MASTER

Design of a typical section model for the study of flutter behaviour of airfoils

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Design of a typical section model for the study of flutter behaviour of airfoils

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ABSTRACT:

Within the framework of the European project VortexCell 2050 we consider dynamical behaviour of non-conventional airfoils. In this project the flutter behaviour of airfoils will be investigated. For this purpose we have designed a set-up, in which a two-dimensional rigid airfoil segment is elastically suspended in a flow produced in a wind tunnel. This kind of 2D rigid wing flutter models are called “typical section models”. The motion of the rigid wing has two degrees of freedom: a rotation about an axis normal to the flow and a translation normal to both the flow and the axis of rotation. These two degrees of freedom simulate the elastic behaviour of a real wing. The set-up is tested with a symmetrical NACA0018 wing profile. On the longer term the aim of the research is to investigate the flutter behaviour of non-conventional wing profiles with a cavity with a trapped vortex (vortex cell). Our short term goal was to develop a set-up to study flutter behaviour and understand some of the basic theoretical aspects of flutter.

Above a critical flow speed the wing starts a self-sustained oscillation, which is called flutter. Two types of flutter can occur: Firstly, flutter caused by hysteresis in the lift force as a function of the angle of attack of the wing with respect to the flow direction. This type of flutter due to flow separation is called stall flutter. Stall flutter involves only one mechanical degree of freedom, either rotation or translation. The second type of flutter is called classical flutter, which is due to aerodynamical coupling of the two mechanical degrees of freedom, translation and rotation.

In the present set-up stall flutter associated with the rotation of the wing appears at a lower flow speed than classical flutter. Classical flutter theory based on a linear approximation predicts indeed a higher critical flow velocity for classical flutter than the critical velocity observed during measurements. Furthermore blocking the translational degree of freedom does not strongly modify the flutter behaviour: The measured critical flow speed is hardly affected. This provides a confirmation that the observed flutter motion is stall flutter and not classical flutter.

As classical flutter is the most relevant type of flutter for an aircraft wing we seek for a set-up in which classical flutter prevails. Classical flutter theory is used to propose a modification of the set-up allowing a classical flutter speed lower than the observed stall flutter speed. Further we consider additional measurements of the influence on the stall flutter speed when the motion of the wing is initiated by perturbations (mechanical impact or main stream turbulence).

Keywords: aerodynamics, aeroelasticity, flutter, classical flutter, stall flutter, critical flutter speed, 2D typical section model.
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1 Introduction

1.1 General introduction to flutter

Flutter is a self-sustained oscillation of an elastic body placed in a flow that is steady in the absence of the body. This oscillation is a result of the interaction of aerodynamic and mechanical forces with the elasticity of the body. Typical and well known examples of flutter are the “Tacoma Narrows Bridge” which collapsed in 1940 (see photographs 1 and 2) and more recently the problems with the “Erasmus Brug” in Rotterdam (the problem has been solved). This report will be restricted to the flutter behaviour of airfoils.

Photograph 1: Flutter of the Tacoma Narrows Bridge (“The dancing bridge”) [1].

In the beginning of the 20th century a lot of airplanes crashed as a consequence of flutter behaviour. Photograph 3 shows an airplane which was able to land successfully after encountering flutter, which is exceptional. Basic research to flutter behaviour has been done in the first decades of the 20th century by Theodorsen and Sears [4]. More recent research has been done by Hemon [3] and Scheewe [6]. The amount of fatal incidents has decreased due to a better understanding of the subject and flutter research is still important for the design of airplanes.
Photograph 3: Rear view of an airplane which successfully landed after encountering flutter. This is exceptional, most of the planes which encounter flutter crash [15].

1.2 Introduction to the present research project

Within the framework of the European project VortexCell 2050 we consider dynamical behaviour of non-conventional airfoils. The key idea of this project is to design a thick wing profile with a cavity. By trapping a vortex in this cavity flow separation should be prevented. On the longer term the aim of the research is to investigate the influence of such a non-conventional wing profile with a cavity on the flutter behaviour. The short term goal of the work presented in this report was to develop a set-up to study flutter behaviour and understand some of the basic theoretical aspects of flutter. For this purpose we have designed a set-up, in which a two-dimensional rigid airfoil is elastically suspended in a flow produced in a wind tunnel. These kind of 2D rigid wing flutter models are called “typical section models” and are described in literature [1][2][3][5]. The motion of the rigid wing has two degrees of freedom: a rotation about an axis normal to the flow and a translation normal to both the flow and the axis of rotation. These two degrees of freedom simulate the elastic behaviour of a real wing. The set-up is tested with a symmetrical NACA0018 wing profile [14]. The NACA0018 wing profile is rather thick and therefore suitable for implementation of the vortex cell concept. Above a critical flow speed the wing starts a self-sustained oscillation, which is called flutter. Two types of flutter can occur: Firstly, flutter caused by hysteresis in the lift force as a function of the angle of deflection of the wing. This type of flutter due to flow separation is called stall flutter. Stall flutter involves only one mechanical degree of freedom. The second type of flutter is called classical flutter, which is due to aerodynamical coupling of the two mechanical degrees of freedom. For this type of flutter the flow is not separated which results in different characteristics. As classical flutter is the most relevant type of flutter for an aircraft wing we seek for a set-up in which classical flutter prevails. Another phenomenon closely related to these two types of
flutter behaviour is called divergence. Divergence is the static elastic deformation of an object under influence of the aerodynamic forces. For our set-up this corresponds to a static change of the angle of attack and the height of the wing due to the flow.

1.3 Overview of this report

In chapter 2 the flutter set-up will be presented. In fact two set-ups have been used, called the first set-up and the improved set-up. The improved set-up is the same as the first set-up apart from a change of the construction of the rotational springs. Both the set-ups will be discussed in this report because comparing the characteristics and behaviour of these two versions of the set-up shows relevant information for further development of the set-up. In chapter 3 theoretical background information will be presented with an emphasis on linear theory to describe classical flutter and the prediction of the critical flutter speed for classical flutter. Stall flutter and divergence will also be discussed in chapter 3. The parameters of the first set-up will be measured and determined in chapter 4, followed by the results of the experiments using the first set-up presented in chapter 5. The parameters for the improved set-up will be discussed in chapter 6 and the measurements using the improved set-up will be treated in chapter 7. An overview of the results with the conclusions and recommendations will be presented in chapter 8. Finally chapter 9 contains the appendices (including a nomenclature) and chapter 10 is a list of references.
2 Flutter set-up

In order to study flutter experimentally an elastically suspended wing is placed in the jet flow produced by an open wind tunnel. In this section this set-up will be presented.

2.1 Typical section

Figure 1 gives an example of a 2D rigid wing flutter model and provides an impression of the basic aspects of our 2D flutter set-up. Models as shown in Figure 1 are described in elementary books about flutter [1][2][3] and are called “2D typical section model”. Figure 1 will be used to define some of the basic parameters used in this report. A list of parameters can be found in the nomenclature presented in appendix 9.12. The set-up consists of a rigid wing, elastically connected to a rigid frame and placed in a uniform airflow produced by an open wind tunnel. The motion of the wing has two degrees of freedom: a translational and a rotational one. For the translational motion a spring with spring constant $K_h$ [N/m] has been used and for the rotation a spring with spring constant $K_\theta$ [Nm/rad]. The center of mass of the wing is indicated as CM and the axis of rotation as P. The distance between CM and the leading edge of the wing is indicated $(1 + e)b$ where $b$ is the semi chord length of the wing. The chord length $2b$ is the distance between the leading edge and the trailing edge of the profile. Therefore, if the dimensionless parameter $e < 0$, CM is upstream of the middle of the chord and if $e > 0$ CM is downstream of the middle of the chord. The same convention is used for the
dimensionless parameter $a$, referring to the position of the axis of rotation (P). In this report the dimensionless variable $x_\theta = (e - a)$ will be used as a measure for the distance between CM and P.

If $x_\theta > 0$ P is upstream of CM.
If $x_\theta < 0$ P is downstream of CM.

2.2 First set-up

The height of the wing has been measured at two positions at the chord of the wing, height $h_1$ [m] at position $x_1$ (upstream of the axis of rotation) and height $h_2$ [m] at position $x_2$ (downstream of the axis of rotation), see Figure 2. The translation of the axis of rotation ($h$ [m]) and the angle of deflection of the wing $\theta$ [rad] can be determined from $h_1$ [m] and $h_2$ [m]. Two different measurement systems have been used for measuring the height, a camera and a laser system. Because the laser system gives much better results this system has been used for all the measurements presented in this report. The measurement method and the results obtained using the first set-up will be treated in chapters 4 and 5. The calibration of the laser sensors will be presented in appendix 9.1.

Figure 2: Schematic representation of the first set-up used in this study.

Figure 2 presents a schematic representation of the first set-up used in this study, which was the first trial set-up that showed some of the characteristics (like flutter) that we wanted to study. An improved set-up has been built, having a better rotational spring system. The improved set-up will be presented in section 2.3. The first set-up we used is
different in some details from the typical 2D section model presented in Figure 1. For the translation leaf springs (with spring constant $K_t$) are used. Also the spring for the rotational motion (with spring constant $K_\theta$) is not a real torsional spring, as can be seen in Figure 3. However, this is simply a choice of how to build the set-up and does not affect the characteristics, which are the same as the typical set-up presented in Figure 1.

Figure 3: Picture of the first set-up in more detail with the parts numbered. To reduce boundary effects on the edges of the wing the plates (2) on both sides are added.

Figure 3 is a photograph of the first set-up, with some individual parts numbered which will be explained now:
-1: The wing is a NACA 0018 profile. This is a symmetric profile (no camber) with chord length $c=80$ mm (semi chord length $b=40$ mm) and a thickness of 18% of the chord length. The span width of the wing is 15.0 cm.
-2: plate to reduce boundary effects on the side edges of the wing
-3: wing holder (this part just translates, it does not rotate)
-4: leaf springs (translation). These spring are made of 0.5 mm thick brass, have a length of 0.277 m and a width of 20 mm.
-5: springs (rotation)
-6: screws to attach the leaf springs to the wing holder
-7: block to attach the rotation springs to the rotational axis
-8: axis of rotation
The picture of the set-up in Figure 3 shows there is an essential difference between the parts of the set-up that rotate and the parts that only translate. The parts numbered 1, 2, 7 and 8 will experience rotational and translational motion. Their total mass will be called $m_{rot}$. The mass of all the moving parts together is $m$. These are the parts which rotate together with the parts that only make a translational motion (parts 3, 6 and 4) and do not rotate. More about $m$ and $m_{rot}$ will be explained in chapter 4 and appendix 9.5. It is clear that it is important not to confuse $m$ and $m_{rot}$.

Figure 4 provides an overview of the set-up (indicated 2) with the wind tunnel (indicated 1) and the surroundings. The set-up is mounted on a frame of rigid aluminium bars to a massive block of concrete (indicated 3), put on a rubber slab to avoid unwanted oscillations of the set-up. The width of the opening of the wind tunnel and the span-wise length of the wing are both 0.15 m.

Figure 4: Photograph of the first set-up with surroundings. The typical section is indicated 1, the wind tunnel 2 and the block of concrete 3.
2.3 Improved set-up

Figure 5 shows the improved set-up with the new rotational springs. Apart from the rotational springs the first and the improved set-up are identical, there have not been any other changes. Comparing Figure 5 and Figure 3 shows the difference between the rotational springs of the improved and the first set-up.

Figure 5: Detail of the improved set-up with new springs for the rotation. Apart from that the first and the improved set-up are identical. The wing is suspended directly in front of the outlet of the wind tunnel. The flow direction is from the right to the left.

Finally Figure 6 shows the set-up with some parts indicated by a number:
1: Outlet of the wind tunnel used to produce the flow. The outlet has been extended to place the wing closer in front of the opening.
2: Wing (NACA0018)
3: Laser sensors (Omron ZS-LD200) used to measure the heights $h_1$ and $h_2$.
4: Grid. For some measurements a grid has been used to disturb the flow and make it more turbulent (see section 7.3.2). Most measurements have been done without this grid.
The flow speed in the test section of the wind tunnel can be varied from 10 m/s up to 30 m/s and has been measured using an anemometer. The laser system (Omron ZS-LD200) can measure distances between 15 and 25 cm with a resolution of 5 µm. Most of the measurements have been done in a range between 15 and 20 cm.

![Figure 6: The whole set-up with the wind tunnel (with grid), the wing and the laser systems to measure the motion of the wing.](image)

As mentioned in the introduction one of the basic things we wanted to measure was the critical flow velocity above which flutter occurs (flutter speed). In chapter 3 the theory to predict the flutter speed will be presented. To work with this theory, all the relevant parameters of the set-up need to be determined, like the translating mass, moment of inertia, the spring constants (rotational and translational), eigenfrequencies for the rotational and translational motion etc. The determination of the parameters for the improved set-up will be presented in chapter 6. The other measurements and results will be presented in chapter 7.
2.4 Some shortcomings of the set-up

The problem with the rotational springs of the first set-up has been solved in the improved set-up. However, the improved set-up still has some shortcomings that need to be improved. The problems of the set-up will be indicated here:

-1: The left and right side of the set-up are attached to each other only by the axis of rotation. Therefore the left and right side can make a small independent motion which causes extra friction and stress on the axis of rotation. This extra out of phase translational mode of motion and the restriction on the freedom of motion of the axis of rotation are of course unwanted.

To avoid the independent motion of the left and right side of the set-up, the left and right side of the set-up should be joined in a rigid way, without disturbing the flow. The axis of rotation will then also be able to move more freely.

-2: The axis of rotation has a clearance, which causes non linear behavior. When the problem above is solved, this clearance to make the axis of rotation able to move in the present set-up is not necessary anymore and the clearance can be reduced.

It is also possible to build a completely different set-up to study flutter. This will be discussed in the recommendations in section 8.5.
3 Flutter Theory

In this chapter the different types aeroelastic behaviour, divergence, stall flutter and classical flutter will be discussed. Most attention will be paid to the theory of classical flutter because this is the most relevant type of flutter for wings of an aircraft. In general stall flutter is avoided in aircrafts. The theory described here is partially based on literature [1][2][5][13][15][16].

3.1 Divergence

Divergence is the unbounded static elastic deformation of an object under influence of the aerodynamic forces and was the first type of aeroelastic instability that was recognized and understood [5]. For our rigid wing set-up divergence corresponds to a static change of the angle of attack and the height of the wing due to the flow. Figure 7 shows a typical section model with the relevant parameters and points for divergence indicated. An initial angle of attack $\alpha_0$ in absence of flow will tend to increase $\alpha$ and the lift force $L$ for a given flow. As long as the spring is strong enough to resist there is no problem. But as the flow speed increases the aerodynamic forces become larger while the spring stiffness is unchanged. At a certain flow speed $U_{\text{divergence}}$, the angle of attack $\alpha$ will increase without bound in response to an initial angle of attack $\alpha_0$, as shown in Figure 8.

![Figure 7: Model of the set-up with the relevant parameters and points to explain divergence indicated. P indicates the axis of rotation and AC is the aerodynamic center of the wing where the lift force acts on the wing. Mac is the moment about the aerodynamic center. The height of the wing is not relevant for divergence.](image-url)
Figure 8: Angle of deflection $\alpha$ as a function of the flow speed $U$. At a critical flow speed $U_{divergence}$ $\alpha$ will increase without bound. This phenomenon is called divergence.

The divergence speed $U_{divergence}$ can be found by considering the balance of torques for a single degree of freedom (rotation). Inertial effects will be ignored. All the torques about the axis of rotation ($P$) need to be determined. The aerodynamic forces are assumed to be proportional to the dynamic pressure $q$:

$$q = \frac{1}{2} \rho U^2,$$

where $\rho$ is the air density and $U$ is the flow speed. The lift force $L$ in this static case is equal to:

$$L = C_L qS = \left( \frac{\partial C_L}{\partial \alpha} \right) \alpha qS,$$  \hfill (2)

because for a symmetric profile the lift force vanishes when the angle of attack $\alpha$ is zero. We assume a linear dependence of $L$ with $\alpha$ for small angle of attack. $C_L$ is the lift coefficient, $S$ is the area of the wing and $q$ is the dynamic pressure. The torque due to the lift is $Ld$. Summing the moments about the axis of rotation gives:

$$\sum M = \frac{\partial C_L}{\partial \alpha} (\alpha_0 + \theta) qSD - K_\theta \theta$$

$$= \left[ \frac{\partial C_L}{\partial \alpha} (\alpha_0 + \theta) \right] qS - K_\theta \theta,$$  \hfill (3)

where $\theta = \alpha - \alpha_0$. For equilibrium the sum of the torques should vanish ($\sum M = 0$).

Solving for the deflection angle $\theta$ we find:

$$\theta = \frac{qS \left( \frac{\partial C_L}{\partial \alpha} \right)}{K_\theta \left( \frac{qS \partial C_L}{\partial \alpha} \right)} \left( 1 - \frac{qS D \partial C_L}{K_\theta \partial \alpha} \right).$$  \hfill (4)

The angle of deflection $\theta$ will diverge when the denominator of the fraction vanishes, which corresponds to:
\[ q_{\text{divergence}} = \frac{K_\theta}{Sd} \frac{\partial C_L}{\partial \alpha}, \]  
\( (5) \)

or

\[ U_{\text{divergence}} = \sqrt{2K_\theta \rho Sd} \frac{\partial C_L}{\partial \alpha}. \]  
\( (6) \)

Note that when \( d > 0 \) (AC upstream of the axis of rotation) divergence only can occur if \( \frac{\partial C_L}{\partial \alpha} > 0 \). It is however possible that \( \frac{\partial C_L}{\partial \alpha} < 0 \), this has to do with stall flutter and will be explained in the next section. The condition \( d < 0 \) (AC downstream of the axis of rotation) is unconditionally stable for \( \frac{\partial C_L}{\partial \alpha} > 0 \).

If we assume that the wing is a flat plate and the flow is 2D and incompressible (that means \( U \ll c \), where \( c \) is the speed of sound) we can further simplify the former equations using the following approximation for small angle of attack \( \alpha \):

\[ \frac{\partial C_L}{\partial \alpha} = 2\pi \]  
\( (7) \)

### 3.2 Stall flutter

As the name implies, stall flutter is a phenomenon which occurs with partial or complete separation of the flow from the airfoil occurring periodically during oscillation, in contrast to classical flutter (next section) where the flow remains attached to the wing [16]. Stall flutter is caused by hysteresis in the lift force as a function of the angle of deflection of the wing. Stall flutter involves only one mechanical degree of freedom.

Within a certain range of angles of deflection the flow will stay attached to the wing over the whole length of the wing and separate at the sharp trailing edge (Kutta condition [5]). However, for a certain critical angle of attack the flow will start to separate from the wing before the trailing edge. For the NACA0018 profile that we used the angle where flow separation starts is about 12°, strong stall occurs about 16° [14].
Figure 9: Example of a flow that stays attached to the wing over the whole length of the wing and a separated flow. At a certain angle of attack \( \alpha \) flow separation starts as shown in Figure 10. Due to flow separation the lift force \( L \) will be lower than the lift for an ideal attached flow at the same angle of attack. The lift coefficient \( C_L \) is defined as:

\[
C_L = \frac{L}{\frac{1}{2} \rho U^2 S} = \frac{L}{qS}
\]

(8)

Where \( L \) is the lift force, \( q \) the dynamic pressure and \( S \) the surface area of the wing. When the angle of attack is increased further the lift coefficient \( C_L \) and the lift \( L \) decreases.

Figure 10: The lift \( L \) as function of the angle of deflection \( \alpha \). Flow separation starts from a certain angle of attack \( \alpha \), causing the \( L \) curve to differ from the ideal curve for an attached flow.

When stall flutter occurs the angle of deflection goes into the region where flow separation occurs. When the wings motion is towards a larger angle of attack the point
where the flow separates from the wing is moving upstream from the trailing edge to the leading edge of the wing. When the wing is moving towards a smaller angle of attack the point of separation is moving downstream as shown in Figure 11. Because there is some time delay between the motion of the wing and the adjustment of the point of separation, the point of separation can be at two different positions for the same angle of attack, depending on the direction of the motion of the wing.

![Figure 11: The motion of the point of separation in relation to the motion of the wing. Because there is a time delay (hysteresis) the point of separation can have two different positions for the same angle of attack \( \alpha \), depending on the direction of the motion of the wing.](image)

Therefore the lift force can have two different values for the same angle of attack, depending on whether the angle of attack is increasing or decreasing, as shown in Figure 12. That means that the wing can gain energy from the flow. The energy gained per oscillation \( W = \int M d\alpha = \int (Ld) d\alpha \) corresponds with the area enclosed by the \( (\alpha, Ld) \) curve of the motion, where \( Ld = M \) is the torque.

![Figure 12: Schematic representation of the hysteresis in the lift force \( L \) as a function of the angle of attack \( \alpha \). Due to a hysteresis the wing can gain energy from the flow per cycle, corresponding to the shaded area.](image)

For a stationairy stall flutter oscillation (limit cycle oscillation) the energy gained each cycle from the flow equals the energy dissipated per cycle due to mechanical friction.
3.3 Classical flutter: prediction of the critical flutter speed

In contrast with stall flutter in classical flutter the flow remains attached. Classical flutter occurs by coupling of two mechanical degrees of freedom with aerodynamic forces. Due to aerodynamic forces the translational and rotational resonance frequencies will change until they merge at a critical velocity $U_{\text{critical}}$ (see Figure 13). Above this velocity the wing will flutter. In this section the theory of classical flutter will be studied to find the critical flow speed above which flutter occurs and the corresponding oscillation frequency. First the equations of motion and expressions for the forces will be proposed. These equations will be linearized and some simplifying assumptions will be made. This has the advantage that for some of the problems analytical solutions can be found. Some of the assumptions will be verified experimentally.

The simplifying assumptions are:
A:-The aerodynamic forces of the flow on the wing will be calculated assuming that the wing is a thin rigid flat plate.
B:-The flow is incompressible, because the speed of sound is much larger than the flow velocity.
C:-The springs for the translational and the rotational motion behave linearly.
D:-The flutter motion is harmonic.

Using the theory finally the critical flutter speed for the set-up is predicted.

![Figure 13: The translational and rotational frequency change due to the aerodynamic forces of the flow, until they will merge at a velocity $U_{\text{critical}}$. Above this flow speed the wing will flutter.](image_url)
3.3.1 Equations of motion, mechanical part: potential and kinetic energy

The equations of motion will be obtained by using the formalism of Lagrange. To accomplish this, expressions for the potential energy (\(P\)) and the kinetic energy (\(K\)) of the system must be calculated. Furthermore an expression for the forces acting on the wing caused by the flow (aerodynamic load) is needed. The expressions for these forces will be presented in section 3.3.4.

