MASTER

Beam emission spectroscopy Motional Stark effect diagnostic on TEXTOR

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Master thesis

TU/e Applied Physics – Fusion

Beam emission spectroscopy
Motional Stark effect diagnostic on TEXTOR

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Abstract

The motional Stark effect (MSE) is an essential technique for determining the magnetic field inside a tokamak. The most common method of performing MSE is measuring the polarization angle by using photo-elastic modulators (PEMs), because of the high time resolution. A disadvantage is the fact that not the total spectrum is recorded, but only a single line. We present 3 different techniques that are an alternative to this method, and that do make use of the total spectrum. We can reconstruct the magnetic pitch angle (the ratio of the magnetic fields) therefrom. We will look at the intensity ratio of the different polarized lines in the MSE spectrum, the splitting of the lines in the MSE spectrum, and the actual polarization of these lines. The polarization is measured using linear polarizers or switchable quarter-wave plates. Simulations using these 4 alternative methods (line splitting, ratio and 2 techniques for measuring the polarization) are performed to determine their sensitivity of pitch angle reconstruction. The methods using only the intensity profile (the splitting and intensity ratio) are less effective for determining a realistic pitch angle, due to geometry and broadening effects. When polarizers and/or wave plates are added, the 4 element \([I, Q, U, V]\) Stokes vector spectrum can be reconstructed. The intensity spectrum \(I\) still contains the line ratio and line splitting information, whereas the polarization angle can be determined using the \(U\) and \(Q\) components. The method using 2 switchable quarter-wave plates is a little more sensitive than the method using 4 static polarizers. However, this method has a lower intrinsic time resolution. Nonetheless, a setup using 2 switchable quarter-wave plates was designed and built, because of its spatial advantages. This setup is deployed at the TEXTOR tokamak in Jülich, Germany. The first results show that the setup is capable of measuring the Stokes vector of polarized light, and constructing a magnetic pitch angle with a low time resolution. Although proof-of-principle is given, the performance should be improved in order to make real-time pitch angle and \(q\)-profile reconstructions on TEXTOR.
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Chapter 1

Introduction

Energy demand is a large issue we encounter in the world nowadays. The common used fossil fuels are running out of reserves and with an increasing world population it is likely that the global energy demand will also increase. Greenhouse gases, like carbon dioxide are expelled on combustion of these fossil fuels. Nuclear power is a non-renewable energy source but can supply energy without creating greenhouse gases. However, this is also not favorable because of the long-living radioactive waste that is created by that process. In order to decrease the dependence on these kind of fuels we look for long-lasting energy solutions. These so-called renewable energy sources, e.g., solar-, hydro- and wind energy, can account for a small part of the energy demand nowadays, and it is hard work to increase the participation of these energy sources on the energy market. A new promising renewable energy source is fusion energy. The fusion process does not create long-living radioactive waste nor greenhouse gases, and the fuel supply is very large because it uses isotopes of the most abundant element in the universe, hydrogen.

1.1 Fusion and Plasmas

Fusion creates energy by fusing light-weight particles. Nowadays the most used fusion reactants are deuterium and tritium, both isotopes of hydrogen (see Equation 1.1). Energy is created because the mass after the reaction is less than the mass before.

\[ ^2\text{D} + ^3\text{T} \rightarrow ^4\text{He} \ 3.5 \text{ MeV} + \text{n} \ 14.1 \text{ MeV} \]  

In order to let these particles fuse, they have to overcome the Coulomb repulsion that is present, i.e., have a high enough temperature. The reaction rate versus the temperature is shown for different fusion reactions in Figure 1.1. It can be seen that the reaction with the peak at the lowest temperature (100 keV) is the D-T reaction. However, this does not mean that we need
the temperature to be 100 keV, at a much lower temperature the reaction rate is sufficient to sustain fusion. This is because energy losses also increase with temperature and an optimum is found around 15 keV (175 million Kelvin), or more or less 10 times the core temperature of the Sun. At these temperatures the particles are in the plasma state, charged ions and electrons are not bound to each other any more. Despite the power required to obtain high temperatures it is still possible to generate an excess of power, making fusion an interesting alternative.

To keep the plasma at high temperature it needs to be kept away from the material walls this is called confinement. There are two concepts of confining the plasma, i.e., make sure the hot plasma does not get in touch with the reactor wall: magnetic- and inertial confinement. The focus of this project will lie on the magnetic confinement device. The magnetic confinement device uses a magnetic field to confine the plasma and keep it from the walls. The charged plasma particles will follow these magnetic field lines inside the reactor vessel because of the Lorentz force.

One such a magnetic confinement device is a tokamak and it is the most used fusion reactor for research of fusion as an energy source, see Figure 1.2. The tokamak uses an external toroidal field \( B_{\text{tor}} \), generated by external field coils. However, with only a toroidal magnetic field the plasma is not stable enough to achieve high confinement times. This is because particle drifts occur and the particles are lost to the wall. Therefore, a plasma current \( j \) needs to be induced, which is generated by a large transformer, this current generates a poloidal magnetic field \( B_{\text{pol}} \) and together with \( B_{\text{tor}} \) a helical magnetic field is created. The particle drifts are cancelled out with this resultant helical magnetic field, thus stability is increased.

The \( q \)-profile is a measure for the helicity of the magnetic field lines along the large radius. \( q \) is the ratio a magnetic field line has to travel in the toroidal and poloidal direction before it is on its starting point. This can also be determined by dividing the poloidal by the toroidal flux surfaces, \( \Delta \psi_{\text{tor}} / \Delta \psi_{\text{pol}} \). When the tokamak is assumed to be cylindrically symmetrical, the safety factor \( q \) can be approximated by \( q(r) \), only a function of the inner-radius \( r \) [2],

\[
q(r) = \frac{r B_{\text{tor}}}{R B_{\text{pol}}}. \tag{1.2}
\]

Here, \( R \) is the major radius of the tokamak and respectively \( B_{\text{pol}} \) and \( B_{\text{tor}} \) are the poloidal and toroidal magnetic field.
With this helical field, instabilities due to perturbations can still be present in the tokamak. There are several phenomena not yet fully understood in the present fusion research on tokamaks. For example, plasma transport and transport barriers, or magnetic island formation that may occur in the reactor [2]. These phenomena tend to grow larger on low order $q$ flux surfaces, a so-called reversed $q$-profile helps to improve this [3, 4]. In order to come to grips with these phenomena it is useful to have a recording of the magnetic topology and reconstruct the current profile $j$ therefrom. This can be done by measuring the magnetic field pitch angle $\gamma_p$ and determine the safety factor profile $q$, thus magnetic topology, from this angle [5]. The magnetic pitch angle can be described by the ratio of the poloidal to the toroidal magnetic field,

$$\tan \gamma_p(r) = \frac{B_{pol}(r)}{B_{tor}(r)}.$$  \hspace{1cm} (1.3)

By this definition we see that the magnetic pitch angle and the $q$-profile are related.

In this report we will investigate what techniques are available to determine the pitch angle. Therefore, we need to answer for each method: What is the sensitivity of the method to determine the pitch angle profile with respect to a changing magnetic field, i.e., what changes in the magnetic field, or $q$-profile, can we measure using this technique? This is an important question since the $q$-profile changes throughout the plasma discharge, and it is a parameter we want to know very precisely for constructing the current distribution inside the plasma. Since, the instabilities mostly occur at low $q$, i.e., $q = 1, 3/2$ and 2, we can constrain the sensitivity of
the method. We want to be able to determine the \( q \)-profile with an accuracy of 0.2. There are several methods to do this measurement, which will be explained in the next part.

### 1.2 Diagnostics

The most common method nowadays for measuring the magnetic topology is the motional Stark effect diagnostic (MSE). This is an optical diagnostic, which uses the light that comes from neutral beam atoms (hydrogen), used for heating, upon interaction with a plasma. This light is coming from excited hydrogen and can be observed as the so-called Balmer-\( \alpha \) line \( (\text{H} \alpha) \) at 656.28 nm. The emitted light is polarized and Stark split because of the magnetic field that is present (see Chapter 2). The emission of the beam is split in several Stark components. When a spectrum is recorded as function of wavelength we can extract the following characteristics.

- The splitting of the \( \pi \) and \( \sigma \) lines can be related to the magnitude of the magnetic field.
- The ratio of intensities of the \( \sigma \)-peak and the \( \pi \)-peak can give the direction of the Lorentz electric field and thus the magnetic field [6].
- The ratio of polarization of the linear polarized components of the Stokes vector give the direction of the Lorentz electric field [7].

The first two methods can be performed with only a spectrometer, since only the intensity is needed as a parameter. But the third point gives the most direct link between light emission and magnetic field directions (see Chapter 2). However, techniques for measuring polarization are more complicated.

The most used technique for analyzing the polarized light is by using PEMs (photo-elastic modulators). This method was reported in 1989 for the first time by Levinton [8]. By using PEMs we can measure the polarization of the emitted light. The polarization is then ‘encoded’ into an intensity modulation, which can be measured, by adding a linear polarizer (analyzer) [8, 9]. This method has a high time resolution, a sample frequency of more around 1 kHz. A disadvantage of this technique is that the detectors will collect small amounts of light; a spectrometer cannot be used. An interference filter is placed before the PEMs to select the region of interest in the emission. Because we select only one wavelength, we throw away spectral information, the intensity ratio and split of the lines. Another disadvantage is that it is rather costly (several thousands of dollars). For example, the total neutral beam spectroscopy system of a large tokamak in South-Korea, K-STAR, is even determined to cost 2 million dollars [10] (50% budget for fiber bundles).
The method described, using PEMs, is good performing on current fusion devices. However, obtaining reliable data at future fusion devices will be a challenge, due to the constraints imposed by the harsh nuclear environment. For safety reasons there has to be thick walls stopping the neutrons created during fusion. A mirror labyrinth is created to lead the light through the wall without creating weaknesses in the wall. These mirrors will change the polarization of the light, and over time the properties of the mirrors will change due to influences of the plasma. The inside of the device has a Beryllium wall which reflects light. The direct reflection of the neutral beam is a minor issue, however the light in the tokamak created by Bremsstrahlung gives polarized light when it is reflected on the wall. These two issues, a polarized background and mirrors used to lead the light through the wall, make it more difficult to use PEMs. Extracting all possible information from a single measurement is therefore very desirable, for calibration, correction and direct measurement. We are looking into alternative methods to determine the magnetic pitch angle using the whole available MSE spectrum to circumvent these issues.

By using 4 polarizers we can also measure polarization as function of wavelength. This method is used in the Large Helical Device (LHD) [11]. In the past Soetens implemented this technique on the TEXTOR tokamak, with an added circular polarizer [7]. By using only analyzers, the circular polarization cannot be determined. This is no problem when the optical setup does not involve mirrors [12]. However, the circular polarization can give information about the radial electric field present. The disadvantage of this method is, in order to get the same resolution, on each analyzer channel 4 times as many fibers are necessary. This makes the method more complicated and costlier, and the time resolution is limited by the CCD camera of the spectrometer.

We are looking for a different approach, to circumvent the use of more fibers, and diminish the used space – a hybrid. We will develop a technique which uses instead of two PEMs, two switchable quarter-wave plates. This method is more or less identical to the method of 4 analyzers: a spectrometer will be used to measure the polarization as function of wavelength, but the polarization will be ‘encoded’ into an intensity modulation, with a lower time resolution than PEMs; making it a ‘hybrid’ technique. With these measurements we can reconstruct the Stokes vector using Mueller calculus and determine the pitch angle (see Chapter 2). The advantages of using these are 1) that the costs will be much lower and 2) that not only the polarization of 1 peak will be measured, but also the polarization as function of wavelength.

The aim of this project is to test the sensitivity of the different methods: looking at the split, the ratio of intensity and the polarization using the 4 static polarizers and hybrid case. We will look at which method can reconstruct an acceptable $q$-profile ($\Delta q < 0.2$) from MSE data. The results are presented in Chapter 4 & 5.

We will also develop a system using two quarter-wave plates and determine the performance regarding noise levels of the signal and Stokes vector reconstruction. The system will be tested
at the TU/e and implemented on the TEXTOR tokamak in Jülich, Germany. This is presented in Chapter 6.

In the next chapter the in-depth theory behind this MSE technique will be explained as well as the theory of each method.
Chapter 2

Theory

This chapter will explain the theory behind the motional Stark effect diagnostic (MSE), the theory necessary for building the setup and the reconstruction of the pitch angle profile. First an overview of the emission of light by atoms will be given, and how the Stark and Zeeman effect come about. The motional Stark effect and the methods available for determining the pitch angle will be explained thereafter. At the end the calculations that are needed for simulations and reconstruction of the setup will be presented.

