrees of freedom that are associated with displacements perpendicular to the fluid boundary. The same applies for the fluid degrees of freedom, i.e. only pressure nodes at the structure are considered. For this reason most elements in matrix $A_{aa}$ are zero.

### 2.3.4 Compressibility

Compressibility of the fluid is represented by the mass matrix $M_{aa}$, equation 2.22. This can be seen from the relation between the fluid density and the wave velocity. The fluid density $\rho^a$ is related to the wave velocity $c_a$ by the bulk modulus according to equation 2.32.

$$c_a^2 = \frac{B}{\rho^a}$$
Chapter 3

Solution possibilities

There are several options to solve the coupled system. The most common method is the frequency response function (FRF) method. Peaks in the frequency response function determine the eigenfrequencies of the coupled system. A second method is the addition of extra degrees of freedom to make the system symmetric and subsequently it is solved by a eigenvalue subroutine. This method is applied in [9, Roozen], a disadvantage is the increase in the number of degrees of freedom and therefore not applied here. The coupled system results in a non-symmetric system therefore a non-symmetric eigenvalue solver is a useful method. In case of a non-compressible fluid model the symmetric eigenvalue subroutine can be used to solve the coupled problem.

With respect to numerical models for the finite element program DIANA 5.1 only solutions are discussed which can be applied here. Currently, there is no non-symmetric eigenvalue subroutine available in DIANA 5.1, however, one is included in a recently developed pilot version of the fluid structure interaction module. The direct solution method incorporates some major disadvantages. It solves the coupled system for only one frequency. Determination of eigenfrequencies is therefore highly time-consuming. In spite of these disadvantages, the direct method will be discussed in section 3.2.1 because it shows clearly the added mass and added damping effect on the structure caused by the fluid.

3.1 Non compressible fluid volume

Assuming that the fluid is non-compressible and using the ‘p=0’ boundary condition, the fluid interaction problem is simplified. This assumption is made to get a first impression of the effect of fluid on the structure. Both the acoustic mass matrix and the acoustic damping matrix disappear in the system. The coupled system of equation 2.31 gives the possibility to eliminate the acoustic pressure $P_a$.

$$ p_a = -[K_{as}^{-1}]^T \rho^a A_{as} \ddot{u}_s $$  \hspace{1cm} (3.1)

Substitution into the structure part of equation 2.31 results in an symmetric eigenvalue problem.

$$ (M_{as} + \tilde{M}_a)\ddot{u}_s + C_{as} \dot{u}_s + K_{as} u_s = 0 $$ \hspace{1cm} (3.2)

The fluid acts like an added mass to the structure $\tilde{M}_a$ and is defined as:
Solution possibilities

\[ \dot{M}_s = \rho^s A_{ss}[K_{ss}]^{-1} A_{sa}^T \]  

(3.3)

3.2 Compressible fluid volume

In case of a compressible fluid volume, solutions are more complex compared to that described in the previous section. The coupled system has to be solved by either a direct method or a non-symmetric eigenvalue subroutine.

3.2.1 Direct solution method

Consider a periodic forcing function of equation 2.31, written as:

\[ F_s(t) = \hat{f}_s e^{i\omega t} \]  

(3.4)

For linear problems, a steady-state solution has the same form, thus

\[ U_s = \hat{u} e^{i\omega t} \quad \text{and} \quad P_s = \hat{p} e^{i\omega t} \]  

(3.5)

By substituting the solution in equation 2.31, a single matrix is obtained.

\[
\begin{bmatrix}
-\omega^2 M_{ss} + K_{ss} + i\omega C_{ss} & -A_{sa} \\
-\rho^s \omega^2 A_{sa}^T & -\omega^2 M_{aa} + K_{aa} + i\omega C_{aa}
\end{bmatrix}
\begin{bmatrix}
\hat{u} \\
\hat{p}
\end{bmatrix} =
\begin{bmatrix}
\hat{f}_s \\
0
\end{bmatrix}
\]  

(3.6)

With this matrix, the complex values of amplitudes \( \hat{u} \) and \( \hat{p} \) are found. Pressure values \( \hat{p} \) are eliminated to obtain a single equation for \( \hat{p} \),

\[ \hat{p} = -\rho^s \omega^2 \begin{bmatrix} -\omega^2 M_{aa} + K_{aa} + i\omega C_{aa} \end{bmatrix}^{-1} A_{sa}^T \hat{u} \]  

(3.7)

where \( H(\omega) \) is the frequency response function. The frequency response function is complex and dependent on excitation frequency. Equation 3.7 is shortened to:

\[ \hat{p} = -\rho^s \omega^2 H_a(\omega) A_{sa}^T \hat{u} \]  

(3.8)

The forcing function can now be written as a function of \( \hat{u} \). This causes an additional fluid matrix \( \tilde{K}_I \).

\[ \tilde{K}_I = -\omega^2 \rho^s A_{sa} H_a(\omega) A_{sa}^T \]  

(3.9)

The additional fluid matrix is split up in a real part and a imaginary part. The real part provides an added mass matrix, the imaginary part provides an added damping matrix. This is denoted in the next equations.

\[ \begin{align*}
\tilde{M}_a &= \rho^s \Re \left( A_{sa} H_a(\omega) A_{sa}^T \right) \\
\tilde{C}_a &= \omega \rho^s \Im \left( A_{sa} H_a(\omega) A_{sa}^T \right)
\end{align*} \]  

(3.10)

Equation 3.9 is simplified to:

\[ \tilde{K}_I = -\omega^2 \tilde{M}_a - i\omega \tilde{C}_a \]  

(3.11)

The structure is now characterized by equation 3.12.

\[ \begin{bmatrix} -\omega^2 (M_{ss} + \tilde{M}_a) + i\omega (C_{ss} - \tilde{C}_a) + K_{ss} \end{bmatrix} \hat{u} = \hat{f}_s \]  

(3.12)

