MASTER

Wind tunnel experiments on wake-vortex decay in external turbulence

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Abstract

Wake vortices behind aircraft may persist for long times and therefore represent a safety concern for following aircraft. It is well known from numerical studies that external turbulence promotes the decay of wake vortices, either by increased vorticity diffusion or by triggering 3D instabilities that lead to their destruction. So far, however, very few quantitative laboratory studies have been devoted to the issue of wake-vortex decay.

In this study, we present an experimental investigation of the evolution of both single and double wing-tip vortices in a wind tunnel with and without a turbulence generating grid, where $Re_{\Gamma}=4 \times 10^4$ typically. Cross-sectional velocity fields were obtained by means of particle image velocimetry at several locations downstream of the airfoil. Special attention was given to resolve the vortex core, since this has often been a region of difficulty in most experimental investigations. The single vortex was found to exist of a small viscous core and an outer region with nonzero vorticity. The decay of the maximum azimuthal velocity is close to laminar, both with and without grid. The main effect of the grid turbulence is to increase the diffusion of vorticity in the outer region and to enhance the wandering of the vortex center. The wandering amplitude is inversely proportional to the strength of the vortex, indicating mutual interaction between the vortex and external turbulence.

For the double wing-tip vortices a split wing configuration was used with three different tip spacings. The vorticity distribution of the double vortices is more compact than that of the single vortex but decays due to cross-diffusion. The cross-diffusion is without grid significantly larger than viscous diffusion and enhanced by grid turbulence. Grid turbulence is also found to promote the onset of the Crow instability. The instability causes the vortex centers to oscillate with an angle of approximately $33^\circ$ to the horizontal. The growth rate of the oscillation amplitude is approximately half the growth rate predicted by inviscid theory. The Crow instability is followed by the phenomenon of vortex linking, which results in a rapid destruction of the vortices. At very strong turbulence levels, core oscillations and decay of circulation are significantly present, though the Crow instability is no longer observed.
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Chapter 1

Introduction

As a result of the flow around the wing of an airplane, a lift is created on the wing necessary for the airplane to fly. Simultaneously, this flow also causes an inevitable side effect on the wing tips: the generation of a pair of counter-rotating tip vortices. The pressure difference over the airfoil causes the flow at the wing tips to rotate as is indicated in figure 1.1a, resulting in two counter-rotating tornado-like structures left behind the wing tips (see figure 1.1b), which may persist for quite a long time. As the pressure difference generating this rotation is related to the lift and thus the mass of the aircraft, it can be understood that these ‘tip vortices’ have great strength and impose a potential hazard for following aircraft. Fortunately due to viscous effects, vortex interaction and atmospheric disturbances, the strength of the vortices decreases. Due to condensation of water vapor inside the vortices, the structures can often be seen across the sky as is shown in figure 1.1c.

Another important property of the tip vortices is that they determine the induced drag. The tip vortices get longer as the plane proceeds and require kinetic energy to be supplied to generate these vortices. This causes an additional drag force to be supplied to the wing, which is called induced drag. The magnitude of the induced drag depends on the properties of the vortices and determines the fuel consumption. Control of the properties of the vortices is therefore an issue in aircraft design.

The prediction and control of the life time of tip vortices is especially important in air traffic control. The vortices directly influence the flight of following aircrafts having similar flight paths and therefore specific spacing rules have to be obeyed. These spacing rules are defined by wake-turbulence constraints, which depend on the weights of the leader and follower aircraft. There are a small number of weight classes and a “matrix” of separation distances given by the International Civil Aviation Organization as shown in figure 1.2 [1]. A light plane following a heavy plane should keep more distance than the other way around.

However, these aircraft spacings are based on limited flight tests, rather than on actual flow field measurements. Optimizing these spacing rules could significantly increase the capacity of an airport and decrease delays. However, an analytical approach of the wake-turbulence problem faces two major obstacles. The first problem is that numerical simulations suffer from limited resolution and the difficulty of turbulence modeling. However, the most important problem is the effect of the changing of the environment (winds, turbulence, stratification etc.), which strongly influences the motion and persistence of the vortices [1]. It causes difficulties to determine the life time of a pair of tip vortices with large confidence and to optimize spacing rules.
Figure 1.1: Due to a pressure difference over the airfoil a shear layer or vortex sheet is created behind the airfoil, which rolls up at the tips into two counter-rotating vortices (a). These vortices are sometimes visible in clouds (b) or as contrails (condensation trails) behind the airplane (c).
Figure 1.2: ICAO’s weight classes and separation distances for following aircrafts given in nautical miles (1.85 km).

By studying the condensation trails across the sky, various mechanisms of decay can be distinguished. Due to diffusive processes vorticity is distributed over a larger area increasing the vortex size as can be seen in figure 1.3a. Next to diffusion, the vortices exhibit long wave instabilities as is shown in figure 1.3b. The long wave or Crow instability is a sinusoidal oscillation which eventually results in linking of the vortex tubes. Several other instabilities may arise, and can lead to a sudden collapse or ‘bursting’ of a vortex, which is shown in figure 1.3c. The atmospheric conditions determine which mechanisms are dominant and thus determine the life time of the vortices.

At the moment, the physics underlying the evolution of wing-tip vortices is not sufficiently understood and experimentally investigated to create a practical model predicting the evolution of the wing-tip vortices in different environmental conditions. Therefore, several experimental investigations are required to obtain more insight in the decay mechanisms of the vortices. In this report, the focus is on the influence of turbulence on the decay of the wing-tip vortices. The goals are to quantify the influence of turbulence on the decay of a single vortex, to get more insight in the mutual interaction between wing-tip vortices which lead to their destruction, and to investigate whether turbulence can increase mutual interaction and thus the decay of the vortices.

The remainder of the report is organized as follows. First, some concepts concerning the evolution vortices are introduced as well as the possible influences of turbulence in chapter 2. Subsequently, the experiment is introduced in chapter 3 and the particle image velocimetry measurement method and its evaluation are discussed. In chapter 4, the measurements on an isolated vortex and double wing-tip vortices in both low and strong turbulent environments are presented. We finish the report by making a comparison and discussion of the results in chapter 5.
Figure 1.3: Several decay mechanisms of tip vortices are observed in the sky. In panel (a), the vortex expansion due to diffusive processes is shown, while in panel (b) the development of the Crow instability is depicted taken from the original paper of Crow [2]. Panel (c), shows a sudden collapse of a single vortex tube also referred to as ‘bursting’ taken from Spalart [1].
Chapter 2

Theory

Wing-tip vortices are strongly rotating structures. In order to quantify these structures we introduce the vorticity which is defined as the rotation of the velocity field \( \vec{\omega} = \vec{\nabla} \times \vec{v} \). As a result, the divergence of the vorticity is zero:

\[
\vec{\nabla} \cdot \vec{\omega} = 0.
\] (2.1)

Integrating the latter equation over a volume (e.g. a box) and applying Stokes theorem gives

\[
\int_V \vec{\nabla} \cdot \vec{\omega} dV = \oint_A \vec{\omega} \cdot \vec{n} dA = C_1 = 0,
\] (2.2)

where \( A \) is the surface enclosing the volume \( V \) and \( \vec{n} \) is the normal to this surface. The equation states that the total flux of vorticity through a volume \( V \) with boundary \( A \) is zero in absence of sources or sinks (\( C_1 = 0 \)). In other words the flux of vorticity into a volume equals the flux of vorticity out of the volume. The flux of vorticity through a surface is called the circulation \( \Gamma \) and is defined as

\[
\Gamma = \oint_C \vec{\omega} \cdot \vec{n} dA = \oint_C \vec{v} \cdot d\vec{l},
\] (2.3)

where \( C \) is a contour enclosing an area \( A \) and \( d\vec{l} \) is an element of this contour. If the contour encloses all vorticity in the plane, \( \vec{\omega} \cdot \vec{n} \), the total circulation is obtained. Following Kelvin's theorem (e.g. [3]) the total circulation is conserved in time over the whole fluid containing domain.

Wing tip vortex are often assumed to be slender, which means that the axial component of the vorticity is much larger than the perpendicular components. It is therefore useful to consider the vorticity in a plane perpendicular to the vortex axis. We define \( x \) as the axial coordinate, while \( y \) and \( z \) are located perpendicular. The first three moments of vorticity in the \( yz \)-plane are defined as [4]

\[
\Gamma = \int_A \omega dA,
\] (2.4)

\[
\Gamma_{1,y} = \int_A y \omega dA,
\] (2.5)

\[
\Gamma_{1,z} = \int_A z \omega dA,
\] (2.6)

\[
J = \int_A (y^2 + z^2) \omega dA.
\] (2.7)
As mentioned before, the zeroth moment of vorticity or circulation is conserved. Since the vortices are slender, the problem can be considered two-dimensional. Using the two-dimensional vorticity equation it can be shown that in an infinite domain with bounded vorticity the first moments of vorticity, $\Gamma_{1,y}$ and $\Gamma_{1,z}$, are also conserved. The second moment of vorticity, $J$, satisfies

$$\frac{dJ}{dt} = 4\nu\Gamma,$$

(2.8)

where $\nu$ is the kinematic viscosity, and is therefore constant for inviscid fluids and flows with net zero circulation [4]. The constancy of these integrals was exploited by Betz to establish the so-called Betz-invariants [5], which are widely used for vortex roll-up processes behind wings. An extension of the integrals for different boundary conditions can be found in Zheng [4].

In three dimensions, only the total circulation in a plane is conserved, and the other conservation laws only apply when three-dimensional effects are small. Three-dimensional effects include instabilities, which might occur during the life time of the vortices, but this is discussed later on. In the initial stages of life time, the two dimensional approach is useful as is shown in the next sections.
2.1 Structure of wing-tip vortices

Even though the present study focuses on wing-tip vortices in the far wake, a brief introduction is given on the lift on an airfoil and the roll-up of the vortex sheet in the near wake. The roll-up of the vortices is essentially important, since it may both affect the final vortex structure and hence the interaction between the vortices.

Due to the sharp edge of an airfoil, also called the trailing edge, circulation is created around the airfoil, which depends on the geometry of the airfoil. A well-known mathematical airfoil model is the Zhukovski-airfoil, which creates circulation equal to

$$\Gamma = 2 \pi U c \sin(\alpha + \beta)$$

(2.9)

where $U$ is the main flow velocity. The angle of attack, $\alpha$, is the angle between chord line of the airfoil, with length $c$, and the main flow as is shown in figure 2.1a. The angle $\beta$ is the camber angle. The lift per unit span generated on a two-dimensional body in an irrotational flow with density $\rho$ is $L = \rho U \Gamma$, and is referred to as the Kutta-Zhukovski lift theorem. The derivation of these relations can be found in many textbooks (e.g. Kundu [3]), but here they serve to give the reader an indication of the relationship between the quantities. The disturbance created by an airfoil in a uniform flow $U$ is in many ways similar to that created by a vortex filament, and therefore the airfoil can theoretically be replaced by a ‘bound’ vortex, which is located on the lifting line of the airfoil. This concept is generally referred to as the lifting line theory of Prandtl and Lancaster [3].

The lift on a single wing decreases from its maximum value at the wing root to zero at the tip (see figure 2.1b). Since the lift theorem states that $L = \rho U \Gamma$, any change in lift corresponds to a change of strength of the bound vortex. Applying eq. (2.2) on a box enclosing a part of the bound vortex as is shown in figure 2.1b, it can be seen that the change of circulation over the airfoil results in a trailing vortex filament of strength $d\Gamma$. In fact a vortex sheet, with strength per unit length $\kappa = d\Gamma/dy$, is created behind the airfoil, which rolls up into two counter-rotating tip vortices. The airfoil geometry (like flaps) can change the lift distribution and result in multiple vortices, but here this case is not considered.

In the following only the one side of the wing is discussed and thus the conservation laws in eq. (2.4) are applied to a single vortex. The first moment of vorticity states that the center of vorticity is conserved during roll-up. The location of the tip vortex is thus determined by the lift distribution on the wing. Since $d\Gamma/dy$ is largest near the wing tip, the center of vorticity of the vortex sheet will be close to the tip. The roll-up of the vortex sheet is extensively (mostly numerically) studied in literature, but in this report we restrict ourselves to an asymptotic approximation of Kaden [6] [7] to make an estimate of the vorticity profile. Assume that the circulation distribution of the airfoil, with $y = 0$ at the wing root, is given by

$$\Gamma = \Gamma_{\text{root}} \left(1 - \left(\frac{2y}{s}\right)^{1/(1-n)}\right)^{1-n}, 0 < n < 1$$

(2.10)

where $s$ is the span width. This circulation distribution is given in figure 2.2a for several values of $n$. The limiting cases $n = 0$ and $n = 1$, correspond to a delta wing loading and a square loading, respectively. The case $n=1/2$, corresponds to elliptical loading and this is a particularly interesting case since it can be shown that the induced drag is minimum (see e.g. Ashley and Landahl [8]) when the circulation is elliptic.
Figure 2.1: In panel (a), a side view of the airfoil is shown. The circulation distribution on a wing leads to trailing vortices behind the airfoil with strength $d\Gamma$ as is shown in panel (b). The trailing vortices rapidly roll-up into two separate tip vortices.

Figure 2.2: Panel (a) shows the circulation distribution over a single wing of span $s/2$ as given in eq. (2.10) for $n=0$, 0.5 and 0.9. In panel (b), vorticity profiles are depicted, which are obtained from an asymptotical approximation of the circulation distribution in panel (a).
For mathematical convenience, the circulation distribution is only considered near the tip. Expanding eq. (2.10) as a Taylor series near \( y = \frac{s}{2} \) gives to first order
\[
\Gamma = \Gamma_{\text{root}} \left( \frac{1}{1 - n} \left( 1 - \frac{2y}{s} \right) \right)^{1-n}.
\] (2.11)

The strength of the vortex sheet per unit length behind the airfoil is then given by
\[
\kappa = -\frac{2(1 - n)}{s}\Gamma_{\text{root}} \left( 1 - \frac{2y}{s} \right)^{-n}.
\] (2.12)

Kaden assumed that this vortex sheet rolls up into infinitely tightly wound spirals around the edge of the vortex sheet at the center of vorticity and that these spirals are circular in the center. Assuming the vortex to be axisymmetric, it follows from dimensional considerations that \( s/2 - y = \lambda r \), where \( r \) is the radius of the vortex and \( \lambda \) is a dimensionless constant, which represents a compression factor. This similarity is more profoundly deduced by Betz, who considered the inviscid roll-up of a vortex sheet with constant \( \Gamma_0, \Gamma_{i,y} \) and \( J \) for an elliptically loaded wing [5], and estimated \( \lambda = 1/(2 - n) \) [7]. The azimuthal velocity around these vortices then becomes for small \( r/s \):
\[
v_\theta(r) = \frac{\Gamma(r)}{2\pi r} \approx \frac{\Gamma_{\text{root}} \lambda}{2\pi s} \left( \frac{\lambda r}{s} \right)^{-n}, 0 < n < 1.
\] (2.13)

The parameter \( n \) can be fitted from experimental velocity profiles and is often found to be close to 0.5 [9] [10] [11], the value for an elliptical loading. The azimuthal velocity has a singularity at \( r = 0 \) because the analysis above is inviscid. Viscous effects will lead to \( v_\theta(r = 0) = 0 \) and a maximum velocity \( v_{\theta,\text{max}} \) at \( r = r_1 \). Note that mutual interaction for double wing-tip vortices is neglected. This could cause different results for an isolated vortex and double vortices. Despite this problem, we continue the present analysis of the velocity profile above.

From eq. (2.13), the vorticity profile can be obtained since \( \vec{\omega} = \vec{\nabla} \times \vec{v} \) which gives for an axisymmetric vortex
\[
\omega(r) = \frac{1}{r} \frac{\partial}{\partial r} (v_\theta r) = \frac{\Gamma_{\text{root}} (1 - n)\lambda^2}{2\pi s^2} \left( \frac{\lambda r}{s} \right)^{-(n+1)}, n < 1.
\] (2.14)

The vorticity profile has also a singularity at \( r = 0 \), which is removed by viscous effects. The case \( n = 1 \) results in a vortex with infinite vorticity in an infinitely small area, \( \omega = \Gamma_{\text{root}} \delta(r) \), which is called the point vortex. Even though the point vortex is not a real vortex, because this would mean a vortex with infinite energy, it is a useful mathematical concept as will be shown in the next section. Vorticity profiles for other values of \( n \) are shown in figure 2.2b

The vorticity profiles introduced above are based on an asymptotical approximation of a specific family of wing loading, but due to the singularity at \( r=0 \) of little practical use. Viscous effects remove the singularity by creating a core, where the azimuthal velocity increases from zero to its maximum value. The radius of peak velocity, \( r_1 \), is determined by the strength of viscous effects in comparison to advectional (roll-up) effects. The viscous time typically scales as \( r^2/\nu \), while the rotation time scales as \( r/v_\theta \sim r^2/\Gamma \). The relative strength of the advectional effects to the viscous effects is given by the flight Reynolds number
\[
Re_\Gamma = \frac{\Gamma}{\nu}.
\] (2.15)
This Reynolds number is large for aircrafts, and therefore \( r_1 \to 0 \), which means that a significant part of the vorticity is outside of the core as can be seen in figure 2.2b. Only in the case \( n \to 1 \), the major part of the vorticity will be in the core. To remove the singularity of infinite vorticity, two approaches are often used as a mathematical model. The first is the assumption that the vorticity is uniform and finite within the core and zero outside the core, which is called a Rankine vortex. The other approach assumes a Gaussian vorticity distribution and is called the Lamb-Oseen vortex.

Under typical flight conditions the vorticity contained in the vortex core is generally less than 30\% of the total vorticity[1], while for the latter two profiles this is 100\% and 72\%, respectively. Wing-tip vortices therefore exists of a small viscous core with radius \( r_1 \) and an inviscid\(^1\) ring of vorticity with radius \( r_2 \). While the first length scale is determined by viscous effects, the physical meaning of the second length scale is not clear [10]. The length scale might be determined by the kinetic energy of the vortex. Because the length scales are decoupled, a two-length-scale vortex model is needed. Examples of two-length-scale vortex models, which are often derived from empirical fits, are given by de Bruin & Winckelmans [12]. Decay mechanisms, such as viscous diffusion, tend to deform the vortex profile. In the next section, the evolution of the vortex due to various decay mechanisms is discussed.

\(^1\)viscous effects can be neglected here
2.2 Decay of wing-tip vortices

In order to describe the decay of tip vortices, a distinction can be made between the decay of the vortex itself, by means of viscosity and turbulence, and the decay induced by the mutual interaction between the vortices. The latter is a result of instabilities, such as the Crow instability. The interaction might also be triggered by external conditions, leading to a very rich and complex decay behavior of the vortices.

Diffusion

Viscous effects have already been briefly mentioned and their influence on vorticity is governed by the diffusion equation

\[ \frac{\partial \vec{\omega}}{\partial t} = \nu \nabla^2 \vec{\omega}, \quad (2.16) \]

with kinematic viscosity \( \nu \). As becomes clear from the form of eq. (2.16), viscous diffusion of vorticity only acts in a plane, since it does not exchange vorticity in other directions. The diffusion of vorticity is therefore a two-dimensional process which describes the redistribution of the concentration \( \vec{\omega} \cdot \vec{n} \) in a plane perpendicular to \( \vec{n} \). The most fundamental solution to eq. (2.16) is the decay of a point vortex, an infinite amount of vorticity in an infinitely small area.