2.1 Hydrogen Line Emission, Stark & Zeeman Effect

Atoms are excited when they undergo collisions. The electron gains energy in this collision and occupies a higher energy level: The energy of an electron in a certain orbit of hydrogen can be described by:

\[ E = \frac{E_{Ry}}{n^2}, \quad (2.1) \]

where \( E_{Ry} \) is the energy of the highest level an electron can have in a hydrogen atom, \(-13.6 \) eV, and \( n \) is the principal quantum number, which describes the orbit the electron is in; the equation doesn’t take into account fine structures. \( n \) can run from 1 to \( \infty \) (see Figure 2.1). The energy, or wavelength, of the emission is described by:

\[ h\nu = E_n - E_m, \quad (2.2) \]

where, \( E_n \) and \( E_m \) are the energies of the levels where the electron relaxes from and to, \( h \) is Planck’s constant and \( \nu \) is the frequency in Hz. For example the emission of \( \langle n = 3 \rightarrow 2 \rangle \) gives light with a wavelength of 656 nm. This is visible light and therefore the most convenient for use with optical applications.

An external field (or perturbation) will induce a (partial) separation of energy levels. Such fields, can be electric or magnetic and the changes in atomic energy levels are explained as the
Stark and Zeeman splitting [13, Chapter 13]. In this report we focus on the Stark splitting. In presence of an electric field, the energy level splits as shown in Figure 2.1, and as a result also the emission line splits in several emission lines as Stark level splits in (2.3).

\[ \Delta \lambda_S = \frac{3e a_0}{2hc} \lambda_0^2 m |E| = 2.7574 \cdot 10^{-8} m |E| \ [\text{nm}], \tag{2.3} \]

where \( \Delta \lambda_S \) is the changed wavelength in nanometers due to the Stark effect, \( m \) is the difference in magnetic quantum number (for \( \pi_{+3} \) to \( \pi_{-3}, m = 6 \)) and \( E \) is the Stark electric field in V/m. The constants \( e, a_0, h, c \) and \( \lambda_0 \) are the electron charge, Bohr-radius, Planck’s constant, speed of light and center wavelength, respectively. For our case, using Balmer-α light, \( \lambda_0 = 656.28 \) nm. The nine peaks are spaced equally with wavelength \( \Delta \lambda_S \) around the Doppler shifted central wavelength.

The light that is emitted by the electron transition is polarized. The polarization state is determined by which transition the electron makes, see Figure 2.1. Lines with \( \Delta m = 0 \) are called \( \pi \) lines, because their polarization is parallel to the electric field. Lines with \( \Delta m = \pm 1 \) are called \( \sigma \) lines, because their polarization is perpendicular (German: \textit{senkrecht}) to the magnetic field. This means that the two polarizations are distinguishable and resolvable on measurement.

Figure 2.1 – On the left, the energy levels of the Lyman and Balmer series. On the right, the degeneracy of the different orbitals. As can be seen 15 lines are possible, but only 9 lines have enough strength for visibility. \( \sigma \) polarization occurs when \( \Delta m = \pm 1 \), \( \pi \) polarization when \( \Delta m = 0 \). Adapted from [14] and [15].
Their relative intensity depends on the direction of the electric field with respect to the line-of-sight.

Also in presence of a magnetic field the lines split, this is called the Zeeman effect. When both a magnetic and electric field are present the polarization and splitting depend on their mutual orientation and strength.

2.2 Motional Stark Effect

Tokamaks have neutral beam injectors installed for heating and diagnosing the plasma. These injected particles are moving at a certain known velocity \( v \). Once they are injected into the tokamak the particles feel the presence of a magnetic field used to confine the plasma, see Figure 2.2. The particles, in their frame of reference, see this magnetic field as an electric field:

\[
E = v \times B + E_s
\]

The Lorentz field, \( E_L = v \times B \),

The Lorentz field \( E_L \) will induce the Stark effect on moving particles, hence the name motional Stark effect. This relation shows that an electric field is present for the plasma particles because they are moving through a magnetic field \( B \) with a velocity \( v \). Therefore, the direction of the local magnetic field can be known since the beam’s velocity direction is known [17]. Because the electric field emerges from the magnetic field, the polarization, the split and the intensity ratio are dependent on the magnetic field.

Figure 2.2 shows that the motional Stark effect not only depends on the Lorentz field but also a radial electric field \( E_{\text{static}} \), this is the electric field the plasma particles exert on the neutral particles. This additional electric field emerges from the plasma velocity and the present pressure gradient. However, the static electric field is typically an order of magnitude smaller than the Lorentz electric field in the beam’s frame of reference and can be neglected in most cases [5].
Also, the contribution of splitting on the energy levels due to the present magnetic field (Zeeman effect) is much smaller than the motional Stark splitting in a tokamak plasma [2]. This means that MSE is dominated by only the Lorentz electric field, $E_L$.

A typical MSE spectrum is shown in Figure 2.3. We can distinguish the spectra because of a Doppler shift due to the neutral particle beam: the full, half and third energy component arise due to the fact that not only $H_1^+$ is accelerated but also $H_2^+$ and even $H_3^+$. These fall apart into H but with less energy [2], and because these hydrogen atoms have less energy they have a different Doppler shift.

![Figure 2.3](image-url) – A typical MSE spectrum with blue-shifted peaks. The internal structure clearly shows the splitting between the $\pi$ and $\sigma$ components. Image adapted from [18].

### 2.3 Magnetic Field Information from the Motional Stark Effect Spectrum

This section will look into three methods to determine the magnetic field from an MSE spectrum.

- The magnitude of the Stark splitting
- The ratio of intensities of the observed $\pi$ and $\sigma$ lines
- The polarization of the $\pi$ and $\sigma$ lines

We will specifically look at the TEXTOR case, where the beam and line-of-sight are in the equatorial plane. Figure 2.4 shows the relevant vectors in the equatorial plane of the tokamak. $\phi$ is the toroidal magnetic field vector, which determines the direction of $B_\phi$. $\mathbf{v}$ is the vector describing the direction of the neutral beam, this vector is known a priori. $\mathbf{k}$ is the vector defining
the line-of-sight. The vector $z$ is perpendicular to the equatorial plane. $l$ is perpendicular to $z$ and $k$, which will be used later on in calculations.

![The different relevant vectors in the tokamak. $k$ is the view vector, $l$ is perpendicular to $k$, $v$ is the velocity vector and $\phi$ is the toroidal magnetic field vector. It has to be noted that all coordinates are in the mid plane of the tokamak, except $z$ which is perpendicular to the mid plane.](image)

**Figure 2.4**

2.3.1 Stark Splitting

The magnitude of the Stark splitting ($\pi$ and $\sigma$) is a consequence of the present electric field (2.4). The larger the electric field, the stronger the splitting of lines, see (2.3). Inverting this relation gives us the magnitude of the electric field, which is related to the magnetic field. For the TEXTOR case $\Delta \lambda_s$ is expressed as,

$$\Delta \lambda_s = 2.76 \cdot 10^{-8} m |v \times B| = 2.76 \cdot 10^{-8} m |vB_\phi (- \sin \alpha + \tan \gamma_p)|.$$  \hspace{1cm} (2.5)

This shows that $\Delta \lambda_s$ is dependent on two functions of the magnetic field, $B_\phi$ and $\gamma_p$. This complicates the inspection of the magnetic field, the measurement has two unknowns. One option is to neglect the diamagnetic (or paramagnetic) contribution of the plasma to $B_\phi$. In that case, $B_\phi$ is approximately the known vacuum field, $B_\phi \propto 1/R$. 

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2.3. Intensity Ratio

When looking at an MSE profile (see Figure 2.3) there is a difference in intensity between the \( \pi \) and \( \sigma \) line(s). This difference can be explained by the fact that the irradiance of Stark’s \( \sigma \) and \( \pi \) components is anisotropic [6]. The ratio between the different lines is determined by atomic physics, and by the direction of the Lorentz electric field with respect to the line-of-sight. Therefore, we can use the anisotropic property of the Stark lines,

\[
I_\pi \propto \sin^2(\theta) f_1(n), \quad I_\sigma \propto (1 + \cos^2(\theta)) f_2(n),
\]

(2.6)

where \( \theta \) is the angle between the Lorentz electric field \( \vec{E}_L \) and the line-of-sight \( \vec{k} \) [6]. The factors \( f_1(n) \) and \( f_2(n) \) are determined by collisional radiative modeling (see next section). \( \theta \) is dependent on \( \vec{E}_L \) and hence the magnetic field components \( B_R, B_Z \) and \( B_\phi \). The \( \pi/\sigma \) ratio gives us the direction of the electric field,

\[
\frac{I_\sigma}{I_\pi} = \frac{1 + \cos^2(\theta)}{\sin^2(\theta)} \cdot f(n) = (1 + 2 \cot^2(\theta)) f(n),
\]

(2.7)

Solving this equation for \( \theta \), and using Figure 2.4 gives us,

\[
\tan^2(\theta) = \frac{2I_\pi}{I_\sigma f^{-1}(n) - I_\pi}
\]

(2.8)

\[
= \frac{(\vec{E} \cdot \hat{e}_z)^2 + (\vec{E} \cdot \hat{e}_l)^2}{(\vec{E} \cdot \hat{e}_k)^2}.
\]

(2.9)

The intensities can be related to the pitch angle \( \gamma_p \) by the following relation,

\[
\frac{I_\sigma}{I_\pi} = \left(1 + 2 \frac{(\vec{E} \cdot \hat{e}_k)^2}{(\vec{E} \cdot \hat{e}_z)^2 + (\vec{E} \cdot \hat{e}_l)^2}\right) f(n)
\]

(2.10)

\[
= \left(1 + 2 \frac{\sin^2(\beta) v^2 B_z^2}{\sin^2(\alpha) v^2 B_z^2 + \cos^2(\beta) v^2 B_z^2}\right) f(n)
\]

(2.11)

\[
= \left(1 + \frac{2 \sin^2(\beta) \tan^2(\gamma_p)}{\sin^2(\alpha) + \cos^2(\beta) \tan^2(\gamma_p)}\right) f(n)
\]

(2.12)

where \( \tan \gamma_p = B_z/B_\phi \). We neglect here the static energy component, and the radial magnetic field is set 0 because we look in the mid plane of the reactor. There is a still an uncertainty because of the term that depends on the density \( f(n) \) in the intensity ratio. This term depends on the level populations of the \( \pi \) and \( \sigma \) lines. We want to know the pitch angle the density term is available from other measurement techniques. There are also models which can predict these level populations, see for example Gu [19]. Mandl shows that the optical components have a different sensitivity for \( \sigma \) and \( \pi \) lines, which also decreases the accuracy of this method [6]. How well we can reconstruct the pitch angle will be explained in Chapter 4. The next section will go into more detail about the density dependent term \( f(n) \).
2.3.3 Statistical Population

The ratio of intensities is dependent on the energy level distribution of the excited electrons. This distribution gives us the characteristic emission of MSE. Collisions can redistribute the population. When we are at a high density ($> 10^{20} \text{ m}^{-3}$) we have lots of collisions and we can assume that all energy levels are equally filled, this is called a statistical distribution. However, at low densities the energy levels that predominantly cause $\pi$ emission tend to be favored. At tokamaks nowadays we have densities of the order $10^{19} \text{ m}^{-3}$, this means that we are in between extremes of energy distributions. The method using intensity ratios is dependent on a density term $n_{\pi}/n_{\sigma}$. There are models which calculate the population distribution for different densities [19]. A plot using data from these models is shown in Figure 2.5, the TEXTOR regime is around $10^{19} \text{ m}^{-3}$. We also see that the factor approaches 1 for higher densities (statistical distribution). Because of these densities ($10^{19} \text{ m}^{-3}$) the measurement of the polarization angle on the $\pi$ line is favored.

![Figure 2.5](image)

**Figure 2.5** - The density function $f(n)$ calculated from theory as function of density $n$, the common TEXTOR density regime is between $1 \cdot 10^{18} - 1 \cdot 10^{20} \text{ m}^{-3}$ [19].

2.3.4 Polarization

The third option makes use of the polarization of the individual emission lines. By adding polarizers to the setup we can distinguish better between the $\pi$ and $\sigma$ lines, since the two polarizations of the lines are separated by $90^\circ$. The polarization angle ($\gamma$) of the incoming light can be related to the pitch angle ($\gamma_p$) in the following way,

$$\tan \gamma = \frac{\cos \beta}{\sin \alpha} \tan \gamma_p. \quad (2.13)$$
This relation uses the angles described in Figure 2.4 and we see that the pitch angle is directly proportional to the polarization angle, making this method the most direct measurement. However, measuring polarization is more complicated. Techniques for measuring polarizations will therefore be described in the next section.