Solutions of equation 3.12 are found for each separate excitation frequency.
3.2.2 Non-symmetric eigenvalue problem

Define a new column $\vec{z}$ with degrees of freedom to transform the coupled equations 2.31 into a system description.

$$\vec{z} = \begin{bmatrix} U_s & P_a & \dot{U}_s & \dot{P}_a \end{bmatrix}^T$$

(3.13)

Substitution in equation 2.31 leads to:

$$C\ddot{\vec{z}} + D\vec{z} = 0$$

(3.14)

Choose a general solution for differential equation 3.14.

$$\vec{z} = v e^{\lambda t}$$

(3.15)

This results in a non-symmetric eigenvalue problem,

$$[\lambda C + D]v = 0$$

(3.16)

where matrices C and D are defined as:

$$C = \begin{bmatrix} B & M \\ M & 0 \end{bmatrix}, \quad D = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}$$

(3.17)

Matrices for mass, damping and stiffness originate from the coupled system in equation 2.31 and are defined as:

$$M = \begin{bmatrix} M_{ss} & 0 \\ \rho A & M_{aa} \end{bmatrix}, \quad B = \begin{bmatrix} C_{ss} & 0 \\ 0 & C_{aa} \end{bmatrix}, \quad K = \begin{bmatrix} K_{ss} & -A_{sa} \\ 0 & K_{aa} \end{bmatrix}$$

(3.18)

Non-symmetric eigenvalue problems can be solved by, for example, a Lanczos subroutine.

If boundary condition 'p=0' is applied, only the acoustic damping matrix $C_{aa}$ disappears. The problem remains non-symmetric, in contrast to section 3.1 where 'p=0' in combination with a non-compressible fluid volume resulted in a symmetric eigenvalue problem.

3.3 Pilot version fluid structure interaction

TNO has developed a pilot version of DIANA's fluid structure interaction module which contains some new features, [7, Nauta]. The most important feature is the non-symmetric eigenvalue subroutine. Fluid structure analysis with the pilot version conforms to the fluid structure theory explained in chapter 2. An adaptation is made to the radiation boundary elements which model the infinite extent by means of the Sommerfield condition. In DIANA 5.1, the wave propagation velocity was calculated by a frequency dependent recursion equation. This equation is replaced by a constant wave propagation velocity which can be specified in the input file. The acoustic damping matrix is determined by equation 2.23 for the specified boundary elements. The pilot version enables plotting of the structure's eigenmodes after fluid structure analysis. The direct method enables the determination of the pressure at the fluid nodes.
Chapter 4

Numerical results

In this chapter, numerical models will be examined with DIANA according to the fluid structure interaction theory described in chapter 2. The purpose of these calculations is to study in particular the influence of a 'thin' fluid layer on the eigenfrequencies and eigenmodes of the structure. Modeling the fluid part involves the problem where, and more important how to restrict the fluid part. The boundary condition ‘p=0’ and the radiation boundary condition have been chosen to study due to application in DIANA. Information about the numerical model in DIANA are described in appendices A and B. Solutions of the coupled equations of the fluid structure system are calculated by a non-symmetric eigenvalue subroutine. In special cases there is the possibility to use the computationally 'cheaper' symmetric eigenvalue subroutine. The numerical models are solved according to the solution methods described in chapter 3.

The numerical model is based on a two dimensional example of a rectangular plate where at one side a fluid volume is modeled. Because of the fundamental character of this research, it is sufficient to use a simple numerical model. Determination of the eigenvalue problem is therefore relatively time saving compared to more complex models. The rectangular plate is modeled by a row of nine quadratic shell elements in x-direction. The fluid volume contains linear volume elements, with a row of nine elements in x-direction and a column of nine elements in z-direction. The number of elements in y-direction is only one. The two dimensional character of this model appears in the boundary conditions along the x-axes for both plate and fluid. For the fluid holds the rigid boundary condition as for the plate the translations in x-and y-direction and the rotation about the x-axes are suppressed. The rectangular plate is modeled assuming the properties of steel; the fluid incorporates the properties of an acoustic volume. Pressure waves can travel freely perpendicular to the yz-plane, so at this position one of the fluid boundary conditions has to be applied.

This chapter begins with the separate analysis of the fluid system and the rectangular plate. Analysis without fluid structure interaction is called the uncoupled situation. First the boundary conditions ‘p=0’ and the radiation boundary condition are analysed with respect to the eigenfrequencies. The pressure distribution in the fluid layer is then calculated to interpret the fluid boundary conditions. The last point of analysis, the influence of the thickness of the fluid layer between the rectangular plate and the rigid wall is studied.
Figure 4.1: The four numerical models A, B, C and D

Figure 4.1 represents the expansion from a simple fluid structure interaction model to a more advanced model with respect to the 'open' fluid boundaries. Analysis is done according to the strategy from this figure in section 4.2. Model A represents an enclosed fluid volume to analyse the uncoupled situation. Model B represents the rectangular plate with an attached non-compressible fluid volume with boundary conditions 'p=0' at the open ends. Model C is suitable to analyse boundary conditions 'p=0' and the radiation boundary conditions at the open ends. In model D, the open ends have been modeled with elements. The influence of the fluid layer thickness on the eigenfrequencies has been examined in section 4.4. Finally the numerical results will be discussed.

4.1 Check numerical results

In this section, first both uncoupled situations are considered for the fluid volume and the rectangular plate. Uncoupled eigenfrequencies of the rectangular plate have been determined in DIANA by a symmetric eigenvalue subroutine. For a rectangular fluid volume enclosed by rigid walls, like model A in figure 4.1, there exists an analytical solution for eigenfrequencies,
Numerical results

equation 4.1 [12, Van Veen]. A calculation of eigenfrequencies of a separate fluid volume with DIANA is possible with fluid structure interface elements. According to model A, the rectangular fluid volume has been restricted by rigid walls and a rigid rectangular plate. If the stiffness of the rectangular plate is large enough the first eigenfrequencies of the fluid volume will not be influenced by the structure.

\[ f_a = \frac{c_a}{2} \sqrt{\left( \frac{l}{L_x} \right)^2 + \left( \frac{m}{L_y} \right)^2 + \left( \frac{n}{L_z} \right)^2} \]  

(4.1)

Where \( L_x, L_y, L_z \) are the dimensions of the rectangular volume and \( l, m, n \) are integers.

