MASTER

Stability analysis of a single point injection engine at idle

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Stability analysis of a single point injection engine at idle

Master's thesis

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Summary

Hunting is defined as low periodic speed fluctuations in engines at idle. It is rather inconvenient for the driver and can result in engine stall. Mainly multi point injection engines fitted with an after sales single point injection autogas system are sensitive to hunting. A problem with these systems is the use of control signals of the original multi point injection system to control a single point injection system. In earlier research, a model of a multi point engine with a single point autogas system was made and the stability of the linearized form of this model was analyzed. This showed a stable engine behaviour around a stationary point, and therefore it did not explain the hunting phenomenon.

The aim of this report is the investigation of the stability of the nonlinear model of the engine, using analysis of the phase plane. In the phase plane, trajectories of the state variables are analyzed. These state variables are the manifold air pressure and the number of revolutions of the crank shaft. In the phase plane the effect of multi point injection and parameter influences are investigated. These influences are studied by analyzing the direction of the slopes of the trajectories passing through points in the phase plane.

In the phase plane, the stability of the engine results in an attraction area. Trajectories inside this area converge to a stationary point, while trajectories outside this area diverge and result in engine stall. Hunting appears on the boundaries of this area. Stability is influenced by parameters such as manifold volume, inertia of the crank shaft, opening angle of the throttle valve, air/fuel ratio, spark advance angle and disturbance torque. The main cause of hunting is however, a fluctuation in air/fuel ratio due to the use of signals of the original multi point system to control the single point autogas system.

It can be concluded that hunting in the autogas system can be suppressed by an enrichment of the air/fuel ratio. However this has a negative effect on fuel consumption and the quality of the exhaust gasses. The best way to deal with hunting is the use of a multipoint injection autogas system. This results in a stable engine with clean exhaust gasses.
Notation

Symbols:

\( A_{\text{eff}} \)  effective open area of the throttle valve  \([m^2]\)
\( C_d \)  discharge coefficient  \([-]\)
\( D_{\text{th}} \)  throttle body throat diameter  \([m]\)
\( H_o \)  fuel heating value  \([J/kg]\)
\( J_{\text{tot}} \)  total moment of inertia of engine and load  \([kg\cdot m^2]\)
\( L \)  air fuel ratio  \([-]\)
\( L_{st} \)  stoichiometric air fuel ratio  \([-]\)
\( M_{fc} \)  fuel mass flow to the cylinder  \([kg/s]\)
\( N_c \)  number of revolutions of the crank shaft  \([\text{RPM}]\)
\( P_{\text{atm}} \)  atmospheric absolute pressure  \([N/m^2]\)
\( P_{\text{man}} \)  manifold absolute pressure  \([N/m^2]\)
\( Q_{fc} \)  cylinder fuel mass flow  \([kg/s]\)
\( Q_{fi} \)  injector fuel mass flow  \([kg/s]\)
\( Q_{mc} \)  cylinder mixture mass flow  \([kg/s]\)
\( Q_{mt} \)  throttle mixture mass flow  \([kg/s]\)
\( R_{m} \)  specific gas constant of mixture  \([J/kg\cdot K]\)
\( t \)  time  \([s]\)
\( T_{\text{atm}} \)  atmospheric temperature  \([K]\)
\( T_{fp} \)  torque to overcome internal frictional and pumping losses  \([N\cdot m]\)
\( T_i \)  induced mechanical torque  \([N\cdot m]\)
\( T_l \)  torque to overcome external loads  \([N\cdot m]\)
\( T_{\text{man}} \)  manifold temperature  \([K]\)
\( u \)  input vector  \([-]\)
\( V_c \)  stroke volume of a cylinder  \([m^3]\)
\( V_{\text{man}} \)  manifold volume, from throttle plate to intake port  \([m^3]\)
\( z \)  state vector  \([-]\)
\( y \)  output vector  \([-]\)
\( z \)  number of cylinders  \([-]\)
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<td>$\lambda$</td>
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**Abbreviations:**

- AFI: Air Fuel ratio equivalence Influence
- DMU: Dose and Mix Unit
- ECU: Electronic Control Unit
- LPI: Liquid Propane injection
- MAP: Manifold Absolute Pressure
- MPI: Multi Point Injection
- SI: Spark advance Influence
- SPI: Single Point Injection
- LPG: Liquified Petroleum Gas
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Chapter 1

Introduction

In order to control exhaust gasses, fuel consumption, and performance of the engine, Gentec has developed an autogas system for internal combustion engines. This system is equipped with single point injection and is fitted after sales on a petrol fuelled engine. In some circumstances it may cause instabilities at idle. These instabilities are low frequency periodic fluctuations in engine speed, and this phenomenon is called hunting. Especially the engine of a Volvo 440 type B18FP, seems to be sensitive to hunting. This originally multi point injection (MPI) engine, is equipped with a single point (SPI) autogas system. A problem with this configuration could be the use of control signals of the original MPI engine to control the SPI autogas system.

In earlier research, Smits [8] has developed a mathematical model of the engine, in order to find the causes for hunting. Bartsch [1] has tried to describe the stability of the engine and to develop a controller. This has been performed by linearizing the describing equations of the engine model. This research concluded that the amplitude of the fluctuations was too large to allow linearization of the equations.

The initial step of this research report is to describe the stability of the engine with nonlinear mathematical techniques. The complex nature of the problem makes it difficult to apply analytical techniques and therefore a lot of the analysis is done numerically. This research concentrates on analyzing the engine model using the phase plane. State variables in this analysis are the manifold air pressure and the number of revolutions of the crank shaft. In the phase plane the effect of MPI and parameter influences are investigated. These influences are studied by analyzing the direction of the slopes of the trajectories passing through points in the phase plane.

In Chapter 2, the engine system is described. Chapter 3 explains the engine model. Chapter 4 describes the fundamentals of phase plane analysis. In Chapter 5, the stability of the engine is analyzed with use of the phase plane. Here the influence of MPI and of parameters on the stability of the engine is described. In Chapter 6, the main conclusions are drawn and recommendations for further research are given.
Chapter 2

The engine system

2.1 The petrol fuelled engine

The engine to be investigated, a Volvo 440 engine type B18FP, is depicted in Figure 2.1. The engine is originally equipped with a Multi Point Injection (MPI) system which consists of four petrol injectors, each fitted in front of an intake port. The injectors are simultaneously activated by an Electronic Control Unit, the petrol ECU. The amount of petrol to be injected is based on the air flow to the cylinders. This air flow is measured according to the speed-density method [8]. This is an indirect method which is based on measurement of the manifold absolute pressure (MAP) and the engine rotation speed. Together with an estimation of the air temperature in the manifold these two quantities determine the mass flow of air from the manifold to the cylinders. The petrol ECU is programmed, by means of look-up tables with injection timing data as a function of the input variables. The values in these tables are found in an experimental way and are based and calibrated on an MPI petrol system with a speed-density measuring method. The signal produced by the petrol ECU is based on interpolating the input variables. The petrol ECU should guarantee a stable engine operation at any working condition.

