MASTER

Characterization of Q2D rotating turbulence evolving over a no-slip bottom

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Abstract

This study discusses the characterization of Q2D rotating turbulence evolving over a no-slip bottom. Numerical simulations and laboratory experiments showed an asymmetric decay of cyclonic and anticyclonic vortices that constitute the flow. This asymmetry is due to non-linear Ekman effects, which are responsible for the squeezing and, because of conservation of angular momentum, slower damping of anticyclonic vorticity. Furthermore, the numerical simulations showed that although an asymmetry exists, the evolution of the total number of vortices, the mean area of the vortices, and the total area of the vortices are independent of the non-linear Ekman effects. In contrast, these effects modify the evolution of quantities related to the vorticity such as the vorticity skewness, the vorticity kurtosis, and the typical length scale given by the ratio of the enstrophy and the kinetic energy. Moreover, the experiments showed the appearance of meandering currents with a preferential direction, which are not observed in the numerical simulations and may be due to different three-dimensional effects.
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Chapter 1

Introduction

In the world around us, all kinds of fascinating swirling phenomena exist in all possible sizes. You can think of tornados, but also of the vortex structures that can arise on the surface of dishwater that is being disturbed (figure 1.1). Most of these phenomena show three-dimensional (3D) behaviour, but many of the flows in the oceans or the atmosphere, studied in the field of geophysical fluid dynamics, behave in an almost two-dimensional (2D) way. The 2D behaviour results from the stratification in the density structure - in the atmosphere caused by thermal effects and in the ocean by a combination of differences in salinity and temperature - and the rotation of the earth. Also the fact that the ocean and atmosphere are both very thin, compared to the horizontal scales on which the studied phenomena take place, makes it possible to operate in ‘flat-land’. Nevertheless, the behaviour is not completely 2D, for which it is called quasi-2D (Q2D). In particular, the presence of a bottom topography actually produces a 3D effect which causes the 2D behaviour to be disturbed. Nonetheless, the bottom damping effects are so small that they still can be included in a 2D physical model. Q2D flows can also be created in a laboratory environment by applying background rotation, density stratification, a magnetic field or by using a set-up with an almost 2D geometry. Lately, the interest in research on 2D phenomena has grown a lot: the observation of the ocean and atmosphere, by means of satellites, proved the frequent existence of 2D flow structures.

An interesting feature of two-dimensional flows is the process of so-called self-organization, which is a process in which large coherent vortex structures grow out of small structures, in contrast to what is happening in 3D flows, where the opposite process occurs.

The goal of this work is to characterize Q2D rotating turbulence evolving over a no slip bottom. Geophysical turbulence is mostly related to and dependent upon the rotation of the earth, and thus better understanding of the effects of rotation will contribute to better models of geophysical phenomena.

Several studies have been carried out on 2D turbulence. For example, Carnevale et al. [3], studied the stage of the decay of the flow that is characterized by the formation of larger structures and proposed a scaling theory that expresses the time evolution of all statistical properties in terms of a single exponent $\xi$. Tabeling et al. [12] used this scaling theory to compare with the power law exponents of the temporal evolution of various quantities, obtained in experiments on Q2D turbulence in thin layers of electrolyte. In the numerical study, performed by van Bokhoven et al. [2], the influence on the time evolution, global quantities, and vortex statistics of decaying 2D turbulence, of different initial conditions, that were based on the experiments by Tabeling et al. [12], was studied. Morize et al. [10] studied the decay of initially three-dimensional grid generated turbulence in a rotating tank. They already reported effects of non-linear Ekman terms on the evo-
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Figure 1.1: Examples of swirling phenomena in the world around us: (a) hurricane ‘Felix’, (b) coherent structures in a soap film.

olution of turbulent flows. Both numerical and experimental research on the effects of background rotation in the presence of a no-slip bottom on cyclonic monopoles was previously performed by Zavala Sansón and van Heijst [15]. Zavala Sansón [14] numerically studied the effects of non-linear Ekman terms on both cyclonic and anticyclonic monopoles. Furthermore, Zavala Sansón et al. [16] performed a combined numerical and experimental study of a dipolar vortex, subjected to rotation over a no-slip bottom. The work on monopolar and dipolar vortices is used as a starting-point for the work presented in this report in order to first study the effects of rotation on single vortices: the ‘elementary particles’ of 2D turbulence. Then the study on 2D turbulence follows partly the approach that was used in the above mentioned work of van Bokhoven et al. [2].

The results from the work reported here are obtained with three vorticity evolution equations, that were described by Zavala Sansón and van Heijst [15]: a vorticity evolution equation including all the viscous effects, namely, the Ekman advection effects, the lateral viscosity, and the non-linear and linear Ekman stretching effects. Furthermore, an evolution equation only including linear Ekman stretching effects and lateral viscosity, and finally, an equation that just takes the lateral viscosity into account. First some numerical work on both cyclonic and anticyclonic monopoles, and on a dipole was done. Some of these simulations were already previously performed by Zavala Sansón et al. [16] and were repeated in order to check the code. Other simulations were performed to create a toolbox in order to explain the rotation effects on Q2D turbulence. Also some laboratory experiments on a dipole in a non-rotating and a rotating tank were carried out. Finally, both simulations and experiments on 2D turbulence were performed. The obtained results consist of the description of the time evolution of several vortex parameters and physical quantities. With the work on 2D turbulence, also some vortex statistics and scaling behaviour were studied.

The remainder of this report is organized as follows: in chapter 2, the physical model that is used in this study as a basis for the numerical simulations and to explain the results of the performed experiments, is derived, and some theoretical background on coherent vortex structures and 2D turbulence is presented. The performed simulations are described in chapter 3, and the results of these simulations are presented and discussed in chapter 4 (monopole and dipole simulations) and chapter 5 (simulations on 2D and Q2D turbulence). Chapter 6 will discuss the performed experiments, both the set-up and the results, and finally, chapter 7 will present the main conclusions and some recommendations.
Chapter 2

Theory

2.1 The physical model

In this section the model that forms the basis for the performed numerical simulations, and that is used to explain the results of the experiments, is derived. Because this report will discuss 2D and Q2D flow phenomena, the general flow equations have to be reduced from three to two dimensions as presented in the first subsection by combining the lecture notes by van Heijst [7] and the article by Zavala Sansón and van Heijst [15]. In the second subsection, these 2D equations will form the starting point for the derivation of the actual model.

2.1.1 General equations -from 3D to 2D-

The 3D Navier-Stokes equation, which describes conservation of momentum for motions relative to a Cartesian coordinate frame \((x, y, z)\), that rotates at a constant rate is

\[
\frac{Dv}{Dt} + 2\Omega \times v = -\frac{1}{\rho} \nabla P + \nu \nabla^2 v, \tag{2.1}
\]

where \(D/Dt = \partial/\partial t + v \cdot \nabla\) is the material derivative, \(v = (u, v, w)\) the relative velocity vector, \(t\) the time, \(\rho\) the fluid density, \(\nu\) the kinematic viscosity, \(\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\) the Laplacian operator, \(\Omega\) the rotation vector, and \(P\) the reduced pressure, defined as

\[
P = p - \rho \Phi_{grav} - \frac{1}{2} \rho \Omega^2 r^2, \tag{2.2}
\]

where \(p\) is the ordinary pressure, \(\Phi_{grav}\) the gravitational potential, and \(r\) the radial distance to rotation axis. \(\nu\) is assumed to be a constant.

It is assumed that the flow is incompressible, \(D\rho/Dt = 0\), so that the continuity equation \(\partial\rho/\partial t + v \cdot \nabla \rho + \rho \nabla \cdot v = 0\) reduces to

\[
\nabla \cdot v = 0. \tag{2.3}
\]

Equation (2.1) can be written in non-dimensional form using the following non-dimensional variables (marked by a tilde)

\[
\tilde{v} = U\tilde{v}, \quad P = \rho \Omega U L \tilde{P}, \quad t = \tilde{t} T_E, \quad x = L\tilde{x}, \quad y = L\tilde{y}, \quad z = H\tilde{z},
\]

\[
\nabla = \frac{1}{L} \frac{\partial}{\partial \tilde{x}} \hat{i} + \frac{1}{L} \frac{\partial}{\partial \tilde{y}} \hat{j} + \frac{1}{H} \frac{\partial}{\partial \tilde{z}} \hat{k}.
\]

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where \( U \) and \( L \) are typical horizontal velocity and length scales, respectively, \( H \) is a typical vertical length scale, \( \hat{i}, \hat{j}, \) and \( \hat{k} \) are the unit vectors in the direction of \( u, v, \) and \( w, \) respectively, and \( T_E \) is the Ekman time scale, that is defined as

\[
T_E = \left( \frac{2}{f \nu} \right)^{1/2} H, \tag{2.4}
\]

where \( f = 2\Omega \) is the Coriolis parameter. This leads to the following non-dimensional equation:

\[
\frac{1}{2} E^{1/2} \frac{\partial \tilde{v}}{\partial \tilde{x}} + Ro (\tilde{v} \cdot \tilde{\nabla}) \tilde{v} + \hat{k} \times \tilde{v} = -\frac{1}{2} \tilde{\nabla} \tilde{P} + \frac{1}{2} E \delta^2 \tilde{\nabla}^2 \tilde{v}, \tag{2.5}
\]

with \( \tilde{\nabla} = \frac{\partial}{\partial \tilde{x}} \hat{i} + \frac{\partial}{\partial \tilde{y}} \hat{j} + \frac{1}{\delta} \frac{\partial}{\partial \tilde{z}} \hat{k}. \)

\[
Ro = \frac{U}{fL} \tag{2.6}
\]

the Rossby number,

\[
E = \frac{2\nu}{fH^2} \tag{2.7}
\]

the Ekman number, and

\[
\delta = \frac{H}{L} \tag{2.8}
\]

the aspect ratio of the vertical and horizontal length scales. It has to be noted that \( \hat{k} = \Omega/|\Omega|, \)

and therefore, \( \hat{k} \) points in the direction of the rotation axis. The Ekman time scale can now also be written as

\[
T_E = \frac{2}{f E^{1/2}}. \tag{2.9}
\]

When combining equations (2.6), (2.7), and (2.8) another important non-dimensional number, the Reynolds number \( Re, \) defined as

\[
Re = \frac{2}{\delta^2} \frac{Ro}{E}, \tag{2.10}
\]

is recovered.

It is assumed that the flow is (almost) stationary, that \( Ro << 1, \) and that \( E \) is small. Then equation (2.5) becomes

\[
2\hat{k} \times \tilde{v} = -\tilde{\nabla} \tilde{P}, \tag{2.11}
\]

the so-called geostrophic balance. Since \( \hat{k} \) does not change and the flow is assumed to be incompressible, taking the curl of equation (2.11) leads to \( \langle \hat{k} \cdot \tilde{\nabla} \rangle \tilde{v} = 0 \) or,

\[
\frac{\partial \tilde{v}}{\partial \tilde{z}} = 0, \tag{2.12}
\]

with \( \tilde{z} \) parallel to the rotation axis. This equation is known as the Taylor-Proudman theorem which states that geostrophic flows are independent of the axial coordinate.

From the Taylor-Proudman theorem follows that horizontal velocities, perpendicular to the axis of rotation, can be assumed to be depth-independent, and equation (2.1) can therefore be reduced to an equation for 2D motions.
2.1.2 Derivation of the model

The derivation of the governing equations that describe the flow of a homogeneous fluid layer over a flat bottom in a rotating system follows the article by Zavala Sansón and van Heijst [15]. In the vertical direction the motion is limited between $0 \leq z \leq H + \eta \equiv h$ (figure 2.1), where $H$ is the layer depth without any relative motion, and $\eta(x, y, t)$ is the free-surface elevation relative to $H$. $\eta(x, y, t)$ contains both the elevation associated with the parabolic shape of the free surface and the elevation related to the motion of the flow. In the boundary layer close to the bottom ($z = 0$), viscous effects dominate the flow. This boundary layer is called the *Ekman layer* and is generally very thin compared to the total fluid depth. The thickness of the Ekman layer ($\delta_E$) is defined as

$$\delta_E = (\nu/\Omega)^{1/2}. \quad (2.13)$$

If the geostrophic balance dominates the flow, the horizontal momentum equations, being the $x$ and $y$ components of equation (2.5), and the continuity equation are

$$\frac{1}{2} E^{1/2} \frac{\partial \tilde{u}}{\partial t} + Ro \left( \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) - \tilde{v} = -\frac{1}{2} \frac{\partial \tilde{P}}{\partial x} + \frac{1}{2} E \delta^2 \nabla^2 \tilde{u}, \quad (2.14)$$

$$\frac{1}{2} E^{1/2} \frac{\partial \tilde{v}}{\partial t} + Ro \left( \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) + \tilde{u} = -\frac{1}{2} \frac{\partial \tilde{P}}{\partial y} + \frac{1}{2} E \delta^2 \nabla^2 \tilde{v}, \quad (2.15)$$

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{1}{\delta} \frac{\partial \tilde{w}}{\partial z} = 0, \quad (2.16)$$

where $\tilde{z}$ is defined in the same direction as the gravitational acceleration, and the velocity components $\tilde{u}$ and $\tilde{v}$ are assumed to be $\tilde{z}$-independent. In the remainder of this subsection, only non-dimensional equations will be used. To simplify the notation, the tildes used to indicate non-dimensional quantities will be omitted from now on.

The pressure gradient is eliminated from equations (2.14) and (2.15) by taking the $y$-derivative of equation (2.14) and subtracting it from the $x$-derivative of equation (2.15), yielding the following vorticity equation:

$$\frac{1}{2} E^{1/2} \frac{\partial \omega}{\partial t} + Ro \left( \frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y} \right) + \left( \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) (\omega Ro + 1) = \frac{1}{2} E \delta^2 \nabla^2 \omega, \quad (2.17)$$

where $\omega = \partial v/\partial x - \partial u/\partial y$ is the relative vorticity in the $z$-direction. Equation (2.17) can be rewritten in an $\omega$-$\psi$ formulation by deriving suitable expressions to substitute the horizontal velocity components ($u$ and $v$) and the horizontal divergence ($\partial u/\partial x + \partial v/\partial y$). For this, a hydrostatic balance
in vertical direction is assumed, which permits the integration of the continuity equation over the fluid depth. This integration gives the following expression:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \cdot h = - (w_h - w_0),
\]

with

\[
w_h = \frac{Dh}{Dt}
\]

the vertical velocity at the free surface and

\[
w_0 = \frac{1}{2} \delta_E \omega
\]

the so-called \textit{linear Ekman condition}, which states that the vertical velocity on top of the bottom-Ekman layer, \(w_0\), and the relative vorticity in the interior flow, \(\omega\), are proportional.

In figure 2.2 the influence of the Ekman layer is illustrated. If the vorticity \(\omega\) of the interior flow is positive relative to the background rotation, fluid will be pumped from the Ekman layer into the interior. This will cause squeezing of the fluid columns. However, if the interior vorticity is negative relative to \(\Omega\), fluid will be sucked towards the Ekman layer causing stretching of the flow. The 2D character of the flow is thus disturbed by 3D effects that are induced by the no-slip condition at the bottom. If the 2D effects are however predominant it is possible to include the 3D effects in a 2D physical model that is therefore called \textit{quasi-two-dimensional} (Q2D).