The first three eigenfrequencies for the uncoupled fluid both analytical and numerical are given in Table 4.1. The difference in analytical and numerical results is a consequence of the two dimensional modeling. The numerical results show that the first three eigenfrequencies originate from the fluid system. The stiffness of the rectangular plate is further decreased

<table>
<thead>
<tr>
<th>Model A</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>fluid</td>
<td>850.00 Hz</td>
<td>1700.0 Hz</td>
</tr>
<tr>
<td>Uncoupled DIANA</td>
<td>fluid</td>
<td>694.32 Hz</td>
<td>1426.6 Hz</td>
</tr>
<tr>
<td>Coupled DIANA</td>
<td>fluid</td>
<td>751.62 Hz</td>
<td>1425.3 Hz</td>
</tr>
<tr>
<td>Non compressible</td>
<td>fluid</td>
<td>641.94 Hz</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Analytical and numerical results

so that the first uncoupled eigenfrequency is approximately the same as the first uncoupled eigenfrequency of the fluid volume. According to [9, Roozen] and [10, Sandberg] strong coupling between fluid and structure occurs when both eigenfrequencies of the uncoupled systems are approximately the same.

In the coupled situation, distinction is made between fluid dominated eigenmodes and structure dominated eigenmodes. In the first case, the vibration energy is mainly caused by the fluid volume and in the other case mainly by the structure. The structure dominated eigenfrequency is smaller compared to the uncoupled eigenfrequency, the fluid is vibrating in phase with the structure which causes an added mass effect to the structure. The fluid dominated eigenfrequency is larger compared to the uncoupled situation. The fluid is vibrating out of phase to the structure which causes a larger stiffness of the fluid volume.

Solutions for the coupled system show a shift in the first eigenfrequency compared to both uncoupled results, (Table 4.1). The structure dominated eigenfrequency shifts 8% downwards, while the fluid dominated eigenfrequency shifts 8% upwards compared to the uncoupled situation. Numerical results of this model demonstrate that the coupled system provides eigenmodes which originate from the fluid and eigenmodes which originate from the structure. Both fluid and structure experience a large influence in case of strong coupling. The
The first structure dominated eigenfrequency has been represented in nodal displacements in figure 4.2 a), the first fluid dominated eigenmodes supplies the same eigenmode. Both eigenmodes have been represented in nodal pressures at the interface in figure 4.2 b). A negative pressure corresponds with a positive displacement of the plate for the structure dominated eigenmode, thus fluid and plate are vibrating in phase. The fluid dominated eigenmodes supplies a positive nodal pressure which corresponds with a positive nodal displacement, fluid and plate are now vibrating out of phase. Representation of fluid dominated eigenmodes in nodal displacements of the plate is possible due to the fact that there is the relation between structural displacement and pressure at the interface according to equation 2.7.

### 4.2 Analysis of fluid boundary conditions

This section contains the results of analysis of the fluid boundaries. The structural part is in each case a model of a rectangular steel plate vibrating near a rigid wall. The fluid between plate and wall has got the properties of air. The 19 mm thick fluid domain is restricted by either the boundary condition ‘p=0’ or the radiation boundary condition. First the rectangular models B and C are discussed and subsequently attention is paid to model D. The eigenfrequencies have been determined in the range from 0 Hz to 1600 Hz.

The rectangular models B and C are equal for boundary condition ‘p=0’ but there is difference in the applied eigenvalue subroutine. In model B the standard eigenvalue solver has been used and in model C the non-symmetric eigenvalue subroutine has been used. The application of the radiation boundary condition is therefore only possible for model C. The fluid in model B is non-compressible and in model C compressible.
Numerical results

Table 4.2: Rectangular fluid.

<table>
<thead>
<tr>
<th>19 mm air layer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>'vacuum'</td>
<td>4.0792</td>
<td>7.0680</td>
<td>271.88</td>
<td>747.83</td>
<td>1471.4</td>
<td></td>
</tr>
<tr>
<td>Model B</td>
<td>'p=0'</td>
<td>4.0502</td>
<td>7.0575</td>
<td>270.65</td>
<td>747.32</td>
<td>1471.5</td>
<td></td>
</tr>
<tr>
<td>Model C</td>
<td>'p=0'</td>
<td>4.0502</td>
<td>7.0575</td>
<td>270.60</td>
<td>747.18</td>
<td>864.17</td>
<td>1471.2</td>
</tr>
<tr>
<td>'radiation'</td>
<td>3.9924</td>
<td>7.0361</td>
<td>270.59</td>
<td>566.26</td>
<td>747.15</td>
<td>1322.6</td>
<td>1471.2</td>
</tr>
</tbody>
</table>

Model B supplies five eigenfrequencies in the considered frequency range, the corresponding eigenmodes are depicted in appendix C. All of them originate from the structure because the fluid is modeled non-compressible. In table 4.2 it can be seen that there is an added mass effect compared to the eigenfrequencies in 'vacuum' or the uncoupled plate. Determination of the eigenfrequencies of model C with a compressible fluid and boundary condition ‘p=0’ supplies six eigenmodes and even seven eigenmodes if the radiation boundary condition is applied.

Table 4.3: Extended fluid model with elements at the open ends.

