Effect of a discontinuous topography on the self-organization of confined quasi-two-dimensional flow

Jehoel, F.

Award date: 2010

Link to publication
Effect of a discontinuous topography on the self-organization of confined quasi-two-dimensional flow

Frank Jehoel

R-1767-A
25 March 2010
Abstract

The effect of a discontinuous topography on the self-organization process in quasi two-dimensional flows is studied experimentally. Experiments are performed in a rotating tank, in which the flow is quasi two-dimensional. Different tank sizes and bottom topographies were used to study their effect on the evolution of the flow. A step topography with different heights and a ridge topography were used to divide the domain in two equally sized subdomains. The effect of a discontinuous topography was studied by considering the time evolution of the ratio of enstrophy and energy, which is a measure of the size of the vortex structures. This ratio shows a power-law behaviour $t^\alpha$. The power-law coefficient for the case when a step or ridge is present is the same as the power-law coefficient for a tank with a flat bottom and half the size. This shows us that the step acts as a virtual wall on the evolution of the characteristic scale of the vortex structures. The evolution of the velocity directly above the step was studied as well. In the initial stage, vortices are strong enough to cross the step, which can be seen by a horizontal velocity which is normal to the step. After several rotation periods, the velocity normal to the step becomes very small and vortices are no longer crossing the step. Also a relatively high velocity parallel to the step is seen in the case of a step topography, however, in the case of a ridge topography, this parallel velocity is absent.
# Table of contents

1. Introduction ............................................................................................................................................ 2

2. Theory ....................................................................................................................................................... 5
   2.1. Equations of motion in a rotating frame ............................................................................. 5
   2.2. Potential vorticity ......................................................................................................................... 6
   2.3. Self organization ........................................................................................................................... 7
   2.4. Ekman boundary layer ............................................................................................................... 8

3. Experimental setup .............................................................................................................................. 9
   3.1. Basic setup and experimental procedure ............................................................................ 9
   3.2. Tank and bottom topography configurations ........................................................................ 10
   3.3. Image capturing and particle image velocimetry method .......................................... 12

4. Results ..................................................................................................................................................... 13
   4.1. Flat bottom .................................................................................................................................... 13
       4.1.1. Evolution of vorticity ........................................................................................................ 13
       4.1.2. Enstrophy-energy decay ................................................................................................. 14
   4.2. Step topography .......................................................................................................................... 17
       4.2.1. Evolution of vorticity ........................................................................................................ 17
       4.2.2. Enstrophy-energy decay ................................................................................................. 18
       4.2.3. Velocity above domain boundary ................................................................................ 21

5. Discussion and conclusions ............................................................................................................ 28
   5.1. Enstrophy-energy decay .......................................................................................................... 28
   5.2. Velocity above domain boundary ........................................................................................ 29

Appendix ......................................................................................................................................................... 30

References ...................................................................................................................................................... 39
1. Introduction

In three dimensions, turbulence is characterized by the break-up of vortex structures into increasingly smaller structures, finally leading to dissipation at the smallest length scales. In two dimensions, however, this decay process shows a completely different picture. Instead of vortices breaking apart into smaller structures, in 2D turbulence vortices tend to merge, leading to larger vortex structures. This process is generally referred to as the inverse energy cascade.

In nature, two-dimensional flows occur in large-scale oceanic and atmospheric flows. Due to the rotation of the earth and the fact that the horizontal length scales (of order $1000 \text{ km}$ in the atmosphere) are much larger than the vertical length scales (of order $10 \text{ km}$ in the atmosphere), these flows are quasi-two-dimensional. In the atmosphere, the merging of high or low pressure areas, which are essentially very large vortex structures, can be seen regularly in weather bulletins. In the oceans, similar large scale vortex structures can be observed. The Gulf Stream is a strong current of hot water flowing north along the eastern coast of North America. Due to the rotation of the earth, it is deflected eastward, transporting hot water all the way to Europe. A branch of this current is responsible for keeping the sea free of ice all the way up to the Barents Sea and Svalbard. These oceanic currents obviously have a large influence on the climate worldwide.

On a scale of about $100 \text{ km}$ horizontally and $1 \text{ km}$ vertically, mesoscale in oceanographic terms, quasi-2D vortices exist in the Gulf of California at the west coast of Mexico. The gulf is a long and narrow basin between the peninsula of Baja California and the Mexican mainland. It is about $1000 \text{ km}$ long and $150 \text{ km}$ wide. It is connected to the open ocean at the narrow southern end. It can be seen in Figure 1 that vortices exist with scales up to the width of the gulf. The mixing of seawater and therefore the dispersion of nutrients is dependent on this flow. The gulf has a highly varied bottom topography: the depth at the entrance of the gulf is over $3000 \text{ m}$, while the bay ends in the shallow river delta of the Colorado river. When a vortex column moves over a topographic feature, it is stretched or compressed, which influences the dynamics of the flow. The bottom topography has a large influence on the formation of large scale vortices as was shown in numerical experiments by Bretherton & Haidvogel (1976).

Apart from being quasi-2D, the flow in the Gulf of California is determined by two features: it is confined between the peninsula and the mainland and there is a bottom topography. The first condition, i.e. the confinement, was studied by Maassen (2000). This study showed that in a square domain, the final state of the decay process is a circular vortex of about the same size as the domain. When a rectangular domain with a higher aspect ratio is used, a line of still more or less circular vortices will form. The influence of topography was mainly studied by considering the interaction of a dipole with topography, e.g. by Zavala Sansón & van Heijst (2002), Tenreiro, Zavala Sansón & van Heijst (2006) and Zavala Sansón (2007).
A combination of quasi-2D turbulence and (random) bottom topography was studied numerically by Bretherton & Haidvogel (1976) as mentioned before. It was found that the flow tends to a steady state with the streamlines parallel to contours of constant depth. However, this was not verified experimentally. Tenreiro et al. (2010) performed a qualitative experimental study on quasi-2D turbulence above a step topography and also performed numerical experiments on the same setup.