2.4 Measuring Polarization

As seen in the previous section, the polarization is the most direct measurement to determine the pitch angle. However, the polarization cannot be measured directly. No detectors exist that can measure the polarization directly. What can be measured are intensities. Optical elements can change the intensity and change the polarization state of the light into an intensity modulation. Each optical component has influence on the polarization of light. The interaction of optics on the polarization of light can be described by Mueller calculus. The whole polarization state of light can be described by the Stokes vector, $\mathbf{S}$. This vector consists of 4 components: the intensity $I$, linear polarization $Q$, linear polarization with $45^\circ$ rotation $U$ and circular polarization $V$.

2.4.1 Stokes Vectors and Mueller Matrices

The Stokes vector is used to describe the polarization of light. It is the most convenient description of polarization when using incoherent, even partially polarized, light. The equations can be written in vector form,

\begin{align*}
I &= I \\
Q &= Ip \cos (2\chi) \cos (2\gamma) \\
U &= Ip \cos (2\chi) \sin (2\gamma) \\
V &= Ip \sin (2\chi)
\end{align*}

(2.14)

where $I$ is the measured intensity, $p$ the fraction of polarization. The angles $\chi$ and $\gamma$ are the ellipticity angle and orientation angle of the polarization ellipse, respectively. The equations can be made visible using the Poincaré sphere (see Figure 2.6).

Determination of the polarization angle is done by dividing the $U$ and $Q$ component and taking the arctangent,

\[ \gamma = \frac{1}{2} \arctan \left( \frac{U}{Q} \right) \]

(2.15)

The measurement errors of the intensity will propagate through the whole reconstruction process and induce an error on the polarization angle. The calculation of the magnitude of this error will give an estimate of the goodness of the method. The polarization angle is only a
function of \( U \pm \Delta U \) and \( Q \pm \Delta Q \). Therefore we can determine the error in \( \gamma \),

\[
\Delta \gamma = \sqrt{ \left| \frac{\partial f}{\partial U} \right|^2 \cdot \Delta U^2 + \left| \frac{\partial f}{\partial Q} \right|^2 \cdot \Delta Q^2 } \quad \text{where} \quad f = \frac{1}{2} \arctan \left( \frac{U}{Q} \right). \quad (2.16)
\]

The total expression for the error is then,

\[
\Delta \gamma = \sqrt{ \frac{1}{4} \left( \frac{Q^2 \Delta U^2 + U^2 \Delta Q^2}{(Q^2 + U^2)^2} \right) } \propto \frac{N}{S}, \quad (2.17)
\]

which can be interpreted as the noise-signal-ratio.

Different optical elements exist that change the polarization; polarizers and wave plates (see Figure 2.7). These optical components can be described by Mueller matrices \( M \). A linear

---

**Figure 2.6** – The Poincaré sphere is the polarization state representation in three-dimensional space. Every point on the surface of the sphere represents one of the possible polarization states of a monochromatic beam of light. Figure adapted from wikipedia.com.

**Figure 2.7** – The interaction of unpolarized light with a linear polarizer and linear polarized light with a quarter-wave plate. Adapted from apiptics.com.
horizontal polarizer can be described by,

\[ M_{\text{pol}} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \] (2.18)

We see that the incoming intensity is halved, this is intrinsic for polarizers. A quarter-wave plate \( \phi = \pi/2 \) can be described by,

\[ M_{\text{WP}}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & \sin \phi & \cos \phi \end{pmatrix}. \] (2.19)

The wave plate does not reduce the amount of light, it only transforms it. When we want to rotate the optical elements we can apply a rotation transformation. This is described as,

\[ M_{\text{rot}}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \] (2.20)

The rotator only rotates the polarization ellipse \( \gamma \). For example, the definition of a rotated wave plate of \( 90^\circ \) \( (\pi/2) \) will give,

\[ M_{\text{WP,90}}(\phi) = M_{\text{rot}}(-\pi/2)M_{\text{WP}}(\phi)M_{\text{rot}}(\pi/2). \]

A typical description of the Stokes vector \( \mathbf{S} \) upon interaction with optical components looks like this,

\[ \mathbf{S}_{\text{meas}} = M_n \ldots M_2 M_1 \mathbf{S}_{\text{in}}, \] (2.21)

where the optical components have their matrix \( M \) and the outcome Stokes vector is a result of matrix multiplication of an incoming Stokes vector. Because the Stokes vector consists of 4 elements \( (I, Q, U, V) \), it is possible with a set of 4 measurements to solve this total incoming Stokes vector. By using static polarimetry 4 polarizers are used to be able to reconstruct the Stokes vector.

### 2.4.2 4 Polarizers

Since we have 4 unknowns, the whole Stokes vector; we need 4 different measurements, i.e., different intensity measurements from 1 light source. One way of creating 4 different measurements is by setting up 4 polarizers with different orientations in such a way that 4 different intensities are measured. Soetens [7] used this setup on TEXTOR, using 3 linear polarizers at different angles \( (0^\circ, 45^\circ \) and \( 90^\circ ) \) and a circular polarizer (for the \( V \) component),
measuring the 4 components at the same time (see Figure 2.8). The setup can be simulated by Mueller matrices in the following way [20],

\[ S_{\text{meas},i} = M_{\text{pol},i}(\theta)S_{\text{in}}, \]  

(2.22)

where the Mueller matrix of the polarizer can be rotated, to create different intensity measurements. The index \( i \) represents the 4 different polarizers. Only the intensity can be measured so we are only interested in the first element of the Stokes vector, \( S_i[1] \). These relations can be inverted to reconstruct the Stokes vector and represented in a matrix multiplication as follows,

\[ S = \begin{pmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 2 & 2 & -8 & 0 \\ -2 & -2 & 0 & 8 \end{pmatrix} \begin{pmatrix} I_0^\circ \\ I_90^\circ \\ I_{-45^\circ} \\ I_{\text{circ}} \end{pmatrix} \]  

(2.23)

This equation shows that the Stokes vector can be reconstructed from 4 measurements using different optical elements, the used matrices are given in Appendix A.

### 2.4.3 Photo-Elastic Modulators

The widely used technique for performing MSE measurements is by using photo-elastic modulators (PEMs). A PEM is a fast modulated wave plate, where the photo-elastic effect is used to change the birefringence. The birefringence can be modulated by applying a voltage at a frequency that induces a standing wave in the crystal at resonance frequency. A wave plate is an optical device that is able to change the state of polarization of light that passes through it.

**Figure 2.8** – The setup as described in the thesis of Soetens, the incoming light is split for the 4 polarizers and analyzed by the spectrometer. Adapted from [7].
Polarized light that enters the plate is traveling parallel and perpendicular to the optical axis. However, the material is birefringent, thus the phase of the two waves will change due to a different index of refraction in the two directions.

The method uses two PEMs in series at different angles relative to each other and with different frequencies $\Omega_1$ and $\Omega_2$. This frequency is typically about 20 kHz. The polarization angle now gets modulated. When a linear polarizer is placed behind the setup, the polarization modulation is transformed into an intensity modulation. The pitch angle is related to the amplitudes of the second harmonic of the two PEMs ($A_1, A_2$) as following,

$$\tan (2\gamma) = \frac{A_1}{A_2}.$$  \hspace{1cm} (2.24)

Because of the high modulation frequency the technique has a very high temporal resolution. But because of this fast method spectral measurements are hard to obtain. Fast detectors are necessary, e.g., photodiodes or photo-multiplier tubes, since cameras cannot measure at such high frequencies. We could use an array of detectors but these tend to become large and the amount of data for every measurement would be enormous. Furthermore we would have such a short integration time that the signal strength is very low, and because we measure the intensity as function of wavelength the signal strength is even more spread out, weakening the signal even further. Therefore, one detector is used and a filter is applied to part of the spectrum where the polarization either $\sigma$ or $\pi$, the rest of spectral information is lost.

### 2.4.4 Hybrid

A camera can be used when we measure the polarization directly, using polarizers, or using a lower intensity modulation frequency. The latter can be achieved using switchable quarter-wave plates.

PEMs are essentially modulated wave plates, creating high resolution data. When less high requirements of time resolution are required, we can think of a hybrid solution combining the PEM and the static polarizers. We need 4 measurements to create 4 equations with 4 unknowns and solve the Stokes vector, as was the case with the 4 polarizers. This is achieved by making different combinations of birefringent crystals (quarter-wave plates). We will use for this case, 2 switchable quarter-wave plates and a polarizer to create this set.

A quarter-wave plate induces a phase shift of a quarter-wavelength ($\frac{\pi}{2}$). A switchable wave plate is capable of switching its optical axis by 45° ($\frac{\pi}{4}$). This means that the fast-axis of the crystal changes when a voltage of +5 Volt is applied. The crystal’s axis will return to its initial position when a voltage of -5 Volt is applied. One way of creating 4 different measurements is by setting up the wave plates in such a way that upon switching 4 different intensities are measured. One crystal is aligned with the optical axis, and one is rotated 45 degrees, a polarizer has its axis at 22.5 degrees. This is the same setup as ‘traditional’ PEMs use. The setup can be simulated by
Mueller matrices in the following way [20],

\[ S_{\text{meas}} = M_{\text{pol}}(22.5^\circ)M_{\text{wp}}(45^\circ)M_{\text{wp}}(0^\circ)S_{\text{in}}. \]  

(2.25)

The measured intensity, because a CCD can only measure light intensities, of the 4 measurements can be used to construct the whole Stokes vector,

\[
\begin{pmatrix}
I_0 \\
Q_0 \\
U_0 \\
V_0
\end{pmatrix} =
\begin{pmatrix}
2 & 0 & 2 & -2 \\
-2\sqrt{2} & \sqrt{2} & -2\sqrt{2} & 3\sqrt{2} \\
0 & -\sqrt{2} & 0 & \sqrt{2} \\
0 & \sqrt{2} & -2\sqrt{2} & \sqrt{2}
\end{pmatrix}
\begin{pmatrix}
I_1(+5,+5) \\
I_2(+5,-5) \\
I_3(-5,-5) \\
I_4(-5,+5)
\end{pmatrix} \tag{2.26}
\]

where \( I_i(x_1, x_2) \) represent the observed intensity as a function of the state of the two quarter-wave plates (+5, −5 \( \lambda \)). All the observed quantities are function of wavelength. The whole derivation can be found in appendix A. We have to keep in mind that every different waveplate configuration represents a different time frame, this is of no problem in a static situation. However, if we take a more realistic situation, we will have that the intensity and also the magnetic field strength of the plasma will change over time. Thus, the real Stokes vector is a function of time, \( S(t) \); the reconstructed vector is a combination of only 4 observed intensities, \( S_{\text{rec}}(t) = aI_{t-3} + bI_{t-2} + cI_{t-1} + dI_t \). Therefore, the reconstructed \( S \) will be an ‘average’ of 4 successive intensity measurements. We can think of a matrix inversion that will give us the original Stokes vector,

\[
S_{\text{recon}} = A \cdot I_{\text{meas}} \quad \text{with} \quad I_{\text{meas}} = B \cdot S_{\text{real}}
\]

\[
\Rightarrow S_{\text{real}} = B^{-1}A^{-1}S_{\text{recon}}. \tag{2.27}
\]

Since not the whole Stokes vector measured contributes to this reconstruction, but only certain components we cannot get an inverse nor pseudo-inverse of \( B \cdot A \). This means that when the properties of the plasma change too much over 4 time frames we are not able to obtain the real Stokes vector at any time \( t \) with this method. Therefore, in order to obtain a better reconstructed vector we can use a simple moving average algorithm to smooth out the change of plasma properties and determine \( S_{\text{real}} \) more precise. Figure 2.9 shows the performance of a moving average filter, averaging over 4 points in time. It can be seen that the method using polarizers or wave plates (measuring \( U \) and \( Q \) can be applied to a steady state (slowly changing) Stokes vector for best results.

In the next chapters we will use this theory to determine the sensitivity of each method presented.
Figure 2.9 – Moving average filtering applied on a sequence of 4 Stokes vector values which change over time. The used Stokes components are the intensity $I$ (black), polarizations $Q$ (cyan) and $U$ (red). When sudden changes in Stokes values are present, the moving average fails.
Chapter 3

Reconstructing Realistic TEXTOR Motional Stark Effect Data

We have seen that besides the PEM method, different optical methods can be used to determine magnetic field information in fusion reactors using MSE spectra. We want to test the sensitivity of the Stark splitting, intensity ratio and polarization method, described in previous chapter. Therefore we need a Stokes vector. The fusion reactor we use for this project is the TEXTOR tokamak in Jülich, Germany. Unfortunately there is no MSE system installed at TEXTOR; no real Stokes vector could be obtained to perform the simulations with the three methods. Therefore we have to generate our own MSE data by performing computer simulations. This chapter describes the simulation for this case and comparison with measurements that are available. We will use the specifications of TEXTOR to approach the expected MSE profiles.