The solution of the decay of a point vortex in an unbounded domain is given by the axisymmetric Lamb-Oseen vortex [13] (see Carslaw & Jaeger for a number of solutions with boundaries [14]):

\[ \omega = \frac{\Gamma_0}{4 \pi \nu t} \exp \left( -\eta^2 \right), \quad (2.17) \]
\[ v_\theta = \frac{\Gamma_0}{2 \pi r} \left( 1 - \exp(-\eta^2) \right), \quad (2.18) \]

where \( \eta = r/\sqrt{4 \nu t} \) is a similarity variable. Solutions for an arbitrary initial axisymmetric vorticity profile can be found by solving the integral [13]

\[ \omega(r, t) = \frac{1}{2 \nu t} \exp \left( -\frac{r^2}{4 \nu t} \right) \int_0^\infty \omega_0(r') \exp \left( -\frac{r'^2}{4 \nu t} \right) J_0 \left( \frac{ir'r}{2 \nu t} \right) r' dr', \quad (2.19) \]

where \( J_0 \) is the zeroth order Bessel function of the first kind. The vorticity profile approaches the Lamb-Oseen vorticity profile as \( t \to \infty \), and therefore the Lamb-Oseen vorticity profile will serve as a useful model throughout the remainder of the report.

Up to now, only diffusion by viscosity has been considered, but it is well known that turbulence increases diffusion. This is a complex process involving energy exchange between vortex like structures, which are called ‘eddies’. It is common practice to model the diffusive property by introducing an effective viscosity replacing the kinematic viscosity. Squire [15] suggested an effective viscosity, consisting of the kinematic viscosity \( \nu \) and an eddy viscosity \( \nu_\epsilon = a \Gamma_0 \) to describe the decay of a line vortex in a turbulent environment. The evolution of the vortex is then determined by the diffusion equation with effective viscosity \( \nu_{eff} = \nu_\epsilon + \nu \).

The constant \( a \) is called Squire’s constant. A more elegant approach was suggested by Roberts distinguishing several regimes [16]. The model by Squire is widely used in the tip-vortex society, but is a little controversial as will be shown below.
Apart from its diffusive property, turbulence is also characterized by its increased dissipation of energy. It is therefore useful to consider the decay of the vortex in terms of energy. The diffusion equation in terms of velocity becomes

$$\frac{\partial \vec{v}}{\partial t} = \nu \nabla^2 \vec{v}, \quad (2.20)$$

where advection and external forces are assumed to be small and are therefore neglected. It can be shown by taking the inner product with the velocity, that eq. (2.20) becomes

$$\vec{v} \cdot \frac{\partial \vec{v}}{\partial t} = \frac{1}{2} \frac{\partial |v|^2}{\partial t} = \nu \vec{v} \cdot \nabla^2 \vec{v} = -\nu |\omega|^2 + \nu \nabla \cdot (\vec{v} \times \vec{\omega}). \quad (2.21)$$

By integrating over an area $A$, the second term becomes

$$\oint_C (\vec{v} \times \vec{\omega}) \cdot \vec{n} \, dl,$$

where $\vec{n}$ is normal to boundary contour $C$. In an infinite domain, this term vanishes and the decay of kinetic energy is given by $dE_K/dt = -\nu \Omega$, where $\Omega$ is the enstrophy. In an infinite domain, axisymmetric vortices are completely determined by the radius $r$ and circulation $\Gamma_0$. The absence of other length scales than $r$, such as those imposed by the domain boundaries or vortex separation distances, requires self-similarity in the diffusion equation. A dimension analysis gives

$$\omega = \frac{\Gamma_0}{r^2} f(\eta), \quad (2.22)$$

$$\frac{dE_k}{dt} = -2\pi \nu \Gamma_0^2 \int_0^{\infty} \frac{f^2}{r^3} \, dr = -\frac{\pi \Gamma_0^2}{2t} \int_0^{\infty} \frac{f^2}{\eta^3} \, d\eta, \quad (2.23)$$

where $\eta = r/\sqrt{4\nu t}$. Surprisingly, the decay of kinetic energy is independent of the magnitude of $\nu$ (for a Lamb-Oseen vortex it becomes $\Gamma_0^2/(8\pi t)$, see also Uberoi [18]). A similar analysis for the one-dimensional viscous problem results in a decay of kinetic energy per unit of length, which is proportional to $\sqrt{\nu}$. In two dimensions, for a single vortex in a domain without boundaries, increased diffusion does not promote the decay of kinetic energy, in contrast to a one-dimensional situation. The use of a two-dimensional effective viscosity model for turbulence is then at least peculiar. One should expect that the dissipative nature of turbulence destroys the similarity $r^2 \sim \nu_{eff} t$ of the vortex (an interesting discussion of other constraints in Squire’s model is given by Uberoi [18]).

Despite the objections against the Squire assumption, it offers a measure of the turbulence level in terms of diffusion. Other ways of approaching the turbulent diffusion and dissipation, like a minimization of kinetic energy, are even more doubtful [1].

**Mutual interaction**

In the analysis above, the presence of the second oppositely signed vortex has been neglected. The vortex induces a velocity in its counterpart and in contrast to the analysis above advection becomes important. This eventually leads to three-dimensional instabilities, but these will be discussed later on. For now, we continue our two-dimensional analysis, by considering the moments of vorticity.

Since the vortices are rotating in opposite direction and have equal strength, the net circulation over a plane enclosing both vortices is zero. To study the individual vortices, only the circulation in a half plane from the symmetry axis is investigated and referred to as

\footnote{the Rayleigh problem (including boundary terms) or the shear layer problem with parameters $U$ and $y$.}
Γ. This half plane circulation is not conserved and will decay through viscous or turbulent diffusion. The viscous diffusion of a vortex pair can be approximated by the time evolution of a pair of Lamb-Oseen vortices, which in terms of vorticity is given by

\[
\omega(y, z) = \frac{\Gamma_0}{4\pi \nu t} \left( \exp\left( -\frac{(y - b/2)^2 + z^2}{4\nu t} \right) - \exp\left( -\frac{(y + b/2)^2 + z^2}{4\nu t} \right) \right),
\]

(2.24)

where the vortex with positive circulation is located at \(y = b/2\) and the vortex with negative circulation at \(y = -b/2\), with \(b\) the vortex separation distance. The circulation is determined over a half plane and its time evolution can be evaluated as

\[
\Gamma = \int_{y>0} \omega dy dz = \frac{2\Gamma_0}{\sqrt{\pi}} \int_0^{b/(4\sqrt{\nu}t)} \exp(-\eta^2) d\eta = \Gamma_0 \text{erf}(b/(4\sqrt{\nu}t)),
\]

(2.25)

with \(\Gamma_0\) the initial circulation. The model gives a useful decay mechanism to determine cross-diffusion. As \(t \to 0\), the circulation remains approximately constant, indicating an initial stage where the vortices are compact, i.e. negligible exchange of vorticity. The length of this stage can be modeled by introducing a virtual starting radius, \(r = R_0 + \sqrt{4\nu t}\), or starting time, \(r = \sqrt{4\nu(t + t_s)}\), where \(R_0 = \sqrt{4\nu t_s}\). Since cross-diffusion of vorticity describes the exchange of vorticity at the symmetry axis between the vortices, this does not necessarily mean that the vortex itself expands as it does due to viscous diffusion. Therefore the cross-diffusion coefficient is given by \(D_{\text{eff}}\) instead of \(\nu\), to avoid confusion. The decay of the circulation does not provide any information about the shape of the vorticity distribution. Local turbulent spots might enhance diffusion locally but do not affect the rest of the vorticity distribution, while viscosity does.

As long as the evolution is in good approximation two-dimensional, the first moments of vorticity are conserved and given by

\[
\Gamma_{1,y} = \Gamma b = \Gamma_0 b_0, \quad \Gamma_{1,z} = 0.
\]

(2.26, 2.27)

Since the vortices have opposite strength, they form a dipole. The first equation shows that the initial dipole strength, \(\Gamma_0 b_0\), is conserved, which means that as the circulation over a half plane decays due to cross-diffusion, the vortices move apart. The second equation shows that as long as the vortices are of equal strength the dipole will not rotate. As the vortices induce a velocity on each other, the dipole moves downward in a straight line following the \(z\)-axis for a positive dipole strength. For compact vortices the downward velocity in the center of the negative vortex at \(y = -b/2\), is given by

\[
w = \Gamma/(2\pi b).
\]

(2.28)

This gives rise to an advectional time \(t_A = b/w = 2\pi b^2/\Gamma\), while the diffusion time scales as \(t_D \sim b^2/\nu\). Their ratio is the Reynolds number \(\Gamma/\nu\) introduced in the previous section.

The third moment of vorticity, \(J\), is also conserved in two dimensions for a dipole of vortices with equal strength (eq. (2.8), with \(\Gamma = 0\)). If the vorticity profile is antisymmetric, \(J=0\) and will remain so. However, if the vorticity profile is not due to different lift distributions on the airfoil for example, this will also be conserved. The third moment can therefore be thought of as conserving (anti)symmetry of vorticity profile, while vorticity is diffusing.
Instabilities

Even though diffusion of vorticity would eventually destroy vortices, Tombach observed that tip vortices in the atmosphere never decayed due to viscous or turbulent diffusion, but were always destroyed by some form of instability. Two modes of instability were ‘bursting’ of an individual vortex core and the Crow or long-wave instability [19]. The instabilities involve three-dimensional effects such as vortex tilting and stretching and the conservation of the moments of vorticity no longer applies when these arise, except for the conservation of circulation in a plane.

Bursting is an apparently spontaneous crisis of the vortex core, which often travels over the vortex tube [20] [21]. These visualizations reveal smoke or dye contractions and expansions in different parts of the vortex tubes (see figure 2.3), leading to a sudden blow up or burst of the vortex core at a certain location. Bursting is rarely observed simultaneously on both vortices, indicating that the process involves only one vortex. The phenomenon is probably related to intersecting pressure waves as was shown in direct numerical simulations [22]. The pressure waves may arise due to turbulence or vortex linking. In the latter case the pressure waves are related to axial flow inside the cores of the vortices. When the vortices connect, opposite axial flow in both vortices gives rise to a pressure wave, which travels through the vortex tube. If the vortex tube also connects at another location another pressure waves arises which will intersect with the other, resulting in a vortex burst [22].

Vortex linking occurs as a result of the development of the Crow instability. This instability is a sinusoidal perturbation of the distance between the vortex cores, which grows in time until the vortices connect resulting in a train of vortex rings as is shown in figure 1.3b. The mechanism of this instability is described by Crow [2] and therefore this instability is often referred to as Crow instability. The main idea is that slight perturbations occur in the distance between two vortex tubes, while leaving the circulation intact. The displacement perturbations are amplified by three-dimensional mutual induction from the slightly bend vortex tubes. Next to that, self-induction occurs which tends to have a stabilizing effect and the strength of this self-induction depends on the size of the vortex core. The theory requires slender vortices, where the core diameter is much smaller than the vortex separation distance. For a realistic choice of the core diameter, Crow showed that the most unstable wave length of the instability $\lambda = 8.6b$, where $b$ is the vortex separation distance, and that the vortex tubes oscillate with angle $\theta_s = 48^\circ$ to the horizontal [2].

In addition to these relatively long wave lengths it was also shown that vortex tubes are unstable to wave lengths of the order of the vortex core [23]. These instabilities are called short-wave instabilities and arise due to deformation of the vortex core for example by strain. The nature of the instabilities is studied by Fabre et al. [24]. In turbulent environments, however, Crow instability as well as vortex bursting are the most often observed instabilities as can be seen in figure 2.3 taken from Liu [20]. The bursting phenomenon appears to be dominant mechanism for Reynolds numbers typical of towing-tank experiments [25] and is mainly observed in strongly turbulent environments [19] [20] [21]. Therefore we focus on turbulence and its effect on the vortex lifetime.
Instabilities are often dependent on the environmental conditions as is shown in this picture obtained from towing tank experiments by Liu [20]. It shows that in a quiescent environment ($\epsilon^* < 0.01$), vortex decay is dominated by the Crow instability. In a weakly turbulent environment ($0.01 < \epsilon^* < 0.2$) both the Crow instability and vortex bursting occur, while in a strongly turbulent environment ($\epsilon^* > 0.4$) only vortex bursting occurs. The parameter $\epsilon^*$ is a dimensionless parameter describing the relative strength of turbulent velocity fluctuations in comparison with the downward advecting velocity induced by the vortices.
Figure 2.4: The shear of the azimuthal velocity around a vortex tube in \( r \)-direction, causes outward radial components of the vorticity to be tilted and stretched in the negative \( \theta \)-direction. The main result of this process is that the enstrophy and thus the kinetic energy of the radial vortex tubes is increased. The increase of kinetic energy directly results in a loss of kinetic energy from the main vortex tube.

2.3 Influence of turbulence

Turbulence is an extensive subject, and it is not our purpose to give a complete introduction to the subject. For such an introduction one can read several books, e.g. Davidson [26] and Nieuwstadt [27], but here the focus is on the interaction between turbulence and tip vortices. One of the major properties of turbulence is its diffusive and dissipative nature. The two-dimensional Squire model accounts for the diffusive nature, but apparently not for the dissipative nature as was shown in the previous section. A three-dimensional approach might be necessary to account for the influence of turbulence [28]. A key process in three-dimensional turbulence is vortex stretching. Large eddies deform smaller eddies by vortex stretching and thereby transferring energy to these smaller scales. This process of stretching is not present in two-dimensional systems.

As a tip vortex has large strength, it will probably deform the turbulent field as well. Assume that the turbulent field exists of vortex patches which added up have no net circulation. Due to the shear of the azimuthal velocity of the main vortex tube in radial direction, vortex patches (secondary vortex structures) with strong radial components are tilted and stretched giving rise to oppositely signed vortex tubes in azimuthal direction. This process is depicted in figure 2.4 and is given by:

\[
\frac{D\omega_\theta}{Dt} = \omega_r \frac{\partial v_\theta}{\partial r} \approx -\omega_r \frac{\Gamma}{2\pi r^2} = -2\pi \frac{\omega_r}{t_{rot}}. \tag{2.29}
\]

Vortex patches, which have a characteristic time scale that is larger than the rotation time scale, \( t_{rot} = 2\pi R/v_\theta = (2\pi R)^2/\Gamma \), experience a ‘rapid distortion’. Physically this means that the patches have no time to exchange energy during the deformation [27]. As viscosity is neglected, the analysis is restricted to the larger, more energetic patches. The vortex tubes form rib-like structures surrounding the main vortex tube and this behavior is found in numerical simulations [29] [30] [31]. The influence of the rib structures on the main vortex are diverse. Axial velocities are induced in the vortex core, which might result in pressure waves and subsequently vortex bursting as discussed in the previous section. Next to that, they may enhance diffusion by outward advection or amplify existing instabilities. Furthermore, the deformation of the turbulence field and the formation of rib-like structures
might implicate that the time scales of the turbulence become coupled to the rotation time of the main vortex. Due to the stretching, the vorticity of the vortex tubes is increased, so the characteristic time scale is decreased. If the time scale becomes smaller than the rotational time scale, this might mean that the tube is dissipated through the cascade process. The influence of external turbulence might then be restricted to those vortex tubes that have time scales near the rotation time of the vortex tube. The behavior of the main vortex tube due to external turbulence can then turn out to be sensitive to its own rotation time.

The process of vortex stretching causes the enstrophy and the kinetic energy of the stretched vortex to be increased. It thus transfers energy from the larger eddies to the smaller, where it is dissipated also called the dissipative range. The range of length scales from the larger eddies to the dissipative range is called the inertial range and this inertial range is governed by the turbulent dissipation rate, $\epsilon$. As discussed in the previous section, the final decay arises from turbulent bursting and the Crow instability. This observation lead to the theory of Crow and Bate [32] for the life time of a pair of vortices. Below the relevant parameters are introduced.

The deformation work done by the turbulent field results in a cascade of energy for eddies with length scales $L$, which is given by $dE/dt = \epsilon \propto v^3/L$. The turbulent velocities causing deformations at the scale of the vortex separation, $b$, are of magnitude, $v_e \propto (eb)^{1/3}$ [32]. The strength of the turbulence velocities can be correlated to the induced descent velocity of the vortex pair, $w$, giving

$$\epsilon^* = \frac{v_e}{w} = \frac{2\pi \epsilon^{1/3} b^{4/3}}{\Gamma_0},$$

which represents the strength of the turbulence in comparison to the strength of the vortex wake. Crow and Bate suggested that this $\epsilon^*$ is the most important parameter governing the life time of the vortex wake. This lifetime, $T$, is defined as the time where the root-mean-square value of the vortex separation fluctuations becomes $b_0$. The life time will be a function of $\epsilon$, $b$ and $\Gamma$, and possibly other parameters. By dimensional analysis,

$$T^* = \frac{\Gamma}{2\pi b^2} T(\epsilon^*, \ldots),$$

where $T^*$ is the life time of the vortex divided by the downward advection time. The decay of a vortex pair is therefore most often studied as a function of the time, $t^*$, which is the evolution time divided by the advection time, $t^* = t\Gamma/(2\pi b^2)$. As eventually, a theory needs to be developed, which predicts the evolution of the vortices as a function of the initial conditions, $t^*$ is based on the initial parameters $\Gamma_0$ and $b_0$. The same holds for the parameter $\epsilon^*$, which is also based on the initial parameters.

Crow and Bate developed a theory for $T^*$ as function of $\epsilon^*$ [32], neglecting other parameters. They derived that when turbulence is very strong or $\epsilon^* > 0.4$, the lifetime of the vortex wake is given by $T^* = 0.41/\epsilon^*$, assuming local isotropy and that the vortex separation, $b_0$, is within the inertial range. When induction between the vortices is at least as important as advection by atmospheric turbulence, it was deduced that for the most unstable wave lengths, the implicit lifespan formula becomes

$$\epsilon^* = 0.87T^{*1/4} \exp (-0.83T^*).$$

The two limits are shown in figure 2.5. The scatter in flight data is unfortunately too large to be completely predicted by this theory, which seems to give a lower boundary for the
Figure 2.5: Two asymptotic limits of the lifespan formula of the dimensionless lifetime $t^*$ against the dimensionless dissipation rate $\epsilon^*$ suggested by Crow and Bate [32]. The limits intersect at $\epsilon^* = 0.4$ and the combined curve is given as the thick black line. The dashed lines show the remainder of both curves from the point of intersection.

lifetime [1]. This indicates that other parameters might be relevant for the decay as well, like cross-diffusion of vorticity. The latter phenomenon is not accounted for by Crow and Bate, who considered compact vortices. The most obvious effect will be that the circulation decays ($\Gamma(t) < \Gamma_0$) causing the advection velocity of the vortex pair to be smaller than its initial value. The dimensionless lifetime, $T^*$, based on the initial values then increases as $\Gamma_0 > \Gamma$, while the theory is based on the instantaneous circulation, $\Gamma$. The parameter, $\epsilon^*$, also evolves due to changes in the circulation.

Figure 2.5 suggests that for low turbulence levels the life time is $T^* = 5$. Therefore, in order to investigate the evolution of the wing-tip vortices, life times should be measured at least up to $t^* = 5$. This is more elaborately discussed in chapter 3. In the next section, the theory section is concluded with some remarks about the relation between time and space for tip vortices.
Figure 2.6: As every location $x + dx$ of the vortex tube is generated a time interval $dt = dx/U$ later than location $x$, where $U$ is the velocity of the airfoil, the diffusion in time can be pictured as a diverging of vortex lines. Vortex lines are defined as lines in the fluid at which the local vorticity is directed tangentially.

2.4 Relation between evolution time and space

In the previous sections, the decay of the tip vortices has been discussed as a function of time. However, it was not mentioned that as the plane continues to move, new parts of the tip vortices are generated and the vortices grow in direction of the plane. As different locations on the vortices in $x$-direction are generated at different times, they will also be in different stages of their decay, but they form a single object (unless vortex linking causes ring formation).