The goal of this project is to investigate the effect of a discontinuous topography on the self-organization process of quasi-two-dimensional vortices. A quasi-two-dimensional
flow is created in a deep fluid layer on a rotating table, simulating the rotation of the earth. Quantitative data is obtained using tracer particles and particle image velocimetry. The effect of a ridge and a step topography with different step heights is investigated, as well as the effect of the domain size on the evolution of the flow. These topographies are an extreme simplification of the real flow in the Gulf of California, but nevertheless, they give insight in the behaviour of confined quasi-2D flow in the presence of topography.

This report is organized as follows. In Chapter 2, a short introduction to the theoretical background is given. The experimental setup and procedure are explained in Chapter 3. The results obtained with the experiments are discussed in Chapter 4. Finally, in Chapter 5, the conclusions are presented and a comparison is made with other research.
2. Theory

2.1. Equations of motion in a rotating frame

The motion of a fluid in a rotating frame is described by the conservation of mass and the Navier-Stokes equation with Coriolis and centrifugal terms:

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + 2\Omega \times \mathbf{v} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mathbf{v},
\]

(1)

where \( \mathbf{v} \) is the velocity in the rotating frame, \( \Omega \) is the angular velocity of the frame, \( \rho \) the density of the fluid, \( P \) the reduced pressure, which includes the centrifugal force and gravitation, and \( \nu \) the kinematic viscosity. A derivation of this equation can be found in van Heijst (2004), Kundu & Cohen (2004) or Batchelor (1967).

We can make the equation dimensionless by introducing the following variables

\[
\mathbf{v} = U \hat{\mathbf{v}}, \quad P = \rho \Omega UL \bar{P}, \quad t = \frac{\bar{t}}{\Omega}, \quad \mathbf{r} = L \bar{r},
\]

(2)

where \( U \) and \( L \) are a typical velocity and length scale and \( \Omega \) is the angular velocity of the rotating frame. The Navier-Stokes equation now becomes

\[
\frac{\partial \hat{\mathbf{v}}}{\partial \bar{t}} + Ro (\hat{\mathbf{v}} \cdot \nabla) \hat{\mathbf{v}} + 2\mathbf{k} \times \hat{\mathbf{v}} = -\nabla \bar{P} + Ek \nabla^2 \hat{\mathbf{v}},
\]

(3)

where \( \mathbf{k} \) is the unit vector in the direction of \( \Omega \) and the Rossby and Ekman number are given by

\[
Ro = \frac{U}{\Omega L}, \quad Ek = \frac{\nu}{\Omega L^2}.
\]

(4)

If both the Rossby and Ekman number are small and the flow is quasi-stationary, this equation reduces to

\[
2\mathbf{k} \times \hat{\mathbf{v}} = -\nabla \bar{P},
\]

(5)

which is called the geostrophic balance. When we take the curl of this equation, we can derive that

\[
\frac{\partial \mathbf{v}}{\partial z} = 0.
\]

(6)
This is known as the Taylor-Proudman theorem and it states that the flow is independent of the axial coordinate \( z \).

The equations of motion can also be written in terms of the vorticity \( \omega = \nabla \times \mathbf{v} \) by taking the curl of equation 1 and taking \( \omega_a = \omega + 2\Omega \):

\[
\frac{D\omega_a}{Dt} = \omega_a \cdot \nabla \mathbf{v} - \omega_a \nabla \cdot \mathbf{v} + \frac{\nabla \rho \times \nabla P}{\rho^2} + \nu \nabla^2 \omega_a, 
\]

in which \( D/Dt \) is the horizontal material derivative. The vorticity may change due to four mechanisms: tilting and stretching (terms 1 and 2), baroclinic production (term 3) and diffusion (term 4). When the flow is two-dimensional, terms 1 and 2 are equal to zero. If the fluid is also homogeneous, term 3 is also zero. Finally, if the flow is inviscid, the equation reduces to an equation describing the conservation of the absolute vorticity \( \omega_a \).

### 2.2. Potential vorticity

Using the Taylor-Proudman theorem and neglecting viscosity, we can write the components of equation 1 as

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -\frac{1}{\rho} \frac{\partial P}{\partial x}, \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -\frac{1}{\rho} \frac{\partial P}{\partial y}, \\
0 = -\frac{1}{\rho} \frac{\partial P}{\partial z} - g. 
\]

Here \( f = 2\Omega \) is the Coriolis parameter. Cross-differentiating and subtracting the equations for \( u \) and \( v \) gives

\[
\frac{1}{f + \omega_z} \frac{D}{Dt} (f + \omega_z) + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0. 
\]

The divergence term can be eliminated using the continuity equation for the height \( h \) of the water column

\[
\frac{1}{h} \frac{Dh}{Dt} + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, 
\]

which leads to the conservation of potential vorticity

\[
\frac{D}{Dt} \left( \frac{f + \omega_z}{h} \right) = 0. 
\]
If $f$ is a constant, $\omega_z$ decreases if $h$ decreases and vice versa. When, for example, a vortex column moves from a region with a shallow fluid layer to a deeper region, the column is stretched. Since the potential vorticity is conserved, the vorticity of the column increases.

2.3. Self-organization

In order to characterize the flow, we define the energy density and enstrophy density as

$$
E = \frac{\rho}{2LW} \iint (u^2 + v^2) \, dx \, dy,
$$

$$
Z = \frac{\rho}{2LW} \iint \omega_z^2 \, dx \, dy,
$$

(12)

where $L$ and $W$ are the length and width of the surface over which the integral is taken. For an inviscid, two-dimensional flow, the energy and enstrophy are both conserved quantities, see Pedlosky (1979). The energy and enstrophy can also be written in spectral form

$$
E(t) = \int_0^\infty \hat{E}(k, t) \, dk,
$$

$$
Z(t) = \int_0^\infty k^2 \hat{E}(k, t) \, dk.
$$

(13)

Here $\hat{E}(k, t)$ represents the spectral energy density in the wave number range $[k, k + dk]$ at time $t$. As shown in Figure 2, if the initial energy spectrum has a peak around $k_0$, the energy is transferred to other wave numbers due to interactions in the flow. The peak becomes wider, and because of the conservation of energy it will become lower. If the broadening of the spectrum would be symmetric around $k_0$, the total enstrophy would increase. Therefore the maximum of the peak also shifts towards smaller values of $k$, which indicates that the size of the structures $l \sim 1/k$ increases. This self-organisation process continues until the size of the structures is bounded by the size of the domain.

