A numerical and experimental analysis of the motion of a sphere in a cylindrical tube containing a viscoelastic fluid

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Award date:
1994
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W.F.W. Report n°: 94.078

Supervised by: Prof.dr.ir. Frank P.T. Baaijens.
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Summary

The finite element (f.e.) method is used to simulate the unsteady motion of a sphere in a cylindrical tube containing a viscoelastic fluid. Two situations are investigated. First, a sphere falling due to gravity. Second, the motion of a sphere attached to wire with an imposed velocity is considered. For this case not only f.e.-computations, but also experiments are performed.

For the falling sphere problem good agreement with analytical results (for a Newtonian fluid) and numerical results from literature (for a non Newtonian fluid) is obtained.

For the sphere pull experiment, the measured drag force is systematically underpredicted by the f.e. results. It is shown that this may be explained by the neglectance of the wire in the numerical simulations.
List of symbols

\( a \) ........................................ radius sphere [m]
\( B \) ........................................ strain velocity tensor
\( D \) ........................................ deformation rate tensor
\( F_D \) ...................................... total hydrodynamic force on the sphere (drag force) [N]
\( F_P \) ...................................... Poisseulle force [N]
\( F_{St} \) .................................. Stokes force [N]
\( F_w \) ...................................... resultant due to gravitational and buoyancy forces [N]
\( g \) ........................................ gravitational acceleration
\( K \) ........................................ wall resistance factor Stokes flow [-]
\( l \) ........................................ length tube [m]
\( L \) ........................................ velocity gradient tensor
\( m \) ........................................ mass sphere [Kg]
\( p \) ........................................ isotropic component of the stress tensor
\( \bar{p} \) ..................................... pressure field
\( \text{PIB/C14} \) .......................... 5 wt\% poly-isobutylene in tetradecane
\( \text{PTT} \) ................................ Phan-Thien Tanner model
\( q \) ........................................ pressure weighting functions
\( R \) ........................................ radius tube [m]
\( Re \) ........................................ Reynolds number [-]
\( t \) ........................................ time [s]
\( t_{\text{max}} \) ................................ elapsed time to maximum sphere velocity [s]
\( t_\perp \) .................................... traction z-coordinate
\( \bar{u} \) ..................................... velocity field
\( \bar{u}_t \) .................................... tube velocity
\( \bar{v}_i \) .................................... set up velocity of the imposed velocity experiment
\( U \) ........................................ velocity sphere [m/s]
\( U_0 \) ...................................... terminal velocity sphere [m/s]
\( U_{\text{max}} \) ................................ maximum velocity sphere [m/s]
\( \text{UCM.} \) ................................ Upper Convected Maxwell model
\( Wi \) ........................................ Weissenberg number [-]
\( z, r, \theta \) ................................... axis in cylindrical coordinates
\( \Gamma_s \) ........................................ sphere boundary
tube boundary
\( \Gamma_t \) ........................................ non linearity parameter of the PTT model [-]
\( \eta \) ........................................... viscosity fluid [Pa.s]
\( \eta_0 \) ........................................ Newtonian viscosity fluid [Pa.s]
\( \theta \) ........................................... relaxation time fluid [s]
\( \rho_i \) ........................................... density fluid [Kg/m³]
\( \rho_s \) ........................................... density sphere [Kg/m³]
\( \Delta \rho \) ........................................ \( \rho_i - \rho_s \)
\( \sigma \) ........................................... total stress tensor
\( \tau \) ........................................... "extra" stress tensor
\( \Omega \) ........................................... integration domain
\( \mathcal{L} \) ........................................ operator used for abbreviation
\( \mathcal{V} \) ........................................ Truesdell rate
\( \mathcal{V} \) ........................................ Gradient operator
1 Introduction

The falling sphere technique is widely used to measure the viscosity of a liquid by monitoring the terminal speed at which a sphere falls under gravity in a cylindrical tube containing the test liquid. The theory for both steady and unsteady motion of the sphere is well established. Stokes' law, which directly relates the liquid viscosity to the terminal velocity of a sphere falling in an unbounded Newtonian fluid at low Reynolds number, forms the basis for calculation the viscosity of the fluid. Bohlin's approximate theory provides a simple method to make a correction for wall effects. For non-Newtonian fluids, especially for viscoelastic fluids, the problem is considerably more complex and the analysis depends on the particular fluid model adopted.

Several theoretical analyses of non-Newtonian flow past a sphere, for example Kind and Waters, show that the sphere velocity may have an overshoot and even oscillate about the terminal velocity. The maximum velocity, and the period of the oscillation were found to be proportional to the square root of the relaxation time of the fluid.

This result implied a possibility of using the falling sphere technique to determine elastic parameters for liquids. However, these analyses were limited to unbounded fluid media; this was partly due to the mathematical difficulty in the inclusion of wall effects in the problem. This difficulty makes the theory inadequate for the practical application to falling-sphere rheometers.

In this work, the finite element method is used to analyze the unsteady motion of a sphere along the axis of a cylindrical tube containing a viscoelastic fluid. The Phan-Thien Tanner (PTT) model is used in this study. This model can accommodate other differential models as well, such as the Upper Convected Maxwell (UCM) model and the Oldroyd-B model.

First the governing equations are discussed whereafter the numerical solution procedure is presented. Then the f.e.-method is checked for analytical Newtonian solutions and compared with non-Newtonian results from literature.
Recently a falling sphere experiment is studied by Becker and McKinley\textsuperscript{[a]}. This experiment, however, could not be repeated in our laboratory, because with the available high speed video system it was not possible to monitor the motion of a sphere in a glass tube containing viscoelastic liquid. A choice is made here to verify the finite element method with a more simple experiment. The sphere is attached to a thin wire that is clamped in a drawbench. The velocity is imposed and the displacement and force are measured. The resulting experimental velocity is fitted and used as an input for f.e.-calculations. The drag on the sphere is used to compare numerical and experimental results. These are discussed and the concluding remarks presented.
2 Governing equations

Consider a sphere of density $\rho_s$ and radius $a$ in a PTT (Phan-Thien Tanner) fluid with viscosity $\eta_0$, $\eta$, a relaxation time $\theta$, a non-linearity parameter $\epsilon$ and a density $\rho_i$ (see fig.2.1). The equation of motion of the sphere:

$$m \ddot{u} = F_w - F_D,$$  \hspace{1cm} (2.1)

where the mass of the sphere $m$ is defined as

$$m = \frac{4}{3} \pi a^3 \rho_s$$  \hspace{1cm} (2.2)

and $\ddot{u}$ as the acceleration of the sphere. The resultant force acting on the sphere due to gravitational and buoyancy forces $F_w$ (where $g$ is the gravitational acceleration) equals:

$$F_w = \frac{4}{3} \pi a^3 (\rho_s - \rho) g.$$  \hspace{1cm} (2.3)

Further, $F_D$ is the total hydrodynamic force (drag force) on the sphere. The latter force is the integral of the traction over the sphere’s surface. There is no closed-form solution for this force for viscoelastic fluids; finding it requires the full numerical solution to the whole problem.