Since the airplane is flying with a velocity $U$, the similarity $x = Ut$ makes sense, relating the position to the decay time. The atmosphere, however, might just be stationary and have a different influence at each location $x$, influencing its behavior. So next to the global decay, which is related to the time the vortex is generated, there is also an instantaneous behavior at each $x$, which also changes in time. For example suppose that at a certain location, the interaction of turbulence with the vortex causes a pressure wave. There is no reason why this pressure cannot travel towards the airplane. So at the locations closer to the airplane, the pressure wave is observed at shorter time intervals from generation. This concept becomes more tangible when the experimental set up is introduced in the next chapter.

The time similarity between $x$ and $t$ is only valid as long as processes are described, which are not three-dimensional. A good example of such a process is diffusion, which occurs in a plane. In this case, different locations on the vortex tube correspond to different times. Now, take a cylindrical box around the vortex tube as in figure 2.6. Applying eq. (2.2) to this box and note that diffusion redistributes vorticity outward. As can be seen in figure 2.6, diffusion causes vortex lines to diverge and increase the radial components of vorticity, since the flux of vorticity should be zero through this volume. Vortex lines are defined as a line in the fluid at which at any point the local vorticity vector is directed tangentially.

The increase of the radial component can be recovered from the divergence theorem for an axisymmetric vortex

$$\frac{\partial \omega_x}{\partial x} + \frac{1}{r} \frac{\partial (\omega_r r)}{\partial r} = 0,$$

(2.33)

where $\omega_x$ and $\omega_r$ are the axial and radial component of the flow respectively. The similarity $\partial / \partial x = 1 / U \partial / \partial t$ allows a direct coupling to the diffusion equation (eq. (2.16)). Note that strong radial components of vorticity should also occur in the case of vortex bursting.
Figure 2.7: In panel (a), the diffusion is presented in terms of vortex lines. Vortex lines are lines in the fluid at which at every position the local vorticity is directed tangentially. Linking of vortex lines causes the circulation to decay in time. In panel (b), the linking of vortex tubes is shown in terms of vortex lines. The vortex ring has constant circulation, but the manner of taking the contour determines the flux of vorticity and therefore the observed circulation.

The cross-diffusion that occurs between a pair of vortices can then be explained by a linking of vortex lines\(^3\) as is shown in figure 2.7a. The linking of the vortex tubes as a result of the Crow instability can then be pictured in terms of vortex lines as is shown in figure 2.7b. In the latter case, it is not obvious to relate the time and length scale, as the vortex oscillates in time. However, due to large airplane velocities, the instability will not influence the whole vortex, because the time necessary to grow over a length \(x\) towards the airplane is much larger than the difference between the time of generation over this length, which corresponds to \(t = x/U\). Large length scales on the vortex therefore have approximately the same evolution time.

To make things more complicated, in the experiments, the airfoil is kept at a fixed position, while the flow and thus the vortex is moving. To study the evolution of the vortex at a fixed point, one should move with the flow. In this case a fixed location behind the airfoil corresponds to a fixed time from generation of the vortex. Measuring at a fixed location behind the airfoil during a time interval, then gives information how fixed points at the vortex differ from each other. Due to the large flow velocity, \(U\), three-dimensional effects occurring on a fixed location behind the airfoil barely influence the vortex upstream. In the next chapter, the experimental set up is introduced and the procedure and measurements are presented. The results for the evolution of wing-tip vortices in several turbulent environments are introduced in chapter 4.

\(^{3}\)as long as Stokes theorem is valid, otherwise the lines can be ergodic
Chapter 3

Procedure and measurements

As was mentioned in the introduction, an analytical approach to the wing-tip vortex problem faces two major problems [1]. The first problem is that numerical simulations suffer from limited resolution and the difficulty of turbulence modeling. The second problem is that boundary conditions, like wind, atmospheric turbulence are ever changing, making it very hard to create a theory giving the lifespan of the vortices. A comparison of numerical simulations and well-defined experiments is therefore important to validate and improve numerical simulation predictions. In this report, we experimentally investigate wing-tip vortices in a well-defined setting that can be used as input to validate numerical simulations.

To study the evolution of wing-tip vortices in external turbulence, four distinct cases are considered: an isolated vortex in a low turbulent environment, an isolated vortex in a turbulent environment, double wing-tip vortices in a low turbulent environment and double wing-tip vortices in a turbulent environment. The velocities in the flow were measured by applying particle image velocimetry (PIV) at several distances behind an airfoil in a wind tunnel. To generate turbulence, a grid was installed at the beginning of the wind tunnel test section. Turbulence properties were measured with a hot wire probe at several locations in the wind tunnel.

This chapter describes the experimental setting, procedure and measurements. The experimental set-up is introduced in the next section. Features of the PIV technique are discussed and optimal parameter settings are given. We end this chapter with a description of the post-processing of the data.

3.1 Experimental set-up

The experiments were performed in the wind tunnel ‘Goliath’ in the Fluid Dynamics Laboratory, which is schematically shown in figure 3.1. The $x$-axis is defined along the axial direction, while the $y$-axis is defined in the horizontal direction perpendicular to the $x$-axis and the $z$-axis in the vertical direction. We define $x=0$ at the beginning of the test section and $y=z=0$ corresponds to the mid level of the tunnel. The wind tunnel can achieve axial velocities ranging from 0 to 24 m/s (without grid). Most experiments were carried out at approximately 16 to 17 m/s dependent on the measurement location in the wind tunnel test section. The test section is 8 m long and has a cross-sectional area of 1.05 m $\times$ 0.70 m (height $\times$ width). Calibration tests show that the relation between the mean velocity in the center of the tunnel, $U$, the distance downstream from the entrance of the test section, $x$,
Figure 3.1: Schematic side view (panel (a)) and top view (panel (b)) of the wind tunnel test section. A wing tip vortex is shed from the airfoil due to the tunnel velocity $U(x)$, which can be determined from the pressure difference over the contraction. The flow is seeded with tracer particles from the DNW Aerosol Generator (homogeneous) and from a smoke tube (inside vortex core). The seeded particles are illuminated by a double pulsed Nd:YAG laser with frequency $\xi_s = 15$ Hz and are monitored by a CCD camera.

and the pressure difference over the contraction, $\Delta p$, is given by [33]:

$$U(x) = \sqrt{\frac{2}{\rho} \left(0.995 + 0.180 \frac{x}{L}\right) \Delta p}, \quad (3.1)$$

where $L$ is the length of the test section. So $U(x)$ increases with distance (up to about 9% at the end of the test section).

Wing-tip vortices are generated with Clark Y type airfoils. Their centers are located at either $x_a = 0.465$ m (without grid) or $x_a = 3.465$ m (with grid) downstream from the entrance of the test section. The chord length of each airfoil, $c$, is 0.075 m while the thickness is 11.7% of the chord length [34]. Both the span width, $s$ ($\approx 0.30$ m), and the angle of attack, $\alpha$, can be adjusted. The angle of attack is determined with a digital level on a protrusion of the wing, which is leveled on the chord line. The tip of the wing is twisted\(^1\) such that in the presence of another vortex at a distance of $d_0 = 0.10$ m the wing obtains a constant load (see Appendix G for details). No transition trip has been applied to the wing. When studying the isolated vortex, the airfoil is mounted on one of the vertical tunnel walls at height $z=0.01$. To generate a vortex pair, two airfoils are mounted upside-down on opposite sides of the vertical tunnel walls (referred to as split-wing-configuration) at a distance, $d_0$, apart. Because of the mutually induced downward velocity $w$, the wing models are mounted slightly higher at a height of $z=0.165$ m.

When comparing these experimental settings to typical flight conditions (table 3.1) [35], it can be inferred that the flight Reynolds number, $\Gamma/\nu$, is 3 orders of magnitude smaller in the wind tunnel. The difference in Reynolds number affects the maximum obtainable lift coefficient. Furthermore, for a smaller Reynolds number, the effects of viscosity are stronger and expected to result in a larger vortex core. Previous wind tunnel experiments [36] suggest that in a turbulent environment the flight Reynolds number has no or little influence on the

\(^1\)optimized design by A. Elsenaar
decay process of the vortices. This might indicate that the results obtained can be translated to flight situations.

Table 3.1: Experimental Facilities [35]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical flight</th>
<th>‘Goliath’ TUE</th>
<th>DNW-LST</th>
<th>Towing Tank TUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity $U$ (m/s)</td>
<td>75</td>
<td>25</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>span $b$ (m)</td>
<td>40</td>
<td>0.03 - 0.10</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Lift Coeff. $C_L$</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$Re = \Gamma/\nu$</td>
<td>2.4 $10^7$</td>
<td>2 $10^4$</td>
<td>1.9 $10^6$</td>
<td>2.1 $10^6$</td>
</tr>
<tr>
<td>$(x/b)_{max}$</td>
<td>200 - 280</td>
<td>35 - 116</td>
<td>10</td>
<td>160.9</td>
</tr>
<tr>
<td>limitation</td>
<td>-</td>
<td>length</td>
<td>length</td>
<td>height</td>
</tr>
</tbody>
</table>

To obtain the velocity field, particle image velocimetry (PIV) was applied. This method is not intrusive and allows us to determine instantaneous velocities indirectly, with relatively high resolution and over a large area. The basic principle is to add tracer particles to the flow, to illuminate these particles by a light source and to correlate two subsequent images of these particles. The particles have moved in the time span between the images due to the flow and thus give the velocity field. Often a laser sheet is used to illuminate the particles. The delay time between the two light pulses determines the range of velocities that can be measured. An extensive description of the PIV technique is given by Raffel et al. [37], and processing of PIV data is studied by Westerweel [38].

The flow was seeded with DEHS (di-2-ethyl hexyl sebacate) tracer particles at the end of the test section through a DNW Aerosol Generator. This generator is a Laskin Nozzle generator and the density of particles can be adjusted by adjusting the pressure. The vortex rotation forces most of the particles out of the vortex core. Therefore, in addition, smoke tubes were used to inject smoke particles in the core. These smoke particles are created by a chemical reaction, which is initiated by the airflow, and have different seeding and particle size as those created by the DNW generator.

The tracer particles were illuminated by a laser sheet produced by a double pulsed Nd:YAG laser. The pulses were very short, approximately 2 ns, to avoid streaks. The Nd:YAG laser is commonly used in PIV and technical details can be found in various books (e.g. [37]). The Nd:YAG laser creates infrared light with a wave length of 1064 nm, which is transformed by a nonlinear crystal by means of frequency doubling to visible green light with wave length $\lambda_{laser}=532$ nm. The laser sheet thickness, $d_L$, could be adjusted by a positive lens. On the one hand the laser sheet should be as thick as possible to minimize out-of-plane motion due to the axial flow, which decreases correlation between images. On the other hand, the intensity of the laser sheet decreases significantly when increasing laser sheet thickness, which also decreases the signal-to-noise ratio. Changing laser sheet thickness was not a practical procedure, since the optical set up had to be outlined. Therefore, the thickness was fixed at $d_L = 4.4$ mm.

A parameter, which could be changed more easily, is the delay time between the laser pulses, $\tau$. By decreasing the delay time, the out-of-plane loss of particles can be reduced. Furthermore, large velocities in small areas, like the core of the vortex can be correlated well by choosing small delay times. However, in the outer region of the vortex, the velocities are low. These velocities can only be measured accurately if the delay time is sufficiently long.
If short delay times were used, velocities in this region showed significant scatter. Therefore, when interested in the core region delay times of $\tau = 30 - 50 \, \mu s$ were used. When interested in the outer regions, e.g. for determining the circulation, the delay times were set at $\tau = 80 - 100 \, \mu s$.

The flow was monitored by an externally triggered 10-bit CCD camera. The CCD has a resolution of 1008 x 1019 pixel$^2$ and may store image pairs with a frequency $\xi_s=15 \, \text{Hz}$ (this frequency is limited by the data transfer speed). The camera was located at approximately 1.2 to 1.3 m downstream of the laser sheet (with a specific angle, $\alpha_{cam}$, to the mean flow to avoid interaction between camera and vortex). Lenses were used with focal lengths of $f = 28$, 50 (with grid), 55 (without grid) and 105 mm. The largest focal length was used to focus on the core of the single wing-tip vortex and the lens with $f=50$ (with grid) or 55 mm (without grid) was used to determine the circulation for this vortex, because it covers a larger area (also a larger area per pixel). This latter lens was also used for measurements on the double wing-tip vortices. For the largest spacing between the wings, $d_0 = 0.10$ m, a lens with $f=28$ mm was used. In all these cases, the area per pixel was larger than 110 $\mu m \times 110 \, \mu m$, though the mean particle diameter, $d_p$, was smaller than 4 $\mu m$ [34]. Even the image of the particles diameter, $d_{im}$, was smaller than one pixel [37]:

$$d_{im} = \sqrt{(M d_p)^2 + (2.44 \frac{f}{D_a}(M + 1)\lambda)^2} < 60 \mu m \hspace{1cm} (3.2)$$

where $M$ is the magnification (with $M < 10$), $D_a$ the aperture diameter ($f/D_a < 3$), and $\lambda$ the wave length of the scattered light ($\approx \lambda_{\text{laser}}$). This means that the particle sizes were much smaller than the pixel size and this can give rise to the phenomenon of peak locking, which is discussed later on.

The image pairs obtained by the CCD camera were cross correlated with the software PivView3C. The image is divided in a adjustable number of interrogation areas, which contain a number of pixels and cross correlated to an interrogation area on the other image. This cross-correlation gives per interrogation area $y$- and $z$-displacements. On the one hand, the interrogation areas should be as small as possible and hence increase the resolution. On the other hand, when more particles are enclosed in an interrogation area the cross-correlation becomes more accurate. In this experiment the interrogation area was chosen 16$\times$16 pixel$^2$. Since the particles were smaller than the pixel size, this correlation is a correlation of superposed particle intensity profiles and one can argue about the validity of using Gaussian Peak detectors to find displacements as the individual particles cannot be distinguished. The optimal software settings are discussed in the next section. To examine the influence of these parameter settings were found by using synthetic image pairs, which is discussed in the next section.
3.2 PIV settings and features

In the previous section the PIV procedure was already introduced. In this section results are shown from synthetic and experimental images which serve as a basis for the final parameter setting in the software package PivView3C. The synthetic images were generated with synthetic image software developed by Ad Holten. This program uses a Lamb-Oseen vortex profile as input and generates PIV images which can be evaluated with the PivView3C software program. The focus is on discovering biases, which could create additional vorticity, and therefore circulation profiles are calculated and presented. The procedure of determining the circulation and other parameters is discussed in section 3.3. Here, we focus on features observed during the PIV measurements and in previous experiments in the same facility by Kroes [34], Ren [36] and Abadi [39].

3.2.1 Particle size and seeding

As was mentioned in the previous section the mean particle diameter is approximately 4 µm. The small size and aerodynamic properties of the particles were necessary for the particles to follow the mean flow [34]. Since the pixel size was much larger, determining the exact positions and therefore displacement of particles may be troublesome. The images of the particles were also smaller than one pixel and the Gaussian peak detection method might not work well at subpixel level. This effect can bias the displacements toward integral pixel values. This phenomenon is generally referred to as ‘peak locking’. Since the velocities in the outer regions are very small, the peak locking could have significant influence on the circulation. Peak locking was introduced to synthetic image software by setting the particle diameters at approximately 2 to 5 µm and by keeping the seeding low. In figure 3.2, the histogram at subpixel level is given, which shows the peak locking clearly. The circulation profile shows no bias (deviation from Lamb-Oseen circulation profile (black line)) however.

It is interesting to note that peak locking is not only dependent on the particle size, but also on the seeding. Increasing the seeding, decreases the effect of peak locking. Apparently, large clouds of particles increase the subpixel peak detection. Increasing the seeding has also another positive effect, since it increases the amount of light scattered, though too much seeding decreases the contrast.

3.2.2 Rankine-like vortices

An interesting feature, which is found to be related to the seeding, is the phenomenon of the ‘square vortex’ [34] [36] [39]. The ‘square vortex’ refers to the fact that some PIV measurements show edged structures inside the vortex core. Investigation of this phenomenon, in both experiments and simulations, shows that this effect is caused by insufficient resolution in the core.

Insufficient resolution in the core can occur in a number of ways: too little particles, too much particles or too long delay times. The common feature is that the correlation produces outliers, which are replaced by velocity vectors that are interpolated between the outer vectors. The azimuthal velocity profile is cut off before its maximum and the interpolated vectors form a core in solid body rotation. Figure 3.3 shows the velocity profile of a Lamb-Oseen vortex based on synthetic images with an ill chosen delay time. The resulting profile is Rankine-like and has a lower maximum azimuthal velocity than the actual azimuthal velocity.
Figure 3.2: Peak locking can be observed in (a) histograms of subpixel displacement, which are biased to an integral pixel value. In this case the peak locking was large, but it did not affect the circulation profile of the Lamb-Oseen vortex (with $\Gamma_0/(U_a c) = 0.39$) which was created with the synthetic image software. The Lamb-Oseen vortex profile is indicated with a black line.

profile.

Figure 3.3: The velocity profile of a Lamb-Oseen vortex generated with the synthetic image software at an unsuitable delay time. At a certain radius the correlation does not produce valid vectors and hence the vectors are interpolated resulting in a Rankine-like velocity profile. The black line shows the correct Lamb-Oseen velocity profile as a reference.

To distinguish the valid and interpolated vectors, the software program PivView3C uses color indicators as shown in figure 3.4. The green vectors are valid, the blue vectors are related to lower order peaks and the orange vectors represent interpolated vectors. If the correlation does not result in valid vectors, lower order correlation peaks are validated. When the lower order peaks also produce outliers, the vector is interpolated. The example in figure 3.3 shows that care should be taken when calculating parameters in the vortex core.
Figure 3.4: The quality of the cross correlation can easily be checked by color validation as in (a). The green vectors refer to valid highest order correlation peaks. When a vector is marked as an outlier, the software searches for valid lower order peaks (blue vectors). When no correlation can be found the vectors are interpolated (orange vectors). The azimuthal velocity profile (b) can then be assumed to be valid and the maximum azimuthal velocity can be found. The scatter on the profile at larger distances is probably due to the small displacements and small delay time used to visualize the core.

The resolution in the core can be optimized by seeding the core with smoke, by reducing delay times (30 - 50 µs), by downsizing interrogation areas (16x16 pixel²) and by decreasing laser sheet thickness and thereby increasing its intensity. A larger laser sheet intensity creates a better signal to noise ratio. The values between brackets denote the optimal values for these experiments. Reducing these values further results in an increase of noise. Figure 3.4 shows that the core could be resolved with these PIV settings, in contrast to other techniques as Laser-Doppler-Velocimetry and Hot-Wire-Anemometry [10].

3.2.3 Background noise and axial velocity

The presence of noise was already mentioned above, but is discussed more elaborately in this section. Two types of noise can be distinguished, the background or measurement noise and noise due to the processing of the data. The latter is discussed later on, but here we focus on the background noise. The background noise of the images also consists of two parts. The first part arises from the electrical noise in the CCD of the camera or deviations in the intensity of the scattered laser light. This noise may change the overall level of intensity or it may create random fluctuations in the intensity. The background random noise increases the number of outliers, but also reduces peak locking as can be seen in figure 3.5. The noise was not found to create a bias when evaluated with PivView3C, as can be seen in figure 3.5.