Figure 2: Due to interactions in the fluid, the energy, which is initially peaked around $k_0$, is transferred to other wave numbers and the peak becomes wider. Due to the conservation of energy, it also becomes lower. Due to the simultaneous conservation of enstrophy, the peak also shifts towards smaller wave numbers.
The ratio of enstrophy and energy is a measure of the average length scale in the flow, i.e.

\[
\frac{Z}{E} \sim k^2 \sim \frac{1}{l^2}.
\]  

(14)

As the flow organizes itself into larger structures, this ratio will decrease. In previous research by e.g. Maassen (2000) and Tenreiro et al. (2010), this ratio is found to be proportional to \(t^{-\alpha}\). Here \(\alpha\) is a coefficient which may differ for different initial and boundary conditions as discussed by Maassen (2000). A larger value for \(\alpha\) implies a faster growth of the vortex structures.

2.4. Ekman boundary layer

At the bottom of the tank a boundary layer with thickness \(\delta = \sqrt{v/\Omega}\), called the Ekman layer, is formed due to the no-slip boundary condition. Due to the presence of the Ekman layer, the flow decays exponentially with a decay time called the Ekman timescale:

\[
T_{Ek} = \frac{h}{\sqrt{\nu \Omega}}.
\]  

(15)

The Ekman layer produces a vertical velocity in the flow outside the boundary layer, which depends on the sign of the vorticity:

\[
w = \frac{1}{2} \omega \sqrt{E_k},
\]  

(16)

as shown in van Heijst (2004). In a cyclonic vortex, the Ekman layer results in an upward flow, while in an anticyclonic vortex, the flow is downward. Because of this, the Ekman layer induces an asymmetry between cyclonic and anticyclonic vortices. A conventional model for the inclusion of the Ekman decay in the evolution of the relative vorticity is

\[
\frac{D\omega}{Dt} = -\frac{1}{2} \sqrt{E_k} f \omega,
\]  

(17)

Here, only the linear part of the Ekman suction (equation 16) is taken into account. Zavala Sansón & van Heijst (2000) showed that the inclusion of non-linear terms leads to the addition of an extra term \(-1/2 \sqrt{E_k} \omega^2\) to equation 17. This term shows that a cyclonic vortex \((\omega > 0)\) decays faster than an anticyclonic vortex. However, it was shown by Kloosterziel & van Heijst (1991) that anticyclonic vortices in a rotating tank are more unstable than cyclonic vortices. Therefore, in rotating quasi-2D turbulence, coherent cyclonic vortices will dominate the flow.
3. Experimental setup

The experiments were performed in a tank filled with water, which was placed on a rotating table. The flow was generated by the horizontal translation of a vertical grid through the fluid. Several tank configurations and bottom topographies have been used to study their effect on the evolution of the flow. Quantitative information about the flow was obtained by particle image velocimetry (PIV).

3.1. Basic setup and experimental procedure

In Figure 3, the setup is shown schematically. A table rotates about a vertical axis with an angular velocity $\Omega = 0.5 \, \text{rad/s}$. A rectangular tank was filled with water up to a height of $h = 20 \, \text{cm}$ and was placed on the rotating table. A co-rotating camera was mounted above the tank. In order to visualize the flow, the free surface of the water was seeded with tracer particles with a diameter of $250 \, \mu m$. The surface of the water is illuminated with slide projectors mounted on two sides of the tank.

Using an electric motor, a vertical grid is pulled through the tank in the horizontal direction in order to generate vortices. The grid consists of 8 PVC bars with a rectangular cross section with a width of $4 \, \text{cm}$ and a thickness of $5 \, \text{mm}$. The length of the bars is adjusted so that there is a space of about $0.5 \, \text{cm}$ between the lowest end of the bars and the bottom of the tank. In this way, the fluid has the same initial energy density in both the deep part and the shallow part of the tank. The grid is pulled through the tank with a velocity $U_{grid} = 75 \, \text{mm/s}$. When the grid reaches the other side of the tank, it is pulled back again through the fluid, after which it is removed from the tank. The Reynolds number $Re = U_{grid} L/\nu$ based on the width of a bar of the grid is about 3000. For this Reynolds number, a Von Kármán vortex street will form in the wake of the grid bars. The width of the grid bars determines the initial size of the vortices. For this particular grid, vortex structures with an initial diameter of $\sim 5 \, \text{cm}$ are formed. The translation velocity determines the strength of these vortices. Since the energy of the flow decays exponentially due to the Ekman layer, a high initial energy density is preferred. There is, however, a practical limit on the velocity of the grid. When the grid is translated through the tank, bow waves will be formed. When the velocity of the grid is increased, the height of these bow waves will increase. These waves disturb the two-dimensional nature of the flow and should therefore be avoided.

After the tank is filled with water, it is left for at least two days, so that the temperature of the water can adjust to room temperature. If there is a difference between the water temperature and the room temperature, convection cells are formed, which disturb the flow. Before the start of the experiments, the surface of the water is cleaned using a drop of soap and tracer particles are added to visualize the flow. Next, the table rotation is started and the fluid is allowed to spin up to solid body rotation for about 45 minutes. Subsequently, the grid is translated through the fluid and removed from the tank vertically, after which the flow is free to evolve. The evolution of the flow is recorded with the camera for 15 minutes. The experiments are analyzed 1 rotation period.
\[ T = \frac{2\pi}{\Omega} \] after the grid is removed, so that most of the surface waves and other non-two-dimensional motion have damped out.

Figure 3: Schematic side view of setup. A tank filled with water is placed on a rotating table. A grid (gray bars) is pulled through the tank using a cart on rails. After the forcing, the grid is removed to allow the flow to evolve without interference. The surface of the fluid is illuminated using slide projectors on both sides of the tank. Tracer particles on the surface are recorded by a camera above the tank. In order to illuminate only the surface of the tank, black cardboard sheets are used to block the light a few centimetres below the surface. All components shown here are fixed to the rotating table. The inset shows a top view of the tank and the grid. The grid is translated through the tank and back, after which it is removed.

