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A transmission-line model for the Planar Inverted F-Antenna

West, R.C.A.

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A transmission-line model for the Planar Inverted F-Antenna

by R.C.A. West

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Graduation report performed at
TU/e, Electromagnetics and Wireless Section

Supervisors:
Dr.ir. T.E. Motoasca
ir. H.J. Visser (TU/e)
prof.dr.ir. A.B. Smolders

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Abstract

In this thesis, a transmission-line model for the Planar Inverted F-Antenna (PIFA) is developed. The PIFA is a printed circuit type antenna which is often used on the main Printed Circuit Board (PCB) of mobile phones, WLAN and Bluetooth. The PIFA consists of a thin narrow F-shaped strip, printed on one side of a homogeneous dielectric substrate. On the other side of the substrate, a conducting ground plane is used. One edge of the stripline is short-circuited to the ground plane. The total length of the antenna, is about a quarter of the wavelength $\lambda/4$.

The transmission-line model for the PIFA was derived by successively adapting and modifying the transmission-line model of the microstrip antenna, the volumetric PIFA, the Short-Circuit Strip (SCS) antenna and finally arriving at a new model for the PIFA, i.e.:

i. The transmission-line model for the microstrip antenna, which make use of two main radiating slots, was analyzed and implemented in Matlab. A comparison with experimental results obtained from the literature, verified the accuracy and correctness of the implemented model.

ii. The model for the microstrip antenna was modified such that it can be used for the volumetric PIFA. The modification consisted in assuming only one radiating slot and replacing the other slot with an ideal short-circuit.

iii. The transmission-line model for the SCS antenna was developed as a modified version of the model of the volumetric PIFA. The ideal short-circuit in the volumetric PIFA was modelled as an equivalent LC-circuit.

iv. The transmission-line model for the PIFA was developed by partly using the LC-circuit equivalent of the short-circuit in the SCS model. The other parts of the model were developed using a new approach.

An overall verification of the transmission-line model for the PIFA was done by comparing the model with Return Loss measurements. A good agreement was found between the measurements and the new transmission-line model for the PIFA.
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Best regards,

Raymond C.A. West.
# Contents

Abstract 1

Acknowledgements 2

1 Introduction 7

2 Transmission-line model for a microstrip antenna 9
  2.1 Introduction ................................................. 9
  2.2 Working principle: the cavity model .......................... 10
  2.3 Description of the transmission-line model .................... 15
  2.4 Calculation of the model parameters .......................... 17
    2.4.1 Radiation conductance $G_s$ .............................. 17
    2.4.2 The self-susceptance $B_s$ .............................. 24
    2.4.3 The characteristic impedance $Z_c$ and the propagation
          constant $\gamma$ of a microstripline ....................... 26
  2.5 Comparison with published results ................................ 29

3 Transmission-line model for the volumetric PIFA 33
  3.1 Introduction .................................................. 33
  3.2 A modified transmission-line model ........................... 33
  3.3 Comparison with published results ................................ 36

4 Transmission-line model for a SCS antenna 39
  4.1 Introduction .................................................. 39
  4.2 A modified transmission-line model ........................... 39
  4.3 A modified transmission-line model using equivalent LC-circuits 44
  4.4 Comparison with measurements .................................. 48

5 Transmission-line model for the PIFA 53
  5.1 Introduction .................................................. 53
  5.2 Modelling approach ............................................. 53
  5.3 Asymmetrical coupled and coplanar strips ....................... 56
5.4 Comparison between asymmetrical coupled and coplanar strips .  59
5.5 Correction capacitor $C_{corr}$ .................................  62
5.6 Results correction capacitor ..................................  65
5.7 The end-effect susceptance $B_{rad}$ .............................  68
5.8 The radiation admittance $G_{rad}$ ...............................  72
5.9 Comparison with measurement .................................  74

6 Conclusions and recommendations ...............................  77
  6.1 Conclusions ..................................................  77
  6.2 Recommendations ............................................  78
Chapter 1

Introduction

Recently, there has been quite a lot of interest in the modelling of the Planar Inverted F-Antenna (PIFA). The PIFA is a printed circuit type antenna which is often used on the main Printed Circuit Board (PCB) of mobile phones. This type of antenna can also be found in other mobile wireless applications such as WLAN, Bluetooth and Zigbee.