The other part of the background noise is due to the mean flow of which the axial velocity would be the most likely source. In wind tunnel experiments the axial velocity was
not uniform and as such contributes scarcely to the circulation as can be seen in figure 3.6. In that figure, the circulation is given when there is no wing in the wind tunnel and the camera is at an angle of 9.6° with respect to the flow direction. Therefore, the axial flow can be assumed uniform and its contribution to the circulation can be neglected.

Figure 3.6 also rejects the possibility of having a biased electrical noise on the CCD or noise due to the laser (difference in light pulses). This narrows possibilities for biases in the measurements to the camera like deformation of the images.

### 3.2.4 Camera angle and deformation of images

The camera might thus be another source of disruptions, because it can influence the velocity field or cause a bias. The camera was put under an angle with the laser sheet to avoid interaction with the vortex. Interaction can cause shock waves that influence up stream conditions. The camera itself was put at least 1 m behind the laser sheet and mounted on an airfoil standard to reduce up and downstream disruptions of the tunnel flow. At 0.30 m in front of the camera, no change in velocity field was measured.

The angle between the camera and the laser sheet causes the upper and lower side of the image to be slightly out of focus. Since, the angle was small (between 4 and 10 degree) no Scheimpflug correction was made. The Scheimpflug correction basically sets the CCD plate parallel to the laser sheet, so the whole laser sheet is in focus. The extra tool that was available for this correction was coarse and might have influenced the tunnel flow.

Because the camera was put under an angle with the laser sheet, the image of the vortex is deformed. These images can be dewarped by PivMap software. The dewarping was not found to have significant influence on the circulation and thus it is not likely that image deformations bias velocity vectors. The circulation profiles for the same image, dewarped and undewarped, are shown in figure 3.7 and only small differences are observed.

### 3.2.5 PivView3C software settings

In addition to the measurement settings, the settings of the processing software also influence the results obtained. Some settings have already been briefly discussed, but here a complete review is given. The software cross-correlates two images. It divides the image into small areas, which are called interrogation areas and the cross correlation of these areas gives a displacement. In combination with the magnification and the delay time, horizontal and vertical velocities are determined for each interrogation area.

In the present study interrogation areas of 16x16 pixel$^2$ were used. The smaller the interrogation area the larger the resolution of the image, but if the interrogation area is too small, there is too little intensity profile (too little particles) for a good correlation. One way to increase the resolution is to let interrogation areas overlap. When considering velocities only, this procedure is useful, but care should be taken when determining derivatives or integrals (like vorticity and stream function), since vectors become biased [37]. Therefore, no overlap was used for the evaluation of the results. The number of interrogation areas is determined by the region of interest (part of the image evaluated). The region of interest usually consisted of the whole image, but was adjusted when correlations in the upper or lower part were bad. This could be due to insufficient light scattering due to the size of the laser sheet (for the lens with $f=28$ mm) or the fact that the camera was put under an angle with the laser sheet and these parts were out of focus.
To evaluate the picture a multipass algorithm was used with three iterations. This algorithm calculates the displacement of each interrogation area and uses this information as input for the next iteration. In the next iteration, the correlation planes are shifted by an integer number of pixels (calculated in the last iteration) to obtain more matched particles and a more accurate displacement. As peak detection, the least-squares Gaussian fit was used. This detection system uses the eight surrounding neighbor pixels to find the displacement from the intensity peaks at subpixel level using a Gaussian fit. Due to the size of the particles, there is no physical argument for using the Gaussian fit. Still, this peak detection method gave the least scatter on circulation profiles of simulated data.

One of the most important settings is the outlier detection system. This system determines whether a correlation is valid or not. Two criteria were used to validate the vectors: the first is a maximum displacement and the second is a maximum displacement difference. The axial velocity was used as the maximum velocity, which was about 16 m/s. At $f=105$ mm and $\tau =50$ $\mu$s (or $f=55$ mm and $\tau=100$ $\mu$s), this gives a maximum displacement of approximately 7.5 pixel. The displacement difference between two adjoining interrogation areas is not likely to be larger than 3 m/s, which corresponds to displacement differences of 1.5 pixel. When a vector does not satisfy these conditions it is labeled as outlier. In that case, the program searches for lower order correlation peaks and tries to validate these. When there are no valid lower order correlation peaks, the vector is determined by linear interpolation from its neighboring interrogation areas.

Before the images were loaded into PivView3C, they were dewarped with the software program PivMap. PivMap used an image of a picture of known dimensions as calibration to deform images. As calibration image a grid with white dots spaced 15 mm was used, positioned at the location of the laser sheet and the magnification was obtained in this procedure.

With the settings above the velocity field was obtained from the image pairs and processed. This velocity field was used to calculate several properties of the vortex. The calculation and determination of these quantities is dependent on the settings discussed in this section. Therefore, in the next section the post-processing is discussed.
Figure 3.5: Random noise increases the intensity of a pixel on the CCD camera by a random number. The random noise can be simulated with the synthetic image software. The contribution of the synthetic random noise reduces the peak locking as can be seen in (a) and (b), since the subpixel displacement is better spread. In the azimuthal velocity profiles (c) and (d), the synthetic random noise causes more outliers and increases the overall spread at the smaller velocities. This does not create a bias as can be seen in (e) from a histogram of the azimuthal velocities in the interval $0.55 < r/c < 0.6$. 
Figure 3.6: Circulation measured in the wind tunnel in absence of the airfoil. Even though the camera is put under an angle of $\alpha_{\text{cam}} = 9.6^\circ$, the circulation is negligible and the background can be assumed uniform.

Figure 3.7: The circulation determined over square contours over an area of $0.107 \times 0.107 \, m^2$ ($\Delta y = \Delta z = 1.7 \, \text{mm}$ grid point distance) for an image with camera angle of $9.8^\circ$, before and after it was dewarped with a calibration image. The dewarping has no significant influence on the circulation.
3.3 Post-processing

As was explained in the previous section, a recording consists of a number of image pairs. There is a delay time $\tau$ between the images and every image pair is evaluated with the software program PivView3C. This way a velocity field is obtained on a grid, which can be further processed to determine vortex properties as is shown below.

3.3.1 Vortex center

An important parameter that needs to be determined is the center of the vortex. The center of the vortex is important for examining the phenomenon of vortex wandering, but has also significant influence on the azimuthal velocity profile and related quantities. For determining the circulation, the center of the vortex does not have to be known exactly, but ideally the circulation is determined at circles around the vortex center.

There are several ways of determining the vortex center. One can use the vorticity, $\omega$, which is the derivative of the velocity field as well as the stream function, $\psi$, which is an integral of the velocity field. The vortex center can be defined as the position of the center of mass or as the maximum of either the vorticity or the stream function. The center of mass method uses all velocities on the grid, but since a distinction is made between the core and the outer region in terms of delay times and lenses, this method does not give the vortex center accurately as can be seen in figure 3.8. A better way of determining the vortex center is by determining the position of either the maximum vorticity or the maximum stream function.

Because the stream function is an integral quantity, it is more advantageous to use than the vorticity. One should pay special attention when using the stream function, since in practice the flow is not incompressible and therefore the divergence of the velocity is not equal to zero. Nevertheless, the maximum of the stream function provides accurate azimuthal velocity profiles when the resolution of the core is high (i.e. correlations in the core can be found). Therefore, when determining the center of the isolated vortex, the maximum of the stream function was used. In case of the double wing-tip vortices, the maximum and minimum of vorticity were used. The dipole velocity in between the vortices caused the extrema of stream function to show more scatter than the extrema of vorticity.

The disadvantage of using extrema is that their positions are fixed on a grid point, but this can be solved by applying a center of mass method using the nearest neighbors of this maximum. However, this resulted in more scatter at the small radii, and therefore the centers of the vortices were determined using the extrema of stream function (isolated vortex) and vorticity (vortex pairs). In figure 3.8, an azimuthal velocity profile obtained from a wind tunnel experiment is plotted for the four ways of determining the vortex center. The scatter in the tail of the vortex is caused by the fact that the delay times to resolve the core were short, making it difficult to accurately determine the small displacements in the outer region. In appendix A, this can be seen more clearly, where results are shown using different delay times.

3.3.2 Azimuthal velocity

When the vortex center is defined, it is straightforward to calculate the azimuthal velocity from the horizontal and vertical velocity. Transforming from grid to polar coordinates $(r, \phi)$,
the azimuthal velocity becomes

$$v_{\theta} = w \cos(\phi) - v \sin(\phi).$$  \hspace{1cm} (3.3)
Figure 3.9: Simulation of the azimuthal velocity profile of a Lamb-Oseen vortex. The vortex center is determined by the position of maximum stream function. The presence of a uniform axial flow produces a background flow on the azimuthal velocity, when the vortex center does not coincide with the image center. This background flow is small, even for very large displacements of the vortex center and large delay times as can be seen by comparing (a) vortex in the center of the image with (b) and (c) vortex 1 × 1 cm² displaced at delay times $\tau = 50$ and 100 $\mu$s.

Another issue that may significantly influence the azimuthal velocity is the fact that the camera was not put perpendicular to the laser sheet. This was done to avoid interaction between the vortex and the camera and to avoid upstream disruptions. Putting the camera at an angle $\phi_{cam}$ to the axial flow, causes the axial flow to appear as a downward velocity of magnitude $w_D = U(x) \tan(\phi_{cam})$. All images were corrected for this downward velocity. Non-uniformities in the background flow might cause additional effects, but these are assumed to be small.

For the vortex pair, the azimuthal velocity profiles cannot be determined in the ways described before. In this case, a cross section through the centers of the vortices is a more convenient way to proceed. One should note, however, that due to the relatively large viewing area of the camera, the resolution in the core is low. In addition, the velocities decay very rapidly $\sim 1/r^2$ outside the vortex cores and therefore the delay times should be large to detect the velocities in the outer regions. These delay times decrease the core resolution.
even more.

### 3.3.3 Core radius and evolution

A quantity related to the position of the vortex center and the azimuthal velocity is the radius of the vortex core. We define the vortex core radius, \( r_1 \), as the radius where the azimuthal velocity is maximum. Due to scatter on the azimuthal velocity profile, the radius cannot accurately be determined as the point where the azimuthal velocity is maximum. Near the radius of maximum azimuthal velocity \( dv_\theta/dr \) becomes zero, so that the uncertainty in determining the radius is large, even when averaged over a number of image pairs. For that reason the azimuthal velocity profile was least-square fitted with a model velocity profile, which reduced the deviation of the radius over all image pairs significantly. Since the core is expected to be determined by viscous forces, the Lamb-Oseen vortex profile was used to fit \( r_1 \) on the interval \( 0 < r/c < 0.3 \).

The evolution of the radius of maximum velocity, can be obtained by measuring at several distances behind the airfoil. Assuming a Lamb-Oseen vortex profile, the radius of maximum velocity is given by

\[
\begin{align*}
  r_1^2 & \approx 5.02 \nu_{eff} t \approx 5.02 \nu_{eff} \left( \int_{xa}^{xa+\Delta x} \frac{dx}{U(x)} + t_S \right), \quad (3.4)
\end{align*}
\]

where the evolution of the velocity, \( U(x) \), is given in eq. (3.1). The parameter \( t_S \) is the theoretical decay time for a point vortex to the trailing edge. The integral of the axial flow becomes

\[
\begin{align*}
  t & = \int_{xa}^{xa+\Delta x} \frac{dx}{U} \approx 89.3 \frac{0.995 + 0.0224 x_a}{U_a} \left( \frac{\Delta x}{0.995 + 0.0224 x_a} - 1 \right). \quad (3.5)
\end{align*}
\]

For \( \Delta x = 4 \) m, this time interval deviates 2.2% from \( \Delta x/U_a \). The parameter \( \nu_{eff} \) defined by Squire [15] represents the effective viscosity consisting of the kinematic viscosity and an eddy viscosity, \( \nu_{eff} = \nu + \nu_t \). This fit parameter follows from the evolution of both the radius of maximum azimuthal velocity and the maximum azimuthal velocity.

The maximum azimuthal velocity, \( v_{\theta, max} \), is determined by averaging the azimuthal vortex profiles over radius intervals of 50 \( \mu \)m and finding the maximum azimuthal velocity over these averaged values. The deviation of this quantity over all images is much smaller, since \( dv_\theta/dr \sim 0 \). Assuming a Lamb-Oseen vortex, the effective viscosity can be obtained from the expression

\[
\frac{1}{v_{\theta, max}^2} = \frac{26.30 \pi^2 \nu_{eff} t}{\Gamma_0}, \quad (3.6)
\]

when the total circulation \( \Gamma_{\text{total}} = \Gamma_0 \) is determined from the outer region of the velocity profile. Since the maximum azimuthal velocity is obtained independently from the Lamb-Oseen vortex profile, the least-squares fit with eq. (3.6) provides a check on whether the Lamb-Oseen approach for the core is accurate.

### 3.3.4 Circulation

As shown in chapter 2 circulation around a vortex is defined as

\[
\Gamma = \oint_C \vec{v} \cdot d\vec{s}. \quad (3.7)
\]
This quantity can be determined completely independent from the vortex center, but it is more conventional to plot the circulation as a function of $r$. Even though the center of the vortex might not be determined very accurately, this does not influence the circulation at larger distances from the center. The circulation was calculated by summing the velocities along a square contour around the center of the vortex with grid coordinates $(i_c, j_c)$:

$$
\Gamma(k) = \left( \sum_{i=i_c-k}^{i_c+k-1} \frac{1}{2} (v_{i,j_c-k} + v_{i+1,j_c-k} - v_{i,j_c+k} - v_{i+1,j_c+k}) \right) \Delta y + \left( \sum_{j=j_c-k}^{j_c+k-1} \frac{1}{2} (w_{i_c+k,j} + w_{i_c+k,j+1} - w_{i_c-k,j} - w_{i_c-k,j+1}) \right) \Delta z,
$$

(3.8)

where $\Delta y$ and $\Delta z$ are the grid spacings in $y$- and $z$-direction, respectively. The $k$-th contour may be related to a radius by defining a circle with the same area as the square enclosed by the loop:

$$
(2k\Delta y)^2 = \pi r^2,
$$

(3.9)

$$
r = \frac{2k\Delta y}{\sqrt{\pi}},
$$

(3.10)

where we assume that $\Delta y = \Delta z$. An example is given in figure 3.10.

It is more conventional to calculate the circulation at circular contours around the center of the vortex. At a specific point on the circular contour the azimuthal velocity $v_\theta$ is calculated from a center-of-mass interpolation of the four nearest neighbor velocity vectors at the original grid. The circulation can than be determined in a similar way as in eq. (3.8):

$$
\Gamma(r) = 2\pi \frac{r(k)}{8k} \sum_{i=1}^{2k} v_{\theta,(i_c-i-k-1,j_c-k)} + v_{\theta,(i_c+k,j_c-i-k-1)} + v_{\theta,(i_c-i+k+1,j_c+k)} + v_{\theta,(i_c-k,j_c-i+k+1)},
$$

(3.11)

where $(i_c, j_c)$ is again the center of the vortex. Basically, the circulation is determined as an average over the azimuthal velocity at a certain radius. An example is given in figure 3.10. Due to the definition of the contours, the square contours enclose larger areas than the circular contours. Therefore, the square contours enable the calculation of circulation at larger effective radii. Both procedures produce approximately the same circulation profiles as can be seen in figure 3.10 and Appendix B.

The circulation around the vortex pairs is calculated in a similar way as in eq. (3.8). First the centers of the vortices are determined by finding the maximum and minimum of vorticity. The image is cut into two half way between these centers. The circulation is determined by taking rectangular contours around the center of both vortices until it has reached both the left and the right border as shown in figure 3.11.

3.3.5 Dimension scaling and length scales

In order to compare results obtained in the wind tunnel, the parameters should be made dimensionless. The total initial circulation $\Gamma_0$, measured at $\Delta x=0.26$ m, is related to the wing geometry and it is therefore logical to scale this quantity by the product of the flow...
velocity, $U_a$, and the chord length, $c$, which is in fact half the lift coefficient, which is defined as

$$C_L = \frac{L}{1/2\rho U_a^2 c} = \frac{2\Gamma_0}{U_a c}.$$ \hfill (3.12)

When considering the evolution of the circulation, another scaling would be $\Gamma_{\text{total}}(t)/\Gamma_0$, where the circulation is evaluated with respect to the initial circulation. The most logical way to scale the azimuthal velocity is using the axial velocity at the location of the airfoil, $U_a$, which gives an indication of the strength of the vortex in comparison with the flow. Other ways of scaling, like using the maximum azimuthal velocity, $v_{\theta,max}$, can be used to compare vortex profiles. When comparing the azimuthal velocity profiles for different circulations, one can also use the velocity $\Gamma_0/c$.

To characterize the flow, several length scales may be defined (see [10]). For the evolution and interaction between the vortex pair, relevant length scales would include $b$, the separation between the vortices and perhaps even a typical length scale of the turbulence. Other length scales are characteristic length scales of the vortex cores. As was discussed in section 2.1, aircraft vortices are generally assumed to exist of an internal core or viscous core of radius $r_1$ (radius of maximum azimuthal velocity), and an external inviscid core with radius $r_2$ (the position where the total circulation is almost obtained). The vortex profile is likely to be determined by the geometry of the airfoil and the decay of the vortex, but the scaling parameters are not obvious. In this report, the length scale is often normalized on the chord length $c$, but equally valid would be the span $s$, of the airfoil. One could also examine the vortex profile by dividing by $r_1$, and thus investigating the ratio $r_2/r_1$.