For low Reynolds numbers it is allowed to neglect the fluid inertia. The Reynolds number (where $a$ is defined as radius of the sphere and $U_0$ the terminal velocity) is given by:

$$Re = \frac{2a \rho U}{\eta_0} \quad [-]$$  \hspace{1cm} (2.4)

For the fluid investigated in this work $Re \approx 10^1$. The neglect of fluid inertia seems to be reasonable. In this case the momentum equation becomes:

$$\nabla \cdot \sigma = 0$$  \hspace{1cm} (2.5)
For an incompressible fluid the hydrostatic pressure is not defined by means of a constitutive equation. However, the constraint that must be satisfied is
\[ \nabla \vec{u} = 0, \]
with \( \vec{u} \) the fluid velocity. The following initial conditions are used:
\[ U(t=0) = 0, \quad \vec{u}(t=0) = \vec{0}. \] (2.6)

The following boundary conditions are adapted in cylindrical coordinates \((r, \theta, z)\) (see fig. 2.2), where the origin of the coordinates is located at the centre of the sphere at all times and the centre line of the tube coincides with the \( z \)-axis. The boundary conditions in terms of \( \vec{u} \) (velocity) and \( \vec{T} \) (traction) are:
\[ u_r = 0, \quad u_z = U(t) \] on sphere surface;
\[ u_r = 0, \quad u_z = 0 \] on the wall;
\[ u_r = 0, \quad u_z = 0 \] far upstream;
\[ u_r = 0, \quad t_z = 0 \] far downstream;
The boundary condition \( t_z = 0 \) far downstream may and has been replaced by \( u_z = 0 \).

It’s customary to split the total stress tensor \( \sigma \) into a hydrostatic and a deviatoric part:
\[ \sigma = -pI + \tau \] (2.7)
The tensor \( \tau \) is often referred to as the extra stress tensor. The PTT (Phan-Thien Tanner) model is then written as
\[ \nabla \tau + (\frac{1}{\theta} + \frac{\varepsilon}{\eta} tr(\tau)) \tau = \frac{2\eta}{\theta} D, \] (2.8)
where the Truesdell convected derivative is defined as
\[ \nabla \tau = \frac{\partial \tau}{\partial t} + \vec{u} \cdot \nabla \tau - L \tau - \tau \cdot L \tau. \] (2.9)
The Oldroyd-B and Upper Convected Maxwell (UCM) fluids are recovered by selecting $\epsilon=0$ and $\epsilon=\eta_0=0$, respectively, e.g. for Oldroyd-B

$$\tau = 2\eta_0 D + \tau_1$$ (2.13)

while for UCM the solvent viscosity $\eta_0$ is discarded.

Introduce the operator $\mathcal{L}$ as

$$\mathcal{L} = \nabla + \left( \frac{1}{\theta} + \frac{\epsilon}{\eta_1} \text{tr}(\tau) \right) \tau.$$ (2.15)

Then the PTT model can be written as

$$\mathcal{L} \tau = \frac{2\eta}{\theta} D.$$ (2.16)

An initial condition is required for solving the constitutive equation. It is reasonable to set $\tau = 0$ everywhere in the domain at $t=0$. 
Figure 2.1  PTT parameters, dimensions and f.e.m. model.

Figure 2.2  Adapted cylindrical coordinates.
3 Numerical method

3.1 Mixed formulation

The strong forms of the incompressible problem is defined in a mixed setting in the chapter above. Mixed in the sense that not only the displacement field is considered as an unknown, but extra stresses $\tau$ and pressure $p$ as well.

The mixed formulation of the viscoelastic problem is given by:

\[ (s:Q\tau - 2\eta \frac{\partial D_u}{\partial t}) = 0 \quad , \quad (3.1) \]

\[ -(D_v:2\eta_0 D_u) + (\nabla \cdot v)p = 0 \quad , \quad (3.2) \]

\[ (q, \nabla \cdot u) = 0 \quad , \quad (3.3) \]

with

\[ D_\alpha = (\nabla \vec{a} + (\nabla \vec{a})^\theta) \quad , \quad \alpha = \vec{u}, \vec{v} \quad , \]

where (.,.) denotes the appropriate inner product over the domain $\Omega$, with $s$ the stress weighting functions, $\vec{v}$ the velocity weighting functions and $q$ the pressure weighting functions.

The mixed viscoelastic problem is nonlinear as $\mathcal{L}_\tau$ depends nonlinearly on both the velocity and extra stress field.
3.2 Operator splitting and mixed form

The method used here is based on an operator splitting methodology. The material rate in the constitutive equation represents the advective part. During each time step this stress advection is dealt with separately from the remaining part of the constitutive equation.

Introduce \( \mathcal{L} \), that represents the material rate as:

\[
\mathcal{L}_r \tau = \frac{\partial \tau}{\partial t} + \bar{u} \cdot \nabla \tau,
\]

and define \( \mathcal{L}_p \) by:

\[
\mathcal{L}_p \tau = -L \cdot \tau - \tau \cdot L^T + \left[ \frac{1}{\lambda} + \frac{\epsilon}{\eta} \right] \text{tr(}\tau\text{)} \tau.
\]

The material rate of \( \tau \) is formally defined as:

\[
\frac{D\tau}{Dt} = \mathcal{L}_r \tau = \lim_{\delta \to 0} \frac{\tau(x, t + \delta \Delta t) - \tau(p, t)}{\delta \Delta t},
\]

where \( p \) denotes the position at time \( t \) of the particle that is located at position \( x \) at time \( t + \delta \Delta t \). After splitting the time interval \( I \) into \( N \) time steps.

\[
I = \bigcup_{n=1}^{N} I_n, \quad I_n = [t_n^-, t_{n+1}^-],
\]

with

\[
t_n^+ = \lim_{\epsilon \to 0^+} t_n + \epsilon.
\]