Potential energy (translation and rotation):

\[
E_p = \frac{1}{2} K_h (h - h_0)^2 + \frac{1}{2} K_\theta (\theta)^2 \tag{9}
\]

Where \(\theta = \alpha - \alpha_0\), and therefore \(\dot{\theta} = \dot{\alpha}\). The height \(h\) and the angle of deflection caused by the flow \(\theta\), are defined in Figure 1. \(h_0\) is the initial height (and also the mean value of \(h\) during translation). \(\alpha\) is the total angle of deflection of the wing and \(\alpha_0\) is the initial (zero flow) angle of attack between the flow and the wing.

Kinetic energy (translation and rotation):

In general for a rigid body of mass \(m\) the kinetic energy \(E_K\) has a translational term and a rotational term:

\[
E_K = \frac{1}{2} m (\ddot{v}_{cm} \cdot \ddot{v}_{cm}) + \frac{1}{2} I_{cm} \dot{\theta}^2 = \frac{1}{2} m |\dot{v}_{cm}|^2 + \frac{1}{2} I_{cm} \dot{\theta}^2, \tag{10}
\]

where (see Figure 1)

\[
\ddot{v}_{cm} = \dot{h} \hat{e}_y + \dot{\theta} (bx_\theta) (\cos \theta \hat{e}_y - \sin \theta \hat{e}_x) \tag{11}
\]

and therefore \(|\dot{v}_{cm}|^2 = (\dot{h} + \dot{\theta} x_\theta \cos \theta)^2 + (-\dot{\theta} x_\theta \sin \theta)^2 = \dot{h}^2 + 2 \dot{h} \dot{\theta} x_\theta \cos \theta + \dot{\theta}^2 (bx_\theta)^2\)

with \(x_\theta = (e - a)\) the dimensionless parameter for the distance \(bx_\theta\) between the center of mass (CM) and P, the point where the rotational axis is attached to the rotational springs. \(e\) and \(a\) are dimensionless parameters related to the position of respectively CM and P. All these parameters have been explained in Figure 1 in Chapter 2. \(\hat{e}_x\) and \(\hat{e}_y\) are the unit vectors in the x and y direction.

Because there is an essential difference (see Figure 3 in chapter 2) between the mass of the set-up which is just translating, which is the mass of all the moving parts of the total set-up \((m)\) and the mass which is rotating and translating in the set-up \((m_{rot})\) this equation has to be adapted:

\[
E_K = \frac{1}{2} (m - m_{rot}) \dot{h}^2 + \frac{1}{2} m_{rot} (\ddot{v}_{cm} \cdot \ddot{v}_{cm}) + \frac{1}{2} I_{cm} \dot{\theta}^2, \tag{12}
\]

where the subscript \(cm\) refers to the center of mass of the rotating part of the set-up.
Combining the last two equations leads to:

\[ E_K = \frac{1}{2} (m - m_{ro}) \dot{h}^2 + \frac{1}{2} m_{ro} (\dot{h}^2 + b^2 x_\theta^2 \dot{\theta}^2 + 2b x_\theta \dot{h} \dot{\theta} \cos \theta) + \frac{1}{2} I_{cm} \dot{\theta}^2, \quad (13) \]

which can be simplified to:

\[ E_K = \frac{1}{2} m \dot{h}^2 + \frac{1}{2} m_{ro} (b^2 x_\theta^2 \dot{\theta}^2 + 2b x_\theta \dot{h} \dot{\theta} \cos \theta) + \frac{1}{2} I_{cm} \dot{\theta}^2. \quad (14) \]

The moment of inertia \( I_{cm} \) needs to be determined, analytically or experimentally. Because the axis of rotation will not necessarily be at the CM we also want to have an expression for the moment of inertia relative to an arbitrary axis of rotation at point P, \( I_P \) (See Figure 1 and Figure 2). Using the relationship between the moment of inertia relative to the center of mass \( (I_{cm}) \), and the moment of inertia relative to a rotation axis at an arbitrary point P \( (I_P) \), leads to the following result:

\[ I_P = I_{cm} + m_{ro} (x_\theta b)^2, \quad (15) \]

where P is at position \( x_\theta b \) from the CM (see Figure 1).

Substituting this in the expression for \( E_K \) (equation 5) gives:

\[ E_K = \frac{1}{2} m \dot{h}^2 + \frac{1}{2} m_{ro} b x_\theta \dot{h} \dot{\theta} \cos \theta \]

For small \( \theta \) we can use a linear theory which implies that the kinetic energy \( E_K \) can be expanded up to second order terms in \( \dot{\theta}, \dot{h} \) and \( \theta \). This leads to the following expression (for small enough \( \theta, \cos \theta \approx 1 \)):

\[ E_K = \frac{1}{2} m \dot{h}^2 + m_{ro} b x_\theta \dot{h} \dot{\theta} + \frac{1}{2} I_P \dot{\theta}^2 \quad (17) \]

In appendix 9.2 the wing is approximated as a flat plate to make an analytical calculation for the moment of inertia. \( I_P \) can also be determined by measuring the natural rotational frequency \( \omega_\theta \) and the rotational spring constant \( K_\theta \). These experiments will be discussed in section 4.2.

### 3.3.2 Equations of motion, mechanical part: Lagrange formalism

The equations of motion can be found using the formalism of Lagrange. In general the equations of motion have the following form in the Lagrange formalism:

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i \quad (i=1,2,\ldots,n). \quad (18) \]
where $\mathcal{L} = E_k - E_p$ is the Lagrangian, $q_i$ are the generalized coordinates and $Q_i$ are the generalized forces. In this case the forces are the aerodynamic forces acting on the airfoil, this will be treated in section 3.3.4. In section 3.3.6 the effect of mechanical friction will be taken into account. Air friction and gravity will be neglected in this study.

The generalized coordinates $q_i$ are in this case:

$q_1 = h$ and $q_2 = \theta$

The Lagrangian $\mathcal{L}$ is (using equations 1 and 9):

$$\mathcal{L} = \frac{1}{2} m \dot{h}^2 + m_{rot} b x \dot{\theta} + \frac{1}{2} I_p \dot{\theta}^2 - \left\{ \frac{1}{2} K_h (h - h_0)^2 + \frac{1}{2} K_\theta (\theta)^2 \right\}$$  

(19)

The equations of motion according to the Lagrange formalism are:

$$\begin{align*}
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{h}} \right) - \frac{\partial \mathcal{L}}{\partial h} &= Q_h \\
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} &= Q_\theta 
\end{align*}$$  

(20)

From equation (19) we find:

$$\begin{align*}
\frac{\partial \mathcal{L}}{\partial h} &= m \ddot{h} + m_{rot} b \dot{\theta} \\
\frac{\partial \mathcal{L}}{\partial \theta} &= I_p \ddot{\theta} + m_{rot} b x \ddot{h}
\end{align*}$$  

(21)

Finally this gives the following equations of motion (without friction):

$$\begin{align*}
m \ddot{h} + m_{rot} b x \ddot{\theta} + K_h h &= Q_h \\
I_p \ddot{\theta} + m_{rot} b x \ddot{h} + K_\theta \theta &= Q_\theta
\end{align*}$$  

(22)

These equations of motion correspond with the results found by Hemon [3].

### 3.3.3 Equations of motion for harmonic motion

As we consider a linear theory we can focus on a harmonic perturbation around the equilibrium position with a frequency $\omega$. In complex notation this can be written as:

$$h = \hat{h} \exp(i \omega t), \quad \theta = \hat{\theta} \exp(i \omega t)$$  

(25)

Where $\hat{h}$ is the complex amplitude of the translation, $\hat{\theta}$ is the complex amplitude of the rotation and $i^2 = -1$. The aerodynamic forces acting on the wing are also assumed to be harmonic:

$$Q_h = \hat{Q}_h \exp(i \omega t), \quad Q_\theta = \hat{Q}_\theta \exp(i \omega t)$$  

(26)
Note: the fact that the same $\omega$ is used for both the translation $h$ and rotation $\theta$ can only be applied at the critical flutter speed. For a flow speed $U$ below $U_{\text{critical/fluence}}$, the angular frequencies $\omega_{\text{translation}}$ (for $h$) and $\omega_{\text{rotation}}$ (for $\theta$) are generally different.

The real part of $\omega (\text{Re}(\omega))$ is the angular speed of the oscillations and the imaginary part of $\omega (\text{Im}(\omega))$ gives the damping. This can be seen when equation (25) for $h$ is written as:

$$h = \hat{h} \{ \exp(i \text{Re}(\omega)t) \} \{ \exp(-\text{Im}(\omega)t) \}$$

(27)

So far, damping is supposed to be caused by the aerodynamic forces, not by mechanical friction.

Three different situations exist:

1: $\text{Im}(\omega) > 0$ The amplitude increases exponentially with time: This situation is called flutter and is clearly unstable.

2: $\text{Im}(\omega) = 0$ This condition is called neutrally stable. This corresponds to the critical flutter speed. In section 3.3.5 this condition will be applied to predict the flutter speed.

3: $\text{Im}(\omega) < 0$ The motion is damped by the flow and will return to the equilibrium position. This is a stable condition.

Substituting equations (25) and (26) in equations (23) and (24) gives:

$$-m \omega^2 \hat{h} - m_{\text{rot}} \omega^2 bx_{\theta} \hat{\theta} + m \omega \hat{h} = \hat{Q}_h,$$

and

$$-m_{\text{rot}} \omega^2 bx_{\theta} \hat{h} - I_{\text{p}} \omega^2 \hat{\theta} + I_{\text{p}} \omega \hat{\theta} = \hat{Q}_\theta.$$

(28)

where

$$\omega_h = \sqrt{\frac{K_h}{m}}.$$  \hspace{1cm} (30)

and

$$\omega_\theta = \sqrt{\frac{K_\theta}{I_{\text{p}}}}.$$  \hspace{1cm} (31)

$\omega_h$ is the natural translational frequency at zero airspeed and rotational motion is blocked ($\hat{\theta} = 0$). The mass in the above equation is the total translating mass $m$, not $m_{\text{rot}}$.

$\omega_\theta$ is the natural rotational frequency at zero airspeed when translational motion is blocked ($\hat{h} = 0$).

### 3.3.4 Equations of motion, flow forces

The flow acts on the wing and causes aerodynamic forces acting on the wing. This drives the motion of the wing and the motion of the wings influences the flow. In this section
expressions will be found for the forces caused by the interaction of the flow and the motion of the wing. For the 2D model that has been studied the forces acting on the wing are the resulting lift force $L$ for the translation of the wing and the force (or pitching) moment (torque) $M$ relative to the axis of rotation for the rotational part of the motion of the wing.

With the assumption of a simple harmonic motion of the previous section the corresponding lift $L$ and pitching moment (torque) $M$ (relative to an axis of rotation through the turning point $P$) can be written as:

$$L = \hat{L} \exp(i \omega \tau)$$  \hspace{1cm} (32)

$$M = \hat{M} \exp(i \omega \tau)$$  \hspace{1cm} (33)

Substituting these functions in the equations of motions (without friction) yields:

$$- m \omega^2 \hat{h} - m_{rot} \omega^2 bx_0 \hat{\theta} + m \omega^2 \hat{h} = \hat{\Theta}_h = \dot{L}$$  \hspace{1cm} (34)

$$- l_p \omega^2 \hat{\theta} - m_{rot} \omega^2 bx_0 \hat{h} + l_p \omega^2 \hat{\theta} = \hat{\Theta}_\theta = \dot{M}$$  \hspace{1cm} (35)

For $\hat{L}$ and $\hat{M}$ the following expressions, based on a flat plate (infinitely thin aerofoil) approximation of the wing, can be used [1]:

$$\hat{L} = l_{wing} \pi \rho \omega^3 b^3 \left[ l_h(k,M_\infty) \hat{h} + \left[l_\rho(k,M_\infty) \hat{\theta} \right] \right]$$  \hspace{1cm} (36)

$$\hat{M} = l_{wing} \pi \rho \omega^3 b^4 \left[ m_h(k,M_\infty) \hat{h} + \left[m_\rho(k,M_\infty) \hat{\theta} \right] \right]$$  \hspace{1cm} (37)

with $l_{wing}$ as the span-wise length of the wing and $b$ the semi chord length of the wing. The parameters $m_\rho, m_h, l_\rho, l_h$ are functions of the flow Mach number $M_\infty$ and the reduced frequency or Strouhal number $k$, where:

$$k = \frac{b \omega}{U}$$  \hspace{1cm} (38)

As mentioned in the introduction an incompressible flow is assumed.

This is the limit for a very low flow speed, i.e. the Mach number $M_\infty = \frac{U}{c_\infty}$ is very small and we can consider the incompressible limit $M_\infty \rightarrow 0$. This assumption simplifies the above expressions to:

$$\hat{L} = l_{wing} \pi \rho \omega^3 b^3 \left[ l_h(k,0) \hat{h} + \left[l_\rho(k,0) \hat{\theta} \right] \right],$$  \hspace{1cm} (39)
\[ \dot{M} = l_{\text{wing}} \pi \rho_{\text{a}} b^4 \omega^2 \left[ m_h(k,0) \frac{\hat{h}}{b} + [m_h(k,0) \hat{\theta}] \right]. \] (40)

The coefficients \( m_g \), \( m_h \), \( l_g \), \( l_h \) are complex functions depending on Theodorsen’s function \((C(k))\). More about Theodorsen’s function and Theodorsen’s theory can be found in [1],[2],[4]&[5]. To get an impression of the form and structure of these function and the coefficients the formulas are given here without derivation.

Theodorsen’s function:
\[ C(k) = F(k) + iG(k) = \frac{H^{(2)}_i(k)}{H^{(2)}_i(k) + iH^{(2)}_0(k)} \frac{K_i(ik)}{K_j(ik) + K_i(ik)}, \] (41)

where \( H^{(2)}_n(k) \) are the \( n^{\text{th}} \)-order Hankel functions of the 2\(^{\text{nd}}\) kind and \( K_n(ik) \) are the \( n^{\text{th}}\)-order modified Bessel functions [1]&[5].

The coefficients expressed in terms of the function of Theodorsen are:
\[ l_h = 1 - \frac{2iC(k)}{k}, \] (42)
\[ l_\theta = -\frac{2C(k)}{k^2} - \frac{2i(\frac{1}{2} - a)C(k)}{k} - \frac{i}{k} - a, \] (43)
\[ m_h = \frac{1}{2} - (\frac{1}{2} + a)l_h, \] (44)
\[ m_\theta = -\frac{i}{k} + \frac{1}{8} - \frac{a}{2} - (\frac{1}{2} + a)l_\theta, \] (45)

where \( a \) is still the parameter indicating the position of the rotation axis (through point \( P \), see Figure 1). The distance between \( P \) and the leading edge of the wing is marked \((1 + e)b\), where \( b \) is the semi chord length of the wing. The distance \( ab \) is therefore the distance from the middle of the chord to \( P \).

### 3.3.5 Calculation of the critical flutter velocity

Substitution of the expressions for the forces (equation (39) and (40)) into the equations of motion ((34) and (35)) gives a homogeneous system of two linear equations for the translational and rotational amplitudes \( \hat{h} \) and \( \hat{\theta} \). The results are given in equation (46) and (47).
A different way to write these two equations is:

\[ \begin{align*}
- m \omega^2 \ddot{h} + m_{\text{rot}} \omega^2 b x_{\theta} \ddot{\theta} + m \omega^2 \dot{h} &= l_{\text{wing}} \pi \rho \omega^3 \left[ l_h(k) \frac{\dot{h}}{b} + [m_{\phi}(k) \dot{\theta}] \right] \\
- m_{\text{rot}} \omega^2 b x_{\theta} \ddot{h} - I_{\text{rot}} \omega^2 \ddot{\theta} + l_{\text{rot}} \omega^2 \dot{h} &= l_{\text{wing}} \pi \rho \omega^4 \omega^2 \left[ m_h(k) \frac{\dot{h}}{b} + [m_{\phi}(k) \dot{\theta}] \right]
\end{align*} \] (46)

A different way to write these two equations is:

\[ \begin{align*}
\left\{ \frac{m_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^2} \left[ 1 - \left( \frac{\omega_h}{\omega} \right)^2 \right] + l_h(k) \right\} \frac{\dot{h}}{b} + \left\{ \frac{m_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^2} \right\} x_{\theta} \dot{\theta} &= 0, \quad (48) \\
\left\{ \frac{m_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^2} \right\} x_{\theta} + m_h(k) \right\} \frac{\dot{h}}{b} + \left\{ \frac{l_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^4} \left[ 1 - \left( \frac{\omega_h}{\omega} \right)^2 \right] + m_{\phi}(k) \right\} \dot{\theta} &= 0. \quad (49)
\end{align*} \]

These two equations can be rewritten in a simpler form by introducing two dimensionless parameters:

\[ \mu = \frac{m_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^2} \quad \text{(mass ratio),} \]
\[ r = \sqrt{\frac{l_{\text{rot}} + m_{\text{rot}} b^2 x_{\theta}}{m_{\text{rot}} b^2}} = \sqrt{\frac{l_{\text{rot}}}{m_{\text{rot}} b^2}} \quad \text{(mass radius of gyration about P).} \] (51)

Now the two homogeneous equations (48) and (49) can be written in the following form:

\[ \begin{align*}
\left\{ \frac{m}{m_{\text{rot}}} \left[ 1 - \left( \frac{\omega_h}{\omega} \right)^2 \right] \right\} \frac{\dot{h}}{b} + \left\{ \frac{l_{\text{rot}}}{l_{\text{wing}} \pi \rho \omega^2 b^2} \right\} x_{\theta} \dot{\theta} &= 0, \quad (52) \\
\left[ \mu x_{\theta} + m_h \right] \frac{\dot{h}}{b} + \left\{ \frac{\mu r}{\omega_h} \right\} \left[ 1 - \left( \frac{\omega_h}{\omega} \right)^2 \right] + m_{\phi} \right\} \dot{\theta} &= 0. \quad (53)
\end{align*} \]

A new dimensionless parameter \( \sigma \) will be introduced to simplify the notation:

\[ \sigma = \frac{\omega_h}{\omega}. \] (54)

With the introduction of \( \sigma \) we get the following transformation:

\[ \left( \frac{\omega_h}{\omega} \right)^2 = \sigma^2 \left( \frac{\omega_h}{\omega} \right)^2. \] (55)

With this transformation a term explicit in \( \omega \) is available, namely \( \left( \frac{\omega_h}{\omega} \right)^2 \) instead of two different terms as in equations (52) and (53). Substituting this in the equations (52) and (53) gives the following flutter determinant:
\[
\begin{vmatrix}
\mu - \frac{m}{m_{\text{rot}}} \left[ 1 - \sigma^2 \left( \frac{\omega_\theta}{\omega} \right)^2 \right] + l_\theta & \mu x_\theta + l_\theta \\
\mu x_\theta + m_\theta & \mu r^2 \left[ 1 - \left( \frac{\omega_\theta}{\omega} \right)^2 \right] + m_\theta
\end{vmatrix} = \Delta(\omega, k). \quad (56)
\]

This determinant should vanish, \( \Delta(\omega, k) = 0 \) to allow for a non trivial solution. The complex function \( \Delta(\omega, k) = 0 \) corresponds to a set of two real equations \( \text{Re}[\Delta(\omega, k)] = 0 \) and \( \text{Im}[\Delta(\omega, k)] = 0 \) with three unknowns, \( \text{Re}(\omega) \), \( \text{Im}(\omega) \) and \( k \). Since we want to determine the Strouhal number \( k \) for which neutral stability exists, we have the following additional condition:

\[ \text{Im}(\omega) = 0 \] (see section 3.3.3)

In combination with \( \Delta(\omega, k) = 0 \) (i.e. \( \text{Re}(\Delta(\omega, k)) = 0 \) and \( \text{Im}(\Delta(\omega, k)) = 0 \)) we have three equations for the unknowns \( \text{Re}(\omega) \), \( \text{Im}(\omega) \) and \( k \) at neutral stability. The problem can be solved now.

We are looking for the roots of the equation from the flutter determinant:

\[
\Delta \left( \frac{\omega_\theta}{\omega} \right)^2, k) = 0
\]

\[
\left( \frac{m}{m_{\text{rot}}} \mu^2 r^2 \sigma^2 \right) \left( \frac{\omega_\theta}{\omega} \right)^4 + \left\{ \frac{m}{m_{\text{rot}}} \mu^2 r^2 (1 + \sigma^2) + \mu \frac{m}{m_{\text{rot}}} \sigma^2 m_\theta + \mu r^2 l_\theta \right\} \left( \frac{\omega_\theta}{\omega} \right)^2 + \frac{m}{m_{\text{rot}}} \mu^2 r^2 + \mu r^2 l_\theta + \mu \frac{m}{m_{\text{rot}}} m_\theta + l_\theta m_\theta - (\mu x_\theta + l_\theta)(\mu x_\theta + m_\theta) = 0 \quad (57)
\]

This equation from the flutter determinant is of the form:

\[ ax^2 + bx + c = 0 \] (where \( x = \left( \frac{\omega_\theta}{\omega} \right)^2 \)) \quad (58)

and can therefore be solved using the ABC formula. The two solutions are:

\[ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad (59) \]

where
\[ a = \left\{ \frac{m}{m_{rot}} \mu^2 r^2 \sigma^2 \right\}, \quad (60) \]

\[ b = -\left\{ \frac{m}{m_{rot}} \mu^2 r^2 (1 + \sigma^2) + \mu \frac{m}{m_{rot}} \sigma^2 m_\theta + \mu r^2 l_h \right\}, \quad (61) \]

\[ c = \left\{ \frac{m}{m_{rot}} \mu^2 r^2 + \mu r^2 l_h + \mu \frac{m}{m_{rot}} m_\theta + l_h m_\theta - (\mu x_\theta + l_\theta)(\mu x_\theta + m_\theta) \right\}. \quad (62) \]

The two solutions for \( \left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \) still depend on \( k \). The critical value of \( k \), \( k_{\text{critical}} \), which corresponds with the critical flow speed \( U_{\text{critical}} \) above which flutter occurs, can be found by applying the condition for a neutrally stable state, \( \text{Im}(\omega) = 0 \) or \( \text{Im}\left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 (k) = 0 \).

The solution \( k = k_{\text{critical}} \), for which the flutter determinant vanishes and for which \( \text{Im}\left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 (k) = 0 \) cannot be calculated analytically and a zero finding routine in Matlab is used to find \( k_{\text{critical}} \) numerically (See appendix 9.3).