2.2 The gas fuelled engine

Gentec’s gas injection system is fitted after sales on the original MPI engine. The engine is now dual fuelled. It can use petrol or gas as fuel. The driver can select the sort of fuel with a fuel switch.

The engine version to be investigated here has a Single Point Injection (SPI) autogas system, according to Gentec’s Liquid Propane Injection technology (LPI). This system has only one injector unit; the Dose and Mix Unit (DMU). The DMU, which consists of two injectors fitted together closely, is positioned upstream the throttle valve. A computed quantity of fuel is injected twice per engine revolution, in the DMU. This signal is computed by the petrol ECU and is corrected for gas injection by the gas ECU, which also uses the measurements of MAP and engine rotation speed.

In the original MPI petrol engine, the fuel is injected in front of an intake port. However, in
case of the SPI autogas system, the fuel is injected in front of the throttle valve, based on control signals of the original MPI engine. This could influence the stability of the engine.

![Diagram of the engine system](image)

**Figure 2.1: The engine system**

**Explanation:**

1. Intake port
2. Petrol injectors
3. Gas injector
4. Throttle valve
5. Manifold absolute pressure sensor
6. Crank shaft velocity sensor
7. Idle speed controller
8. Spark advance controller
9. \( \lambda \)-sonde

\( \omega_c \): Angular velocity of crank shaft
\( P_{\text{man}} \): Manifold air pressure
\( \Theta \): Opening angle of throttle valve
\( Q_{\text{fi}} \): Injected fuel

\( \lambda \): \( \lambda \) of exhaust gasses
\( \alpha \): Spark advance angle

**2.3 The control systems**

The engine consists of a closed loop control system, designed to function well in a normal operating range. Characteristic engine variables are measured and a feedback loop is used to control other engine variables.

Idle speed is defined as the engine rotation speed below 1350 revolutions per minute in combination with a closed throttle valve. In the idle range there are three active closed loop control systems: the idle speed controller, the spark advance controller and the \( \lambda \)-controller. These
controllers have to keep the idle speed at a constant value.

The idle speed controller consists of a bypass valve with a stepper motor and is fitted parallel to the throttle valve. A variation in engine speed causes a variation in mixture supply. The mixture has to pass through the intake manifold first and this is therefore a slow controller.

The spark advance controller minimizes the fluctuations in idle speed, regardless of the level of its changing mean. It varies the spark advance angle $\alpha$ as a function of the difference between instantaneous and mean idle speed. This has a direct influence on the induced torque, and this is therefore a fast controller.

The $\lambda$-controller keeps the air/fuel mixture at a stoichiometric level ($\lambda = 1$), which is necessary for optimal operation of the three way catalyst. The injected amount of fuel is determined by the composition of air measured by the $\lambda$-sonde. $\lambda$ is the air fuel equivalence ratio. A lean mixture is presented by $\lambda > 1$ and a rich mixture by $\lambda < 1$.

These three controllers are designed to work for a petrol fuelled engine. They are not designed to work for a gas fuelled engine. These controllers should suppress instabilities but under some circumstances they could cause instabilities at idle, called hunting. However even without these controllers a hunting phenomenon seems to be possible. This investigation aims at the stability of the engine without these control systems, the "open loop controlled" engine. The look-up tables in the petrol ECU are not changed. The gas ECU can be used to deal with the instabilities. It can only control one variable: the amount of fuel injected by the DMU.

2.4 Hunting

Hunting can occur under specific conditions and is defined as the periodic low frequency fluctuations of engine rotation speed around a nominal operating speed. Hunting is rather inconvenient for the driver and can lead to engine stall. This idle hunting phenomenon has a frequency of less than 1 [Hz]. The amplitude can be as high as 200 revolutions per minute.
Chapter 3

The engine model

3.1 Subsystems

The engine model to investigate the hunting phenomenon is a mean value model [8]. It can be divided in three dynamic subsystems:

- the manifold subsystem, describes the manifold pressure fluctuation between the throttle valve and the cylinder ports.
- the crank shaft subsystem, describes the transformation of fuel energy into kinetic energy.
- the fuel supply subsystem, defines a certain feedback law.

These subsystems are depicted in Figure 3.1.

Figure 3.1: The engine subsystems

This modular model is applicable to different engine versions. A blockdiagram of the engine model is given in Appendix A. The engine model and it’s different subsystems are described in this chapter.
3.2 The manifold subsystem

The equation for the time derivative of the manifold pressure, $\dot{P}_{\text{man}}$, is based on the law of conservation of mass. If the mixture is seen as an ideal gas, this law results in:

$$\dot{P}_{\text{man}} = \frac{R_m \cdot T_{\text{man}}}{V_{\text{man}}} \cdot (Q_{\text{mt}} - Q_{\text{mc}}) \quad (3.1)$$

where $Q_{\text{mt}}$ is the mixture mass flow past the throttle valve and $Q_{\text{mc}}$ is the mixture mass flow into the cylinder. $Q_{\text{mt}}$ is described in subsection 3.2.1 and $Q_{\text{mc}}$ in 3.2.2.