Eq. (3.7) suggests the following approximation of the material rate during \( I_n \),

\[
\mathcal{L}_r^p \tau = \frac{\tau(x, t_{n+1}^-) - \tau(p, t_n^-)}{\Delta t}.
\]

Suppose, for the time being, that \( \tau(p, t_n) \) is known, then for each time interval \( I_n \), the mixed weak formulation of problem viscoelastic is given as follows.
Given \( \tau(\bar{p},t_n) \), find \((\tau, \bar{u}, p)\) at \( t=t_{n+1} \), such that

\[
\left( s, \xi' \bar{\tau} + \xi' \hat{\tau} - 2 \frac{\eta}{\lambda} D_u \right) = 0 ,
\]

\[
-(D_u, 2\eta_0 D_u + \tau) + (\bar{v} \cdot \bar{v}, p) = 0 ,
\]

\[
(q, \bar{v} \cdot \bar{u}) = 0 .
\]

The remaining problem is to determine \( \tau(\bar{p},t_n) \) for each mode. Eq. (3.10) requires the knowledge of \( \tau_p \) (defined as \( \tau(\bar{p}(x,t_{n+1}),t_n) \)) for all \( x \in \Omega \). This field can be obtained by advecting the stress field at \( t=t_n \), \( \tau_n \) (defined as \( \tau(\bar{x},t_n) \)), by the known velocity field computed from the preceding problem, say \( \bar{u}(\bar{x},t) \), hence by solving

\[
\frac{\partial \tau_p}{\partial t} + \bar{u} \cdot \bar{v} \tau_p = 0 , \quad \tau_p(t_n) = \tau_n , \quad t \in I_n .
\]
3.3 The mesh updating system

While monitoring a moving object in a fixed frame, eventually the object will move out of side. In the numerical problem, the sphere would be travelling through the mesh. To avoid this problem, a mesh updating system is adapted in the used Eulerian method. The tube boundary \( \Gamma_t \) has an imposed velocity which equals the velocity of the sphere. Thus the sphere is fixed in the centre and the tube is moving. After each iteration the tube velocity is updated. The implementation is given in chart 3.1.

The Eulerian formulation is given by:

\[
\frac{\partial \tau}{\partial t} + \vec{u} \cdot \nabla \tau + \frac{1}{\theta} \tau = \frac{2\eta}{\theta} D ,
\]

(3.15)

The mesh updating system is given by:

\[
\frac{\partial \tau}{\partial t} + \vec{(u - \bar{u})} \cdot \nabla \tau + \frac{1}{\theta} \tau = \frac{2\eta}{\theta} D ,
\]

(3.16)

where \( u_t \) is the tube velocity. With \( u_t = 0 \), the Eulerian formulation is obtained.
At $t=0$, $F_D(0)=0$; $U(0)=0$; $\tau(0)=0$.

$t=t+\Delta t$

$U(t+\Delta t)=U(t)+\frac{(F_D-F_w)\Delta t}{m}$

Update bnd. cond.
$U_t(U(t+\Delta t)$ on $\Gamma_t$

Solve the equations by using f.e.m.
$F_D(t+\Delta t)=\int_{\Gamma_s} \tau \vec{n}_z d\Gamma$

Convergence?

$t>t_{\text{end}}$?

Chart 3.1 The adapted mesh updating system.
4 Newtonian test problem

In the Newtonian case, f.e.-solutions can be compared with analytical results for the falling-sphere problem.

The force on a sphere in an unbounded Newtonian liquid (see fig.4.3) falling at a constant velocity is given by the well known Stokes formula:

\[ F_{\text{St}} = 6\pi \eta_d a U . \]  

(4.1)

The resultant due to gravitational and buoyancy forces is given by:

\[ F_w = \frac{4}{3} \pi a^3 \Delta \rho g . \]  

(4.2)

With all the forces known, the acceleration can be calculated:

\[ \dot{U} = \frac{F_w - F_{\text{St}}}{m} , \]  

(4.3)

with:

\[ m = \frac{4}{3} \pi a^3 \Delta \rho . \]  

(4.4)

In order to simplify the notation, \( \alpha \) is introduced:

\[ \alpha = 6\pi \eta_d a . \]  

(4.5)

Then the acceleration is given by:

\[ \dot{U} = \frac{F_w}{m - \frac{\alpha}{m}} U . \]  

(4.6)

This is a first order differential equation which has the following solution:

\[ U = \frac{F_w}{\alpha} \left(1 - e^{-\frac{\alpha}{m}} \right) . \]  

(4.7)
The presence of a cylindrical wall is represented by coefficient $K^{[7]}$ (of the order 6 if the ratio of the tube radius to sphere radius is 2). $K$ (table 4.1, fig. 4.2) must be introduced in $\alpha$ as:

$$\alpha = 6\pi K \eta a$$

(4.8)

With the corrected velocity for wall effects, the velocity and drag force of a falling sphere in a tube containing a Newtonian fluid are known for all time. In fig. 4.4 the f.e.-solution is compared with the analytical result. The dashed line in this figure is the analytical solution. The deviation, the dotted line, is less then 1%. This is very acceptable for the used mesh (mesh 3.3, fig. 4.1).

**Figure 4.1** Used mesh 3.3 for the Newtonian test problem.

<table>
<thead>
<tr>
<th>$a/R [-]$</th>
<th>$K [-]$</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>1.263</td>
</tr>
<tr>
<td>0.2</td>
<td>1.68</td>
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<tr>
<td>0.3</td>
<td>2.371</td>
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<tr>
<td>0.4</td>
<td>3.596</td>
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<td>0.5</td>
<td>5.970</td>
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<tr>
<td>0.6</td>
<td>11.135</td>
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<tr>
<td>0.7</td>
<td>24.955</td>
</tr>
<tr>
<td>0.8</td>
<td>73.555</td>
</tr>
</tbody>
</table>

**Table 4.1**

**Figure 4.2** The Stokes resistance coefficient.
Figure 4.3 Properties and dimensions of the Stokes flow test.

Stokes velocity and difference

Figure 4.4 Numerical Stokes flow compared with the analytical solution.
5 Non Newtonian test problem

The maximum overshoot velocity is a function of both the aspect ratio \( a/R \) and the Weissenberg number. Phan-Thien and Zheng\(^8\) found with the boundary-element method, using the Upper Convected Maxwell model, the maximum overshoot and the elapsed time to maximum overshoot interdependent. The approximated equations are:

\[
U_{\text{max}} = K_u(Wi)^{\alpha_u} \tag{5.1}
\]

\[
t_{\text{max}} = K_t(Wi)^{\alpha_t} \tag{5.2}
\]

these function are plotted in fig. 5.2 and 5.3 with the best fit lines. The values of \( \alpha_t \) and \( \alpha_u \) (table 5.1) are plotted against the ratio \( a/R \) (see fig.5.1). \( U_{\text{max}} \) and \( t_{\text{max}} \) are made nondimensional. For the velocity \( U_{\text{max}} = u(t)/U_0 \):

\[
U_0 = \frac{3\eta_0}{4\pi a \Delta \rho} \tag{5.3}
\]

and for the time, \( t_{\text{max}} = t/(a/U_0) \).