In this thesis a transmission-line model for the PIFA is developed. The work presented here attempts to contribute to a better understanding of the operation of the PIFA antenna.

Several other antennas and models were first studied before actually developing the model for the PIFA. The first among these antennas was the rectangular microstrip antenna. The references for the transmission-line model of the microstrip antenna go back to the 1970’s, when the first model was developed [1]. Our attention, however, was mainly restricted to the transmission-line model proposed by van de Capelle and Pues [7]. This model has shown to be very accurate in the modelling of the input impedance and return loss of the microstrip antenna. Chapter 2 discusses the transmission-line model for the $\lambda/2$ rectangular microstrip antenna.

Chapter 3 presents a transmission-line model for the volumetric PIFA, which in fact is a modified version of the microstrip antenna model of the previous chapter. This is the second step in developing the transmission-line model for the PIFA. The volumetric PIFA is basically a microstrip patch antenna which is fully short-circuited across the width $W$ [3]. The antenna length $L$ is approximately a quarter of the wavelength $\lambda$.

In chapter 4, a transmission-line model for the Short-Circuit Strip (SCS) antenna is developed. The SCS antenna consists of a thin conducting strip above a metal ground plane. One end of the strip is short-circuited to the ground plane, whereas the other end is left open. The new transmission-line model represents the SCS antenna as a transmission-line, short-circuited at
one side and terminated with a load impedance at the other side. The new model is used to calculate the return loss and input impedance of the SCS antenna.

The final part of this thesis discusses the development of the transmission-line model for the PIFA itself. The transmission-line parameters used in the model are derived from the well-known asymmetrical coplanar strips model [4]. The line parameters computed with the model were compared with those obtained from EM simulations in the commercially available tool HFSS.

Another important element in the model is the end load impedance \( Y_L = G_{rad} + jB_{rad} \). This impedance can be seen as an equivalent load which represents the end-effect at the edge the PIFA. Return loss measurements were done on several PIFA’s in the antenna laboratory at the Eindhoven University of Technology (TU/e). An overall verification of the model is done by comparing the results from the measurements with the results obtained from the model. The model can be used to estimate the return loss and input impedance of the PIFA. The new model can be used as a tool to provide antenna designer with resonance frequency, bandwidth and impedance estimations. The model is implemented in Matlab.
Chapter 2

Transmission-line model for a microstrip antenna

2.1 Introduction

This section discusses the transmission-line model for the rectangular microstrip antenna. This is the first step in developing the transmission-line model for the Planar Inverted F-Antenna (PIFA).

The model discussed here, as well as the underlying assumptions and simplifications, is mainly taken from the work of Pues and van de Capelle [2]. The model has shown to be very accurate in the modelling of the input impedance and return loss of the rectangular microstrip antenna. It assumes that the rectangular microstrip antenna can be represented by two radiating main slots and two side slots. The main slots are modelled using a self-admittance and self-susceptance. The self-admittance $G_s$, is obtained from the far-field analysis of the slot. The self-susceptance $B_s$, is computed using an open-circuited transmission-line. The accuracy of the model depends on the transmission-line parameters. Therefore, much attention is paid to these parameters, i.e. the characteristic impedance and propagation constant.

This chapter also attempts to demonstrate that the present model is correctly implemented. Results from published literature are compared with impedance results obtained from our own model.

The main subject of this section is the transmission-line model. Nevertheless, some attention has been paid to the cavity model. The cavity model is used to analyse the electric and magnetic fields within the dielectric substrate, which can give a better understanding of the working principle of the antenna. We start our analysis with the cavity model.
2.2 Working principle: the cavity model

The rectangular microstrip antenna consists of a thin conducting plate (patch) and a conducting ground plane separated by a dielectric substrate. The antenna is generally fed either by a probe through the ground plane or by a microstrip line printed on top of the dielectric substrate. The photograph in Fig. 2.1 shows an example of a probe-fed rectangular microstrip antenna.