3.2. Tank and bottom topography configurations

A number of different tank and bottom topography configurations have been used. An overview is given in Table 1 and in Figure 4. The experiments were performed in both a square \( 1 \times 1 \) tank and in a rectangular \( 1 \times 0.5 \) tank. These tanks were used with either a flat bottom, a step topography with different heights or a ridge. In this way, square or rectangular subdomains are created of sizes \( 1 \times 1 \), \( 1 \times 0.5 \) and \( 0.5 \times 0.5 \). The step or ridge was placed in the tank parallel to the direction of forcing. When a step is used, the tank is divided in a deep and a shallow part. The fluid
depth of the deep part, i.e. subdomain A, is the same in all experiments, viz. 20 cm. In the shallow part, i.e. subdomain B, the fluid depth is 17 cm or 15 cm, depending on which step is used. Both tanks may also be divided in two equal domains by placing a 5 cm high ridge; the fluid depth in both subdomains is then 20 cm. The ridge has a width of 5 mm, so compared to the width of the tank and the size of the vortices, it is negligibly thin.

The different configurations are labelled using the following convention: \{tank, topography\} or \{tank, topography, domain\}. Here ‘tank’ can be 1, 2 or 3 for the \(1 \times 1 m\), \(1 \times 0.5 m\) or \(0.5 \times 0.5 m\) tank, respectively. ‘Topography’ is 0, 3s, 5s, 5r for a flat bottom, 3 cm step, 5 cm step or 5 cm ridge, respectively. ‘Domain’ is either T, A or B to indicate whether we consider data for the total domain, or subdomain A or B.

**Table 1 Overview of different tank and bottom topography configurations used in the experiments.**

<table>
<thead>
<tr>
<th>Name</th>
<th>Tank size (m)</th>
<th>Domain size (m)</th>
<th>Topography</th>
<th>Water height A (cm)</th>
<th>Water height B (cm)</th>
<th>Number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,0}</td>
<td>1 x 1</td>
<td>1 x 1</td>
<td>Flat</td>
<td>20</td>
<td>n/a</td>
<td>13</td>
</tr>
<tr>
<td>{1,3s}</td>
<td>1 x 1</td>
<td>1 x 0.5</td>
<td>3 cm step</td>
<td>20</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>{1,5s}</td>
<td>1 x 1</td>
<td>1 x 0.5</td>
<td>5 cm step</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>{1,5r}</td>
<td>1 x 1</td>
<td>1 x 0.5</td>
<td>5 cm ridge</td>
<td>20</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>{2,0}</td>
<td>1 x 0.5</td>
<td>1 x 0.5</td>
<td>Flat</td>
<td>20</td>
<td>n/a</td>
<td>15</td>
</tr>
<tr>
<td>{2,3s}</td>
<td>1 x 0.5</td>
<td>0.5 x 0.5</td>
<td>3 cm step</td>
<td>20</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>{2,5s}</td>
<td>1 x 0.5</td>
<td>0.5 x 0.5</td>
<td>3 cm step</td>
<td>20</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>{2,5r}</td>
<td>1 x 0.5</td>
<td>0.5 x 0.5</td>
<td>5 cm ridge</td>
<td>20</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>{3,0}</td>
<td>1 x 0.5</td>
<td>0.5 x 0.5</td>
<td>20 cm wall</td>
<td>20</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

![Figure 4: Overview of tank and bottom topography configurations. The top part of the figure shows a top view of different tanks used. The dotted line indicates the position of the step or ridge, if present. In the lower part of the figure, a side view of the different topographies and their placement in the tank are shown. Here, the dotted line indicates the water surface.](image)
3.3. Image capturing and particle image velocimetry method

The technique of particle image velocimetry (PIV) from PIVTec was used to retrieve the horizontal velocity field. Using the method of finite differences, the derivatives of the velocity field are calculated. With the velocity vectors and their derivatives, relevant quantities such as the vorticity, energy and enstrophy can be calculated. The camera captures images at a resolution of 1024 x 1024 pixels and is mounted above the tank so that the largest 1 m x 1 m tank fills the entire image. In this setup, 1 pixel roughly corresponds to 1 mm. PIV analysis is performed using interrogation areas of 32 x 32 pixels and 50% overlap. Since velocities vary over a large range during an experiment, care has to be taken in choosing a suitable frame rate. At the beginning of an experiment, typical maximum velocities are ~50 mm/s, while at the end of the experiment, after about 40 rotation periods, the typical velocity has decreased by a factor of 10. For PIV to work accurately, the displacement of the particles between two sequential frames should be less than the window size, but more than a single pixel. As a compromise, a frame rate of 5 s\(^{-1}\) was chosen, so the displacement of particles between two sequential frames is about 10 pixels at the start of the experiment and about 1 pixel at the end.
4. Results

In this chapter, the results of the experiments are presented and analysed. We first present the results related to the flat bottom. The step and the ridge topography will be discussed in Section 4.2.

4.1. Flat bottom

Three experimental series with different horizontal domain sizes were performed for the flat bottom, i.e. \{1,0\}, \{2,0\} and \{3,0\}. These cases serve as a reference to study the influence of bottom topography on the evolution of 2D vortices.