TEXTOR is a small circular tokamak, its inner radius is 47 cm and it has a major radius of 175 cm. Since the vessel is circular, it is relatively easy to construct a magnetic ‘equilibrium’ using an initial pressure profile and flux functions. From this we calculate the poloidal and toroidal magnetic field. This way we can make a magnetic field configuration which is realistic for TEXTOR.

The realistic emission is created in 3 steps:

- a magnetic equilibrium for TEXTOR is calculated, which approximates the present magnetic fields in the tokamak.

- light emission will be calculated from this TEXTOR equilibrium, where 3D effects are taken into account. These effects cause asymmetry of the spectrum and this influences the polarization angle and intensity of the spectrum.

- the emission is linked to a measured MSE signal at TEXTOR, in order to see if the simulation indeed approximates TEXTOR-like emission.
To create an as realistic possible Stokes vector we use MSE data originally measured by Soetens, using 4 polarizers on TEXTOR [7]. An MSE simulation code, written in IDL, is used to simulate the beam into plasma and emission of the full, half and third components of the beam. The code imports the magnetic field configuration generated by the equilibrium. The output of the code is a Stokes vector as function of wavelength for a specified radius. The spectrum that is obtained from Soetens' TEXTOR shot #82967 is then laid over the simulated spectrum. Therefore we can get an estimate for the separation of the $\pi$ and $\sigma$ components and thus an estimate of the magnetic field magnitudes. Figure 3.1 gives the reconstructed magnetic fields used for all Stokes vector simulations. Figure 3.2 shows the final pitch angle and $q$-profile for the case of Soetens. An overview of the beam in the tokamak can be seen in Figure 3.3.

After the beam emission is simulated, the optical view trajectory is simulated. This includes applying an instrument function to the signal, which broadens the 9 emission lines, see Figure 3.4. The Stark spectrum not only broadens because of optical broadening, but also because of the finite width of the beam (3D). The line-of-sight averages over this beam, thus sampling a volume (including a velocity distribution). We can calculate the spectra for different fiber positions, looking at different beam positions in the plasma. By changing the fiber position we look at different positions in the plasma, and thus experience different Stark splits, since the magnetic field is a function of the radius. The influence of the magnetic field at every point in the plasma on the emission profile can be calculated using the formulas for the Lorentz electric field and the Doppler shift, Eq. (2.4). The geometrical setup is shown in Figure 3.3.

Figure 3.1 – The magnetic fields, $B_z$ and $B_\phi$, determined as equilibrium configuration for the data of Soetens.
Figure 3.2 – The pitch angle and q-profile, determined from the equilibrium magnetic fields, see Figure 3.1.

Figure 3.3 – An overview of TEXTOR with the setup of Soetens (NBI2 and the accompanying viewport). The neutral beam has a divergence of 1° and a width of 10 cm upon entering the vessel. The line-of-sight (red) intersects the center of the beam at \( r = 180 \) cm in this overview.
Figure 3.4 – A typical MSE intensity spectrum. The 9 underlying emission components are broadened by the velocity distribution of the beam, a finite observation volume and the collection angle of the lens. The polarization angle changes due to the broadening.

Figure 3.5 – The Stark spectrum for two points (green (1) and blue lines (2)) on the beam with the same line of sight, 1 and 2 correspond to location 1 and 2 in Figure 3.3. Different magnetic fields and Doppler shift angles cause the spectrum to become asymmetrical.
Table 3.1 – The radii corresponding with the 10 channels used in the MSE simulation.

<table>
<thead>
<tr>
<th>Channel #</th>
<th>Radius (cm)</th>
<th>Channel #</th>
<th>Radius (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179.2</td>
<td>6</td>
<td>200.5</td>
</tr>
<tr>
<td>2</td>
<td>184.3</td>
<td>7</td>
<td>204.2</td>
</tr>
<tr>
<td>3</td>
<td>189.5</td>
<td>8</td>
<td>209.0</td>
</tr>
<tr>
<td>4</td>
<td>191.8</td>
<td>9</td>
<td>213.1</td>
</tr>
<tr>
<td>5</td>
<td>195.5</td>
<td>10</td>
<td>216.6</td>
</tr>
</tbody>
</table>

At TEXTOR, the spectrometer has 10 channels. We simulated these at the locations shown in Table 3.1. With these different channels we can take a radial profile of the pitch angle; and therefore the \( q \)-profile since they are related, as explained in Chapter 2.

3.1 Profile Reconstruction

When we reconstruct the Stokes vector using Soetens’ measurements we use Mueller matrices presented in his PhD thesis, this in order to see if the simulated profile results in the measured profile [7]. These specific matrices are put in Appendix A for convenience. We need to do this to create a TEXTOR-like Stokes vector for the simulations, because we could not measure an MSE spectrum ourselves at TEXTOR.

We can make a first indication at what position the profile is taken, since the measurement only states ‘channel 1’, but no position information is stated. We have two indications for determining the position, 1) the position of the peaks, i.e., the Doppler shift. The Doppler shift depends on the viewing angle between the neutral beam and the viewport, the more to the center, the smaller the shift will be. And 2) the Stark splitting, the splitting of the \( \pi \) peaks. This splitting depends on the Lorentz electric field, since the Lorentz electric field depends on the cross-product \( \mathbf{v} \times \mathbf{B} \) and thus the angle between the neutral beam and magnetic field, we can make a plot of the splitting as function of radius (see Figure 3.6 and 3.7). Figure 3.8 shows the 4 measurements taken at a position of 184.5 cm. It can be seen that the full energy peak is at the right position, but due to a wrong dispersion the half and third energy peaks are displaced.

We expect for every peak to have the same angle with respect to the viewport, thus Doppler shift. But there is a deviation of about 10 cm or 6° in angles. By adjusting this we have to make sure that we do know the exact position of the viewing line. We can try to adjust the position of the peaks to almost every viewing angle we can think of, by optimizing equation (3.1) and solve this equation as a least-square problem,

\[
\min_i \left[ x_i - x'_i \right]^2 \quad \text{with } x'_i = ax_i + b, \tag{3.1}
\]
3.1. Profile Reconstruction

Figure 3.6 – The Stark splitting for different positions along the beam for $B = 2.5$ T. The $y$-values of the horizontal lines are the values of the Stark split from Soetens data.

Figure 3.7 – The Doppler shift for different positions along the beam. The starred points are the measured points from Soetens data, the points are placed on the radius where the Doppler shift matches the calculated Doppler shift. They are expected to be on one line, which is not the case.
3.1. Profile Reconstruction

Figure 3.8 – The simulation for all 4 polarizers, as measured by Soetens [7]. The simulation was taken at $R = 184.5$ cm and $B = 2.5$ T, the simulated instrument function was 1.5 Å.

where $x_i$ is the original axis for each peak, and $x'_i$ is the scaled, new axis. Also the measured Stark split is different from the expected Stark split. It is shown in Figure 3.6 that no measured Stark split can be related to a position. The Stark splitting simulated is too small compared to the measured splitting. We can thus also not trust the Stark splitting to give us an indication of the position in the reactor, because the measured values never intersect the expected values. When (3.1) is performed to minimize both the dispersion in the Doppler shift and the mismatch in measured and simulated Stark split it turns out that with a magnetic field of 2.5 T and a radius of 216.6 cm the measurement can be reproduced best by our simulations, see Figure 3.9.

As can be seen from Figure 3.3 the intersection of the line of sight and the neutral beam occurs at different radii across the beam. This is because the beam has a width of 10 cm and a divergence of $1 - 2^\circ$. Therefore the beam particles won’t experience the same magnetic field across the line-of-sight. Because the divergence of the beam is $1 - 2^\circ$, the angle with the beam velocity will also vary thus the Doppler shift is different for the ‘inside’ and ‘outside’ of the beam (inside of the beam is defined as the side closest to the center viewed from the beam inlet). The sum of these two effects gives rise to a lower wider peak at the right side of the beam, and a taller narrower peak at the outside of the beam (see Figure 3.5). These effects explain that the MSE intensity profile is asymmetric.

When we look qualitatively at Figure 3.3 we see that the Doppler shift at 1 is larger than at point 2. This means that the all Stark components at 1 are shifted more to the blue side of the spectrum than at 2. This effect alone would result in a wider but not asymmetrical spectrum. The asymmetry comes from the fact that the Lorentz field at these points differ. The $\mathbf{v} \times \mathbf{B}$ term is smaller at 2 than at 1, thus the Stark splitting $\Delta \lambda_v$ is smaller at 2 than at 1. The sum of
3.2 Results

When we apply the Mueller matrices (used by Soetens) to the simulated Stokes vector and add an offset of 15 (a.u.) we see that the vertical polarization is missing a factor 2 and that the circular polarization is more or less 4 times too high. It could be that the described setup was not used for the measurement presented. When we look at the individual Stokes components and apply the polarization inequality $I^2 \leq Q^2 + U^2 + V^2$ we see that this does not hold for this measurement, thus the given results of Soetens are off. Figure 3.9 shows the simulation and the measurement for all 4 polarizers, we accounted for a factor of 2 in the mismatch of the vertical intensity. The inverse of the measurement was applied and the Stokes vector was recovered. Figure 3.10 shows the simulated Stokes vector along with the recovered Stokes vector. The reconstruction of the circular component is off, because the intensity measurement was also off. We suspect that this has to do with the quality of the measurement performed by Soetens, but this cannot be confirmed since the available data was not well documented.

In conclusion, a magnetic equilibrium was used for generation of the motional Stark effect spectrum for a TEXTOR configuration. Once the equilibrium was constructed, we simulated data for a magnetic field configuration of 2.25 T, which is common for TEXTOR. This simulation includes 3D effects caused by the observation volume and optical broadening. The
The reconstructed Stokes vector (solid line) as function of wavelength. This reconstruction is obtained by applying the inverse Mueller matrices as used to simulate Figure 3.9. The simulation (crosses) Stokes vector is coming from the IDL program.

Figure 3.10 – The reconstructed Stokes vector (solid line) as function of wavelength. This reconstruction is obtained by applying the inverse Mueller matrices as used to simulate Figure 3.9. The simulation (crosses) Stokes vector is coming from the IDL program.

shape of the emission is qualitatively in agreement with measured MSE data, however there are large differences between the position and intensity of available data. We suspect that this has to do with the quality of the measurement performed by Soetens, since the Stokes vector inequality does not hold for this measurement. The Stokes vector of the MSE emission calculated from these settings is used in the following chapters to determine the sensitivity of the methods.
Chapter 4

Pitch Angle Reconstruction: Stark Splitting & Intensity Ratio

We have seen that the pitch angle profile and \( q \)-profile for TEXTOR can be calculated from TEXTOR relevant magnetic fields and an MSE profile can be simulated. The intensity profile \( I \), from the simulations described in the previous chapter will be fitted and the intensity ratio will be used to determine the pitch angle. Also the magnitude of the Stark splitting will be used because the splitting of the Balmer-\( \alpha \) line depends on the present magnetic field. We will go into more detail about the ability to resolve magnetic fields of these methods. We will use the simulated MSE spectra and look at the sensitivity of both methods.

First we will look at the ideal case, i.e., make only use of the theory and formulas presented in chapter 2 and taking no 3D effects into consideration, i.e., no beam width and no diverging line-of-sight. Thereafter we will look at a more realistic case and use the Stokes vector created in previous chapter, which does include these 3D effects.

4.1 Ideal TEXTOR Case

4.1.1 Stark Split

The distance between the \( \pi_{3+} \) and \( \pi_{3-} \) peak give us the splitting due to the Stark effect, with \( m = 6 \). This will be used to determine the pitch angle from the splitting. In chapter 2 we presented the relation to do this,

\[
\Delta \lambda_s = 2.76 \cdot 10^{-8} m |vB_\phi (-\sin \alpha + \tan \gamma_p)|.
\] (4.1)
Figure 4.1 – Stark splitting as function of radius. The Stark split is calculated from the magnetic fields that were used in the simulation and the $1/R$ vacuum field.