In fig. 5.2 and 5.3 a comparison with the current work and the b.e.m. results of Phan-Thien and Zheng is made. The numerical simulation is done with mesh 3.3 figure 4.1. Satisfactory comparison with the work of Phan-Thien and Zheng is found.
<table>
<thead>
<tr>
<th>$a/R$</th>
<th>$K_u$</th>
<th>$\alpha_u$</th>
<th>$K_t$</th>
<th>$\alpha_t$</th>
</tr>
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<tr>
<td>1/2</td>
<td>0.5572</td>
<td>0.3438</td>
<td>0.1352</td>
<td>0.4240</td>
</tr>
<tr>
<td>1/4</td>
<td>1.1979</td>
<td>0.4104</td>
<td>0.2778</td>
<td>0.4561</td>
</tr>
<tr>
<td>1/6</td>
<td>1.4664</td>
<td>0.4335</td>
<td>0.3340</td>
<td>0.4639</td>
</tr>
</tbody>
</table>

*Table 5.1  Power indices found with b.e.m.*

*Figure 5.1  Power indices $\alpha_u$ and $\alpha_t$ found by Phan-Thein and Zheng with b.e.m.*
Figure 5.2 Best fit of maximum sphere velocity b.e.m. (lines) with maximum sphere velocity found here with f.e.m. (▲), U.C.M. approximation.

Figure 5.3 Elapsed time fit to maximum sphere velocity found with b.e.m. (lines), compared with the here used f.e.m. (▲), U.C.M. fluid model.
6 Falling sphere problem

The falling sphere problem enjoys great attention from today's scientists. The idea exists that this simple experimental method (widely used to encounter the Newtonian viscosity) could be used to measure the material properties on a simple and cheap basis.

The basic idea is a sphere placed in the centre line of a tube (see fig. 6.2). The sphere is suspended by using a fine steel wire (±0.1 [mm]). The assumption is made that the wire does not have a large effect on the force and velocity of the sphere. The wire is clamped above the sphere. The experiment is triggered by loosening the clamp. During the experiment, the sphere is monitored by a video system and the coordinates are measured.

The choice of the density of the sphere and the ratio $a/R$ (resistance due to the tube boundary) defines the Weissenberg number. This number is given by:

$$Wi = \frac{U_{\text{max}}}{a},$$

where $U_{\text{max}}$ is the maximum velocity (first overshoot). The Weissenberg number is important for the numerical solution. The higher the Weissenberg number the more difficult it is to obtain a convergent solution.

Recently Becker and McKinley[91] have carried out this experiment. It was done with a polyisobutylene (PIB) Boger fluid. The overall experiment is shown in fig. 6.2. A long plexiglass cylinder is filled with Boger fluid. The entire cylinder is enclosed in a rectangular box filled with refractive-index matched fluid to eliminate refraction effects. Infrared diodes are used to obtain an independent estimate of the ultimate steady-state settling velocity. A thin short wire (length $a$) is attached to the sphere. This wire is clamped in a centred release mechanism. The shortage of the wire ensures the reduction of the settling velocity is less than 2%-3%.

A ccd-camera records on a Super-VHS videorecorder and the frames can be digitized and stored in memory at a rate of 30 frames per second.
A Numerical solution is obtained with the experimental mesh fig 6.1 for the test fluid. The interesting domain lies in approximate 0.05 [s] and the important first overshoot occurs in 0.025 [s] for the test fluid (fig. 6.3). The fluid properties are given in fig. 7.1. This short time domain requires a resolution of frames of the camera of 200 [Hz] at least.

The available camera in our laboratory, a Hensell video system can do this easily. But the system works with markers. These markers are followed and the coordinates are given as an output. Problems arise with monitoring the markers through the tube and the fluid. Also angle distortions are hard to catch.

Because of these experimental difficulties, chosen is for a more simple sphere pull experiment (see chapter 7).

![Figure 6.1 Used mesh for falling sphere and sphere pull calculations.](image)
Figure 6.2 Schematic diagram of the experimental apparatus, used by Becker et al., for measuring transient and steady motion of a sphere falling through a viscoelastic fluid.

Figure 6.3 Falling sphere in a PTT fluid.
7 Sphere pull problem

A sphere pull experiment is easier to perform than a falling sphere experiment. Using a customary drawbench with a sensitive force transducer the drag on the sphere can be measured.

Here, a steel sphere (from a ball-bearing) is attached to a thin steel thread which is clamped to the drawbench (see fig. 7.1). The dimensions of the experimental design are given in fig. 7.2. The sphere placed at the centre line of the glass tube containing the test liquid, 5 wt% poly-isobutylene in tetradecane (PIB/C14, properties see fig. 7.1). This whole framing is placed in a drawbench. Now the desired velocities can be adjusted and the experiment carried out. The Weissenberg number is defined as:

$$Wi = \frac{U_{\text{max}}}{a},$$

where $U_{\text{max}}$ is the maximum velocity.

Five trials are done. The drawbench, a Zwick 1434, measured the displacement of the sphere (fig. 7.3) and the drag force (fig. 7.7). The terminal velocity as imposed on the draw bench is denoted by $v_t$. Because of a wrinkle in the measured displacement (see fig. 7.4), the calculated velocities give a cloud of points (see fig. 7.5). Through these points a velocity fit is obtained by dividing the real velocity curve into three parts. The first part (I), a straight line, then a third order curve part (II), and part (III) a constant, see figure 7.5. Part (II) is defined by part (I) and part (III) 2 velocities and 2 slopes. The fitted velocity curves are shown in figure 7.6. The obtained fit-data are presented in table 7.1, where the constants should be interpreted as:

$$0 \leq t < t_1 \quad U = a_1 t$$
$$t_1 \leq t \leq t_2 \quad U = b_1 t^3 + b_2 t^2 + b_3 t + b_4$$
$$t > t_2 \quad U = c_1.$$

The fitted experimental velocity curves are used as an input for the f.e.-analysis of the problem. With these input velocities the forces are computed. The input velocity curves are shown in figure 7.6, where the curves $v_i = 0.030$, $v_i = 0.025$, $v_i = 0.020$ are actually used.
The numerically calculated forces are always lower than the experimental forces (an example is given in figure 7.8-10). Further, the resistance \((F_p/U)\) should be constant after a certain period. But there plot has clearly a slope in the measured data (see fig. 7.12).