Substituting equation (2.19) and equation (2.20) into equation (2.18) for the horizontal divergence leads to

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{1}{h} \frac{Dh}{Dt} + \frac{1}{2} \frac{\delta_E}{h} \omega.
\]

This equation can be further simplified if \(h \approx H\) and \(Dh/Dt \approx 0\), while considering \(\eta \ll H\), resulting in

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{2} E^{1/2} \omega,
\]

where \(E^{1/2} = \delta_E/H\). Rewriting of equation (2.22) yields

\[
\frac{\partial}{\partial x} \left( u - \frac{1}{2} E^{1/2} v \right) + \frac{\partial}{\partial y} \left( v + \frac{1}{2} E^{1/2} u \right) = 0.
\]
Now, a streamfunction $\psi$ is defined such that

$$u - \frac{1}{2}E^{1/2}v = \frac{\partial \psi}{\partial y},$$

(2.24)

$$v + \frac{1}{2}E^{1/2}u = -\frac{\partial \psi}{\partial x}.$$  

(2.25)

Since $E \ll 1$, only $O(1)$ - and $O(E^{1/2})$ terms are conserved and the following expressions for the horizontal velocities in terms of the streamfunction are derived:

$$u = \frac{\partial \psi}{\partial y} - \frac{1}{2}E^{1/2}\frac{\partial \psi}{\partial x},$$

(2.26)

$$v = -\frac{\partial \psi}{\partial x} - \frac{1}{2}E^{1/2}\frac{\partial \psi}{\partial y}.$$  

(2.27)

By substituting equations (2.26) and (2.27) in the definition of the relative vorticity ($\omega = \partial v/\partial x - \partial u/\partial y$) it can be verified that

$$\omega = -\nabla^2 \psi.$$  

(2.28)

Finally, substituting the derived expressions for the horizontal velocity components and the horizontal divergence in equation (2.17) yields

$$\frac{1}{2}E^{1/2}\frac{\partial \omega}{\partial t} + RoJ(\omega, \psi) - \frac{1}{2}E^{1/2}Ro\nabla \psi \cdot \nabla \omega = \frac{1}{2}E\delta^2 \nabla^2 \omega - \frac{1}{2}E^{1/2}Ro\omega^2 - \frac{1}{2}E^{1/2}\omega,$$  

(2.29)

where $J$ is the Jacobian operator defined as

$$J(\omega, \psi) = \frac{\partial \omega}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \omega}{\partial y} \frac{\partial \psi}{\partial x}.$$  

(2.30)

The physical Q2D-model of a geostrophic flow in a rotating system including the flat-bottom Ekman damping is now given by the equations relating the horizontal velocities and the streamfunction (equation (2.26) and (2.27)), Poisson’s equation (2.28), and the vorticity evolution equation (2.29). This model will henceforth be referred to as ‘model M1’.

**Linear and non-linear Ekman terms**

The physical Q2D model derived above is actually an extended version of a conventional model including bottom damping, used in many previous studies, e.g. in the work by Orlandi and van Heijst [11]. This conventional model, hereafter referred to as model M2, only takes into account the linear part of the Ekman terms, and it is described by the following vorticity equation:

$$\frac{1}{2}E^{1/2}\frac{\partial \omega}{\partial t} + RoJ(\omega, \psi) = \frac{1}{2}E\delta^2 \nabla^2 \omega - \frac{1}{2}E^{1/2}\omega.$$  

(2.31)

In this model $\omega$ and $\psi$ are again related by Poisson’s equation (2.28), but in equations (2.26) and (2.27), the $O(E^{1/2})$ terms are not taken into account. This model applies for a rotating system but it can also be used for a non-rotating system, since the linear Ekman term is similar to the Raleigh friction term used to simulate bottom friction, as in the work by Clercx et al. [4].
2.2. 2D TURBULENCE

CHAPTER 2. THEORY

The purely 2D model

The importance of the horizontal diffusion, the linear Ekman terms, and the non-linear Ekman terms that are all included in equation (2.29), can be studied separately, by reducing model M1 to the purely 2D-model. This is done by taking only lateral viscous effects into account by leaving out all the Ekman terms, and has the following vorticity evolution equation:

\[ \frac{1}{2} E^{1/2} \frac{\partial \omega}{\partial t} + Ro J(\omega, \psi) = \frac{1}{2} E \delta^2 \nabla^2 \omega. \]  

(2.32)

This model will henceforth be referred to as model M3, and applies for a non-rotating system, but also for a rotating system without a no-slip bottom. It has to be noted that \( E, Ro, \) and \( \delta \) lose their physical meaning in the purely 2D model. However, they can be used in the comparison of flow behaviour for the three different models.

2.2 2D turbulence

As already mentioned in the introduction, many of the flow phenomena in the oceans or in the atmosphere are considered as quasi-2D, including turbulence.

Three-dimensional turbulence, for example visible in the smoke from a burning cigarette, is characterized by big structures falling apart into smaller structures that once more fall apart in smaller structures and so on until dissipation on molecular scale occurs. Two-dimensional turbulence is characterized by exactly the opposite process, called self-organization.

2.2.1 Energy and enstrophy cascades

The main difference between 2D and 3D turbulence is caused by the effects of the mechanism of vortex stretching, defined by \( \omega \cdot \nabla \), that is present in 3D but not in 2D.

In a 2D flow, the flow field \( \mathbf{v} = (u, v, 0) \) resides on one plane; the vorticity-vector \( \omega = \nabla \times \mathbf{v} = (0, 0, \omega) \) is directed in the direction perpendicular to the plane of the flow field, causing \( \omega \cdot \nabla \mathbf{v} = 0 \). Since vortex stretching is absent, vorticity is conserved in non-viscous flows. From the non-viscous vorticity equation, two other conserved quantities can be derived, namely the kinetic energy, \( E_k = (1/2)|\mathbf{v}|^2 \) and the enstrophy \( S = (1/2)\omega^2 \), a measure of the total vorticity of a flow. In the introduction of this chapter, the process of self-organization was already mentioned, and can now be explained by first writing the kinetic energy and enstrophy in spectral form:

\[ \begin{align*}
E_k(t) & := \int \frac{1}{2} |\mathbf{v}(k)|^2 dk, \\
S(t) & := \int \frac{1}{2} \omega^2 dk.
\end{align*} \]

Figure 2.3: Schematic representation of the inverse energy cascade. (a) Energy spectrum at \( t = 0 \). (b) Energy spectrum at \( t > 0 \).
\[ E_k \sim \int_0^\infty \epsilon(k,t)dk = \text{constant}, \tag{2.33} \]

\[ S \sim \int_0^\infty k^2 \epsilon(k,t)dk = \text{constant}, \tag{2.34} \]

where \( \epsilon(k,t) \) is the spectral energy-density in the wavenumber interval \([k, k + dk]\) at time \( t \). Figure 2.3 shows the spectra of the kinetic energy for \( t = 0 \) and \( t > 0 \). Figure 2.3(a), for \( t = 0 \), shows a peak around \( k_0 \). For \( t > 0 \) (figure 2.3(b)), the distribution of energy becomes broader, due to the non-linear interactions in the flow, while the area underneath the curve, the total kinetic energy, remains constant. At the same time, the total amount of enstrophy (in spectral form) must not change. To satisfy both requirements the peak of kinetic energy will shift to smaller wavenumbers and thus to bigger scales, apparently violating the second law of thermodynamics. This process is called the inverse energy cascade and leads to self-organization.

In contrast to the kinetic energy, the enstrophy cascades normally from big scales to small scales. This can be explained from the flow behaviour in real space where the vorticity gradient, and thus the enstrophy gradient, increases in time to meet conservation of mass. In Fourier-space this means that the enstrophy shifts to higher wavenumbers and thus to smaller scales. The direct enstrophy cascade also solves the apparent violation of the second law of thermodynamics, that the inverse energy cascade seemed to cause: the total entropy of the flow system shows a net increase because of the enstrophy processes.

Figure 2.4 shows a schematic representation of both the inverse energy cascade and the enstrophy cascade together with the values of their slopes. Energy and enstrophy are inserted by forcing, at wavenumber \( k_f \). Energy transfers to smaller wavenumbers at a rate \( \epsilon \), while enstrophy is moved to bigger wavenumbers with flux \( \eta \) and is dissipated at wavenumber \( k_d \), illustrated in figure 2.4.

![Schematic representation of the inverse energy cascade and the direct enstrophy cascade.](image)

2.2.2 Coherent vortex structures

Due to the inverse energy cascade in 2D turbulence, coherent vortex structures emerge. ‘A coherent vortex in a two-dimensional flow consists of one or more compact ‘patches’ of continuously distributed vorticity’[6]. In the next two subsections, this definition will be illustrated by getting into more detail on two important coherent vortex structures: the monopolar vortex and the dipolar vortex.
Monopolar vortices

A monopolar vortex, or monopole, consists of a single set of closed streamlines around a common centre. The streamlines are generally circular or elliptical, but they also may appear with a different shape. Monopolar means that the vortex contains only a single centre, but it does not mean that the vortex has to be single-signed; quite often the circularly-symmetric vortices consist of a core with vorticity of one sign that is enclosed by a band of vorticity with opposite sign. These vortices are called shielded or isolated vortices.

For the simulations described in chapter 3, a sink vortex, also called Gaussian or Lamb monopole, was used. This vortex can be created in rotating tank experiments by locally syphoning fluid through a thin tube. It has a single-signed vorticity making it non-isolated. For the case of a flat bottom, the radial distributions of vorticity and azimuthal velocity are described by

\[ \omega_{sink}(r) = \omega_0 \exp \left( -\frac{r^2}{R^2} \right), \] (2.35)

\[ v_{sink}(r) = \frac{R^2 \omega_0}{2r} \left[ 1 - \exp \left( -\frac{r^2}{R^2} \right) \right], \] (2.36)

with \( \omega_0 \) the peak vorticity, \( R \) the radius of the vortex, and \( r \) the radial distance to the center of the vortex. Figure 2.5 shows the radial distributions of vorticity and azimuthal velocity for a sink-vortex with radius \( R = 2.86 \, \text{cm} \) and strength \( \Gamma = 84 \, \text{cm}^2 \, \text{s}^{-1} \), that can be calculated from

\[ \Gamma = \omega_0 \pi R^2. \] (2.37)

Figure 2.5: Typical radial distribution of (a) the normalized vorticity and (b) the azimuthal velocity of a sink vortex.

Monopoles can be either cyclonic, rotating in the same direction as the system, or anticyclonic, counter-rotating with respect to the rotation of the system. An anticyclone is simulated numerically by using \(-\omega_0\) with \( \omega_0 > 0 \).
Dipolar vortices

A dipolar vortex, or dipole, can be described as a symmetric ordering of two regions of vorticity with opposite sign. A dipole has a net linear momentum which causes it to move steadily as a whole in the direction defined by its axis. If the two regions of vorticity are equally strong the dipole is symmetric, and it translates along a straight line. However, if one half of the dipole is stronger, the dipole is asymmetric, and it translates along a curved trajectory [6].

For the simulations, described in chapter 3, a Lamb dipole (Figure 2.6(a)) was used. This is a dipole with a continuous vorticity distribution on a circular region as described by Lamb [8]. He assumed a linear relation between the streamfunction and the vorticity, \( \omega = k^2 \psi \), where \( k \) is a constant. The exterior flow of the dipole, \( r > a \), where \( a \) is a constant, is assumed to be a potential (irrotational) flow. Based on these assumptions, it is possible to analytically solve the streamfunction from equation (2.28). The equation, in polar coordinates \((r, \theta)\), is

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = \begin{cases} -k^2 \psi, & 0 \leq r \leq a, \\ 0, & r > a. \end{cases}
\] (2.38)

Because of the assumption of the linear relation between the streamfunction and the vorticity, the general solution of the first of these two differential equations is

\[
\psi_i(r, \theta) = \sum_n \{A'J_n(kr) + B'Y_n(kr)\} \{C' \sin(n\theta) + D' \cos(n\theta)\}, \quad 0 \leq r \leq a,
\] (2.39)

where \( n \) is an integer and \( A', B', C', \) and \( D' \) are unknown coefficients. \( J_n \) is the n-th order Bessel function of the first kind and \( Y_n \) the n-th order Bessel function of the second kind. For \( n = 1 \) the solution for a dipole is obtained. \( Y_1 \to -\infty \) as \( r \to 0 \). In order to avoid this singularity \( B' = 0 \). The flow in the exterior of the vortex was assumed to be a potential flow which in this case is equivalent to the 2D potential flow around a circular cylinder placed in a stream with uniform speed \( U \) for which the streamfunction \( \psi_e \) is given by a sinusoidal distribution:

\[
\psi_e(r, \theta) = -U \left( r - \frac{a^2}{r} \right) \sin(\theta), \quad r > a.
\] (2.40)
Because of the sinusoidal distribution in the exterior also $\psi \sim \sin(\theta)$ resulting in $D' = 0$. Equation (2.39) is now reduced to:

$$\psi_i(r, \theta) = CJ_1(kr) \sin(\theta), \quad 0 \leq r \leq a, \quad 0 \leq \theta \leq 2\pi. \quad (2.41)$$

The constant $C$ and the wavenumber $k$ result from continuity requirements for the streamfunction $\psi$ and the azimuthal velocity $v_\theta$ on $r = a$, giving the following solution for the interior streamfunction of the Lamb-dipole:

$$\psi_i(r, \theta) = -\frac{2U}{kJ_0(ka)}J_1(kr)\sin(\theta), \quad 0 \leq r \leq a. \quad (2.42)$$

The equation that describes the vorticity distribution in polar coordinates $(r, \theta)$ is

$$\omega(r, \theta) = \begin{cases} \frac{-2Uak}{aJ_0(ak)}J_1(rk)\sin\theta, & 0 \leq r \leq a, \\ 0, & r > a. \end{cases} \quad (2.43)$$

Figure 2.6(b) shows a plot of the streamlines of the Lamb dipole that coincide with contours of vorticity, and figure 2.7 shows some typical vorticity, streamfunction, and velocity radial profiles, obtained from numerical simulations of a Lamb dipole.

![Graphs showing streamfunction and vorticity](image1.png)

Figure 2.7: (a) Typical profile of the vorticity and streamfunction along a line through the extrema of vorticity of a Lamb dipole. (b) Profile of the velocity in the positive x-direction along a line through the extrema of vorticity. The velocity of the dipole as a whole is indicated by the straight horizontal line.

### 2.2.3 Effect of the non-linear Ekman terms on coherent structures

The vorticity evolution equation of model M1 (equation (2.29)), contains non-linear Ekman terms where the terms $-1/2E^{1/2}Ro\nabla\cdot \nabla \omega$ are the corrections to the advective terms $J(\omega, \psi)$, that enable recirculation in the Ekman pumping process, and the terms $-1/2E^{1/2}Ro\omega^2$ represent the non-linear contribution to stretching effects associated with Ekman pumping. When these non-linear Ekman terms are taken into account there are two main differences between cyclonic and anticyclonic vortices that are not observed if only linear Ekman terms are taken into account. The first difference is that fluid is radially advected in a spiral fashion, outward in cyclones and inward in anticyclones.
The second difference is that cyclones decay faster than anticyclones \[14\].

The first difference can be explained by considering the bottom Ekman condition for axisymmetric vortices, from which follows that the weak radial velocity is proportional to the azimuthal velocity \[9\]:

$$v_r = \frac{1}{2} E^{1/2} v_\theta,$$

where \(v_r\) is the radial velocity and \(v_\theta\) the azimuthal velocity. This equation can also be written as:

$$\frac{dr}{dt} = \frac{1}{2} E^{1/2} \frac{r \, d\theta}{dt},$$

from which the following equation for the radial position \(r\) of a fluid element as a function of the angular coordinate \(\theta\) can be derived:

$$r = r_0 \exp \left[ \frac{1}{2} E^{1/2} (\theta - \theta_0) \right].$$

This equation describes a spiral motion by which fluid elements are advected outward if \(\theta\) increases (for cyclonic vortices) and advected inward if \(\theta\) decreases (for anticyclonic vortices). Note that when model \(M2\) or model \(M3\) are used, taking only the advection terms into account, there is no advection of fluid since the Jacobian term vanishes for axisymmetric flows which in both cases causes the material trajectories to be circles at a fixed radius instead of spirals.