For the vortex pair, the initial separation distance between the vortices, $b_0$, might be proposed, however one might argue whether the interaction is already relevant in the early stages of the vortex decay. The scaling remains an interesting problem, which is relevant in order to transform the results obtained, to flight conditions. This is discussed later on. Table 3.2 provides a list of the parameters used in this report.
Figure 3.11: Procedure of determining the circulation for a vortex pair. First the centers of the vortices are determined from the positions of maximum and minimum vorticity and the image is split. Then the circulation is determined by summing velocities along square/rectangular contours as given in eq. (3.8).
Table 3.2: nomenclature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>vortex separation distance</td>
</tr>
<tr>
<td>c</td>
<td>chord length airfoil (=0.075m)</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift Coefficient</td>
</tr>
<tr>
<td>$d_0$</td>
<td>spacing between wing tips</td>
</tr>
<tr>
<td>$D_{eff}$</td>
<td>Cross-diffusion coefficient which estimates the decay of circulation of a vortex pair</td>
</tr>
<tr>
<td>f</td>
<td>focal length camera lens</td>
</tr>
<tr>
<td>L</td>
<td>length test section (=8m)</td>
</tr>
<tr>
<td>r</td>
<td>radius from the center of vortex</td>
</tr>
<tr>
<td>$Re_c = \frac{U_c c}{\nu}$</td>
<td>Reynolds number based on the mean flow around the airfoil and the chord length</td>
</tr>
<tr>
<td>$Re_T = \frac{r_0}{\nu}$</td>
<td>Flight Reynolds number</td>
</tr>
<tr>
<td>$Re_\lambda = \frac{\lambda_T}{\nu} \sqrt{\frac{u'^2}{\nu}}$</td>
<td>Reynolds number characterizing turbulent flows</td>
</tr>
<tr>
<td>$r_2$</td>
<td>fit parameter Jacquin VM2 model where $\Gamma(r_2) = 0.98\Gamma_{total}$</td>
</tr>
<tr>
<td>$r_1$</td>
<td>radius, where azimuthal velocity becomes maximum</td>
</tr>
<tr>
<td>S</td>
<td>standard deviation of center position</td>
</tr>
<tr>
<td>t</td>
<td>time interval from vortex creation (eq. (3.5))</td>
</tr>
<tr>
<td>$t^* = \frac{4r_0}{2\pi v_0^2}$</td>
<td>normalized time interval from vortex creation</td>
</tr>
<tr>
<td>$u'$</td>
<td>velocity fluctuation from mean</td>
</tr>
<tr>
<td>U</td>
<td>mean velocity at position $x$ in wind tunnel</td>
</tr>
<tr>
<td>$v_\theta$</td>
<td>azimuthal velocity of vortex</td>
</tr>
<tr>
<td>x</td>
<td>axial coordinate test section (beginning $x=0$)</td>
</tr>
<tr>
<td>y</td>
<td>horizontal coordinate test section (center $y=0$)</td>
</tr>
<tr>
<td>z</td>
<td>vertical coordinate test section (center $z=0$)</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>angle of incidence / angle of attack</td>
</tr>
<tr>
<td>$\beta$</td>
<td>power exponent fit parameter in Jacquin VM2 model</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>axial distance behind wing (measured from trailing edge)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>turbulence dissipation rate</td>
</tr>
<tr>
<td>$\epsilon^* = 2\pi b_0 \left( \frac{\epsilon_{b_0}}{\epsilon_{\Gamma}} \right)^{1/3}$</td>
<td>dimensionless turbulence dissipation rate</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>circulation</td>
</tr>
<tr>
<td>$\Gamma_{total}$</td>
<td>total circulation determined over the plateau where $\Gamma$ becomes constant</td>
</tr>
<tr>
<td>$\hat{\Gamma}_{total}$</td>
<td>total circulation averaged over image pairs or in case of linking at the plateau in figure 4.14</td>
</tr>
<tr>
<td>$\Gamma_0$</td>
<td>total circulation averaged over image pairs at $\Delta x=0.26$ m</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>Taylor micro scale of turbulence</td>
</tr>
<tr>
<td>$\nu = 1.5 \times 10^{-5}$ m$^2$/s</td>
<td>kinematic viscosity of air [40]</td>
</tr>
<tr>
<td>$\nu_{eff}$</td>
<td>Effective viscosity [15]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>delay time between two matching laser pulses</td>
</tr>
<tr>
<td>$\xi$</td>
<td>frequency</td>
</tr>
<tr>
<td>$\xi_s$</td>
<td>sample frequency of the camera (=15 Hz)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>vorticity</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>initial value, in measurements determined at $\Delta x=0.26$ m</td>
</tr>
<tr>
<td>a</td>
<td>value at the location of the airfoil (center)</td>
</tr>
<tr>
<td>max</td>
<td>maximum value of parameter</td>
</tr>
<tr>
<td>r</td>
<td>in $r$-direction</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square value of parameter (e.g. $u_{RMS} = \sqrt{u'^2}$)</td>
</tr>
<tr>
<td>x</td>
<td>in $x$-direction</td>
</tr>
<tr>
<td>y</td>
<td>in $y$-direction</td>
</tr>
<tr>
<td>z</td>
<td>in $z$-direction</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>ensemble averaged value of a sample $p$</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>fluctuation component of a parameter</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>parameter $p$ averaged over image pairs</td>
</tr>
</tbody>
</table>
3.4 Characterization of turbulence

Turbulence in a wind tunnel is often generated by a grid, and experiments have shown that the grid-turbulence fields are approximately isotropic [41]. The boundary layers on the wall are thin and therefore have little influence on the flow. Behind each grid bar a vortex street is created, which coalesces downstream with other vortex streets to form a turbulent field, provided that the grid parameters are sensitively chosen. Grid turbulence cannot be fully homogeneous as it decays in the downstream direction of the flow. Under the assumption of Taylor’s hypothesis, different locations in the wind tunnel correspond to homogeneous isotropic turbulence at different times [42]. In reality isotropic, homogeneous flows are almost never encountered [43].

To investigate the homogeneity and isotropy of the wind tunnel, measurements of the turbulent field were made with a single hot wire. The hot wire was oriented perpendicular to the axial direction and therefore basically measures the axial velocity over a specific time interval. The essential part of the hot wire is a metal rod with a temperature dependent resistance. By applying a voltage over this rod, it becomes hot and is cooled down by the flow causing its resistance to change. A feed back control circuit adjusts the voltage to keep the current constant. The change in voltage is recorded and translated to a velocity. This way, the fluctuating axial velocity component is measured in time.

The turbulent flow may be characterized by the turbulence intensity, \( q \), and turbulent dissipation rate, \( \epsilon \), which are defined in Einstein-summation-convention as [27]:

\[
q = \frac{1}{2} \frac{u_i'}{\bar{u}^2}, \tag{3.13}
\]

\[
\epsilon = \nu \left( \frac{\partial u_i'}{\partial x_j} \right)^2, \tag{3.14}
\]

where we have used the Reynolds decomposition to characterize the flow by an ensemble average, \( \bar{u}_i \) and a fluctuating part \( u_i' \). For homogeneous, isotropic turbulence it can be shown that these quantities become [27]:

\[
q = \frac{3}{2} \frac{\bar{u}^2}{\bar{u}^2} = \frac{3}{2} \frac{u_{RMS}^2}{\bar{u}^2}, \tag{3.15}
\]

\[
\epsilon = 15 \nu \frac{\bar{u}^2}{\lambda_T^2} = 15 \nu \int_0^\infty k^2 F(k) dk, \tag{3.16}
\]

where \( u' \) is the axial fluctuating component of the velocity, \( \lambda_T \) the Taylor micro scale, \( k \) the wave number and \( F(k) \) the one-dimensional space spectrum defined by \( \bar{u}^2 = \int_0^\infty F(k) dk \) (points are only correlated along the x-axis). The wave number \( k \) is related to the sample frequency \( f_S \) through the Taylor hypothesis, \( k = 2\pi f / \bar{u} \), assuming homogeneity of the turbulence.

3.4.1 Measurements in tunnel without grid

Measurements of the axial velocity in a wind tunnel without the grid show that the axial velocity fluctuates about 1% about its mean value with a frequency of about 1 Hz, see figure 2.

---

\(^{2}\)Measurements and data-processing by Ad Holten, technician at the Fluid Dynamics Laboratory in Eindhoven.
3.12. This fluctuation is quite large for a wind tunnel. The turbulence level could not be measured well, since the noise arising from the hot wire itself was dominant. Therefore, we could not determine the exact dissipation rate, but an integration of the one-dimensional space spectrum (eq. (3.15)) over the same wave number range used for the integration with grid gives the upper limit $\epsilon \approx 2 \times 10^{-6}$ m$^2$ s$^{-3}$. The turbulent fluctuations in terms of $u_{RMS}$ are around 0.5% of the mean axial velocity $\bar{u} = U$.

### 3.4.2 Measurements in tunnel with grid

![Figures 3.12 and 3.13](image)

Figure 3.12: Axial velocity fluctuation in the wind tunnel measured with a single hot-wire for a time interval of 20 s.

Figure 3.13: Several configurations of the stationary grid with different transparencies relative to the transparency of the open grid, viz. (a) 1.00, (b) 0.94, (c) 0.85 and (d) 0.71.

The grid with mesh size $M=0.10$ m is located at the entrance of the test section and has agitators with dimensions $0.067 \times 0.067$ m$^2$ on both its horizontal as vertical bars, see figure 3.13, which can be operated in several modes. In the active modes the agitators are displaced by the rotating bars and hence change the transparency of the grid (open part of cross-section) in time and position. However, these modes increase the turbulence and decrease the mean velocity of the tunnel significantly. For that reason the active mode was unsuitable for vortex measurements. Most measurements were done in the stationary mode, with maximum
transparency (see figure 3.13a). For the largest tip spacing, \(d_0=0.10\) m, the grid was also used with transparencies 0.94, 0.85 and 0.71 relative to the maximum transparency (see figures 3.13b, c and d). Characteristics of these modes can be found in appendix D, but in this section the focus is on the stationary grid with maximum transparency, referred to as the open stationary mode.

In figure 3.14, the one-dimensional spectra \(F(k)\) are shown for the open stationary mode and an active mode. In the latter case the agitators are moving with random velocity in a random pattern. The spectra are determined at several distances behind the grid. Due to the larger length scales involved in the active mode, the inertial range is significantly longer. The length of the inertial range and thus the height of the plateau decays with increasing distance, suggesting that the larger length scales transfer their energy to the smaller length scales in time. As \(k\eta \rightarrow 1\), the inertial range passes into the dissipating range, where the spectra decay exponentially. Comparing these spectra and their corresponding Reynolds number \(Re_\lambda = \lambda_T \sqrt{\frac{u'^2}{\nu}}\) with well known results (which define \(k_D = 1/\eta\)) [42], the results obtained by the open stationary grid seem close to that found in a shear flow (\(Re_\lambda = 130\)), while the results obtained by the active grid seem to be close to measurements of a wake behind a cylinder (\(Re_\lambda = 308\)). Note that \(0.015 < k_0\eta = 2\pi\eta/d_0 < 0.085\), for \(d_0=0.03\) m, 0.05 m, and 0.10 m, which is approximately within the inertial range. This was used as an assumption in the lifespan formula of Crow and Bate [32], eq. (2.32).

![Figure 3.14: One-dimensional spectra for two modes of the turbulence generating grid measured at several distances behind the grid. The quantities \(\epsilon, \nu, \eta\) denote the dissipation rate, kinematic viscosity and the Kolmogorov length scale \(\eta = (\nu^3/\epsilon)^{1/4}\), respectively. For the open stationary grid, \(0.01 < 2\pi\eta/M < 0.025\), where \(M=0.10\) m is the mesh size.](image-url)
The decay of the turbulence intensity and the turbulent dissipation rate for the stationary grid can be found in figure 3.15 and in table 5.1 in Appendix D. The decay of the quantities is close to what is predicted in literature for homogeneous turbulence \[44\]:

\[ q \propto \left( \frac{x}{M} \right)^{-1.3}, \]  

(3.17) 

\[ \epsilon \propto \left( \frac{x}{M} \right)^{-2.3}. \]  

(3.18)

The parameter \( M \) denotes the mesh size of the grid, which is \( M=0.10 \) m. The evolution of both quantities in figure 3.15 seems close to the predicted decay, indicating that the turbulence is approximately homogeneous from \( x=1.5 \) m \((x/L=0.19)\) downstream of the grid.

\[ q = \frac{3}{2} \frac{u'^2}{\bar{u}^2} \] 

\[ \epsilon = \frac{x}{M} \]

Figure 3.15: Evolution of (a) turbulence intensity \( q \) and (b) eddy dissipation rate \( \epsilon \) as a function of the distance behind the grid, \( x \). The grid is located at the entrance of the test section.

To verify whether the flow is isotropic one should ideally measure all three flow components, but in this case only one was measured. A widely used quantity to examine the isotropy and homogeneity of a flow is the skewness, which is defined as:

\[ S = \frac{1}{n} \frac{\sum_i^n u_i^3}{\left( u'^2 \right)^{3/2}}. \]  

(3.19)

This quantity gives information about whether the distribution of the fluctuating component is symmetric \((S=0)\). Even though this is not the ideal way of examining the flow, for a single hot wire measurement it is useful. For an interesting discussion on the skewness see Tresso and Munoz \[43\]. The skewness is shown for the stationary grid in figure 3.16 for several distances behind the grid. At \( x=3.5 \) m \((x/L=0.44)\), the skewness is below 10\%, which is low in comparison to the results by Tresso and Munoz \[43\] and similar to values found by Mouri \textit{et al.} \[45\] at larger distances from the grid. The skewness is positive suggesting that the positive deviations from the mean are spread over a wider range. At the smallest distance behind the grid, the skewness is largest indicating that the turbulence might not yet be approximately homogeneous.
At this small distance behind the grid, the geometry of the grid significantly affects the vertical profile of the axial velocity as is shown in figure 3.17. The crests in the velocity profile correspond to the locations of the grid bars. At $x=3.5$ m the profile is smoothed, suggesting a fully developed turbulent field. Close to the upper tunnel wall the boundary layer seems to coalesce with the wake behind the upper grid bars. These results suggest that from $x=3.5$ m, the turbulence is close to homogeneous and isotropic.

In addition it is useful to consider the parameters relevant for investigating the effect of turbulence on double wing-tip vortices [32], which were introduced in section 2.3. That is,

$$
\epsilon^* = \frac{2\pi \epsilon^{1/3} b_0^{4/3}}{\Gamma_0},
$$

(3.20)
The first parameter describes the relative strength of turbulence velocity fluctuations in comparison with the downward induced velocity from the mutual interaction. Four regimes are distinguished in numerical simulations [46], see table 3.3. In figure 3.18a, the dimensionless turbulent dissipation rate $\epsilon^*$ is shown as a function of the distance from the grid for three wing-tip spacings, i.e. $d_0=$0.03 m, 0.05 m and 0.10 m. The very high turbulent regime is not likely encountered in nature [32], and therefore the airfoils should be mounted relatively far away from the grid. On the other hand, the evolution time of the vortex, $t^*$, which is related to the distance behind the airfoil, should be significantly large. It should be as large as the life time predicted by Crow and Bate for weak turbulence, $T^*=5$ [32] [46]. This requires the airfoil to be as close to the grid as possible. The maximum value that can be obtained for the evolution time is depicted in figure 3.18b for the same three tip spacings, under the assumption that the last location for measuring is at $x=6$ m. During our investigation without grid it was found that at $t^*=12$ instabilities already occur, so measuring for larger $t^*$ is not useful. The optimum location is to mount the wing at $x=3.5$ m, since three regimes of turbulence (low, medium, high) are covered and for both $d_0=0.03$ m and 0.05 m, $t^*>5$ can be obtained.

Figure 3.18: The decay of the dimensionless dissipation rate, $\epsilon^*$, as a function of the distance behind the grid (a) and the maximum obtainable lifetime $t^*$ as a function of the location of the airfoil, $x_a$. The curves are shown for three wing tip spacings (≈ vortex separation distance $b$), i.e. $d_0=$0.03 m, 0.05 m, and 0.10 m where $U_a=16$ m/s and $\Gamma_0=0.6$ m$^2$s$^{-1}$.
In conclusion, measurements of turbulence suggest that from $x=3.5$ m the turbulence is in good approximation isotropic and homogeneous for the stationary grid, and estimates of $t^*$ and $\epsilon^*$, show that $x=3.5$ m is a good location to mount the wing giving levels of sufficiently low turbulence and sufficiently long evolution times. Therefore, the investigation of the effect of turbulence on the evolution of the wing tip vortices was made with the wings mounted at $x=3.5$ m.
Chapter 4

Results

Now that the experimental set up and procedure, as well as the processing of the velocity field, are described in the previous chapter, we proceed by presenting the vortex properties and evolution obtained from the measurements. For this purpose, the chapter is divided into four sections. First, the isolated vortex shed from a single airfoil in a low turbulent environment is described. To investigate the influence of turbulence on the evolution of a vortex, the isolated vortex in a strong turbulent environment is studied in the subsequent section. Mutual interaction between vortices gives rise to different behavior and therefore the third section deals with a vortex pair shed from two airfoils in a low turbulent environment. Triggering effects of turbulence are discussed in the last section, which considers the vortex pair in a strong turbulent environment.

4.1 Isolated vortex without grid turbulence

The isolated vortex in a low turbulent environment serves as the reference case. Measurements of the instantaneous velocity field were made at distances $\Delta x = 0.26$ m, 0.98 m, 3.82 m and 4.72 m downstream of the airfoil with angle of attack $\alpha = 7.5^\circ$. At every distance, a number of image pairs (typically 50 - 200) were captured with frequency $\xi_s = 15$ Hz. For the isolated vortex, distinction is made between two regions. In the core region, delay times ranging from 30 $\mu$s ($\Delta x = 0.26$ m and 0.98 m) to 50 $\mu$s ($\Delta x = 3.82$ m) and a lens with focal point $f = 105$ mm were used. In the outer region, a delay time of 100 $\mu$s and a lens with focal point $f = 55$ mm were used. The evolution in time was examined at several locations downstream of the airfoil and at a fixed location for a certain time interval. The latter was used to study the wandering of the vortex core.

4.1.1 Core of the vortex

The phenomenon of vortex wandering is a concern in wind tunnel experiments, especially when measuring with fixed probes or hot wires (e.g. [10] [47]). Four explanations are considered by Jacquin et al. [10]: (1) Interference of the vortex with wind tunnel unsteadiness, (2) excitation of perturbations of the core by turbulence in the surrounding shear layer (as it rolls up around the core), (3) co-operative instabilities (such as the Crow instability) and (4) propagation of unsteadiness from the model. For an isolated vortex, co-operative instabilities can be neglected.
In figure 4.1, the vortex core positions are given at several distances behind the airfoil. The vortex core positions are defined as the positions of maximum stream function. The

\[
\Delta x = 0.26 \text{ m (250 image pairs)}
\]

\[
\Delta x = 0.98 \text{ m (249 image pairs)}
\]

\[
\Delta x = 3.82 \text{ m (449 image pairs)}
\]

\[
\Delta x = 4.72 \text{ m (149 image pairs)}
\]

Figure 4.1: The panels above show vortex core positions of a single wing-tip vortex at several distances \(\Delta x\) downstream of the airfoil. The vortex core is defined as the position of maximum stream function. The coordinates of the core are given in the horizontal plane (with distance between grid points \(\Delta y = \Delta z = 1.7 \text{ mm}\)), while the number of counts are given in vertical direction. Every count represents an image pair and the frequency of the counts is 15 Hz.

square boxes in figure 4.1 represent the grid of interrogation areas whereas the bars indicate the number of counts at a certain grid point. Although the time interval is restricted, the vortex core position does not deviate much from its mean position, though the deviation seems to increase (slightly) with the distance behind the airfoil. This suggests that unsteadiness arising from the airfoil or the wind tunnel are small and that the turbulence in the shear layer is low.

The azimuthal velocity profiles, which are shown in figure 4.2a, are averaged over the corresponding time interval and over radial intervals of length 50 \(\mu\text{m}\). Typical profiles obtained from a single image pair are shown in figure 3.8c and Appendix A for several
The radius of maximum azimuthal velocity $r_1$ and the maximum azimuthal velocity $v_{\theta,\text{max}}$ were determined as described in section 3.3 for the same distances behind the airfoil and their values were averaged over the corresponding time interval of measurement (see figure 4.1). Assuming an (effective) viscous decay, it is possible to determine an effective viscosity based on the time evolution of the radius and the maximum azimuthal velocity as is shown in figure 4.2. The error bars correspond to the standard deviation of the corresponding quantities.

![Graphs](image)

(a) azimuthal velocity profiles  
(b) evolution core radius  
(c) evolution azimuthal velocity

Figure 4.2: The azimuthal velocity profiles are shown in panel (a). The squared radius $r_1^2$ (b) and the squared maximum azimuthal velocity $v_{\theta,\text{max}}^2$ (c) as a function of the distance behind the airfoil (averaged over 250, 249, 449 and 149 image pairs). The solid lines correspond to a least-squares fit which yield effective viscosities of $\nu = 2.4 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ and $\nu = 3.7 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, respectively. For air $\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ [40].

The values found for the effective viscosity are relatively close to the kinematic viscosity of air, i.e. $\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ [40], whereas in most turbulence experiments the effective viscosity is typically of the order $10^{-2} \text{ m}^2 \text{ s}^{-1}$ [1] or $10^{-4} \text{ m}^2 \text{ s}^{-1}$ [48]. This suggests that the evolution of the viscous vortex core is close to laminar, which is consistent with other experiments on tip vortices [47].
4.1.2 Outer region of the vortex

The core encloses only a part of the vorticity, and therefore the outer region of the vortex has to be examined as well. This region is often an order of magnitude larger than the core region [1] [10], so that a lens with focal length $f=55 \text{ mm}$ was used. Since velocities are small, the delay time was set at $\tau=100 \text{ } \mu\text{s}$ (limited by the axial velocity). The core could not be resolved with these settings so that the results in the approximate interval $0 < r/c < 0.2$ are not correct (vectors are interpolated) as can be seen in figure 4.3.