![Fig. 2.1: A photograph of a probe-fed rectangular microstrip antenna.](image)

In the cavity model, the electromagnetic fields within the dielectric substrate can be determined using Maxwell's equations in the frequency domain. When the material of the substrate is characterized by a permittivity \( \varepsilon = \varepsilon_0 \varepsilon_r \) and a permeability \( \mu = \mu_0 \), these fields in a source-free region can be written as

\[
\nabla \times \vec{E} = -j\omega \mu_0 \vec{H} \quad (2.1)
\]

\[
\nabla \times \vec{H} = j\omega \varepsilon_0 \varepsilon_r \vec{E} \quad (2.2)
\]

where \( \vec{E} \) denotes the electric field and \( \vec{H} \) is the magnetic field, both in frequency domain.

An expression for the electric field \( \vec{E} \) can be found if \( \vec{H} \) in Eq.(2.1) is substituted into Eq.(2.2), i.e.:
2.2 Working principle: the cavity model

\[ \nabla \times \left( \frac{1}{j \omega \mu_0} \nabla \times \vec{E} \right) = -j \omega \varepsilon_0 \varepsilon_r \vec{E} \]

\[ \nabla \times (\nabla \times \vec{E}) = \omega^2 \varepsilon_0 \mu_0 \varepsilon_r \vec{E}. \]  \hfill (2.3)

If the wavenumber is defined as \( k_0 = \omega \sqrt{\varepsilon_0 \mu_0} \), then Eq.(2.3) can also be written in the form

\[ \nabla \times (\nabla \times \vec{E}) - k_0^2 \varepsilon_r \vec{E} = \vec{0}. \]  \hfill (2.4)

In the expression above, we recognize the vector operation \( \nabla \times (\nabla \times \vec{E}) \). Using vector algebra it can be shown that \( \nabla \times (\nabla \times \vec{E}) = (\nabla \cdot \vec{E}) \nabla - (\nabla \cdot \nabla) \vec{E} \). Therefore, Eq.(2.4) becomes

\[ (\nabla \cdot \vec{E}) \nabla - (\nabla \cdot \nabla) \vec{E} - k_0^2 \varepsilon_r \vec{E} = \vec{0}. \]  \hfill (2.5)

Since in a source-free region \( \nabla \cdot \vec{E} = 0 \), Eq.(2.5) becomes

\[ \nabla^2 \vec{E} + k_0^2 \varepsilon_r \vec{E} = \vec{0}. \]  \hfill (2.6)

Eq.(2.6) is the Helmholtz equation for the electric field \( \vec{E} \).

Now, to solve Eq.(2.6) the following assumptions about the electric field distribution are used. First, Fig. 2.2 shows the microstrip antenna in the cavity model. The current density \( \vec{J}_c \) on the top patch has only a component along the patch edge [5]. This means that at the cavity walls, the magnetic field \( \vec{H}_a \) can be represented by

\[ \vec{H}_a = \vec{u}_z \times \vec{J}_c. \]  \hfill (2.7)

From Eq.(2.7) it is clear that the magnetic field \( \vec{H}_a \) is always perpendicular to cavity sidewalls and thus,

\[ \vec{n}_a \times \vec{H}_a = \vec{0}. \]  \hfill (2.8)
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA

Here represents \( \vec{n}_a \) the normal vector to the cavity sidewalls.

From Eq. (2.8) it is concluded that the cavity sidewalls can be represented by Perfect Magnetic Conductors (PMCs). The top patch and the ground plane are assumed to be Perfect Electric Conductors (PECs).

![Diagram of a microstrip antenna](image)

Fig. 2.2: (a) Top view of the microstrip antenna in a coordinate system 
(b) Side view of the microstrip antenna.