In order to explain the second difference, the azimuthal component of the 2D Navier-Stokes equation in cylindrical coordinates \((r, \theta, z)\) is considered, while the flow is assumed to be axisymmetric:

$$\frac{D v_\theta}{Dt} + f v_r + \frac{v_r v_\theta}{r} = \nu \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r v_\theta)}{\partial r} \right),$$

where \(D/Dt = \partial/\partial t + v_r (\partial/\partial r)\) is the material derivative. The inviscid form of this equation is given by:

$$\frac{D v_\theta}{Dt} + f v_r + \frac{v_r v_\theta}{r} = 0,$$

which implies material conservation of the absolute angular momentum:

$$\frac{D}{Dt} \left( v_\theta r + \frac{1}{2} f r^2 \right) = 0.$$

The angular momentum of an object that is rotating about some reference point is the measure of the extent to which the object will continue to rotate about that point unless an external torque is acting on it. Defining the absolute angular momentum for a particle \(i\): \(c_i = v_i r_i + 1/2 f r_i^2\), where \(v_i\) is the azimuthal velocity of the particle and \(r_i\) the radial position, and considering that \(c_i\) is conserved \((v_i = v_{i0} \text{ and } r_i = r_{i0})\), where the subindex 0 indicates the velocity and position at \(t = 0\), the following equation for the change in azimuthal velocity can be derived:

$$\Delta v = -\frac{\Delta r}{r_{i0}} (f r_{i0} + v_{i0}),$$

where \(\Delta r\) is a small change in the radial position. This means that the increase or decrease in azimuthal velocity of a particular fluid element is directly related to its net radial displacement.
Two situations are now possible:

\[ fr_{i0} + v_{i0} > 0, \]  
\[ fr_{i0} + v_{i0} < 0. \]  

Fluid elements in cyclones for which \( v_{i0} > 0 \), and in weak anticyclones, for which \( v_{i0} < 0 \), satisfy equation (2.51). For cyclonic vortices applies \( \Delta r > 0 \) which, following from equation (2.50), causes the azimuthal velocity of outward moving fluid elements to decrease. For anticyclonic vortices applies \( \Delta r < 0 \), causing \( \Delta v > 0 \) for inward moving fluid elements, resulting also in a decrease \((−v_{i0} + \Delta v)\) of the azimuthal velocity. By writing out equation (2.50) for both anticyclonic and cyclonic vortices and taking the absolute value of the ratio of the two equations it is verified that:

\[ \frac{|\Delta v_A|}{|\Delta v_C|} = \frac{fr_{i0} - v_{i0}}{fr_{i0} + v_{i0}} < 1. \]  

Thus the decay of the azimuthal velocity, caused by radial displacement, is smaller in the anticyclone \((A)\) than in the cyclone \((C)\) and therefore causing the cyclones to decay faster.

For strong anticyclones both equation (2.51) and equation (2.52) apply, depending on the initial position of the fluid elements: if fluid elements are placed far from the peak velocity equation (2.51) applies, causing again a decrease in the azimuthal velocity. However, if the fluid elements are placed close to the peak value they satisfy equation (2.52) causing that the azimuthal velocity may be intensified. This causes strong anticyclones to be potentially unstable.

The faster decay of cyclones compared to anticyclones can also be seen from model \( M1 \) when only Ekman stretching terms are taken into account:

\[ \frac{\partial \omega}{\partial t} = -\frac{1}{2} E^{1/2} \omega (\omega + f). \]  

When only the linear part is considered the vorticity decay does not depend on the sign of \( \omega \). However when the non-linear term is included the decay is larger for \( \omega > 0 \) than for \( \omega < 0 \).

The asymmetric decay of cyclones and anticyclones also has implications on the behaviour of dipolar vortices as described in the paper by Zavala Sansón et al. [16]: due to the non-linear Ekman effects the anticyclonic part of the dipole decays slower compared to the cyclonic part, causing the dipole trajectory to be deflected in the direction of the anticyclonic part. The asymmetrical evolution of the vortex is related with the advection of fluid from the cyclonic part to the ambient, leaving a tail behind the dipole.

### 2.2.4 Characterizing 2D and Q2D turbulence

This subsection describes the quantities and methods that are used in this study to characterize the evolution of two-dimensional turbulence.

**Enstrophy, kinetic energy, and their ratio**

The enstrophy \( S \) and the kinetic energy \( E_k \) were already defined in subsection 2.2.1. The ratio of the enstrophy and the kinetic energy \( S/E_k \) was previously used by Clercx et al. [4], and represents an estimate of the characteristic length scale \( l \) in the flow: \( \sqrt{S/E_k} \propto 1/l \). Thus if the ratio decreases, the characteristic length scale grows.
Skewness and kurtosis

The skewness is a statistic measure of the asymmetry of a data distribution around the mean value. If the skewness is negative there is more data to the left of the mean value, and if the skewness is positive the distribution of data is more to the right. The skewness of a perfectly symmetric distribution is zero. The skewness of a distribution of a quantity $x$ is defined as

$$sk(x) = \frac{\mu(x - x)^3}{\sigma^3}, \quad (2.55)$$

where $\bar{x}$ is the mean value of $x$, $\sigma$ the standard deviation of $x$, and $\mu$ the expected value of the argument.

The kurtosis is a statistic measure that is used to indicate the 'peakedness' of a distribution. A high value of the kurtosis indicates a distribution of data with a strong peak, which means that a relatively big part of the variance of the distribution is due to infrequent extreme deviations. A small kurtosis means a flat distribution of data, where the variance is mostly due to a bigger part of less extreme values. The kurtosis is defined as:

$$kur(x) = \frac{\mu(x - x)^4}{\sigma^4}. \quad (2.56)$$

Vortex statistics

Previously, Carnevale et al. [3], among others, performed a study of the evolution of vortex statistics in 2D turbulence. In order to describe the time evolution of 2D and Q2D turbulence, e.g. the evolution of the number of vortices or average area of the vortices can be determined. To be able to apply these statistics, a vortex census technique is needed to recognize coherent vortex structures. In the work by van Bokhoven et al. [2] a physical approach of vortex census was used: flow structures that approximately conform to the idealized shape of an isolated coherent vortex are identified. To distinguish between a vortex core and the halo of filaments that surrounds the cores of continuous vortices, the Weiss function $Q$ as described by Weiss [13] is used. This function is defined as

$$Q = tr(A)^2 = \psi_{xy}^2 - \psi_{xx}\psi_{yy}, \quad (2.57)$$

with $A = \nabla \hat{\psi}$ the stress tensor. The Weiss function is by definition proportional to the rate of strain squared, $S^2$, minus the vorticity squared $\omega^2$:

$$Q \equiv \frac{1}{2}(S^2 - \omega^2). \quad (2.58)$$

Regions where $|S| > |\omega|$ have a positive value for $Q$ and contain structures that are dominated by strain, such as vorticity filaments. Regions where $|S| < |\omega|$ however, have a negative value for $Q$ and contain structures that are dominated by rotation, such as coherent vortices. For axisymmetric vortices the sign of $Q$ changes where the azimuthal velocity has a maximum making it possible to associate a region of negative $Q$ with the core of a vortex. When using the Weiss function as a vortex census technique the following conditions have to hold: The Weiss function is negative everywhere in a simply connected region about a single local minimum of significant amplitude, typically smaller than $Q_{\text{min}}/10$, with $Q_{\text{min}}$ the global minimum Weiss function. Also the size of this region must be at least $30 \ l^2/N^2$, where $l$ is the size of the side of the domain that is defined to investigate and $N$ is the equivalent number of grid points. Finally the shape of the region must satisfy the restriction $I/I_c < 6$, where $I$ is the inertia of the region and $I_c$ the inertia of an equally sized circular region assuming that each point of the region has the same ‘weight’.
Scaling theory

The decay of a 2D turbulent flow from random initial conditions can be characterized by three different stages as described by e.g. van Bokhoven [2]: the first stage is characterized by self-organization of the fluid into coherent vortex structures, that contain most of the vorticity in the flow field. In the intermediate state coherent vortices dominate the flow, and it is governed by the process of nearly conservative mutual advection of the vortices when they are well separated, and the process of dissipative interaction of vortices when they become close. The final state contains just a single dipole that decays due to diffusion only.

The intermediate state, the state of interest in this study, shows self-similarity, which implies that the structure of the turbulence is independent of the scale at which it is observed. Physical phenomena that show self-similar behavior can be represented by a power-law spectrum. Carnevale et al. [3] proposed a scaling theory in which conservation of both the kinetic energy $E_k$ and the average absolute vorticity extremum $\zeta_{ext}$ are assumed. The absolute vorticity extremum is the vorticity amplitude in the vortex core, and is therefore shielded from the deformation, cascade, and dissipation, that all occur on the edges and outside of the vortices [13]. From the conserved quantities, a length scale $L \equiv \sqrt{E/\zeta_{ext}}$, and a time scale $T \equiv \zeta_{ext}^{-1}$ are defined. On dimensional grounds it follows that $\rho = L^{-2}g(t/T)$, and it is then assumed that $g \sim t^{-\xi}$. Now all scaling exponents can be given in terms of the single exponent $\xi$: e.g., the enstrophy scales as $S(t) \sim T^{-2}[t/T]^{-\xi/2}$. It has to be noted that the conservation of energy, and average vorticity extremum can only be assumed if $Re \to \infty$, making this scaling theory an infinite-Re theory.
Chapter 3

Numerical simulations

The numerical simulations were performed with the finite difference code \textit{NSevol}. This code uses the Navier-Stokes equations in the vorticity-streamfunction formulation to compute the evolution of a certain initial vorticity distribution. It is based on a code that was originally written by R. Verzicco and P. Orlandi (Rome, Italy) and was adapted and extended to be able to set more different initial vorticity distributions and boundary conditions, to follow the motion with passive tracers, and to include background rotation and/or a topography if needed. More details on \textit{NSevol} can be found in the manual, written by van Geffen [5]. The code was used before in studies by Zavala Sansón [14], Zavala Sansón and van Heijst [15], and Zavala Sansón \textit{et al}. [16], among others.

3.1 Monopolar vortices

Simulations on monopolar vortices were previously performed by Zavala Sansón [14] and Zavala Sansón and van Heijst [15]. The initial vortex parameters of their simulations partly correspond with those used in laboratory experiments on the spin-down of non-isolated cyclonic vortices in a rotating tank, which were performed in the same study [15].

In this study, some of the simulations, performed in previous studies ([14], [15]), were reproduced to check if the code was working properly. Different from the previous work, also simulations with aspect ratios $\delta = 3.15$ and $\delta = 0.28$, and simulations on anticyclones with model M2 and M3, were performed. The main goal of the simulations on monopoles was to gain more insight in the basic differences between a single vortex in a rotating and a non-rotating flow.

In all the simulations, a sink vortex (see subsection 2.2.2) was used as the initial vorticity distribution. A numerical square domain, with sides equal to 14 times the radius of the monopole, and with stress free boundaries, was discretized by 128 $\times$ 128 grid points. Stress-free boundary conditions were used since the total circulation in the domain was not equal to zero, so that periodic boundary conditions were not allowed. The monopole was placed in the center of the domain. The performed simulations are listed in table 3.1 and can be characterized by the Rossby number, the Ekman number, and the aspect ratio of the vertical and horizontal length scales.

For the simulations performed here, it was more convenient to use a different definition of the Rossby number: $Ro = Ro_\omega = \omega_0/f$, derived from equation (2.6) by using equation (2.37), and defining the characteristic velocity as $U = \Gamma/\pi R$. The choice for this definition, that measures the strength of the vortex with respect to the background rotation, can be explained from equation (2.29), written in dimensional form:

$$ \frac{\partial \omega}{\partial t} + J(\omega, \psi) - \frac{1}{2} E^{1/2} \nabla \psi \cdot \nabla \omega = \nu \nabla^2 \omega - \frac{1}{2} E^{1/2} \omega (\omega + f). $$

(3.1)
Table 3.1: Parameters of the simulations of monopolar vortices. Simulations 1 and 4 are characterized by the same non-dimensional numbers as the simulations performed by Zavala Sansón [14] and Zavala Sansón and van Heijst [15].

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$Ro_\omega$</th>
<th>$E$</th>
<th>$\delta$</th>
<th>$Re_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.26</td>
<td>$6.17 \cdot 10^{-5}$</td>
<td>6.29</td>
<td>2671</td>
</tr>
<tr>
<td>2</td>
<td>3.26</td>
<td>$2.47 \cdot 10^{-4}$</td>
<td>3.15</td>
<td>2660</td>
</tr>
<tr>
<td>3</td>
<td>3.26</td>
<td>$3.13 \cdot 10^{-2}$</td>
<td>0.28</td>
<td>2657</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$6.17 \cdot 10^{-5}$</td>
<td>6.29</td>
<td>819</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>$2.47 \cdot 10^{-4}$</td>
<td>3.15</td>
<td>816</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>$3.13 \cdot 10^{-2}$</td>
<td>0.28</td>
<td>815</td>
</tr>
</tbody>
</table>

The last term in this equation can be rewritten as $-\frac{1}{2}E^{1/2}\omega f(1 + \omega/f)$, where at $t = 0$, $Ro_\omega$ appears inside the parentheses. For simulations where anticyclones are present, $Ro_\omega$ should not become smaller than $-1$, because this will result in an unlimited increase of negative vorticity causing the solution to explode. Hence, in order to monitor the stability of the solutions obtained from the simulations, $Ro_\omega$ is a more useful definition.

For both cyclonic and anticyclonic vortices, simulations on two different Rossby numbers, $Ro_\omega = 3.26$, and $Ro_\omega = 1$ were performed. While keeping these Rossby numbers constant, the aspect ratio was varied. The representative length scale in horizontal direction $L$ was defined to be the radius of the vortex $R$, and was a constant in all the simulations. The representative length scale in vertical direction $H$ was defined to be the fluid depth and was varied, causing the Ekman number, $E$, to vary at the same time. For every combination of $Ro_\omega$, $E$, and $\delta$, simulations for all three models, $M1$, $M2$, and $M3$, described in subsection 2.1.2 were performed. The simulations were performed for times of order $T_E$, during which bottom friction effects become manifest. Table 3.1 also lists the Reynolds number $Re_\omega$, that is defined as in equation (2.10), but now calculated with $Ro_\omega$ instead of $Ro$.

Before running all the simulations, a suitable size for the time steps was determined. This size is related to the Rossby number in such a way that when the Rossby number increases, the speeds become bigger, and the time steps have to decrease to show the same accuracy. To find the right size for the time steps, several simulations with decreasing time step were performed at the highest initial Rossby number, and it was checked if the results did not change. It was found that there was no difference in results between simulations with time step $dt = 1 \cdot 10^{-1}s$, and time step $5 \cdot 10^{-2}s$, so the rest of the simulations were performed with time step $dt = 1 \cdot 10^{-1}s$. Also the effect of the spatial resolution was checked. This is important because, when the resolution is too low, the solution will have a large error. The resolution that is needed to obtain good results, also depends on the velocities of the flow: if the speeds become smaller, the resolution has to be big enough to ensure that the changes in the studied quantities take place within the grid size. To find the right resolution, several simulations with increasing resolution were performed, and the resulting outputs were compared. It was found that when the resolution was increased from $128 \times 128$ to $256 \times 256$, there was no change in the results so that in the remaining simulations resolution $128 \times 128$ could be used.

The time evolution of the radius of maximum velocity, the maximum velocity, the maximum vorticity, and the circulation were studied.
Table 3.2: Parameters of the simulations of dipolar vortices. Simulation 1 is characterized by the same non-dimensional numbers as were used in the simulations performed by Zavala Sansón et al. [16]. Simulations 2–4 are characterized by the same non-dimensional numbers as in some of the simulations on monopoles performed in this study, and simulations 5 and 6 are characterized by the same non-dimensional numbers as in the simulations on 2D turbulence performed in this study.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$Ro_\omega$</th>
<th>$E$</th>
<th>$\delta$</th>
<th>$Re_\omega$</th>
</tr>
</thead>
<tbody>
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<td>1.8</td>
<td>10005</td>
</tr>
<tr>
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<td>1</td>
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<td>0.9</td>
<td>9997</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$3.13 \cdot 10^{-2}$</td>
<td>0.08</td>
<td>9984</td>
</tr>
<tr>
<td>5</td>
<td>0.99</td>
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<td>1.6</td>
<td>854</td>
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<tr>
<td>6</td>
<td>0.99</td>
<td>$9.06 \cdot 10^{-5}$</td>
<td>1.6</td>
<td>8537</td>
</tr>
</tbody>
</table>

### 3.2 Dipolar vortices

Simulations on dipolar vortices were previously performed by Zavala Sansón et al. [16]. As in their work, here also a Lamb dipole (see subsection 2.2.2) was chosen to be the initial vorticity distribution. As with the monopole simulations, some of these previously performed simulations were reproduced, in order to check the code. The other simulations that were performed in this study, were simulations with the same characteristic numbers as used in some of the monopole simulations, and simulations with the same characteristic numbers as were used in the simulations on 2D turbulence, see section 3.3. A numerical rectangular domain, with sides equal to 10 and 15 times the radius of the dipole, was chosen. It was discretized by $128 \times 128$ grid points. The boundaries were periodic since the total amount of circulation in the domain was zero. The spatial resolution and time step, $5 \cdot 10^{-2}$ s, were determined in the same way as was described in subsection 3.1 on the monopole simulations. The dipole was initially placed at position $(5a, 5a)$. The angle $\theta$, as depicted in figure 2.6, was set 0. The performed simulations are listed in table 3.2, and are characterized by the Ekman number $E$, the Rossby number $Ro_\omega$, and the aspect ratio of the vertical and horizontal length scales $\delta$. The representative length scale in horizontal direction $L$ was the radius of the dipole ($a$), and the representative length scale in vertical direction $H$ was the fluid depth. The in table 3.2 listed simulations were performed for all three models: $M1$, $M2$, and $M3$.