3.2.1 The throttle flow

The mixture mass flow past the throttle valve ($Q_{\text{mt}}$) depends on the pressure ratio $\gamma$,

$$\gamma = \frac{P_{\text{man}}}{P_{\text{atm}}} \quad (3.2)$$

where $P_{\text{man}}$ is the pressure in the manifold and $P_{\text{atm}}$ is the atmospheric pressure. The so-called critical pressure ratio $\gamma_{cr}$ is defined by:

$$\gamma_{cr} = \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa - 1}{\kappa + 1}} \quad (3.3)$$

where $\kappa$ is the adiabatic exponent of the mixture. If $\gamma > \gamma_{cr}$ then $Q_{\text{mt}}$ is subsonic and given by [1] and [4]:

$$Q_{\text{mt}} = C_d(P_{\text{man}}) \cdot A_{\text{eff}}(\Theta) \cdot \frac{P_{\text{atm}}}{\sqrt{R_m \cdot T_{\text{atm}}}} \cdot \left[ 2 \cdot \kappa \cdot \left( \frac{P_{\text{man}}}{P_{\text{atm}}} \right)^{\frac{1}{\kappa}} - \left( \frac{P_{\text{man}}}{P_{\text{atm}}} \right)^{\frac{\kappa + 1}{\kappa}} \right] \quad (3.4)$$

where $R_m$ is the gas constant of the mixture, $T_{\text{atm}}$ is the atmospheric temperature, $C_d$ is the throttle body or discharge coefficient. This coefficient compensates for flow losses and variations of the effective throttle area. $A_{\text{eff}}(\Theta)$ is the effective open area of the throttle valve. This area depends on the opening angle $\Theta$ of the valve and is given by:

$$A_{\text{eff}}(\Theta) = \frac{\pi}{4} \cdot D_{\text{th}}^2 \cdot \left( 1 - \frac{\cos(\Theta_0 + \Theta)}{\cos(\Theta_0)} \right) \quad (3.5)$$

It is assumed that $C_d$ only depends on the manifold pressure and can be approximated by:

$$C_d(P_{\text{man}}) = c_5 + c_6 \cdot P_{\text{man}} \quad (3.6)$$

If $\gamma < \gamma_{cr}$ then $Q_{\text{mt}}$ is supersonic and given by:

$$Q_{\text{mt}} = C_d(P_{\text{man}}) \cdot A_{\text{eff}}(\Theta) \cdot \frac{P_{\text{atm}}}{\sqrt{R_m \cdot T_{\text{atm}}}} \cdot \left[ \kappa \cdot \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa + 1}{\kappa}} \right] \quad (3.7)$$

For the mixture under consideration, $\kappa$ is approximately equal to 1.4. and the critical pressure ratio $\gamma_{cr}$ has the value of 0.53. At idle applies, $\gamma > \gamma_{cr}$. 

3.2.2 The cylinder flow

The mixture mass flow into the cylinder is given by:

\[ Q_{mc} = \eta_v \cdot V_c \cdot \frac{P_{man} \cdot z \cdot \omega_c}{R_m \cdot T_{man} \cdot 4 \cdot \pi} \]  

(3.8)

where \( T_{man} \) is the manifold temperature, \( \eta_v \) is the volumetric efficiency, \( z \) is the number of cylinders (i.e. \( z=4 \) for the B18FP engine), \( V_c \) is the stroke volume (=cylinder volume) and \( \omega_c \) is the angular velocity of the crank shaft. It is assumed that \( \eta_v \) only depends on \( \omega_c \) and \( P_{man} \) and is given by

\[ \eta_v = c_1 + c_2 \cdot \omega_c + c_3 \cdot \omega_c^2 + c_4 \cdot P_{man} \]  

(3.9)

The fuel mass flow into the cylinder is given by:

\[ Q_{fc} = \frac{Q_{mc}}{1 + \lambda_c \cdot L_{st}} \]  

(3.10)

where \( L_{st} \) is the stoichiometric air fuel ratio, i.e. the ratio of air and fuel for which perfect combustion takes place. For LPG, this ratio is 15.81. The quantity \( \lambda_c \) is the air fuel equivalence ratio i.e. the real air fuel ratio \( \lambda \) divided by the stoichiometric air fuel ratio \( L_{st} \) in the cylinder:

\[ \lambda_c = \frac{L_c}{L_{st}} = \frac{1}{L_{st}} \cdot \frac{Q_{ac}}{Q_{fc}} \]  

(3.11)

where \( Q_{ac} \), the air mass flow to the cylinder, follows from,

\[ Q_{ac} = Q_{mc} - Q_{fc} \]  

(3.12)

If \( \lambda_c < 1 \) the mixture is rich and if \( \lambda_c > 1 \) the mixture is lean.

The fuel mass flow to the cylinder is given by:

\[ m_{fc} = \frac{Q_{fe} \cdot 4 \cdot \pi}{\omega_c \cdot \pi} \]  

(3.13)

3.3 The crank shaft subsystem

The equation for the time derivative of the angular velocity of the crank shaft speed is given by:

\[ \omega_c = \frac{1}{J} \cdot [T_i - T_{fp} - T_i - T_d] \]  

(3.14)

Here \( T_i \) is the induced torque,

\[ T_i = \eta_t \cdot H_o \cdot \frac{z}{4} \cdot \pi \cdot m_{fc} \]  

(3.15)

with the fuel heating value of combustion \( H_o \) and thermal efficiency \( \eta_t \),

\[ \eta_t = \eta_{opt} \cdot AFI(\lambda_c) \cdot SI(\alpha) \]  

(3.16)
The optimal thermal efficiency \( \eta_{opt} \) has a constant value between 0.2 and 0.4. AFI is the Air Fuel ratio equivalence Influence and SI is the Spark advance Influence. AFI accounts for the dependence of \( T_i \) on \( \lambda_c \), SI for the dependence on \( \alpha \):

\[
AFI(\lambda_c) = c_7 + c_8 \cdot \lambda_c + c_9 \cdot \lambda_c^2 + c_{10} \cdot \lambda_c^3
\]

\[
SI(\alpha) = c_{11} + c_{12} \cdot \alpha + c_{13} \cdot \alpha^2
\]

The quantity \( T_{fp} \) in Eq. 3.14 is the dissipative torque due to frictional and pumping losses. It depends on \( P_{man} \) and \( \omega_c \) and it is assumed that:

\[
T_{fp} = c_{16} + c_{17} \cdot \omega_c + c_{18} \cdot \omega_c^2 + c_{19} \cdot P_{man} + c_{20} \cdot \omega_c \cdot P_{man}
\]

\( T_i \) is a continuously present load torque. At idle this is the drive torque of the dynamo. This torque depends on the angular velocity of the engine. For \( T_i \) it is assumed that:

\[
T_i = \frac{1}{\sqrt{c_{14} + c_{15} \cdot \omega_c}}
\]

\( T_d \) is a possible disturbance torque on the dynamo. This torque occurs due to electrical appliances like air blowers or powersteering.

### 3.4 The fuel supply subsystem

The fuel is injected just in front of the throttle valve in order to get the best possible mixing of fuel and air. The two state variables \( P_{man} \) and \( \omega_c \) are fed back to the input variable \( Q_{fi} \) by means of a feedback strategy based on the look up tables in the petrol ECU. This version adds fuel to the cylinder flow, according to,

\[
Q_{fi} = \frac{Q_{mc}}{1 + Lst \cdot \lambda_{ecu}}
\]

here \( \lambda_{ecu} \) is the preferred \( \lambda_c \). This value depends on the strategy of the ECU.