The distance between the $\pi$ peaks can be determined for all 10 channels, or all 10 expected Stark splits. When we plot the expected Stark split as function of the radius, obtained from the simulated magnetic field ($B_z/B_\phi$), we get the ideal sensitivity, see Figure 4.1. We also use instead of the real toroidal magnetic field, the vacuum magnetic field which goes as $1/R$. The dia- or paramagnetic contributions on the toroidal field are small and as an initial guess the vacuum field can be used to extract the pitch angle. By doing this the equation reduces to 1 unknown, $\gamma_p$. Figure 4.2 shows the sensitivity of the pitch angle as function of the Stark split. This graph is the mapping of the radial dependence onto the pitch angle dependence. An increase in split corresponds to a different radius, thus we are able to resolve the pitch angle radially. To overcome $6^\circ$ pitch angle we have a difference in splitting of 0.79 Å. This is a change of 36%. The spectrometer at TEXTOR has a reported dispersion of 0.12 Å/px. This means we have an uncertainty of at most $0.9^\circ$ to determine the pitch angle. The peak can be measured with sub pixel accuracy and be determined to 0.2 px, which gives an uncertainty of 0.18°.

### 4.1.2 Intensity Ratio

The pitch angle can be determined by dividing the magnitude of the $\sigma$ and $\pi$ peak and its relation to the angle $\theta$. When we use relationship 2.12,

$$\frac{I_\sigma}{I_\pi} = \left( 1 + \frac{2 \sin^2(\beta) \tan^2(\gamma_p)}{\sin^2(\alpha) + \cos^2(\beta) \tan^2(\gamma_p)} \right) f(n) \quad (4.2)$$
we can calculate the expected intensity ratio from a given equilibrium \( \gamma_p \). The intensity ratio changes 1% over a 6° change in pitch angle, making this method insensitive to changes in the pitch angle profile, see Figure 4.3. Figure 4.4 shows the intensity ratio as function of pitch angle with the same sensitivity. Both graphs are also corrected with the density term \( f(n) \) from literature, for \( n = 1 \cdot 10^{19} \text{ m}^{-3} \), \( f(n) \) is 0.86.
4.1. Ideal TEXTOR Case

Figure 4.3 – The intensity ratio as function of radius with and without $f(n)$. The ideal ratio is calculated using the equilibrium $\gamma_p$ from equation 2.12.

Figure 4.4 – The intensity ratio as function of pitch angle with and without $f(n)$. The ideal ratio is again calculated using the equilibrium $\gamma_p$ from equation 2.12. $\gamma_p$ is taken at the different radii.
4.2 Realistic TEXTOR Case

In this section we will investigate the spectra we made with simulations. First we will look into the ability of the Stark splitting to resolve the pitch angle, thereafter the intensity ratio method will be investigated.

4.2.1 Fitting the Spectrum

The Stark spectrum consists out of 9 emission lines (see Chapter 2). These lines can be fitted to Gaussians, resulting in 18 free fitting parameters. We use the fact that the $\pi$ and $\sigma$ lines are split by the same wavelength, using Eq. (2.3). Despite the 9 visible lines, instead of trying to fit 3 (sub) lines for each of the 3 peaks, we assume the shape of these combined peaks to be Gaussian. This is done to reduce the free parameters of the fit to 9. These parameters can be reduced when the intensity of the $\pi$ polarized peaks are coupled. It is possible that these 9 lines are fitted, (see for example Figure 1 in [21]). But by using less free parameters this increases the fit quality at the cost of losing physics, e.g., the Doppler broadening of each single peak. We use the Stokes vector for a magnetic field of 2.25 T and 10 fiber channels. These channels correspond with radii shown in Table 3.1. A Matlab routine is written, which is based on a least-square fit routine. The Gaussian fit parameter $w$, the width of the peak is not the same for all peaks, due to optical geometry (see chapter 3). The positions of the peaks are fitted for the center peak $\sigma_0$ and the of center lines of the $\pi$-polarization, $\pi_{3+}, \pi_{3-}$ of the full energy component. Once a fit is made we put the intensity values, coming from the magnitude of the separate peaks.

A fitted intensity profile is shown in Figure 4.5, 3 Gaussian curves are fitted to the $\pi$ and $\sigma$ peaks. The goodness of a profile fit is shown in Figure 4.5. Here the three single Gaussians fitted to the dotted profile are shown and their resulting profile (solid line). The dotted profile is the simulated profile. The second graph shows the residual of the fit as function of wavelength.

4.2.2 Stark Splitting

We will now look at the simulated (fitted) profiles (including 3D effects) and their splitting. Figure 4.6 shows the measured Stark splitting and the corresponding pitch angle. It can be seen in Figure 4.6 that the measured splitting is larger than it was for the ideal case. This could be due to broadening of the signal caused by Doppler broadening. This means that the pitch angle will not correspond to the real pitch angle, since every other condition, e.g., magnetic field, stayed the same. We cannot construct $\gamma_p$ using the simple formula for the Stark splitting in this case. However, if we can improve the determination of the Stark split, we can use a $1/R$ relation
Figure 4.5 – The three fitted Gaussians to the simulated MSE profile. The graph below shows the relative error of the fit. This specific spectrum shows the full energy component at channel 5 with a magnetic field of 2.25 T.
4.2. Realistic TEXTOR Case

Figure 4.6 – Stark split as function of radius for the Stark split method. The Stark split is obtained from the distance between 2 fitted Gaussian π peaks of the simulated intensity profile. The difference in Stark split results in a pitch angle profile, which differs from the ideal calculation, because of the accounted 3D effects in the simulated spectrum.

for $B_\phi$ and obtain realistic pitch angle values. For example a lookup table could be created, mapping the measured Stark split to the ideal case, of which a pitch angle profile could be constructed; or the splitting should be fitted to 9 peaks instead of 3. We cannot obtain realistic values for the $q$-profile from this simulated graph since we have not created such a table. It is positive for this method that the splitting does not change due to reflectivity or polarization of mirrors making this method useful with the right calibration.

4.2.3 Intensity Ratio

It is possible to fit the $\pi$ and $\sigma$ lines in the MSE spectrum, see Figure 4.5. The fit of the intensity ratio gives us the values of the intensities. The $\Sigma I_\pi$ and $\Sigma I_\sigma$ are shown in Figure 4.7. It turns out that we cannot reconstruct the pitch angle because the term in the square root (see equation 2.12) is negative for the measurement.

This could lead back to the intensity ratio, where a function of density of states was needed to give the correct value. We see that the intensity ratio varies 30% over the tokamak’s radius. This is not due to a change in pitch angle, since the expected ratio change is the ideal line, but due to 3D effects accounted for in the simulated spectrum. The beam has a finite width, we are thus sampling a collection of different magnetic fields and directions, which complicates the measurement towards the center. Thus, when trying to reconstruct a pitch angle from the intensity ratio we obtain imaginary results; we cannot use the simple formula for determining
the pitch angle from the intensity ratio. The fact that the ratio gives imaginary pitch angles means that the geometry used for the simulations is not ideal for this measurement. For this setup, the angle $\beta$ is not large enough causing the pitch angle to be imaginary. In order to make the pitch angle real, one needs to look outside the equatorial plane, adding other geometrical terms.

4.3 Conclusion

We have seen that we are able to fit an MSE intensity profile using 3 Gaussians. Using the intensity ratio and Stark splitting of the MSE profile we can obtain information about the magnetic field present. The conditions in which these techniques can be applied have to be investigated, e.g., the geometry of the viewing lines and the dispersion of the spectrometer. The Stark splitting method has a finite accuracy in determining the splitting of the peaks which results in an error of $0.3^\circ$ on the determination of the pitch angle. We have seen that 3D effects influence the determination of the pitch angle from the Stark split. To account for this effect, one can think of a lookup table of Stark splits measured in plasmas and the simulated Stark split using the same parameters of the measured plasma. By doing this a measured Stark split profile can be translated 1-to-1 to an already present pitch angle profile.

The intensity ratio is insensitive to changes in the pitch angle for this current viewing configuration. 3D effects worsen the determination of the intensity ratio. The determination of the pitch angles has to become more certain, both methods we were not able to construct a $q$-profile, or even a realistic pitch angle profile. In the next chapter we will look into the polarized part of the spectrum, by doing this we try to get more detailed information from the magnetic field.
Chapter 5

Pitch Angle Reconstruction: Four Polarizers & Two Quarter-Wave Plates

This chapter will look into the pitch angle- and q-profile reconstruction using a method using two quarter-wave plates and a method using 4 polarizers. The simulated Stokes vector will be put through the Mueller matrices to create 4 intensity measurements, for each specific method. By using these we can use the whole spectrum to determine the pitch angle. We will show that the pitch angle changes as function of wavelength and radius.

5.1 Method

We will use the simulated MSE spectra to derive the $U$ and $Q$ spectra and use those to reconstruct the polarization angle; and from that using (2.13) the pitch angle $\gamma_p$ and the $q$-profile. We will use the polarizer- and wave plate matrices to generate 4 intensity measurements. Because the Stokes vector is a function of wavelength (see Figure 5.1), we get the polarization angle as a function of wavelength. The change in polarization angle as function of wavelength comes from the fact that we have a finite collection volume thus averaging over different locations in the beam. The change in polarization angle per peak can be averaged over every peak to decrease the wavelength dependence. Therefore we average over each peak to be able to interpret the data with a larger certainty, also when noise is applied. However, we will only look at the average of the $\pi$ peaks, since they have less polarization mixing.

We apply a noise term to the intensities to resemble a more realistic measurement. The noise in the used measurements of Soetens [7], which means they are realistic for TEXTOR, can be
5.2 Results

Figure 5.1 – The $Q$ and $U$ components of the Stokes vector. The polarization angle $\gamma$ (---) is calculated by dividing both components (without noise) and take the arctangent.

to generate the Stokes vector. Not only the $U$ and $Q$ components of the Stokes vector will be measured but also $V$. This term is not used in the $q$-profile reconstruction.

An example of a reconstructed $Q$ and $U$ is shown in Figure 5.2. The Stokes vector is obtained by averaging over every peak to diminish the effect of the changing polarization angle over wavelength. This can be done because the ratio of the $U$ and $Q$ peak will remain the same. For this case we will sum over the $\pi$ peaks and use the RMS of the signal as error. Distinguishing between $\pi$ and $\sigma$ lines can be done based on the wavelength separation and/or based on sign of the polarization. For example a certain threshold could be used above which the values are the peaks. However, for small signals this method is not performing well, therefore in this simulation we hardcoded the position of the peaks. In real-time applications this could be a problem due to fluctuations of the peak positions.

5.2 Results

First we will look at the pitch angle reconstruction using polarizers, then we will look at the results of the method using two quarter-wave plates.
5.2. Results

Figure 5.2 – The $Q$ and $U$ profile reconstructed from the polarizations measurement shown in Figure 5.3.

5.2.1 Polarizers

Since the pitch angle can be calculated from only $U$ and $Q$ of the Stokes vector, we can start right away by dividing these values for the reconstructed vector.

We will now apply a noise term to the simulation. Once this noise is applied we calculate the Stokes vector back using the inverse matrix, see Figure 5.2. The $Q$ and $U$ components are then processed as shown in Chapter 2.

When we look at the reconstructed pitch angle for $B = 2.25$ T (Figure 5.4), with and without noise applied, we see that the band of error values is around 0.2° wide. The error bar is obtained by taking the RMS of the noise on a range of wavelengths where no peaks are present. The corresponding $q$-profile can be seen in Figure 5.5. An error of $\Delta q = 0.07$ is obtained with an error in the measurements of 1%. The 3D effects on these measurements are much smaller than is the case with the previous presented methods. Towards the plasma center the reconstruction of $q$ differs because of Shafranov shift of the plasma where $r \rightarrow 0$.

We can conclude that the method using 4 polarizers is capable of tracking changes in the $q$-profile within the accepted error of 0.2, given a S/N ratio of 100.
5.2. Results

Figure 5.3 – The simulated signal is put through the Mueller matrices, this way 4 measurements are produced and 1% noise was added.

Figure 5.4 – The magnetic pitch angle along the channels for $B = 2.25$ using the method of 4 polarizers. The error bars are obtained by taking the RMS of the signal with noise added. The solid line is the pitch angle of the magnetic field used to perform the Stokes vector simulation.
Chapter 5  5.2. Results

5.2.2 Wave Plates

Now we will use the same approach, but this time using wave plates instead of polarizers. The simulated data is put through the Mueller matrix to create 4 measurements. We will again add 1\% noise to the simulated Stokes vectors. Figure 5.6 shows the results of the simulation and the error of the pitch angle profile. We see that the error bars extend to 0.15° for wave plates, whereas they went up to 0.2° in the case of the 4 polarizers. This gives a $\Delta q = 0.05$. Therefore, we can say that the method of wave plates is slightly more sensitive in determining the pitch angle (and thus the $q$-profile) than the method of polarizers. The corresponding $q$-profile is shown in Figure 5.7. When a signal-to-noise ratio of 10 is simulated, the error in the pitch angle is 1.7° and the error in $q$ gives $\Delta q = 0.2$, see Figure 5.8 and 5.9. The small error bars on several data points are due to the fact the error is determined, and not calculated. The RMS of the error is taken at a position where no MSE peaks are present. We see that the error in the pitch angle scales to noise as expected from (2.16). The error in $q$ however, scales as the square root of the noise.