We assume that the substrate is very thin \((h \ll \lambda_{\text{mat}})\), where \(\lambda_{\text{mat}}\) is the wavelength in free space. The consequence of this is that the electric field
2.2 Working principle: the cavity model

is constant along the $z$-direction, i.e. $\partial_z E_z = 0$. Furthermore, the boundary conditions at the top plate and at the ground plane state that the tangential components of the electric field are equal to zero, that is $E_x = E_y = 0$. Now, using $\partial_z E_z = 0$ and $E_x = E_y = 0$ we can write Eq.(2.6) as,

$$(\partial_z^2 E_z + \partial_y^2 E_z)\bar{u}_z + k_0^2 \varepsilon_r E_z \bar{u}_z = 0.$$ 

Finally, we assume that the electric field is distributed along the $x$-axis in an uniform fashion. This implies that $\partial_x E_z = 0$ and therefore we can write for the electric field that

$$\partial_y^2 E_z + k_0^2 \varepsilon_r E_z = 0. \quad (2.9)$$

In this thesis we will limit our investigation to the first resonance of the antenna. Therefore we will only look at the fundamental mode that can exists in the cavity. Thus, the solution to Eq.(2.9) can be written as [6],

$$E_z(y) = E_e \sin(k_0\sqrt{\varepsilon_r} y) + E_o \cos(k_0\sqrt{\varepsilon_r} y). \quad (2.10)$$

The electric field is now described by sinusoidal functions. The subscripts $e$ and $o$ are used to denote even and odd waves, respectively.

Next, we will discuss the boundary conditions. If Eq.(2.1) is written in the Cartesian coordinate system we obtain

$$\partial_y E_x - \partial_z E_y = -j\omega \mu_0 H_x \quad (2.11)$$

$$\partial_z E_x - \partial_x E_z = -j\omega \mu_0 H_y \quad (2.12)$$

$$\partial_x E_y - \partial_y E_x = -j\omega \mu_0 H_z. \quad (2.13)$$

Because of the assumed electric field distribution between the antenna plates we have $\partial_z E_y \to 0$, $\partial_z E_z \to 0$, $\partial_x E_z \to 0$, $\partial_x E_y \to 0$ and $\partial_y E_x \to 0$. For Eqs.(2.11), (2.12) and (2.13) we consequently write,

$$H_x = -\frac{1}{j\omega \mu_0} \partial_y E_z \quad (2.14)$$

$$H_y = 0$$

$$H_z = 0.$$
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA

Because the cavity model assumes that the microstrip antenna has PMC sidewalls, it is concluded that the tangential magnetic field components must be zero at the cavity walls. This means that for \( y = 0 \), \( H_x \) equals zero. Using Eq.(2.14) we then write

\[
H_x|_{y=0} = 0
\]

and thus,

\[
\partial_y E_z|_{y=0} = 0. \tag{2.15}
\]

If Eq.(2.10) is substituted into Eq.(2.15) we obtain

\[
k_0 \sqrt{\varepsilon_r} E_e \cos(k_0 \sqrt{\varepsilon_r} y) - k_0 \sqrt{\varepsilon_r} E_o \sin(k_0 \sqrt{\varepsilon_r} y) = 0|_{y=0}. \tag{2.16}
\]

Which yields

\[
k_0 \sqrt{\varepsilon_r} E_e = 0
\]

\[
E_e = 0. \tag{2.17}
\]

The electric field within the substrate has therefore the form

\[
E_z(y) = E_o \cos(k_0 \sqrt{\varepsilon_r} y) \tag{2.18}
\]

or

\[
\vec{E} = E_o \cos(k_0 \sqrt{\varepsilon_r} y) \vec{u}_z. \tag{2.19}
\]

A sketch of the electric field distribution within the dielectric substrate is shown in Fig. 2.3.
2.3 Description of the transmission-line model

![Diagram of perfect electric conductor](image)

Fig. 2.3: Electric field distribution within the substrate of a microstrip antenna.