To obtain clean exhaust gasses \( \lambda_{ecu} \) should have value 1. However, to achieve optimal engine performance, also in situations of acceleration and deceleration, enrichment or enleanment of the mixture might be necessary. The controlled system has in those cases a dynamic feedback. At idle we assume \( \lambda_{ecu} = 1 \) and a possible dynamic feedback in the petrol ECU is left out of consideration.

With this feedback strategy, a MPI engine is always provided with a stoichiometric mixture. In the case of MPI there is no delay time and \( \lambda_i = \lambda_c \). However in case of SPI, some delay time \( \tau_{man} \) elapses between the moment of injection of fuel and the moment this fuel reaches the cylinder. In this case \( \lambda_c(t) = \lambda_i(t - \tau_{man}) \). This \( \tau_{man} \) is modelled as,

\[
\tau_{man} = \frac{2 \cdot \pi \cdot V_{man}}{2 \cdot \eta_v \cdot V_c \cdot \omega_c}
\]

\( \tau_{man} \) varies with \( \omega_c \). At idle, where \( \omega_c \) is low, \( \tau_{man} \) is large.
3.5 State description

The equations for the engine can be written in a state space form as

\[ \dot{z} = F_3(z, u, w) \]  (3.23)

where \( w \) is a disturbance (the disturbance torque \( T_d \)), \( z \) is the state vector and \( u \) is the input vector:

\[ z^T = [P_{\text{man}} \ \omega_c]; \quad u^T = [Q_{fi} \ \Theta \ \alpha] \]  (3.24)

\( Q_{fi}, \Theta, \alpha, \) are input variables in the model because they can be externally adjusted.

The measured variables are put in a vector \( y \), the output:

\[ y^T = [P_{\text{man}} \ \omega_c \ \lambda_{\text{exh}}] \]  (3.25)

\( y \) can be used to determine \( u \). As mentioned in section 2.3 hunting can occur if the inputs \( \Theta \) and \( \alpha \) are not changed. \( Q_{fi} \) is determined by the gas ECU, which also uses the measurements for \( P_{\text{man}} \) and \( \omega_c \). In the sequel we consider the "open loop controlled" engine at idle where the input \( Q_{fi} \) is determined by Eq. 3.21, and where \( \Theta \) and \( \alpha \) are fixed. For input \( u \) applies,

\[ Q_{fi} = F_2(z) \]  (3.26)

The relevant equation for this system can now be written as,

\[ \dot{z} = F_3(z, u) \]  (3.27)

Note that in Eq. 3.27 \( \tau_{\text{man}} \) is implicitly present.
Chapter 4

Phase plane analysis

4.1 Introduction

An important aspect in the design of a controller is to guarantee the stability of the controlled system. It is therefore important to know which parameters influence stability. Bartsch [1] linearized the engine model and determined the poles of the system. The real part of the poles of the linearized system were negative, implying that the linearized model is asymptotically stable. This research did not explain the hunting phenomenon. It is assumed that the linearization process has influence on the occurrence of hunting and therefore the system has to be analyzed in its nonlinear form.

Phase plane analysis is a technique to investigate stability of a nonlinear system [5],[7]. Phase plane analysis is a graphical method for studying systems with two state variables. The basic idea of the method is to generate motion trajectories corresponding to various initial conditions, in the state space of such a system, i.e. in a two-dimensional plane called the phase plane. Such an illustration is called a phase portrait. The qualitative features of the trajectories can then be examined and information on stability and other characteristics of the system can be obtained. The engine system can be described with two coupled first order differential equations and therefore the phase plane method is applicable. Phase plane analysis visualizes the behaviour of a nonlinear system starting from various initial conditions, without having to solve the nonlinear equations analytically. It is not restricted in the degree of nonlinearity. However it is restricted by $\tau_{\text{max}}$.

4.2 Phase plane description

The phase plane method is concerned with the graphical representation of autonomous systems described by:

$$\dot{z} = F(z); \quad z^T = [x_1 \; x_2]$$

(4.1)

where $x_1$ and $x_2$ are the state variables and $F$ is a nonlinear function of $z$. The phase plane is the geometrical representation of the state space, i.e. the set of all states $z$.

For each set of initial conditions $z(0) = z_0$, Eq. 4.1 defines a solution $z$ which can be represented as a trajectory, in the phase plane.
4 Phase plane analysis

The state space model for the engine (Eq. 3.27) is in the form of 4.1 with $\mathbf{z}$ defined by

$$\mathbf{z}^T = [P_{man} \quad \omega_c] \quad (4.2)$$

However, this is true only if the disturbance $T_d = w$ is constant.

### 4.3 Equilibrium points

A state $\mathbf{z}_s$ is an equilibrium state (or equilibrium point) of the system if $\mathbf{z}(t_0) = \mathbf{z}_s$ implies $\mathbf{z}_s = \mathbf{z}_s$ for all $t \geq t_0$. Mathematically, this means that $\mathbf{z}_s$ has to satisfy:

$$0 = F(\mathbf{z}_s) \quad (4.3)$$

### 4.4 Linear systems

Bartsch [1] linearized the state equations around the equilibrium point:

$$\dot{\mathbf{z}}_s + \delta \dot{\mathbf{z}} = F(\mathbf{z}_s, w) + \frac{\delta F(\mathbf{z}_s, w)}{\delta \mathbf{z}} \cdot \delta \mathbf{z} \quad (4.4)$$

With the use of,

$$\dot{\mathbf{z}}_s = F(\mathbf{z}_s, w) \quad (4.5)$$

this can be written in the form:

$$\delta \dot{\mathbf{z}} = A(\mathbf{z}_s, w) \cdot \delta \mathbf{z}; \quad A_{ij} = \frac{\delta F_i}{\delta z_j} \quad (4.6)$$

The equilibrium point is stable if all eigenvalues of $A(\mathbf{z}_s, w)$ have negative real parts. In appendix B phase portraits of linear systems are depicted and, furthermore, the complex plane is depicted with the poles. It was shown that the real parts of the eigenvalues of $A(\mathbf{z}_s, w)$ are negative, so the engine system is stable and hunting is not explained.