Figure 5.5 – The $q$-profile reconstruction from the pitch angle shown in Figure 5.4. The solid line is again from the magnetic field used to simulate the Stokes vector.
5.2. Results

Figure 5.6 – The magnetic pitch angle along the channels for $B = 2.25$ T. The error bars are obtained by taking the RMS of the signal with noise added. The solid line is the pitch angle of the magnetic field used to perform the Stokes vector simulation.

Figure 5.7 – The reconstructed $q$-profile for $B = 2.25$ T, from Figure 5.6. The solid line is again from the magnetic field used to simulate the Stokes vector.
5.2. Results

Figure 5.8 – The magnetic pitch angle along all channels for $B = 2.25$ T. The error bars are obtained by taking the RMS of the signal with noise added (10%). The solid line is the pitch angle of the magnetic field used to perform the Stokes vector simulation.

Figure 5.9 – The reconstructed $q$-profile along the channels for $B = 2.25$ T, from Figure 5.8. The solid line is again from the magnetic field used to simulate the Stokes vector.
5.3 Conclusion

It is shown that it is possible to reconstruct a $q$-profile from MSE measurements of light, using polarizers or wave plates. The main advantage of this method is that the 3D effects do not have a major impact on the determination of the pitch angle profiles, making it much easier to determine the pitch angle ‘out-of-the-box’. The method using 4 polarizers and 2 wave plates perform equally well, but the sensitivity is in favor of the wave plates. This is due to the fact that the wave plate setup only has 1 polarizer while the 4 polarizer setup has a polarized beam splitter to split the light in 4 beams to measure simultaneously, thus losing intensity and increasing errors. To improve the SNR ratio of a measurement, the analysis can be performed on all 3 components to obtain more signal. Also, using this method the background polarization on future fusion devices like ITER (see Introduction) can be calibrated real-time on wavelengths away from the MSE peak positions, thus increasing the sensitivity of the method.
Chapter 6

Proof-of-Principle of an Alternative MSE Diagnostic on TEXTOR

As discussed in the previous chapters, we want to measure MSE on a tokamak using emission of neutral beam particles. This chapter reviews the setup that is developed. We want to measure MSE as function of wavelength to capture as much information as possible. We will now build a setup that has to capture the polarization as function of wavelength. The constraints on this setup will be that the available spectrometer and fiber bundle must be used to record the outgoing light. And also, the setup must occupy little space, because of the available view port at TEXTOR (see Figure B.1).

Based on these requirements we based our setup on two switchable quarter-wave plates, because it takes less space than a setup with 4 different polarizers, and achieves equal results (see chapter 5). Also less incoming light is lost, resulting in lower noise, and only 1 spectrometer channel is needed. For this setup we use a view port which is installed on the TEXTOR tokamak at the Forschungszentrum Jülich, Germany. Therefore, when the device will be designed and built we have to take the housing into consideration. There are already a grating spectrometer and fiber bundles available at TEXTOR. This means that, in order to build a working setup, we only need to design the part that encodes the polarization of the light, the wave plates.

6.1 Design

We have seen with simulations that wave plates can resolve the magnetic pitch angle. The whole wave plate setup has been be built into a view port at TEXTOR. The design of the optical system was determined with simulations, taking into account lenses and viewing lines on the neutral beam. Since we had to use an existing view port we had to take into account
the housing properties. The housing has an inner diameter of 78 mm. Drawings of the view port are placed in Appendix B. The company of Micron technologies offers custom made wave plates\(^1\) with an outer diameter of 65 mm and inner diameter of 45 mm. These wave plates with a center wavelength of 650 ± 50 nm will be fitted in the housing because we want to use as much light as possible. The error in central wavelength of the quarter wave plate is at most 7%. Therefore it could be that the wave plate does not have a quarter wave delay but slightly less, or more. However it turns out that this effect is small, when looking at equation (A.2), the 7% change only is significant in the terms of single \(\cos \phi \), but not more than 10%. With a ray tracing procedure we simulated the light from the tokamak’s neutral beam into the housing and this confirmed that the beam is visible from the magnetic axis of the plasma to the outer edge of the plasma (see Figure 6.1).

![Ray trace of viewport on TEXTOR towards NB11](image)

**Figure 6.1** – A ray trace procedure is used to determine the line of sight and the location of the viewport.

Wave plates are rather sensitive and delicate systems, even outside the tokamak there is a rather harsh environment, high magnetic fields, high temperatures and vibrations. We will look into more detail of these possible issues. The magnetic field of the tokamak extends the vacuum vessel outside the tokamak, the magnetic field at the position of the viewport is determined to be around 1 Tesla. Because we use fluid liquid crystals in the wave plates, one can think of alignment of the crystals when magnetic fields are applied, however according to the manufacturer of the wave plates, such a magnetic field is of little to no influence on the prescribed operation. The temperature however could be of influence. The contrast of the wave plate, the difference in intensity between the two states, decreases when the temperature increases. This means that

\(^{1}\)The product page is located at: [http://www.micron.com/products/flcos-microdisplays/photonics-rotators](http://www.micron.com/products/flcos-microdisplays/photonics-rotators)
when the housing gets too warm the measurement will fail, because of lack of contrast. The rotator has a contrast ratio of 1800:1 at 21°C with a FWHM of 16°C, see Figure 6.2. Thus when the waveplates become too hot the contrast ratio decreases rapidly. Although when we do not use the waveplates as optical shutters, the contrast ratio is of less importance; only intensity is lost since the waveplates ‘act’ as glass for the light. For example at 35° a contrast ratio of 100:1 is achieved, thus for 1% of the light the wave plate does nothing, while still 99% can be used.

Vibrations of the optical setup could cause the projection of light on the optical fibers to change over time, therefore broadening the signal (sampling over a large volume). However, this is not determined.

6.1.1 Driver

The wave plates are active components and need to be driven. This is done by applying a voltage to the wave plates. The voltage applied to the wave plates is a square wave with an amplitude of 5 V and a mean of 0 V. The wave plate’s optical axis is 0° when −5 V is applied, and rotated 45° when +5 V is applied. The wave plates will make 4 intensity measurements before one can construct the Stokes vector.

We want the switch intervals to be the most efficient in combination with spectrometer recordings. Since the system will use a spectrometer at TEXTOR, the exposure time of the spectrometer and the switching interval of the wave plates need to be tuned. TEXTOR uses a trigger system that sends pulses every time a plasma shot is up front. The pulses we use are $T_{-21}$ and $T_0$. The first pulse is given to prepare the systems, while $T_0$ is given at the plasma start.

![Figure 6.2](image-url) - The temperature dependence of the wave plate. At 35°C the contrast ratio drops down to 100:1.
The spectrometer starts acquiring data at $T_0$, at TEXTOR this means typically every 50 ms a trigger pulse is sent to the CCD. The wave plates also need to be configured the same way as the CCD, this in order to record the images the most optimal way. The signal diagram is shown in Figure 6.4.

The wave plates are driven by two DR-95 wave plate controllers. A device that is capable of switching fast between $-5$ and $+5$ V. Since we need to drive two wave plates out of phase, we developed a controller for these controllers, which can set the switching time to match the integration time of the CCD. There is also a time offset included, so the timing can be fine-tuned. When there are no measurements, the wave plates need to be in standby state, otherwise the crystals could build up ionic charges and that could make them function not as advertised. To prevent this, a gate is applied to the wave plate controllers, which puts the wave plate in standby-mode after a `shot’ is fired. Figure 6.3 shows the whole schematic overview of the setup at TEXTOR.

![Figure 6.3](image)

*Figure 6.3 –* A schematic overview of the setup used at TEXTOR. Light from the tokamak goes through the wave plates into the spectrometer, which is read out by the PC. The wave plates are driven by the wave plate controllers, which are driven by the TueDAC.

### 6.2 Testing the System in the Lab

Before we install the setup at TEXTOR, we test the setup at the university. These tests will measure the polarization of polarized light sources, with known polarization state. By doing this we can see how well the polarization measurement performs. We use a laser pen and a neon gas discharge lamp as light source and measure the Stokes vector of these sources.
The orientation of the wave plates was determined by placing two linear polarizers around the wave plate. By doing this a shutter is created, passing light in the +5 V state, and blocking light in −5 V. First we tested the determination of polarization on light coming from a laser pen. The center wavelength of the laser was around 650 nm. It is possible that this is not the center wavelength of the wave plates, which imposes a different retardance. A change of 7% in retardance gives an error in the Stokes vector this error is at most 10% (\(\cos(\pi/2(1 \pm 0.07))\)). The laser was focussed on a photodiode, the voltage of the diode was read out using a multimeter. Once the orientations were determined we build the setup and try to measure the Stokes vector of the laser. The results are shown in Table 6.1. We have assumed that the laser produces vertically polarized light (Stokes vector \([1, -1, 0, 0]\)). When the laser is rotated 45° there seems to be some mixing in polarizing states, Table 6.2. It could also be the case that the laser was not perfect linear polarized, as can be seen in the results from Table 6.1, the circular component \(V\) is not 0. The \(Q\) and \(U\) value are a factor 2 larger than expected, when transforming the measurement shown in Table 6.2, this is an error since the inequality \(I^2 \geq Q^2 + U^2 + V^2\) for every polarized source is not satisfied. This could be due to an accidental change to the setup, thus altering the amount of collected light during the measurement. The 4 sets were created triggering the wave plates manually, making the measurement slow and prone to errors. For the real measurement on TEXTOR this triggering is automated, so we do not expect this problem to occur (see the next section for investigation of this at the tokamak).

Figure 6.4 – The TTL pulses, \(T_{-21}\) and \(T_0\), the trigger for reading the CCD and the stop TTL pulse. The wave plates have a clock that is twice the length of the CCD because of the 4 configurations that we need. The on-state (switched optical axis) requires +5 Volt, the off-state (normal) −5 Volt.
### Table 6.1  
Stokes vector reconstruction of laser pen. Measured with a photodiode and the wave plate setup

<table>
<thead>
<tr>
<th>Wave plate state</th>
<th>Measured ( V )</th>
<th>Expected Stokes vector ( I, Q, U, V )</th>
<th>Reconstructed Stokes vector (normalized to ( I ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+5,+5)</td>
<td>0.20</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>(+5,-5)</td>
<td>0.08</td>
<td>-1</td>
<td>-0.76</td>
</tr>
<tr>
<td>(-5,-5)</td>
<td>0.046</td>
<td>0</td>
<td>-0.03</td>
</tr>
<tr>
<td>(-5,+5)</td>
<td>0.076</td>
<td>0</td>
<td>0.26</td>
</tr>
</tbody>
</table>

### Table 6.2  
Stokes vector reconstruction of laser pen. Measured with a photodiode and the wave plate setup

<table>
<thead>
<tr>
<th>Wave plate state</th>
<th>Measured ( V )</th>
<th>Expected Stokes vector ( I, Q, U, V )</th>
<th>Expected from transforming Table 6.1</th>
<th>Reconstructed Stokes vector (normalized to ( I ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+5,+5)</td>
<td>0.24</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(+5,-5)</td>
<td>0.056</td>
<td>0</td>
<td>0.31</td>
<td>0.62</td>
</tr>
<tr>
<td>(-5,-5)</td>
<td>0.13</td>
<td>1</td>
<td>0.69</td>
<td>1.31</td>
</tr>
<tr>
<td>(-5,+5)</td>
<td>0.26</td>
<td>0</td>
<td>0.26</td>
<td>0.36</td>
</tr>
</tbody>
</table>

An alternative measurement to show the working of the system is measuring the Stokes vector of the Zeeman splitting. The Zeeman effect was created using a neon lamp, because it has bright lines around 650 nm, see Table 6.3 and a magnet. For this case we use the line of 640.2 nm, because it does not coincide with higher or lower order diffraction of different lines. This is still in the range where the wave plates should function. In this case we looked at 4th order, the higher order refractions increase the dispersion, so we obtained a higher resolution.