Finally, from Eq. (2.14) the magnetic field is computed as

\[
H_x = \frac{1}{j\omega \mu_0} k_0 \sqrt{\varepsilon_r} E_o \sin(k_0 \sqrt{\varepsilon_r} y)
\]

or

\[
\tilde{H} = \frac{1}{j\omega \mu_0} k_0 \sqrt{\varepsilon_r} E_o \sin(k_0 \sqrt{\varepsilon_r} y) \tilde{u}_x.
\]

2.3 Description of the transmission-line model

The following sections present the transmission-line model for a microstrip antenna. The model adopted in this thesis is based on the model for microstrip antennas developed in [2, 7]. The model describes the microstrip antenna as a four-slot system (two main slots and two side slots). The analysis of the two side slots is not included in the thesis. Fig. 2.4(a) shows the four-slot antenna system.

The antenna consists of a conducting patch, a dielectric substrate and a conducting ground plane. The patch has a resonant length L, a width W and a thickness t.
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA

Other important parameters of the patch are the conductivity \( \sigma_s \) and the patch surface error \( \Delta_s \). The parameters \( \sigma_s \) and \( \Delta_s \) are important because they are later used to compute the conducting losses in the patch.

The dielectric substrate has a thickness \( h \), a relative permittivity \( \varepsilon_r \) and a loss tangent \( \delta_s \) and it is assumed to have infinite dimensions in the plane of the patch.

Similarly to the substrate, the ground plane is infinitely long and wide in dimension. It is further characterized by the conductivity \( \sigma_g \), the surface error \( \Delta_g \) and the thickness \( t_g \).

The transmission-line model represents the antenna by a transmission-line terminated at both ends by admittances. In this model, the self-admittance

---

Fig. 2.4: (a) Four-slots system of a microstrip antenna, (b) Transmission-line model for the microstrip antenna.
2.4 Calculation of the model parameters

$Y_s$ is used to model the two main radiating slots. The self-admittance $Y_s$ in Fig. 2.4(b) consists of a real and an imaginary part. The real part of $Y_s$ is the radiation conductance $G_s$ and the imaginary part is the self-susceptance $B_s$. Two voltage-dependent current sources ($Y_m V_1$ and $Y_m V_2$) are used to model the mutual coupling between the two main slots. Fig. 2.4(b) shows the transmission-line model for the rectangular microstrip antenna.

The key elements in the model are: the self-admittance $Y_s$, the mutual admittance $Y_m$, the complex propagation constant $\gamma$ and the characteristic admittance $Y_c$. The authors in [2] have done much effort in developing expressions for these parameters. These parameters have shown to be sufficiently accurate for most design work [8]. A more detailed explanation of the parameters is given in the next sections.

2.4 Calculation of the model parameters

In this section a more detailed explanation on the transmission-line parameters: $Y_s$ (the self-admittance), $\gamma$ (the complex propagation constant) and $Y_c$ (the characteristic admittance) is given. The mutual admittance $Y_m$ is for simplicity neglected.

2.4.1 Radiation conductance $G_s$

This section explains the derivation of the radiation conductance $G_s$ using the far-field analysis. We start with some explanation about the open-end edges of the antenna.

An open-ended microstrip shows stray fields at the end of the conducting patch, see Fig. 2.5. In the case of the rectangular microstrip antenna, the stray fields extend beyond the end of the microstripline. In these type of antennas, the stray fields are the sources of the radiation.

The idea behind this is that the stray fields can be represented by two horizontal slots along the two open-ends of the patch. These slots are the main radiating slots in the model. The tangential stray fields uniformly illuminate the aperture of the main slots. This is similar to the cavity model of the microstrip antenna where there is an uniform electric field in a vertical slot (see Fig. 2.6). In fact, these vertical slots are replaced by equivalent horizontal slots.