### 4.5 Limit cycles in nonlinear systems

The behaviour of nonlinear systems is much more complex than that of linear systems. A nonlinear system can have multiple equilibrium points and limit cycles can occur. A limit cycle is an oscillation of fixed amplitude and fixed period without external excitation. In the phase plane a limit cycle is an isolated closed curve. The trajectory has to be both closed, indicating the periodic nature of the motion, and isolated, indicating the limiting nature of the cycle with nearby trajectories converging or diverging from it. Depending on the motion patterns of the trajectories in the vicinity of the limit cycle, three kinds of limit cycles can be distinguished [7]:

1. Stable limit cycles: all trajectories in the vicinity of the limit cycle converge to it for $t \to \infty$ (Fig. 4.1.a)
2. Unstable limit cycles: all trajectories in the vicinity of the limit cycle diverge from it for $t \to \infty$ (Fig. 4.1.b)

3. Semi-stable limit cycles: some of the trajectories in the vicinity converge to it, while others diverge from it for $t \to \infty$ (Fig. 4.1.c)
Chapter 5

Stability of the engine system

5.1 Introduction

In this chapter the stability of the engine is analyzed using the phase plane. For the engine system this results in an attraction area. The influence of MPI and of parameters on the stability of the engine is examined.

5.2 The attraction area

The stability of the engine system can now be analyzed by performing simulations with initial conditions $x_0$ that differ from the equilibrium state $x_*$ i.e. the undisturbed state in idle. For these initial conditions the next bounds are chosen:

- $P_{man}$ between 0 and $P_{srm}$
- $N_c$ between 450 and 1500. If $N_c \leq 450$ then the engine stalls. The upper bound is chosen more or less arbitrarily.

$\omega_c$ is changed to $N_c$, because in practice this quantity is used. $N_c$ is the number of revolutions of the crank shaft,

$$N_c = \frac{60}{2 \cdot \pi} \cdot \omega_c \quad (5.1)$$

The simulations have been done for $0 \leq t \leq 15$ sec. For $T_d = 0$ Nm, the relevant detailed area is focused on in Fig. 5.1.

The simulations result in the definition of an attraction area: trajectories within this area (trajectory C) are attracted to the center of the area, the equilibrium point $x_*$. Trajectories outside this area are unstable and result in engine stall (trajectory D). The attraction area is in fact the effect of an unstable limit cycle (Fig. 4.1.b). The time history of several quantities in case of hunting is given in Fig. C.1 through C.6.
Stability of the engine system

Figure 5.1: Attraction area for $T_d = 0$ with converging (C) and diverging (D) trajectories

The research of Bartsch can now be put in the right perspective. It was determined that the real parts of the poles of the linearized engine system around the equilibrium point $\mathbf{z}_e$ (for $T_d = 0$) were negative. From figure 5.1 it can be seen that the conclusions concerning stability are correct: the system is locally stable around $\mathbf{z}_e$. It is, however, not globally stable.

The magnitude of the disturbance $T_d$ greatly influences the attraction area, through

- the location of the attraction area
- the size of the attraction area
- the stability of the limit cycle

The first two phenomena can best be explained by Fig. 5.2. The limit cycles in this figure were computed for $T_d = 0$ till 3 Nm.

When $T_d$ increases, the equilibrium point tends to move to higher values of $P_{\text{man}}$ and lower values of $N_c$. The size of the attraction area decreases. The stability of the limit cycle increases for an increasing $T_d$. Convergence of trajectories inside the attraction area to the equilibrium point and divergence of the trajectories outside the area are slower. For a certain $T_d$ no attraction area exists anymore. In this case the real parts (Re(s)) of the poles of the linearized system should become positive. In Fig. 5.3 the rootloci of the linearized engine system are shown as function of $T_d$. (Bartsch used an extra state variable $\lambda_c$ to deal with the delay time [1], for the linearization proces resulting in 3 poles)
5 Stability of the engine system

Figure 5.2: Attraction area as function of $T_d$

explanation:
0: $T_d = 0$ Nm
1: $T_d = 1$ Nm
2: $T_d = 2$ Nm
3: $T_d = 3$ Nm

Figure 5.3: Rootloci of the linearized system as function of $T_d$

explanation:
1: $T_d = 0$ Nm
2: $T_d = 20$ Nm
3: $T_d = 21$ Nm
4: $T_d = 25$ Nm
The real parts of the poles become positive for $T_d \approx 21 \text{Nm}$, but this value for $T_d$ will not occur in real life applications. However, in Fig. 5.1 the attraction area does not exist anymore for smaller values of $T_d$. This could be the result of the linearization process.

5.3 Stability of a MPI autogas system

For the engine system simulations were carried out without delay time ($\tau_{\text{man}}=0$) and $T_d = 0$. In this case the fuel is injected exactly in front of an intake port. In fact, the SPI engine is changed into a MPI engine. Some of the trajectories are depicted in Fig. 5.4.

![Phase portrait of a MPI autogas system](image)

Figure 5.4: Phase portrait of a MPI autogas system

This is a phase portrait of a stable system. Trajectories converge fast to the equilibrium point. When this figure is compared with Fig. 5.1, it can be concluded that a MPI system increases the stability of the engine.

In Fig. C.7 through C.12, time histories are depicted for one of these trajectories. When Fig. C.11 is compared to Fig. C.5 it can be seen that for a MPI engine $\lambda_i = 1$, whereas for the SPI engine $\lambda_i$ fluctuates. This fluctuation in $\lambda_i$ can be explained as follows: in case of MPI the fuel is injected exactly in front of an intake port. $Q_{fi}$ is based on $Q_{mc}$ to get a stoichiometric mixture ($\lambda_i = 1$). However for a MPI engine with an after sales fitted SPI autogas system, this fuel is not injected in front of an intake port but in front of the throttle valve. In case of hunting, $Q_{mt}$ is more or less constant (supersonic), but $Q_{mc}$ and thus $Q_{fi}$ fluctuate (Fig. C.6, C.3 and C.4). $Q_{fi}$ is injected to a constant $Q_{mt}$ causing a fluctuation in $\lambda_i$, $T_i$, and therefore in $N_c$.

It can be concluded that the signals to control the original MPI engine cause hunting in a SPI autogas system. Preferably, in case of SPI, $Q_{fi}$ should be injected on a stoichiometric
5 Stability of the engine system

basis with $Q_{mt}$. However this requires another measurement system and is therefore not recommended.