For this \( k_{\text{critical}} \) we find two different values for \( \left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \) namely \( \left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \). We are looking for a solution \( \left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \) with \( \text{Re}(\omega) > 0 \). If both the solutions have a positive real part the one with the smallest \( \omega \) is the relevant one, because this value of \( \omega \) corresponds to the lowest flow speed \( U \) above which flutter occurs. The other solution for \( \left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \) is physically not interesting. When \( k_{\text{critical}} \) is found, together with the corresponding value of \( \text{Re}\left( \frac{\omega_\theta}{\omega} \right)_{1,2}^2 \), \( \omega_{\text{critical}} \) can be calculated (\( \omega_\theta \) is a known property of the set-up). With \( \omega_{\text{critical}} \) and \( k_{\text{critical}} \) the critical flutter speed can be calculated. By definition of \( k \) we have:

\[ U = \frac{b \omega}{k} \quad \text{and therefore} \quad (63) \]

\[ U_{\text{flutter critical}} = \frac{b \text{Re}(\omega_{\text{critical}})}{k_{\text{critical}}} \quad (64) \]
3.3.6 Calculation of the critical flutter velocity with friction

We still assume harmonic motion but now an extra term representing the friction will be introduced. As a first estimate the friction is assumed to be proportional with the velocity. In this case the equations of motion change to:

\[
\begin{align*}
    m\ddot{h} + m_{rot}b\dot{x}_h\dot{\theta} + R_h\dot{h} + K_h\dot{h} &= Q_h = L, \quad (65) \\
    I_\rho\ddot{\theta} + m_{rot}b\dot{x}_\theta\dot{h} + R_\theta\dot{\theta} + K_\theta\dot{\theta} &= Q_\theta = M, \quad (66)
\end{align*}
\]

where \( R_h \) is the friction coefficient for the translational motion and \( R_\theta \) is the friction coefficient for the rotational motion. Recall: \( \omega_h = \sqrt{\frac{K_h}{m}}, \omega_\theta = \sqrt{\frac{K_\theta}{I_\rho}} \) and therefore \( K_h = m\omega_h^2, K_\theta = I_\rho\omega_\theta^2 \) and \( k = \frac{b\omega}{U} \) (reduced frequency or Strouhal number)

Again we assume harmonic motion and therefore (like in the former section):

\[
\begin{align*}
    h &= \hat{h}\exp(i\alpha t), \quad \theta = \hat{\theta}\exp(i\alpha t), \quad L = \hat{L}\exp(i\alpha t), \quad M = \hat{M}\exp(i\alpha t), \quad (67)
\end{align*}
\]

where (recall)

\[
\begin{align*}
    L &= l_{\text{wing}} \pi \rho \omega^3 \omega^2 \left[ I_h(k) \frac{\hat{h}}{b} + \left[ m_h(k) \frac{\hat{\theta}}{\omega^2} \right] \right], \quad (68) \\
    M &= l_{\text{wing}} \pi \rho \omega^3 \omega^2 \left[ m_h(k) \frac{\hat{h}}{b} + \left[ m_\theta(k) \frac{\hat{\theta}}{\omega^2} \right] \right]. \quad (69)
\end{align*}
\]

Substituting these expressions for \( L \) and \( M \), analog to the former section, the equations of motion become for the translational part:

\[
\begin{align*}
    -\omega^2 mh - \omega^2 m_{rot}b\dot{x}_h\dot{\theta} + i\omega k_\theta\dot{h} + m\omega_h^2 h &= \dot{Q}_h = \dot{L} \\
    = l_{\text{wing}} \pi \rho \omega^3 \omega^2 \left[ I_h(k) \frac{\hat{h}}{b} + \left[ m_h(k) \frac{\hat{\theta}}{\omega^2} \right] \right] \quad (70)
\end{align*}
\]

and for the rotational part the equations of motion become:

\[
\begin{align*}
    -m_{rot}\omega^2 b\dot{x}_h\dot{\theta} - I_\rho\dot{x}_\theta\dot{\theta} + I_\rho\omega_\theta^2 \dot{\theta} + i\omega R_\theta \dot{\theta} &= \dot{Q}_\theta = \dot{M} \\
    = l_{\text{wing}} \pi \rho \omega^3 \omega^2 \left[ m_h(k) \frac{\hat{h}}{b} + \left[ m_\theta(k) \frac{\hat{\theta}}{\omega^2} \right] \right] \quad (71)
\end{align*}
\]

These equations can be written as:
Translation:
\[
\begin{align*}
\begin{cases}
\frac{m_{rot}}{l_{wing} \pi \rho_{w} b^{2}} m_{rot} \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] + l_{h}(k) - \frac{i \omega R_{h}}{l_{wing} \pi \rho_{w} b^{2} \omega^{2}} \frac{\dot{h}}{b} + \\
\frac{m_{rot}}{l_{wing} \pi \rho_{w} b^{2}} x_{\theta} + l_{\theta}(k) \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(72)

Rotation:
\[
\begin{align*}
\begin{cases}
\frac{m_{rot}}{l_{wing} \pi \rho_{w} b^{2}} x_{\theta} + m_{h}(k) \frac{\dot{h}}{b} + \\
\frac{l_{\theta}}{l_{wing} \pi \rho_{w} b^{4}} \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] + m_{\theta}(k) - \frac{i \omega R_{\theta}}{l_{wing} \pi \rho_{w} b^{4} \omega^{2}} \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(73)

where (recall): \( \mu = \frac{m_{rot}}{l_{wing} \pi \rho_{w} b^{2}} \) (mass ratio) and \( r = \sqrt{\frac{l_{cm} + m_{rot} b^{2} x_{\theta}}{m_{rot} b^{2}}} = \sqrt{\frac{l_{\theta}}{m_{rot} b^{2}}} \) (mass radius of gyration about P).

Now the two homogeneous equations (71) and (72) can be written in the following form:
\[
\begin{align*}
\begin{cases}
\mu \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] + l_{h} - \frac{i \omega R_{h}}{l_{wing} \pi \rho_{w} b^{2} \omega^{2}} \frac{\dot{h}}{b} + [\mu x_{\theta} + l_{\theta}] \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(74)

and
\[
\begin{align*}
\begin{cases}
[\mu x_{\theta} + m_{h}] \frac{\dot{h}}{b} + \left\{ \mu r^{2} \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] + m_{\theta} - \frac{i \omega R_{\theta}}{l_{wing} \pi \rho_{w} b^{4} \omega^{2}} \right\} \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(75)

When the friction related terms are also expressed in the chosen dimensionless variables this equations can be further simplified to:
\[
\begin{align*}
\begin{cases}
\mu \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] - \frac{i R_{h}}{m \omega} \frac{\dot{h}}{b} + [\mu x_{\theta} + l_{\theta}] \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(76)

and
\[
\begin{align*}
\begin{cases}
[\mu x_{\theta} + m_{h}] \frac{\dot{h}}{b} + \left\{ \mu r^{2} \left[ 1 - \left( \frac{\omega_{h}}{\omega} \right)^{2} \right] - \frac{i R_{\theta}}{l_{\theta} \omega} \right\} + m_{\theta} \frac{\dot{\theta}}{b} = 0
\end{cases}
\end{align*}
\]
(77)

Because these equations are (again) homogeneous and linear, the determinant \( \Delta(\omega, k) \) of the coefficients matrix must be zero to allow for a non-trivial solution, which leads to the following flutter determinant:
\[ \mu \frac{m}{m_{\text{rot}}} \left[ 1 - \sigma^2 \left( \frac{\omega_{b}}{\omega} \right)^2 - \frac{iR_{b}}{m\omega} \right] + l_{b} \mu \omega_{\theta} + l_{\theta} \mu x_{\theta} + m_{h} \mu \left[ 1 - \left( \frac{\omega_{b}}{\omega} \right)^2 - \frac{iR_{b}}{l_{\omega}} \right] + m_{\theta} = \Delta(\omega, k) \]  

(Recall: the parameter \( \sigma = \frac{\omega_{b}}{\omega} \) has been introduced \( \{ \left( \frac{\omega_{b}}{\omega} \right)^2 = \sigma^2 \left( \frac{\omega_{b}}{\omega} \right)^2 \} \))

Using the additional condition for a neutrally stable state (for the critical flutter speed), \( \text{Im}(\omega) = 0 \) (or \( \text{Im} \left( \frac{\omega_{b}}{\omega} \right)^2 \) \( k = 0 \)), the critical flutter speed with friction can be predicted using \( U_{\text{flutter}} = \frac{b \text{Re}(\omega_{\text{flutter}})}{k_{\text{flutter}}} \) (equation (64)). The Matlab file for this calculation is given in appendix 9.4.

The equation from the flutter determinant has additional frictional terms. It cannot be written in the form \( ax^2 + bx + c = 0 \) (where \( x = \left( \frac{\omega_{b}}{\omega} \right)^2 \)) used before. Because the roots of the equation from the flutter determinant cannot be found with the ABC formula, the Matlab code finding the roots of this complex function is different from the non-frictional one. This code can (of course) also handle the former equation of the flutter determinant without friction, because that is the limit when the friction coefficients \( R_{b} \) and \( R_{\omega} \) vanish. The solution without friction is used as a starting point for the iterative procedure finding solutions with friction (as can be seen in the code in appendix 9.4). In the iteration the friction coefficients are gradually increased from zero up to the given value. At each increment of the friction coefficient a convergent solution is obtained within bounds chosen by the user.
4 Determination of the parameters for the first set-up.

In this section the parameters needed as input for the classical flutter theory of chapter 3 are determined. Some parameters can be measured directly and others have to be calculated from other (measured) parameters. If possible, parameters are determined in more than one way so that the accuracy of the results can be estimated. All theoretically determined parameters will of course be compared with the experimental results.

Figure 14 shows a detailed picture of the first set-up as presented in chapter 2. The determination of the parameters for the first set-up will be treated now. Other measurements such as the flutter velocity will be presented in chapter 5. The determination of the parameters of the improved set-up will be treated in chapter 6, as far as the parameters are different from the first set-up. The other measurements on the improved set-up will be presented in chapter 7. The parameters that have been measured (for the first set-up) are listed in the table below, with their values. The measurements and determination of the parameters will be discussed after the table in the remaining part of this chapter:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$(0.448 \pm 0.09),kg$</td>
<td>Total effective mass for the first set-up. The uncertainty is mainly due to the leaf springs. More about this in appendix 9.5.</td>
</tr>
<tr>
<td>$m_{\text{rot}}$</td>
<td>$0.224,kg$</td>
<td>Rotating mass.</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.040,m$</td>
<td>The chord length of the wing is 80mm and $b$, the semi chord is therefore 0.040m. The wing is a symmetric (no camber) NACA 0018 profile, which means that the thickness is 18% of the chord length.</td>
</tr>
<tr>
<td>$a$</td>
<td>$-0.1$</td>
<td>where $(1 + a)b$ is the distance from the leading edge of the wing to the rotational axis, which goes through point P (see Figure 1). We find $(1 + a)b = 36,mm$ and therefore $a = -0.1$.</td>
</tr>
<tr>
<td>$e$</td>
<td>$-(0.075 \pm 0.025)$</td>
<td>where $(1 + e)b$ is the distance from the leading edge of the wing to center of mass of the wing (CM) (see Figure 1). $(1 + e)b = 37 \pm 1,mm$. The axis of rotation goes almost through the center of mass, the difference in order of 1mm is difficult to measure exactly. The CM has been determined by putting the wing several times on a sharp edge and carefully moving it till it is balanced and marking that position.</td>
</tr>
<tr>
<td>$x_\theta$</td>
<td>$(e - a) = (0.025 \pm 0.025)$</td>
<td>Parameter indicating the distance between CM and rotation axis (P).</td>
</tr>
<tr>
<td>$l_{\text{wing}}$</td>
<td>$0.150,m$</td>
<td>is the length (span width) of the wing.</td>
</tr>
<tr>
<td>$K_\theta$</td>
<td>$(0.36 \pm 0.03),\frac{Nm}{\text{rad}}$</td>
<td>Rotational spring stiffness. Details of the measurements of this parameter are given in section 4.1.</td>
</tr>
<tr>
<td>$K_h$</td>
<td>$(82 \pm 4),\frac{N}{m}$</td>
<td>Translational spring stiffness. See further in section 4.1.</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>$(12.47 \pm 0.05)[\text{rad} / \text{s}]$</td>
<td>Measured natural frequency at zero flow when the rotational motion is negligible. When $\omega_n$ is calculated from the measured values of $K_h$ and $m$ the result is $\omega_n = \sqrt{\frac{K_h}{m}} = (13.5 \pm 0.5)[\text{rad} / \text{s}]$. More about the measurements of this parameter in section 4.2.2.</td>
</tr>
<tr>
<td>$\omega_\theta$</td>
<td>$(48 \pm 5)[\text{rad}/\text{s}]$</td>
<td>Measured natural rotational frequency at zero flow when the translation is blocked. See further in section 4.2.1.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\frac{\omega_n}{\omega_\theta} = (0.26 \pm 0.02)$</td>
<td>Coefficient of the natural translational and rotational frequencies.</td>
</tr>
<tr>
<td>$I_p = (1.5 \pm 0.3) \times 10^{-4} [kgm^2]$</td>
<td>Moment of inertia relative to the axis of rotation.</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>$M_\infty \rightarrow 0$</td>
<td>$M_\infty = \frac{U}{c_\infty}$ is the Mach number (dimensionless). The index $\infty$ means free stream conditions far away from the wing where the flow is not disturbed. An incompressible flow is assumed ($U_{flow} \ll c_{sound}$) and therefore $M_\infty \rightarrow 0$.</td>
<td></td>
</tr>
<tr>
<td>$\rho_m = 1.2 \frac{kg}{m^3}$</td>
<td>Air density.</td>
<td></td>
</tr>
<tr>
<td>$m_\theta, m_h, l_\theta, l_h$</td>
<td>These coefficients determine the expressions for the lift (L) and moment (M) caused by the flow and are already given in equations (42) till (45).</td>
<td></td>
</tr>
<tr>
<td>$R_\theta = (0.01 \pm 0.002)[kg/s] (= [Ns/m])$</td>
<td>Mechanical friction coefficient for the translational motion measured when the rotational motion is negligible. More about the measurements of this parameter in section 4.3.1.</td>
<td></td>
</tr>
<tr>
<td>$R_\theta = (1.1 \pm 0.7) \times 10^{-3}$ [Nms/rad] (= $kgm^2/s$)</td>
<td>Mechanical friction coefficient for the rotational motion measured when the translation is negligible. More about the measurements of this parameter in section 4.3.2.</td>
<td></td>
</tr>
</tbody>
</table>

### 4.1 Spring stiffnesses

#### 4.1.1 Measuring the translational spring constant $K_h$

Measuring $K_h$, the translational spring stiffness is quite straightforward (see Figure 15 and Figure 16) A load is put on the wing holder screws. This is done symmetrically on both sides. The result of one of these calibration measurements is given in Figure 16.
Figure 15: Model of the set-up for measuring the translational (leaf) spring stiffness, $K_h$. The springs are loaded with a known mass and the deflection of the springs has been measured.

Figure 16: The deflection of the translational springs as a function of the load. The slope is the spring stiffness $K_h$.

Figure 16 shows that the translational springs are linear. The results are easy reproducible (no hysteresis) and the result from several calibration measurements is:

$$K_h = (82 \pm 4) \frac{N}{m}$$

for the translational spring stiffness

(79)
4.1.2 Measuring the rotational spring constant $K\theta$

Measuring $K_\theta$, the rotational spring stiffness is more complex than measuring $K_h$. The wing holder has been fixed to prevent translation. Only rotational motion is possible. The load is attached to a rope, fixed on the wing and exerting a force on the sharp trailing edge of the wing.

![Figure 17: Model of the set-up for measuring the rotational spring stiffness, $K\theta$. The torsional spring system is loaded with a known load.](image)

The angle of deflection has been measured with a digital goniometer and checked with the laser system. An aluminium strip, which exactly follows the wing profile on one side and with a flat upper side (See Figure 18) has been used to make measuring more easy and consistent. This angle of deflection also had to be used to calculate the lever arm of the load which is $0.044 \times \cos(\theta)$ [m].

![Figure 18: Wing profile and digital goniometer to measure the angle of deflection $\theta$.](image)
The result of one of the calibration measurements is given in Figure 19. The fit is not going through the origin because the zero flow (and zero load) angle of attack $\alpha_0$ is not equal to zero.

\[
K_\theta : \text{ spring stiffness (rotation) } [\text{Nm/rad}]
\]

\[
y = 0.3583x + 0.0073
\]

\[
\theta \quad 0 \quad 0.02 \quad 0.04 \quad 0.06 \quad 0.08 \quad 0.1 \quad 0.12 \quad 0.14 \quad 0.16 \quad 0.18 \quad 0.2
\]

\[
x \quad 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6
\]

**Figure 19**: The angle of deflection as a function of the torque. The slope is the rotational spring constant $K_\theta$.

Several calibration measurements have been done in the same way and the result for the rotational spring stiffness is:

\[
K_\theta = (0.36 \pm 0.03) \left[ \frac{\text{Nm}}{\text{rad}} \right] \tag{80}
\]

An important remark about all the measurements considering the rotational motion of the wing for the first set-up is that the position (angle of deflection) of the wing is not very reproducible and a deviation in the order of some degrees (or about 0.03 rad) is possible. The first set-up is clearly suffering from hysteresis. The wing can return to another angle of deflection after moving the wing or putting a load on it. This has to do with the construction of the set-up, especially the construction of the rotational springs and friction of the axis of rotation in its bearing. For all these reasons an improved set-up has been built with a different construction of the rotational springs. Apart from that the set-up is the same. The measurements will be presented in chapter 6 and 7.

An example of this hysteresis for the rotation of the first set-up is given in Figure 20. This is a measurement of the height of a certain point at the wing, which is equivalent to the angle of deflection of the wing. The voltage represents the height of this point. During this measurement two times a tap is given on the wing as can be seen in the height variation (there is just rotational motion). It is clear that the equilibrium position is three times different. This demonstrates the hysteresis problem for the rotation.
4.2 Natural frequencies and moment of inertia

In this section the natural frequencies for the rotation ($\omega_\theta$) and for the translation ($\omega_h$) will be measured. From $\omega_\theta$ the moment of inertia relative to the axis of rotation $I_p$ will be calculated using $I_p = \frac{K_\theta}{\omega_\theta^2}$. The measurement of $\omega_h$ gives a check for the measured values for $m$ and $K_h$ because $\omega_h = \sqrt{\frac{K_h}{m}}$.

4.2.1 Natural rotational frequency $\omega_\theta$ and moment of inertia $I_p$

To measure $\omega_\theta$ the translational motion had been blocked with a bar just like the measurements for $K_\theta$ (See last section, Figure 17). To initiate a rotational motion a ball was dropped from a rail on the trailing edge of the wing as can be seen in Figure 21. It is very difficult for the set-up to calculate $I_{cm}$ or $I_p$ because the set-up consists of several rotating parts with complex shapes. An approximation of the moment of inertia of the individual parts with (for example) a flat plate is likely to be unrealistic. Therefore the moment of inertia will be calculated from the measurements of the rotational spring stiffness $K_\theta$ (see section 4.1) and the measurements of the natural rotational
frequency $\omega_0$ which will be presented in this section. $I_p$ can be calculated using the formula $I_p = \frac{K_p}{\omega_0^2}$. When $I_p$ is known $I_{cm}$ can also be calculated with $I_p = I_{cm} + m_{cm} (x_0 b)^2$ (see section 3.3). However, $I_{cm}$ is not a relevant parameter in fact.

Figure 21: Photograph of the set-up from above: A ball is dropped from a rail on the trailing edge of the wing to initiate a motion. The translational mode is blocked and there is no flow in order to measure the natural rotational frequency $\omega_0$.

An example of a measurement of the natural rotational frequency $\omega_0$ is given in Figure 22. These frequency measurements have also been checked in the frequency domain.
Figure 22 : The heights h1 and h2 measured by the two laser sensors, representing the rotational motion of the wing. The translation is blocked and there is no flow. Therefore the wing is oscillating with the natural rotational frequency $\omega_\theta$.

The rotation for the first set-up is damped (by mechanical friction) to zero within a few oscillations. However the period $T_\theta$ can be measured. $T_\theta = (0.13 \pm 0.01)s$ which corresponds to $f_\theta = (7.6 \pm 0.8)[Hz]$ ($f = \frac{1}{T}$). The natural rotational frequency is:

$$\omega_\theta = \frac{2\pi}{T_\theta} = (48 \pm 5)[rad/s] \quad (81)$$

In section 4.1 we already found that $K_\theta = (0.36 \pm 0.05)[Nm/rad]$ and therefore

$$I_p = \frac{K_\theta}{\omega_\theta^2} = (1.5 \pm 0.3)10^{-3}[kgm^2] \quad (82)$$

### 4.2.2 Natural translational frequency $\omega_h$

To measure the natural translational frequency $\omega_h$, it is not necessary to block the rotational motion, because this damps out in about 0.4s as shown in Figure 22. The translation lasts much longer as can be seen in Figure 23 (all time scales are in seconds).
Figure 23: Determination of the natural translational frequency $\omega_h$ (with 0 flow).

The 1st graph gives the height ($h_1$) measured by sensor 1, the 2nd graph gives the height ($h_2$) measured by sensor 2 and the 3rd graph gives these results together in one plot over the whole time domain.

The 3rd of the three graphs gives the signal of the two sensors together over the entire measurement time. The amplitude is decreasing exponentially, due to friction. Friction will be treated in the next section. The 1st and 2nd graph give the heights measured by sensor 1 and sensor 2. Both the graphs are (as can be seen) on a different time scale and are zoomed in on a different spot to show the following things:

The 1st graph shows that indeed the “start” effect of the rotation is damped out within a second and that then a pure translational, harmonic motion persists. The 2nd graph shows that after quite a long period (more than a minute) this well defined, harmonic oscillation still exists (only with decreased amplitude due to friction). The following results can be concluded from several of these measurements:

$$T_h = (0.504 \pm 0.002)s \quad \text{or} \quad f_h = 1.98\ Hz,$$

and therefore:
When $\omega_h$ is calculated from the measured values of $K_h$ and $m$ the result is:

$$\omega_h = \sqrt{\frac{K_h}{m}} = (13.5 \pm 0.5) [\text{rad/s}].$$

It is clear from Figure 22 and Figure 23 that there is a significant mechanical friction, for both the translational as the rotational mode since the amplitudes decay with time. The friction will be discussed now.

### 4.3 Friction coefficients

As we saw in the last section the motion is damped. The measurements of the last section will now be used to determine the (zero flow) friction coefficients $R_h$ (translation) and $R_\theta$ (rotation). Because there is no flow the damping is totally caused by mechanical friction. Air friction will be neglected. Damping caused by the flow (which occurs for every flow speed below the flutter speed because $\text{Im}(\omega) < 0$, as explained in section 3.3.3) will be treated in chapter 5, section 5.3. The equations of motion with friction are (Equation (65) and (66))

$$\begin{align*}
 mh + m_{\omega} bx_\omega \ddot{\theta} + R_\theta h + K_h h &= Q_h = L \\
m_{\omega} bx_\omega \ddot{h} + I_\theta \ddot{\theta} + R_\theta \dot{\theta} + K_\theta \dot{\theta} &= Q_\theta = M
\end{align*}$$

Because we measure without flow we assume that $Q_h = L = 0$ and $Q_\theta = M = 0$.

Furthermore to measure the damping coefficient for the rotation, $R_\theta$, the translational mode has been blocked (see Figure 22). Therefore equation 75 simplifies in this situation to:

$$I_\theta \dot{\theta} + R_\theta \dot{\theta} + K_\theta \dot{\theta} = 0$$

If we look again at Figure 23 and Figure 22 we see that the rotational motion damps out in about 0.4 s. The translation persists much longer and is still oscillating after 90s. Therefore it is reasonable to state that the rotation has no affect on the damping of the translational mode and therefore no effect on the (mechanical) friction coefficient $R_h$.