<table>
<thead>
<tr>
<th>19 mm air layer</th>
<th>1, 7</th>
<th>2, 8</th>
<th>3, 9</th>
<th>4, 10</th>
<th>5, 11</th>
<th>6, 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model D1</td>
<td>'p=0'</td>
<td>4.0255</td>
<td>7.0464</td>
<td>270.59</td>
<td>644.57</td>
<td>747.15</td>
</tr>
<tr>
<td>r=100 mm.</td>
<td></td>
<td>1318.3</td>
<td>1471.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'radiation'</td>
<td>3.9674</td>
<td>7.0327</td>
<td>0.0000</td>
<td>270.59</td>
<td>680.65</td>
<td>747.16</td>
</tr>
<tr>
<td></td>
<td>1359.4</td>
<td>1471.2</td>
<td>1492.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model D2</td>
<td>'p=0'</td>
<td>4.0191</td>
<td>7.0442</td>
<td>270.59</td>
<td>522.98</td>
<td>639.75</td>
</tr>
<tr>
<td>r=200 mm.</td>
<td></td>
<td>818.07</td>
<td>1271.4</td>
<td>1327.5</td>
<td>1345.0</td>
<td>1471.1</td>
</tr>
<tr>
<td>'radiation'</td>
<td>3.9723</td>
<td>7.0336</td>
<td>0.0000</td>
<td>270.59</td>
<td>642.33</td>
<td>747.16</td>
</tr>
<tr>
<td></td>
<td>731.02</td>
<td>794.12</td>
<td>1314.6</td>
<td></td>
<td>1367.9</td>
<td>1471.2</td>
</tr>
</tbody>
</table>

The extra eigenmodes found in the frequency range originate from the fluid domain. Model C with the radiation boundary condition supplies damped eigenmodes. The damping for the structure dominated eigenmodes is negligible but the fluid dominated eigenmodes on the contrary incorporate a rather large damping. There is considerable difference between the two boundary conditions for the fluid dominated eigenfrequencies. The structure dominated eigenfrequencies show small differences between models B and C.

In model D the fluid domain is extended at the open ends, model D1 with a radius r=100 mm and model D2 with a radius r=200 mm. The boundary conditions have been placed at distance from the structure compared to the rectangular models B and C. Model D1 supplies eight and, nine eigenmodes for the 'p=0' and radiation boundary conditions, respectively (Table 4.3). The number of eigenmodes increases to twelve for model D2. The fluid dominated eigenfrequencies of model D with the radiation boundary condition are damped, the third eigenfrequency is a complete real eigenvalue. Extension of the fluid volume leads to an increase in the number of fluid dominated eigenfrequencies in the considered frequency range.
Fluid dominated eigenfrequencies are clearly influenced by the extension of the fluid volume. Structure dominated eigenfrequencies are almost identical for all models.

![Normalized pressure distribution in fluid layer: model C, \(p=0\)](image1)

![Normalized pressure distribution in fluid layer: model D, \(p=0\)](image2)

**4.3 Pressure distribution in the fluid layer**

The pressure distribution in the 19 mm thick rectangular fluid layer between plate and rigid wall has been calculated for models C, D1 and D2 with both boundary conditions. Calculations have been done for the first fluid dominated eigenfrequency. Appendix C contains the six results for the pressure distributions. In this section, only figures are depicted for models C and D1 with boundary condition \(p=0\).

Figure 4.3 and 4.4 show the normalised pressures in the rectangular fluid part. It can be seen that boundary condition \(p=0\) in model C does not give a realistic pressure distribution compared to model D, which has the same boundary condition but placed at a distance from the structure. The pressure value at the edges of the rectangular plate (x-position 0 and 200 mm.) is approximately 40% of the maximum pressure. Therefore it is not allowed to put the \(p=0\) boundary condition directly at the structure’s edge. It must be noted that the pressure values perpendicular to the plate in z-direction are practically constant.

The other models supply pressure distributions comparable to figure 4.4. Model C with radiation boundary condition gives good results although the boundary conditions are placed at the edge of the plate. Calculations in the radiation boundary elements show a tendency of the pressure to zero if the fluid boundary is located further away from the structure. This is in accordance with [10, Sandberg].
4.4 Results in relation to the fluid layer thickness

The eigenfrequencies are influenced by the thickness of the fluid layer. Models C and D1 with 'p=0' and radiation boundary conditions, are compared to each other. Only one structure dominated eigenmode and one fluid dominated eigenmode are discussed. For additional results, please refer to appendix C. Figure 4.5 shows the third fluid dominated eigenfrequency as function of the fluid layer thickness for the different acoustic models C and D1. A decrease of the fluid layer thickness results in an added mass effect which is similar for all models. Figure 4.6 shows the large increase of the eigenfrequency at decreasing thickness of the fluid layer.

The first fluid dominated eigenfrequencies as function of the fluid layer thickness are considerably different for thick fluid layers, however, the curves converge at thin fluid layers in figure 4.6. With model D1 and the radiation boundary condition as reference, there is good agreement with model D1 and 'p=0'. Model C with the radiation boundary condition shows a larger eigenfrequency shifting and model C with 'p=0' is only little influenced by the fluid layer thickness. The eigenfrequency of models with the radiation boundary condition are damped. Appendix C shows that the damping has the same characteristic as the eigenfrequency related to the fluid layer thickness.

![Figure 4.5: 3rd structural eigenfrequency.](image1)

![Figure 4.6: 1st fluid eigenfrequency.](image2)

4.5 Discussion

It is clear that in fluid interaction problem two types of eigenmodes have to be distinguished; first the eigenmodes which originate from the structure and second the eigenmodes from the fluid. The analysis of models B, C, D1 and D2 do not show the strong coupling effect as has been described in section 4.1. From the point of view of the structure, the fluid will interact only as an added mass to the structure. Excitation of the structure at the fluid dominated
Numerical results

eigenfrequencies results in relatively large pressure alternations. The physical meaning of fluid dominated eigenmodes is the increase of noise. Fluid dominated eigenmodes have to be avoided in noise reduction problems.