Two remarks can be made. First, the fluid properties were not measured from the actually used PIB/C14 fluid. Second, the wire, which is not taken into account in the computations, has a large effect on the drag force. This explains immediately the slope in the measured drag force.

The slope of the drag force seems to be proportional with the velocity (see fig. 7.14). In table 7.2 are the data of the fits given. The fits on the slope should interpreted as: \(F = a_1t + a_2\).

De Mestre and Katz\(^{[1]}\) have obtained the drag force of a sphere attached to a thin threat in an unbounded Newtonian fluid. This result can be used to estimate the error in the f.e.-computation due to the exclusion of the threat in the model. The forces on the isolated sphere \(F_{\text{sphere}}\) and the isolated wire \(F_{\text{wire}}\) are given by:

\[
F_{\text{sphere}} = 6\pi \eta_0 a K U, \tag{7.3}
\]

\[
F_{\text{wire}} = 4\pi \eta_0 l_w U (\epsilon_w + 0.807 \epsilon_w^2), \tag{7.4}
\]

where \(K\) is the Stokes wall resistance factor (table 4.1), \(l_w\) the length of the wire and \(\epsilon_w\) is defined as:

\[
\epsilon_w = \left[ \ln \left( \frac{2l_w}{R_w} \right) \right]^{-1}, \tag{7.5}
\]

with \(R_w\) the radius of the wire.

With a few assumptions of the properties at the time the sphere reached it’s maximum velocity:

\(l_w = 100\) [mm]
\(a = 5\) [mm]
\(R_w = 0.05\) [mm]
\(\eta_0 = 0.9821\) [Pa.s],

the difference in the solution due to the wire is approximately 30\%. Clearly the assumption that the wire does not affect the force velocity curve is wrong. The forces \(F_{\text{sphere}}\), \(F_{\text{wire}}\), \(F_{\text{sphere}} + F_{\text{wire}}\) and the maximum experimental forces are given in table 7.3.
Equation 7.6 gives the approximate relative difference in the solution due to the neglectance of the wire in the numerical computations.

\[
\kappa = \frac{\Delta F}{F_{tot}} = \frac{F_{wire}}{F_{sphere} + F_{wire}} = \frac{1}{1 + \frac{2L}{3ak}(\varepsilon_w + 0.807\varepsilon_w^2)} \tag{7.6}
\]

This difference is given as a function of the design parameters in figure 7.15. To minimize the difference it is of most importance to select a large sphere radius and a thin wire with respect to the length of the wire.
Figure 7.1 Experimental design of the sphere pull experiment and properties of the PTT-fluid PIB/C14 used for the experiment.

<table>
<thead>
<tr>
<th>PTT fluid properties MIT-fit</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_i$ [Kg/m$^3$]</td>
<td>790</td>
</tr>
<tr>
<td>$\theta$ [s]</td>
<td>0.06</td>
</tr>
<tr>
<td>$\eta$ [Pa.s]</td>
<td>1.424</td>
</tr>
<tr>
<td>$\varepsilon$ [-]</td>
<td>0.1</td>
</tr>
<tr>
<td>$\eta_0$ [Pa.s]</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Figure 7.2 Dimensions of the experimental apparatus.
Figure 7.3 Experimentally measured displacement.

Figure 7.4 Enlargement of the experimentally measured displacement.
Figure 7.5  Velocity fit with experimental data (▲) and filtered experimental data. The experimental curve is divided in three parts: I straight line, II third order, III constant.

<table>
<thead>
<tr>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$c_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.40</td>
<td>0.03926</td>
<td>0.20131</td>
<td>-0.24460</td>
<td>0.09906</td>
<td>-0.00414</td>
<td>0.00922</td>
</tr>
<tr>
<td>0.21</td>
<td>0.50</td>
<td>0.04452</td>
<td>0.16163</td>
<td>-0.24889</td>
<td>0.12767</td>
<td>-0.00798</td>
<td>0.01383</td>
</tr>
<tr>
<td>0.32</td>
<td>0.55</td>
<td>0.04763</td>
<td>0.37866</td>
<td>-0.59768</td>
<td>0.31382</td>
<td>-0.03639</td>
<td>0.01841</td>
</tr>
<tr>
<td>0.40</td>
<td>0.60</td>
<td>0.04557</td>
<td>-0.053368</td>
<td>-0.03387</td>
<td>0.09828</td>
<td>-0.01225</td>
<td>0.02300</td>
</tr>
<tr>
<td>0.40</td>
<td>1.00</td>
<td>0.04499</td>
<td>0.03592</td>
<td>-0.11292</td>
<td>0.11809</td>
<td>-0.01347</td>
<td>0.02762</td>
</tr>
</tbody>
</table>

Table 7.1  Acquired data for the used velocity fit.
Figure 7.6  The aquired velocity fits from the experimentally measured displacement.

Figure 7.7  Experimentally measured forces.
Figure 7.8 Experimentally acquired and f.e.m. determined drag force.

Figure 7.9 Experimentally acquired and f.e.m. determined drag force.
Figure 7.10 Experimentally acquired and f.e.m. determined drag force.

Figure 7.11 Forces found with f.e.m simulations.
Figure 7.12 The experimentally acquired resistance, imposed velocity ($F_d/U$).

Figure 7.13 The f.e.m. resistance imposed sphere velocity ($F_d/U$).

34
Figure 7.14 Slopes of the measured drag forces.

Table 7.2 Slopes data of the measured drag forces.

<table>
<thead>
<tr>
<th>velocity [m/s]</th>
<th>slope $a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02762</td>
<td>-0.0005855</td>
<td>0.01942</td>
</tr>
<tr>
<td>0.02300</td>
<td>-0.0004696</td>
<td>0.01673</td>
</tr>
<tr>
<td>0.01841</td>
<td>-0.0002697</td>
<td>0.01371</td>
</tr>
<tr>
<td>0.01383</td>
<td>-0.0002106</td>
<td>0.01075</td>
</tr>
<tr>
<td>0.009224</td>
<td>-0.00009424</td>
<td>0.006982</td>
</tr>
</tbody>
</table>
Relative deviation due to the wire

Figure 7.15 The approximate deviation by the neglectance of the wire in the numerical simulations.