This simplifies equation (85) to:

$$m \ddot{h} + R_h \dot{h} + K_h h = 0$$

Now these equations of motion enable the deduction of $R_h$ and $R_\theta$ from the measurements like the ones presented in Figure 22 and Figure 23.

#### 4.3.1 Translational friction coefficient $R_h$

We assume the amplitude of $h$ decreasing exponentially and substitute this in the equation of motion to verify:
\[ h = \hat{h} \exp(A t) \]  

Where \( A \) is a negative parameter representing the damping. Substituting this in equation of motion (88) gives:

\[ mA^2 + R_h A + K_h = 0 \]  

(89)

The solution is:

\[ A = \frac{-R_h \pm \sqrt{R_h^2 - 4K_h m}}{2m} = \frac{-R_h \pm i \sqrt{4K_h m - R_h^2}}{2m} \]  

(90)

In our case: \( R_h^2 - 4K_h m < 0 \).

Recall: \( \omega_h = \sqrt{\frac{K_h}{m}} \). The expression for \( A \) can be written as:

\[ A = \frac{-R_h}{2m} \pm i \sqrt{\frac{K_h}{m} - \left( \frac{R_h}{2m} \right)^2} = \frac{-R_h}{2m} \pm i \sqrt{\omega_h^2 - \left( \frac{R_h}{2m} \right)^2} = \frac{-R_h}{2m} \pm (i \omega_h^* \tau_h) \]  

(91)

Where \( \omega_h^* \) represents the zero flow natural translation frequency with friction. This frequency with damping \( \omega_h^* \) is in fact the measured frequency in section 4.2.2 of course.

Because \( \frac{\omega_h^* 2m}{R_h} = O(10^2) \) the relative deviation between \( \omega_h^* \) and \( \omega_h \) is of the order of \( 10^{-4} \). When we substitute the expression for \( A \) into equation (89) we get:

\[ h = \hat{h} \exp(-\frac{R_h}{2m} t) \exp(\pm i \omega_h^* t) = \hat{h} \exp(-\frac{t}{\tau_h}) \exp(\pm i \omega_h^* t) \]  

(92)

Where \( \frac{1}{\tau_h} = \frac{R_h}{2m} \) is the damping time for the translational mode. Indeed an oscillation with frequency \( \omega_h^* \) is to be expected with an amplitude decreasing with \( \exp(-\frac{R_h}{2m} t) \). This expression corresponds with the measurements presented in Figure 23. Now the decreasing exponential has to be fitted from the graph to determine \( R_h \). Matlab has been used to make fits with a general formula for a damping oscillation:

\[ h(t) = \hat{h} \exp(-t/\tau_h) \sin(\omega_h^* t) + h_0 \]  

(93)

All these variables are already explained except \( h_0 \) which is the mean value of the height \( h \). These fits were good in general, an example is given in Figure 24. In this figure a plot is given of the measured height \( (h_t) \) of the wing after a ball is dropped on it (like Figure 23 in the former section) together with a fit for this damped oscillation. The fit is so good that the two curves almost cannot be distinguished.
Figure 24: The measured height (h1) as a function of time (t) together with a fit of this damped oscillation.

The coefficients found by fitting the measurement are:

\[ \hat{h} = 0.005633 \text{ [m]} \text{ (amplitude)} \]

\[ -1/\tau_h = 0.01023 \text{ [1/s]} \text{ (damping, } \frac{1}{\tau_h} = \frac{R_h}{2m} \text{)} \]

\[ \omega_h = 12.5 \text{ [rad/s]} \text{ (frequency)} \]

\[ h_0 = 0.1844 \text{ [m]} \text{ (offset)} \]

The frequency \( \omega_h \), found with this fit corresponds exactly with the measured frequency in the former section 4.2.2.

Although these fits in general are good, fitting these exponential functions remains difficult. For some measurements the damping is stronger in the beginning of the oscillation, just after the impact of the ball and the damping decreases a little bit at the end of the oscillation. This indicates a non-linearity which probably has to do with the clearance of the axis of rotation. The result from several measurements is:

\[ \frac{1}{\tau_h} = (1.1 \pm 0.2) \times 10^{-2} \text{ [1/s]} \]

which corresponds to:

\[ R_h = (1.0 \pm 0.2) \times 10^{-2} \text{ [kg/s]} \] \[ ([\text{kg/s}] = [\text{Ns/m}]) \]
4.3.2 Translational friction coefficient $R_\theta$

In exactly the same way as for the translation an expression can be derived for the rotational motion. All equations are exactly the same, only $R_\theta$ has to be replaced by $R_\theta$ and $m$ by $I_\rho$. The decaying exponential is: $\exp\left(-\frac{R_\theta}{2I_\rho}t\right)$ with $\frac{1}{\tau_\theta} = \frac{R_\theta}{2I_\rho}$.

The decaying exponential has to be determined from the measurements to calculate $R_\theta$. Because the rotational motion is damped within a few oscillations as already showed in Figure 22 the value of $R_\theta$ is less accurate than the value for $R_\theta$.

The results of the fits are not as beautiful and unambiguous as the one showed in Figure 24 for the translation. An example is given in Figure 25. In appendix 9.6 several other fits will be presented. The amplitude is not really decreasing exponentially and therefore the damping is non-linear.

The measured frequency $\omega_\theta$ of section 4.2.2 corresponds with the frequency found in the fits (see appendix 9.6) and the results found for the damping (as a consequence of mechanical friction) are:

$$\frac{1}{\tau_\theta} = (3.5 \pm 1.5)[1/s]$$

(97)

and therefore:

$$R_\theta = (1.1 \pm 0.7) \times 10^{-3} \quad [\text{Nms/ rad}] (= \text{kgm}^2/\text{s})$$

(98)

Now all relevant parameters to predict the flutter speed are known. In the next chapter the flutter speed will be predicted and the other measurements using the first set-up will be presented.
5 Flutter speed calculations and measurements using the first set-up

In this chapter the flutter speed with and without friction will be predicted. These results will be compared with the measured flutter speed. Finally some frequency response measurements will be presented.

5.1 Flutter speed prediction without friction

In chapter 3 (section 3.3.5) the theory to predict the flutter speed without friction is explained and in chapter 4 the relevant parameters have been determined. Now the flutter speed can be predicted, using the MATLAB code given in appendix 9.3. This appendix also contains some additional information regarding the calculations. The results without friction are:

\[ k_{\text{critical}} = 0.015 \]

\[ \omega_{\text{critical}} = 19 \text{ [rad/s]} \quad (f_{\text{critical}} = 3.1\text{[Hz]}) \]

\[ U_{\text{critical}} = 49.5 \text{ [m/s]} \]. This is the predicted flutter speed without friction.

When the uncertainty of the parameters is taken into account this leads to:

\[ U_{\text{critical}} = (50 \pm 10)[m/s] \] (without friction)

Before comparing this speed with measurements we first calculate the effect of the (mechanical) friction.

5.2 Flutter speed prediction with friction

In chapter 3 (section 3.3.6) the theory to predict the flutter speed with friction has been explained and in chapter 4 all the relevant parameters including the coefficients \( R_h \) (translation) and \( R_r \) (rotation) for the mechanical friction, have been determined. Now the flutter speed with friction can be predicted, using the MATLAB code given in appendix 9.4. In this appendix is also some additional information regarding the calculations. The results are:

\[ k_{\text{critical}} = 0.013 \]

\[ \omega_{\text{critical}} = 16.2 \text{ [rad/s]} \]

\[ U_{\text{critical}} = 49.9 \text{ [m/s]} \]

The friction coefficients hardly affects the flutter speed (less then 1 %)

However note that both \( k_{\text{critical}} \) and the frequency \( \omega_{\text{critical}} \) have changed significantly.

Because their ratio hardly changed the flutter speed \( U \) is almost unaffected

(Recall: \( U_{\text{critical}} = \frac{b \Re(\omega_{\text{critical}})}{k_{\text{critical}}} \) (equation (64))).
When both $R_h$ and $R_g$ are made 10 times larger than the measured values of section 4.3 the effect on the flutter speed is small. The results are:

$k_{\text{critical}} = 0.0107$

$\omega_{\text{critical}} = 13.3 \text{ [rad/s]}$

$U_{\text{critical}} = 49.5 \text{ [m/s]}$

Note again that the flutter frequency $\omega_{\text{critical}}$ has changed significantly. The friction does not lead (in the used model) to a significant change of the critical flutter speed and when the uncertainty of the parameters is taken into account the result remains:

$U_{\text{critical}} = (50 \pm 10) \text{[m/s]}$ (with and without friction)

Apparently the (mechanical) friction forces and torque acting on the wing and wing axis are very small, compared with the forces and torque caused by the flow, at the flutter speed. Now these predictions will be compared with measurements of the flutter speed.

### 5.3 Measurement of the flutter speed

In this section measurements of the motion of the wing in the time domain will be presented at different flow speeds to determine the flutter speed. In the next section measurements of the frequency response will be presented. The measured spontaneous flutter speed for the first set-up is approximately 20 m/s which is 2.5 times lower than the flutter speed predicted by theory. Measurements at a range of speeds will be presented in Figure 26 till Figure 29. When the motion of the wing is initiated by tapping the wing for example the wing starts to flutter at a flow speed of about 18.5 m/s. An explanation for the difference between the theory and the experiment can be the non linearity introduced by the clearance the rotational axis has. Another non linear effect can be the construction of the rotational springs. Another possibility is the fact that the left and right side of the set-up sometimes have an unwanted out of phase translational mode of motion which in itself influences the flutter speed and also makes it difficult for the axis to rotate freely.

Figure 26 shows the motion for a flow speed of 15.7 m/s. The first graph shows the height measured by sensor 1, the second the height measured by sensor 2 and the third shows the two sensors together in one plot over the entire measurement time. The graphs are all zoomed in on a different time scale to show as much information about the motion as possible. There is no flutter and the only motion seen is the translation at about 2 Hz. Rotational motion does not occur till the flutter speed is (almost) reached. The amplitude of the motion varies, the flow triggers the motion but also damps it immediately. The amplitude is about 0.5mm.
Figure 26: The motion of the wing at 15.7 m/s flow. The 1\textsuperscript{st} graph gives the height (h1) measured by sensor 1, the 2\textsuperscript{nd} graph gives the height (h2) measured by sensor 2 and the 3\textsuperscript{rd} graph gives these results together in one plot over the whole time domain.

Figure 27: The motion of the wing at 19.8 m/s flow. The 1\textsuperscript{st} graph gives the height (h1) measured by sensor 1, the 2\textsuperscript{nd} graph gives the height (h2) measured by sensor 2 and the 3\textsuperscript{rd} graph gives these results together in one plot over the whole time domain.
Figure 27 shows the motion of the wing for a flow speed of 19.8 m/s. This is just below the flutter speed and it can be seen (especially in the second graph) that apart from the translational frequency (about 2Hz) sometimes another higher frequency (of about 8 Hz) occurs. This frequency is from the rotation which starts when the wing is fluttering. The flow speed is not high enough for a constant flutter. Several times the flow initiates the wing to flutter. However, after a few oscillations the wing stops at maximum angle of attack. Because this “blocked” maximum angle of attack the flow then blows the wing to a maximum translational height, the wing turns back to the initial angle of attack and afterwards the translational motion is damped by the flow as can be seen in the 1st and 3rd graph. The maximum oscillation amplitude is in the order of 1.5 cm which is 3 times larger than for 15.7 m/s flow speed.

When the flow speed is increased further the wing remains fluttering most of the time as can be seen in Figure 28. For a flow speed of 20.8 m/s the wing is almost constantly fluttering (8 Hz) as can be seen in the first graph. The 1st and 2nd graph show that occasionally the translational mode (2Hz) also occurs, with the rotational mode of 8Hz together.

Further increase of the flutter speed gives constant flutter as can be seen in Figure 29 (22m/s). The rotational mode (8 Hz) is the most important and there is almost no translation. The 2nd graph shows that there is however also some translation at a much lower frequency than the flutter frequency. To investigate the frequencies in more detail in the next section measurements of the frequency response will be presented.

Figure 28 : The motion of the wing at 20.8 m/s flow. The wing is fluttering at about 8 Hz however the translational mode (about 2Hz) also shows up. The 1st graph gives the height (h1) measured by sensor 1, the 2nd graph gives the height (h2) measured by sensor 2 and the 3rd graph gives these results together in one plot over the whole time domain.
Figure 29: The motion of the wing at 22 m/s flow. The wing is clearly fluttering at a frequency about 8 Hz. The 1st graph gives the height (h1) measured by sensor 1, the 2nd graph gives the height (h2) measured by sensor 2 and the 3rd graph gives these results together in one plot over the whole time domain.

5.4 Frequency response measurements

A summary of all frequency response measurements is given in Figure 30. Figure 30 shows that the translational frequency is independent of the flow speed for this set-up. The mean frequency is $\omega_h \approx 1.97$ Hz. This is in almost perfect agreement with the $\omega_h$ measured without flow (see section 4.2.2 and section 4.3.1, the result there is $\omega_h \approx 1.97$ Hz). Some frequency response measurements for different flow speeds are presented in appendix 9.7. It is interesting to compare these frequency response measurements with the time domain measurements of the last section. It can be seen that the translational mode (about 2Hz) decreases in strength with increasing flow speed and almost vanishes for flow speeds higher than the flutter speed. The rotational (flutter) motion gets stronger with increasing flow speed and above the critical flutter speed mainly this mode remains. The rotational frequency is dependent on the flow speed. Without flow $f_\theta = (7.6 \pm 0.8)$ Hz as shown in chapter 4, section 4.2.1. This corresponds with the (rotational) frequency at the critical flutter speed as can be seen in Figure 30. When a peak or a maximum in the frequency response is not obvious, i.e. not very sharp and much lower (weaker) than other observed peaks, this frequency is indicated as “dubious frequency” in the legend.
Figure 30: The measured frequencies for different flow speeds. The translational frequency is independent of the flow speed and equal to the natural translational frequency $\omega h$. The flutter frequency is close to the natural rotational frequency $\omega \theta$. 
6 Determination of the parameters for the improved set-up

In this section the parameters of the improved set-up will be determined. The other measurements using the improved set-up will be presented in chapter 7. Some of the parameters are the same for the improved set-up as for the first set-up, others are different because the rotational springs and the way they are attached has been changed. Figure 31 shows a detailed picture of the improved set-up as presented in chapter 2. Comparing Figure 31 with a picture of the first set-up like Figure 14 for example clearly shows the differences (and resemblances).

The parameters that have been measured because they are different are listed in the table below, with their values. The measurements and determination of the parameters will be discussed after the table in the remaining part of this chapter. Because the measurement procedures and remarks are the same as for the first set-up described in chapter 4 there will be no additional explanation in general. The parameters not listed below have the same value as for the first set-up:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>((0.591 \pm 0.009),kg)</td>
<td>Total effective (translating) mass for the improved set-up. More about this in appendix 9.5.</td>
</tr>
<tr>
<td>(m_{rot})</td>
<td>((0.313 \pm 0.001),kg)</td>
<td>Rotating mass of the improved set-up.</td>
</tr>
<tr>
<td>(K_{\theta})</td>
<td>((0.13 \pm 0.01),\frac{Nm}{rad})</td>
<td>Rotational spring stiffness. More about the measurements of this parameter in section 6.1.2.</td>
</tr>
<tr>
<td>(K_h)</td>
<td>((85 \pm 2),\frac{N}{m})</td>
<td>Translational spring stiffness. This parameter is the same as for the first set-up. More about the measurements of this parameter in section 6.1.1.</td>
</tr>
<tr>
<td>(\omega_h)</td>
<td>((11.00 \pm 0.04),[rad/s]) ((f_h = 1.75,\text{Hz}))</td>
<td>Measured natural translational frequency at zero flow. When (\omega_h) is calculated from the measured values of (K_h) and (m) the result is (\omega_h = \sqrt{\frac{K_h}{m}} = (11.96 \pm 0.23), [rad/s]). More about the measurements of (\omega_h) in section 6.2.1.</td>
</tr>
<tr>
<td>(\omega_\theta)</td>
<td>((39 \pm 1),[rad/s]) ((f_\theta = 6.2,\text{Hz}))</td>
<td>Measured natural rotational frequency at zero flow. More about the measurements of (\omega_\theta) parameter in section 6.2.2.</td>
</tr>
<tr>
<td>(I_p)</td>
<td>((8.6 \pm 1.1)\times10^{-5},[kgm^2])</td>
<td></td>
</tr>
<tr>
<td>(R_h)</td>
<td>((0.012 \pm 0.001),[kg/s] = [Ns/m])</td>
<td>Mechanical friction coefficient for the translational motion (measured at zero flow). More about the measurements of this parameter in section 6.2.1.</td>
</tr>
<tr>
<td>(R_\theta)</td>
<td>((4.4 \pm 1.4)\times10^{-4},[Nm/rad] = [kgm^2/s])</td>
<td>Mechanical friction coefficient for the rotational motion (measured at zero flow). More about the measurements of this parameter in section 6.2.2.</td>
</tr>
<tr>
<td>(x_\theta)</td>
<td>((e - a) = (0.075 \pm 0.025))</td>
<td>Dimensionless parameter describing the distance between center of mass (CM) and the axis of rotation (P). (a = -0.10, e = (-0.025 \pm 0.025)).</td>
</tr>
</tbody>
</table>

### 6.1 Spring stiffnesses

#### 6.1.1 Translational spring constant \(K_h\)

The (translational part of the) improved set-up is heavier than the first set-up and in equilibrium position the leaf springs are more loaded. However this should not effect the
translational spring stiffness $K_h$, unless the load is too heavy for the leaf springs and they are loaded into their (probably) non-linear range. We check here whether we still have the same spring stiffness or not.

$$y = 84.59x - 0.0006$$

Figure 32: Check of the rotational spring stiffness, $K_h$ for the improved set-up. $K_h$ should be (and is) the same as for the first set-up.

From this check and the earlier found result for the first set-up it can be concluded that:

$$K_h = (85 \pm 2) \frac{N}{m}$$ is the translational spring stiffness

This result corresponds with the result found for the first set-up (section 4.1.1) and therefore the leaf springs are still linear.

### 6.1.2 Rotational spring constant $K_\theta$

During the measurements of the rotational spring stiffness it was already clear that the new rotational springs of the improved set-up functioned much better than the first set-up. The results, like the angle of deflection for a certain load were much better reproducible and also the effect of hysteresis is much smaller. The results of the calibration are shown in Figure 33.
From Figure 33 it can be concluded that:

\[ K_\theta = (0.13 \pm 0.01) \left( \frac{Nm}{rad} \right) \]

is the rotational spring stiffness \( K_\theta \) of the improved set-up.

That is about 3 times lower than for the first set-up.

### 6.2 Natural frequencies, moment of inertia and friction coefficients

To measure these parameters, again a ball is dropped from a rail on the trailing edge of wing to initiate a motion, like showed in Figure 21 in section 4.2.1. Because these measurements are done without flow there is no effect of the flow on the motion and we measure the natural rotational \( \omega_\theta \) and translational \( \omega_h \) frequencies. To measure the rotational natural frequency the translation is blocked like explained in section 4.2.1. Because the rotational frequency is damped to zero within 1s and the translational motion lasts for minutes it is not necessary to block the rotation to measure the translational frequency. This is showed already in section 4.2.2 in Figure 23.

#### 6.2.1 Translation: natural frequency \( \omega_h \) and friction coefficient \( R_h \)

The motion of the wing after dropping a ball from the rail (see Figure 21) on the trailing edge of the wing is damped as showed in Figure 34, together with a fit for this motion. The damping is caused by mechanical friction (air friction is neglected). The parameters \( R_h \) and \( R_\theta \) for the damping by friction are determined in the same way as for the first set-up (see section 4.3), by fitting the measurements (using MATLAB) with a function of the form:

\[ h(t) = \hat{h} \exp(-t / \tau_h) \sin(\omega_h t) + h_0 \]
Figure 34: Example of a measurement of the translational motion of the wing together with a good fit. The motion is damped by mechanical friction. $t/\tau_h = 0.010 \,[\text{I/s}]$.

Figure 34 shows that the fitting method can work very well. These fits automatically give a value for $\omega_h$ as well so this parameter can be determined with $R_h$ simultaneously. The results for the frequency $\omega_h$ is:

$$\omega_h = (11.00 \pm 0.04) [\text{rad} / \text{s}] \text{ (at zero flow)}$$  \hspace{1cm} (101)

or $f_h = 1.75 \text{ Hz}$. $\omega_h$ can also be predicted from the measured values of $K_h$ and $m$, the result is:

$$\omega_h = \sqrt{\frac{K_h}{m}} = (11.96 \pm 0.23) [\text{rad} / \text{s}]$$

Determining the friction coefficient $R_h$ showed some problems with non-linear behavior, despite the almost perfect fits. The determination of $R_h$ will be treated in more detail in appendix 9.8. As explained in section 4.3.1, the damping term is: $\exp(-\frac{R_h}{2m}t)$ and

$$\frac{1}{\tau_h} = \frac{R_h}{2m}.$$ The result is (as long as the behavior is linear):

$$R_h = (0.012 \pm 0.001) \,[\text{Ns/m}] \text{ (at zero flow)}$$  \hspace{1cm} (102)
But in the actual measurements $R_h$ can vary from (0.01-0.03) [Ns/m] because of possible extra friction on the axis of rotation caused by an unwanted mode of translational motion of the (independent) left and right side of the set-up and the clearance the axis of rotation has. However, it is interesting to know that this parameter has hardly any effect on the predicted flutter speed (as shown already for the first set-up in appendix 9.4) and therefore this problem is less painful.

### 6.2.2 Rotation: natural frequency $\omega_\theta$ and friction coefficient $R_\theta$

Figure 35 shows the rotational motion of the wing at zero flow after the heavy ball (30.5g) has been dropped on the leading edge from the rail (see Figure 21), together with an exponential fit of the motion. The translation is blocked to make sure there is just one degree of freedom. The results are much better than for the first set-up and it takes about 8 oscillations before the amplitude of the motion has almost vanished to zero. $1/\tau_\theta=2.0$ Hz.

![Damped rotation (zero flow) with fit](image)

**Figure 35**: Example of a measurement of the rotational motion at zero flow to determine the natural frequency $\omega_\theta$ and the friction coefficient, together with a fit of the motion. $1/\tau_\theta=2.0$ Hz.

Figure 36 shows the rotational motion of the wing at zero flow after the light ball (8g) has been dropped on the leading edge from the rail, together with an exponential fit of the motion. It takes about 6 oscillations before the motion is damped to zero and $1/\tau_\theta=2.8$ Hz. In this case the damping of the motion goes faster for the light ball than for the heavy ball, in contrary with the translational motion in the last section where the heavy ball gave a higher friction coefficient (at least in the beginning of the motion, just after the impact of the ball).
Figure 36: Example of a measurement of the rotational motion at zero flow to determine the natural frequency $\omega_0$ and the friction coefficient, together with a fit of the motion. $1/\tau_\theta=2.8$ Hz.