Due to the aforementioned small velocities, the best way of representing the vortex is by calculating the circulation as described in section 3.3.4. Typical circulation profiles obtained from single image pairs are shown in Appendix B. Figure 4.3, shows the circulation profiles averaged over a specific time interval (to reduce scatter) at several distances downstream of the wing. Also the high-resolution data corresponding to the vortex core region are included in figure 4.3 so that an overall picture of the circulation profile is obtained. The data obtained in the core region is accurate, but because of the limited area that was investigated, it was not possible to obtain the total circulation. The total circulation can be determined from the data in the outer region. Unfortunately, with the setting used for the outer region it was not possible to resolve the core. This is caused by the fact that with both the settings in the core region and the outer region, only a specific range of velocities can be determined accurately. This is more elaborately explained in section 3.2.2.

At the smallest distance behind the airfoil, $\Delta x=0.26 \text{ m}$ (figure 4.3a), the vortex sheet might not have rolled up yet. Close to the vortex center a region $0 < r/c < 0.2$ is present with strong vorticity, leading to a strong increase of circulation. For slightly larger radii, there is a region of zero vorticity, with constant circulation. This region of zero vorticity is surrounded by a ring of nonzero vorticity which is apparent from the increase of circulation. This could suggest that the roll up process was not finished yet at this distance. At $\Delta x = 0.98 \text{ m}$ (figure 4.3b), the region of strong vorticity is still present, but the region of zero vorticity is almost absent. At $\Delta x=3.82 \text{ m}$ (figure 4.3c) and $\Delta x=4.72 \text{ m}$ (figure 4.3d), the vorticity increases even more gradually, which may be caused by viscous or turbulent diffusion. The total circulation, $\Gamma_{\text{total}}$, can be determined in the plateau region starting at $r/c = 0.8$, and as is expected the total circulation remains constant (within uncertainty) with respect to $\Delta x$. At $\Delta x=0.98 \text{ m}$ and $\Delta x=4.72 \text{ m}$, the circulation profile appears to have no plateau, but this is probably caused by inaccuracy near the edges of the image, due to small velocities and a relatively thin and less intense laser sheet. Both the small velocities and the laser sheet decrease the signal to noise ratio. The total circulation is obtained in these case as the average over the intervals $0.8 < r/c < 1$ and $0.9 < r/c < 1.1$, respectively.

The circulation profiles show the necessity of using a two-length scale model to characterize the vortex for numerical simulations. The core region contains approximately 60% of the vorticity, while the remaining part of the vorticity is spread over a significantly larger region as is shown in figure 4.3. The circulation profiles in this figure do not resolve the core correctly and can therefore not be fitted accurately. To solve this problem, the circulation profiles obtained in the core region and those obtained in the outer region are linked together at $r/c = 0.3$. To obtain an accurate fit, the circulation profiles are determined in a different manner as in figure 4.3. The azimuthal velocity is averaged over radial intervals of $50 \text{ } \mu\text{m}$ (see figure 4.2a), and the circulation is then given by $\Gamma = 2\pi rv_\theta$. This way more points are used in the fit, increasing its accuracy. The best fit was found for the Jacquin VM2 model.
Figure 4.3: The circulation determined over square and circular contours as described in section 3.3.4 and averaged over 49, 48, 4 and 198 images, respectively. The quantity $\Gamma_0$ is the total circulation at the first measurement position behind the wing $\Delta x = 0.26$ m, which is determined as an average over the last 8 points of its circulation profile. In addition the circulation profiles obtained with the lens with focal point $f = 105$ mm and delay times $\tau = 30$ - 50 $\mu$s are shown. Typical single image pair circulation profiles are given in Appendix B.

\[ \frac{\Gamma}{\Gamma_{total}} = \frac{(r/r_2)^2}{((r_1/r_2)^4 + (r/r_2)^4)^{1/4} \left(1 + (r/r_2)^4\right)^{1/4}} \]  

where $\Gamma_{total}$ is determined from averaging the plateau and $r_1$ is found from the analysis of the core region. The fit parameters, $r_2$ and $\beta$, denote the region enclosing almost all vorticity and the power exponent of the azimuthal velocity (for $r_1 < r < r_2$, $v_\theta \sim r^{-\beta}$), respectively.

The strength of the model is shown by the fact that approximately the same values for $\beta$ are found at all stations as is shown in figure 4.4, suggesting that the typical shape of the circulation profile is conserved in time. However, from the figure it is clear that the Jacquin VM2 model overestimates the circulation profile in some regions, so the model does...
not represent the physical situation. The length scale $r_2$ still remains interesting, because it represents the size of the region with nonzero vorticity. However, the parameter is very sensitive to slight changes in $\Gamma_{total}$ and $r_1$. It is therefore difficult to draw conclusions from its evolution. Though it is clear that it is approximately an order of magnitude larger (at $r = r_1$, $\Gamma/\Gamma_{total} = 0.6$). The exponent $\beta$ is also fitted with a power law in two different regimes as is shown in figure 4.4e, which is independent of the Jacquin VM2 model and approximately the same value is found at all distances behind the airfoil. This suggests that this parameter is characteristic for the vorticity distribution and may be related to the lift distribution on the airfoil, because it barely changes in time.

A parameter related to the total circulation is the lift coefficient, $C_L$, which is defined in eq. (3.12). The variation of the lift coefficient $C_L$ with the angle of attack $\alpha$ is characteristic for the wing model. The lift coefficient was obtained at $\Delta x=0.26$ m as a function of the angle of attack, $\alpha$, as is shown in figure 4.5. Also shown in this figure are measurements on the same model provided by Veldhuis [49]. For larger angles of incidence ($\alpha > 6^\circ$), the curves seem to follow a similar trend. The difference between the curves might be due to either viscous effects or the twisted wing tip (see Appendix G). Flow around the airfoil with a smaller Reynolds number causes the viscous boundary layer to be thicker and thus decreases the circulation. This can also be seen by comparing the slopes of the lift curves to the theoretical inviscid Zhukovski airfoil (with no camber). The smaller angles ($\alpha < 6^\circ$) show large lift coefficients in comparison to the Zhukovski airfoil and the data provided by Veldhuis, which is probably due circulation around the twisted tip. Because of this twist, the tip of the airfoil has an angle to the main flow, even if the rest of the airfoil has not. If the angle of incidence becomes larger than $13^\circ$, the boundary around the airfoil separates and wing is said to ‘stall’. The vortex becomes unstable and oscillates significantly.

The previous results serve as a reference for both the isolated vortex in a turbulent environment and the vortex pair. In the next section, the isolated vortex is studied in an environment with strong turbulence.
Figure 4.4: Least-squares fit of the circulation profile with the Jacquin VM2 model, eq. (4.1). The circulation profiles are obtained by linking profiles determined in the core region to those determined in the outer region at $r/c = 0.3$ (dotted line) and are averaged over a time interval. The core region shows less scatter, because it was averaged over a larger time interval than the outer region. In panel (e), the azimuthal velocity profiles are shown for different $\Delta x$ in a log-log plot. The power law exponents are determined with power law fits.
Figure 4.5: The lift coefficient \( C_L = 2\Gamma_0/(Uc) \) at \( \Delta x = 0.26 \) m behind the airfoil as a function of the angle of incidence, \( \alpha \). The solid line shows the relation between the lift coefficient and angle of incidence for a Zhukovski airfoil (without camber). The circulation is determined as an average over 4 or 5 image pairs. For each image pair the circulation was determined over square contours around the vortex center and the mean was calculated over the last 15 contours. The standard deviation was used as weight.
4.2 Isolated vortex with grid turbulence

In order to investigate the behavior of the isolated vortex in a turbulent environment, the turbulence generating grid is put at the beginning of the test section. The airfoil is located at $x=3.5$ m from the grid, where the dissipation rate of the grid turbulence is $\epsilon=1.01$ m$^2$ s$^{-3}$. Again, a distinction is made between the core region, which is studied with $f=105$ mm and delay times of 30 and 50 $\mu$s, and the outer region. The outer region is captured with a lens with $f=50$ mm and delay times of 80 and 100 $\mu$s and is mainly interesting for evaluation of the circulation. In the next subsection, the focus is on the evolution of the vortex core.

4.2.1 Core of the vortex

In a similar way as introduced in the previous section, the evolution of the core can be studied by means of the maximum azimuthal velocity, $v_{\theta,\text{max}}$, and the radius of maximum azimuthal velocity, $r_1$. Azimuthal velocity profiles were obtained for three different angles of attack at four different stations and the corresponding values were averaged over 150 images see figure 4.6. Assuming a diffusive decay, it is possible to determine an effective viscosity as is shown in figure 4.7. Based on the evolution of the radius of maximum azimuthal velocity the effective viscosity for the three different angles of attack is close to the value of the kinematic viscosity of air, $\nu=1.5 \times 10^{-5}$ m$^2$ s$^{-1}$ [40], though the uncertainty in the values is around 20%. This suggest that turbulence inside the core is small, probably due to the high rotational velocities inside the core. The difference in the magnitude of the effective viscosity for $v_{\theta,\text{max}}$ and $r_1$ might be related to the assumption that these parameters evolve similar to the Lamb-Oseen vortex. Still it seems that both with and without grid the vortex core region may be considered a slender rod in solid-body rotation. The values of $r_1$ with grid are smaller than the corresponding values without grid and the former are approximately the same for all three values of $\alpha$. The ratio of the maximum azimuthal velocity $v_{\theta,\text{max}}$ to the circulation $\Gamma_0$ is approximately the same for the cases with and without grid.

It is interesting to investigate whether higher levels of turbulence and lower values of the angle of attack influence the evolution of the core. One would expect that at a certain ratio of the turbulence (e.g. in terms of $u_{\text{RMS}}$) and the vortex strength (e.g. in terms of $v_{\theta,\text{max}}$), the decay of the vortex will be much more rapid as turbulence might enter the core. Perhaps there is a critical ratio which represent a transition where the diffusive decay goes from laminar to turbulent. However, within the current parameter range this could not be verified.

Apparently the external turbulence does not increase the turbulence inside the core, but it can still influence the core movement. In figure 4.8a, the standard deviations of the mean center position in both $y$ and $z$-direction are shown as a function of dimensionless time. The standard deviation significantly increases up to the order of magnitude of the core radius and shows no direction of preference. The center positions are defined as the position of maximum stream function, and as the core region showed similar values for $r_1$ and $v_{\theta,\text{max}}$, the maximum stream function will also be approximately the same. Therefore inaccuracies in the determination of the vortex center due to noise may be neglected. Considering the four explanations of vortex wandering discussed in section 4.1, unsteadiness due to the wind tunnel and the airfoil seem unlikely because these were not found in the case without grid. Therefore, the wandering with grid is probably due to turbulence in the shear layer surrounding the core.
As was shown above, the vortex core can be represented by a slender rod with radius \( r_1 \), and the main effect of the fluctuating turbulent field is to move this rod in arbitrary directions. If the rod can be modeled as a particle cloud in a stationary and homogeneous turbulent field, it can be shown that the standard deviation of the particle cloud becomes for \( t \gg \tau_{turb} \) [3]

\[
S_x = u_{RMS} \sqrt{2\tau_{turb} t},
\]

(4.2)

where \( \tau_{turb} \) is the turbulent correlation time. Because of the shear in the azimuthal velocity the surrounding turbulence is deformed and we anticipate that the turbulent correlation time is coupled to the vortex rotation time \( t_{rot} \sim r_{1,0}^2/\Gamma_0 \). These assumptions can be used to scale the dimensionless time and the radial standard deviation of the vortex center, \( S_R = \sqrt{S_x^2 + S_y^2} \), and the data is plotted in figure 4.8b. The model fits the data well and seems to predict the right correlation. No specific frequency for the oscillation

Figure 4.6: Azimuthal velocity profiles averaged over 250 image pairs and over radial intervals of 50 µm for three different angles of attacks.
Figure 4.7: The squared radius $r_1^2$ (a) and the squared maximum azimuthal velocity $v_{θ,max}^2$ (b) as a function of the distance behind the airfoil (averaged over 150 image pairs). Obtained values of the effective viscosity are close to the kinematic viscosity of air, $ν = 1.5 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$ [40].

could be found, but this might be due to the low sampling frequency $ξ_S \ll t_{rot}^{-1}$. A similar correlation has been observed in previous studies [50] [51].

Figure 4.8: The standard deviations in $y$ and $z$-direction of the mean center position normalized by the core radius at $Δx=0.26$ m, defined as the position of maximum stream function, over 150 images as a function of the distance behind the airfoil in panel (a). In panel (b) the radial standard deviation is depicted as a function of the time normalized by the strength of the vortex $Γ_0$ and the root mean square velocity of the turbulence, $u_{rms}$.

4.2.2 Outer region of the vortex

Core wandering is most likely influenced by the presence of external turbulence outside the core, and therefore the outer region is considered in this section. The region is captured with
a lens with focal point $f=50$ mm and delay times of 80 and 100 $\mu$s. It is possible that due to the turbulence, the airfoil becomes unsteady and that the lift distribution changes. This would drastically alter the vorticity profile and the circulation and thus the lift coefficient. As is shown in figure 4.9a, the lift coefficient with and without grid almost coincide, indicating that the airfoil itself does not become unsteady.

Figure 4.9: (a) The lift coefficient $C_L = 2\Gamma_0/(Uc)$ at $\Delta x =0.26$ m behind the airfoil as a function of the angle of incidence $\alpha$ both with (circles) and without grid (squares). (b) The amount of vorticity in the core for three angles of attack with grid and without grid as a function of time.

Again a large part of the circulation is outside of the core as can be conducted from the ratio of the circulation at the core radius to the total circulation in figure 4.9b. This ratio also shows that the core contains less vorticity in the presence of external turbulence. The circulation profiles can be fit with Jacquin VM2 profiles, defined in eq. (4.1), as was done in section 4.1. Two circulation profiles are linked together at $r/c=0.3$, the first profile determined with $\tau=30$ $\mu$s and $f=105$ mm, while the second profile was obtained with $\tau=100$ $\mu$s and $f=50$ mm. The fits can be found in Appendix C and give the radius $r_2$ and the exponent $\beta$, which are plotted as a function of time in figure 4.10. No clear trend was observed for both $r_2$ and $\beta$, though the exponent seems to decrease a little in time. The mean value of $r_2$ with grid is somewhat smaller than the value without grid, but this can be explained by a somewhat lower total circulation in the case with grid than in the case without grid as can be seen in figure 4.9. The fitting parameters are quite sensitive to the input parameters $r_1$ and $\Gamma_{\text{total}}$ and the uncertainty in the fitted parameters might be even larger than shown.

The uncertainty in the fits makes it difficult to quantify the differences between the cases with and without grid. However, a qualitative comparison can be done by comparing the circulation profiles with and without grid as is shown in figure 4.11. Since the circulation profiles for the three different angles of attack coincide outside of the core, the shape of the circulation profiles is independent of the total circulation. The thick black line denotes the circulation profile without the grid and it is clear that the vorticity is concentrated closer to the core without grid. In the case with grid a more rapid diffusion takes place (see Appendix F), distributing the vorticity over a larger area resulting in more gradual circulation profiles,
though initially (for small $\Delta x$) the circulation profiles are much alike both with and without grid.

The isolated vortex apparently exists of a rapidly rotating vortex core, which is close to laminar, surrounded by a region with a significant amount of the vorticity that is governed by turbulent diffusion of vorticity. The turbulence surrounding the vortex core triggers the wandering of the vortex center. The latter is interesting in relation to cooperative instabilities arising from the mutual interaction between vortices, such as the Crow instability. Therefore, in the next two sections, double wing-tip vortices are considered both with and without turbulence generating grid.
Figure 4.11: For three different angles of attack the circulation profiles (averaged over 100 image pairs) over square (squares) and circular contours (circles) with grid are shown. Also shown are the circulation profiles without grid (thick black line) at the same distance (for $\Delta x=1.78$ m and $\Delta x=2.82$ m, the profile at $\Delta x=3.82$ m is taken). In addition the small black dots denote the circulation profile profile without grid in the core (with $f=105$ mm and $\tau=30$ - 50 $\mu$s). See Appendix C for more information about the fits.
4.3 Double wing-tip vortices without grid turbulence

In this section we consider the dynamics of a pair of wing-tip vortices in an environment with low turbulence. The vortices were created with a split wing configuration, as was shown in section 3.1, with tip separation distances, $d_0 = 0.03$ m, 0.05 m and 0.10 m and angle of attack $\alpha = 7.7^\circ$. In order to be able to capture both vortices at the same time the resolution of the measured velocity field was necessarily reduced by the use of lenses with a smaller focal length. The vortex cores could therefore not be resolved. For that reason only a comparison with the results related to the outer region of the isolated vortex is made.

4.3.1 Instabilities of the vortices

In figure 4.12, the wandering of the vortex centers (defined as the extrema of vorticity) is shown for 50 image pairs ($\Delta t = 3.3$ s) both for the isolated vortex and the vortex pairs. Note that the lower resolution in the core effectively results in a larger deviations from the mean vortex center positions, compared to figure 4.1a. For all cases, the vortex wandering seems alike suggesting that the presence of a second vortex has minor influence on the vortex core in the early stages of the evolution.

As the evolution proceeds (at larger distances behind the airfoil), the wandering effect becomes more pronounced but is still similar in all cases. At $\Delta x = 3.82$ m, however, for $d_0 = 3$ cm, a strong oscillation of the vortex core in a specific direction is observed as is shown in figure 4.13a. Figure 4.13b reveals a strong correlation between the vertical positions of both vortex centers, which is characteristic for the Crow instability as discussed in section 2.2. The cores oscillate with a mean angle of $33^\circ$ to the horizontal, where Crow predicts an

![Figure 4.12: Positions of the vortex centers (extrema of vorticity) for 50 image pairs for (from top to bottom) the isolated vortex and the vortex pair with tip separations $d_0 = 0.03$ m, 0.05 m and 0.10 m, respectively, at $\Delta x = 0.26$ m behind the airfoil. The images were captured with a lens with focal point $f = 55$ mm and puls delay times $\tau = 100 \mu$s. The black squares are located at grid positions and the images were shifted and put into one image for comparison.](image-url)
Figure 4.13: Panel (a) shows the vortex core positions (extrema of vorticity) at $\Delta x = 3.82 \text{ m}$ for a vortex pair with tip separation $d_0 = 0.03 \text{ m}$. The vortex cores oscillate with a mean angle of $33^\circ$ to the horizontal. The left vortex core positions are indicated by squares, while the right vortex cores are indicated by crosses. In panel (b), the vertical positions of the left and right vortex are plotted against. The symmetry of the movement and the vortices is shown by the fit with slope 1.

angle of $48^\circ$ [2]. The difference in angle would suggest that the instability has a different wave length than that predicted, $\lambda = 8.6 b_0$, which corresponds to a frequency of $\xi = U/\lambda = 62 \text{ Hz}$. Because the sampling frequency $\xi_S$ is 15 Hz, the instability was undersampled and the frequency could not be determined. To increase the sampling frequency, either the axial velocity should be decreased, but then also the strength of the vortices is decreased (increasing the life time of the vortices), or the separation distance should be increased, but then the life time of the vortex is increased as well (probably beyond the length of the test section). In addition, the disagreement may also be attributed to the low core resolution. The vectors in the core are interpolated which may lead to a direction of preference [34].