4.1.1. Evolution of vorticity

In Figure 5 the time evolution of the vorticity is given for an experiment in the series of \{1,0\}. The vorticity distribution is given at six moments in time. The first plot, $t = 0$, shows the vorticity distribution at one rotation period $T$ after the grid is removed. It can be seen that in this initial stage of the evolution, the flow is characterized by vortex structures smaller than ca. 10 cm. Subsequently, these structures merge and grow in size. It can be seen that at $t = 7 T$, the smallest structures have already merged into larger structures. Also note that the structures with positive vorticity are more coherent than those with negative vorticity, as discussed in Section 2.4. During the first stage of the experiment, the vortices move around due to interactions with other vortices. Because the energy of the vortices decreases in time, the mobility of the structures decreases. After about $t = 30 T$, the structures are more or less frozen in place. It can be seen that in this 'final' state, the number of vortices has reduced by ca. a factor 10, while the characteristic length scale of the vortices has increased about a factor 3. After $t = 40 T$, the structures have become so weak that they can hardly be distinguished from the background disturbances. Due to the formation of convection cells, the structures fall apart.
4.1.2. Enstrophy-energy decay

The decay of energy and enstrophy for the case \{1,0\} is shown in Figure 6. The values plotted here are an ensemble average over all experiments in the series of \{1,0\}. The energy and enstrophy decay almost three orders of magnitude. At the end of the experiments the decrease of especially the enstrophy comes to a halt. This is when the background flow, mainly convection cells, starts to dominate. The convection cells impose a lower limit on the energy and enstrophy.
Figure 6: Energy and enstrophy decay for the case \{1,0\}. Both energy and enstrophy are normalized at \( t = 0 \). Time is measured in rotation periods, where \( t = 0 \) corresponds to 1 rotation period after the grid is removed.

As described in Section 2.3, the ratio between the enstrophy and the energy is an important parameter to characterize the global flow. In Figure 7, the enstrophy-energy ratio is shown for the case \{1,0\}. For the total tank, \( \{1,0,T\} \), the decay of the ratio \( Z/E \) is small initially and this decay becomes stronger as the self-organization process comes into play. Between \( t \sim 5 \, T \) and \( t \sim 45 \, T \) the ratio \( Z/E \) shows a power law behaviour \( t^{-\alpha} \), represented by a straight line in a log-log plot. It is difficult to determine the power-law exponent \( \alpha \) very precisely, since the decrease of \( Z/E \) gradually becomes larger over the entire range. For the case \{1,0\}, the power-law exponent \( \alpha \approx 0.50 \). After \( t \sim 45 \, T \), the convection cells start to dominate as discussed before and the flow which we are interested in cannot be distinguished anymore. This last stage is characterized by an increase of the ratio \( Z/E \), which indicates that the size of the structures decreases.

In Figure 7, the ratio \( Z/E \) is shown for \( \{1,0,T\} \), \( \{1,0,A\} \) and \( \{1,0,B\} \). In this case, subdomains A and B represent two equal halves of the tank. It can be seen that the ratio \( Z/E \) for subdomains A and B is not equal, but varies around the ratio \( Z/E \) corresponding to the total tank T. Above a flat bottom, vortices are free to move from one side of the tank to the other. When in one experiment a strong vortex crosses from one side to the other, this will show up in the ensemble average. It can be expected that when an infinite number of experiments is performed, the ratio \( Z/E \) related to A, B and T will be equal. Therefore, in the rest of this report, whenever we consider a case with a flat bottom, the ratio \( Z/E \) corresponding to the total domain is considered.
Later in this report, the ratios for different cases will be compared. Although the forcing of the flow is always performed in the same way, the initial enstrophy and energy density slightly differ between experiments. Therefore, the ratio $Z/E$ is normalized at $t = 2T$. This normalization has no influence on the power-law exponent. Since we are mainly interested in this exponent, the normalization can be chosen arbitrary. For $t \leq 2T$, the topography has no large influence on the flow, which will be shown later, whereas for $t \geq 2T$, the large fluctuations, e.g. due to surface waves, seen in the first stages of the experiment have damped out.

Three series of experiments have been performed with a flat bottom, each with a different tank size. In Figure 8, the enstrophy-energy ratio for the ensemble average of each of the series is plotted. It can be seen that the power-law exponents for the different tank sizes are very different, where the highest decay rate corresponds to the largest tank, i.e. $\{1,0\}$. For $\{1,0\}$, which was already discussed before, the power-law exponent is $\alpha = 0.50$. Series $\{2,0\}$ and $\{3,0\}$ give $\alpha = 0.38$ and $\alpha = 0.17$, respectively.

Since the scale of the structures $l \sim \sqrt{E/Z}$, a stronger decay of the ratio $Z/E$ indicates a faster growth of the vortex structures and vice versa. In a smaller tank, vortex structures will encounter the walls sooner than in a large tank, which hinders the
growth of these structures. This could explain the smaller decay rates for smaller sized tanks.

In a different reasoning, the same conclusion can be drawn. Due to the no-slip condition at the wall, a positive vortex near a wall will generate a patch of negative vorticity between the vortex and the wall and vice versa. Thus enstrophy is produced along the walls. Since a smaller tank has relatively more length of wall compared to its surface area, the enstrophy decay rate will be smaller for smaller tanks. Since energy is dissipated along the walls, the energy decay rate will be larger in a smaller tank. This results in a decay rate of the enstrophy-energy ratio which is smaller for a small tank.

![Figure 8](image_url)

*Figure 8: Normalized enstrophy-energy ratio for three different tank sizes and a flat bottom. The three straight lines correspond to the slope of the curve: $t^{-0.50}$ for $\{1,0,T\}$, $t^{-0.38}$ for $\{2,0,T\}$ and $t^{-0.17}$ for $\{3,0,T\}$.***

### 4.2. Step topography

For both tank 1 and 2, experiments were performed with three different discontinuous topographies, namely a 3 cm step, a 5 cm step and a 5 cm ridge. In this section, the evolution of the vorticity, the ratio $Z/E$ and the velocity above the step will be discussed and compared to the flat bottom cases.

#### 4.2.1. Evolution of vorticity

In Figure 9, a similar picture as Figure 5 is shown, but now for an experiment in the \{1,3s\} series. The decay process starts similar as in the case with a flat bottom, but after some time, two separate subdomains become visible. Before $t \sim 10 \ T$, vortices are still strong enough to cross the step dividing the deep (left) and shallow (right) part. Due to
the Ekman decay, the vorticity decreases. Since vortices with a low vorticity have a smaller Rossby number, the Taylor-Proudman theorem holds more strongly. Starting from $t = 14 T$, a boundary between the deep and shallow part becomes visible at $x = 0.5 \, \text{m}$. This boundary becomes even more clear when looking at movies of the time evolution of the vorticity, since the vortex structures will not cross this line.