5.4 Parameters influencing stability

In literature [6],[8],[10] it is stated that the following parameters influence the stability of the engine,

1. Decrease of stability for decreasing $\Theta$.
2. Decrease of stability for increasing $\lambda_{ecu}$.
3. Decrease of stability for increasing $V_{man}$.
4. Decrease of stability for decreasing $J_{tot}$.
5. Spark advance, $\alpha$.

An increase of stability is achieved for an opposite change of these parameters. In appendix D the influence of these parameters on the rootloci of the linearized system and for the attraction area in the phase plane are shown. It can be concluded that the tendency of the rootloci and the location and size of the attraction area correspond well to what is stated in literature.

Note that for an after sales autogas system $V_{man}$ and $J_{tot}$ cannot be changed. $\Theta$ and $\alpha$ are difficult to adjust, leaving $\lambda_{ecu}$ as the only parameter to stabilize the system.

5.5 Use of the phase plane to explain parameter influences

In this section the parameter influences stated in section 5.4 will be investigated, by studying the phase plane in more detail.

5.5.1 Slopes in the phase plane

In Fig. 5.1 a slope of a trajectory passing through a point $(P_{man}, N_c)$ in the phase plane is given by:

$$\beta = \frac{dN_c}{dP_{man}} = \frac{\dot{N}_c}{\dot{P}_{man}} = \frac{2\pi}{60} \frac{\dot{\omega}_c}{\dot{P}_{man}}$$

(5.2)

where $\dot{\omega}_c$ is given by Eq. 3.14 and can be written as:

$$\dot{\omega}_c(t) = F_1 (P_{man}(t), P_{man}(t - \tau_{man}), \omega_c(t), \omega_c(t - \tau_{man}))$$

(5.3)

$\dot{P}_{man}$ is given by 3.1 and can be written as:

$$\dot{P}_{man}(t) = F_2 (P_{man}(t), \omega_c(t))$$

(5.4)

Eq. 5.2 can not be determined for $\tau_{man} \neq 0$. In that case the slope in a point $(P_{man}, N_c)$ depends on the initial conditions. For $\tau_{man} = 0$ the slope is unambiguously determined in a point $(P_{man}, N_c)$. For a MPI engine applies $\tau_{man} = 0$. Therefore, to explain parameter influences, this analysis is based on a MPI engine.
5.5.2 Explain parameter influences

For the engine system the slopes in the phase plane are calculated for the case depicted in Fig. 5.4. This is shown in Fig. 5.5.

![Phase portrait for MPI, with slopes](image)

Figure 5.5: Phase portrait for MPI, with slopes

This phase portrait can be divided in four parts. This is shown in Fig. 5.6.

part 1 : \( \dot{p}_{\text{man}} > 0 \land \dot{\omega}_c < 0 \)
line 1 : \( \dot{p}_{\text{man}} > 0 \land \dot{\omega}_c = 0 \)
part 2 : \( \dot{p}_{\text{man}} > 0 \land \dot{\omega}_c > 0 \)
line 2 : \( \dot{p}_{\text{man}} = 0 \land \dot{\omega}_c > 0 \)
part 3 : \( \dot{p}_{\text{man}} < 0 \land \dot{\omega}_c > 0 \)
line 3 : \( \dot{p}_{\text{man}} < 0 \land \dot{\omega}_c = 0 \)
part 4 : \( \dot{p}_{\text{man}} < 0 \land \dot{\omega}_c < 0 \)
line 4 : \( \dot{p}_{\text{man}} = 0 \land \dot{\omega}_c < 0 \)

The slopes in part 1 are most important. These slopes determine the stability of the engine. If the slopes in this part are very steep, the engine stalls fast, leaving no or only a small attraction area. The engine can be stabilized by decreasing the steepness of the slopes in part 1. According to Eq. 5.2 this can be achieved by making \( \dot{\omega}_c \) less negative or \( \dot{p}_{\text{man}} \) more positive. The equations for these state variables are given by Eq. 3.14 and Eq. 3.1. The parameters that influence these equations therefore influence stability. Here it must be noted that the slopes in the other parts influence stability too. Preferably the slopes in part 3 are less steep and steeper in part 2 and 4, however the slopes in part 1 are determinative. Now the phenomena stated in section 5.4 can be explained as follows:

- \( V_{\text{man}} \): if the manifold volume is increased, according to Eq. 3.1 \( \dot{p}_{\text{man}} \) will decrease and
the slopes $\beta(P_{\text{man}}, N_c)$ will be steeper in part 1. The stability of the engine decreases. This is shown in Fig. E.1.

- $J_{\text{tot}}$: if the moment of inertia is decreased, according to Eq. 3.14, $\dot{N}_c$ increases. The slopes $\beta(P_{\text{man}}, N_c)$ in part 1 will be steeper. This leads to a decrease of stability (Fig. E.2).

- $\Theta$: if the throttle valve angle is decreased, according to Eq. 3.5, $A_{\text{eff}}$ decreases. This results in a reduce of $Q_{\text{mt}}$ and thus $\dot{P}_{\text{man}}$. The slopes $\beta(P_{\text{man}}, N_c)$ are steeper in part 1. The stability of the engine decreases Fig. (E.3).

- $\lambda_{\text{ecu}}$ and $\alpha$: these items influence the induced torque $T_i$ and therefore $\dot{N}_c$. The slopes in the phase plane change, however this change is not clear. The influence of these items will later be explained on the basis of the AFI- and SI-function.

The influence of disturbance torque $T_d$ can be explained as follows: If $T_d$ increases, $\dot{N}_c$ will be more negative and the slopes in part 1 will be steeper. This destabilizes the engine (Fig. E.4). It must be noted that the location of the equilibrium point changes for changing $T_d$, just as for changing $\Theta$, $\lambda_{\text{ecu}}$, and $\alpha$.

The AFI-function is depicted in Fig. F.1. For $\lambda_c \approx 1$ optimal combustion takes place. A leaner or a richer mixture causes a worse combustion resulting in a lower $T_i$. The parameters in the AFI-function were determined on basis of measurements done by Smits [8]. The measurements have taken place for values of $\lambda_c$ in a range of 0.82 to 1.15.
For $\lambda_{\text{ecu}} = 1$, a fluctuation $\Delta \lambda_c$ will cause a larger fluctuation $\Delta AFI$ (Fig. F.2) than for $\lambda_{\text{ecu}} = 0.97$. (Fig. F.3). Therefore a little decrease in $\lambda_{\text{ecu}}$ will stabilize the engine. For $\lambda_{\text{ecu}} > 1$ the stability of the engine is reduced.