The time evolution of the peak negative and positive vorticity, the negative and positive circulation, the energy, the enstrophy, the ratio of enstrophy and energy, the skewness, and the kurtosis were studied.

The peak vorticity $\omega_0$, used in the definition of $Ro_\omega$, was calculated from the equation for the vorticity distribution of the Lamb dipole (equation (2.43)). To find the value for $r$ where the vorticity reaches the maximum value, the derivative $\partial / \partial r$ of this equation was equated with zero. The maximum is reached at the first zero of $J'_1$ ($rk = 1.8412$), and thus $rk$ in equation (2.43) was substituted by this value. For a Lamb dipole, $J_1$ has to be cut off at its first non-zero root to obey the boundary conditions of the streamfunction, resulting in $ak = 3.8317$. With equation (2.43), it has to be noted that $U$ is the speed at which the dipole as a whole moves, indicating the strength of the dipole.
3.3 2D and Q2D turbulence

3.3.1 Method and performed simulations

Based on the work by van Bokhoven et al. [2], a checkerboard-shaped array of 100 equally sized Gaussian monopoles (50 cyclones and 50 anticyclones) was used as the initial condition. A numerical square domain with sides equal to 40 times the radius of the monopoles was chosen. The domain was discretized by $1024 \times 1024$ grid points. The boundaries were defined to be periodic since the total amount of circulation in the domain was zero.

Because of the characteristics of turbulence, different initial distributions can give completely different results. To avoid this problem several initial distributions of vortices with a random variation of $\pm 10\%$ in the $x$- and $y$ coordinates of the positions of the vortices and a random variation in the sign of the strength of the vortices were created. At the same time, the amount of both cyclones and anticyclones was kept at 50. Then, the results of the simulations that were performed with these different initial distributions were averaged.

Two different Reynolds numbers were chosen: a small $Re_\omega = 854$ and a large $Re_\omega = 8537$. The small Reynolds number was chosen in the same order of magnitude that is found in experiments, performed on thin fluid layers. On the other hand, the large Reynolds number was chosen in order to be able to apply scaling theory, which was proven to be possible in the case of sufficiently large Reynolds numbers [2], and to compare with previous research on 2D turbulence, e.g. the work by van Bokhoven et al. [2]. For the large Reynolds number the average over four different simulations was taken, and for the small Reynolds number the average over six different simulations was taken. A variable time step was used in order to save computation time: when, during the evolution of the flow, the velocities become smaller the time steps can be increased, and with higher speeds, the time steps will decrease to keep accurate results. When using a variable time step it is important to check whether the results stay the same when a different Courant number ($CFL$) is used. This number is a measure of the maximum velocity on the grid and is defined as

$$CFL = \max \left\{ \frac{|u|}{\Delta x} + \frac{|v|}{\Delta y} \right\} \Delta t,$$

(3.2)

where $u$ and $v$ are the 2D velocities at the grid points, $\Delta x$ is the mesh size in $x$ direction, $\Delta y$ is the mesh size in the $y$ direction, and $\Delta t$ is the time step that, when variable, is calculated from this equation. The criterion for stability of the numerical code used is linked to this Courant number by indicating the method as stable for $CFL \leq \sqrt{3}$. Here, $CFL = 1$ was chosen. Before running all the simulations with $CFL = 1$, a simulation with $Re = 8537$ and $\delta = 1.6$ was run for both $CFL = 1$ and $CFL = 0.5$, and the results were compared. The results for the two different Courant numbers were the same, so it was safe to use $CFL = 1$ in the remaining simulations.

The performed simulations are listed in Table 3.3 and are characterized by the Ekman number $E$, the Rossby number $Ro_\omega$, and the aspect ratio of the vertical and horizontal length scales $\delta$. The representative length scale in horizontal direction $L$ was the radius of the monopoles $R$, and the representative length scale in vertical direction $H$ was the radius of the monopoles $R$.

### Table 3.3: Parameters of the simulations of 2D turbulence.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$Ro_\omega$</th>
<th>$E$</th>
<th>$\delta$</th>
<th>$Re_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99</td>
<td>9.06 $\cdot 10^{-4}$</td>
<td>1.6</td>
<td>854</td>
</tr>
<tr>
<td>2</td>
<td>0.99</td>
<td>9.06 $\cdot 10^{-5}$</td>
<td>1.6</td>
<td>8537</td>
</tr>
</tbody>
</table>
representative length scale in vertical direction $H$ was the fluid depth. The listed simulations were performed for all three models, $M_1$, $M_2$, and $M_3$.

In the simulations on 2D turbulence some global quantities were determined: the average energy, the average enstrophy, the ratio of enstrophy and energy, the kurtosis, and the skewness. Also some vortex statistics were examined: the number of vortices, the average vortex area, and the total vortex area.

**Note:** In the work of van Bokhoven the Reynolds number $Re^*$ was calculated differently:

$$Re^* = \frac{1}{\nu} \left( \frac{E_{k,0}}{2} \right)^{1/2} \quad (3.3)$$

where $E_{k,0}$ is the initial kinetic energy. If the small and large Reynolds number, used in this study, are calculated according to this equation, they respectively become $Re^* = 8050$, and $Re^* = 80500$, differing a factor $3\pi$ with the Reynolds numbers calculated from equation (2.10), using $Ro_\omega$.

### 3.3.2 Vortex census

In order to make vortex statistics possible, vortices have to be recognized from the data as separate features. This is done by using a vortex census. Selection of features is based on vorticity, Weiss function, and feature size, that are all limited by a certain tolerance. The tolerance for the vorticity was fixed at $\omega = \exp(-1)$, following from the definition of a Gaussian monopole. The other tolerances, for the minimum and maximum size of the features, and for the Weiss function, were adjusted to the best fit: the quality of the vortex census was checked by trying different values of the tolerances and comparing the calculated accepted features with manual counted features in different simulations for different time steps. An example of a vortex census output is depicted in figure 3.1, where the contours in both the vorticity and Weiss function distribution, include the regions that are accepted by the census. The figure shows the vorticity field, the Weiss function and the accepted features for a simulation with $Re = 8537$, using model $M_1$ at $t = 0.17T_E$. 

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Figure 3.1: Example of the vortex census applied on a vorticity field, obtained from a simulation on an array of monopolar vortices with $Re = 8537$, using model M1: (a) vorticity field at $t = 0.17 \, T_E$, (b) same vorticity field, with contours placed on the vorticity tolerance value, (c) Weiss function with contours placed on the Weiss tolerance value, (d) accepted features as result of vortex census.
Chapter 4

Results of simulations of monopoles and dipoles

The discussion of the results will be based on the three models that were introduced in subsection 2.1.2. These models make it possible to independently study the effects of the different viscous terms: model $M_1$ includes all the viscous effects, namely the Ekman advection effects, the lateral viscosity, and the non-linear and linear Ekman stretching effects. Model $M_2$ only includes linear Ekman stretching effects and lateral viscosity, and model $M_3$ only takes the lateral viscosity into account.

4.1 Monopolar vortices

In this subsection the results from the monopole simulations, as listed in table 3.1, will be discussed. Figure 4.1 through 4.4 illustrate part of the obtained results: the time evolution of the radius of maximum velocity $R_{\text{max}}$, the maximum velocity $v_{\text{max}}$, the maximum vorticity $\omega_0$, and the circulation $\Gamma$. All these quantities, except the radius, are normalized by the initial value. The time is non-dimensionalized by the Ekman time scale. It should be noted that, when the simulations are performed with different values of $\delta$, the Ekman time scale is different too.

The first simulations performed are simulations that were previously done by Zavala Sansón [14] and Zavala Sansón and van Heijst [15]. The time evolution of the vortex’s parameters that is obtained here does not differ from those previously determined. Consequently, it can be concluded that the model works properly.

The most important conclusions drawn from the results of the simulations and experiments on non-isolated cyclones performed by Zavala Sansón and van Heijst [15] are: i) the results obtained with model $M_1$ fit the experimental results best. ii) Obviously, the more viscous terms are included, the faster velocity and vorticity decrease. iii) The radius of maximum velocity, $R_{\text{max}}$, increases due to the secondary flow in the radial direction, succeeding the non-linear Ekman stretching effects driven by the Ekman layer at the bottom; the relative vorticity in the vortex core is positive, and thus the vertical velocity induced by the Ekman layer is also positive. As a result fluid is pumped upwards and recirculates. This recirculation results in an expansion of the vortex. Lateral viscous effects also cause expansion of the vortex like the nonlinear advection effects driven by the Ekman layer, but the linear Ekman terms only contribute to the vortex decay, without any radial advection of $v_{\text{max}}$. iv) In the case of axisymmetric flows, as with a monopole, the Jacobian term in the models vanishes. As a result, model $M_2$ and $M_3$ do not contain terms that cause outward advection of
fluid and the material trajectories to be circles. Conversely, model M1 contains Ekman advective terms, which cause material particles to move outward spiralling.

Differently from the work by Zavala Sansón [14], simulations on anticyclonic monopoles were also performed with model M2, and M3. The results of these simulations are illustrated in figure 4.1 together with the results obtained using model M1. Obviously, the results that are obtained with model M2 and M3 do not differ from the results from the simulations on cyclonic monopoles: the non-linear Ekman terms that cause asymmetric decay of cyclones and anticyclones are absent. Although \( Ro_\omega < -1 \) in these simulations, they remained stable during their run-time: initially the small Ekman number was able to repress the effects of the large Rossby number. The results of the simulations on anticyclonic vortices where model M1 is used suggest that i) the initial decrease of \( R_{max} \) is caused by the nonlinear Ekman effects that are initially stronger than the counteracting radial diffusion, ii) the vorticity \( \omega_0 \) initially increases, because of the decreasing radius and the conservation of angular momentum, iii) the decrease of \( v_{max} \) is initially slower than for the cyclone due to the initial increase of the vorticity, that according to equation 2.36 causes the velocity to increase too.

The following performed simulations, 2 and 3, as listed in table 3.1 are simulations where \( \delta \) was reduced to respectively 3.15 and 0.28, causing the Ekman number \( E \) to increase to \( 2.47 \times 10^{-4} \) and \( 3.13 \times 10^{-2} \), respectively. Figure 4.2 illustrates the resulting time evolution of the vortex parameters obtained from simulation 3 on a cyclonic monopole. With the results it can be noticed that the velocity, vorticity, and vortex strength decrease slower while the radius increases slower, with all models. This can be explained from the reduction of \( \delta \), causing a huge decrease in the importance of the lateral viscosity term in equation (2.29), \( 1/2E\delta^2\nabla^2\omega \), compared to the other terms. Therefore, the time evolution of the vortex’s parameters in the simulation where only lateral viscosity is taken into account (model M3) hardly changes. It was not possible to perform simulation 3 for

![Figure 4.1](image-url)

**Figure 4.1:** *Time evolution of the vortex parameters of a strong anticyclonic monopole, characterized by \( Ro_\omega = -3.26, E = 6.17 \times 10^{-5}, \) and \( \delta = 6.29: \) (a) radius of maximum velocity \( R_{max} \), (b) maximum velocity \( v_{max} \), normalized by the initial value, (c) maximum vorticity \( \omega_0 \), normalized by the initial value (d) vortex strength \( \Gamma \), normalized by the initial value.*
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4.1. MONOPOLAR VORTICES

Figure 4.2: Time evolution of the vortex parameters of a strong cyclonic monopole, characterized by $R_{o\omega} = 3.26$, $E = 3.13 \cdot 10^{-2}$, and $\delta = 0.28$: (a) radius of maximum velocity $R_{\text{max}}$, (b) maximum velocity $v_{\text{max}}$, normalized by the initial value, (c) maximum vorticity $\omega_0$, normalized by the initial value, (d) vortex strength $\Gamma$, normalized by the initial value.

an anticyclone. This is caused by the high values for both the Rossby and Ekman numbers, which causes a solution where the vorticity grows indefinitely.

The results of the simulations where the Rossby number is decreased to $R_{o\omega} = 1$, simulations 4, 5, and 6, as listed in table 3.1, only differ from the simulations with $R_{o\omega} = 3.26$, if model $M1$ is used. This can be explained from equation (2.29). As was already mentioned above, the Jacobian vanishes in the case of axisymmetric flows. For simulations where model $M2$ and $M3$ are used, also all the non-linear terms disappear out of the equation, which makes the equation and the results of the simulations independent of the Rossby number. Thus for the simulations where model $M2$ and $M3$ are used, there is no difference in results if only the Rossby number is changed.

Figure 4.3 shows the time evolution of the vortex parameters of a weak anticyclonic monopole ($R_{o\omega} = -1$ and $\delta = 6.29$) for all models. As explained above, the time evolution with model $M2$ and $M3$ does not differ from those obtained from simulations on the cyclone and the anticyclone, if the same $\delta$ and $E$ are used. The time evolution of the vortex parameters with model $M1$ was already explained by Zavala Sansón [14]. An important remark is that, in contrast to the simulations with the strong anticyclone, now the radial diffusion is stronger compared to the non-linear Ekman effects, causing $R_{\text{max}}$ to increase and $\omega_0$ to decrease.

Finally, figure 4.4 depicts the time evolution of the vortex parameters of a weak anticyclonic monopole with $\delta = 0.28$ (simulation 6 in table 3.1). Although the Ekman number $E$ is large and $\delta$ is small, this simulation could be run because $R_{o\omega} = -1$, which is preventing the solution to explode. The time evolution of the vortex parameters for model $M2$ and $M3$, is the same as was presented in figure 4.2 for the simulation of the cyclonic monopole with the same small $\delta$. With the time evolution of the parameters with model $M1$ can be noticed that, although there are hardly lateral viscous effects to compete with the non-linear Ekman effects, $\omega_0$ does not initially increase. This can be explained by comparing the time evolution of $\omega_0$ with the time evolution of $R_{\text{max}}$. The evolution of $\omega_0$ is not completely determined by the evolution of $R_{\text{max}}$, because $\omega_0$ already starts...
4.1. MONOPOLAR VORTICES  

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Figure 4.3: Time evolution of the vortex parameters of a weak anticyclonic monopole, characterized by $Ro_\omega = -1$, $E = 6.17 \cdot 10^{-5}$, and $\delta = 6.29$: (a) radius of maximum velocity $R_{\text{max}}$, (b) maximum velocity $v_{\text{max}}$, normalized by the initial value, (c) maximum vorticity $\omega_0$, normalized by the initial value, (d) vortex strength $\Gamma$, normalized by the initial value.

Figure 4.4: Time evolution of the vortex parameters of a weak anticyclonic monopole, characterized by $Ro_\omega = -1$, $E = 3.13 \cdot 10^{-2}$, and $\delta = 0.28$: (a) radius of maximum velocity $R_{\text{max}}$, (b) maximum velocity $v_{\text{max}}$, normalized by the initial value, (c) maximum vorticity $\omega_0$, normalized by the initial value, (d) vortex strength $\Gamma$, normalized by the initial value.
decreasing while $R_{\text{max}}$ is still decreasing; therefore, the initial growth of $\omega_0$ is prevented by the linear Ekman terms that contribute to the vortex decay. Also, the time evolution of $R_{\text{max}}$ shows a decrease until $t = 0.75T_E$, and then continues as a constant. $t = 0.75T_E$ thus indicates the time at which the non-linear Ekman terms lose effect.

The decay of the strength, $\Gamma$, in all simulations discussed in this section, is dominantly affected by the linear Ekman terms, which cause an exponential decay for both model $M2$ and $M1$. Obviously, with model $M3$, that does not contain linear Ekman terms, the strength is almost constant. The strength is affected by a change of $E$: a smaller $\delta$ accompanied by a larger $E$, causes a slightly smaller decay.

### 4.2 Dipolar vortices

In this subsection, the results from the dipole simulations will be discussed. As with the monopole simulations, the model was first tested by running some simulations with the same initial conditions used by Zavala Sansón et al. [16] (simulation 1 in table 3.2). The results of these simulations were equal to those previously obtained, again confirming a properly working numerical code.