![Figure 9](image_url)

Figure 9: Vorticity evolution in time of an experiment in the \{1,3s\} series. Axes are the same as in Figure 5. The left part of the figure is the deep part, while the right part is shallow. The step is located at $x = 0.5 \, \text{m}$.

### 4.2.2. Enstrophy-energy decay

The ratio $Z/E$ for the \{1,3s\} series is plotted in Figure 10. When we compare these results to those of the flat bottom case in Figure 7, two major differences can be seen. First, the power-law exponent, which was $\alpha \approx 0.50$ in the flat bottom case is notably different, i.e. $\alpha \approx 0.38$. The only change in setup between these experiments was the addition of a 3 cm step topography. The size of the tank and the initial flow are the
same. Another difference is seen when comparing the ratio $Z/E$ in the two subdomains A and B. In the case of $\{1,0\}$ there was no actual division in subdomains and the ratio $Z/E$ for the two ‘subdomains’ could be seen varying around the ratio for the total tank $T$. In the present case, the water depth in subdomain A is 20 cm, while the water depth in subdomain B is 17 cm. In the initial stage of the experiment, the decay in these subdomains is identical, which can be seen because the ratios for both subdomain A and B coincide with that for the total tank $T$. After about $t = 10T$, the shallow part (B) shows a slightly stronger decay of the ratio $Z/E$ than the deep part (A).

Figure 10: Enstrophy-energy ratio in a log-log plot against time for $\{1,3s\}$. $T$ gives the decay of the total tank, while A and B give the decay of, respectively, the deep and shallow subdomains. The power law $t^{-0.38}$, which represents the slope of the $Z/E$ ratio between $t = 10T$ and $t = 40T$, is also plotted.

In order to compare the results for a step or ridge topography with the results for a flat bottom, the evolution of $Z/E$ for the cases with a step or ridge topography is plotted together with the evolution of $Z/E$ for the flat bottom case, which was already shown in Figure 8. For $\{1,3s\}$, $\{1,5s\}$ and $\{1,5r\}$, i.e. a square tank divided into two rectangular subdomains of $1 m \times 0.5 m$, the ratio $Z/E$ is plotted in Figure 11. In Figure 12, similar plots are shown for $\{2,3s\}$, $\{2,5s\}$ and $\{2,5r\}$, i.e. a rectangular tank divided into two square subdomains of $0.5 m \times 0.5 m$. For the step configurations (3s and 5s), only the evolution of $Z/E$ for the deep part (A) is plotted, so that the fluid depth is the same for all graphs. In the case of a ridge topography (5r), the fluid depth is the same in both subdomains. Since this setup is symmetric, the ratio for the total tank (T) is used.
It can be seen that for a fixed tank geometry, there is no large difference in the decay of $Z/E$ between the different topographies. The ratio $Z/E$ shows the same power-law behaviour for a small step (3s), a large step (5s) or a ridge (5r). There is, however, a significant difference compared to the evolution of $Z/E$ in the same tank with a flat bottom. This is the case for both the square tank (1) and the rectangular tank (2).

A remarkable result is that when a step or ridge is placed in the tank, the $Z/E$ ratio behaves as if there was a fixed boundary at the location of the step. The power-law exponent for {1,3s}, {1,5s} and {1,5r} is $\alpha \approx 0.38$. This does not agree with the flat bottom case in the same tank {1,0}, but it does agree with the flat bottom case in the half sized tank {2,0}. In other words, the evolution of $Z/E$ is the same as if there was a wall at the location of the step. Similarly, the ratio $Z/E$ for {2,3s}, {2,5s} and {2,5r} shows the same power-law behaviour as {3,0}, which is again half the size of tank 2. Although there is no physical boundary between the two subdomains, the vortices are unable to cross the step. The evolution of the size of the vortex structures is influenced by this virtual wall as if it was a real wall. Therefore the ratio $Z/E$, which is actually a measure of the size of the vortex structures, behaves as if the tank was only half the size.

Figure 11: Enstrophy-energy ratio for {1,3s,A}, {1,5s,A} and {1,5r,T} together with the cases shown in Figure 8.
4.2.3. Velocity above domain boundary

Another way to investigate the influence of a step or ridge topography on the evolution of the flow is by measuring the velocity above the step. In the horizontal velocity field, we can distinguish between two velocity components, one in the direction normal to the step ($u$) and the other parallel to the step ($v$). In the following, these velocity components will be referred to as the ‘normal velocity’ and the ‘parallel velocity’, respectively. The Taylor-Proudman theorem dictates that the fluid in a vertical column has the same velocity over the entire height of the column. When fluid in the deep part is moving towards the step, i.e. in the positive $u$-direction, it will be blocked by the step, i.e. the normal velocity will be 0 at the location of the step. Due to the Taylor-Proudman theorem, this does not only hold for the flow along the bottom of the tank, but for all heights. The same holds for the velocity parallel to the step. Due to the no-slip condition along the wall of the step, the parallel velocity above the step should also be 0. Of course this is only true in the ideal case when both the Rossby and Ekman number are equal to zero.

If an ensemble average of the velocity components is calculated, opposite velocities in different experiments would cancel out. Therefore the root mean square (RMS) values of the velocity components over the ensemble are used. Since we take the RMS of the velocity components, this only gives the magnitude of the velocity component, not the sign. Because this is done separately for the velocity components in the $x$- and $y$-direction, we can still distinguish between the magnitude of the parallel velocity and the
normal velocity. Since the energy decays over almost a factor $10^3$, as shown in Figure 6, the RMS velocity components are divided by the square root of the energy, i.e. the average velocity, at that time. In this way, the average normalized velocity in the entire tank equals 1, so it can be expected that the average for one component of the normalized velocity is 0.7.