A magnet of 0.85 T was placed around the lamp to induce the Zeeman effect and split the emission lines. When these lines are split the light goes through our two quarter-wave plates and an analyzer into a spectrometer. The intensities are captured with a CCD camera. Figure 6.5 shows the reconstructed Stokes vector as function of wavelength. We see that the splitting is visible in all Stokes components, thus the light polarization as function of wavelength is

### Table 6.3  
Observed wavelengths for neon in air around 650 nm. Obtained from the NIST atom spectra database.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Observed wavelength (Å)</th>
<th>Relative intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne I</td>
<td>6402.248</td>
<td>20000</td>
</tr>
<tr>
<td>Ne I</td>
<td>6506.527</td>
<td>15000</td>
</tr>
<tr>
<td>Ne I</td>
<td>6532.882</td>
<td>1000</td>
</tr>
<tr>
<td>Ne I</td>
<td>6598.953</td>
<td>10000</td>
</tr>
</tbody>
</table>
observable. $U$ is the largest polarization component, which means most of the light is polarized at 45° with respect to the optical axis. This means that the optical and magnetic axis are not aligned with each other.

With these measurements we have shown that we are able to reconstruct a Stokes vector from a set of 4 measurements using 2 switchable quarter-wave plates and thus we are able to detect polarized light. The error on the Stokes vector, due to different retardance of the observed wavelength, should be at most 10% for both tests. The setup behaves qualitatively as expected; we are able to record multiple intensity spectra with the spectrometer and the linear polarization of the spectrum can be determined. The alignment of the optical axis cannot be exactly determined, and therefore we are not able to determine the Zeeman effect quantitatively from the polarization.

![Figure 6.5](image.png)

**Figure 6.5** – The reconstructed Stokes vector as function of wavelength. A magnetic field of 0.85 T is applied to a neon lamp.

### 6.3 TEXTOR Measurements

This section will describe all the measurements and calibrations performed on the TEXTOR tokamak and the outcome.

#### 6.3.1 Preparations at TEXTOR

First, we will look at the signal-to-noise ratio (SNR) of the spectrum that comes out of the tokamak, without the wave plate setup installed between the lens and fibers, to see if the signal
is usable. Furthermore, we can determine the dispersion of the spectrometer and look at the intensity changes of the spectrum through time.

On 30 November 2011 there was a shot program of beam emission spectroscopy at TEXTOR, relevant to us (neutral beam injection enabled). We made measurements with two available spectrometers, k5 and k6. This, to look at the beam emission in TEXTOR and determine its spectral shape (is it as expected) and the signal-to-noise level of the measurement. k5 will be the spectrometer we use for recording spectra and magnetic pitch angle extraction. It should be noted that this spectrometer is using a CCD camera that has a uncalibrated quadrant. Therefore, we had to rotate the grating in such a way that we could record the spectral emission line of the Balmer-$\alpha$ with a Doppler shift (around 652 nm). The grating had an angle of 24.4°. The computer connected to this spectrometer uses LabView to process the data. The camera integration time was 50 ms and 180 frames were captured. This data was uploaded to the TEXTOR Web Umbrella. There were some problems with the reliability of the PC and only a few usable images of shots were captured. The shot #116220 gives the best signal of that day, which shows a part of the Balmer-alpha spectrum\(^2\). The data that we recorded shows the MSE peaks of the half and third energy beam and not the full component, due to wrong grating rotation (see Figure 6.6). It turns out that of the 10 channels present only the data of #6–10 are of use. Channel #1–5 only have half of the CCD available for recording and the alignment was not good, due to limited time. The intensity fluctuations are 10% during the beam injection, which does not enable us to follow these changes using moving averages, making it hard to reconstruct a Stokes from 4 subsequent data points (see Figure 6.7). However the quality of the signal is low.

The dispersion of the spectrometer, the ability to resolve the wavelength, can be estimated by looking at the amount of pixels between two known wavelengths in an observed spectrum. We have the deuterium and hydrogen line at 656.1 and 656.28 nm respectively and a carbon line at 657.8 nm. It turns out that the dispersion is 0.01228 nm/px. This is close to the reported dispersion\(^3\) of $0.01265 \pm 1.8975 \cdot 10^{-4}$ nm/px. It can be seen that the recorded intensity is low, we decreased the noise by averaging over 30 frames (see Figure 6.6). The SNR is 20 for this averaged measurement, this is close to the lower limit we want for the SNR, which is at least 10, see previous chapter. A lack of light coming through the view port could be the cause. From this exploration of the signal quality we could achieve we proceeded on building the setup and perform measurements on TEXTOR.

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\(^2\)for shot info see: http://ipptwu.ipp.kfa-juelich.de/textor/all/116220
\(^3\)For dispersion data of k5 see: http://ipptwu.ipp.kfa-juelich.de/textor/all/111418/cxrs/calibration/k5/dispersion.bulk
6.3.2 Installation of the Setup

Before measurements were performed we improved the transmission of the port by cleaning it during the opening of TEXTOR in the summer (see Figure 6.8). While being inside, the alignment of the fibers and the beam was confirmed.

![Figure 6.6](image)

**Figure 6.6** – A plot of the recorded profile, the setup consists only of a lens and a fiber bundle. The profile is averaged over 30 time frames in order to reduce the noise. As can be seen, the full energy component is not recorded due to bad alignment of the spectrometer.

![Figure 6.7](image)

**Figure 6.7** – The time trace of the center peak (1 px) of the $E/2$ component shown in Figure 6.6. The signal intensity does change 10% every other frame, however the intensity of the signal is very small and prone to errors.
After the opening of TEXTOR we measured enough signal with the right alignment. Figure 6.9 shows the reconstructed Stokes vector from the measurements. The intensity has an offset of 16000 which is due to background radiation. The 3 polarized components do not show this offset, so there is no polarized background in TEXTOR. This is expected since the light is directly measured, i.e., no mirrors polarize the background radiation. We see the 3 energy components are measured and the shape of the $Q$ and $U$ component are as expected, see Figure 5.2. The SNR is 10 at the edge. However, towards the center the ratio goes to 3.

However, despite the signal being promising there are problems determining the position of the channels in the plasma. This is due to the dispersion of the spectrometer, which is determined with calibration. It turns out that a small change in dispersion gives large changes in the radial position of the measurement. This is a problem, since the factor to transform the polarization angle to a pitch angle is dependent on the radial position. However, it seems that the reported dispersion of 0.01265 nm/px is correct in our case. It has to be noted that the data was averaged over 30 frames of 50 ms each (or 8 full wave plate cycles), just as was the case of the first measurement (see Figure 6.6). The error bars towards the center of the plasma are rather large, since the S/N ratio for the first few channels is around 3 for the $Q$ component and around 10 for the $U$ component. The quality of $Q$ is worse than $U$ should be expected since $Q$ depends on 4 successive measurements, while $U$ only depends on 2 measurements, see Figure 6.8 – The dirty window on top, the cleaned window in bottom place at the inside of the vessel.
equation 2.26.

It is noted that upon installation of the device there was a rotation in the view port due to external constraints. The optical axis could not be aligned with the equatorial plane of the tokamak. The estimate is that this rotation angle was around 20°. This measured polarization angle can be seen in Figure 6.10. It is possible to remove this offset and obtain the real pitch angle profile. This can be done because we know what polarization angle we expect for this plasma, from simulations. However, for our purpose we want to know the polarization angle at the inside of the tokamak therefore we also need to perform measurement.

When we determine how large the magnetic fields \((B_z, B_\phi)\) should be at the edge using the vacuum magnetic field \(1/R\) relationship and the total plasma current, we can calculate the corresponding pitch angle at that position. In our case the expected pitch angle at \(R = 2.25\) m, is \(\gamma_p = 4.9^\circ\) with \(B_\phi = 1.75\) T and \(B_z = 0.15\) T at \(R\). The expected polarization angle can then be obtained from relation (2.13) using angle \(\alpha\) and \(\beta\) for \(R = 2.25\) m. The corresponding polarization angle is \(\gamma = -6.8^\circ\). We extracted a polarization angle of \(17.2 \pm 1.1^\circ\) at \(R = 2.23\) m from the measurement. This means that we have to subtract 24° from the measurement to compensate for the offset of the viewport, see Figure 6.10. Now we have obtained the expected polarization angle we can calculate the pitch angle and therefrom a \(q\)-profile. The result can be seen in Figure 6.11. In the figure the expected profiles from simulations for this particular shot are shown as dotted lines. The measured values are plotted on top.

The error \(\Delta q\) on the last 4 channels is 0.4, this is a factor 2 too large to meet our requirement. However, the reconstruction of \(q\) in this measurement can be related to the simulations from

![Reconstructed Stokes vector](image)

**Figure 6.9** – The reconstructed Stokes vector as function of pixel number. The intensity (black line) has an offset of 16500 counts, while the polarized components have 0 offset.
The SNR is equal for the outside channels, 10. The reconstruction of $q$ performs consistent when looking at the simulation with a SNR of 10. This means that in order to reconstruct usable $q$-profiles, the SNR has to be increased to around 100. The time resolution is low, 1.5 seconds for 1 reconstruction. The agreement on the last points is due to the fact we corrected for the offset using the plasma current and magnetic field at the edge. The signal to noise ratio should be increased before this setup can be useful. From this measurement we cannot extract the physics we are interested in, the $q$ and $\gamma_p$ profile.

The effect of the temperature on the performance is not known, since it is unknown what temperature the wave plates had during operation, however this is certainly above room temperature.

Although no final conclusions can be drawn, the $q$-profile we measure is consistent with what we expect from simulations. The setup can measure the motional Stark effect spectrum at TEXTOR as function of wavelength.

- the setup can resolve polarization as function of wavelength.
- a Stokes vector is reconstructed from polarized light measurements using wave plates at 4 different configurations.
- the Stokes components behave as expected from simulations, different polarizations of $\pi$ and $\sigma$ lines.
• no polarization mixing is observed; no polarized background is present apart from the MSE signal.

• the measured pitch angle profile has large errors due to a bad SNR.

Improvements can be made, a better alignment could improve the signal and correct the offset of the view port, increasing the signal. The temperature of the device could be controlled or insulated to keep the device around room temperature, however impact on performance is not determined. These optimizations are key to improve the pitch angle reconstruction at TEXTOR.

Figure 6.11 – The pitch angle (red) and the $q$ (black) from the reconstructed Stokes vector for shot #117474, averaged over frame 30–60. And the expected simulated profiles. The noise on the first channels is large, and the dispersion is also very sensitive to different locations.
Chapter 7

Conclusion

The motional Stark effect is an essential technique for determining the configuration of magnetic fields inside a fusion reactor. We have shown that besides the common technique for determining the magnetic pitch angle (by using PEMs), three other methods can be used. Namely, making use of the intensity ratio of $\pi/\sigma$ lines of a recorded MSE profile, measuring the split of the $\pi$ lines and by using polarizers (active or passive) with Mueller matrices to determine the ratio of the Stokes vector’s $U$ and $Q$.

The main advantage of these alternative techniques in comparison with PEMs is the fact that they record the whole spectrum, which is advantageous for future tokamak research. Where PEMs need to select the wavelength of interest, these methods can record a whole range, thus being able to determine polarization mixing of the signal. By using these, also removing the necessity of making a selection of a specific point with a narrow-band filter, which is also sensitive to misalignment. This increases the flexibility of these methods when it comes down to changing neutral beam parameters (velocity, particles). However, the time resolution of the methods using a spectrometer with a CCD will be lower than is the case with photodiodes. Also, more info can be extracted from the whole recorded MSE spectrum. For example the position where the channel is looking from Doppler shift.

To obtain realistic MSE data for the TEXTOR tokamak we simulated the beam emission using an IDL simulation. This simulation was fitted to real MSE data measured on TEXTOR by Soetens. By doing this we know that the parameters we used for magnetic field, current and pressure parameters are realistic for the TEXTOR tokamak.

The performance of the alternative methods is determined by using the simulated Stokes vector (only $I$ or the full vector) and reconstruct the magnetic pitch angle profile. We have looked at the intensity ratio of the $\pi$ and $\sigma$ lines, the splitting of these lines and the Stokes vector, to determine the pitch angle profile. The methods using only the intensity spectrum (intensity
ratio and splitting) have not performed well, in our simulations.

The intensity ratio method:

- needs to take into account the population distribution of the energy levels, making it hard to predict the right pitch angle profile.
- to increase the sensitivity of the method, a proper geometry is needed.
- a change in pitch angle of 6° results in a variation of intensity of 1% making it insensitive to a change in pitch angle and difficult with noisy signals.
- when 3D effects are taken into account the method is not usable using the viewing geometry of TEXTOR, the relation between the pitch angle and the intensity ratio is lost towards the center.

The Stark splitting method:

- depends on both the magnetic field and the pitch angle, making it one equation with two unknowns, rendering this method less convenient. However, a magnetic field approximation of $1/R$ can be used to circumvent this.
- the difference between the calculated and measured splitting is large due to broadening of the signal caused by 3D effects, but the method is still sensitive to the pitch angle.