Different from the work by Zavala Sansón et al., in this study also the time evolution of the ratio of enstrophy and energy, the skewness, and kurtosis of the vorticity distribution are determined. This was done in order to associate the results of the dipole simulations with the studied quantities of the simulations on 2D and Q2D turbulence, that will be described in chapter 5.

#### 4.2.1 Comparison with the simulations of monopolar vortices

The simulations that are described in this subsection, listed in table 3.2 as simulation 2, 3, and 4, have initial conditions that are characterized by the same characteristic numbers as in the simulations on the monopole with $Ro = 1$. The idea behind these simulations is to be able to explain the behaviour of a dipole, by comparing with the behaviour of the ‘individually evolving parts’, represented by the cyclonic and anticyclonic monopoles. Figures 4.5 through 4.9 present the time evolution of the studied quantities for the three models, $M1$, $M2$, and $M3$, for different initial combinations of characteristic numbers. The time is again normalized by the Ekman time.

Figures 4.5, 4.6, and 4.7 show results of the same quantities, only obtained with different initial values for $E$ and $\delta$. Subfigures $a$ depict the time evolution of the maximum vorticity and subfigures $b$ the time evolution of the strength. Both vorticity and strength are determined separately for the cyclonic and anticyclonic half of the dipole. Subfigures $c$ and $d$ depict the time evolution of the kinetic energy and enstrophy, respectively. All quantities, except the time, are normalized by the initial value.

With the time evolution of the vorticity, it can be noted that with model $M2$ and $M3$ the vorticity of the cyclonic and anticyclonic half of the dipole decay at the same rate. The overall vorticity decays faster with model $M2$, which includes more viscous terms than model $M3$. All this was already mentioned with the simulations of the monopole. With model $M1$, the cyclonic and anticyclonic half decay differently: the cyclonic part decays faster than with model $M2$ and the anticyclonic part decays slower than with model $M2$, but faster than with model $M3$. This can be explained in the same way as was done with the cyclonic and anticyclonic monopole simulations, for which the same evolution occurred.

The strength for all three models decays at the same rate for both the cyclonic and the anticyclonic half, because the decay of the vortex strength is dominated by the linear Ekman terms.
4.2. DIPOLAR VORTICES

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Figure 4.5: Results of simulations of a dipole with the characteristic numbers based on the simulations of the monopole with $R_{\omega} = 1$, using model $M_1$, $M_2$, and $M_3$. $E = 6.17 \cdot 10^{-5}$ and $\delta = 1.8$: (a) positive and negative vorticity $\omega_{+/\pm}$, normalized by the initial value, (b) positive and negative circulation $\Gamma_{+/\pm}$, normalized by the initial value, (c) kinetic energy $E_k$, normalized by the initial value, (d) enstrophy $S$, normalized by the initial value.

Figure 4.6: Results of simulations of a dipole with the characteristic numbers based on the simulations of the monopole with $R_{\omega} = 1$, using model $M_1$, $M_2$, and $M_3$. $E = 2.47 \cdot 10^{-4}$ and $\delta = 0.9$: (a) positive and negative vorticity $\omega_{+/\pm}$, normalized by the initial value, (b) positive and negative circulation $\Gamma_{+/\pm}$, normalized by the initial value, (c) kinetic energy $E_k$, normalized by the initial value, (d) enstrophy $S$, normalized by the initial value.
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Figure 4.7: Results of simulations of a dipole with the characteristic numbers based on the simulations of the monopole with $Ro_\omega = 1$, using model $M_1$, $M_2$, and $M_3$. $E = 3.13 \times 10^{-2}$ and $\delta = 0.08$: (a) positive and negative vorticity $\omega_{+/-}$, normalized by the initial value, (b) positive and negative circulation $\Gamma_{+/-}$, normalized by the initial value, (c) kinetic energy $E_k$, normalized by the initial value, (d) enstrophy $S$, normalized by the initial value.

This can be derived from the equal decay with model $M_1$ and $M_2$, and hardly any decay with $M_3$. Also, with model $M_1$ and $M_2$ the decay is again exponential: if the results are plot on a log-scale, they become straight lines.

In the work by Zavala Sansón et al. [16], it was already mentioned that the energy decay is similar with model $M_1$ and $M_2$. This behaviour is also shown in the results obtained here. Zavala Sansón et al. [16] explained this by an equal time evolution of the total velocity with these two models: in their work was looked at the time evolution of the velocity components of the dipole center, $(u_d, v_d)$, which is different for model $M_2$ and $M_1$. That is, with model $M_2$ there is only one velocity component, $u_d$, and with model $M_1$ the velocity consists of two components: next to $u_d$, there is also an increasing $v_d$ component, resulting from the non-linear Ekman terms. However, the magnitude of the translation velocity $U = (u_d^2 + v_d^2)^{1/2}$ as a function of time behaves alike for model $M_1$ and $M_2$. The similar time evolution for the energy decay for model $M_1$ and $M_2$ can now be easily understood from the way the kinetic energy is defined.

The asymmetric decay of the cyclonic and anticyclonic half of the dipole with model $M_1$, can thus not be recognized from the decay of the energy, but it can be seen in the decay of the enstrophy, especially in figure 4.7 where the smallest $\delta$ is used. The difference between the results with model $M_1$ and $M_2$ can be explained from the definition of the enstrophy, $S = (1/2)\omega^2$. Since the decay of the vorticity with model $M_1$ is smaller for the anticyclonic part, the total decay will also be smaller than with model $M_2$, where both sides decay almost as fast as the cyclonic part with model $M_1$. This effect is most pronounced when the smallest $\delta$ is used, because the non-linear Ekman terms are much more important than the lateral viscous term, which is multiplied by $\delta^2$.

The most important effect of the decreasing of $\delta$ from $\delta = 1.8$, in figure 4.5, to $\delta = 0.08$ in figure 4.7, accompanied by an increase in $E$, is the slower decay of vorticity, strength, energy, and enstrophy for all models.
Figure 4.8a, b, and c depict the time evolution of the ratio of the enstrophy and the kinetic energy \( S/E_k \) as a function of the time that is non-dimensionalized by the Ekman time scale. As was already explained in subsection 2.2.4, this ratio represents an estimate of the characteristic length scale in the flow. In order to find the characteristic length scale, figure 4.8 also depicts the vorticity distribution, obtained with model \( M_1 \) at time \( t = 1.3T_E \), with the same \( E \) and \( \delta \) as in the corresponding plots for \( S/E_k \). The distribution with model \( M_1 \) is chosen because the ratio of the enstrophy and the kinetic energy, and thus the length scale obtained with this model, shows big differences for the three \( E-\delta \) combinations, while the evolutions with model \( M_2 \) and \( M_3 \) differ just a bit: it is easier to see what exactly is changing, if the changes are big enough. Combining the information, obtained from the time evolution and the vorticity distribution, it is assumed that the characteristic length scale is represented by the ‘vorticity-weighted’ size of the dipole: parts of the dipole where the vorticity is stronger, have a bigger weight factor times the size of this part, than parts with a small vorticity.

With the results presented in figure 4.8, it can be remarked that the time evolution of the ratio \( S/E_k \) evolves the same with model \( M_2 \) and \( M_3 \) and different from these two models with model \( M_1 \) and is therefore independent of the linear Ekman terms, but dependent on the non-linear Ekman terms. This again can be explained from what is seen for the time evolution of the radius with the monopole simulations, since the size of a vortex is determined by its radius: the time-evolution of the radius of the vortex was also the same, when obtained with model \( M_2 \) or \( M_3 \), and also evolved differently with model \( M_1 \), which was explained in detail in the former section. Also, with model \( M_2 \) and \( M_3 \) the ratio decreases, and decreases less with increasing \( \delta \), while the ratio with model \( M_1 \) decreases for \( \delta = 1.8 \), decreases less for \( \delta = 0.9 \), but increases for \( \delta = 0.08 \). The course of the evolution of the ratio with model \( M_2 \) and \( M_3 \) can also be explained with the same arguments that were used to explain the time evolution of the radius with the monopole simulations. The evolution with model \( M_1 \) can be explained from the vorticity distributions at \( t = 1.3T_E \): while \( \delta \) decreases and \( E \) increases, the Ekman terms become more important: the dipole covers a smaller distance in the domain due to the larger linear Ekman terms and there is a stronger asymmetry in the decay of the cyclonic and the anticyclonic part, caused by the non-linear Ekman terms.

Figure 4.9 shows the time evolution of the skewness, and kurtosis of the vorticity distribution. The skewness is, and stays, obviously zero if model \( M_2 \) and \( M_3 \) are used, independent of the characteristic numbers, because of the symmetric decay for the cyclonic and anticyclonic part of the dipole. The skewness with model \( M_1 \) grows negative in time, caused by the non-linear Ekman terms, that induce the stronger decay of the cyclonic half of the dipole. The strongest negative increase in time occurs with the smallest \( \delta \). This can be explained from the fact that Ekman effects act more effectively for lower depths.

Remarks that can be made with the time evolution of the kurtosis are that the kurtosis for model \( M_2 \) and \( M_3 \) decreases in time, and decreases the same for these two models. Apparently the decay is independent of the linear Ekman terms, which can be explained from the fact that these terms do not affect the size of the vortex, but only contribute to the vortex decay. The lateral viscous effects cause expansion of the dipole, and thus the kurtosis, defined as the ‘peakedness’ of a distribution (see subsection 2.2.4) will decrease. This is because the decaying vorticity is spread out over a bigger area, causing the peaks to lower. Furthermore, the kurtosis with model \( M_2 \) and \( M_3 \) decreases less, with the decrease of \( \delta \) from 1.8 to 0.9, and with \( \delta = 0.08 \), even remains at the same value. This can be explained by the large decrease of \( \delta^2 \), causing the lateral viscosity term in equation (2.29) to be less important and therefore the dipole to expand less. Moreover the kurtosis obtained with model \( M_1 \) increases in time, which is due to the non-linear Ekman terms, that become more important with smaller \( \delta \), causing the strongest increase with the smallest \( \delta \).
Figure 4.8: Results of simulations of a dipole with the characteristic numbers based on the simulations on the monopole with $R_o \omega = 1$, using model $M1$, $M2$, and $M3$: (a), (b), and (c) ratio of the enstrophy and the kinetic energy $S/E_k$ as a function of time, normalized by the Ekman time; ad(a), ad(b), and ad(c) the corresponding vorticity distribution on the full computational domain, obtained with model $M1$ at time $t = 1.3T_E$. 

(a) $E = 6.17 \cdot 10^{-5}$, $\delta = 1.8$. 

(b) $E = 2.47 \cdot 10^{-4}$, $\delta = 0.9$. 

(c) $E = 3.13 \cdot 10^{-2}$, $\delta = 0.08$. 

ad (a) 

ad (b) 

ad (c)
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(I) $E = 6.17 \cdot 10^{-5}$, $\delta = 1.8$.

(II) $E = 2.47 \cdot 10^{-4}$, $\delta = 0.9$.

(III) $E = 3.13 \cdot 10^{-2}$, $\delta = 0.08$.

Figure 4.9: Results of simulations of a dipole with the characteristic numbers based on the simulations on the monopole with $Ro_\omega = 1$, using model M1, M2, and M3: (a) skewness, and (b) kurtosis of the vorticity field as a function of time, normalized by the Ekman time.
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Figure 4.10: Typical time evolution of the vorticity field for a simulation of a dipole with $Re_\omega = 854$ and model $M_1$ (a–c), model $M_2$ (d–f), and model $M_3$ (g–i). Each column shows a different time step, that is indicated at the bottom of the figure. Depicted is the full computational domain.

The non-linear terms cause the vorticity to concentrate in the shrinking anticyclonic part, while the cyclonic part grows and therefore spreads out its vorticity. The result of this process is a vorticity distribution with only one extreme deviation, represented by the anticyclonic part of the dipole, which according to the definition of the kurtosis leads to a bigger value of this quantity. Finally, the time evolutions of the kurtosis, obtained with model $M_2$ and $M_3$, show some small dips in subfigure I and II. These dips occur in the time interval in which the dipole passes the periodic boundary of the domain and are therefore probably caused by an error in the way the skewness is calculated.

4.2.2 Comparison with the simulations of 2D turbulence

This subsection will discuss the results of numerical simulations of a dipole, with the same initial conditions that are chosen in the simulations on 2D and Q2D turbulence, that will be discussed in chapter 5. These simulations can be characterized by two different Reynolds numbers, $Re_\omega = 854$ and $Re_\omega = 8537$, and are listed in table 3.2 as simulation 5 and 6.

Figures 4.10 and 4.11 show typical time evolutions of the vorticity distribution for a dipole with initial Reynolds number $Re_\omega = 854$ (figure 4.10) and $Re_\omega = 8537$ (figure 4.11). In both figures, the subfigures in the first column depict the vorticity field at $t = 0$, the subfigures in the second and third column, the vorticity field at $t = 1.2T_E$ and $t = 2.4T_E$, respectively, as indicated at the
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Figure 4.11: Typical time evolution of the vorticity field for a simulation of a dipole with \( \text{Re}_\omega = 8537 \) and model M1 (a–c), model M2 (d–f), and model M3 (g–i). Each column shows a different time step, that is indicated at the bottom of the figure. Depicted is the full computational domain.

With these results it can be remarked that in the time evolution, obtained with the large Reynolds number and model M1, the dipole does not follow a straight line but moves on a curved trajectory, caused by the non-linear Ekman terms that produce a stronger decay of the cyclonic vortex. This behaviour cannot be noticed in the time evolution with model M1, obtained with the small Reynolds number. This can be explained by the fact that the dipole, in this case, hardly moves from the spot where the simulation starts running, which is also the case for the simulation with small Reynolds number and model M2. With model M2, only for the large Reynolds case, and M3, the dipole moves on a straight line, caused by the symmetric decay of the cyclonic and anticyclonic part of the dipole.

Furthermore, with model M1 and M2 the dipole has already stopped moving in the domain at time \( t = 1.2T_E \) and has then covered about the same distance for the two models. With model M3 the dipole continues moving on a straight line and, in the case of the large Reynolds number, even passes the periodic boundary. From this it becomes clear that the linear Ekman terms reduce the distance that is covered by the dipole, and that the non-linear terms just effect the way vorticity is divided over the cyclonic and anticyclonic half of the dipole.

Moreover, subfigure b and c show the spreading of the vorticity for the cyclonic part and the
concentration of the vorticity in the anticyclonic part of the dipole. Also the ‘leaking’ of the cyclonic part, that causes a tail behind the dipole, as mentioned at the end of subsection 2.2.3 is clearly visible in these figures.

Also, for the time evolution, obtained with the small Reynolds number and model $M_1$, it can be noticed that at time $t = 1.2T_E$ there is an asymmetric decay of vorticity for the cyclonic and anticyclonic part of the dipole, but at time $t = 2.4T_E$ this difference has become a lot smaller. Apparently the non-linear terms loose their effect after some time. This was also noticed in the results of the monopole simulations where a comparable Reynolds number was used (simulation 6 in table 3.1).

Finally, in the simulations with the small Reynolds number, the dipole increases a lot in size, compared to the simulations with the large Reynolds number. When comparing the vorticity fields for the three different models at the same time steps, there is hardly any difference in the size of the dipole, from which it can be concluded that the lateral viscosity is the dominant term that causes the dipole to grow. Also the strength of the vorticity decreases very fast, which also can be explained by the strong lateral viscosity effects.

Figure 4.12 depicts the time evolution of several global quantities, obtained with the three models, $M_1$, $M_2$, and $M_3$, where the time is again normalized by the Ekman time. The left picture shows the results that are obtained with the initial small Reynolds number $Re_\omega = 854$ and the right picture shows the time evolutions, obtained with $Re_\omega = 8537$. Subfigures $a$ show the time evolution of the maximum vorticity and subfigures $b$ the time evolution of the strength, where both vorticity and strength are determined separately for the cyclonic and anticyclonic half of the dipole. Subfigures $c$ and $d$ depict the time evolution of the kinetic energy and enstrophy, respectively. All quantities in figure 4.12, except the time, are normalized by the initial value.

With these results can be remarked that the vorticity, obtained with the small Reynolds number, decays almost the same for the cyclonic and anticyclonic part of the dipole, with all three models. In the results obtained with the large Reynolds number, there is a asymmetry in the decay with model $M_1$, as expected from the presence of the non-linear Ekman terms. This difference for the two Reynolds numbers can be explained from the remarks that were made with figure 6.3 regarding the dominance of the lateral viscosity in the simulations with small Reynolds number. The vorticity decay, obtained with both the small and the large Reynolds number, are ‘squeezed’ by the lateral viscosity: the decays with model $M_1$, $M_2$, and $M_3$ differ little.