In Figure 13, the normalized RMS velocity components at $x = 0.5\, m$, i.e. directly above the step, are plotted for the case $\{1,3s\}$. First we take a look at the normal velocity $u_{\text{step}}$, i.e. the top part of the figure. It can be seen that in the initial stage of the experiment, the velocity ranges from about 0.5 to 0.8. Therefore the normal velocity above the step at the beginning of the experiment is about average, i.e. 0.7 as discussed above. This means that there are vortices directly above the step and that fluid is crossing the step. After about $t = 10\, T$, the normal velocity above the step is almost 0. In this stage of the experiment, vortices are unable to cross the step and we have two separate subdomains. As said before, after $t \sim 40\, T$, the flow has become so weak that the background flow is of the same order of magnitude as the flow which is a result of the forcing. Due to wind friction at the surface, a large clockwise motion can be seen even if the setup is left spinning for hours. This large scale rotation has only a small absolute velocity, but since the average velocity in the tank is very small, the normalization factor is also very small. This rotation due to wind friction is visible as two bands of high normalized velocity at the edges of the domain. At the top of the domain, i.e. at $y = 1\, m$, there is a flow in the positive $x$-direction, while at the other side there is a flow in the negative $x$-direction.

The lower part of Figure 13 shows the normalized parallel velocity above the step. At $t = 0$, the parallel velocity is in the same range as the normal velocity, about 0.5 to 0.9. However, in contrast with the normal velocity, the parallel velocity increases in time. When the normal velocity reaches its minimum around $t = 20\, T$, the parallel velocity is above average over nearly the entire length of the step. When reviewing the individual experiments, it can be seen that there is a tendency towards a flow in the positive $y$-direction, so a flow with the shallow part of the tank on its right hand side. This result agrees with the findings of Tenreiro et al. (2010). However, it does not agree with the prediction made earlier that the parallel velocity should be 0 due to the no-slip condition along the step in combination with the Taylor-Proudman theorem.

In Figure 14, a similar plot is shown for the case $\{1,5s\}$. The plots look very similar and the analysis made before still holds. There is, however, one important difference. The time at which the normal velocity almost disappears is sooner than in the case $\{1,3s\}$. The strong parallel flow also appears sooner. This can be explained by the fact that a higher step can only be crossed by stronger vortices, because they are less influenced by the Taylor-Proudman theorem. Since the initial condition for both experiments is the same, the vortices will become too weak to cross the step sooner in the case of a 5 cm step than for a 3 cm step.
Figure 13: Normalized RMS velocity at $x = 0.5 \, m$ for $\{1,3s\}$. The vertical axis is the $y$-coordinate, the horizontal axis is time in rotation periods. The colour scale gives the RMS velocity divided by the square root of the energy in the entire tank. The top picture gives the velocity normal to the step, while the lower plot is the velocity parallel to the step.

Figure 14: Similar to Figure 13, but now for $\{1,5s\}$. 
The velocity above the step can also be viewed in a different way. Instead of plotting the local (RMS) velocity as a function of the \( y \)-coordinate and time, we now plot the RMS velocity averaged over all \( y \)-coordinates as a function of the \( x \)-coordinate and time. So if we would take an average over the \( y \)-coordinate of the previous graphs, this gives us an average velocity at \( x = 0.5 \). This process is repeated for all \( x \)-coordinates. This is shown in Figure 15 for \{1,3s\} and in Figure 16 for \{1,5s\}.

If we look at the normal velocity \( \frac{\partial u}{\partial x} \), we can see that there are two bands of high velocity separated by an area of low velocity at \( x = 0.5 \), where the step is located. At the sides of the domain, the velocity goes to 0 due to the presence of the solid walls. The left (deep) side of the domain shows higher velocities than the right (shallow) side, because the Ekman decay is weaker for a deeper fluid layer. This is especially clear in Figure 16, where the difference in height between the two subdomains is largest. If we focus on the area around \( x = 0.5 \), i.e. above the step, it can again be seen that the velocity at the start of the experiment is about the same as the average velocity and then decreases to almost 0. It can be seen that the decrease for \{1,5s\} is faster than for \{1,3s\}.

In the right part of Figure 15 and Figure 16, the parallel component of the velocity is plotted. Again, we can see that at the step, there is a high velocity parallel to the step. This 'jet' appears to be in a narrow band just above the step. Another point of interest is the large scale rotation due to wind friction after \( t \sim 40 \). Two bands of high velocity at the side of the domain can be seen in the last part of the experiment. We can also see again that the velocity at the shallow side decays faster than at the deep side. Especially for \{1,5s\}, there is a clear distinction between the shallow and deep side.
Figure 15: Normalized RMS velocity in the tank averaged over the $y$-direction as a function of the $x$-coordinate and time. The horizontal axis is the $x$-coordinate, with the step at $x = 0.5$, the vertical axis is the time in rotation periods. The colour scale gives the RMS velocity averaged in the $y$-direction and divided by the square root of the energy. The left plot gives the $x$-component of the velocity, which is the velocity normal to the step. The right plot gives the $y$-component.

Figure 16: Similar to Figure 15, but now for $\{1,5s\}$. 
In Figure 17 and Figure 18, similar velocity plots are shown for the \( \{2,5r\} \) case. In Figure 17, it can be seen that the normal velocity above the ridge is even lower than when a high step is used. Note that both the ridge and the high step have a height of 5 cm, so the step height cannot be the cause of the difference. The parallel velocity shows an even larger difference between a step and a ridge. Instead of a high parallel velocity above the step, the ridge case shows a low parallel velocity. This is especially clear in Figure 18, where there is an area of low velocity visible between two bands of high velocity not only for the normal, but also for the parallel velocity.

This difference can be explained by realizing that the ridge topography actually consists of two steps, one up and one down. The parallel jet which was discussed before has a tendency to keep the shallow side on its right hand side as shown by Tenreiro et al. (2010). In the case of a ridge, the top of the ridge can be seen as the shallow side for both subdomains. In the left subdomain, the jet would be in positive \( \frac{\partial}{\partial y} \)-direction, as in the case of a step topography. However, in the other subdomain a jet will be formed in the opposite direction, in order to keep the shallow side, i.e. the ridge, on the right hand side. This could explain why in contrast to the case of a step topography, the parallel velocity in the case of a ridge topography is low.
Figure 17: Similar to Figure 13 for series \(\{2,5r\}\)

Figure 18: Similar to Figure 15 for series \(\{2,5r\}\)
5. Discussion and conclusions

The goal of this project was to investigate the influence of a discontinuous topography on the evolution of quasi-2D turbulence. This influence has been investigated in two ways: (1) by considering the time evolution of the ratio $Z/E$, which is a measure of the characteristic length scale of the vortex structures, and (2) by considering the horizontal velocity above the step.