The overlap of the vortex core positions suggests that vortices ‘link’. This is supported by figure 4.14a, which shows the total circulation around a single vortex evaluated in a plane perpendicular to the $x$-axis, $\Gamma_{\text{total}}$, as a function of the instantaneous vortex separation, $b$. Since for a closed box around one of the vortex tubes, we must have $\Gamma_{\text{in}} = \Gamma_{\text{out}}$ (see figure 4.14b), it is clear that the decrease in $\Gamma$ in a plane perpendicular to the $x$-axis implies linking between the vortex tubes. It can be shown that the linking in terms of the total circulation around a single vortex is mathematically equivalent to a 2D vortex overlap by noting that both shaded areas in figure 4.14b contain the same circulation. As the shape of the shaded area or the direction of the vortex tube are not important for the magnitude of the circulation the overlap can be approached by simple vortex models. For a Gaussian vortex, the circulation as a function of the separation distance can be fitted with a formula similar to eq. (2.25): $\Gamma_{\text{total}} = \Gamma_{\text{tube}} \text{erf}(b/(2R))$, indicated by the solid line in figure 4.14a. The parameter $\Gamma_{\text{tube}}$ is the total circulation of the vortex tube, and $R$ is the radius of a circular region containing 68% of the vorticity. The fits suggest that $R$ is of the order of half the initial vortex spacing. Since the value of $R$ is determined by the slope from $\Gamma_{\text{total}} = 0$ to $\Gamma_{\text{total}} = \Gamma_{\text{tube}}$, the smaller vortex separation distances are dominant. At these small vortex
separation distances the vortices are linked, and $R$ thus denotes the radius of the vortex tube close to the point of linking. The magnitude of $R$ might be related to instabilities such as pressure waves occurring due to the linking and enhancing the core size. The circulation does not provide information about the direction of the vortex tubes and can therefore not be related to the wave length of the instability. As the Crow instability is only observed at a single location for a single configuration no information could be obtained about the growth of the instability. However, since the typical oscillation of the vortex centers in figure 4.13 was observed at this location only, the onset of the instability may occur at a certain crisis time, eventually leading to the decay of the vortices by linking. The evolution of the vortices before the linking is discussed below.

4.3.2 Circulation profiles and evolution

As was already shown above, the vortex core motion is only slightly influenced by mutual interaction in the early stages of the vortex lifetime. In contrast, the vorticity distribution is more likely to be influenced by the presence of a second vortex, since half the vortex separation $d_0/2$ is smaller than the radius of vorticity $r_2$ found for the isolated vortex. Also the split wing configuration induces extra lift on the airfoil due to the vortex created on the other wing. Since the wings were mounted up-side down, an incline of the wind tunnel was also important, which effectively changes the angle of incidence. The vortex dipole is characterized by a separatrix, which encloses the volume of fluid descending downward with

Figure 4.14: The total circulation $\Gamma_{\text{total}}$ for $d_0 = 0.03$ m at $\Delta x = 3.82$ m, is determined as an average over the last 11 points of the circulation profile obtained over square contours around the center and is plotted in panel (a) against the distance between the vortex centers, $b$, normalized by the total circulation $\Gamma_0$ and vortex separation $b_0$ at $\Delta x = 0.26$ m. In panel (b), it is shown that the circulation around one of the vortices in a plane perpendicular to the flow is independent of the direction and the shape of the vortex cross section by noting that both shaded areas enclose the same vortex lines. The circulation obtained in the plane at the point of linking is mathematically equivalent to 2D vortex overlap. Therefore, the profiles in (a) can be fitted with a Gaussian model similar to eq. (2.25) because both the shaded areas contain the same circulation.
the vortices. The vorticity that is outside of the separatrix, e.g. remnants of the roll-up process, is left behind.

The circulation profiles of the vortex pairs are shown in figure 4.15. The circulation profile for $d_0 = 0.10$ m, could be determined within the range $0 < r/c < 0.5$ as a result of the limited viewing area corresponding to the lens with $f=55$ mm. The lens with $f=28$ mm has a larger viewing area, but the resolution of the core region becomes even lower. The double wing-tip vortices seem more compact (larger percentage of the vorticity in core region) than the isolated vortex, especially for a small tip separation distance as can be seen at $\Delta x=0.26$ m (figure 4.15a). This suggests that the presence of a second vortex speeds up the roll-up process, probably due to the large velocities in between the vortices. In addition, it should be noted that the circulation around each vortex is larger than that around the isolated vortex as can be seen at the larger distances from the core. The resolution in the region $0 < r/c < 0.2$ depends on the local velocities and delay times used. In figure 4.15 the circulation profiles are shown for delay times of $\tau = 70 \text{ - } 100 \mu$s. Note that with these delay times the core may not be completely resolved.

![Figure 4.15](image)

**Figure 4.15:** Circulation profiles of the isolated vortex (solid line: $\tau=100 \mu$s) and vortices with tip spacings $d_0 = 0.03$ m, 0.05 m and 0.10 m at (a) $\Delta x=0.26$ m with $\tau=70 \mu$s and (b) $\Delta x=0.98$ m with $\tau=100 \mu$s. In addition the core data of the isolated vortex is shown (dots) to give an estimate of the region where the outer region data is approximately valid.

At $\Delta x=0.98$ m (figure 4.15b), it is apparent that vorticity is more concentrated around the vortex core. Also the circulation profiles coincide for separation distances $d_0 = 0.03$ m and 0.05 m. This indicates that mutual interaction not yet determines the vortex evolution at this stage, and that the vortex separation distance would not be the relevant parameter governing the time evolution at this stage.

To study the time evolution, the integral quantities $\Gamma_{total}$ and $\Gamma_y$ as defined in eq. (2.4) are shown in figure 4.16. These quantities are plotted as a function of $t^* = t\Gamma_0/(2\pi b_0^2)$, where $\Gamma_0$ and $b_0$ are the total circulation and vortex separation distance at $\Delta x=0.26$ m. This dimensionless time is often used in literature and is based on the assumption that the circulation and vortex separation distance are the important parameters governing the decay of the vortices. At $t^*=12$ vortex linking was observed and the instantaneous circulation is
no longer determined by diffusion as can be seen in figure 4.14a. However, it is assumed that the plateau in figure 4.14a is governed by diffusion.

![Graphs showing circulation, vortex separation, and dipole strength over time](image)

Figure 4.16: The evolution of (a) the total circulation $\Gamma_{total}$, (b) the vortex separation $b$, and (c) the dipole strength, $\Gamma_{total}b$, normalized with the values measured at $\Delta x=0.26$ m, as a function of the dimensionless time $t^*$. The total circulation was determined by calculating the mean over the last 8 points of the circulation profiles and averaging over 250 image pairs (both left and right vortices gave approximately the same value). The vortex separation distance was calculated from the vortex core positions and was also averaged over 250 image pairs. At $t^*=12$ (i.e. $\Delta x = 3.82$ m and $d_0 = 0.03$ m), the total circulation was determined from the plateau in figure 4.14a, $\Gamma_{total}/\Gamma_0=0.85$ and the corresponding dipole strength was determined as the mean of the circulation determined in the plane perpendicular to the $x$-axis multiplied by the instantaneous separation distance. The error bars indicate the standard deviation of the mean values. In panel (a), a fit based on eq. (2.25) was used to estimate the cross-diffusion, which is based on viscous decaying Lamb-Oseen vortices. The cross-diffusion coefficient $D_{eff}$ is found to be relatively large in comparison with the kinematic viscosity $\nu=1.5 \times 10^{-5}$ m$^2$s$^{-1}$ [40].

The total circulation is more or less constant in the early stages of the evolution and then decays with 10% around $t^*=5$ until 20% around $t^*=12$. The decay of circulation may be
attributed to cross-diffusion of vorticity. The fact that the circulation is constant for a considerable time suggests that the vortices are compact during the initial stage of the evolution. As an approximation, the model of two overlapping Lamb-Oseen vortices in eq. (2.25) can be used to estimate the cross-diffusion, i.e. $\Gamma_{\text{total}}/\Gamma_0 = \text{erf} \left( 1/4 \sqrt{D_{\text{eff}} (t^* + t_S^*) 2\pi/\Gamma_0} \right)$. Here, $D_{\text{eff}}$ denotes the cross-diffusion coefficient, while $t_S^*$ corresponds to the time it would take for a point vortex to acquire a radius $\sqrt{8\pi D_{\text{eff}}/\Gamma_0 t_S^*}$. A least squares fit with this model reveals that $D_{\text{eff}} = 4 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ which is an order of magnitude larger than the kinematic viscosity of air ($\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ [40]). The large cross-diffusion coefficient suggest the presence of a region with significant turbulence in between the vortices. A similar observation was made in experiments by Devenport et al. [52].

As discussed in section 2.2, the dipole strength $\Gamma_{\text{total}} b$ is conserved in a two-dimensional flow. Hence, since the circulation of the vortices decreases, the vortices should move apart if the flow is close to 2D, which can be observed in figure 4.16b. Figure 4.16c shows that the dipole strength is within an uncertainty of 15% indeed constant. At some crisis time, in the interval $7 < t^* < 12$, the Crow instability sets in and leads to vortex linking and eventually to the decay of the vortices. In this regime the conservation of dipole strength is no longer valid as can be seen from the large error bars at $t^* = 12$. Still, it is assumed that in this regime the cross-diffusion of vorticity is determined by the same $D_{\text{eff}}$.

Comparing the vortex pairs with the isolated vortex, the mutual interaction was found to slightly increase the core oscillations until the occurrence of the Crow instability. The main difference is found in the vorticity distributions, which are more compact for the vortex pairs. The decay of circulation for $t^* > 2$ is governed by turbulent diffusion, and this might be enhanced in a turbulent environment. In the next section, the evolution of the vortices in a turbulent environment is described.
4.4 Double wing-tip vortices with grid turbulence

In the previous sections, the influence of turbulence and the mutual interaction between the vortices were investigated separately. Here, both effects are studied by considering a pair of wing-tip vortices in grid-generated turbulence. Measurements were performed at four different positions behind the airfoil, $\Delta x = 0.26$ m, 0.98 m, 1.78 m and 2.82 m, while the airfoils were located at $x_a = 3.5$ m and the grid at $x = 0$. Velocity fields were determined for three different tip spacings $d_0 = 0.03$ m, 0.05 m and 0.10 m, with 5, 3, 3, and 2 different angles of attack at the subsequent locations, respectively. For the largest tip spacing, the vortices were also examined for three different configurations of the grid at the first three locations. The configurations were all stationary (agitators), but had transparencies relative to the open stationary grid of 0.94, 0.85 and 0.71. Turbulence characteristics for these configurations can be found in Appendix D. The dimensionless dissipation rate $\epsilon^*$ is based on the value of $\epsilon$ at the location of the airfoil. Measurements were done with delay times of 80 and 100 $\mu$s and a lens with $f = 50$ mm. For the largest tip spacing, a lens with focal length $f = 28$ mm was used as well.

4.4.1 Lift distribution

When studying the influence of turbulence on the vortex pairs, it is important to verify whether the initial conditions of the vortices with and without grid are the same. As was shown in section 4.2, the lift coefficient of the airfoil measured at $\Delta x = 0.26$ m behind the grid was virtually the same both with and without grid. In figure 4.17a, the lift coefficient is shown for both the isolated vortex and the vortex pairs with grid. For the two larger tip spacings, $d_0 = 0.05$ m and $d_0 = 0.10$ m, the lift coefficient curves are only slightly lower than that of the isolated vortex but show the same increase with $\alpha$. The curve for the smallest tip spacing, $d_0 = 0.03$ m, is significantly higher than that of the isolated vortex. The extra induced lift is due to the velocities generated by the other vortex on the airfoil. For smaller values of the lift coefficient, the difference becomes smaller. This is supported by figure 4.17b, which shows that for the smallest tip spacing, $d_0 = 0.03$ m, the vortex separation is reduced due to mutual interaction. The left airfoil creates a velocity field behind the right airfoil, which is largest near the tip of the airfoil ($1/r$ dependence of the azimuthal velocity). The airfoil becomes more tip-loaded (see section 2.1 with $n \rightarrow 1$) and since the center of vorticity is conserved during the roll-up, the vortex centers become closer to the tips and thus closer to each other. The results for the other tip spacings do not show this behavior this clear.

The lift coefficient was also determined for the grid configurations with transparencies, 0.94, 0.85 and 0.71, and is given in figure 4.17. For decreasing transparency, the lift coefficient decreases as well. This is probably due to a Reynolds number effect, since by blocking a part of the flow, the mean velocities in the tunnel decreased, causing the Reynolds number $Re_c$ to decrease. A decrease of Reynolds number implies an increase of the boundary layer thickness on the airfoil, which reduces the circulation and thus the lift coefficient. The values of the circulation were used as the initial values in the rest of the section. We first turn our attention to the core and the center movement of the vortices.
Figure 4.17: The lift coefficient $C_L = 2\dot{\Gamma}_0/(Uc)$ at $\Delta x = 0.26$ m behind the airfoil as a function of the angle of incidence, $\alpha$, for both the isolated vortex as well as the double wing-tip vortices for different tip spacings, $d_0$, with grid transparency 1 and three other grid transparencies (diamonds) in panel (a). Panel (b) shows the vortex separation as a function of the angle of attack with grid transparency 1, showing that for the smallest tip spacing the mutual interaction reduces the vortex separation by increasing the lift near the tip of the airfoils.

### 4.4.2 Instabilities of the vortices

Despite the limited resolution in the core, the Crow instability was observed in the vortex core movement (see figure 4.13) for several vortex spacings at several distances behind the airfoil for the open grid. Since the initial conditions of the vortices are similar both with and without grid, it can be concluded that the Crow instability is triggered by external turbulence. By plotting the total circulation around a single vortex as a function of the instantaneous separation distance like figure 4.14, it could be determined whether linking took place at different locations downstream of the airfoil. In addition, we were able to record top view smoke visualizations of the vortex tubes as is shown in figure 4.18.

Since Crow and Bate predict [32] that the relative strength of the turbulence, $\epsilon^*$ (defined in eq. (2.30)), governs whether Crow instability is important for the decay, it is useful to plot the dimensionless times $t^*$ where Crow instability was observed against $\epsilon^*$. This plot is shown in figure 4.19a for grid transparency 1 only, because for the three other grid configurations no Crow instabilities could be observed. Within the range of $\epsilon^*$ considered, three regimes can roughly be distinguished: $0 < t^* < 1$, initial stage of the vortices with no cooperative instabilities, $1 < t^* < 4$, onset and evolution of the Crow instability, and $t^* > 4$, vortex linking. Note that without grid the onset of the Crow-instability does not occur before $t^* = 7$. In figure 4.19b, the measurement locations in terms of $\epsilon^*$ and $t^*$ are given both with and without grid and the inviscid linking time, $T^*$, of Crow and Bate is indicated by the black line. The measurement locations without grid are based on the upper limit of $\epsilon$ given in section 3.4.1 and seem consistent with the theory. In the presence of the grid, $\epsilon$ decays with the distance from the grid and the minimum value for $\epsilon^*$ is given by the error bars attached to the measurement locations with grid, since we used the value of $\epsilon$ at the location of the airfoils. The nine extreme right points correspond to the configurations of the grid with...
transparencies 0.94, 0.85 and 0.71 relative to the open grid. The main effect of viscosity will be to decrease the circulation and hence change both $t^*$ and $\epsilon^*$ in the inviscid theory of Crow and Bate. This can be accounted for by displacing the black line, along a line where $\epsilon^* t^* = \epsilon^{1/3} t / b^{2/3}$ is constant (dotted line), because there is no dependence on $\Gamma$ along this line. Even with these considerations, the theoretical line only gives a lower boundary. Next to that, vortex linking is only observed for tip spacing $d_0=0.03$ m and we would like to know whether the time of linking $T^*$ is independent (for constant $\epsilon^*$) of $b$ as suggested by the theory. Therefore, in the following the core movement and the Crow instability are studied in more detail.

Since the theory of the Crow instability [2] predicts that the most unstable wave length, $\lambda = 8.6b_0$, has a preferred angle of 48° with the horizontal, it makes sense to determine the mean angle of oscillation. As is shown in figure 4.20a, the mean angle of oscillation to the horizontal averaged over both vortices grows initially as a function of $t^*$ to a constant value, which remains significantly smaller than 48°. Note that the images are corrected for the fact that the camera is oriented under an angle to the laser sheet (7.8°), and the angle of the camera could therefore not be the explanation. Without the grid, virtually the same mean angle 33° was observed, see figure 4.13, and the angle of oscillation would therefore not be related to the external turbulence. Also note that in the final stage of the evolution (before vortex linking) the angle remains constant in time $t^*$ and is virtually the same for all separation distances $b_0$ in figure 4.20a. Viscous effects would be noted in a time dependence
Figure 4.19: For the open stationary grid all measurement locations are given in terms of the dimensionless parameters $t^*$ and $\epsilon^*$ (defined in eqs. (2.31) and (2.30)). In panel (a), the locations where Crow instability occurs are marked (open circles) as well as the locations where linking is found (black triangles). Panel (b) shows the same locations on a log-log scale and also includes the data without grid based on the upper limit of $\epsilon$. The nine extreme right points correspond to the three other grid configurations with transparencies of 0.94, 0.85, 0.71 relative to the open grid. The error bars indicate the minimum value of $\epsilon^*$ with grid at $\Delta x = 2.82$ m. The black line is the time of linking predicted by the inviscid theory of Crow and Bate [32]. Diffusive effects cause a decay of circulation and hence change both $t^*$ and $\epsilon^*$. Assuming that $b$ is approximately unaffected by cross-diffusion, diffusive effects can be accounted for by translating the data points along lines, where $\epsilon^*t^*$ is constant (dotted line).

The growth of the instability is exponential with the period of the most unstable wave length $\lambda = 8.6b_0$ given by $\tau_{\text{Crow}} = 1.21(2\pi b_0^2)/\Gamma_0$ [2]. To investigate the growth rate of the oscillation the vortex spacing, $b$, is a useful parameter, but as it oscillates we follow Crow and Bate [32] to examine the root-mean-square vortex separation, $b_{\text{RMS}}$, as is shown in figure 4.20b. The observed growth is exponential with amplification rate $a(k) = 0.41 \Gamma_0/(2\pi b_0^2)$, which is only half the amplification rate predicted by Crow ($a_{\text{Crow}} = 1/\tau_{\text{Crow}} = 0.83 \Gamma_0/(2\pi b_0^2)$) [2]. Note that the strength of the vortices has decayed significantly already since the onset of the instability as is shown in figure 4.21. This figure shows that during both the first and second regime, cross-diffusion takes place. The decrease of circulation also decreases the growth rate $a$ but this does not account for half the amplification rate. Furthermore, diffusion would make the growth rate time dependent as is shown in numerical simulations [53] [54] which is not apparent from the observation. The growth rate seems also independent of the vortex separation suggesting that the time of linking $T^*$ for $d_0 = 0.05$ m is larger than the linking time for $d_0 = 0.03$ m even though $\epsilon^*$ increases. This larger life time for $d_0 = 0.05$ m contradicts the theory of Crow and Bate [32].

The theory of the Crow instability does not reject smaller angles and growth rates, but these are not related to the most unstable wave number $k_m$. The value of $k_m$ depends upon the ratio of the core radius to the vortex separation, but larger ratio’s should increase the angle of oscillation [2]. A dependence on the ratio of the core radius and the vortex separation
Figure 4.20: Panel (a) shows the mean angle of oscillation averaged over both vortices to the horizontal as a function of $t^*$. In panel (b), the growth of the root-mean-square vortex separation is shown. The symbols correspond to the distance between the tips, $d_0$.

would also make the angle and growth rate dependent on the vortex separation, which is not apparent in figure 4.20.