Note: the AFI function must strictly be used in the range $0.82 < \lambda_c < 1.15$!

In Fig. F.4 the AFI-function is calculated for a larger range of $\lambda_c$. For $\lambda_c < 0.74$, AFI > 1. This implies a combustion efficiency over 100 %, which is physically not possible. For $\lambda_c > 1.35$ the AFI-function becomes negative, also not possible. In literature [2] and [3] the AFI-function is given for a larger range of $\lambda_c$, this is shown Fig. F.5). It can be seen that stabilizing the engine by decreasing $\lambda_{\text{ecu}}$ only applies for small changes in $\lambda_c$. For a large decrease of $\lambda_{\text{ecu}}$ the stability of the engine is reduced. It is recommendable to perform more experiments to determine AFI for a larger range of $\lambda_c$.

SI is depicted in Fig. F.6. In normal operation, $\alpha$ is set on 8°. This is the setting for petrol fuel. However for gas the SI-function changes: for $\alpha = 8^\circ$, SI=0.7. When this $\alpha$ is increased, this value tends to larger values of SI. The optimum is in the neighbourhood of $\alpha = 37^\circ$. This results in a rise of the stationary value of $N_c$ and therefore an increase of the kinetic energy. In this case a fluctuation in $T_d$ has less effect on the stability of the engine. This increase of efficiency could also be used to decrease fuel consumption. For these two reasons it is recommendable to investigate the possibilities to increase $\alpha$.

5.6 Physical interpretation

The results of this chapter can be interpreted as follows: hunting is the result of a fluctuation in $\lambda_i$ caused by the use of MPI control signals to control a SPI autogas system. Hunting is an exchange of potential energy in the manifold subsystem and kinetic energy of the crank shaft subsystem. This can be explained as follows: $P_{\text{man}}$ reacts on a change of $N_c$. This is determined by the pressure reaction time, $\tau_p$. This $\tau_p$ follows from the first order differential equation for $P_{\text{man}}$, Eq. 3.1. When a disturbance torque occurs, the following processes can be distinguished:

1. $N_c$ decreases: $Q_{mc}$ decreases, but $Q_{mt}$ is more or less constant. As a result $P_{\text{man}}$ increases (the velocity of this increase is determined by $\tau_p$).

2. $P_{\text{man}}$ increases: $Q_{mc}$ increases and therefore $N_c$ increases.

3. $N_c$ increases: $Q_{mc}$ increases, $Q_{mt}$ is constant and $P_{\text{man}}$ decreases.

4. $P_{\text{man}}$ decreases: $Q_{mc}$ decreases, and $N_c$ falls off. The process starts once more at the first point.

Hunting is a periodic oscillation of these processes. The effect of parameters is:

- $J_{\text{tot}}$: when $J_{\text{tot}}$ is decreased, fluctuations in $T_i$ cause larger fluctuations in $N_c$, this destabilizes the system.

- $V_{\text{man}}$: when $V_{\text{man}}$ is increased, $\tau_p$ will be larger. $P_{\text{man}}$ will react slower on a change of $N_c$. This has a negative effect on stability.
5 Stability of the engine system

- Θ: when Θ is reduced, $Q_{mt}$ reduces and $\tau_p$ is larger causing a decrease of stability. Moreover, reducing Θ will result in a decrease of the stationary value of $N_c$ and thus kinetic energy. A disturbance torque $T_d$ will cause larger fluctuations in $N_c$ and destabilizes the system.

- $\alpha$: An increase of $\alpha$ will result in an increase of the stationary value of $N_c$ and kinetic energy. This results in an increase of stability.

- $\lambda_{ecu}$: A little decrease of $\lambda_{ecu}$ reduces fluctuations in AFI. This has a stabilizing function.

5.7 Discussion

The analysis of the slopes and trajectories in the phaseplane must be seen as an expedient to describe parameter influences. With phase plane analysis the stability can be investigated. It can be concluded that the use of control signals of an originally MPI system to control a SPI autogas system is the cause of hunting.

$\lambda_{ecu}$ is the only adjustable variable to suppress hunting. This can be performed by decreasing $\lambda_{ecu}$. However this only works for a small decrease, and has a negative effect on fuel consumption and the quality of exhaust gasses. The best solution to deal with hunting is the use of a MPI autogas system. This stabilizes the engine and has no effect on fuel consumption and quality of exhaust gasses.
Chapter 6

Conclusions and recommendations

6.1 Conclusions

The aim of this research was to determine the stability of the engine and find the causes for hunting. On the basis of this research the next conclusions can be drawn,

- Hunting is a limit cycle in the engine system. This limit cycle results in an attraction area in the manifold pressure - number of revolutions phase plane.
- Linearizing the system gives local stability. For a larger working area linearization is not suitable anymore.
- Phase plane analysis describes most of the parameter influences, it is a proper technique to analyze systems with two state variables and two first order differential equations.
- The use of control signals of a multi point injection engine to control a single point autogas system causes a fluctuation in the air/fuel ratio. This is the cause of hunting.
- Hunting can be suppressed by decreasing $\lambda_{ecu}$. However, this only works for a small change in $\lambda_{ecu}$ and has a negative effect on fuel consumption and exhaust gasses.
- A MPI autogas system stabilizes the engine and has no effect on fuel consumption and exhaust gasses.
6.2 Recommendations

Recommendations for further research are,

- Extensive experimentation with an engine, to,
  - Identify parameters and evaluate the model especially, determine the AFI-function for a larger range of $\lambda_c$.
  - Evaluate the influence of several parameters described in this report.

- Investigate the possibilities to increase the spark advance angle. In this way the efficiency of the engine could be improved, and a decrease of fuel consumption can be achieved.