Furthermore, in the time evolution of the strength, non-linear Ekman terms do not play a role: there is no difference in decay with model $M_1$ and $M_2$. The evolution, obtained with the small Reynolds number, is also initially dominated by the lateral viscosity. This can be derived from the fact that, if the strength obtained with model $M_1$ and $M_2$ is plotted on a log-scale, the decay starts to follow a straight line from $t = 0.5T_E$, indicating that the linear Ekman terms become more important from that time on. The decay with model $M_3$, if plotted on a log-scale, is non-linear during the whole simulation time, but is becoming less important quite fast. In the simulations with the large Reynolds number, the strength, if plotted on a log-scale, decays linear with all three models during the whole time evolution. Therefore, the time at which the linear Ekman terms start to dominate the decay, cannot be indicated.

Finally, the time evolution of the energy and enstrophy are comparable to the time evolutions that were found in the simulations of the dipole, with the characteristic numbers corresponding with those used in simulations of the monopole, as depicted in figure 4.5 through figure 4.7. The course of these quantities is already explained in the corresponding subsection 4.2.1. Nonetheless, it has to be noticed that the decay in the simulations discussed in this subsection evolves much faster, which is again caused by strong lateral viscosity.
Figure 4.13 illustrates the ratio of the enstrophy and the kinetic energy, $S/E_k$, as a function of time normalized by the Ekman time, which, as discussed in subsection 2.2.4, represents an estimate of the characteristic length scale $l$ in the flow. Subfigure $a$ shows the evolution, obtained with the small Reynolds number, and subfigure $b$, the evolution obtained with the large Reynolds number.

It can be remarked that the decay in ratio, thus the increase in characteristic length scale, is stronger, when obtained with the small Reynolds number. This could already be concluded from looking at the time evolution of the vorticity distribution, as depicted in figure 6.3 obviously, the vorticity weighted size of the dipole increases more in the distributions obtained with the small Reynolds number, which was explained in detail in this same subsection. Furthermore, the decay in ratio, obtained with the small Reynolds number, is the same for all three models. With the large Reynolds number, the evolution obtained with model $M1$ only deviates a little bit from the decay obtained with model $M2$ and $M3$, that decay in the same way. This can be explained by comparing with the corresponding vorticity distributions in figure 6.3: with model $M2$ and $M3$ the dipole grows symmetrically and with the same amount in time, for both models. With model $M1$ the cyclonic part of the dipole grows, especially in the simulation with the large Reynolds number, and the anticyclonic part grows too, but a lot less than the cyclonic part. Because the characteristic length scale is vorticity weighted, the asymmetric growth is compensated (almost completely with the large Reynolds number) by the asymmetric distribution of the vorticity, causing the vorticity weighted dipole size, obtained with model $M1$, to increase in the same way as with model $M2$ and $M3$.

Figure 4.14 shows the skewness (subfigures $a$) and kurtosis (subfigures $b$) of the vorticity distribution as a function of time, normalized by the Ekman time. The upper pictures present the results obtained with the small Reynolds number and the lower pictures, the results obtained with the large Reynolds number.

The skewness with model $M2$ and $M3$ obviously stays zero, because of the absence of the non-linear Ekman terms. The skewness, obtained with model $M1$ and the small Reynolds number, shows only a small dip, compared to the evolution obtained with model $M1$ and the large Reynolds number. This is also something that can be derived from the remarks that were made with the results depicted in figure 6.3 with which was mentioned that the non-linear Ekman terms lose effect after some time. This can now also clearly be seen in the evolution of the skewness.

The kurtosis, obtained with the small Reynolds number, just decreases in time, for all three models. This is directly related to the decay of the ratio $S/E_k$: the more vorticity is spread by the increase of the size of the dipole, the more the peakedness of the distribution will decrease. The time evolution, obtained with the large Reynolds number and model $M3$, shows a small dip around $t = 1.2T_E$, which, as can be checked in subfigure $h$ in figure 6.3, is due to the dipole passing the periodic boundary of the domain. The evolution obtained with model $M1$ initially increases, before it starts decaying in the same way as with model $M2$ and $M3$. This deviation arises due to the non-linear Ekman terms that are responsible for the redistribution of vorticity over the two dipole parts: in the beginning of the simulation the two parts still have the same size, and the redistribution of vorticity then causes an increase in the peakedness of the vorticity field, but as soon as the vortices increase more in size, the vorticity is spread out again and the kurtosis will decrease. The deviation for model $M1$ was also already noticed in the evolution of the ratio $S/E_k$. 

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Figure 4.12: Results of simulations of a dipole with the same dimensionless numbers as will be used in the simulations on 2D and Q2D turbulence: (a) positive and negative vorticity \( \omega^{+/-} \), normalized by the initial value, (b) positive and negative circulation \( \Gamma^{+/-} \), normalized by the initial value, (c) kinetic energy \( E_k \), normalized by the initial value, (d) enstrophy \( S \), normalized by the initial value.

\[
\begin{align*}
\text{(a)} & \quad \text{Ro}_\omega = 0.99, \quad E = 9.06 \cdot 10^{-4}, \quad \text{and} \quad \delta = 1.6 \\
& \quad (Re_\omega = 854). \\
\text{(b)} & \quad \text{Ro}_\omega = 0.99, \quad E = 9.06 \cdot 10^{-5}, \quad \text{and} \quad \delta = 1.6 \\
& \quad (Re_\omega = 8537).
\end{align*}
\]

Figure 4.13: Results of simulations of a dipole with the same dimensionless numbers as are used in the simulations on 2D and Q2D turbulence: ratio of the enstrophy, and the kinetic energy \( S/E_k \) as a function of time, normalized by the Ekman time.

\[
\begin{align*}
\text{(a)} & \quad \text{Ro}_\omega = 0.99, \quad E = 9.06 \cdot 10^{-4}, \quad \text{and} \quad \delta = 1.6 \\
& \quad (Re_\omega = 854). \\
\text{(b)} & \quad \text{Ro}_\omega = 0.99, \quad E = 9.06 \cdot 10^{-5}, \quad \text{and} \quad \delta = 1.6 \\
& \quad (Re_\omega = 8537).
\end{align*}
\]
$Ro_\omega = 0.99, \ E = 9.06 \cdot 10^{-4}, \ and \ \delta = 1.6 \ (Re_\omega = 854)$. 

$Ro_\omega = 0.99, \ E = 9.06 \cdot 10^{-5}, \ and \ \delta = 1.6 \ (Re_\omega = 8537)$. 

**Figure 4.14:** Results of simulations of a dipole with the same dimensionless numbers as are used in the simulations on 2D and Q2D turbulence: (a) skewness and (b) kurtosis of the vorticity field as a function of time, normalized by the Ekman time.
Chapter 5

Results of simulations of 2D and Q2D turbulence

Taking the results of the monopole and dipole simulations, as discussed in the former chapter, into account, in this section the most important simulation results with respect to the main goal of this study will be discussed: the results of the simulations on 2D and Q2D turbulence. The discussion will, as in the former chapter, be based on the three models $M_1$, $M_2$, and $M_3$, that were explained in subsection 2.1.2. The performed simulations are characterized by the numbers that are listed in table 3.3 and can be ‘summarized’ in the Reynolds numbers $Re_\omega = 854$ and $Re_\omega = 8537$.

In the first section the time evolution of the vorticity field for a typical run will be discussed. The following section will describe the time evolution of some interesting global quantities. Then, the results of vortex statistics will be discussed. Finally, the last section will deal with power law behaviour.

5.1 Vorticity field evolution

Figure 5.1 and figure 5.2 depict a typical time evolution of the vorticity field for simulations on 2D turbulence, using the three different models, $M_1$ (upper row), $M_2$ (middle row), and $M_3$ (lower row), for a flow with initial Reynolds number $Re_\omega = 854$ (figure 5.1) and $Re_\omega = 8537$ (figure 5.2). The time was scaled with the Ekman time scale. The main difference between the simulations, obtained with small and large Reynolds number, is that in the simulations where $Re_\omega = 854$ was used the Ekman number $E$ is larger, and therefore all the damping terms in equation (2.29) become stronger, causing the process of self-organization to slow down. This causes a smaller difference between the vorticity distributions at the different time steps than in the simulations with $Re_\omega = 8537$.

At time $t = 0.45T_E$, all the simulations are still dominated by strain, and also merging starts to occur. Many of the vortex interactions involve more than two vortices. At the end of the simulation time, $t = 2.27T_E$, the vorticity fields for the different models differ considerably, particularly the ones obtained with the large Reynolds number: the simulations where model $M_1$ was used, show a clear asymmetry between cyclones and anticyclones: the anticyclones are relatively small and strong, and the cyclones are relatively large and weak. In the simulations where model $M_2$ was used this asymmetry is absent, but the vortex merging is progressed in about the same amount as in the simulations with model $M_1$. The simulations with model $M_3$ also show a symmetric time evolution of the vorticity field, and when simulations with model $M_3$ are compared with simulations with model $M_2$, merging has evolved a lot more in the first ones. The final state of 2D turbulence has not yet been reached since, instead of the presence of only one dipole, there are still plenty...
Figure 5.1: Typical time evolution of the vorticity field for a simulation of an array of vortices with $\text{Ro}_\omega = 0.99$, $E = 9.06 \cdot 10^{-4}$, and $\delta = 1.6$ ($Re_\omega = 854$), for model M1 (a–c), model M2 (g–i), and model M3 (m–o). Time increases from left to right. Depicted is the full computational domain.
Figure 5.1: (continued) for model M1 (d–f), model M2 (j–l), and model M3 (p–r).
5.1. VORTICITY FIELD EVOLUTION  

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Figure 5.2: Typical time evolution of the vorticity field for a simulation of an array of vortices with $Ro_\omega = 0.99$, $E = 9.06 \cdot 10^{-5}$, and $\delta = 1.6$ ($Re_\omega = 8537$), for model M1 (a–c), model M2 (g–i), and model M3 (m–o). Time increases from left to right. Depicted is the full computational domain.
CHAPTER 5. RESULTS OF SIMULATIONS OF 2D AND Q2D TURBULENCE

5.1. VORTICITY FIELD EVOLUTION

Figure 5.2: (continued) for model M1 (d–f), model M2 (j–l), and model M3 (p–r).
5.2 Global quantities

In this section the time evolutions of some global quantities are discussed. The first paragraph will describe the evolution of the energy, the enstrophy, and the ratio of these two quantities, \( S/E_k \), in time. In the second paragraph the time evolution of the skewness and kurtosis of the vorticity will be discussed.

Figure 5.3 shows the time evolution of the kinetic energy (subfigures a) and the enstrophy (subfigures b) for \( Re_\omega = 854 \) (upper pictures) and \( Re_\omega = 8537 \) (lower pictures).

The energy, obtained with model \( M3 \), hardly decreases in time, while the enstrophy is, after some time, strongly decreasing. This is typically due to the self-organization process as a result of vortices present. The time evolution of the size and amount of the vortices will be discussed in more detail in section 5.3 on vortex statistics.

5.2 Global quantities

Mean results of 6 simulations with \( Ro_\omega = 0.99, E = 9.06 \cdot 10^{-4}, \text{ and } \delta = 1.6 \) (\( Re_\omega = 854 \)).

Mean results of 4 simulations with \( Ro_\omega = 0.99, E = 9.06 \cdot 10^{-5}, \text{ and } \delta = 1.6 \) (\( Re_\omega = 8537 \)).

Figure 5.3: Mean results of simulations of 2D and Q2D turbulence for which model M1, M2, and M3 were used: (a) kinetic energy \( E_k \), normalized by the initial value, and (b) enstrophy \( S \), normalized by the initial value.
of the merging of vortices and did therefore not occur in the simulation of the monopole and the dipole. As explained in the report of van Bokhoven [1], the vortex size increases during the process of self-organization as a result of vortex merging. In the merging process a small part of the energy is dissipated, while the largest amount of energy ends up in the larger vortex, thus causing the energy to transfer to the larger length scales. This causes that less energy is contained by the smallest scales at which dissipation occurs, which is the reason for the slow decrease of energy. As discussed in subsection 2.2.1, the enstrophy cascades to smaller scales where it is dissipated, explaining the faster decrease of enstrophy compared to the energy.

With model $M_1$ and $M_2$, both the energy and the enstrophy decay strongly in time. This is due to the linear Ekman terms, that cause the vortices to slow down much faster as with model $M_3$. Something comparable occurred with the simulations of the dipole, where the linear Ekman terms were responsible for the decrease of the distance that was covered by the dipole. If the velocity is reduced faster, so is, by definition, the kinetic energy. The faster decay of the vorticity obtained with model $M_1$ and $M_2$ compared to the decay with model $M_3$, can also be explained by the linear Ekman effects that also contribute to vortex decay.

The energy decays the same with model $M_1$ and $M_2$ for both Reynolds numbers. This was also found as a result from the simulations on the dipole, where this was explained by an equal time evolution of the total velocity for the two models. Apparently, here the same thing happens.

When the large Reynolds number is used, the amount of decrease of energy in time is a bit smaller with all three models. This can be explained by the weaker horizontal damping terms that are also responsible for the faster merging in the simulations with large Reynolds number.

In the time evolution of the enstrophy, obtained with the large Reynolds number, there is a small but noticeable difference in decay obtained with model $M_1$ and $M_2$: after some time, the decay with $M_1$ is a bit slower than the decay with model $M_2$. This was also found in the results of the simulations of the dipole, discussed and explained in subsection 4.2.1.

Figure 5.4 depicts the ratio of the enstrophy and the kinetic energy $S/E_k$ as a function of time, normalized by the Ekman time.

![Figure 5.4](image-url)
5.2. GLOBAL QUANTITIES

CHAPTER 5. RESULTS OF SIMULATIONS OF 2D AND Q2D TURBULENCE

(I) Results of 6 simulations with $\text{Ro}_\omega = 0.99$, $E = 9.06 \cdot 10^{-4}$, and $\delta = 1.6$ ($\text{Re}_\omega = 854$)

(II) Mean results of 6 simulations with $\text{Ro}_\omega = 0.99$, $E = 9.06 \cdot 10^{-4}$, and $\delta = 1.6$ ($\text{Re}_\omega = 854$)

(III) Mean results of 4 simulations with $\text{Ro}_\omega = 0.99$, $E = 9.06 \cdot 10^{-5}$, and $\delta = 1.6$ ($\text{Re}_\omega = 8537$).

Figure 5.5: Results of simulations of 2D and Q2D turbulence for which model $M1$, $M2$, and $M3$ were used: (a) skewness and (b) kurtosis of the vorticity field as a function of time, normalized by the Ekman time.
obtained with both small and large Reynolds number.

It also applies for both Reynolds numbers, that the more viscos terms are added, the less the ratio of the enstrophy and energy decreases, thus the less the characteristic length scale in the flow increases. The largest increase in characteristic length scale is obtained with model $M_3$ and can be explained from vortex merging, that is dominantly occurring with this model, compared to with the other two models. The ratio $S/E_k$, obtained from the simulations of the dipole, showed no difference for model $M_2$ and $M_3$. The appearance of the difference between these two models in the simulations on 2D and Q2D turbulence, respectively, is due to the linear Ekman terms, that cause a decrease of the velocity of the vortices and in this way prevent some of the vortices to reach the critical distance to other vortices, at which they can merge. This also explains the absence of the difference in the dipole simulations where, since there is only one vortex in the domain, merging of course cannot occur anyway.

With model $M_1$ the increase of the characteristic length scale is slower compared to the increase with the other two models due to the non-linear Ekman terms, that cause the anticyclones to decrease in size, and the cyclones to increase in size. This results in anticyclones that move around and therefore merge more easily than the cyclones, that become more unwieldy. Thus merging, with the emphasis on the smaller vortices, results in a smaller increase of the characteristic length scale.