5.1. Enstrophy-energy decay

It is shown that the time evolution of the ratio $Z/E$ depends on the horizontal geometry of the tank. For a $1 \times 1$ tank, the ratio is proportional to the power law $t^{-0.50}$. For a $1 \times 0.5$ tank, this power law is $t^{-0.38}$ and for a $0.5 \times 0.5$ tank, it is proportional to $t^{-0.17}$. An explanation for these differences is that because of the no-slip condition, there is enstrophy production along the walls of the tank. Since a smaller tank has more length of wall in comparison with the surface area, this production term will result in a slower decay of the enstrophy. The energy decay, however, will be larger for small tanks, since energy is dissipated along the walls. Therefore, the ratio $Z/E$ will decay slower in a small tank than in a large tank. Since the characteristic scale of the structures $l\sim\sqrt{Z/E}$, this means that structures in a large tank grow faster than structures in a small tank.

It is seen that when a discontinuous topography is placed in a tank, the ratio $Z/E$ behaves as if there was a fixed wall at the location of the step. When a step or ridge topography is placed so that a $1 \times 1$ tank is divided in two equally sized subdomains, the ratio $Z/E$ is proportional to $t^{-0.38}$. This is the same power law as for a $1 \times 0.5$ tank, which is the size of the subdomains in a $1 \times 1$ tank. The same trend holds for the smaller tank. If the $1 \times 0.5$ tank is divided in two subdomains by a step or ridge, it shows the same power law as a $0.5 \times 0.5$ tank. Due to the Taylor-Proudman theorem and the conservation of potential vorticity, vortices are unable to cross the step, especially those with low vorticity. Therefore, the growth of the vortex structures is hindered by the presence of the step.

Maassen (2000) performed numerical simulation on 2D turbulence in domains of different aspect ratios with no-slip walls. Different power-law coefficients $\alpha$ were found for different initial conditions. For a square domain, power-law coefficients of $\alpha = 0.63$ and $\alpha = 0.48$ were found, where the last coefficient corresponds to experiments with an initial condition which contained larger sized structures. For domains with aspect ratio 1, 2 and 3, the same power-law coefficient $\alpha = 0.48$ was found. However, for the experiments with aspect ratio 3, the initial condition consisted of larger vortex structures than for aspect ratio 2. In the experiments presented in this report, the (absolute) initial size of the vortex structures was the same for all experiments. This could explain why here, a difference is seen between domains with different aspect ratios. The trend that a smaller domain size, i.e. a larger relative initial size of the
structures, leads to smaller values for the power-law coefficient is consistent with the results from Maassen for a square domain.

5.2. Velocity above domain boundary

The horizontal velocity above the step, normal to the step is very low compared to average velocity in the tank. Thus fluid particles are unable to move from one subdomain into the other. This confirms the theory that the step acts as a virtual wall. It is also seen that for a higher step, this effect is reached sooner than for a low step. This indicates that the vortices need to be stronger in order to cross a high step. When we look at the velocity parallel to the step, it is seen that there is a relatively strong flow parallel to the step, keeping the shallow side on the right hand side. This phenomenon is discussed before in e.g. Tenreiro et al. (2010). This shows that, although there is a virtual wall between the two subdomains, there is still interaction between the subdomains, i.e. the virtual wall does not behave as a no-slip wall. However, when a ridge topography is used instead of a step, the parallel velocity above the ridge is low, this is in contrast with the step case. This can be understood by viewing the ridge as a, very narrow, shallow region. In both deep regions, a parallel velocity with the shallow region on the right hand side would form. These velocities are in opposite directions in the two subdomains and therefore cancel out.
Appendix

In this appendix, the results for all experiments are given. Both the ratio of enstrophy over energy and the velocity plots are given. The $E/Z$ plots are plotted in the same way as Figure 7, the velocity plots are presented the same as Figure 13 and Figure 15.

Enstrophy-energy decay plots

{1,0}
See Figure 7.

{1,3s}
See Figure 10.

{1,5s}
Note that there is an asymmetry already of the start of the experiment, the cause for this is unknown.

{3,0} is actually tank 2 with a 20 cm wall in the centre of the tank. Domain A and B thus both represent a 0.5 m x 0.5 m domain.
Velocity above step

\{1,0\}

See Figure 13 and Figure 15.

\{1,3s\}
The ridge used for \{1,5r\} was made of a reflective material and therefore showed bright in the recorded images. For the bulk of the fluid, this was not a problem, however, the PIV software had problems in calculating the velocity directly above the step, because of
the low contrast between the particles and the ridge. It is expected that this has little influence on the ratio $E/Z$. The velocity plots however are unreliable in the region above the step. This shows especially in the first velocity plot.

\{(2,0)\}
\{2,3s\} - u_{\text{step}} (T) \\
\begin{align*}
\text{\begin{tabular}{c c c c c c c c c c c}
0 & 10 & 20 & 30 & 40 & 50 & 60 \\
\hline
0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1
\end{tabular}}
\end{align*}

\{2,3s\} - v_{\text{step}} (//) \\
\begin{align*}
\text{\begin{tabular}{c c c c c c c c c c c}
0 & 10 & 20 & 30 & 40 & 50 & 60 \\
\hline
0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1
\end{tabular}}
\end{align*}

\{2,3s\} - u (T) RMS in y \\
\begin{align*}
\text{\begin{tabular}{c c c c c c c c c c c}
0 & 5 & 10 \\
\hline
0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 & 50
\end{tabular}}
\end{align*}

\{2,3s\} - v (//) RMS in y \\
\begin{align*}
\text{\begin{tabular}{c c c c c c c c c c c}
0 & 5 & 10 \\
\hline
0 & 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 & 50
\end{tabular}}
\end{align*}
See Figure 17 and Figure 18.

The velocity plots have not been made for this case, but they are similar to those for \{1,0\} and \{2,0\} since there is no step present.
References