- An originally equipped MPI engine should be equipped a MPI autogas system. Therefore it is recommendable to develop a MPI autogas system.
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Appendix A

Blockdiagram of the engine

In the blockdiagram the following parameters are used:

\[
\begin{align*}
D_{ch} &= 0.051 \quad [m] \\
H_o &= 45.6 \cdot 10^6 \quad [J/kg] \\
J_{tot} &= 0.15 \quad [kg \cdot m^2] \\
\kappa &= 1.4 \quad [-] \\
L_{st} &= 15.81 \quad [-] \\
P_{atm} &= 1.013 \cdot 10^5 \quad [Pa] \\
R_m &= 281 \quad [J/kg \cdot K] \\
T_{atm} &= 296 \quad [K] \\
T_{man} &= 328 \quad [K] \\
V_c &= 0.431 \quad [m^3] \\
V_{man} &= 3.79 \quad [m^3] \\
Z &= 4 \quad [-] \\
\alpha &= 8 \quad [-] \\
\eta_{opt} &= 0.305 \quad [-] \\
\Theta &= 4.27 \quad [-] \\
\Theta_o &= 5.5 \quad [-] \\
\end{align*}
\]

\[
\begin{align*}
c_1 &= 0.6773 \\
c_2 &= -0.1439 \cdot 10^{-2} \\
c_3 &= 0.8836 \cdot 10^{-5} \\
c_4 &= -0.7331 \cdot 10^{-6} \\
c_5 &= 0.8330 \\
c_6 &= -0.8484 \cdot 10^{-6} \\
c_7 &= 0.1146 \cdot 10^2 \\
c_8 &= -0.3438 \cdot 10^2 \\
c_9 &= 0.3823 \cdot 10^2 \\
c_{10} &= -0.1385 \cdot 10^2 \\
c_{11} &= 0.5038 \\
c_{12} &= 0.2726 \cdot 10^{-1} \\
c_{13} &= -0.3752 \cdot 10^{-3} \\
c_{14} &= -0.1561 \\
c_{15} &= 0.3323 \cdot 10^{-2} \\
c_{16} &= 0.1437 \cdot 10^3 \\
c_{17} &= -0.1487 \cdot 10^1 \\
c_{18} &= 0.4382 \cdot 10^{-2} \\
c_{19} &= -0.1643 \cdot 10^{-2} \\
c_{20} &= 0.1230 \cdot 10^{-4}
\end{align*}
\]
*1: $Q_{mt}$

In the blockdiagram for $Q_{mt}$ applies,

$$\gamma = \frac{P_{man}}{P_{atm}}$$  \hspace{1cm} (A.1)

$$\gamma_{cr} = \left(\frac{2}{\kappa + 1}\right)^{\left(\frac{\kappa}{\kappa - 1}\right)}$$  \hspace{1cm} (A.2)

If $\gamma > \gamma_{cr}$, $Q_{mt}$ is given by:

$$Q_{mt} = C_d(P_{man}) \cdot A_{eff}(\Theta) \cdot \frac{P_{atm}}{\sqrt{R_m \cdot T_{atm}}} \cdot \sqrt{\frac{2 \cdot \kappa}{\kappa - 1}} \cdot \left(\frac{P_{man}}{P_{atm}}\right)^{\left(\frac{2}{\kappa - 1}\right)} - \left(\frac{P_{man}}{P_{atm}}\right)^{\left(\frac{2 + 1}{\kappa - 1}\right)}$$  \hspace{1cm} (A.3)

If $\gamma < \gamma_{cr}$, $Q_{mt}$ is given by:

$$Q_{mt} = C_d(P_{man}) \cdot A_{eff}(\Theta) \cdot \frac{P_{atm}}{\sqrt{R_m \cdot T_{atm}}} \cdot \sqrt{\kappa \cdot \left(\frac{2}{\kappa + 1}\right)}$$  \hspace{1cm} (A.4)

With,

$$A_{eff}(\Theta) = \frac{\pi}{4} \cdot D_{th}^2 \cdot \left(1 - \frac{\cos(\Theta_0 + \Theta)}{\cos(\Theta_0)}\right)$$  \hspace{1cm} (A.5)
Appendix B

Phase portraits of linear systems

![Phase portraits of linear systems](image)

Figure B.1: Phase portraits of linear systems
Appendix C

Time histories of some trajectories

In this Appendix the time histories of two trajectories are given:

1. Fig. C1 trough C6: Hunting in a SPI autogas system (Limit cycle in Fig. 5.1).

2. Fig. C7 trough C12: A stable MPI autogas system (Fig. 5.4, trajectory with initial conditions: $N_c = 1000$ and $P_{man} = 3.5E5$)
C Time histories of some trajectories

Figure C.1: \( N_{o,1} \), for hunting SPI system

Figure C.2: \( P_{\text{man},1} \), for hunting SPI system

Figure C.3: \( Q_{\text{mc},2} \), for hunting SPI system

Figure C.4: \( Q_{f,1} \), for hunting SPI system

Figure C.5: \( \lambda_{1,2} \), for hunting SPI system

Figure C.6: \( Q_{m,t} \), for hunting SPI system
C Time histories of some trajectories

Figure C.7: $N_c$, for stable MPI system

Figure C.8: $P_{\text{man}}$, for stable MPI system

Figure C.9: $Q_{mc}$, for stable MPI system

Figure C.10: $Q_{fi}$, for stable MPI system

Figure C.11: $\lambda_i$, for stable MPI system

Figure C.12: $Q_{mt}$, for stable MPI system
Appendix D

Parameter influences on poles and attraction area

In this appendix, parameter influences are investigated. In each figure only one variable is changed while the others are kept on the constant values given in Appendix A.
D Parameter influences on poles and attraction area

Figure D.1: Rootloci of the linearized system as function of inertia $J_{tot}$

Figure D.2: Attraction area as function of $J_{tot}$
D Parameter influences on poles and attraction area

Figure D.3: Rootloci of the linearized system as function of $V_{man}$

Figure D.4: Atraction area as function of $V_{man}$
Figure D.5: Rootloci of the linearized system as function of $\Theta$

Figure D.6: Attraction area as function of $\Theta$
D Parameter influences on poles and attraction area

Figure D.7: Rootloci the linearized system as function of $\lambda_{ecu}$

Figure D.8: Atraction area as function of $\lambda_{ecu}$
Appendix E

Parameter influences on the phase plane

In this Appendix parameter influences are showed for $\tau_{man} = 0$. Only one variable is changed while the others are kept constant at values given in Appendix A.

Figure E.1: $V_{man} = 11$ L

For $V_{man} = 11$ L, hunting is possible again for a MPI system. However this parameter is not adjustable.
Figure E.2: $J_{tot} = 0.05$

Figure E.3: $\Theta=3.7^\circ$
Figure E.4: $T_d=5$ Nm
Appendix F

AFI- and SI-function

Figure F.1: AFI – function

Figure F.2: AFI fluctuating for $\lambda_{ecu} = 1$

Figure F.3: AFI fluctuating for $\lambda_{ecu} = 0.97$

Figure F.4: AFI for $0.6 < \lambda_c < 1.4$
AFI- and SI-function

(Figure F.5: AFI in literature, [2])

(Divide Air to fuel ratio through 14.7 to obtain $\lambda$)

(Figure F.6: SI-function)