The ratio, obtained with the large Reynolds number and model $M_1$, shows a bend in direction around $t = T_E$, from which it starts to decay slower. The change in direction of the time evolution of $S/E_k$ occurs around the same time where the enstrophy decay, obtained with model $M_1$, starts to decay slower than the decay, obtained with model $M_2$, as depicted in figure 5.3. This change in the decay of the enstrophy was also found in the results of the dipole simulations, discussed in subsection 4.2.1 and well depicted in subfigure d of figure 4.7. With these dipole simulations, this was explained by the effects of the non-linear Ekman terms on the total vorticity in the flow. In the simulations on Q2D turbulence the asymmetry in vorticity distribution, caused by this non-linear terms, is more pronounced in the results obtained with the large Reynolds number, where the anticyclones have a stronger vorticity and are smaller at the different time steps, compared to the small Reynolds case. From the results of the dipole simulations it was assumed that the characteristic length scale is represented by the ‘vorticity-weighted’ size of the dipole. On the analogy of this assumption, in the case of 2D and Q2D turbulence the characteristic length scale in the flow is assumed to be the sum of the vorticity weighted size of all vortices in the flow domain. With use of this assumption the bend in the ratio can be explained by the anticyclones that weight stronger in the total size, and because they become smaller in time, are responsible for the smaller increase in total size.

Figure 5.5 illustrates the time evolution of the skewness and kurtosis of the vorticity field. The results depicted in subfigures II and III are averages of respectively 6 and 4 quite differing runs. To illustrate this, subfigure I shows the skewness and kurtosis, obtained with the small Reynolds number, before the results were averaged. Comparing subfigure I and II makes clear that the averaged results should be treated as a rough estimation, just indicating a trend in the course of the quantity.

For the skewness the time evolution with model $M_2$ and $M_3$ shows some variations around zero. This can be explained from the process of merging where cyclones and anticyclones merge at different times, causing the skewness to shift to the positive or negative side, depending on which vorticity has the majority. The skewness obtained with model $M_1$ shows an obvious decay, caused by the stronger anticyclones, due to the non-linear Ekman terms. Around $t = 1.5T_E$ with the small Reynolds number, the non-linear terms already start to become less important, as can be
concluded from the minimum in skewness with model $M1$. This can be explained by looking at equation (2.29) where the non-linear Ekman terms, responsible for the stretching and squeezing of the vortices, are multiplied by $\omega^2$ and therefore reduce fast when the vorticity decreases. As was already mentioned in the section on the evolution of the vorticity field, the vorticity decreases stronger in the simulations with the small Reynolds number, due to the larger Ekman number.

The kurtosis, obtained with model $M1$, shows an increase in time, only initially in the small Reynolds case, that is again related to the asymmetry caused by the non-linear Ekman terms: the anticyclones are smaller and stronger than the cyclones, causing a small part of the vorticity in the flow domain to be more peaked, resulting in an increase in kurtosis. Caused by the fact that the non-linear terms in the small Reynolds case start to become less important after some time, the kurtosis also decreases. With model $M2$ and the big Reynolds number, the kurtosis increases just a little bit compared with the evolutions obtained with the other two models. With model $M2$ and the small Reynolds number, there is even only a decrease in kurtosis, where the evolution obtained with model $M3$, follows for some time, but then starts to increase. The smallest increase in the simulations with the large Reynolds number can be explained from the absence of the non-linear Ekman terms and less merging, due to the linear Ekman terms, that cause a faster decay of the vortices. The decrease in kurtosis with the small Reynolds number is due to stronger damping of the flow in general, caused by the larger Ekman number. The lateral viscosity causes the vortices to grow and thus become less peaked, combined with the linear Ekman terms that partly prevent vortices to merge by slowing them down. The decrease in velocity and vorticity in the simulations with model $M3$ is less than with model $M2$ because there are no linear Ekman terms, making merging easier. Merging explains the sudden increase of the evolution with model $M3$.

5.3 Vortex statistics

Before discussing the results of the vortex statistics, first some notes on the vortex census, concerning the different tolerances that were set to find the accepted features in the simulations performed in this study, are made.

5.3.1 Notes on vortex census

As was already mentioned in subsection 3.3.2 a vortex census is needed to recognize vortices as separate features in order to make vortex statistics possible. The acceptance of features is based on tolerance values that form the limit between a region of the vorticity field, defined as vortex, and the regions, not being a vortex. In the numerical code, the features are first detected, based on the threshold values of Weiss function and vorticity. The full computational domain is scanned, and for each grid-point is determined whether the values of Weiss function and vorticity meet the requirements ($Q \leq tol_q \cdot q_{\min}$) and ($\omega \geq tol_\omega \cdot \omega_{\max}$) or ($\omega \leq tol_\omega \cdot \omega_{\min}$), where $q_{\min}$, $\omega_{\min}$, and $\omega_{\max}$ are the minimum Weiss function, the minimum vorticity, and the maximum vorticity, respectively. The tolerance value for the Weiss function, that was adjusted to the best fit, is $tol_q = 1 \cdot 10^{-6}$. The tolerance of the vorticity was fixed at $tol_\omega = \exp(-1)$, as explained in subsection 2.2.4.

The fine tuning is based on the size of the accepted features by rejecting small and extended features: ($I_0 \leq tol_{I_0}$) and ($area \geq tol_{area}$), where $I_0$ is the normalized moment of inertia, and $area$ is the area of the accepted features. The tolerance values for $I_0$ and $area$ that were adjusted to the best fit, are $tol_{I_0} = 5$, and $tol_{area} = 0.1$, respectively.

Errors in the amount of vortices that are determined by the census mainly occurred because of the periodic boundaries of the domain: while a vortex passes these boundaries, it can be recognized as vortex on both sides of the domain and can therefore be counted twice.
5.3.2 Results

Figure 5.6 depicts the results of the vortex statistics, the first column obtained with $Re_\omega = 854$, and the second column obtained with $Re_\omega = 8537$. Determined are the total number of vortices $N_{vor}$, the mean area of the vortices $A_{mean}$, and the total vortex area $A_{tot}$.

Because of the asymmetric evolution for cyclonic and anticyclonic vortices, that is caused by the non-linear Ekman terms, figure 5.7 shows vortex statistics obtained with model $M_1$, for which the evolution for the cyclones and anticyclones, is depicted separately.

When taking an overall look at the results, illustrated in figure 5.6, it is noticed that with model $M_1$ and $M_2$, the different determined quantities have almost the same time evolution. This is quite remarkable, because figure 5.7 shows a big asymmetry in the behaviour of cyclones and anticyclones, that is somehow compensated when looking at the statistical behaviour of all the vortices, without the distinction between cyclones and anticyclones. Apparently the non-linear Ekman terms cause a redistribution of the statistical vortex quantities over the cyclones and anticyclones, keeping the totals constant.

When taking a closer look at the different statistics, the following can be mentioned: the total number of vortices decreases more with model $M_3$ than with the other two models, both with small and large Reynolds number. This is obviously due to the faster merging occurring with model $M_3$. From figure 5.7 it follows that the number of anticyclones decreases faster than the number of cyclones. This can be explained from what was already mentioned in section 5.2: the non-linear Ekman terms cause the anticyclones to decrease and the cyclones to increase in size, resulting in anticyclones that move around and therefore merge more easily than the cyclones that become more unwieldy. With the big Reynolds number the decay in $N_{vor}$ is exponential, which will be discussed in more detail in the section on power law behaviour 5.4.

The area, both the mean and the total, does not only change due to merging, as with the number of vortices, but is also influenced by strain in the flow, the non-linear Ekman terms, and by lateral viscosity.

The evolution of the mean area, for both small and large Reynolds number, shows an initial dip. This is due to strain, that dominantly affects the flow as it starts to evolve: the strain causes the vortices to deform by stretching, which results, due to the way the vortex census is defined, in smaller accepted features. After some time, merging is getting more important, causing the mean area to increase. The evolution of the mean area, obtained from the simulations with small Reynolds number, is about equal for all three models. This can be explained by comparing with the evolution of the vorticity field, depicted in figure 5.1: with model $M_3$ there is a lot of strain, resulting in a few relatively small vortices that compensate for the large vortices, that arise due to merging. Therefore, the evolution of the mean area is somewhat the same as is found with model $M_1$ and $M_2$, that, as with the number of vortices, show an equal evolution, although the individual evolution of cyclones and anticyclones, depicted in figure 5.7 shows a clear asymmetry. The mean area, obtained with model $M_3$ and the large Reynolds number, shows a strong increase at the end of the evolution, compared to the other two models. This can again be explained by looking at the evolution of the vorticity field for this case, depicted in figure 5.2: subfigure r shows some really large vortices, some vortices equally sized as the initial vortices, and a few really small vortices, that will not be accepted by the vortex census and thus will not count in the calculation of the mean area. This is causing the increase of the mean area above the initial value. The vorticity field evolutions for both model $M_1$ and $M_2$ show less extrema, and the mean area will therefore increase less fast.
Figure 5.6: Results of vortex statistics of simulations with \( Re_\omega = 854 \) (a, c, and e) and \( Re_\omega = 8537 \) (b, d, and f), for which model \( M1, M2, \) and \( M3 \) were used: total number of vortices \( N_{\text{vor}} \) (a, and b), mean area of the vortices \( A_{\text{mean}} \), normalized by the initial value (c, and d), and total vortex area \( A_{\text{tot}} \), normalized by the initial value (e, and f).
The evolution of the mean area, depicted in figure 5.7, illustrates that the evolution of the anticyclones, obtained with the small Reynolds number, shows an initial decrease, then some leveling, and then an increase. The initial decrease is due to a combination of non-linear Ekman terms and the strain in the flow. Then slowly the lateral viscosity and merging take over, causing the mean area to increase in the end. The cyclones initially have a constant mean area and then increase in size. The constant evolution is due to a balance between the strain in the flow, that causes the area to decrease, and the non-linear Ekman terms and the lateral viscosity, causing the vortices to increase. When merging starts to occur, the mean area starts to grow. The evolution of the mean area, for both the cyclones and the anticyclones, obtained with the large Reynolds number, shows an initial decrease, but after some time starts to level for the anticyclones and to increase for the cyclones. The initial decrease, which is a bit stronger for the anticyclones, due to the non-linear Ekman terms, is caused by the strain in the flow. The increase of the mean area for the cyclones, that occurs after some time, is again mainly due to merging, but also caused by the lateral viscosity and the non-linear Ekman terms. The same things that cause the increase in mean area for the cyclones are also responsible for the leveling in the evolution of the anticyclones, but now the non-linear Ekman terms balance the effects that would cause an increase of the mean area.

Finally, the time evolution of the total area is discussed. With the small Reynolds number, the total area obtained with model $M_1$ and $M_2$, does hardly change in time, although the mean area increases above the initial value. Taking merging into account will make this seeming contradiction disappear: the total area is being covered by less, but growing, vortices. In the case of the results, obtained with model $M_3$, the increase of the area of the vortices cannot keep up with the merging process, causing the total area, covered by vortices, to decrease in time. With the large Reynolds
number, the total area initially decreases with all three models, but after some time starts leveling with model M1 and M2, but keeps decreasing with model M3. The initial decrease can be explained from the, compared to the small Reynolds case, initially much stronger decrease in mean area, combined with the stronger decrease in total number of vortices, indicating faster merging. The leveling that sets in with model M1 and M2 after some time, is due to the increase in mean area and not too fast merging. The total area, obtained with model M3, keeps on decreasing, as with the small Reynolds number, due to the strong merging process.

With the evolution of the total area, where the evolution of cyclones and anticyclones is studied separately (figure 5.7), it is noticed that in the results obtained with the small Reynolds number the total area of the anticyclones decreases and after some time has a leveling evolution, while the cyclones start with a constant evolution and end up with an increase in total area. The initial behaviour of both types of vortices is the same as the evolution of the mean vortex size, as depicted in subfigure c of the same figure. The increase in total area, that then follows for the cyclones, is a lot less than the increase determined for the mean area. This can again be explained by the merging process that also explains the fact that the evolution with the anticyclones, at the end, does not increase anymore. The same things as described for the small Reynolds case also happen with the large Reynolds number, and the evolution of the total area can therefore be explained in the same way.

### 5.4 Power-law behaviour

The evolution of the ratio $S/E_k$ illustrated in subfigure b of figure 5.4 and the evolution of the vortex statistics depicted in subfigures b, d, and f of figure 5.6 show power law behaviour. The associated algebraic decay rates, $\alpha$, at which the presented quantities evolve are also indicated and are determined in the time interval that is bounded by the lines that visualize the decay rates. In this section this power law behaviour will be discussed.

In subsection 2.2.4 was already explained that physical phenomena that show self-similar behaviour can be represented by a power-law spectrum. Also a scaling theory, proposed by Carnevale et al. [3], was mentioned. This scaling theory is based on the assumption of constant energy and average absolute vorticity extremum, that occur if $Re \to \infty$. Since these conditions are not met in the work that is described in this report, the scaling theory by Carnevale cannot be used.

Van Bokhoven et al. [2] numerically studied the time evolution of both global quantities and vortex statistics of evolving 2D turbulence, by applying model M3. The power-law behaviour that is reported in their work should therefore more or less agree with the behaviour that is found in the results, obtained with model M3 in this study. In the work by van Bokhoven et al. a Reynolds number $Re^* = 34500$ was used as initial condition. It was already mentioned in section 3.3 that the Reynolds number in this study is calculated differently: the large Reynolds number, when calculated in the same way as in the work by van Bokhoven et al. becomes $Re^* = 80500$. These two Reynolds numbers are of the same order of magnitude, making the results obtained in this study comparable with those previously determined [2]. In their study, van Bokhoven et al. did not determine the time evolution of the ratio $S/E_k$ but calculated the decay rate for the normalized enstrophy. Since the decay in energy is neglectable, the obtained decay rates for the ratio $S/E_k$ and the normalized vorticity should at least be comparable. In the work by Bokhoven et al. a decay rate $\alpha = -0.8$ was obtained for the enstrophy, whereas in the present study the ratio $S/E_k$ decays at a rate $\alpha = -0.96$. The difference in these two values can be explained by the larger Reynolds number that is used in this study causing a faster decrease. The influence of a larger Reynolds number can also be seen by comparing the decay in subfigure a of figure 5.4, which is obtained with a smaller Reynolds number, with the decay in subfigure b that is found to be larger. In the work by van Bokhoven et
al., instead of the time evolution of the number of vortices, the evolution of the normalized vortex density was obtained. Because these two quantities imply the same thing, they should evolve in the same way, and also their decay rates should be similar. The paper by van Bokhoven et al. reported $\alpha = -0.9$, while in this study $\alpha = -0.88$ was found and thus these decay rates nicely agree. The evolution of the mean area obtained in this work by applying model $M3$ does not show clear power-law behaviour due to the large amount of noise on the results. For the time evolution of the total area a decay rate of $\alpha = -0.42$ was found, which is larger than the value obtained by van Bokhoven et al. ($\alpha = -0.3$), but the difference can again be explained by the large noise. It also has to be noted that the power law exponents are very dependent on the time interval where you determine them.

Clercx et al. [4] also found power-law behaviour in the time evolutions of obtained quantities. They studied Q2D turbulence in shallow fluid layers, both numerical and experimental. Their numerical work can be compared with the numerical simulations with model $M2$, performed in this study, as already mentioned in subsection 2.1.2. Unfortunately a real comparison can not be made because the Reynolds number is calculated differently but it can be mentioned that both the present study and the work by Clercx et al. showed power-law behavior in the evolution of the ratio $S/E_k$ and the vortex density. The exponents that are obtained in this report, are $\alpha = -0.54$, for the decay rate of the ratio $S/E_k$, $\alpha = -0.33$ for the decay rate of the number of vortices, and $\alpha = 0.28$ for the rate of increase of the mean area. With the values for the vortex statistics it has to be noticed again that these are based on rather noisy results.

The results obtained with model $M1$ also evolve exponentially, making it possible to determine power-law exponents. The ratio $S/E_k$ shows two different regimes: one with a decay rate $\alpha = -0.45$ and the other with decay rate $\alpha = -0.33$. The appearance of these two regimes was already explained in the discussion of the results. From the vortex statistics the exponents that can be determined are equal to those obtained with model $M2$. 
Chapter 6

Experiments

This chapter describes the laboratory experiments that were carried out in a rotating tank, to see flow behaviour in the ‘real world’ and to be able to make a qualitative comparison with the simulations. A one to one comparison with simulations can not be made: in experiments it is not possible to create periodic boundary conditions. Also, you will always have a bottom. Furthermore, the initial condition is created by forcing. Moreover, while rotating, the free surface is deformed. Finally, the Reynolds number that is used in the simulations is not the same as the one used in the experiments. Despite all of this, it is still worth while to take a look at vorticity evolutions, subjected to experimental conditions.