The stray fields can be used to compute the radiated power $P_T$ associated with the slot aperture.
The radiated power $P_r$ is then used to determine the radiation conductance $G_s$ of the slot. The radiation conductance itself is written as,

$$ G_s = \frac{2P_r}{|V_s|^2} \quad (2.22) $$

where $V_s$ is the voltage across the slot,

$$ |V_s|^2 = b^2|E_c|^2. \quad (2.23) $$

In the following set of formulas, a derivation of the radiated power $P_r$ is given. Fig. 2.7 gives an illustration of the slot in a Cartesian coordinate system. We
2.4 Calculation of the model parameters

Fig. 2.7: Uniform illuminated slot in coordinate system.

assume that the field distribution in the slot is uniform. Using Fig. 2.7 we can write the electric and the magnetic field in the slot aperture as

\[
\begin{align*}
\vec{E}_a & = E_c \hat{u}_y \quad \text{for} \quad -\frac{a}{2} < x < \frac{a}{2}, \quad -\frac{b}{2} < y < \frac{b}{2}. \quad (2.24) \\
\vec{H}_a & = \frac{E_c}{Z_o} \hat{u}_x \quad \text{for} \quad -\frac{a}{2} < x < \frac{a}{2}, \quad -\frac{b}{2} < y < \frac{b}{2}. \quad (2.25)
\end{align*}
\]

where \(E_c\) is a constant and \(Z_o\) is the wave impedance in free space. The unit vectors \(\hat{u}_x\) and \(\hat{u}_y\) are defined in the Cartesian coordinate system. The width \(a\) and the length \(b\) define the dimension of the radiating slot.

Using the electric and the magnetic fields in the slot aperture, one can compute the corresponding equivalent surface currents as,

\[
\begin{align*}
\vec{J}_e & = \vec{n} \times \vec{H}_a \\
\vec{J}_m & = \vec{E}_a \times \vec{n},
\end{align*}
\]

where \(\vec{n}\) is the normal vector of the slot surface.

Since \(\vec{n} = \vec{u}_z\) we have

\[
\begin{align*}
\vec{J}_e & = -\frac{E_c}{Z_o} \vec{u}_z \times \vec{u}_x = -\frac{E_c}{Z_o} \vec{u}_y \\
\vec{J}_m & = E_c \vec{u}_y \times \vec{u}_z = E_c \vec{u}_x.
\end{align*}
\]

Now, a PEC plate is placed below the upper patch. We can do this because it does not change the far-field of the antenna. A more detailed explanation for
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA

This working method is given in [9, pp. 818]. In the PEC patch, the surface electric current $\vec{J}_e$ is equal to zero. Therefore, the equivalent surface currents become,

\begin{align}
\vec{J}_e & = 0 \quad (2.30) \\
\vec{J}_m & = E_c \vec{u}_x. \quad (2.31)
\end{align}

Using the image theory, the conducting plate is now removed. This is taken into account by doubling the magnetic current. This leaves us with,

$$\vec{J}_m = 2E_c \vec{u}_x. \quad (2.32)$$

Using $\vec{J}_m$ in Eq.(2.32), the electric field in the far-field region ($R \ll \lambda_w$) can be written as,

$$\vec{E}(\vec{r}) = \frac{jk_o e^{-jk_o r}}{4\pi r} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \vec{e}_r \times \vec{J}_m e^{jk_o(\vec{r}_0 \cdot \vec{e}_r)} d\vec{x}_0 d\vec{y}_0$$

$$= \frac{jk_o e^{-jk_o r}}{4\pi r} L_m, \quad (2.33)$$

with $L_m$ defined as,

$$L_m = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \vec{e}_r \times \vec{J}_m e^{jk_o(\vec{r}_0 \cdot \vec{e}_r)} d\vec{x}_0 d\vec{y}_0. \quad (2.35)$$