<table>
<thead>
<tr>
<th>velocity</th>
<th>( F_{\text{sphere}} )</th>
<th>( F_{\text{wire}} )</th>
<th>( F_{\text{tor}} )</th>
<th>( F_{\text{exp max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02762</td>
<td>0.01526</td>
<td>0.004510</td>
<td>0.01977</td>
<td>0.0182</td>
</tr>
<tr>
<td>0.02997</td>
<td>0.01271</td>
<td>0.003755</td>
<td>0.01646</td>
<td>0.0158</td>
</tr>
<tr>
<td>0.01841</td>
<td>0.01753</td>
<td>0.003007</td>
<td>0.01318</td>
<td>0.0134</td>
</tr>
<tr>
<td>0.01383</td>
<td>0.007644</td>
<td>0.002259</td>
<td>0.009902</td>
<td>0.0104</td>
</tr>
<tr>
<td>0.009224</td>
<td>0.005097</td>
<td>0.001506</td>
<td>0.006603</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

Table 7.3 Forces due to the sphere, wire, summized and the maximum experimental forces.
8 Concluding remarks

The numerical method is compared with several Newtonian and non Newtonian solutions and is found to give accurate results. The mesh size and form is optimized for this problem.

The falling ball experiment seems to be very simple but problems arise due to high frequency of the desired video-system and to problems with markers. An experimental analyses of the falling sphere technique is postponed. Nevertheless Becker et al.\cite{12} have recently carried out this experiment with succes.

The sphere pull experiment is carried out on an drawbench. This is a more simple method for finding a velocity-force curve. Here is a velocity imposed, which is held constant as fast as possible. The experimentally obtained terminal force was always higher than the calculated one. The best explanation can be found in the exclusion of the thread in the f.e.-model. A Newtonian analysis of the influence of the thread on the drag force was made. This resulted in a deviation of the f.e.-computed drag force of about 30%.

The by Phan-Thien found equations for the velocity on the first overshoot in an U.C.M. are not usable for PTT-fluid analysis, for reasons that the non-linear PTT-parameter $\varepsilon$ is not included.
9 Recommendations

1) The sphere pull experiments should be done on a drawbench with more power. This to minimize start effects (slow acceleration at the beginning of the sphere pull experiment) and to increase the viscoelastic effect.

2) The material properties should be based on the fluid that is actually used.

3) The drawbench should be calibrated with known Newtonian fluids.

4) A good choice of the sphere-thread geometry must be made. This to permit the exclusion of the wire in the numerical model.

5) More study must be done to collect valid relations between the Weissenberg number, force and velocity for a PTT-fluid (inclusion of $\varepsilon$ in the model).
Appendix A: Newton iteration

Define the columns
\[
\begin{bmatrix}
  s_{11} \\
  s_{22} \\
  s_{12}
\end{bmatrix}, \quad
\begin{bmatrix}
  d_{11} \\
  d_{22} \\
  d_{12}
\end{bmatrix}, \quad
\begin{bmatrix}
  u_1 \\
  u_2
\end{bmatrix}
\]

(A.1)

and the PTT matrix \( L_1 \) (\( \mathcal{L} \tau \rightarrow L_1 \mathcal{L} \))
\[
L_1 = U_i \frac{\partial}{\partial x_i} I + \mathcal{C} + \left( \frac{1}{\theta} + \frac{\epsilon}{\eta} (\tau_{11} + \tau_{22}) \right) I
\]

(A.2)

where \( I \) the 3*3 unit matrix, and
\[
\mathcal{C} = 
\begin{bmatrix}
  -2L_{11} & 0 & -2L_{12} \\
  0 & -2L_{22} & -2L_{21} \\
  -L_{21} & -L_{12} & -L_{11} - L_{22}
\end{bmatrix}
\]

(A.3)

the material matrix \( D \)
\[
D = \frac{2\eta}{\theta} 
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 0.5 
\end{bmatrix}
\]

(A.4)

the strain-velocity matrix \( B \)
\[
B = 
\begin{bmatrix}
  \frac{\partial}{\partial x} & 0 \\
  0 & \frac{\partial}{\partial y} \\
  \frac{\partial}{\partial y} & \frac{\partial}{\partial x}
\end{bmatrix}
\]

(A.5)

the stress divergence matrix \( S \),

the pressure divergence matrix \( P \)
\[ S = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \]  
(A.6)

\[ P = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} \end{bmatrix} \]  
(A.7)

and, finally, the velocity divergence matrix \( V \)

\[ V = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \]  
(A.8)

With this notation, functionals \( F, G \) and \( H \) can be defined as

\[ F(s, \tilde{v}, q; \tau, \tilde{\mu}, p) = \int_\Omega \frac{T_L}{T} (L_T y - D B y) d\Omega \]  
(A.9)

\[ G(s, \tilde{v}, q; \tau, \tilde{\mu}, p) = \int_\Omega -y^T B^T \frac{T}{T} + y^T V^T p d\Omega \]  
(A.10)

\[ H(s, \tilde{v}, q; \tau, \tilde{\mu}, p) = \int_\Omega q V^T y d\Omega \]  
(A.11)

To compute the Jacobian, the directional derivative of a functional \( A \) into the direction \( \delta \tilde{u} \) is defined as

\[ \delta \tilde{u} A(s, \tilde{v}, q; \tau, \tilde{\mu}, q; \delta \tilde{u}) = \lim_{\theta \to 0} \frac{A(s, \tilde{v}, q; \tau, \tilde{\mu}, q; \theta \delta \tilde{u}, p) - A(s, \tilde{v}, q; \tau, \tilde{\mu}, q; \delta \tilde{u}, p)}{\theta} \]  
(A.12)
Derivatives with respect to $\tau$ and $p$ ($\delta_\tau A$ and $\delta_p A$, respectively), are defined analogously.