The growth rate in figure 4.20b does not change in time, indicating that the circulation in the expression for $a$ is constant. As the total circulation decays, this might mean that there is a strong core dominant in the development of the Crow instability which barely decays. The core can only be dominant, when vorticity is concentrated in this region giving the largest contribution to the induced velocities. The presence of a strong core is found in the case of the isolated vortex and contains approximately 54 to 60% of the vorticity ($0.04 < r_1/b_0 < 0.17$ based on the core radius of the isolated vortex). Neglecting the contribution of the outer region to the development of the Crow instability, an effective growth rate can be introduced, based on the effective part of the circulation $\Gamma_{eff} = 0.54 \Gamma_0$. The growth rate then becomes $a = 0.76\Gamma_{eff}/(2\pi b_0^2) = 0.41\Gamma_0/(2\pi b_0^2)$, which is closer to the value 0.83 predicted by Crow [2]. This does not explain the different angle of oscillation, but that might be related to the vorticity distribution. The theory is described using a Rankine vortex for the vorticity distribution and is linearized, which means that the radius containing the vorticity is much smaller than the vortex separation distance. Since cross-diffusion takes place, the radius containing the major part of the vorticity and the vortex separation distance may be of the same order of magnitude.

### 4.4.3 Circulation evolution

Since the Crow instability is already observed for $t^* > 1$, conservation of dipole strength is no longer a relevant quantity. In the case of the isolated vortex, the diffusion of vorticity in the outer region of the vortex was increased in time. It is therefore interesting to consider the decay of circulation as a function of the time. These plots are given in figure 4.21a, b and c, which show a more rapid decay with grid than without grid as could be expected. For the large values of $\epsilon^*$, the circulation decays very rapidly. Note that the curves for $\epsilon^*=0.76$ and 0.77 almost coincide, though they have different $\epsilon$ and $\Gamma_{total}$. On the other hand, the curves for $\epsilon^*=0.28$ and $d_0 = 0.03$ m and 0.05 m do not.
Figure 4.21: In panel (a), (b) and (c): The evolution of the total circulation, normalized on its initial values at $\Delta x = 0.26$ m for various values of $\epsilon^*$ with grid transparency 1 and vortex separations $d_0 = 0.03$ m, 0.05 m and 0.10 m, respectively. Evolution of the circulation without grid is included in panel (a). The three other grid configurations are also given in panel (c). Panel (d) shows the cross-diffusion coefficient $D_{eff}$ as a function of the relative strength of the turbulence. Fits were made with eq. (2.25) and can be found in Appendix E.

Again it is possible to fit the decay with eq. (2.25) to find cross-diffusion coefficients. The individual fits can be found in Appendix E. In figure 4.21d, the cross-diffusion coefficients are plotted as a function of $\epsilon^*$. There seems to be a positive correlation between the cross-diffusion of vorticity and the relative strength of the turbulence. It is not clear, however, whether $\epsilon^*$ is the correct parameter defining the relation.
Chapter 5

Discussion and conclusions

The goal of the report is to investigate the influence of turbulence on the decay of wing-tip vortices, to gain insight in the mutual interaction between the vortices and to examine whether turbulence can trigger instabilities arising from mutual interaction. In order to investigate these subjects, experiments were performed with an isolated vortex in both a low and strong turbulent environment and vortex pairs in a low and strong turbulent environment. Vortices were shed from an airfoil in the wind tunnel and velocity fields were obtained with particle image velocimetry at several locations behind the airfoil. The low level of core oscillations showed no unsteadiness of the wind tunnel and the airfoil and that the turbulence levels were low, suggesting that the experimental configuration is suitable for accurate measurements of the wing tip vortices.

In order to both resolve the core and outer regions three different lenses were used with focal points $f = 105$ mm, 55 mm and 28 mm and delay times varying from $\tau = 30 \mu s$ to 100 $\mu s$. This slightly complicates comparisons made between the various cases, especially when focusing at properties related to the cores of the vortices.

The isolated vortex was found to consist of a concentrated laminar, viscous core region and a larger region containing little vorticity, which is thought to be related to an unrolled-up remainder of vortex sheet. The lift coefficient, determined closely behind the airfoil was not in complete agreement with previous measurements at a smaller Reynolds number. The difference between the lift curves may be related to the twist of the wing tip. The presence of external turbulence does not influence the decay of the core, which remains close to laminar. In the region surrounding the core, vorticity seems to diffuse more rapidly which is enhanced by external turbulence. The turbulence increases the wandering of the core, and was found to be inversely proportional to the rotation time of the vortex core. This might indicate that the vortex deforms the turbulence field in such a manner that the characteristic time scales of the turbulence become coupled to the rotation time scale of the vortex.

Core oscillations for the vortex pairs were only slightly increased by mutual interaction, but no dependence was found on the vortex separation distance. For $t^* = 12$, vortex core motions showed a direction of preference and vortex linking could be observed, what is related to the Crow instability. Presence of external turbulence promotes the onset of the Crow instability, though if the turbulence is too large this instability could no longer be observed. The growth factor of the instability could be determined and seems to be a factor 2 smaller than predicted by inviscid theory. The angle of oscillation was found to become approximately 33° with and without grid turbulence, which is also smaller than the angle predicted by inviscid theory. There seems to be no time or vortex separation distance.
dependence in the growth rate of the instability and the angle of oscillation.

Vortex profiles for the vortex pairs seem to be more compact, what might be due to the presence of another vortex core. The circulation remains constant in the initial stages, but then decays due to cross-diffusion of vorticity. This cross-diffusion could well be modeled by assuming a two-dimensional decay of a pair of Lamb-Oseen vortices. The diffusion coefficient is quite large, suggesting a turbulent region in between the vortices, and increases in a more turbulent environment. A positive correlation could be found between the relative strength of the turbulence to the strength of the vortices and the magnitude of this diffusion constant.

The influence of turbulence on the decay of tip-vortices may thus be found in two contributions. First, there is an increase of cross-diffusion, which is related to the strength of the turbulence and the strength of the vortices. Second, the Crow instability is triggered eventually leading to linking of the vortex tubes, which rapidly destroys the vortices. Both processes occur at the same time creating a complex decay mechanism. This mechanism is not yet fully understood, but the authors expect to gain more insight by employing different turbulence intensities in future measurements. Still, we expect that the results presented in this report will prove to be useful in wake vortex control.
Acknowledgements

The completion and quality of this report had not been possible without the help of a number of people. I would like to use this opportunity to thank all the people of the Fluid Dynamics Department for providing a both stimulating and pleasant environment to work in. I would like to thank GertJan van Heijst, Bernard Geurts, Willem van de Water, Anton de Bruin, Ergün Çekli and Menno Lauret for many illuminating discussions as well as for their interest in me. I would also like to thank my room mate Bert Lodewijks for sharing his vision on subjects beyond the scope of this project.

Next to that, there are some people that I like to thank in particular for their special contribution to my work. In the first place, my supervisors Bram Elsenaar and Ruben Trieling. Bram, your physical intuition and critical reviews of my results, greatly improved my understanding of the subject and stimulated me to examine the subject more thoroughly. Ruben, your logical thinking, sense for priorities and suggestions during the analysis, helped to solve many issues without loosing the focus of the project. At last, but certainly not least I would present my special thanks to Ad Holten. The quality and pace of my work on the project is largely contributed to your assistance and support. Your help with the PIV set up determined the quality of the measurements, your simulation program gave us great possibilities to investigate the method and by doing the hot-wire measurements for the characterization of turbulence you took away a large amount of work required for this investigation. Next to this enormous amount of work you have invested in the project, you are also a very pleasant person to work with.

Finally I would also like to thank to my parents, friends and family, who have stimulated and supported me during the whole course of my study.
Bibliography


Appendix A: Azimuthal velocity profiles of isolated vortex

Figure A-1: Typical azimuthal velocity profiles of the single wing-tip vortex (1 image) without grid. The profiles are given at three different distances $\Delta x$ behind the wing. The azimuthal velocity is determined by transforming the grid to polar coordinates around the vortex center, defined as the maximum of the stream function. Delay times were set at respectively 30, 30, 50 and 50 $\mu$s and a lens was used with focal point $f=105$ mm. Increasing the delay time reduces the scatter for $r/c > 0.3$ as can be seen by comparing panel (a) and (b) to (c). Panel (d) shows more scatter in this region which is probably related to the quality of the measurements at this station.
Figure A-2: Typical azimuthal velocity profiles of the single wing-tip vortex (1 image) with grid. The profiles are given at three different distances $\Delta x$ behind the wing. The azimuthal velocity is determined by transforming the grid to polar coordinates around the vortex center, defined as the maximum of the stream function. Delay times were set at respectively 30, 30, 50 and 50 $\mu$s and a lens was used with focal point $f=105$ mm. The velocity profile in panel (d) shows again more scatter than the profile in panel (c), which might be related to the strength of the turbulence and the strength of the vortex. The measurements at this station might also be of less quality.
Appendix B: Circulation profiles of isolated vortex without grid

Figure B-1: Typical circulation profiles at distances $\Delta x$ behind the wing. The circulation is obtained as described in eq. (3.11) (black circles), and as in eq. (3.8) (black squares) in section 3.3.4.
Appendix C: Jacquin VM2 fits of circulation profiles of isolated vortex with grid

Figure C-1: Circulation profiles at $\Delta x=0.26m$, composed of core data (diamonds), with delay times of 30$\mu$s and $f=105$ mm, and data obtained with $f=50$ mm and 100$\mu$s delay times (asterisks). Fits were made with the Jacquin VM2 model (eq. (4.1)), where the data was linked at $r/c=0.3$. The core radius $r_1$ and the total circulation $\Gamma_{\text{total}}$ were used as input and the fits determine the exponent $\beta$ and radius of vorticity $r_2$ as outer radius.
Figure C-2: Circulation profiles at $\Delta x=0.98m$, composed of core data (diamonds), with delay times of 30$\mu$s and $f=105$ mm, and data obtained with $f=50$ mm and 100$\mu$s delay times (asterisks). Fits were made with the Jacquin VM2 model (eq. (4.1)), where the data was linked at $r/c=0.3$. The core radius $r_1$ and the total circulation $\Gamma_{\text{total}}$ were used as input and the fits determine the exponent $\beta$ and radius of vorticity $r_2$ as outer radius.
Figure C-3: Circulation profiles at $\Delta x = 1.78\text{m}$, composed of core data (diamonds), with delay times of 30$\mu$s and $f=105$ mm, and data obtained with $f=50$ mm and 100$\mu$s delay times (asterisks). Fits were made with the Jacquin VM2 model (eq. (4.1)), where the data was linked at $r/c=0.3$. The core radius $r_1$ and the total circulation $\Gamma_{\text{total}}$ were used as input and the fits determine the exponent $\beta$ and radius of vorticity $r_2$ as outer radius.
Figure C-4: Circulation profiles at $\Delta x = 2.82m$, composed of core data (diamonds), with delay times of 30$\mu$s and $f = 105$ mm, and data obtained with $f = 50$ mm and 100$\mu$s delay times (asterisks). Fits were made with the Jacquin VM2 model (eq. (4.1)), where the data was linked at $r/c = 0.3$. The core radius $r_1$ and the total circulation $\Gamma_{total}$ were used as input and the fits determine the exponent $\beta$ and radius of vorticity $r_2$ as outer radius.
Table 5.1: Turbulence Characteristics Stationary grid

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<th>Skewness (eq. (3.19))</th>
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The data for the grid with transparency 0.94 are peculiar, since it is the only one with a negative skewness, has rather high velocities compared with its transparency and velocity fluctuations seem to increase with the distance behind the grid. The negative skewness indicates that fluctuations of the velocity in direction of the flow are generally smaller than those opposite to the flow. It suggests that there might be regions of back flow in time. The large, but constant RMS velocity indicates that the turbulence is not fully developed yet. The large velocities suggest that the flow is accelerated in the center, this could be due to the special configuration of the bars, leading to a region with a strong jet in the center and low velocities near the walls.

For the other three grid configurations (see figure 3.13), the mean velocities are approximately proportional to the transparency. The RMS velocities are given in figure D-1a, and are close to the predicted relation [44]. In figure D-1b, the decay of the dissipation rate is given which is also close to eq. (3.17).
Figure D-1: The decay of (a) the rms velocity $u_{RMS}$ and (b) the dissipation rate $\epsilon$ as a function of the distance behind the grid, $x$ normalized on the length of the test section, $L$. Both quantities are normalized on the value measured at $x = 3.43$ m.
Appendix E: Circulation decay fits with grid

Figure E-1: Fits of the decay of circulation with eq. (2.25) for tip spacing $d_0=0.03 \text{ m}$.
Figure E-2: Fits of the decay of circulation with eq. (2.25) for tip spacing $d_0=0.05$ m.
Figure E-3: Fits of the decay of circulation with eq. (2.25) for tip spacing $d_0=0.10$ m.
(a) $\epsilon^* = 0.77$

$D_{\text{eff}} = 0.0715 \pm 0.0199 \text{ m}^2 \text{ s}^{-1}$

$R_0 / b_0 = -0.33 \pm 0.07$

(b) $\epsilon^* = 1.55$

$D_{\text{eff}} = 0.2135 \pm 0.1069 \text{ m}^2 \text{ s}^{-1}$

$R_0 / b_0 = -0.51 \pm 0.18$

(c) $\epsilon^* = 1.80$

$D_{\text{eff}} = 0.7775 \pm 0.6452 \text{ m}^2 \text{ s}^{-1}$

$R_0 / b_0 = -0.92 \pm 0.44$

Figure E-4: Fits of the decay of circulation with eq. (2.25) for tip spacing $d_0 = 0.10$ m but the three other grid configurations (transparencies 0.94, 0.85 and 0.71.)
Appendix F: Change of kinetic energy in the outer region of the isolated vortex

The core region of the vortex is close to laminar for the isolated vortex both with and without grid. In section 4.2 figure 4.11, it is apparent that the outer region diffuses more rapidly with grid than without grid. To quantify this process, the change of kinetic energy is calculated. The circulation is determined at radii $r/c = 0.2, 0.3, 0.4$ and $0.5$ and the change in kinetic energy is defined as:

$$
\Delta E_k = E_k(t = 0) - E_k(t) = 2\pi \int_{0.2c}^{0.5c} u_0^2 r dr \approx \sum_{i=1}^{4} \frac{c\Gamma_0^2 \Gamma(r_i, t = 0)^2}{2\pi r_i} \left( 1 - \frac{\Gamma(r_i, t)^2}{\Gamma(r_i, t = 0)^2} \right) \Delta r/c,
$$

where $r_1/c = 0.2$, $r_2/c = 0.3$, $r_3/c = 0.4$, $r_4/c = 0.5$ and $\Delta r/c = 0.1$. The circulations $\Gamma_0$, $\Gamma(r_i, t = 0)$ and $\Gamma(r_i, t)$ denote the initial total circulation measured at $\Delta x=0.26$ m, the initial circulation at $r_i$ measured at $\Delta x=0.26$ m and the circulation at $r_i$, respectively. The change in kinetic energy is shown in figure F-1 and shows that the change of kinetic energy in the interval $0.2 < r/c < 0.5$ is indeed significantly larger with grid than without grid.

![Figure F-1](image)

Figure F-1: The change of kinetic energy $\Delta E_k$ in the interval $0.2 < r/c < 0.5$ as a function of the time $t$ both with and without grid.
Appendix G: Optimization of the wake generator model

Figure G-1: The ‘basic element’ also used to generate an ‘isolated’ vortex. Two such elements, mounted on the left and right tunnel walls, are used to generate a vortex pair. The wing is twisted such that a constant lift over the span is obtained.

This appendix describes the optimization of the airfoil by means of a wing twist as designed by A. Elsenaar [55]. The basic element of the wake generator is a slender rectangular straight wing (high aspect ratio) with a constant load in spanwise direction (see figure G-1). This wing is mounted at the tunnel side wall. Since the tunnel wall acts as a reflection plane, the model can also be regarded to represent one side of a rectangular model with span b and chord c. For such a configuration all vorticity is shed ideally at the wing tip with a strength \( \Gamma \) defined by:

\[
\Gamma = \int \omega_x dA = \frac{C_L U s}{2 A R b_D},
\]

where \( \omega_x \) is the vorticity component in \( x \)-direction and \( C_L \) the lift coefficient, which is for this particular case of a straight rectangular wing with constant load equal to the local sectional lift coefficient \( c_L \). The parameter \( U \) represents the tunnel velocity and \( s \) the wing span (including its mirrored image part), \( c \) is the chord length of the model and \( A R \) is the aspect ratio defined as \( s^2/S \) with \( S \) the wing surface. For a straight rectangular wing \( A R = s/c \). The parameter \( b_D \) is the non-dimensional distance between the vortices shed from the wing tips and defined by \( b_D = \Gamma_{1,y}/(0.5s \Gamma) \) where \( \Gamma_{1,y} \) defined in eq. (2.4). In the ideal case when all vorticity is shed from the wing, \( b_D = 1 \). In this particular case \( \Gamma \) reduces to

\[
\Gamma = \frac{C_L U s}{2 A R b_D} = \frac{c_L U s}{s/c b_D} = \frac{1}{2} c_L U c
\]

The concentrated vortices at the wing tips introduce a flow angle at the model location. This induced flow angle can be expressed in the case of a single wing as (see figure G-1 and
also figure G-2) as
\[ \alpha_i = \frac{w(r)}{U} = \frac{1}{U} \left( \frac{\Gamma}{4\pi r} - \frac{\Gamma}{4\pi(s - r)} \right), \quad (G-3) \]

with \( r \) the distance from the wing tip. Note that a factor of \( 1/4\pi \) is used since only half of the vortex line (the downstream part) has to be considered at the model location; further downstream the induced velocities follow from \( \Gamma/(2\pi r) \).

To obtain a constant lift distribution on the (half)model wing, these induced flow angles should be compensated by twisting the wing over the same amount \( \alpha_{\text{twist}} = -\alpha_i \). To obtain the lift coefficient \( c_L \) the wing has to be set at an angle of \( \alpha \). For the wing section a CLARK-Y for which data are available at \( Re_c = 60000 \). The airfoil has a rather straight lift curve up till \( c_L = 1.0 \) (info: Leo veldhuis). The local lift coefficient \( c_L \) follows from:
\[ c_L = \frac{d c_L}{d\alpha} (\alpha - \alpha_0), \quad (G-4) \]

with \( \alpha_0 = -1.4 \) degrees and \( d c_L / d\alpha = .10 \) /degree. Lift values have been measured at the available Reynolds number \( Re_c = 60000 \) till \( c_L = 1.2 \).

\[ \alpha_i = \frac{w(r)}{U} = \frac{-\Gamma}{4\pi U} \left( \frac{1}{r} + \frac{1}{s - r} - \frac{1}{s + 2d_0 + r} - \frac{1}{d_0 + r} - \frac{1}{s + d_0 - r} + \frac{1}{s + d_0 + r} \right). \quad (G-5) \]

The wings are optimized for wing tip spacing \( d_0 = 0.10 \) m. The values for \( \alpha_i \) are depicted in figure G-3.

Theoretically, the twist angle will be infinite at the wing tip. To eliminate this a constant twist is applied between the wing tip and a position 0.25 chord inboard from the tip. Three-dimensional effects and viscosity are expected to modify the wing tip flow anyhow.
Figure G-3: The estimated twist angle for the isolated wing and the wing pair. The figure shows that the twist distribution for the two models are very different from the isolated vortex even if the lift value for the isolated wing is halved. However, a small variation in the distance between the wing tips has only a minor effect. This allows small changes in the distance d between the wing tips during in the wind tunnel tests to refine the optimization.