Results obtained with the non-symmetric eigenvalue subroutine may be confusing because there is no direct distinction between structure dominated and fluid dominated eigenmodes. The fluid dominated eigenmodes are plotted by DIANA as displacements of the structure due to the relation between pressure and acceleration at the structural nodes at the interface. Therefore care has to be taken in assessing the numerical results with respect to structure dominated eigenmodes and fluid dominated eigenmodes.

The numerical results obtained with the computationally 'cheap' symmetric eigenvalue solver are quite accurate, concerning the added mass on the structure when there is no strong coupling. If the important structural eigenfrequencies are lower then the first fluid eigenfrequency, the strong coupling effect does not occur, thus it may be justified to apply this method. However, if strong coupling occurs the coupled eigenfrequency is not correctly predicted. It should be noted that this method is not suitable for acoustic purposes.

The structure dominated eigenfrequencies are practical equal for model B,C,D1 and D2. All models describe the added mass effect due to the fluid on the structure correct. The influence of the acoustic mass matrix is small and the influence of the acoustic damping matrix can be neglected with respect to structure dominated eigenfrequencies. The added mass effect is caused by the acoustic stiffness matrix and the coupling matrix. However, acoustic mass and damping matrices are of major importance for the fluid dominated eigenfrequencies. For this reason, the influence of fluid volume size and boundary conditions is rather large.

The effect of structure dominated eigenfrequency shifting to smaller values due to a decreasing thickness of the air layer is a phenomenon which is not caused for reasons of strong coupling. This effect is merely caused by a relative higher pressure variation in the smaller fluid layer which causes a larger added mass effect on the structure. A logical consequence of a thinner fluid volume is the increase of the fluid dominated eigenfrequencies because of the increasing stiffness.

Damping in models C and D is introduced by the radiation boundary condition. The damping related to the fluid layer thickness has the same characteristic as the eigenfrequency related to the fluid layer thickness. The quantitative assessment of damping can not be done because of the lack of comparison possibilities. However, a model where damping is included gives a more realistic description of the fluid model, because the pressure distribution gives reasonable results if the boundary condition is located directly at the structure in contrast to the 'p=0' boundary condition. For acoustic analysis, both boundary conditions 'p=0' and 'radiation' have to be located at sufficient distance of the structure. This is concluded from the results of model B and D1 for the eigenfrequencies related to the thickness of the fluid layer.
Chapter 5

Conclusions and recommendations

5.1 Conclusions

The non-symmetric eigenvalue subroutine enables acoustic analysis with DIANA according to the fluid structure interaction theory. For a correct assessment of the numerical results, a clear distinction should be made between fluid dominated eigenfrequencies and structure dominated eigenfrequencies. Acoustic analysis requires accurate determination of the applied fluid model. For dynamic analysis of the structure, a simple fluid model is sufficient to determine the eigenfrequency shifting due to the added mass effect. Specific conclusions of this study are:

With respect to structure dominated eigenfrequencies

- Structure dominated eigenfrequencies are only influenced due to the added mass effect of the fluid. The fluid models with different sizes and boundary conditions supply practically identical results for the structure dominated eigenfrequencies. The influence of the acoustic damping matrix is negligible and the influence of the acoustic mass matrix is very small.

- The structure dominated eigenfrequencies are described correctly with all discussed compressible fluid models. If there is interest in the behaviour of the structure only, the rectangular fluid model C with boundary condition ‘p=0’ is already sufficient.

- If there is no strong coupling between fluid and structure the computationally ‘cheap’ standard symmetric eigenvalue subroutine may be used to determine the added mass effect. The fluid model is non-compressible with boundary condition ‘p=0’. This method is only convenient if the important eigenfrequencies of the structure are smaller than the first eigenfrequency of the fluid. It should be noted that this method is not suitable for acoustic analysis purposes.

- The influence of a decreasing fluid layer on the structure dominated eigenfrequencies is a decrease in eigenfrequency due to the added mass effect.
With respect to fluid dominated eigenfrequencies

- The rectangular fluid model C where 'p=0' boundary condition is placed directly at the edges of the structure is not a useful model for acoustic analysis. The pressure distribution in the fluid layer with 'p=0' at the edges is not correct because the pressure at the edges is approximately 40% of the maximum pressure according to the more advanced fluid models.

- The radiation boundary condition introduces damping which gives a more realistic fluid model. If the radiation boundary condition is located directly at the structure edge there is a pressure rise at the fluid boundary in contrast to the 'p=0' boundary condition.

- The fluid dominated eigenfrequencies as function of a decreasing fluid layer show that the boundary conditions have to be placed at a distance of the rectangular plate. The rectangular fluid model is therefore not correct with respect to the fluid dominated eigenfrequencies.

- The structural behaviour is not influenced by fluid dominated eigenfrequencies. Fluid dominated eigenmodes are important in noise reduction problems.

5.2 Recommendations

- For specific acoustic applications with DIANA, validation of numerical results with existing problems and experiments is recommended.

- Experiments should be done with a model which can be modeled accurately in DIANA to compare the results. With experiments in vacuum and in air, it is possible to determine the influence of air.

- The fluid structure interaction module (FLUSTR) should be split up in an acoustic analysis module with the non-symmetric eigenvalue subroutine and the present interaction module with the symmetric eigenvalue subroutine.

- The pilot version should have a built-in routine to detect fluid and structure dominated eigenfrequencies to avoid mistakes. The structure dominated eigenmodes should be represented in displacements, and the fluid dominated eigenmodes in contour plots of the pressure at the surface.
Bibliography


Appendix A

Fluid structure interface

Interface elements present coupling between structure elements and fluid elements. So, fluid structure interaction actually takes place at the interface. This results in a coupled set of equations.

A.1 Problem

To obtain fluid structure elements in a finite element model is a problem. This element naturally requires nodes connected to structure and to fluid. But strange enough it is not allowed for these nodes to have the same number, as though the nodes have the same coordinates. For this reason it is not possible to generate a mesh of the fluid structure coupling. Until now it had to be done completely by hand, which is quite a problem for large models.