The linearised form of the set of equations (3.12), (3.13) and (3.14) is given by

\[
\begin{bmatrix}
\delta_\tau F & \delta_p F & \delta_p G \\
\delta_\tau G & \delta_\tau G & \delta_\tau H \\
\delta_\tau H & \delta_p H & \delta_p H
\end{bmatrix}
\begin{bmatrix}
\delta_\tau \tau \\
\delta_\tau u \\
\delta_\tau p
\end{bmatrix}
= F + G + H
\]  
(A.13)

The linearised form of $F(.;.)$ is given by

\[
\delta_\tau F = \int_\Omega g^T (L_2 + L_3 - DB) \delta_\mu d\Omega
\]  
(A.14)

with

\[
L_2 = \begin{bmatrix}
\frac{\partial \tau_{11}}{\partial x} & \frac{\partial \tau_{11}}{\partial y} \\
\frac{\partial \tau_{22}}{\partial x} & \frac{\partial \tau_{22}}{\partial y} \\
\frac{\partial \tau_{12}}{\partial x} & \frac{\partial \tau_{12}}{\partial y}
\end{bmatrix}
\]  
(A.15)

\[
L_3 = \begin{bmatrix}
2\tau_{11} \frac{\partial}{\partial x} + 2\tau_{12} \frac{\partial}{\partial y} & 0 \\
0 & 2\tau_{12} \frac{\partial}{\partial x} + 2\tau_{22} \frac{\partial}{\partial y} \\
\tau_{12} \frac{\partial}{\partial x} + \tau_{22} \frac{\partial}{\partial y} & \tau_{11} \frac{\partial}{\partial x} + \tau_{12} \frac{\partial}{\partial y}
\end{bmatrix}
\]  
(A.16)

\[
\delta_\tau F = \int_\Omega g^T (L_1 + L_2) \delta_\tau d\Omega
\]  
(A.17)
with
\[ L_\delta = \frac{e}{\eta} \begin{bmatrix} \tau_{11} & \tau_{11} & 0 \\ \tau_{22} & \tau_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

\[ \delta_p F = 0 \quad \text{(A.19)} \]

The linearised form of \( G(.,.) \) is given by
\[ \delta_u G = 0 \quad \text{(A.20)} \]

\[ \delta_\tau G = \int_\Omega -\gamma^T B \delta_\tau d\Omega \quad \text{(A.21)} \]

\[ \delta_p G = \int_\Omega \gamma^T \mathcal{V} \delta p d\Omega \quad \text{(A.22)} \]

while the linearised form of \( H(.,.) \) follows from
\[ \delta_u = \int_\Omega q \mathcal{V} \delta u d\Omega \quad \text{(A.23)} \]

\[ \delta_\tau H = 0 \quad \text{(A.24)} \]

\[ \delta_p H = 0 \quad \text{(A.25)} \]
The iteration process can be described as follows:

\[ i=0; \text{ initial } u^0, z^0, p^0 \]

\[ i=i+1 \]

Given \( u^{i-1}, z^{i-1}, p^{i-1} \) find \( \delta u^i, \delta z^i, \delta p^i \) such that

\[
\begin{bmatrix}
    \delta F \\
    \delta u F \\
    \delta p F \\
    \delta z G \\
    \delta u G \\
    \delta p G \\
    \delta z H \\
    \delta u H \\
    \delta p H \\
\end{bmatrix} = F + G + H
\]

\[ u^i = u^{i-1} + \delta u^i; \quad z^i = z^{i-1} + \delta z^i; \quad p^i = p^{i-1} + \delta p^i \]

Convergence?

\textit{Chart A.1} The Newton iteration process.
Appendix B: Newtonian test problem Poiseuille flow

A quadratic velocity field must be adapted (see fig.B.1, \( u_m \) is the mean velocity of the fluid):
\[
 u = 2u_m \left(1 - \frac{r^2}{R^2}\right).
\]  
(B.1)

The shear stress at the wall is given by

\[
\tau_w = \frac{4u_m \eta_0}{R}.
\]  
(B.2)

A summation over the wall gives

\[
F_p = \int \tau_w \, dT = 2\pi RL\tau_w.
\]  
(B.3)

Substituting formula B.2 in formula B.3 gives

\[
F_p = 8\pi L u_m \eta_0.
\]  
(B.4)

Now the boundary forces are known for a given velocity field. This field can be forced upon the fluid via the boundary conditions. The used material and design properties are:
velocity \( u_m = .1 \) [m/s]
length \( L = .1 \) [m]
radius \( R = .01 \) [m]
viscosity \( \eta_0 = 1.2 \) [Pa.s].

Then \( \tau_w \) becomes \((4*.1*1.2)/.01=48\text{ [Nm}^2]\) and \( F \) becomes \(8*\pi*.1*.1*1.2=0.302\text{ [N]}\). The variation of the force was less then 3%, which is for the used mesh (fig B.2) very acceptable.
**Figure B.1** Velocity field, dimensions and properties Poisseulle flow.

- $F_p$: Poisseulle force [N]
- $U$: velocity [m/s]
- $\eta_0$: viscosity [Pa.s]

**Figure B.2** Used mesh for the Poisseulle flow test problem.
Appendix C: Mesh specification

The mesh was generated by SEPRAN. The optimization is split in three parts.
1) Find the optimal mesh shape.
2) Find the optimal mesh size.
3) Find the optimal length of the tube.

C.1 Mesh shape optimization

When elements are distorted from the shape of the parent they are found to be less accurate[13]. As the distortion increased the greater the error on the element behaviour. In setting up the mesh the elements must be as near to the basic shape of the parent as possible. The four possible forms of element distortion are:

1. Aspect ratio distortion (elongation of the element)
2. Angular distortion of the element, where any included angle between edges approaches either 0° or 180° (skew and taper).
3. Volumetric distortion of the element, where the mapping between the real element space and the non-dimensional basis space is such that the volume transformation may tend to zero at some point.
4. Mid node position distortion.

Examples of distorted elements are given in fig.C.1.

The construction is axisymmetric for loadings and structure. These structures are best solved by means of axisymmetric elements. Solving 3D problems only by a two dimensional section gives a considerable saving in the cost of the analysis. On experience in former work, there is chosen for a axisymmetric quadratic rectangle.

The simplest mesh, generated for the flow past a sphere in a tube is given in fig.C.2. The mesh is divided in four areas (I,II,III,IV) where mesh is generated. The areas where problems occur are indicated with a letter (see fig.C.2).