From several measurements it can be concluded that the result for the natural rotational frequency is fortunately independent on which ball has been used to initiate the motion. The result is:

$$\omega_0 = (39 \pm 1) \text{[rad/s]} \text{ (at zero flow)}$$

or $f_\theta=6.2$ Hz. This result is valid as long as the behavior of the set-up is linear. Now

$$I_p = K I_p \omega_0^2$$ can be calculated, the result is:

$$I_p = (8.6 \pm 1.1) \times 10^{-5} \text{[kgm}^2\text{]}$$

(103)

The damping term for the rotational motion is: $\exp(-R_\theta/2I_p t)$ with $1/\tau_\theta = R_\theta/2I_p$, as explained in section 4.3.2. The result from several measurements is $1/\tau_\theta = (2.5 \pm 0.5)$ Hz and therefore:

$$R_\theta = (4.4 \pm 1.4) \times 10^{-4} \text{[Nms/rad]} \text{ (at zero flow)}$$

(105)

All these results are measured with the translation blocked in two directions. During actual measurements the friction can get much higher because the motion of the axis of rotation can be blocked by deformation of the set-up caused by (for example) the extra unwanted mode of motion of the independent left and right side of the set-up. Further the axis of rotation has a clearance, causing non-linear behavior. The consequences for the rotational frequency during actual measurements with flow and $R_\theta$ will be discussed in appendix 9.9. The result is: $\omega_\theta(\text{non-linear}) = (35 \pm 4) \text{[rad/s]}$ or $f_\theta(\text{non-linear}) = 5.6$Hz. The extra friction lowers the rotational frequency. In chapter 7 the measured rotational frequency with flow will be compared with $\omega_\theta$ and the results with extra friction for $\omega_\theta(\text{non-linear})$. The conclusion is that the frequencies of actual measurements correspond better to the lower $\omega_\theta(\text{non-linear})$ values with extra friction. Now all the relevant parameters of the improved set-up have been determined and the flutter speed can be predicted. This will be presented in chapter 7, together with the other measurements using the improved set-up.
7 Measurements using the improved set-up

In this section the experiments using the improved set-up will be presented. First the critical flutter speed will be determined and compared with the predicted critical flow speed, using classical flutter theory. Further the influence of the individual parameters on the flutter speed will be discussed. In section 7.2 frequency response measurements will be presented. The response of the system to two different kinds of perturbations, mechanical impact and main stream turbulence, will be discussed in section 7.3. Section 7.4 presents experimental results of the influence on the flutter speed of the position of the center of mass relative to the axis of rotation. Finally in section 7.5 will be discussed how the critical flutter speed can be lowered, based on classical flutter theory.

7.1 Critical flutter speed

The experiments using the first set-up showed that the translational mode is dominant for flow speeds below the critical flutter speed. When flutter occurs, there is almost no translation and the motion is mainly rotational. This is also the case for the improved set-up as shown in Figure 37, Figure 38 and Figure 39.

![Figure 37: Angle of deflection (mean and amplitude) as a function of the flow speed. At the critical flutter speed the motion is changing.](image)

![Figure 38: Mean height of the center of mass as a function of the flow speed. At the critical flutter speed the motion is changing.](image)
Figure 39: Amplitude of the translation of the center of mass as a function of the flow speed.
At the critical flutter speed the motion is changing.

Figure 37 shows the mean angle of deflection and the oscillation amplitude of the angle of deflection as a function of the flow speed. Figure 38 shows the mean height of the center of mass as a function of the flow speed and Figure 39 shows the amplitude of the translation oscillation of the center of mass as a function of the flow speed. The motion of the wing is spontaneous and caused by the flow only, no forcing has been used to initiate the motion. It is clear in all these figures, especially in Figure 37, that the motion is dramatically changing at a flow speed of about 19 m/s, which is the critical flutter speed above which flutter occurs.

The fact that the (static) mean angle of deflection and the (static) mean height are increasing with increasing flow speed suggests divergence (section 3.1). Using equations (6) and (7) we find $U_{divergence} = 13\text{ m/s}$, assuming the aerodynamic center of the wing to be positioned at $\frac{1}{4}$ chord length ($c$) from the leading edge. However, the angle of deflection is not observed to grow without bound (see Figure 37). We observe flutter for a flow speed of about 19 m/s. The mean angle of deflection approaches 12° (Figure 37), at which flow separation starts for the NACA0018 profile that we used (section 3.2 and [14]). This is an indication that the observed flutter behaviour is stall flutter and not classical flutter.

To check whether the observed behaviour is stall flutter or not, the flutter speed has been measured with the translation blocked, allowing only rotation. As explained in section 3.2 and section 3.3, classical flutter needs two coupled degrees of freedom (translation and rotation), stall flutter needs just one degree of freedom (rotation). When the translation is blocked the measured flutter speed is about 23 m/s which is close to the measured flutter speed with translation (19 m/s). Hence probably stall flutter is observed instead of classical flutter. Nevertheless, the experimental results will still be compared with the prediction (according to classical flutter theory) in the following table:

<table>
<thead>
<tr>
<th>Measurement:</th>
<th>Prediction without friction:</th>
<th>Prediction with friction:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{critical} = (5.6 \pm 0.3)\text{Hz}$ (section 7.2)</td>
<td>$f_{critical} = 2.13\text{Hz}$</td>
<td>$f_{critical} = 1.99\text{Hz}$</td>
</tr>
<tr>
<td>$U_{critical} = 19-20 \text{ m/s}$ (spontaneous)</td>
<td>$U_{critical} = 29.0 \text{ m/s}$</td>
<td>$U_{critical} = 27.8 \text{ m/s}$</td>
</tr>
</tbody>
</table>
When the uncertainty of the parameters (determined in chapter 6) is taken into account the theoretical result for the critical flutter speed and the critical flutter frequency is:

\[
U_{\text{critical}} = (28 \pm 5)[m/s] \\

f_{\text{critical}} = (2.02 \pm 0.13)Hz
\]

This prediction is much better than for the first set-up regarding the flutter speed. However the prediction for the flutter frequency is bad, the measured flutter frequency is between 5 and 6 Hz as will be shown in the section 7.2. The predicted frequency corresponds with the translational frequency, not with the flutter frequency. The deviation between theory and experiment will be discussed in section 7.4, as well as the possibility that we observe stall flutter.

The influence of some of the individual parameters will be discussed in appendix 9.10. The moment of inertia \(I_p\) and the distance between the center of mass (CM) and the axis of rotation (P), represented in \(x_p = (e - a)\) appear the most important. Experiments where these two parameters have been varied will be discussed in section 7.4. The translational friction coefficient \(R_h\) is not important (less than 1 % change). The rotational friction coefficient \(R_\theta\) is more important than \(R_h\) (about 3% change of the flutter velocity). The parameters \(R_h\) and \(R_\theta\) cannot be varied and therefore there will be no further experiments to investigate their effect on the flutter velocity.

### 7.2 Frequency response measurements.

In this section frequency response measurements will be presented at different flow velocities. The motion of the wing is spontaneous and caused by the flow only, no forcing has been used to initiate the motion. The results of the measurements are summarized in Figure 40. Figure 40 shows the frequency response of the set-up, measured for increasing flow speed. For flow speeds till about 12 m/s there is just a translation with a frequency about 1.8 Hz. For higher flow speeds there is also a rotational mode of about 3.1 Hz when there is no flutter and between 5.3 and 5.9 Hz when there is flutter. The critical flutter speed is about 19 [m/s] which corresponds with the result found in the previous section.
There is hysteresis: When the set-up flutters the flow speed can be reduced till about 16 m/s before flutter stops. This is shown in Figure 41, which shows the frequency response measured for increasing and decreasing flow speed. If the flow speed is increased above the critical flutter speed the flutter frequency decreases from about 5.7 Hz to 5.3 Hz. If these results are compared with the measured natural rotational frequency \( f_\omega = 6.2 \) Hz in section 6.2.2, the measured frequencies in Figure 40 and Figure 41 correspond better with \( \omega_\theta \) for the non-linear range with extra friction (\( f_\theta_{(non-linear)} \approx 5.6 \) Hz, see appendix 9.9). The influence of the flow speed on the measured frequencies is small in general.
Figure 41 gives the impression that the translational frequency of about 1.8 Hz is independent of the flow speed. Classical flutter theory however predicts that this frequency should increase with increasing flow and eventually merge with the rotational natural frequency, when flutter occurs (section 3.3). In Figure 42 the same results as in Figure 41 for the translational frequency are shown on a different scale.

![Graph showing translational frequency vs flow speed](image)

Figure 42: Translational spontaneous oscillation frequency without flutter with linear fit, as a function of the flow velocity. There seems to be some correlation, corresponding to the predictions of classical flutter theory. However the natural frequency for zero flow does not support this.

The translational frequency seems to increase a little bit with increasing flow speed, corresponding with classical flutter theory. However the change in frequency is small. To illustrate the results presented in Figure 40, Figure 41 and Figure 42 two examples of a measured frequency response will be shown. When there is no flutter the translational mode is again dominant and the translational frequency is about 3.1 Hz as shown for a flow speed of 16.8 m/s in Figure 43. There is still no indication of the flutter frequency. If the flow speed is further increased a peak will come up at the flutter frequency between 5 Hz and 6 Hz and the rotational peak at about 3.1 Hz will decrease in strength and eventually vanish. The translational peak remains approximately at the same frequency but decreases in strength when the flow speed is increased.

![Graph showing frequency response for translation and rotation](image)

Figure 43: Frequency response of the improved set-up for a flow speed of 16.8 m/s. The translation is dominant to the rotation. The translational frequency is almost independent of the flow speed and is 1.8 Hz. The rotational frequency is 3.1 Hz when there is no flutter.
Figure 44 shows the frequency response for the translation and the rotation at a flow speed of 21.5 m/s. The set-up is fluttering now without falling back into the mainly translational non-fluttering mode. The motion mainly consists of a rotation at the flutter frequency when flutter occurs. The rotational peak at about 3 Hz has totally vanished. The translational frequency of about 1.8 Hz persists but there is also a new translational peak at the flutter frequency. The translation has a very small amplitude compared with the rotation and the motion is quite irregular.

![Frequency response for translation and rotation](image)

Figure 44: Frequency response of the set-up for a flow speed of 21.5 m/s. The flutter frequency is most clearly seen as a rotation however there is also a very small and irregular translation with two frequency components: The flutter frequency and the natural translational frequency.

### 7.3 Response of the system to perturbations

#### 7.3.1 Frequency response measurements with initiated motion

The measurements so far showed the spontaneous response of the wing to a steady flow, without external perturbations such as the impact of a ball nor turbulence of the flow. Figure 45 shows the results of the spontaneous frequency response experiments, that were already shown in Figure 41 together with the measurements using a ball to initiate the motion. There is no difference in the measured frequencies. Flutter occurs above a flow speed of 16 m/s, which corresponds to the spontaneous critical flow speed when hysteresis is taken into account. The only remarkable thing is that the heavy ball seems to trigger a frequency of about 5 Hz for flow speeds below 17 m/s. For higher flow speeds the frequency response of the heavy ball (30.5 g) measurements is the same as for the light ball (8g) and the spontaneous measurements.
As an illustration the frequency response using the heavy ball for a flow speed of 11 m/s is given in Figure 46. It shows that there is clearly a peak at about 1.8 Hz for the translation. About 5 Hz is not really a peak, however there is a maximum in the response. Similar results are found for all measurements using the heavy ball for flow speeds up to 17 m/s. This kind of “broad” peaks or maxima does not occur for other measurements, there the peaks are all narrow. Apart from this there are no further differences between forced and spontaneous motion.
7.3.2 Frequency response measurements with disturbed flow (grid in flow)

In order to disturb the flow and make it more turbulent some experiments have been done with a grid in front of the wind tunnel as shown in Figure 47. Measurements have been done without initializing motion and by dropping a ball from a rail on the leading edge of the wing to initiate motion, like explained earlier.

Figure 47: Front view of the improved set-up, with a grid in front of the opening of the wind tunnel. For the measurements in this section a grid has been used to disturb the flow and make it more turbulent.

The results are given in Figure 48. Just as for the experiments without a grid, there is not a big difference between the measurements where the motion is initiated with a ball and the spontaneous measurements. The measured frequencies are the same, the only difference is that the flutter mode is triggered for a lower flow speed when using a ball, corresponding to the flow speeds for spontaneous measurements when hysteresis is taken into account. To compare the measurements with grid to the measurements without grid all the results of Figure 48 (with grid) and Figure 45 (without grid) are plot in one graph given in Figure 49.
Figure 48: Frequency response with a grid to disturb the flow. All the measurements (spontaneous, light ball, heavy ball) are put together in one graph, because the measured frequencies are almost the same. The frequency of about 1.8 Hz corresponds to a translation, the frequency about 4 Hz to a rotation and the highest frequency about 6 Hz is the flutter frequency.

Figure 49: Frequency response with and without grid together in one plot. The critical flutter speed is higher when using a grid in the flow and the rotational frequencies are higher when using a grid. The translational frequency is the same as long as the flow speed is below the critical flutter speed.
From Figure 49 it can be concluded that the flutter speed is higher when the grid is used to disturb the flow. The flutter speed without grid is about 19 m/s and with grid between 21 m/s and 22 m/s. This is strange, one would expect that the turbulence in the flow caused by the grid would initiate motion in some way and lower the flutter speed. The translational frequency is the same with and without grid as long as the flow speed is below the critical flutter velocity. Strange is that all frequencies for both the rotation and the translation increase when the flow speed approaches the critical flutter velocity when a grid is used. That is not the case for the measurements without grid. Figure 50 shows the translational frequency as a function of the flow speed in more detail. The frequency increases with increasing flow, corresponding to classical flutter behaviour (section 3.3). The changes of the frequency as a function of the flow speed are significant larger than without a grid (compare with Figure 42). It would be interesting to predict the translational (and rotational) frequency for a given flow speed $U$. This will be discussed in section 8.5.3.

![Figure 50: Translational frequency response using a grid to disturb the flow. For higher flow speeds there is a difference in the frequency between the spontaneous measurements and the measurements where the motion is initiated with the light ball or the heavy ball. This difference is not noticed in the experiments not using a grid.](image-url)
7.4 Changing the position of the CM relative to the axis of rotation

In this section the influence on the critical flutter speed of the position of the center of mass (CM) relative to the rotation axis (P) will be investigated. This is one of the most important parameters for the flutter speed. The distance between CM and P is $bx_a = b(e - a)$ (see Figure 1). For our set-up the axis of rotation is fixed at a distance $(1 + a)b$ from the leading edge and cannot be changed easily. Therefore the position of CM, at a distance $(1 + e)b$ will be changed by attaching additional two weights (19.0 g each) to the wing as shown in Figure 51.

![Image](image.png)

Figure 51 : Weights (indicated m) are added to change the position of the center of mass relative to the (fixed) position of the axis of rotation.

By changing the distance of the weights to the axis of rotation, the position of CM changes and the parameter $e$ (and therefore $x_a = (e - a)$) varies. For each position of the additional weights, the center of mass has been determined, as well as the moment of inertia $I_p$ which also changes. The mass $m$ and the rotational mass $m_{rot}$ change of course by the amount of the added mass. Therefore in fact four parameters have been changed. However, the relative change of the parameter $x_a$ is much larger than the relative change of the parameters $m$ and $m_{rot}$ and therefore differences in the measured and predicted...
flutter speed will be mainly due to a change of \( x_\theta \) and \( I_p \). \( I_p \) is also an important parameter. Unfortunately it is not possible to vary \( e \) and \( I_p \) independent of each other. The added weights both have a mass of 19.0 g and are symmetrically attached to the wing at some different positions:

<table>
<thead>
<tr>
<th>Position of the weights relative to the axis of rotation:</th>
<th>Measured parameter values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.5 mm downstream</td>
<td>( x_\theta = 0.16 \pm 0.03 )</td>
</tr>
<tr>
<td></td>
<td>( f_\theta = 4.2 \pm 0.3 \text{Hz} ) (natural frequency)</td>
</tr>
<tr>
<td></td>
<td>( I_p = (1.9 \pm 0.2) \times 10^{-4} \text{kgm}^2 )</td>
</tr>
<tr>
<td>30 mm downstream</td>
<td>( x_\theta = 0.14 \pm 0.03 )</td>
</tr>
<tr>
<td></td>
<td>( f_\theta = 4.7 \pm 0.5 \text{Hz} )</td>
</tr>
<tr>
<td></td>
<td>( I_p = (1.5 \pm 0.3) \times 10^{-4} \text{kgm}^2 )</td>
</tr>
<tr>
<td>30 mm upstream</td>
<td>( x_\theta = -0.13 \pm 0.03 )</td>
</tr>
<tr>
<td></td>
<td>( f_\theta = 4.4 \pm 0.7 \text{Hz} )</td>
</tr>
<tr>
<td></td>
<td>( I_p = (1.8 \pm 0.5) \times 10^{-4} \text{kgm}^2 )</td>
</tr>
</tbody>
</table>

The results for the weights 30mm upstream are not as convincing and accurate as for downstream added weight, the motion of the wing is irregular. The measured values for \( I_p \) are approximately two times the value for \( I_p \) without added weight (chapter 6).

Because the values for \( I_p \) are approximately the same for the 3 different positions where the weight is added, the effect on the flutter speed will mainly be caused by the change of \( x_\theta = (e - a) \).

Figure 52 and Figure 53 show the mean angle of deflection and the amplitude of the angle of deflection as a function of the flow speed for the weights added at 30mm and 52.5 mm downstream of the axis of rotation, respectively. In both graphs the predicted angle, according to equation (4) (an approximation valid for small angles) has been added. The motion of the wing is spontaneous and not initiated.

Figure 52: Angle of deflection (mean and amplitude) as a function of the flow speed for weights added 30mm downstream of the axis of rotation \( (x_\theta = 0.14) \). The prediction is an approximation for a small angle of deflection. The motion is spontaneous and not initiated.
It is clear that the motion is dramatically changing at a flow speed of about 18 m/s in Figure 52 and 14 m/s in Figure 53. This change corresponds with the beginning of flutter: As explained earlier, the flutter motion is mainly rotational, which means a large amplitude for the rotation. The rotational motion is almost zero when there is no flutter, which corresponds with earlier mentioned observations of the motion of the wing. The fact that the (static) mean angle of deflection is increasing with increasing flow speed suggests divergence (section 3.1). The flutter starts when the mean angle is 18° (Figure 52) and 20° (Figure 53), both larger than 16°, the angle at which severe flow separation begins for the NACA0018 profile that we used (section 3.2 and [14]). This suggests that the flutter behaviour observed is stall flutter and not classical flutter.

To check whether the observed behaviour is stall flutter or not, one of the measurements of the flutter speed has been repeated with the translation blocked, allowing only rotational motion. As explained in section 3.2 and section 3.3, classical flutter needs two coupled degrees of freedom (translation and rotation), stall flutter needs just one (rotation). Figure 53 shows that the flutter speed is about 14 m/s for the weights added at 52.5 mm downstream of the axis of rotation. When the translation is blocked the measured flutter speed is about 16 m/s, close to 14 m/s, which indicates that the observed behaviour is indeed stall flutter.

The results for the measured flutter speed (for spontaneous and initiated motion) as a function of $x_\phi$ (weights at different positions) for al the measurements are shown in Figure 54. The examples given in Figure 52 ($x_\phi = 0.14; U_{\text{critical}} = 18 [m/s]$) and Figure 53 ($x_\phi = 0.16; U_{\text{critical}} = 14 [m/s]$) are indicated with a square.

Figure 53: Angle of deflection (mean and amplitude) as a function of the flow speed for weights added 52.5 mm downstream of the axis of rotation ($x_\phi = 0.16$). The prediction is an approximation for a small angle of deflection. The motion is spontaneous and not initiated.
Figure 54: The flutter velocity as a function of $x_\theta$ (indicating the distance between the center of mass and the axis of rotation). Adding weight downstream increases $x_\theta$ and clearly lowers the flutter speed. The flutter speed without added weight is indicated with a triangle.

It is clear that the flutter speeds decreases for increasing $x_\theta$, which corresponds with expectations for classical flutter. However, probably the cause is that the initial zero flow angle of attack increases with adding weight (compare Figure 52 and Figure 53) and comes closer to the critical angle where flow separation starts and stall flutter can occur. The motion is also more stable and regular when mass is added downstream than without the added weight. The motion starts really smooth with a small amplitude in stead of the rather sudden start of the motion for the set-up without added weight.

Figure 55 shows an example of the frequency response when the weights are added at 52.5 mm (from the axis). The motion is initiated by softly tapping the wing using a finger. Flutter already starts at a flow speed of 11 m/s (!).

The flutter frequency is approximately 4 Hz and is the same for all the measurements with added weight (regardless of the position of the weight). This corresponds to the
natural rotational frequencies given in the table above. This is essentially lower than the flutter frequency without added weight which is between 5Hz and 6 Hz (see Figure 40). The translational frequency (for all measurements regardless of the position of the weight) is between 1.7 Hz and 1.75 Hertz which corresponds to the natural translational frequency. The frequency is just a little bit lower than without added weight (Figure 40), which is logical, because the added weights of 38 g is 6% of the mass \( m \) of the set-up (0.591 kg, see chapter 6). The natural frequency should therefore decrease about 3\% (\( \omega_h = \sqrt{\frac{K_h}{m}} \)). Again there is no large difference between initiated and spontaneous motion.

The experimental results will be discussed and compared with classical flutter theory is appendix 9.11. The conclusion in this appendix is that the agreement is poor, both for the critical flutter velocity and the critical frequency. This is not strange: We apparently observe stall flutter. In that case classical flutter theory cannot be applied.