The remainder of this chapter will be as follows: in the first section the experimental set-up will be described. In the next section, the data processing by means of PIV will be discussed, and the last section will discuss the performed experiments and the obtained results.

6.1 Experimental set-up

For the experiments, the set-up presented in figure 6.1 was used. It is composed of a container, made out of plexi-glass, with inner dimensions $52 \times 52 \text{ cm}$, an outer height of $4 \text{ cm}$, and a PVC bottom of $1 \text{ mm}$ thickness. Underneath the container a PVC plate with 100 disc-shaped holes is placed. This plate serves as a mold to hold permanent magnets and to keep them separated. The magnets that are placed in the mold have a magnetic field at the center, directly above the magnet, of approximately $0.2 \text{ T}$, a diameter of $25 \text{ mm}$, and a thickness of $5 \text{ mm}$. The distance between the magnets, when placed in the mold, is also $25 \text{ mm}$. The distance between the magnets at the boundaries of the domain and the edge of the container is $22.5 \text{ mm}$. The container is filled with a sodium chlorine solution of 12% Brix, and on both sides of the container, a titanium electrode is placed. When a voltage potential is placed over the two electrodes, the $Na^{+}$ ions move to the negative electrode, and the $Cl^{-}$ ions to the positive electrode, thus a current will arise in the solution. This current will interact with the magnetic fields $\mathbf{B}$ above the magnets. As illustrated

Figure 6.1: Schematic representation of the experimental set-up and cross-section
in figure 6.2, the result of this interaction is a Lorentz force, defined as $F_L = qv \times B$, where $q$ is the charge of the ions and $v$ the velocity of the ions. The Lorentz force is pointing in the same direction for both the positive and the negative ions, because of the opposite sign of their velocities. When a short voltage pulse is given, a linear momentum is induced in the flow, causing the appearance of a dipole above each magnet.

![Diagram of the Lorentz force](image)

Figure 6.2: Diagram of the Lorentz force

The whole set-up, is placed on a rotating table facility with adjustable velocity, both in magnitude and direction, to be able to apply rotation. In order to visualize the flow, seedings of diameter $250\mu m$ were added to the solution and for illumination four slide projectors were placed around the set-up. A high resolution camera with a speed of $15\, fps$ connected to a computer was placed above the tank.

### 6.2 Data processing

In order to be able to obtain velocity fields from the pictures taken during the evolution of the flow in the experiments, Particle Image Velocimetry (PIV) is used.

The procedure to obtain the velocity fields is as follows: first a calibration grid is placed in the experimental set-up, from which a picture is taken. After the calibration grid is removed, the tank is filled with the sodium chlorine solution, and the surface of the fluid is seeded with tracer particles, small enough to follow the fluid closely. Four slide projectors are placed around the set-up. Finally the experiments are performed and recorded with a PC-based VideoSavant system. The obtained data is transferred to a UNIX system and processed with FPTVWIZ-software to obtain the velocity fields.

PIV starts from an equidistant time-sampled sequence of images of particles. These images are divided into rectangular interrogation windows, and the corresponding windows of two subsequent images are cross correlated, from which the average displacement $\Delta \vec{x}$ of the particles between the two images is obtained. The local average velocity in the interrogation windows is found by dividing the displacement vector $\Delta \vec{x}$, by the time $\Delta t$ between two subsequent images. It requires only two subsequent images to produce a velocity field. Since the pictures of the particles are in pixel coordinates, so are the velocities, obtained from these pictures. To transform the coordinates of these velocities into physical coordinates, the recorded calibration grid is used, from which the mapping functions, used to transform pixel coordinates to physical coordinates, are calculated.

When performing PIV, in order to obtain sufficient correlation, the amount of particle images in an interrogation window must be sufficiently high. Also the maximum average particle displacement should be no more than $1/4$ of the size of a window, and the velocity gradient in a window should not be too high, but a minimal displacement of two pixels is recommended.
6.3 Initial conditions, and results

Both experiments with only one magnet, creating a dipole, and experiments with an array of 100 magnets with alternating direction of the magnetic field, creating a turbulent flow, are performed. The simulations were performed without and with background rotation ($\Omega = 0.25s^{-1}$ for the dipole experiment, and $\Omega = 0.7s^{-1}$ in the experiments with the array). The rotation direction of the tank was chosen anticlockwise, in all the experiments.

Experiments with a single magnet

The experiments with the single magnet were performed with a fluid depth of $H = 1.06 \, cm$, and during 1 s, a current of 6.5 $A$ was applied. Figure 6.3 shows the results from the experiments without (upper two rows) and with background rotation (lower two rows). Subfigures a–c and g–i, show streakline pictures obtained by adding 12 subsequent pictures making the path along which the particles move visible. The streakline pictures are created around the times placed at the bottom of the figure, by using 6 pictures before and 6 pictures after the indicated times. Because the amount of pictures that is used to obtain the streakline pictures is the same for all time steps, it is possible to see the evolution of the magnitude of the velocity of the particles: smaller velocities can be recognized by shorter streaklines. From the streakline pictures it is not possible to know the direction in which the particles move. Therefore PIV is applied to obtain velocity fields, that are depicted in subfigures d–f and j–l. Since the resolution of the obtained velocity field is very low, it was not useful to obtain more data from the experiments and therefore the discussion of the results will be only qualitative.

The results obtained from the experiments on the dipole show an evolution, similar to those obtained in the simulations: in the non-rotating experiment the dipole has equally sized parts and is moving on a straight line, and in the rotating set-up the dipole evolves unequally for its two parts and moves therefore on a curved trajectory. As can be clearly seen in the velocity field depicted in subfigure l, the anticyclonic part ends up much stronger than the cyclonic part of the dipole. There is however also a difference between the experiments and the simulations: instead of a non-isolated dipole, which was the initial vorticity condition in the simulations, an isolated dipole is created above the magnet. This causes the appearance of an extra dipole-like structure at the top of the flow domain, where the dipole starts to move from.

Experiments with an array of 100 vortices

Figure 6.4 and figure 6.5 display time evolutions obtained from experiments with 100 magnets. In both figures the upper two rows show results from non-rotating experiments, and the lower two rows show results from experiments where the set-up was rotated. Subfigures a–f and m–r show streakline pictures, and subfigures g–l and s–x show the corresponding velocity fields obtained with PIV. The time steps at which the streakline pictures and velocity fields are determined are depicted at the bottom of the figures. The streakline pictures were again obtained by using 6 pictures before and 6 pictures after the indicated times. When forcing is applied, this results in the appearance of dipolar vortices above each magnet. Due to mutual interactions between neighboring dipole halves with equal signs, the dipoles flip over an angle of 90 degrees and the equally signed dipole halves merge. This mechanism leads to an array of monopolar vortices, that are located in between the magnets. When looking at the whole domain, the 100 dipoles are transformed into 90 large monopoles and 20 small monopoles, where the large monopoles result from merging of two dipole halves, and the small monopoles are single dipole halves at the boundaries of the domain. The
array of monopolar vortices is the initial distribution for the studied time evolution and exists as soon as the forcing is stopped.

The results, illustrated in figure 6.4, are obtained from experiments with a fluid depth of \( H = 0.6 \, \text{cm} \). During 1 s, a current of 5 A was forced over the electrodes. In the experiments with rotation, the speed of the rotating table was set at \( \Omega = 0.7 \, \text{s}^{-1} \). With these experiments, only a relatively small part of the domain was suitable to study due to poor light distribution.

With the time evolution of the flow for the non-rotating experiment can be noticed that at time \( t = 0.1T_E \) (subfigures a and g), the fluid is still being forced, and therefore the flow is still being arranged into the initial distribution. Both the forcing and the data storage are started manually, which can cause the measurements to start before the forcing has ended due to reaction time. At time \( t = 0.45T_E \), the flow distribution clearly shows a monopolar vortex distribution and is therefore a better starting point to study the evolution of the flow. Following the time evolution of the flow, it can be observed that the evolution is dominated by merging, resulting in a few large vortices, both cyclonic and anticyclonic, at the last depicted time step.

The time evolution of the flow for the rotating experiment shows some interesting features. First, the streamline picture, obtained at time \( t = 0.1T_E \), nicely illustrates the merging of the dipole halves: the separate halves are still visible in the monopolar shapes. Then, when looking at the continuation of the flow evolution, also merging starts to occur, but now only the anticyclonic vortices are retained and are smaller than the vortices that appeared with the non-rotating experiments. This difference between rotating and non-rotating case was also mentioned with the simulations. Another difference between the rotating and non-rotating experiments is the faster velocity decay in the former, which can be explained by the presence of more damping terms due to the Ekman layer, that only exist in the rotating case. This faster decay can be recognized by the short streaklines in subfigure r, compared to the streaklines in subfigure f. A final remark that can be made with these results, especially with the last pictures (subfigures r and x), is the appearance of connections between the remaining vortices, resulting in meandering currents.

The results that are illustrated in figure 6.5, are obtained from experiments with a fluid depth of \( H = 0.8 \, \text{cm} \). During 1 s, a current of 6 A was applied. In the experiments with rotation, the speed of the rotating table was again set at \( \Omega = 0.7 \, \text{s}^{-1} \). With these experiments it was possible to obtain useful results from almost the full flow domain. The remarks that were made with the results of the experiments, depicted in figure 6.4, also apply to these results. Again, the meandering currents appear, but because of the larger domain it is now more visible how the flow continues on these paths. When looking at the results, obtained from the non-rotating set-up, also meandering currents are visible, but the flow it is just moving around randomly. In contrast, in the rotating case, the flow seems to have a preference for a certain direction, which is in this case from the right side to the left side of the domain. The preference for a certain direction is also visible in figure 6.4, where subfigure x shows flows from the top to the bottom of the domain.

The two rotating experiments showed fluid transport in a preferred direction. In order to get some more support for the existence of this phenomenon, figure 6.6 shows streamfunctions that are obtained from four different simulations on an array of vortices at the end of the computation time. Subfigures a and b are obtained with model M2, which can be used to describe flow evolution in a non-rotating system in presence of a no-slip bottom, as with the experiments with the non-rotating set-up. Subfigures c and d are obtained with model M1, which comes closest to describe the flow in the rotating set-up. The streamfunctions that are obtained with model M1, both show flows over the full domain, indicated by the white band of constant streamfunction in subfigure c and the black band in subfigure d. Such strong paths do not appear in subfigures a and b. This result adds to the assumption that these transport bands appear structurally when rotation is present.
Figure 6.3: Time evolution of the flow in experiments with one magnet, and fluid depth 1.06 cm: a–c streakline pictures for experiments without rotation ($\Omega = 0$), d–f the matching velocity field, g–i streakline pictures for experiments with rotation ($\Omega = 0.25 \text{s}^{-1}$), and j–l the matching velocity field. Time increases from left to right. Only a part of the flow domain is depicted.
Figure 6.4: Time evolution of the flow in experiments with 100 magnets, with alternating magnetic field and fluid depth 0.6 cm: a–c streakline pictures for an experiment without rotation ($\Omega = 0$), g–i the matching velocity field, m–o streakline pictures for an experiment with rotation ($\Omega = 0.7 s^{-1}$), and s–u the matching velocity field. Time increases from left to right. Only a part of the flow domain is depicted.
Figure 6.4: (continued) d–f streakline pictures for an experiment without rotation ($\Omega = 0$), j–l the matching velocity field, p–r streakline pictures for an experiment with rotation ($\Omega = 0.7s^{-1}$), and v–x the matching velocity field.

$t = 1.36T_E$

$t = 1.82T_E$

$t = 2.27T_E$
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Figure 6.5: Time evolution of the flow in experiments with 100 magnets, with alternating magnetic field and fluid depth 0.8 cm: a–c streakline pictures for an experiment without rotation ($\Omega = 0$), g–i the matching velocity field, m–o streakline pictures for an experiment with rotation ($\Omega = 0.7s^{-1}$), and s–u the matching velocity field. Time increases from left to right. Only a part of the flow domain is depicted.
Figure 6.5: (continued) d–f streakline pictures for an experiment without rotation ($\Omega = 0$), j–l the matching velocity field, p–r streakline pictures for an experiment with rotation ($\Omega = 0.7s^{-1}$), and v–x the matching velocity field.
Figure 6.6: Streamfunctions, obtained from simulations using model M2 (a and b) and model M1 (c and d).
Chapter 7

Conclusions and recommendations

The goal of this study is to characterize Q2D rotating turbulence evolving over a no-slip bottom. The approach used was both numerical and experimental, where the experiments only showed qualitative results. The discussion of the results is based on the three models $M_1$, $M_2$, and $M_3$, described in subsection 2.1.2. In previous work by Zavala Sansón and van Heijst [15], it was already determined that model $M_1$ is the most complete model to describe the evolution of a monopole and a dipole in a rotating system with a no-slip bottom, because it fits best with experimentally obtained flow behaviour. This model is also used in the simulations on an array of vortices, and therefore, the possible effects of rotation arising in the simulations, are mainly due to the terms that are defined in this model: the Ekman advection effects, the non-linear and linear Ekman stretching effects, and the lateral viscosity. Of course, because more vortices are involved, also the interaction between them plays an important role in the behaviour of the flow.

A remarkable effect on the flow due to applying model $M_1$ in the simulations performed in this study, is the asymmetric decay for cyclones and anticyclones. When studying all the results obtained in this work, it is noticed that this asymmetry is only visible in the evolution of quantities related to the vorticity, which can be explained from the fact that the non-linear Ekman effects, that are responsible for the asymmetry, only cause a redistribution of the vorticity. An example is the equal decay of energy and a different decay in enstrophy in the simulations with model $M_1$ and $M_2$. Also the vortex statistics do not show differences in the time evolutions obtained with model $M_1$ and $M_2$. However, the differences do appear when looking explicitly at the individual evolution for cyclones and anticyclones.

Furthermore, simulations with model $M_2$ are analogous to the simulations, performed by Clercx et al. [4], in which bottom friction is added. In their work it is stated that due to bottom friction, any nonlinearity is rapidly depleted, and the inverse energy is virtually halted. The flow dynamics will be frozen by bottom friction before the larger structures are able to emerge. In this study this is exactly what happens in the simulations with $M_2$.

In previous work by Clercx et al. [4] the ratio $S/E_k$ was already defined as a characteristic length scale of the flow. From the results, obtained in the simulations of the dipole, it is assumed that this length scale is represented by the vorticity-weighted size of the dipole. On the analogy of this assumption, in the simulations on 2D turbulence this length scale is assumed to be the sum of the vorticity weighted size of all vortices in the flow domain.

In addition, a scaling behaviour of different obtained quantities is observed. Power-law behaviour by applying model $M_2$ and $M_3$ was already found before in work by Clercx et al. [4] and van Bokhoven et al. [2], respectively, and appeared again in the results discussed in this report. In this study it is also found that if model $M_1$ is used, power law behaviour also appears.
In the experiments, also a remarkable effect of rotation was found: the time evolution of the flow, obtained with background rotation showed the appearance of meandering currents with fluid transport in a preferred direction. Although, in experiments without applied rotation, meandering currents also appeared, these were moving around randomly. More support for the existence of these direction-selective flows, was found in plots of the streamfunction obtained from the simulations with model $M_2$. As was already mentioned in the introduction, better understanding of the effects of rotation will contribute to better models of geophysical phenomena. When applying the possible existence of meandering currents with preferred directions, on the flows that exist in the oceans and the atmosphere, this could contribute to a better understanding of transport of mass and heat around the globe.

Some dips appeared in the evolution of the kurtosis with the dipole simulations due to vortices that pass the periodic boundaries. Obviously, because the periodic boundaries in the simulations on 2D turbulence are really close, some effects are expected: it can explain the rather unpredictable course of the evolution of skewness and kurtosis but also be responsible for the distortion on the results of the vortex statistics, next to errors occurring due to failing vortex census.

**Recommendations** A suggestion for further research is to perform more simulations of 2D and Q2D turbulence before determining the ensemble average. The results for skewness, kurtosis, and vortex statistics showed a large noise on their evolution in time. This also negatively affected the reliability of the power law exponents that were obtained for the vortex statistics.

Another point for attention is the quality of the experimental set-up. The experiments only produced qualitative results. In order to obtain quantitative results, the light distribution has to be improved, which can be achieved by using a laser to illuminate the flow. Also, the rotating set-up should be changed to flatten the surface of the fluid, which was curved due to rotation, disturbing 2D behaviour.
References