A.2 Solution

To solve this problem a simple program is developed in MATLAB. First modelling is done by means of an external preprocessor like I-DEAS. In I-DEAS it is possible to create a universal file of the FEM model. The program 'sptdia' converts the universal file into a DIANA input file. The program's purpose is to create fluid structure elements from an existing set of node numbers. As input the MATLAB program requires:

- mesh size x,y
- node number edge fluid element at surface
- node number edge structure at surface

The results are written to a data file which can be added to the original file which contains the model data for DIANA.

A.2.1 Limitations

The program can be seen as a temporary solution, because there are some limitations. Errors may occur when a different node numbering sequence is applied. Furthermore, only mapped meshes can be applied.
A.3 INTGEN.m

% INTGEN.m
% Program to generate interface elements at interface
% between fluid and structure.
% Condition: nodes plate and fluid available in data-file.
% Mesh X Y Z.
% Written by Luuk de Boe, for Matlab 4.0
% Date: 10 maart 1994

clc, clear;
fprintf('
****************************FLUID STRUCTURE INTERFACE GENERATOR****************************
');
fprintf('******************************
');
fprintf('**** Fluid structure interaction with DIAREA ******
');
fprintf('**** Interface element BQ2454 for coupling ******
');
fprintf('**** quadratic shell to linear volume elements ******
');
fprintf('******************************
');

n=1:

X Y :

1:

M(1) M(2)

face=zeros(M(1)+1,M(2)+1);
mesh(face);
title('Your selected mesh');

Nodes connected to FLUID elements.
Start at element number one.

pt(1)=input('Give node at right edge FLUID element nr.1 : ');
pt(2)=pt(1)+1;
pt(4)=pt(2)+M(1);
pt(3)=pt(4)+1;
fprintf('
********************
');
fprintf('Nodes interface element nr.1 FLUID PART :
');
fprintf('********************
');

g(3)=pt(1)+M(1);
g(4)=g(3)-1;
g(1)=pt(1)-2;
g(2)=pt(1)-1;
elem(M(1)+M(2)+1,:)=g;

ghost element number

loop first element next row
for nr= M(1)+M(2):-M(1):1
k=nr+1;
punt=elem(k,3)+M(1);
elem(nr,:)=punt+2*M(1) punt+1+M(1);
end

loop rest of row
for i=nr:-1:(nr-M(1)+2)
k=i+1;
elem(i,:)=elem(k,2) elem(k,3)+1 elem(k,3)+1 elem(k,3)];
end
end

clear i k nr pt xx
elem(M(1)+M(2)+1,:)=[];

Nodes connected to SHELL elements
Start at element number one
Right hand rule, thumb in negative z-direction

pt(1)=input('Give node at right edge SHELL element nr.1 : ');
pt(2)=pt(1)+1;  
pt(3)=pt(2)+1;  
pt(4)=pt(2)+((M(i)+1)*2);  
pt(5)=pt(1)+((M(i)+1)*4);  
pt(6)=pt(5)-1;  
pt(7)=pt(6)-1;  
pt(8)=pt(4)-2;

fprintf('nNodes interface element nr.1 STRUCTURE PART :\n');

% ghost element number
v1=M(i);  
v2=M(2)+1;

for nr=v1:v2
   punt=shell(k,5)-N(1)*2;  
   hlo=(M(i)+1)*2;  
   hlp=(M(i)+1)*4;  
   shell(nr,:)=[punt punt+l punt+2 punt+l+hlo punt+hlp ...  
                 punt+hlp-1 punt+hlp-2 punt-1+hlo ];
end

% loop first element next row
for nr=(M(i)+M(2)):-M(1):1
   k=nr+1;  
   punt=shell(k,5)-N(1)*2;  
   hlo=(M(i)+1)*2;  
   hlp=(M(i)+1)*4;  
   shell(nr,:)=shl(k,3)+shell(k,3)+1 shell(k,3)+2 shell(k,4)+2 ...  
                  shell(k,5)+2 shell(k,5)+1 shell(k,4);  
end

% loop rest of row
for i=nr:-1:(nr-M(1)+2)
   k=i+1;  
   shell(i,:)=shell(k,3)+shell(k,3)+1 shell(k,3)+2 shell(k,4)+2 ...  
               shell(k,5)+2 shell(k,5)+1 shell(k,4);  
end

clear i k nr pt punt hlo hlp g

% Supply output
k=input('Give start nr. elements BQ2454 : ');  
filename=input('Give name to data file : ','s');  
file=[filename '.dat'];  
fid=fopen(file,'w');  
fprintf(fid,':', FLUID STRUCTURE INTERFACE \n');

% loop reverse sequence
for i=(M(i)+M(2)):-1:1
   fprintf('%d %d BQ2454 %d %d %d %d %d %d',k,shell(i,1),shell(i,2));
   fprintf('%d %d %d %d %d
',shell(i,3),shell(i,4),shell(i,5));
   fprintf('%d %d %d %d %d
',shell(i,6),shell(i,7),shell(i,8));
   fprintf('%d %d %d %d %d',shell(i,9),elem(i,1),elem(i,2));
   fprintf('%d %d %d %d
',elem(i,3),elem(i,4));
end

% Output to screen
fprintf(fid,' %d %d BQ2454 %d %d %d %d %d %d',k,shell(i,1),shell(i,2));
fprintf(fid,' %d %d %d %d %d %d %d %d',shell(i,3),shell(i,4),shell(i,5));
fprintf(fid,' %d %d %d %d %d %d %d %d',shell(i,6),shell(i,7),shell(i,8));
fprintf(fid,' %d %d %d %d %d %d %d %d',shell(i,9),elem(i,1),elem(i,2));
fprintf(fid,' %d %d %d %d %d',elem(i,3),elem(i,4));

fclose(fid);

clear i k M fid ans

% End program
Appendix B

Input files DIANA

Numerical analysis with DIANA requires a command file and a data file. The data file is often made by means of a preprocessor. Only part of the data file is listed in this appendix to give an impression of the numerical model used in this research project. The command file enables the user to select the desired subroutines in DIANA. The first command file calls the eigenvalue subroutine and the non-symmetric eigenvalue subroutine. The second command file has been used for the pressure calculations. For more detailed information is referred to [11, User's manual DIANA].