### 7.5 Lowering the critical flutter speed below the observed stall flutter speed

We can conclude that we are in fact observing stall flutter instead of classical flutter. Therefore it would be nice if we could lower the predicted classical flutter speed to a value lower than the measured (stall) flutter speed. Maybe classical flutter can be measured then. An option to is to make the leaf springs for the translation more stiff, which will also increase the natural translational frequency (\( K_h = m\omega_h^2 \)). The theoretical prediction using classical flutter theory is shown in Figure 56. This prediction is for the wing without added weights. By stiffening of the springs the natural translational frequency (1.75 Hz for the present set-up) moves towards the natural rotational frequency (6.2 Hz) which makes it easier to flutter at one and the same frequency (see section 3.3, Figure 13). Of course the spring stiffness should not be changed so much that \( \omega_h > \omega_y \). In that case the frequencies will not get closer but diverge with increasing flow speed. From Figure 56 it can be concluded that when the natural translational frequency would be multiplied by a factor of 2.5 (that would mean that the leaf springs should be made 6.25 times stronger which is possible), the critical flutter speed would become about 13 m/s, which is lower than the actual measured (stall) flutter speed (19 m/s). The translational natural frequency would become about 4.5 Hz which is indeed much closer (and still below) to the rotational natural frequency. Unfortunately, at the time of writing this report the necessary leaf springs were not available.
Figure 56: The predicted critical flutter speed (classical flutter) as a function of the natural translational frequency $\omega_n$. $\omega_{h0}$ is the natural frequency of the present set-up (1.75 Hz). $\omega_n$ can be changed by changing the springs stiffness of the leaf springs ($K_h = m \omega_n^2$).
8 Conclusions and recommendations:

8.1 Critical flutter speed

Flutter is a self-sustained oscillation due to coupling between aerodynamic and mechanical behaviour of an object in a flow. Long term goal of our research is to investigate the influence of a cavity in a wing on its flutter behaviour. We have built a “typical section” model of an aircraft wing. In this model the wing is a two dimensional rigid airfoil section. The wing can rotate along an axis perpendicular to the flow and can translate in a direction normal to both the flow direction and the axis of rotation. Springs are used to maintain the wing in an equilibrium position (chapter 2). Two types of flutter behaviour are expected, classical flutter and stall flutter. Classical flutter is the most relevant one in aircraft design. For classical flutter the aerodynamical and aeroelastic forces are essentially linear. This flutter involves two mechanical degrees of freedom, rotation and translation. Due to aerodynamic forces the mechanical resonance frequencies converge towards a single frequency. Classical flutter occurs above the critical flow velocity at which the two resonance frequencies merge. When stall flutter occurs only one degree of freedom is involved and the flow is separated in contrast to classical flutter where the flow remains attached to the wing. Stall flutter due to the rotational degree of freedom is driven by the hysteresis in the lift force as a function of the angle of attack of the wing. Stall flutter occurs above a critical value of about 12° for the NACA0018 profile we used (section 3.2 and [14]). At 16° there is strong stall and the lift curve has a negative slope.

In this report we discuss the prediction of classical flutter based on the aerodynamical model of Theodorsen for an infinitely thin flat plate (chapter 3). In the following chapter (4) we have determined the model parameters (spring constants, mass, moment of inertia, etc) for a first set-up. In chapter 5 we compared the prediction of the critical velocity above which flutter occurs with experimental observations. Theory predicts flutter above a velocity of 50 m/s at an oscillation frequency of 3 Hz. In experiments one observes oscillations above 20 m/s which are determined by the rotational mode around 8 Hz. This indicates that we have stall flutter rather than classical flutter. A problem with the first set-up is that the rotational movement is strongly damped. Furthermore results do not reproduce well because the springs could change position (hysteresis). Both problems were solved by improving the construction of the rotational springs. This resulted in a second, improved set-up.

In chapter 6 the parameters of the improved set-up are discussed. In chapter 7 we consider the flutter behaviour. The predicted dynamical flutter behaviour is a critical flow speed of 28 m/s and a frequency of 2 Hz, close to the translational natural resonance frequency. We observe flutter above 19 m/s and again the oscillation occurs close to the rotational natural resonance frequency of 6 Hz. There is a hysteresis: when flutter occurs the flow speed can be decreased to about 16 m/s before the flutter stops.

When the translational degree of freedom is blocked we find a critical flow speed of 23 m/s close to that with two degrees of freedom (19 m/s). This is a confirmation that the
flutter behaviour is stall flutter and not classical flutter. Looking at the angle of deflection of the wing as a function of the flow speed, we see that it approaches the critical angle of 16° above which the lift characteristic has a negative slope and stall occurs [14]. We therefore assume stall flutter is dominating in most experiments.

When the center of gravity of the wing is moved away from the rotational axis by adding weights to the wing downstream the axis of rotation, the critical flow speeds and the frequency decrease. But the frequency remains close to the natural rotational frequency. The measured flutter speed is almost the same when the translation is blocked, which confirms stall flutter.

We consider a modification of the set-up to achieve a classical flutter speed below the observed stall flutter speed. The easiest parameter to change is the spring stiffness of the leavesprings for the translation. Theory predicts (chapter 7, Figure 56) that increasing the translational spring stiffness by a factor 6 should decrease the classical flutter speed from 28 m/s down to 13 m/s. At the time of writing this report the necessary springs were not available.

8.2 Oscillation behaviour

Above the critical flow speed the motion of the wing is a steady oscillation. When considering stall flutter the amplitude of the oscillation is determined by a balance between the energy gain due to hysteresis of the lift force and dissipation of energy due to friction (section 3.2). The measurements of the critical flutter speed showed a hysteresis: flutter starts spontaneously at a flow speed of about 19 m/s. However, when there is flutter the flow speed can be decreased to about 16 m/s before the flutter stops (section 7.2). In the case of classical flutter the linear theory does not predict a finite amplitude. Above the critical flutter speed the predicted oscillation amplitude grows exponentially in time. If a finite amplitude is observed for classical flutter, this is due to non-linear effects.

8.3 Response of the system to perturbations

When the motion of the wing is initiated by impact of a ball the measured frequencies are the same as for non-initiated motion. Flutter occurs above a flow speed of 16 m/s, which corresponds to the spontaneous critical flow speed when hysteresis is taken into account (section 7.3). There are no big differences between the light ball (8g) and the heavy ball (30.5g) regarding the flutter speed and frequency measurements (section 7.3.1).

When a grid is used to disturb the flow and make it more turbulent, flutter starts at a flow speed of 22 m/s which is higher than without grid (19 m/s). One would expect a lower flow speed because it seems logical that flow perturbations initiate motion more easy than a steady uniform flow. We do not understand the increase of the critical flutter speed induced by turbulence. We also observe, when a grid is used that the flutter frequency increases with increasing flow speed. This effect is not observed without grid (section 7.3.2).
8.4 Shortcomings of the set-up

Ideally the set-up should only have two mechanical degrees of freedom. As we used independent leaf springs for the left and the right side of the set-up, a relative motion of these two springs tends to block the axis of rotation (section 2.4). This problem has been solved by reducing the diameter of the axis of rotation. As a result of this, there is a clearance which can induce non-linear effects.

The left and right side of the set-up should be rigidly attached to each other to avoid the unwanted oscillation mode. This would also allow the axis of rotation to move more freely. The clearance can be reduced.

8.5 Recommendations

8.5.1 Recommendations regarding stall flutter

We did not intend to study stall flutter. We therefore seek for an improved set-up which displays classical flutter. Modifications of the present set-up are possible, for example:
- the stiffness of the translational springs can be increased by making them shorter and/or thicker. This will move the natural translational frequency $\omega_\theta$ closer to the natural rotational frequency $\omega_\phi$. These frequencies are a factor 3 different for the present set-up.

When these frequencies are closer it is more likely that the translational and rotational motion will be forced by the aerodynamic forces to flutter at (just) one and the same frequency (section 3.3, Figure 13). Theory predicts that it will also lower the critical flow speed above which classical flutter occurs below the critical stall flutter flow speed (chapter 7, Figure 56).

8.5.2 Recommendations regarding problems of the set-up

The problems with the extra unwanted mode of translational motion due to the independence of the lefts and right side should be solved, as well as the clearance of the axis of rotation. An option is:
- The left and right side of the set-up should be attached rigidly in order to avoid the unwanted translational mode. This will also help the axis to move more freely.

Another option is:
- Building a set-up in which the wing is attached to a single elastic rod, providing both the translational and rotational degrees of freedom.

8.5.3 Recommendations regarding things to investigate

Below the critical flutter velocity, the rotation and translation mode have complex frequencies. The imaginary part of these frequencies corresponds to the damping of the oscillation. The real part of these frequencies corresponds to the oscillation frequencies which we can determine from the frequency response of the system to a broad band perturbation such as turbulence in the flow. In order to use the theory presented in section
3.3 For the prediction of this response below the critical flutter speed one has to reformulate the equations of motion and the lift and torque contributions (equations (36) and (37)) in terms of two independent oscillation frequencies, \( \omega_{\text{trans}} \) for the translation and \( \omega_{\text{rot}} \) for the rotation. The fact that a linear approximation is used allows to consider the contributions of these two modes to the lift and torque as independent.
9 Appendices

Figure 57: The whole set-up, including the wing, the wind tunnel and the two laser systems to measure the height.

9.1 Calibration of the laser system (chapter 2)

As explained in chapter 2 the height of the wing is measured with two laser sensors at two points on the chord of the wing, one upstream of the axis of rotation and one downstream of the axis of rotation. The height of the center of mass and the angle of deflection $\theta$ can be calculated from these two heights. The laser sensor gives a voltage as output for the computer and this is the quantity that is measured and recorded. Therefore this voltage has to be related to the measured distance and the calibration is given in Figure 58. A simple set-up has been made for these measurements. The distance and angle between the surface of the reflecting body and the laser beam could be varied. The angle has been measured using a digital goniometer and the measured distance could be checked using the measured angle and simple goniometric calculations. Both the laser sensors are perfectly linear. Because we are interested in the translation of the wing and not in the
actual height itself, the offset is not relevant. However the slope was for different measurements on both the sensors exactly 0.00250 and therefore the laser system perfectly works for our purposes.

![calibration of the laser sensor](image)

Figure 58: Calibration of the laser sensor, giving the height $h$ [m] of the wing as a function of the voltage output of the sensor.

An important note is that the angle of the laser beam, relative to the reflecting surface also has been varied to see if this had any effect on the output of the sensor. However, this had no effect at all. This is an important result because the angle of the beam relative to the wing is continuously changing when the wing is in motion.

### 9.2 Moment of Inertia calculation for a flat plate (section 3.3)

If the wing is thin enough it can be approximated by a flat plate. To have a first estimate for the moment of inertia an infinitely thin aerofoil (a flat plate) will be assumed for now. This estimate will be verified and judged by measurements (results in chapters 4 and 6).

The moment of inertia relative to an axis of rotation through the center of mass, $I_{cm}$ is defined by:

$$ I_{cm} = \int r^2 \, dm^* $$

(106)

Where $r^2 = (x - x_{cm})^2 + (y - y_{cm})^2$ and $dm^*$ is an infinitesimal small part of mass ($^*$ is just added to avoid confusing $m$ and $dm$)
The moment of inertia for a flat plate of length 2b is (per unit length):

$$I_{cm} = \int_{-b}^{b} x^2 \, dm^* = \frac{m_{rot}}{2b} \int_{-b}^{b} x^2 \, dx = \frac{m_{rot}}{2b} \left[ \frac{1}{3} x^3 \right]_{-b}^{b} = \frac{1}{3} \cdot m_{rot} b^2$$  \hspace{1cm} (107)

Where $x = 0$ is chosen to be at the middle of the wing chord (with length 2b) and

$$dm^* = \frac{m_{rot}}{2b} \, dx$$  \hspace{1cm} (108)

is an infinitesimal part of mass. Substituting this expression for $I_{cm}$ in $E_K$ (equation (14)) gives:

$$E_K = \frac{1}{2} \dot{m} h^2 + \frac{1}{2} \frac{m_{rot}}{b^2} (b^2 x_\theta^2 \dot{\theta}^2 + 2bx_\theta \dot{h} \theta \cos \theta) + \frac{1}{2} \frac{1}{3} m_{rot} b^2 \dot{\theta}^2$$

$$= \frac{1}{2} \dot{m} h^2 + \frac{1}{2} m_{rot} b^2 (x_\theta^2 + \frac{1}{3}) \dot{\theta}^2 + m_{rot} bx_\theta \dot{h} \theta \cos \theta$$  \hspace{1cm} (109)

Because the axis of rotation will not necessarily be at the CM we also want to calculate the moment of inertia relative to an arbitrary axis of rotation at point P ($I_p$)

(See Figure 1).

Using the relationship between the moment of inertia relative to the center of mass ($I_{cm}$), and the moment of inertia relative to a rotation axis at an arbitrary point P ($I_p$), we find:

$$I_p = I_{cm} + m_{rot} (x_\theta b)^2$$  \hspace{1cm} (110)

$$I_p = \left( \frac{1}{3} + x_\theta^2 \right) m_{rot} b^2$$  \hspace{1cm} for a flat plate  \hspace{1cm} (111)

Where P is at a distance $x_\theta b$ from CM (see Figure 1) Substituting this in $E_K$ gives:

$$E_K = \frac{1}{2} \dot{m} h^2 + \frac{1}{2} I_p \dot{\theta}^2 + m_{rot} bx_\theta \dot{h} \theta \cos \theta$$  \hspace{1cm} (112)

For small $\theta$ the kinetic energy $E_K$ can be linearized to the following expression:

$$E_K = \frac{1}{2} \dot{m} h^2 + m_{rot} bx_\theta \dot{h} \theta + \frac{1}{2} m_{rot} (\frac{1}{3} + x_\theta^2) b^2 \dot{\theta}^2 = \frac{1}{2} \dot{m} h^2 + m_{rot} bx_\theta \dot{h} \theta + \frac{1}{2} I_p \dot{\theta}^2$$  \hspace{1cm} (113)

(used is lim(cos$\theta$) = 1 when $\theta$ → 0)

The kinetic energy of the rotating (and also translating) part can also be found directly (without first calculating the moment of inertia) by calculating the following integral (see Figure 1):

$$E_K = \frac{1}{2} \int_{x=-b}^{x=b} \left( (\dot{h} + (x_\theta b - x) \dot{\theta} \cos \theta)^2 + ((x_\theta b - x) \dot{\theta} \sin \theta)^2 \right) dm^*$$  \hspace{1cm} (114)
\[
\frac{1}{2} \frac{m_{rot}}{2b} \int_{x=-b}^{x=b} \left( \ddot{h}^2 + 2(x_0b - x)\ddot{h}\dot{\theta}\cos\theta + (x_0b - x)^2 \dot{\theta}^2 (\cos^2\theta + \sin^2\theta) \right) dx
\]
\[
= \frac{1}{2} \frac{m_{rot}}{2b} \int_{x=-b}^{x=b} \left( \ddot{h}^2 + 2(x_0b - x)\ddot{h}\dot{\theta}\cos\theta + (x_0b - x)^2 \dot{\theta}^2 \right) dx
\]
Etc

Where \( x = 0 \) is again chosen to be at the middle of the wing chord (with length 2b) and \( dm^* = \frac{m_{rot}}{2b} dx \) is an infinitesimal part of mass. With the assumption that \( \theta \) is small enough to approach (linearize) \( \cos \theta \approx 1 \) this gives the same result for \( E_k \) as found in equation (113). (Of course the term \( \frac{1}{2} (m_{rot})\ddot{h}^2 \) needs to be added for the part of the set-up that only translates). However in this calculation the moment of inertia \( I_p \) does not show up directly. Because \( I_p \) is an important parameter in the following discussion and more physically intuitive than \( \frac{1}{3} (x_0^2) m_{rot} b^2 \) often \( I_p \) will be used in this report instead of the expression for \( I_p \), \( (\frac{1}{3} + x_0^2) m_{rot} b^2 \). Moreover \( I_p \) can also be determined by measuring the natural rotational frequency \( \omega_\theta \) and the rotational spring constant \( K_\theta \) \( \left( \frac{K_\theta}{\omega_\theta^2} = I_p \right) \). The results of the measurements for determining these parameters will be presented in chapter 4. The result for \( I_p \) from the measurements in chapter 4 is:

\[ I_p = \frac{K_\theta}{\omega_\theta^2} = (1.5 \pm 0.3) \times 10^{-3} [kgm^2]. \]

And the result the flat plate approach gives is:

\[ I_p = (\frac{1}{3} + x_0^2) m_{rot} b^2 = (\frac{1}{3} + x_0^2) \cdot 0.224 \cdot (0.040)^2 = 1.2 \cdot 10^{-4} [kgm^2] \]

\( (x_0 := (\varepsilon - a) = (0.025 \pm 0.025) \). However the effect is so small compared with the \( \frac{1}{3} \) that this does not affect the significant numbers of \( I_p \). This result seems ok and not to be excluded by the measured value. However, this flat plate approximation assumes all the rotating mass to be “spread” in an equal mass distribution in a flat plate with the dimensions of our wing. For the wing itself that might be plausible. But when we look at Figure 3 again for the other rotating parts with complex forms it is probably better to use the (indirectly) measured value for \( I_p \) in further calculations.

9.3 Calculation of the critical flutter velocity (section 3.3.5)

When predicting the critical flutter speed a lot of parameters need to be determined as has been done in chapter 4. Therefore it can be assumed that the parameters \( m, I_p, \varepsilon, a \) and
the spring stiﬀnesses and natural frequencies \( \omega_h, K_h, \omega_\theta, K_\theta \) are known for a specific situation. An incompressible flow was assumed already and therefore \( M_\infty \to 0 \). We assume the air density is a constant: \( \rho_\infty = 1.2 \frac{kg}{m^3} \), corresponding to typical room conditions (temperature) and therefore also \( \mu = \frac{m}{l_{wing} \pi \rho_{\infty} b^2} \) is fixed. This means that indeed all the parameters except \( \omega \) and \( k \) are known. Now we get to the MATLAB file calculating the flutter speed with as input all the mentioned parameters \( m, I_p, \)

\( e, a, \omega_h \) (or \( K_h \)), \( \omega_\theta \) (or \( K_\theta \)), \( M_\infty = 0 \) (incompressible) and \( \rho_\infty = 1.2 \frac{kg}{m^3} \).

What the program does is “trying” \( k \) values (by a convergent iterative procedure) till a solution, \( k \) critical, is found for which the earlier mentioned two conditions apply:

1-the flutter determinant has to vanish and

2- \( \text{Im} \omega(k) = 0 \) (which is an equivalent expression for \( \text{Im}(\frac{\omega_\theta}{\omega})^2 (k) = 0 \))

What the MATLAB file does (schematically) is the following procedure (which can also be done by hand when you don’t need to much significant numbers):

1: choose some values for \( k \) (typical values are between 0.001 and 1)
2: For each \( k \) calculate the functions \( m_\theta, m_h, l_\theta, l_h \) (see section 3.3.4 these functions determine the flow forces and torque)
3: The flutter determinant has to vanish for each value of \( k \), this gives (complex) expressions for \( \left( \frac{\omega_\theta}{\omega} \right)^2 \). The real part of these expressions is an approximation for \( \left( \frac{\omega_\theta}{\omega} \right)^2 \) and the imaginary part gives the damping.

4: Interpolate \( k \) to find the \( k \) for which the imaginary part of \( \left( \frac{\omega_\theta}{\omega} \right)^2 \) is zero (a non zero imaginary part means either the flutter will be damped ( \( \text{Im}(\omega) < 0 \) ) to zero or the amplitude will increase without bound ( \( \text{Im}(\omega) > 0 \) ) (like explained in section 3.3.3). The corresponding real value of \( \left( \frac{\omega_\theta}{\omega} \right)^2 \) gives the value of \( \omega_{\text{critical}} \). \( \omega_\theta \) is a known parameter and is just used in this calculation to simplify notation). Now the 2 unknown parameters \( \omega \) and \( k \) are determined.

5: \( U = \frac{b \omega}{k} \) and in this case \( U_{\text{flutter}} = \frac{b \text{Re}(\omega_{\text{flutter}})}{k_{\text{flutter}}} \) can be calculated.

The program has been tested on exercises from a book [2] and gave the correct answers (as given in the book), with just a very small deviation. This very small deviation is caused by an approximation of the Bessel functions in the Theodorsen’s function.
(equation (41)) used in the book. The MATLAB file below does not use this approximation.

The output of the MATLAB code, using the measured parameters is:

\[
\begin{align*}
k_{\text{critical}} & = 0.015 \\
\omega_{\text{critical}} & = 19.2 \ \text{rad/s} \quad f_{\text{critical}} = 3.1[\text{Hz}] \\
U_{\text{critical}} & = 49.5 \ [\text{m/s}]
\end{align*}
\]

MATLAB code:

```matlab
function [U] = flutterestimateVERSLAGfirstset-up()
%function for estimating the flutter speed without friction using linear
%theory and the classical flutter approach, for 2D flow/motion.
%The parameters are determined by measurements on the present set-up and
%verified theoretically. The flow forces and torque acting on the wing are
%based on a flat plate (infinitely thin aerofoil) approximation. Because
%these forces and torque are per unit length normally the expressions
%have been adjusted for our specific wing (NACA 008).

%INPUT PARAMETERS FOR THE first SET-UP:

m = 0.448;  \% (0.439-0.457) mass [kg] of all the translating parts of the set-up
% deviation caused by the mass of the translational springs
% (see appendix about mass)
mrot = 0.224; \% mass [kg] of the parts of the set-up that rotate (see mass appendix)
%NOTE: the predicted flutter speed is not very sensitive for changes of m and m_rot
a = -0.1;  \% a (dimensionless) indicates the location of the pivoting (turning) point P.
% distance leading edge to P is (1+a)*b = 36 mm, therefore a = -1/10
e = -0.05; \% e = (-0.075 +/- 0.025). e (dimensionless) indicates center of mass (CM) location.
% distance leading edge to CM is (1+e)*b
xt = e-a;  \% xt = xtheta = (e-a): indicates distance between P (turning point) and CM
% Note: the predicted flutter is sensitive for changes of the
% dimensionless parameters a and e
b = 0.040; \% semi chord length [m] (The wing is a NACA 008 profile)
length= 0.15; \% length of the wing

kt=0.36;  \% (0.36 +/- 0.03)[Nm/rad]: torsional spring stiffness
kh = 82; \% (82 +/- 4)[N/m]: translational spring stiffness
% Note: the spring stiffnesses kh and kt have been used to determine other parameters, however
% they are now not directly necessary as input here
rho = 1.21; \% air density [kg/m^3] (an incompressible flow is assumed)
mu = (mrot/(length^2*pi*rho*b^2)); \% mass ratio: dimensionless parameter to simplify notation

omegah = 12.47; \% (12.47 +/- 0.05)[rad/s]: measured eigen frequency of the translational
% mode (bending)
% omeagah can also be calculated using omeagah = sqrt(kh/m) (see chapter parameters
determination)
omegat=48; \% (48 +/- 5)[rad/s]: measured eigen frequency for the rotational mode
% omeagat has been used to calculate I_p, using omegat = sqrt(kt/I_p) (see chapter parameters
determination)
```

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\( I_p = 0.00015; \quad \% (1.5 +/- 0.3 \times 10^{-4}) \) [kg\( \cdot m^2 \)] measured moment of inertia about rotation axis

\( I_p = I_{cm} + m_{rot} b^2 x t^2 \) (gives relation between \( I_p \) and \( I_{cm} \))

\( r = \sqrt{I_p/(m_{rot}b^2)}; \quad \% \) mass radius of gyration about P (the turning point)

\( \% \) dimensionless parameter introduced to \( \% \) simplify notation

\( \sigma = \omega_{h}/\omega_{t}; \quad \% \) ratio of the eigen frequencies for the translational and rotational mode.

\( \% \) END OF THE INPUT OF THE PARAMETERS

\( \% \) FUNCTION for giving the COEFFICIENTS OF THE FLUTTER DETERMINANT.

\( \% \) Here theodorsens function (see theory) and the expressions for \( \% \) the coefficients \( l_h, l_t (= l_{theta}) \), \( m_h \) and \( m_t (= m_{theta}) \) will be used.