A mapping of the measured broadening to the expected broadening could be made to improve the pitch angle reconstruction. This method is dependent on the accuracy of the spectrometer. In using this method one should take into account that the Stark splitting is small so the dispersion needed is high, but it is promising if it can be used in combination with a polarization ($\gamma$) measurement. For example measure the pitch angle from a polarization measurement and use this pitch angle for simulation of the spectrum. From this spectrum the split can be fitted and tested to the real measurement.

The method using the polarizations performs better. We tested two possible configurations: 4 static polarizers, and 2 ‘active’ switchable quarter-wave plates. Both static and dynamic polarizers are able to reconstruct the Stokes vector, from which we can calculate the pitch angle:

- the 3D effects are of much less influence on the pitch angle reconstruction.
- when noise is applied to both methods we see that the dynamic method performs little better when it comes to narrowing down the change in pitch angle due to a magnetic field.
- the static method shows an error of 0.2° while the active method shows an error bar of 0.15° for our TEXTOR simulations. For a $q$-profile this gives an error of 5% for wave
plates and 7% for static polarizers, making it possible to reconstruct a good $q$-profile with large confidence for the TEXTOR tokamak.

- the SNR ratio should be larger than 10 to obtain a $q$ with an error smaller than 0.2 on TEXTOR.

An MSE setup was designed and built and pilot measurements on TEXTOR showed that this setup is capable of measuring the motional Stark effect as function of wavelength. The wave plates are tested at the university of Eindhoven at determining Stokes vectors of polarized light sources. The setup is capable of determining the Stokes vector’s polarized elements of a laser pen. Also the wavelength dependence of the polarization can be described using the Zeeman effect. We are able to measure the Stokes vector as function of wavelength on TEXTOR and proof of principle is delivered. Due to a small SNR the $q$-profile cannot be determined within our constraints of 0.2, but the results are consistent with simulations at low SNR. The SNR was 10 at the edge and as low as 3 at the center of the tokamak. The determination of the view location on the beam is an issue, and necessary to obtain usable information. The influence of the temperature on the performance of the wave plates during operation is not determined, but thought of as no ‘show-stopper’. An offset due to rotation of the view port can be effectively determined, using the plasma current and toroidal magnetic field, and be corrected for. No polarized background was observed, as is expected for small tokamaks.
Outlook

The SNR ratio should be increased of the intensities measured. Optimization of incoming light through the wave plate setup at TEXTOR should be performed; a better alignment of the lenses and light collection of the fibers and diminishing signal losses from fiber to spectrometer. Improving the total throughput of the system should improve the quality of the signal, which will yield better results. Also the influence of temperature on the performance of the device should be investigated, this could be of low importance but this is not tested. Once there are convincing results the setup can be fine-tuned to obtain higher time resolutions (<50 ms) by decreasing the recording time and be able to obtain ‘real-time’ measurements, while maintaining a high SNR (≈ 100). Modeling of the intensity ratio and splitting should be improved to create a model which can relate measurements to pitch angles. The accuracy of the Stokes vector spectrum measurement could be improved when all techniques are combined in one measurement (integrated data analysis), since the data is already there.

In future fusion research devices (like ITER and DEMO) time resolution is of less importance, due to the long current diffusion time scales (which determine $q$). However, on these devices the polarization is changed by mirrors, which lead the light out of the vessel, and polarization mixing of background light and the signal occurs. These alternative techniques can determine the polarized background and by collecting more light on all 3 energy components the accuracy will be much higher than by using PEMs, which select a very small wavelength; Therefore the full spectrum measurement will give more information, making them a viable alternative to PEMs on these devices.
Appendix A

Mueller Matrices

Wave Plates

The polarization state of light can be described by the Stokes vector,

$$S \equiv (I, Q, U, V)$$ \hspace{1cm} \text{(A.1)}

The 4 components are the intensity $I$, linear polarization $Q$, linear polarization with $45^\circ$ rotation $U$ and circular polarization $V$. When light is put through, e.g., a polarizer, the polarization state changes. This changing component can be represented by a matrix multiplication on the Stokes vector. Mueller calculus was invented for describing optical components in terms of matrices. This allows us to represent the setup as a matrix multiplication. By using these, the calculations are getting much easier. The Mueller matrix representation of a rotated wave plate is [20],

$$M_{wp} (\theta, \phi) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos[2\theta]^2 + \cos[\phi] \sin[2\theta]^2 & \cos[2\theta](1 - \cos[\phi]) \sin[2\theta] & \sin[2\theta] \sin[\phi] \\
0 & \cos[2\theta](1 - \cos[\phi]) \sin[2\theta] & \cos[2\theta]^2 \cos[\phi] + \sin[2\theta]^2 & -\cos[2\theta] \sin[\phi] \\
0 & -\sin[2\theta] \sin[\phi] & \cos[2\theta] \sin[\phi] & \cos[\phi]
\end{pmatrix}$$ \hspace{1cm} \text{(A.2)}

where $\theta$ is the physical rotation of the wave plate with respect to the optical axis, and $\phi$ is the rotation of light.

A rotated polarizer is represented by,

$$M_{pol} (\theta) = \frac{1}{2} \begin{pmatrix}
1 & \cos[2\theta] & \sin[2\theta] & 0 \\
\cos[2\theta] & \cos[2\theta]^2 & \cos[2\theta] \sin[2\theta] & 0 \\
\sin[2\theta] & \cos[2\theta] \sin[2\theta] & \sin[2\theta]^2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$ \hspace{1cm} \text{(A.3)}
where $\theta$ is the physical rotation of the polarizer with respect to the optical axis. Since we use switchable quarter-wave plates we can set $\phi = \pi/2$, the quarter-wave. The switchable part means that the fast-axis of the crystal changes by $45^\circ$ ($\pi/4$) when a voltage of $+5$ Volt is applied. The crystal’s axis will return to its initial position when a voltage of $-5$ Volt is applied. For the equation this means that $\theta$, the optical axis, will switch between $\theta_{-5V}$ and $\theta_{+5V}$. When we equate the matrices and the polarizer accordingly we get,

$$S_{\text{meas}} = \mathbf{M}_{\text{pol}}(22.5^\circ) \cdot \mathbf{M}_{\text{QWP}}(\theta_2) \cdot \mathbf{M}_{\text{QWP}}(\theta_1) \cdot \mathbf{S}_{\text{in}}$$  \hspace{1cm} (A.4)

where $\theta_1$ and $\theta_2$ are the states of the first and second wave plate respectively. The set of 4 measurements are created by applying $+5$ and $-5$ Volt to the two wave plates, that is changing $\theta_1$ and $\theta_2$ in (A.4),

$$S_1 = \mathbf{M}_{\text{pol}}(-22.5^\circ)\mathbf{M}_{\text{wp}}(\pi/4)\mathbf{M}_{\text{wp}}(0)\mathbf{S}_{\text{in}}$$
$$S_2 = \mathbf{M}_{\text{pol}}(-22.5^\circ)\mathbf{M}_{\text{wp}}(\pi/4)\mathbf{M}_{\text{wp}}(-\pi/4)\mathbf{S}_{\text{in}}$$
$$S_3 = \mathbf{M}_{\text{pol}}(-22.5^\circ)\mathbf{M}_{\text{wp}}(0)\mathbf{M}_{\text{wp}}(-\pi/4)\mathbf{S}_{\text{in}}$$
$$S_4 = \mathbf{M}_{\text{pol}}(-22.5^\circ)\mathbf{M}_{\text{wp}}(0)\mathbf{M}_{\text{wp}}(0)\mathbf{S}_{\text{in}}$$  \hspace{1cm} (A.5)

Measurement $S_1 - S_4$ reflect the combinations of the wave plates, $(+5 + 5, +5 - 5, -5 - 5, -5 + 5)$ Because of the CCD camera we only measure intensities, the first elements are of importance,

$$\begin{pmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}$$  \hspace{1cm} (A.6)

If we take the inverse we obtain the combinations which are needed to reconstruct the Stokes vector,

$$\begin{pmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2 & -2 \\ -2\sqrt{2} & \sqrt{2} & -2\sqrt{2} & 3\sqrt{2} \\ 0 & -\sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & -2\sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}$$  \hspace{1cm} (A.7)

The polarization angle is equal to $\gamma = \frac{1}{2} \arctan(U/Q)$ we can write the $U$ and $Q$ in terms of measured intensities,

$$\gamma = \frac{1}{2} \arctan \left( \frac{-\sqrt{2}I_2 + \sqrt{2}I_4}{-2\sqrt{2}I_1 + \sqrt{2}I_2 - 2\sqrt{2}I_3 + 3\sqrt{2}I_4} \right)$$  \hspace{1cm} (A.8)

This shows that we can reconstruct the polarization angle from a set of 4 intensities modulated with two quarter-wave plates in this particular setup.
Non-ideal Components

For now we have assumed that all the optical components needed for constructing the Stokes vector were assumed to be ideal. Also the equilibrium in the Tokamak is assumed to be static, the interaction of the neutral beam and the plasma are dynamic. Therefore the intensity of light that is emitted by the hydrogen in the beam is changing over time, and even the polarization can change due to interactions of the plasma.

When we talked about the fact that we can construct the Stokes vector, we needed 4 measurements. Now we know that the intensity is dynamic, we will have an error induced, because the frames that we’ve captured are non-steady state. We need to take into account the error in the intensity measurement.

The analyzer we used for the simulations was ideal. The full expression is [20],

\[
M_{pol} = \frac{1}{2} \begin{pmatrix}
  p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\
p_x^2 - p_y^2 & p_x^2 + p_y^2 & 0 & 0 \\
  0 & 0 & 2p_xp_y & 0 \\
  0 & 0 & 0 & 2p_xp_y
\end{pmatrix}
\] (A.9)

where, \( p_x \) and \( p_y \) are the absorption coefficients defined in the amplitude domain. The contrast ratio of the polarizer is defined as, \( \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \). The result of these extra diagonal terms are negligible. It turns out that a finite polarization introduces an uncertainty to the results; for a sheet polarizer with a 500:1 ratio of less than 0.4%. The wave plates we use aren’t ideal either. But for this project we haven’t determined the real Mueller matrix for them.

Soetens’ Polarizers

The non-ideal Mueller matrices used by Soetens to measure the data presented in Chapter 5.

The ideal matrices,

\[
M_{hor} = \frac{1}{2} \begin{pmatrix}
  1 & 1 & 0 & 0 \\
  1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0
\end{pmatrix}
\] (A.10)

\[
M_{ver} = \frac{1}{2} \begin{pmatrix}
  1 & -1 & 0 & 0 \\
  -1 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0
\end{pmatrix}
\] (A.11)
\[ M_{-45^\circ} = \frac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]  \hspace{1cm} (A.12) \\
\[ M_{\text{circ}} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]  \hspace{1cm} (A.13) \\

And their non-ideal components. All matrices are taken from [7].

\[ M_{\text{hor}} = \begin{pmatrix} 0.208 & 0.207 & 0 & 0 \\ 0.207 & 0.208 & 0 & 0 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0.02 \end{pmatrix} \]  \hspace{1cm} (A.14) \\
\[ M_{\text{ver}} = \begin{pmatrix} 0.175 & -0.175 & 0 & 0 \\ -0.175 & 0.175 & 0 & 0 \\ 0 & 0 & 3.84 \cdot 10^{-3} & 0 \\ 0 & 0 & 0 & 3.84 \cdot 10^{-3} \end{pmatrix} \]  \hspace{1cm} (A.15) \\
\[ M_{-45^\circ} = \begin{pmatrix} 0.073 & 1.17 \cdot 10^{-4} & -0.073 & 0 \\ 5.87 \cdot 10^{-3} & 1.46 \cdot 10^{-3} & -5.87 \cdot 10^{-3} & 0 \\ -0.073 & 0 & 0.073 & 0 \\ 0 & 0 & 0 & 1.45 \cdot 10^{-3} \end{pmatrix} \]  \hspace{1cm} (A.16) \\
\[ M_{\text{circ}} = \begin{pmatrix} 0.101 & 1.49 \cdot 10^{-4} & 0 & 0.101 \\ 7.49 \cdot 10^{-3} & 2.02 \cdot 10^{-3} & 0 & 7.49 \cdot 10^{-3} \\ -0.101 & 0 & 0 & -0.101 \\ 0 & 0 & 2.02 \cdot 10^{-3} & 0 \end{pmatrix} \]  \hspace{1cm} (A.17) 

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Appendix B

Drawings

This appendix contains the drawings used for this project. The drawings are made by H.M.M. de Jong using Inventor.
Figure B.1 - View port - detailed drawing.
Figure B.2: Polarizer holder - detailed drawing.
Figure B.3 - Wave plate holder - detailed drawing
Bibliography