B.1 Command file for eigenmodes

*FILOS
  INITIA MA=8000000
*INPUT
*FLUSTR
  INITIA ANALYSIS MODAL
END INITIA
  EXECUTE
    ANALYSIS MODAL
    EIGEN NV=24
    FREQUEN 50. /
    NORESP
END

: Call of the non-symmetric eigenvalue subroutine
*eigen
  segment evalua/ev30i
  execute
eigen nv=20
  :perform qrtran ec=1.e-6
  perform lanczos nt=50 ec=1.e-6
  option mass
  option fsiact
  end
:
*GRAPH1
  MODEL EV=500. 500. 500.
  SELECT ELEMEN 1-9 /
  END MODEL
LAYOUT
  MODEL. LI=.. MODES. DATA.D LI=-
  END LAYOUT
SELECT
MODES 1-23(2) /
END SELECT
OUTPUT EIGEN
DISPLACEMENTS
END OUTPUT
*END
*END

B.2 Command file for pressure calculation

*FILOS
INITIA MA=8000000
*INPUT
*FLUSTR
INITIA
ANALYSIS total
option damping
END INITIA
SELECT
: only structure part
nodes 1-19 /
END SELECT
output print
displa
end output
EXECUTE
FREQUENCY 644.57 /
RESPONSE P LOAD(1)
EXCITATION HARMON
END response
END execute
: Pressure calculation for the excitation frequency
*STEADY
file *steady theta/flpres
select
: only fluid part
nodes 125-134 145-154 165-174 185-194 205-214
end select
output print
potent
end output
*END
*END

B.3 Data file

2D model, fluid volume rectangle; 19mm; mesh 9x1x9; Radiation Boundaries
'C00BDI`
: coordinates of the nodes
: Structure
1 0.00000E+00 0.00000E+00 0.00000E+00
2 5.00000E+00 0.00000E+00 0.00000E+00
3 1.00000E+01 0.00000E+00 0.00000E+00
4 2.28571E+01 0.00000E+00 0.00000E+00
5 3.57143E+01 0.00000E+00 0.00000E+00
: : :
56 1.05000E+02 1.00000E+01 0.00000E+00
57 2.00000E+02 1.00000E+01 0.00000E+00
### Fluid

<table>
<thead>
<tr>
<th>Element</th>
<th>Cavity Temperature</th>
<th>Wall Temperature</th>
<th>Gap Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>0.00000E+00</td>
<td>0.00000E+00</td>
<td>-1.90000E+01</td>
</tr>
<tr>
<td>59</td>
<td>1.00000E+01</td>
<td>0.00000E+00</td>
<td>-1.90000E+01</td>
</tr>
<tr>
<td>60</td>
<td>3.57143E+01</td>
<td>0.00000E+00</td>
<td>-1.90000E+01</td>
</tr>
</tbody>
</table>

### CONNECTIVITY

#### Structure elements

1. **CQ40F**
   - 1: 2, 3, 22, 41, 40
   - 39: 20
2. **CQ40F**
   - 2: 3, 4, 5, 24, 43, 42
   - 41: 22
3. **CQ40F**
   - 3: 6, 7, 26, 45, 44
   - 43: 24
4. **CQ40F**
   - 4: 7, 8, 9, 28, 47, 46
   - 45: 26
5. **CQ40F**
   - 5: 9, 10, 11, 30, 49, 48
   - 47: 28
6. **CQ40F**
   - 6: 11, 12, 13, 32, 61, 50
   - 49: 30
7. **CQ40F**
   - 7: 13, 14, 15, 34, 53, 52
   - 51: 32
8. **CQ40F**
   - 8: 15, 16, 17, 36, 55, 54
   - 53: 34
9. **CQ40F**
   - 9: 17, 18, 19, 38, 57, 56
   - 55: 36

#### Fluid elements

10. **HX8HT**
    - 58: 68, 88, 78, 59, 69
    - 89: 79
11. **HX8HT**
    - 59: 69, 89, 79, 60, 70
    - 90: 80
12. **HX8HT**
    - 60: 70, 90, 80, 61, 71
    - 91: 81

#### Radiation boundary elements

91. **BQ4HT**
    - 68: 88, 78, 58
92. **BQ4HT**
    - 88: 108, 98, 78
93. **BQ4HT**
    - 108: 128, 118, 98

#### FLUID STRUCTURE INTERFACE ELEMENTS

107. **BQ4HT**
    - 207: 217, 237, 227
    - 217: 237
108. **BQ4HT**
    - 227: 237, 257, 247

---

*Input files DIANA*
Input files DIANA

3008 BQ2454 15 16 17 36 55 54 53 34 245 246 256 255
3009 BQ2454 17 18 19 38 57 56 55 36 246 247 257 256

: spring elements plate
4001 SPITR 20
4002 SPITR 38

: 1 structure elements
: 2 radiation boundary elements
: 3 fluid elements
: 4 fluid structure interface elements
: 5 spring elements

MATERI
/ 1-9 / 1
/ 91-108 / 2
/ 10-90 / 3
/ 3001-3009 / 4
/ 4001-4002 / 5

GEOMET
/ 1-9 / 1
/ 10-90 / 3
/ 3001-3009 / 4
/ 4001-4002 / 5

: Material properties
: Dimensions in kg, mm, m²

'MATERI'
1 YOUNG 1.925200E+08
POISON 0.3
DENSIT 7.604000E-06
2 RADIAT
CSOUND 3.4040E+05
GRAVAC 9.81E+03
3 CONDUC 1.
CSOUND 3.4040E+05
4 DENSIT 1.2250E-09
5 SPRING 10

'GEOMET'
1 THICK 2.000000E+00
3 FLUID 0. 0. 0. 1.
4 NORMAL 0. 0. 0. -1.
5 AXIS 0. 0. 0. 1.