\( \% \) these expressions normally are per unit length and had to be adjusted \( \% \) for our wing with length \( 0.15m \).

\( \text{function} \quad S = \text{solution}(k) \)

\( C_k = \text{besselk}(1,i*k)./(\text{besselk}(0,i*k)+\text{besselk}(1,i*k)); \)

\( \% \) here are the expressions for the coefficients per unit length

\( \% l_h = (-2'i*C_k/k + 1); \)

\( \% l_t = (-2*C_k/k^2 -2^*(1/2-a)*i*C_k/k -i/k -a); \)

\( \% m_h = (1/2) -(1/2+a)^*l_h; \)

\( \% m_t = (-i/k +1/8 -a/2) -(1/2+a)^*l_t; \)

\( \% \) here, by introducing the parameters \( l_{h_{correct}}, l_{t_{correct}} \) etc the correction for \( \% \) the wing length \( 0.15m \) is done.

\( l_{h_{correct}} = 0.15^*(-2'i*C_k/k + 1); \)

\( l_{t_{correct}} = 0.15^*(-2'C_k/k^2 -2^*(1/2-a)*i*C_k/k -i/k -a); \)

\( m_{h_{correct}} = 0.15^*(1/2) -(1/2+a)^*l_{h_{correct}}; \)

\( m_{t_{correct}} = 0.15^*(-i/k +1/8 -a/2) -(1/2+a)^*l_{t_{correct}}; \)

\( l_h = l_{h_{correct}}; \)

\( l_t = l_{t_{correct}}; \)

\( m_h = m_{h_{correct}}; \)

\( m_t = m_{t_{correct}}; \)

\( \% \) Now the \( A, B \) and \( C \) of the \( ABC \) formula will be given:

\( \% A = (m/m{rot})^2*\mu^2*\sigma^2*r^2; \)

\( S(1) = (m/m{rot})^2*\mu^2*\sigma^2*r^2; \)

\( S(2) = -(m/m{rot})^2*\mu^2*\sigma^2*r^2+mt -mu^2*r^2*((m/m{rot})^2*mu+lh); \)

\( S(3) = ((m/m{rot})^2*mu+lh).*((mu*r^2+mt) -mu^2*xt+lt).*((mu*r^2+mt)}; \)

\( \% \) function for \((-B+sqrt(B^2-4AC))/(2A)\)

\( \text{function} \quad F1 = \text{flutdet1}(k) \)

\( S = \text{solution}(k); \)

\( F1 = \text{imag}(-S(2)+sqrt(S(2)^2-4*S(1)*S(3)))/(2*S(1)) ; \)

\( \% \) end
% function for (-B-sqrt(B^2-4AC))/(2A)
function F2 = flutdet2(k)
    S  = solution(k);
    F2 = imag( (-S(2)-sqrt(S(2)^2-4*S(1)*S(3)) )./(2*S(1)) );
end

function F3 = flutdet3(k,solu_nr)
    S  = solution(k);
    if   solu_nr==1
        F3 = real( (-S(2)+sqrt(S(2)^2-4*S(1)*S(3)) )./(2*S(1)) );
    elseif solu_nr==2
        F3 = real( (-S(2)-sqrt(S(2)^2-4*S(1)*S(3)) )./(2*S(1)) );
    else
        disp(['error, wrong input for solu_nr: ' num2str(solu_nr)]);
        return;
    end
end

% function for checking the exit flag of zero finding function, this might
% tell you something about the correctness of the value.
function c = checkexitstatus( exitflag )
    disp('----- exitflagstatus -----');
    switch exitflag
        case {1}, disp('  1: Function converged to a solution, OK');
        case {-1}, disp(' -1: Algorithm was terminated by the output function');
        case {-3}, disp(' -3: NaN or Inf function value was encountered during search for an
           interval containing a sign change');
        case {-4}, disp(' -4: Complex function value was encountered during search for an interval
           containing a sign change');
        case {-5}, disp(' -5: fzero might have converged to a singular point');
    end
    disp('-------------------------');
    c = exitflag;
end

options = optimset('Display','iter','FunValCheck','on');
options = optimset('Display','none');
[ k0  fval0 exflag0 ] = fzero( @flutdet1, 0.5, options );
[ k02 fval2 exflag2 ] = fzero( @flutdet2, 0.5, options );

disp(['status of solution 1:']);
if ( (checkexitstatus(exflag0)~=1) || (k0<0.0) || (abs(fval0)>1e-9) )
    disp(['NO value found in flutdet1!']);
    k0 = -1.0;
end

disp(['status of solution 2:']);
if ( (checkexitstatus(exflag2)~=1) || (k02<0.0) || (abs(fval2)>1e-9) )
    disp(['NO value found in flutdet2!']);
    k02 = -1.0;
end
% check if we have a solution
if ( k0==1.0 && k02==-1.0 )
    disp(['NO solution found in flutter determinant!']);
    return;
end

% we have a solution, now pick the smallest positive solution
if ( k02>k0 && k0>0 )
    kcrit = k0;
    solu = 1;
else
    kcrit = k02;
    solu = 2;
end
omega = sqrt( omegat^2./flutdet3(kcrit,solu));
U = b*omega./kcrit;

disp(['----------------------------------']);
disp(['k_critical = ' num2str(kcrit)]);
disp(['omega_crit = ' num2str(omega) ' rad/s = ' num2str(omega/(2*pi)) ' Hz']);
disp(['U_critical = ' num2str(U) ' m/s']);
disp(['omega_t = ' num2str(omegat) ' rad/s = ' num2str(omegat/(2*pi)) ' Hz']);
disp(['omega_h = ' num2str(omegah) ' rad/s = ' num2str(omegah/(2*pi)) ' Hz']);

9.4 Calculation of the critical flutter velocity with friction
(section 3.3.6)

All parameters except \( \omega \) and \( k \) are known, just like the situation without flutter. Different is that for the case with friction there are two parameters more, \( R_h \) and \( R_\theta \). The determination of these parameters is given in chapter 4.

These two parameters represent the damping caused by friction. The force caused by this friction is assumed to be proportional with the velocity. (See equation ( 65 ) ). The Matlab procedure solving this is a little bit different from that for the situation without friction, because the ABC formula cannot be applied. The equation from the flutter determinant has a different form. (there is an \( i \omega R_h \) and an \( i \omega R_\theta \) term in the equation of motion as can be seen in equations ( 73 ) and ( 74 )). But the idea of finding a \( k \) for which:

1- the flutter determinant vanishes and
2- \( \text{Im} \omega(k) = 0 \) (which is an equivalent expression for \( \text{Im} \left( \frac{\omega_\theta}{\omega} \right)^2 (k) = 0 \))

Is the same as in the situation without flutter.
The situation without friction is used to start the iterative process and the \( \omega_{\text{critical}} \) and \( k_{\text{critical}} \) without flutter will be used as input parameters. This is of course also the first check to find out whether this code was working (or not). The results without friction from this code are:

\[
\begin{align*}
 k_{\text{critical}} &= 0.015 \\
 \omega_{\text{critical}} &= 19.2 \text{ rad/s} \quad f_{\text{critical}} = 3.1[Hz] \\
 U_{\text{critical}} &= 49.5 [\text{m/s}] 
\end{align*}
\]

These results are the same as the result from the code without friction. When the flutter speed with friction is predicted using the measured values for \( R_h \) and \( R_\theta \) the results are:

\[
\begin{align*}
 k_{\text{critical}} &= 0.013 \\
 \omega_{\text{critical}} &= 16.2 [\text{rad/s}] \\
 U_{\text{critical}} &= 49.9 [\text{m/s}] 
\end{align*}
\]

The friction coefficients hardly affect the flutter speed (less then 1 %)! Also when both \( R_h \) and \( R_\theta \) are made 10 times bigger, the effect on the flutter speed is almost zero. The results are:

\[
\begin{align*}
 k_{\text{critical}} &= 0.0107 \\
 \omega_{\text{critical}} &= 13.3 [\text{rad/s}] \\
 U_{\text{critical}} &= 49.5 [\text{m/s}] 
\end{align*}
\]

Note: However, \( k \) and the flutter frequency \( \omega_{\text{critical}} \) have changed significantly due to the friction! Because their ratio hardly changed the flutter speed \( U \) is almost unaffected.

Recall: \( U_{\text{flutter}} = \frac{b \text{Re} (\omega_{\text{flutter}})}{k_{\text{flutter}}} \) (equation (64)).

MATLAB CODE:

```matlab
function [] = flutterestimateVERSLAGfrictionfirstset-up ()
%gives an estimate of the flutter speed with friction
%Note: the measured friction coefficients are much to small to have any
%influence on the predicted flutter speed

%START VALUES FOR THE ITERATION:
%the k critical and omega critical of the flutter speed prediction
%without friction have to be given here and serve as start values
%for the iteration to predict the flutter speed with friction:

kCritical=0.015; %k_critical = 0.015126 (from flutter speed without friction)
omegaCritical=19; %omega_crt = 19.6227 rad/s = 3.123 Hz (without friction)

%Note:the input for omega here must be in RADIANS/s, not in Hz!

%FRICTION COEFFICIENTS (see report, chapter parameter determination):
```

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%The following 2 parameters introduce the mechanical friction as explained in the theory:
% Note: these parameters are measured at zero flow speed to avoid confusing damping
% by mechanical friction with damping by the flow (see theory)
Rh = 0.010;  % (0.010 +/- 0.002) [kg/s=Ns/m] translation damping coefficient
Rtheta = 0.0011;  % (0.0011 +/- 0.0007) [Nms/rad=kgm^2/s] This is the rotational damping coefficient
%Note: making both these friction coefficients 10 times higher hardly
% affects the predicted flutter speed (less than 1% !!!!)
% al die moeite voor fitten e machten wordt niet echt beloond dus....
% Rh will be called gammah and Rtheta will be called gammat in this code

% END OF INPUT WHICH IS DIFFERENT FROM THE SITUATION WITHOUT FRICTION.
% THIS CODE IS DIFFERENT FROM THE CODE WITHOUT FRICTION BECAUSE THE
% ABC FORMULA CANNOT BE USED. HOWEVER, THIS CODE HAS THE OTHER CODE AS THE
% LIMIT WHEN THE FRICTION COEFFICIENTS GO TO ZERO. THIS CODE USES THE
% RESULTS OF
% THE OTHER CODE AS AN INPUT PARAMETER TO START THE ITERATION.

function [ A ] = f( var )

k = var(1);
omega = var(2);
% Note the k critical and the omega critical from the flutter speed without flutter
% serve as the start values for the iteration process for predicting the
% flutter speed with friction. These values are needed as input and are
% already given, together with the friction/damping coefficients.

m = 0.448;  % (0.439-0.457) mass [kg] of all the translating parts of the set-up
% deviation caused by the mass of the translational springs
% (see appendix about mass)
mrot = 0.224;  % mass [kg] of the parts of the set-up that rotate (see mass appendix)
% NOTE: the predicted flutter speed is not very sensitive for changes of m and m rot
a = -0.1;  % a (dimensionless) indicates the location of the pivoting (turning) point P.
% distance leading edge to P is (1+a)*b = 36 mm, therefore a = -1/10
e = -0.05;  % e = (-0.075 +/- 0.025) e (dimensionless) indicates center of mass (CM) location.
% distance leading edge to CM is (1+e)*b.
x_t = e-a;  % x_t = xtheta = (e-a) indicates distance between P (turning point) and CM
% Note: the predicted flutter is sensitive for changes of the
% dimensionless parameters a and e
b = 0.040;  % semi chord length [m] (The wing is a NACA 008 profile)
length = 0.15;  % length of the wing

kt=0.36;  % (0.36 +/- 0.03) [Nm/rad] torsional spring stiffness
kh = 82;  % (82 +/- 4) [N/m] bending/translational spring stiffness
% Note: the spring stiffnesses kh and kt have been used to determine other parameters, however
% they are now not directly necessary as input here
rho = 1.21;  % air density [kg/m^3] (an incompressible flow is assumed)
\[ \mu = \frac{m_{\text{rot}}}{(\text{length} \cdot (\pi \cdot \rho \cdot b^2))} \] 

Mass ratio: dimensionless parameter to simplify notation.

\[ \omega_{\text{ah}} = 12.47; \quad (12.47 \pm 0.05) \text{[rad/s]} \] 

Measured eigen frequency of the translational mode (bending).

\[ \omega_{\text{ah}} \] can also be calculated using \[ \omega_{\text{ah}} = \sqrt{\frac{k_h}{m}} \] (see chapter parameters determination).

\[ \omega_{\text{at}} = 48; \quad (48 \pm 5) \text{[rad/s]} \] 

Measured eigen frequency for the rotational mode.

\[ \omega_{\text{at}} \] has been used to calculate \( I_p \), using \[ \omega_{\text{at}} = \sqrt{\frac{k_t}{I_p}} \] (see chapter parameters determination).

\[ I_p = 0.00015; \quad (1.5 \pm 0.3 \times 10^{-4}) \text{[kg \cdot m^2]} \] 

Measured moment of inertia about rotation axis.

\[ r = \sqrt{\frac{I_p}{m_{\text{rot}} \cdot b^2}} \] 

Mass radius of gyration about \( P \) (the turning point).

Dimensionless parameter introduced to simplify notation.

\[ \sigma = \frac{\omega_{\text{ah}}}{\omega_{\text{at}}} \] 

Ratio of the eigen frequencies for the translational and rotational mode.

% END OF THE INPUT OF THE PARAMETERS

Here follow the coefficients \( l_h, l_t (= \theta) \) etc, which determine the lift (\( L \)) and and torque (pitching moment (\( M \)) on the wing. As explained in the theory these coefficients depend on Theodorsens function \( C(k) \). Again these coefficients have to be adapted for the length (0.15m) of the wing.

First Theodorsens function is given:

\[ C_k = \frac{\text{besselk}(1, i \cdot k)}{\text{besselk}(0, i \cdot k) + \text{besselk}(1, i \cdot k)}; \]

% here are the expressions for the coefficients per unit length:

\[ l_h = \left( -2 \cdot i \cdot C_k \cdot k + 1 \right); \]

\[ l_t = \left( -2 \cdot C_k \cdot k^2 - 2 \cdot (1/2 - a) \cdot i \cdot C_k \cdot k - i/k - a \right); \]

\[ m_h = \left( 1/2 - (1/2 + a) \right) \cdot l_h; \]

\[ m_t = \left( -i/k + 1/8 - a/2 - (1/2 + a) \right) \cdot l_t; \]

Here, by introducing the parameters \( l_{h\text{correct}}, l_{t\text{correct}} \) etc the correction for the wing length (0.15m) is done.

\[ l_{h\text{correct}} = 0.15 \cdot (-2 \cdot i \cdot C_k / k + 1); \]

\[ l_{t\text{correct}} = 0.15 \cdot (-2 \cdot C_k / k^2 - 2 \cdot (1/2 - a) \cdot i \cdot C_k / k - i/k - a); \]

\[ m_{h\text{correct}} = 0.15 \cdot (1/2 - (1/2 + a)) \cdot l_{h\text{correct}}; \]

\[ m_{t\text{correct}} = 0.15 \cdot (-i/k + 1/8 - a/2 - (1/2 + a)) \cdot l_{t\text{correct}}; \]

% here follows the equation from the flutter determinant with friction:

\[ \text{outp} = \left( \mu \cdot m / m_{\text{rot}} \right) \left( 1 - \sigma^2 \right) \left( \omega_{\text{at}}^2 - i \cdot \gamma_{\text{at}} / \omega_{\text{at}}^2 \right) \left( \omega_{\text{at}} \right)^2 \left( 1 - \omega_{\text{at}} \left( \omega_{\text{at}}^2 - i \cdot \gamma_{\text{at}} / \omega_{\text{at}}^2 \right) \right) \]

\[ A = \left[ \text{real(outp)} \quad \text{imag(outp)} \right]; \]
function c = checkexitstatus( exitflag )
    disp('----- exitflagstatus -----');
    switch exitflag
        case { 1}, disp(['  1: Function converged to a solution x.']);
        case { 2}, disp(['  2: Change in x was smaller than the specified clearance.']);
        case { 3}, disp(['  3: Change in the residual was smaller than the specified clearance.']);
        case { 4}, disp(['  4: Magnitude of search direction was smaller than the specified clearance.']);
        case { 0}, disp(['  0: Number of iterations exceeded options.MaxIter or number of function evaluations exceeded options.FunEvals.']);
        case {-1}, disp([' -1: Algorithm was terminated by the output function.']);
        case {-2}, disp([' -2: Algorithm appears to be converging to a point that is not a root.']);
        case {-3}, disp([' -3: Trust radius became too small.']);
        case {-4}, disp([' -4: Line search cannot sufficiently decrease the residual along the current search direction.']);
    end
    disp('-------------------------');
    c = exitflag;
end

N   = 25;  %number of iterative steps to find solution
%as a starting point for the iterative process the solution without friction will be used
%in the iteration the friction coefficients will gradually (in 25 steps)
%increase from zero friction till the input friction coefficient is reached.
%in every step of the iteration it will be checked if there is still a
%convergent solution. If not the process will be terminated.

start = [kcritical omegacritical];  %The iteration process starts with the values for
%kcritical and omegacritical of the
%flutter speed prediction without friction
%these values are given in the
%beginning of the code

for j=1:N,
    gammah = (gammaht*(j-1))/N;
    gammat = (gammatt*(j-1))/N;
    options=optimset('Display','iter');
    [sol,fval,exflag] = fsolve(@f,start,options);
    % if exflag~=1
    %     checkexitstatus( exflag );
    %     return
    % end
    start = sol;
end

disp(['function value = ' num2str(fval)]);
disp(['solution     = ' num2str(sol)]);
disp('--------------------------------------------');
disp(['k = ' num2str(sol(1))]);
disp(['omega = ' num2str(sol(2)) ' [rad/s]']);
disp(['U = ' num2str(sol(2)*b/sol(1)) ' [m/s]']);
end

9.5 Mass of the individual parts of the set-up (chapter 4 and 6)

- The total mass of all the translating parts of the first set-up is 0.405 kg without the four leaf springs. With springs $m = (0.448 \pm 0.09)kg$
- The total mass of all the rotating parts of the first set-up is $m_{rot} = (0.224 \pm 0.01)kg$
- The total mass of all the translating parts of the improved set-up is 0.548 kg without the four leaf springs. With springs $m = (0.591 \pm 0.09)kg$
- The total mass of all the rotating parts of the improved set-up is $m_{rot} = (0.313 \pm 0.01)kg$
- The mass of the four leaf springs for the translational motion (together) is 0.103 kg. If the form of the leaf springs is exactly known the effective translating mass they contribute to $m$ can be calculated. This is complicated and does not have a very big influence on the end result for $m$. Therefore this effective mass is estimated with a calculated guess to be between $\frac{1}{3}$ and $\frac{1}{2}$ of the total mass of the springs which results in an effective contribution to $m$ of $(0.043 \pm 0.09)kg$.

All the individual parts of the first set-up in Figure 3 are numbered. The table below gives the masses of these parts:

<table>
<thead>
<tr>
<th>Part nr</th>
<th>Name of the part</th>
<th>Mass ($10^{-3} kg$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>wing</td>
<td>76.5*</td>
</tr>
<tr>
<td>2</td>
<td>2 plates to avoid boundary effects+ 2 blocks (not on picture, are inside the wing) to attach the axis to the wing</td>
<td>35.7*</td>
</tr>
<tr>
<td>3</td>
<td>2 wing holders (including 2 rings to fix the axis)</td>
<td>102.5+11</td>
</tr>
<tr>
<td>5</td>
<td>2 rotational springs</td>
<td>1,0</td>
</tr>
<tr>
<td>6</td>
<td>4 screws to attach the leaf springs to the wing holder</td>
<td>67</td>
</tr>
<tr>
<td>7</td>
<td>2 blocks to attach the rotational springs to the axis</td>
<td>46*</td>
</tr>
<tr>
<td>8</td>
<td>rotation axis</td>
<td>65.3*</td>
</tr>
<tr>
<td>4</td>
<td>four leaf springs</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>Effective contribution of the 4 leaf springs is between $\frac{1}{2}$ and $\frac{1}{3}$ of their mass</td>
<td>34-52</td>
</tr>
</tbody>
</table>

Notes:
- All the numbered parts translate.
- The parts marked with a * also rotate.

General remarks:
- The clearance the axis has in its motion is a source of (non linear) problems. It would be a good idea to reduce this clearance and avoid the possible (non linear) consequences.
-the contribution to $m_{rot}$ of the mass of the rotational springs (1.0 g) of the first and improved set-up can be neglected.

-Further it is arguable that the rings in the wing holder to fix the rotation axis also rotate or at least move a little bit. That would add 11 g (0.011 kg) to $m_{rot}$. This has no significant influence on the predicted flutter speed (less than 1%).

9.6 **Measuring $R\theta$ for the first set-up (section 4.3.2)**

As already explained and showed in 4.3.2 and 4.2.2, the rotational motion damps out in just a few oscillations for the first set-up. Therefore it is difficult to determine the damping coefficient $R\theta$. (Recall the damping term is $\exp(-\frac{R\theta}{2I_{\theta}}t)$ and $\left(\frac{1}{\tau_{\theta}} = \frac{R\theta}{2I_{\theta}}\right)$).

These bad results of non exponential damping and non linear behaviour provided an important argument for improving the rotational spring system, which resulted in the improved set-up. The rotational spring system of the improved set-up is indeed much better (see section 6.2.2). However, in this appendix we try to find the best possible value for $R\theta$ of the first set-up.

In the following figures some examples of fits of the rotational motion are given with different values of $\tau_{\theta}$. The frequency $\omega_{\theta}$ from these and a lot of other fits of measurements is $\omega_{\theta} = 53 \pm 3 [rad/s]$, which correspond with the $\omega_{\theta}$ found by measuring the period time T in section 4.2.1.

![Figure 59: Fit of the rotational motion. $(1/\tau_{\theta})=2.1$ Hz which seems to low.](image1)

![Figure 60: Fit of the rotational motion. $(1/\tau_{\theta})$ is about 3 Hz.](image2)
$1/\tau_\theta = 3\text{Hz}$ still seems to low, however this is difficult to judge because the damping is not really an $e$ power which suggests non linearities for the rotational springs and the motion of the rotational axis.

![Figure 61: Fit of the rotational motion. ($1/\tau_\theta$) = 3.9 Hz which seems not unrealistic.](image1)

It is difficult to judge which fit, with $1/\tau_\theta = 3$ Hz or 3.9 Hz or (the next) 4.3 Hz is the best. Also the numerical values that Matlab gives for the quality of the plot are about the same for all of them.