This design seemed not to be appropriate (see Mesh size optimization). In the problem areas were to big. Therefore there a new mesh design was created (see fig.C.3).
The problem area with this approach are much smaller. The element is in the most important area as good as possible shaped as the parent (a rectangle). For the keeping of the rectangle the elements must be placed, optimized shape along the sphere boundary, for optimal elements. With \( \alpha = 45^\circ \) and \( R = 2a \) gives:

\[
\frac{8 + \pi}{8n} m^2 - \left( \frac{8 + \pi}{8n} 2\sqrt{2} - 1 \right) m + (2\sqrt{2} - 1) = 0.
\]

(C.1)

for the factor on the long side \( f_{ml} \) and the short side \( f_{ms} \) are the formulae:

\[
f_{ml} = \frac{8n}{\pi m} - 1, \quad f_{ms} = \frac{8n(2\sqrt{2} - 1)}{\pi m} - 1
\]

(C.2)

The resulting number of elements on the area boundaries are listed in table II. The multiplication factors \( f_{ml} \) (long side with \( m \) elements (see fig.C.3)) and \( f_{ms} \) (short side) are used in the SEPRAN mesh generator. They mean (with the choice of ratio = 1) that the last element boundary on the edge is \( f \) times longer than the first. A few examples of meshes generated with these values are shown in fig.C.6-C.9.
Figure C.1  Examples of distorted elements.

Figure C.2  Areas where mesh distortion problems occur, a,b,c,d are declared in fig.C.1
Figure C.3  New mesh-design with fewer distorted areas, \( m \) and \( n \) are the number of elements on the boundaries.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( m )</th>
<th>( f_{ms} )</th>
<th>( f_{ml} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.55</td>
<td>3.66</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.55</td>
<td>3.66</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1.55</td>
<td>3.66</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.04</td>
<td>2.72</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>1.12</td>
<td>2.88</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>1.18</td>
<td>2.99</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>1.23</td>
<td>3.07</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1.04</td>
<td>2.72</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>1.08</td>
<td>2.81</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>1.12</td>
<td>2.88</td>
</tr>
</tbody>
</table>

Table C.1  Number of elements (see fig.B.3) and multiplication factors.
C.2 Mesh size optimization

There have been a few test meshes created. First with the simple mesh (see fig.C.6). The expected problems did occur. They are tested with the known Stokes flow. In order to have the greatest meaning for this problem the following physical quantities have been used for all the tests:

\[
\begin{align*}
\Delta \rho &= 1700 \text{ [kg/m}^3]\text{]} \\
\eta_0 &= 1.2 \text{ [Pa.s]} \\
a &= 10 \text{ [mm]} \\
R &= 20 \text{ [mm]} \\
l &= 100 \text{ [mm]} \\
g &= 9.81 \text{ [m/s}^2]\text{]} \\
\text{simulation time } t_s &= 0.04 \text{ [s]} \\
\text{time step } \Delta t &= 0.001 \text{ [s]}
\end{align*}
\]

For these quantities the Wall resistance factor \( K \), end velocity \( U_0 \) and Reynolds number are:

\[
\begin{align*}
K &= 5.970 \text{ [-]} \\
U_0 &= 0.0517 \text{ [m/s]} \\
Re &= 0.862 \text{ [-]}
\end{align*}
\]

The tests with mesh 1 are done in various amounts of elements; 9 test meshes have been used with 24; 36; 60; 80; 100; 160; 208; 248; 350 elements. A examples of the tested meshes 1 are shown in figures C.6-C.9. The difference in the wall resistance factor \( K \) calculated by FEM and the known wall resistance factor \( K \) are shown in fig.C.4. It is clear that for mesh 2 the fault was very high. This was because of the Jacobian matrix tending to zero along the sphere boundary. The integration points were almost put upon each other (see fig.C.7). Then the more elements used seemed not to lead to convergence. Therefore test mesh 2 was created.

Tests proved that mesh 2 was much more stable. The meshes were generated with the factors of table II. Most of the tested meshes 2 are shown in fig.C.10-C.13. The resulting difference of the wall resistance factor \( K \) is given in fig.C.5.

For
calculation purposes (fine meshes used an enormous amount of calculation time) is chosen for a rather coarse mesh. Mesh 2.3 (fig.C.11) seemed to be useful, coarse enough, but the convergence was reasonable good (see fig.C.5).

**Figure C.4** Difference between f.e.m. and analytical solution with varying number of elements, mesh class 1 (fig.C.6-9).

**Figure C.5** Difference between f.e.m. and analytical solution with varying number of elements, mesh class 2 (fig.C.10-13).
Figure C.6  Mesh no. 1.1; 30 elements.

Figure C.7  Mesh no. 1.2; 60 elements.

Figure C.8  Mesh no. 1.6; 180 elements.

Figure C.9  Mesh no. 1.9; 450 elements.
Figure C.10 Mesh no. 2.1; 28 elements.

Figure C.11 Mesh no. 2.3; 112 elements.

Figure C.12 Mesh no. 2.4; 170 elements.

Figure C.13 Mesh no. 2.5; 266 elements.
C.3 Tube length optimization

Now the mesh size and shape have been optimized, the only mesh problem left is the length of the tube. If the length is taken to short, the assumptions on the boundary are incorrect. If the tube length is too long, the amount of time taken for calculation will be too high. Therefore the tube length must be as short as possible, with respect to the accuracy.

As a starting point is chosen for mesh 2.1. The basis (the mesh around the sphere (areas II, III, IV, V see fig.C.3)) is kept the same, while the tube length is varied (areas I and VI fig.C.3). The density of the mesh is kept approximately the same. The mesh optimization is plotted in fig.C.14.

Convergence check

Figure C.14 Mesh and tube length optimization, serie 1 (fig.C.6-9), serie 2 (fig.C.10-13) and serie 3 (fig.C.15-18).
Figure C.15 Mesh no. 3.1; 104 elements.

Figure C.16 Mesh no. 3.2; 112 elements.

Figure C.17 Mesh no. 3.3; 128 elements.

Figure C.18 Mesh no. 3.5; 176 elements.
Samenvatting

De eindig elementen methode f.e.m. is gebruikt voor het simuleren van een bol in een cilinder die gevuld is met een viscoëlastische vloeistof. Twee situaties zijn onderzocht. Ten eerste, een vallende bol in een zwaartekracht veld. Ten tweede, de beweging van een bol met daaraan een draad bevestigd. Met deze draad kan een snelheid opgelegd worden. Voor deze situatie zijn niet alleen eindige elementen berekeningen gedaan, maar ook experimenten.

Voor de vallende bol situatie is goede overeenstemming gevonden met analytische resultaten (Newtonse vloeistof) en numerieke resultaten die in de literatuur gevonden zijn (voor viscoëlastische vloeistoffen).

Bij het experiment waarbij de bol getrokken wordt was de gemeten weerstand systematisch onderschat door de f.e.-resultaten. Gepresenteerd is dat dit verklaard kan worden door het weglaten van de draad in de numerieke berekeningen.
References


5. F.P.T. Baaijens, Applied Computational Mechanics 2, internal lecture notes of the Eindhoven University of Technology.