'DIRECT'
1 1.000000E+00 0.000000E+00 0.000000E+00
2 0.000000E+00 1.000000E+00 0.000000E+00
3 0.000000E+00 0.000000E+00 1.000000E+00

: Boundary condition 'p=0'

'FIXPOT'
: / 58-238(20) 68-248(20) 77-257(2) 67-247(20) /
/ 68 77 /

'SUPPOR'
/ 238-257 / pr
/ 1-19 39-57 / RD 1 TR 1 TR 2

'LOADS'
CASE 1
NODAL 32 F 3 1.E+03

: Dummy load for pressure analysis
: Case number must be twice the number of excitation frequencies

'BOUNDA'
CASE 2
NODAL 135 P 0.0

'END'
Appendix C

Numerical results with DIANA

This appendix contains the numerical results of the fluid structure interaction models. First the eigenmodes of both structure and fluid have been represented in displacements of the rectangular plate. Subsequently the pressure distribution in the fluid layer has been analysed. The last section contains the result of eigenfrequencies and damping related to the fluid layer thickness.

C.1 Eigenmodes

DIANA draws both fluid and structure dominated eigenmodes in nodal displacements of the structure.

Structure dominated eigenmodes

The first five eigenmodes of the rectangular plate after fluid structure interaction analysis are identical to the eigenmodes of the rectangular plate without the influence of the acoustic medium. The eigenmodes of the rectangular plate correspond to the bending modes of a beam. Figures C1 to C5 represent the structure dominated eigenmodes.

Fluid dominated eigenmodes

The first five fluid dominated eigenmodes have been represented in figures C6 to C10. The displacements of the rectangular plate give little information about the pressure distribution in the fluid at the interface due to the relation between displacement and pressure. These results should not be confused with structure dominated eigenmodes.

C.2 Pressure distribution

The pressure distribution has been calculated in the fluid layer between rectangular plate and rigid wall for models C, D1 and D2. The calculations have been done with the direct method, the excitation frequency is for each model the first fluid dominated eigenfrequency. Both boundary conditions ‘p=0’ and radiation have been applied which results in six pressure distributions. The length of the rectangular plate is 200 mm. The thickness of the fluid layer is 19 mm for all models. The rectangular plate is situated at z=0 mm and the rigid wall is situated at z=19 mm. In figures C11, C13 and C15 the pressure distribution has been depicted.
Numerical results with DIANA

for models C, D1, D2 with boundary condition ‘\(p=0\)’. In figures C12, C14 and C16 this has been done for the radiation boundary condition.

C.3 Influence fluid layer thickness

Both fluid and structure dominated eigenfrequencies are dependent on the thickness of the fluid layer between the rectangular plate and the rigid wall. Models C and D1 are analysed with both boundary conditions.

Structure dominated eigenfrequencies

In figures C17 to C21 the first five structure dominated eigenfrequencies have been shown as function of the fluid layer thickness. The range of the thickness is 1 mm to 80 mm. The four considered models show hardly no difference in results.

Fluid dominated eigenfrequencies

The first five fluid dominated eigenfrequencies have been depicted in figures C22 to C26. A logical consequence of a decreasing fluid layer is the increase of the eigenfrequency. The decrease in the fourth eigenfrequency of model D1 with the radiation boundary condition is caused by a change in the eigenmode. The third and fourth eigenfrequencies do not exist for model C.

Damping

Damping is introduced by the radiation boundary condition. Eigenfrequency and damping of models C and D1 have been represented in one figure for the first five fluid dominated eigenfrequencies. In figures C27 to C31 has been shown that the damping characteristic is equal to the characteristic of the eigenfrequency as function of the fluid layer.
Numerical results with DIANA

Figure C.1: Eigenmode SD 1
Figure C.2: Eigenmode SD 2
Figure C.3: Eigenmode SD 3
Figure C.4: Eigenmode SD 4
Figure C.5: Eigenmode SD 5
Numerical results with DIANA

Figure C.6: Eigenmode FD 1

Figure C.7: Eigenmode FD 2

Figure C.8: Eigenmode FD 3

Figure C.9: Eigenmode FD 4

Figure C.10: Eigenmode FD 5
Numerical results with DIANA

Figure C.11: $f=864.17$ Hz

Figure C.12: $f=644.57$ Hz

Figure C.13: $f=522.98$ Hz

Figure C.14: $f=566.26$ Hz
Numerical results with DIANA

Figure C.15: $f=680.65$ Hz

Figure C.16: $f=642.33$ Hz

Figure C.17: $1^{st}$ SD eigenfrequency

Figure C.18: $2^{nd}$ SD eigenfrequency
Numerical results with DIANA

Figure C.19: 3\textsuperscript{rd} SD eigenfrequency

Figure C.20: 4\textsuperscript{th} SD eigenfrequency

Figure C.21: 5\textsuperscript{th} SD eigenfrequency
Numerical results with DIANA

Figure C.22: 1st FD eigenfrequency

Figure C.23: 2nd FD eigenfrequency

Figure C.24: 3rd FD eigenfrequency

Figure C.25: 4th FD eigenfrequency
Numerical results with DIANA

Figure C.26: 5th FD eigenfrequency

Figure C.27: 1st FD eigenfrequency

Figure C.28: 2nd FD eigenfrequency
Numerical results with DIANA

Figure C.29: 3\textsuperscript{rd} FD eigenfrequency

Figure C.30: 4\textsuperscript{th} FD eigenfrequency

Figure C.31: 5\textsuperscript{th} FD eigenfrequency