![Figure 62: Fit of the rotational motion. ($1/\tau_\theta$) = 4.3 Hz which is not unrealistic.](image2)

![Figure 63: Fit of the rotational motion. ($1/\tau_\theta$) = 9.0 Hz which is clearly too high.](image3)

From all these fits it can be concluded that it is difficult to determine the value of $1/\tau_\theta$ because the behavior is non linear and the decay is not really exponential. An approximation is $(I_p = (1.5 \pm 0.3)10^{-4}[kgm^2]) 1/\tau_\theta = 3.5 \pm 1.5[1/s]$, which results in: $R_\theta = (1.1 \pm 0.7)10^{-3} \text{[Nms/rad]} = [kgm^2/s]$
9.7 **Frequency response measurements first set-up (section 5.4)**

In this section some measurements of the frequency response of the first set-up will be presented at different flow speeds. The results of all the measurements together have been presented in section 5.4 in Figure 30. It is interesting to compare these measurements with the corresponding time domain measurements presented in section 5.3.

Figure 64 shows the frequency response of the system at 18.7 m/s, just below the flutter speed. The translational oscillation is independent of the flow speed and has a frequency of 1.97 Hz. For a flow speed below 18.7 m/s this is the only measured frequency and there is no rotation at all. At 18.7 m/s a small rotational motion is starting at a frequency between 6 Hz and 7 Hz.

![18.7 m/s frequency response](image)

**Figure 64 : Frequency response for a flow speed of 18.7 m/s.**

In Figure 65 and Figure 66 the flow speed is 19.4 m/s and 19.8 m/s respectively. The frequency response pattern is almost similar to that of 18.7 m/s flow, however for a flow speed of 19.8 m/s a broader range of frequencies from about 6-8 Hz seems to be triggered. This indicates the start of the (mainly rotational) flutter motion. However the translational peak at 2.0 Hz is still dominant.

![19.4 m/s flow frequency response](image)

**Figure 65 : Frequency response for a flow speed of 19.4 m/s.**
A further increase of the flow speed crosses the critical flutter speed as can be seen in Figure 67. For a flow speed of 20.8 m/s the set-up is most of the time fluttering. However sometimes flutter stops and restarts later. When the set-up temporarily stops fluttering the translation gets a much bigger and more regular amplitude than during flutter. When the set-up is fluttering the motion is mainly rotational and the translation is very small and a little bit irregular.

For 21.3 m/s flow speed (Figure 68) the set-up is almost constantly fluttering and does hardly fall back into the translational mode. The flutter peak is now much stronger than the translational peak and has moved to a lower frequency of 7.7 Hz. For some reason an additional (rotational) peak shows up at a frequency of 3.95 Hz, which is almost half the flutter frequency.
For 22 m/s flow speed the set-up is constantly fluttering and does not at all fall back into the translational mode. The flutter peak (7.1 Hz) has again increased in strength compared with the other two peaks (1.9 Hz for the translation) and the “new” peak at 3.57 Hz. This is especially clear in the first graph which shows the frequency response on a linear scale. The second graph shows the frequency response at a logarithmic scale (like all earlier showed results). The third graph shows the frequency response over the whole frequency range for both the laser sensors on a logarithmic scale.

![Graph showing frequency response](image)

**Figure 69:** Frequency response for a flow speed of 22 m/s. The flutter frequency has further decreased to 7.1 Hz. The flutter motion has however increased in strength, compared with the translation at 1.9 Hz. The additional has moved to 3.57 Hz.

Finally, the MATLAB code used to make these frequency responses will be given (The file has been tested on some simple periodic functions before using it on the measurements and seems to work perfectly):

```matlab
function fft_simple500HZ(fname)
fid = fopen(fname);
[a] = textscan(fid,'%n%n', 'CommentStyle','%');
% just read relevant information, skip comments
y1 = a{1}*0.0025+0.175;
% watch the difference between {} and [] and ()
y2 = a{2}*0.0025+0.175;
% here the voltage of the laser sensors is transfered to the height

tmax = 1/100 * length(y1); % adjust for sample frequency
t = linspace(0,tmax,length(y1));
% choose t (time) min and t max

fsam = 500; % used sample frequency

N = length(y1);
y1 = y1(1:N);
```

98
y2 = y2(1:N);
C1 = fft(y1,N);
C2 = fft(y2,N);
P1(1)  = abs(C1(1))^2;
P1(N/2+1) = abs(C1(N/2+1))^2;
P2(1)  = abs(C2(1))^2;
P2(N/2+1) = abs(C2(N/2+1))^2;
for k = 2:N/2
    P1(k) = 2 * abs(C1(k))^2;
    P2(k) = 2 * abs(C2(k))^2;
end
f1= fsam/2 * linspace(0,1,length(P1));
f2= fsam/2 * linspace(0,1,length(P2));
P1 = N/fsam * P1;
P2 = N/fsam * P2;

%some print options for the graphs : linear scale (plot) or semilogy
%plot(f,P);
%semilogx(f,P);
%this the print option we want:semilogy(f,P1);x linear,ylog scale
%loglog(f,P);
subplot(3,1,1); plot(f1,P1);
ylim([1e-3,1e4]);
subplot(3,1,2); semilogy(f2,P2);
ylim([1e-3,1e5]);
subplot(3,1,3); semilogy(f1,P1,f2,P2);
ylim([1e-3,1e4]);

%end

9.8  Measuring $R_h$ for the improved set-up (section 6.2.1)

Despite the almost perfect fits for the exponentially damped motion for every single measurement the problem is that the measured friction coefficients depend on the weight of the ball, used to move the wing and also differ a little from measurement to measurement. The initial amplitude of the oscillation is, also for the heavy ball entirely in the linear range of the leaf springs. Even bigger amplitudes should be possible. Therefore the different results are probably not caused by a non linearity in the translational leaf springs. A more appropriate explanation for this non-linear behavior is the existence of a third degree of freedom caused by an out of phase motion of the left and right side of the set-up (section 2.4). This unwanted mode of translational motion also causes extra friction and stress on the axis of rotation. Further the clearance of the rotation axis introduces non linear behavior which can also influence the damping of the translation.
The friction coefficient $R_h = 0.012 \pm 0.001 [kg/s] (= [Ns/m])$ dropping the light ball (8.0g) on the trailing edge of the wing. The friction coefficient $R_h$ varies from $(0.01 - 0.03)[kg/s] (= [Ns/m])$ using the heavy ball (31.5g). During the first 20 seconds with a heavy ball measurement $R_h = (0.2 - 0.3) [kg/s]$ and it decreases (for some measurements) with time to (approximately) $R_h = 0.01 [kg/s]$ (the result of the light ball).

Regarding these results, it is difficult to say what the friction coefficient will be during an actual measurement for which the conditions are different each time. Maybe it is tempting to assume that $R_h = 0.012 [kg/s] (= [Ns/m])$ is the right answer because this is what the light ball shows, which affects probably less non linear behavior than the heavy ball. However, the fits in general are so good (also for the heavy ball) that these results cannot be neglected. The question remains what the friction is during actual measurements in which both rotational and translational motion are involved.

An important note is that in the end, these friction coefficients, especially $R_h$ for the translation have no significant effect on the estimated flutter speed as shown earlier.

### 9.9 Measuring $\omega_\theta$ and $R_\theta$ for the improved set-up (section 6.2.2)

Notes on measuring $\omega_\theta$:

The result $\omega_\theta = (39 \pm 1)[rad/s]$ for the improved set-up has been measured with the translation blocked in both directions, up and down. However when the translation is just blocked in one direction (down, as shown in section 4.1.2 in Figure 17), which possibly introduces non linear behavior, a wider range of frequencies $\omega_\theta = (35 \pm 4)[rad/s]$ has been measured. There is no visible translational motion seen when the translation is just blocked in one direction. The only (visible) difference is that the rotational motion vanishes quicker to zero in about 2 oscillations in stead of 6-8 oscillations for the translation blocked up and down. The question is what happens during actual measurements in which both rotational and translational motion are involved. The measurements in chapter 7 show that the rotational frequency (with flow) varies from 5.3-5.9 Hz corresponding to $\omega$ varying from 33 up to 37 $[rad/s]$ which is closer to the “non linear frequencies” of $\omega_\theta$ than to the linear frequencies.

Notes on measuring $R_\theta$:

Fortunately the new rotational springs of the improved set-up work much better than those of the first set-up. However, for these measurements the same problem occurs as for the determination of $R_h$ as described in the former appendix, the damping is dependent on the weight of ball used to initiate the motion. However the differences are much smaller than for $R_h$ and in this case the light ball gives a stronger damping than the
heavy one as shown in section 6.2.2 in Figure 35 and Figure 36. There is in this case no argument to prefer the measurements with the light ball to that of the heavy one. Therefore the given value $R_\theta = (4.4 \pm 1.4) \times 10^{-4}$ covers all the measurements. However, all these measurements are with the translation blocked in two directions. When the translation is blocked in just one direction (down) the motion damps out within 2 oscillations and the decay is much stronger and (like the first set-up) essentially non linear as shown in Figure 70 and Figure 71.

![Diagram of damped rotation (zero flow)](image)

**Figure 70**: The rotational motion with translation blocked in just one direction (down), after impact of the heavy ball. The behavior is non linear, which is probably also the case for other measurements with rotational and translational motion.

![Diagram of damped rotation (zero flow)](image)

**Figure 71**: The rotational motion with translation blocked in just one direction (down), after impact of the light ball. The behavior is even more non linear than for the heavy ball in Figure 70. This behavior is probably also realistic for other measurements with rotation and translation.

The conclusion is that estimating a realistic value for $R_\theta$ during actual measurements involving translation and rotation is difficult, because of non linear behavior of the set-up.
### 9.10 Critical flutter velocity for the improved set-up (section 7.1)

In this appendix the critical flutter speed $U_{\text{critical}}$, together with $k_{\text{critical}}$ and $\omega_{\text{critical}}$, will be predicted (using the Matlab code shown in appendix 9.4), varying the parameters determined in chapter 6, within their uncertainties. The predicted results are shown in the table below, the conclusions of these results are already discussed in the beginning of chapter 7. When the value of a parameter is indicated in the table below, this means that the parameter is varied (within the uncertainty) and the value of the parameter is different from the “mean” value. When nothing is indicated, the parameters have the “mean” values shown in the table in the beginning of chapter 6.

<table>
<thead>
<tr>
<th>without friction</th>
<th>$R_h = 0.03$ (small effect), $R_\rho = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{\text{critical}} = 0.018542$</td>
<td>$R_h = 0.03$, $R_\rho = 0.00044$ (measured value)</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.018612$</td>
<td>$R_h = 0.012$, $R_\rho = 0.00044$ (these are the mean measured values)</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.017948$</td>
<td>$R_h = 0.03$, $R_\rho = 0.00044$</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.017958$</td>
<td>$R_h = 0.00058$, $R_\rho = 0.012$</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.018014$</td>
<td>$R_h = 0.012$, $R_\rho = 0.0003$</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.016978$</td>
<td>$I_p = 0.00097$</td>
</tr>
<tr>
<td>$k_{\text{critical}} = 0.019225$</td>
<td>$I_p = 0.00075$ (big effect)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\omega_{\text{critical}} = 13.4261 \text{ rad/s}$</th>
<th>$\omega_{\text{critical}} = 13.5094 \text{ [rad/s]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{\text{critical}} = 28.9635 \text{ m/s}$</td>
<td>$U_{\text{critical}} = 29.0335 \text{ [m/s]}$</td>
</tr>
<tr>
<td>$\omega_{\text{critical}} = 12.4294 \text{ [rad/s]}$</td>
<td>$\omega_{\text{critical}} = 12.4595 \text{ [rad/s]}$</td>
</tr>
<tr>
<td>$U_{\text{critical}} = 27.6838 \text{ [m/s]}$</td>
<td>$U_{\text{critical}} = 27.7593 \text{ [m/s]}$</td>
</tr>
<tr>
<td>$\omega_{\text{critical}} = 12.5028 \text{ [rad/s]}$</td>
<td>$\omega_{\text{critical}} = 12.5028 \text{ [rad/s]}$</td>
</tr>
<tr>
<td>$U_{\text{critical}} = 27.8639 \text{ [m/s]}$</td>
<td>$U_{\text{critical}} = 27.8639 \text{ [m/s]}$</td>
</tr>
<tr>
<td>$\omega_{\text{critical}} = 12.6799 \text{ [rad/s]}$</td>
<td>$\omega_{\text{critical}} = 12.6799 \text{ [rad/s]}$</td>
</tr>
<tr>
<td>$U_{\text{critical}} = 28.1549 \text{ [m/s]}$</td>
<td>$U_{\text{critical}} = 28.1549 \text{ [m/s]}$</td>
</tr>
<tr>
<td>$\omega_{\text{critical}} = 12.6806 \text{ [rad/s]}$</td>
<td>$\omega_{\text{critical}} = 12.6806 \text{ [rad/s]}$</td>
</tr>
<tr>
<td>$U_{\text{critical}} = 29.8761 \text{ [m/s]}$</td>
<td>$U_{\text{critical}} = 29.8761 \text{ [m/s]}$</td>
</tr>
</tbody>
</table>

$R_h = 0$.

$p_I = 0.000097$.

$p_I = 0.000075$ (big effect).
<table>
<thead>
<tr>
<th>$k_{\text{critical}}$</th>
<th>$\omega_{\text{critical}}$</th>
<th>$U_{\text{critical}}$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.018815</td>
<td>12.3687 [rad/s]</td>
<td>26.2948 [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>0.017099</td>
<td>12.5567 [rad/s]</td>
<td>29.374 [m/s]</td>
<td>-0.05 (big effect)</td>
</tr>
<tr>
<td>0.019315</td>
<td>12.0809 [rad/s]</td>
<td>25.0185 [m/s]</td>
<td></td>
</tr>
<tr>
<td>0.020492</td>
<td>12.0011 [rad/s]</td>
<td>23.4265 [m/s]</td>
<td></td>
</tr>
<tr>
<td>0.021233</td>
<td>11.9249 [rad/s]</td>
<td>22.4647 [m/s]</td>
<td></td>
</tr>
<tr>
<td>0.016098</td>
<td>13.1479 [rad/s]</td>
<td>32.6695 [m/s]</td>
<td></td>
</tr>
<tr>
<td>0.016449</td>
<td>13.025 [rad/s]</td>
<td>31.6728 [m/s]</td>
<td></td>
</tr>
</tbody>
</table>

$R_{\theta} = 0.000058, \ I_{p} = 0.000075$

$R_{\theta} = 0.0058, \ I_{p} = 0.000075, e=0$

$R_{\theta} = 0.0003, \ I_{p} = 0.000097, e = -0.05$

$R_{\theta} = 0.0003, \ I_{p} = 0.000097, e = -0.05$
### 9.11 Changing the position of CM relative to the axis of rotation (section 7.4)

Just 3 positions have been measured because this already showed the desired effect, when compared with the situation without added weight. For these measurements the set-up has to be dismantled and put together each time to be able to measure the position of the center of mass (apart from all the other parameters that have to be determined) and therefore the number of measurements is also reduced to a minimum. The weights added both have a mass of 19.0 g and are symmetrically attached to the wing. In the following table the measured and predicted results (classical flutter theory) will be compared:

<table>
<thead>
<tr>
<th>Position of the weights relative to the axis of rotation:</th>
<th>Measured parameter values:</th>
<th>Predicted parameter values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>52.5mm downstream</td>
<td>$k_{\text{critical}}$ = 0.014</td>
<td>$k_{\text{critical}}$ = 0.014</td>
</tr>
<tr>
<td></td>
<td>$f_{\text{critical}}$ = 4.0 [Hz]</td>
<td>$f_{\text{critical}}$ = 2.2 [Hz]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 14.4 [m/s] (spontaneous)</td>
<td>$U_{\text{critical}}$ = 39.1 [m/s]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 11.1 [m/s] (initiated)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e$ = 0.063±0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_p$ = $(1.9±0.2) \times 10^{-4} km^2$</td>
<td></td>
</tr>
<tr>
<td>30 mm downstream</td>
<td>$k_{\text{critical}}$ = 0.015</td>
<td>$k_{\text{critical}}$ = 0.015</td>
</tr>
<tr>
<td></td>
<td>$f_{\text{critical}}$ = 3.9 [Hz]</td>
<td>$f_{\text{critical}}$ = 2.1 [Hz]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 18.3 [m/s] (spontaneous)</td>
<td>$U_{\text{critical}}$ = 35.1 [m/s]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 14.5 [m/s] (initiated)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e$ = 0.038±0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_p$ = $(1.5±0.3) \times 10^{-4} km^2$</td>
<td></td>
</tr>
<tr>
<td>30 mm upstream (results not as convincing and accurate as for downstream added weight)</td>
<td>$k_{\text{critical}}$ = 0.011</td>
<td>$k_{\text{critical}}$ = 0.011</td>
</tr>
<tr>
<td></td>
<td>$f_{\text{critical}}$ = 3.5 [Hz]</td>
<td>$f_{\text{critical}}$ = 2.5 [Hz]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 22 [m/s] (spontaneous)</td>
<td>$U_{\text{critical}}$ = 57.2 [m/s]</td>
</tr>
<tr>
<td></td>
<td>$U_{\text{critical}}$ = 20-21 [m/s] (initiated)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e$ = -0.23±0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_p$ = $(1.8±0.5) \times 10^{-4} km^2$</td>
<td></td>
</tr>
</tbody>
</table>

The agreement between theory and experiment is poor, both for the critical flutter velocity and the critical frequency. This is not strange because the observed flutter behaviour is apparently stall flutter, in which case classical flutter theory cannot be applied. It is however also possible that the shortcomings of the set-up as explained in section 2.4 (extra unwanted mode of motion due to independence of the left and right side
of the set-up and the clearance of the axis of rotation) and the consequential non-linear behaviour partially cause the bad agreement. The fact that the axis of rotation cannot move freely for example affects the measurement of $\omega_\theta$ (the motion damps out in just a few oscillations) and therefore also affects the determination of the moment of inertia $I_p$ ($I_p = \frac{K_\theta}{\omega_\theta}$), which is not very accurate. To be sure about this these problems with the set-up should be solved.

However, also when the poor agreement is (completely) due to the fact that the observed behaviour is stall flutter, these problems with the set-up should be solved, to get better reproducibility and better results.

It would also help if the critical flutter speed for classical flutter could be changed to a value lower than the observed flutter speed by changing parameters of the set-up. This will be discussed in section 7.5.
9.12 Nomenclature

- CM: center of mass of the wing
- P: turning point or centre of pitch, where the rotation axis of the wing is attached to the set-up and the rotational spring.
- \( l_{\text{wing}} \) [m]: the length of the wing (0.150 m is this survey)
- \( m \) [kg]: (total) mass of the parts of the set-up that translate (See chapter 4 and appendix 9.5)
- \( m_{\text{tot}} \) [kg]: mass of the wing and other parts of the set-up that rotate (See chapter 4 and appendix 9.5)
- \( c \) [m]: chord of the wing (used in aerodynamics)
- \( b \) [m]: semi chord of the wing (used in aero elasticity ) (\( b = c / 2 \))
- \( e \): dimensionless parameter for the distance of the CM to the middle of the chord (see fig1)
- \( a \): dimensionless parameter for the distance of the turning point P to the middle of the chord (see fig1)
- \( x_0 := (e - a) \): dimensionless parameter for the distance between the CM and the rotational axis that goes through P. e and a are defined in figure 1. (typical section model)
- \( d \): distance between the aerodynamic center of the wing (AC) and the axis of rotation(P)
- \( \hat{e}_x, \hat{e}_y \): unity vectors in the x and y direction
- \( q \): the dynamic pressure, \( q = \frac{1}{2} \rho U^2 \)
- \( h \) [m]: translation of the rotation axis in the y-direction
- \( h_1 \) [m]: height measured by the first sensor (distance sensor to a certain position at the chord of the wing)
- \( h_2 \) [m]: height measured by the second sensor (distance sensor to wing)
- \( h_0 \) [m]: initial rest value of \( h \) (and mean value of \( h \))
- \( \alpha_0 \) [rad]: The initial, zero flow, angle of attack
- \( \theta \) [rad]: The angle of deflection caused by the flow, relative to \( \alpha_0 \)
- \( \alpha := \alpha_0 + \theta \) [rad]: is the total angle of deflection with respect to the x-axis.
- \( I_{\text{cm}} \) [kgm^2]: moment of inertia relative to the center of mass for the rotating parts of the set-up. (See Figure 1)
- \( I_p \) [kgm^2]: moment of inertia relative to a rotation axis though an arbitrary point P for the rotating part of the set-up. (See Figure 1)
- \( K_g \) [Nm/rad]: torsional spring stiffness
- \( K_h \) [N/m]: translational (bending) spring stiffness
- \( \omega_n = \sqrt{\frac{K_h}{m}} \) [rad/s]: natural translation frequency at zero airspeed without friction
- \( \omega_n^* \) [rad/s]: (measured) natural zero flow translation frequency (with friction).
\[ \omega_0 = \sqrt{\frac{K_2}{I_p}} \text{ [rad/s]: natural rotation frequency at zero airspeed} \]

- \( \omega_0 \) [rad/s]: (measured) natural zero flow rotational frequency (with friction).
- \( \omega \) [rad/s]: frequency (in general)
- \( \sigma = \frac{\omega_0}{\omega} \): dimensionless parameter for the ratio of the rotational and translational natural frequencies (spring stiffnesses)

- \( R_R \) [Ns/m]: mechanical friction coefficient for the translational motion (measured at zero flow)
- \( R_\theta \) [Nms/rad]: mechanical friction coefficient for the rotational motion (measured at zero flow).
- \( E_p \) [Nm]: potential energy
- \( E_\theta \) [Nm]: kinetic energy
- \( L \) [N]: resulting lift force acting on the wing (flat plate approximation)
- \( M \) [Nm]: pitching moment (torque) acting on the wing (flat plate approximation) relative to an axis of rotation through the turning point \( P \)
- \( \mathcal{L} = E_k - E_p \) [Nm]: Lagrangian

- \( M_p = \frac{U}{c_m} \) is the Mach number. The index \( \infty \) refers to free stream conditions far away from the wing. An incompressible flow is assumed, therefore \( M_p \to 0 \).
- \( \rho_\infty = 1.2 \frac{kg}{m^3} \) is the air density far away from the wing. We assume this to be a constant.

- \( k = \frac{b \omega}{U} \): reduced frequency or Strouhal number (dimensionless)
- \( \mu = \frac{m_{rot}}{I_{rot} \rho_b b^2} \): mass ratio (dimensionless)
- \( r = \sqrt{\frac{I_{cen} + m_{rot} b^2 x_g}{m_{rot} b^2}} = \sqrt{\frac{l_p}{m_{rot} b^2}} \) mass radius of gyration about \( P \) (dimensionless)

- \( m_g, m_h, l_g, l_h \): dimensionless coefficients determining the expressions for the lift \( L \) and the moment \( M \). These coefficients are complex and involve Bessel functions.

- \( Q_g, Q_h \) [N]: Generalized forces as occurring in the Lagrange formalism.
- \( \tau_h = \frac{R_h}{2m} \) [s]: critical damping time for the translational mode. This is the time to reduce the amplitude to a certain fraction of the amplitude at \( t=0 \).
- $\tau_\theta = \frac{R_\theta}{2I_p}$ [s]: critical damping time for the rotational mode.

- $T$: period time [s]
- $f$: frequency [1/s]
10 Literature and references


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