The far-field is now computed using the procedure described in [5, pp. 74-75]. The integration in Eq.(2.35) is simplified by first rewriting the cross and dot product terms in spherical coordinates,

\begin{align}
\vec{e}_r \times \vec{e}_x & = \vec{e}_r \times (\vec{e}_\phi \cos \theta \cos \phi - \vec{e}_\theta \sin \phi) \\
& = \vec{e}_\phi \cos \theta \cos \phi + \vec{e}_\theta \sin \phi \quad (2.36) \\
(\vec{r}_0 \cdot \vec{e}_r) & = (x_0 \vec{e}_x + y_0 \vec{e}_y) \cdot (\vec{e}_x \sin \theta \cos \phi + \vec{e}_y \sin \theta \sin \phi + \vec{e}_z \cos \theta) \\
& = x_0 \sin \theta \cos \phi + y_0 \sin \theta \sin \phi = x_0 u + y_0 v. \quad (2.37)
\end{align}
2.4 Calculation of the model parameters

where \( u = \sin \theta \cos \phi \) and \( v = \sin \theta \sin \phi \).

Using Eq. (2.36) and Eq. (2.37) it follows that the integral \( L_m \) is,

\[
L_m = 2E_c(\vec{e}_\phi \cos \theta \cos \phi + \vec{e}_\theta \sin \phi) \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} e^{jk_o(x_o u + y_o v)} dx_o dy_o \tag{2.38}
\]

\[
= ab2E_c(\vec{e}_\phi \cos \theta \cos \phi + \vec{e}_\theta \sin \phi) \frac{\sin k_o u a/2 \sin k_o v b/2}{k_o u a/2 \ k_o v b/2}. \tag{2.39}
\]

The electric field in the far-field region is obtained by substituting Eq. (2.39) in Eq. (2.34),

\[
\vec{E}(r, \theta, \phi) = \frac{jk_o e^{-jk_o r}}{4\pi r} L_m
\]

\[
= \frac{jk_o e^{-jk_o r}}{2\pi r} abE_c(\vec{e}_\phi \cos \theta \cos \phi + \vec{e}_\theta \sin \phi) \times \frac{\sin k_o u a/2 \sin k_o v b/2}{k_o u a/2 \ k_o v b/2}. \tag{2.40}
\]

The next step in the analysis is to find the total radiated power \( P_r \) from the slot aperture. Using the far-field electric field given in Eq. (2.40) it is found that,

\[
P_r = \frac{1}{2Z_o} \int_0^{2\pi} \int_0^\pi |\vec{E}(r, \theta, \phi)|^2 r^2 \sin \theta d\theta d\phi
\]

\[
= \frac{1}{2Z_o} \frac{k_o^2 (ab)^2 |E_c|^2}{(2\pi r)^3} \int_0^{2\pi} \int_0^\pi (\cos^2 \theta \cos^2 \phi + \sin^2 \phi)^2 \times \left[ \frac{\sin k_o u a/2}{k_o u a/2} \right]^2 \left[ \frac{\sin k_o v b/2}{k_o v b/2} \right]^2 r^2 \sin \theta d\theta d\phi \tag{2.41}
\]

To carry out the integration in Eq. (2.41), the following substitution is applied:

\[
k_x = k_o \sin \theta \cos \phi = k_o u,
\]

\[
k_y = k_o \sin \theta \sin \phi = k_o v,
\]

\[
k_z = k_o \cos \theta.
\]
The radiated power is therefore expressed as

\[
P_r = \frac{1}{2Z_o} \frac{k_o^2 (ab)^2 |E_c|^2}{(2\pi r)^2} \int_0^{2\pi} \int_0^\pi (1 - u^2) \left[ \frac{\sin k_x a/2}{k_x a/2} \right]^2 \times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 r^2 \sin \theta d\theta d\phi
\]

which is (2.42)

and thus

\[
P_r = \frac{1}{2Z_o} \frac{b^2 |E_c|^2}{(2\pi r)^2} \int_0^{2\pi} \int_0^\pi \frac{(k_o^2 - k_x^2)}{k_x^2} \left[ \sin k_x a/2 \right]^2 \times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 r^2 \sin \theta d\theta d\phi.
\]

Furthermore, we have \(dk_x dk_y = k_o^2 \sin \theta \cos \theta d\theta d\phi\) and the radiated power is therefore written as,

\[
P_r = \frac{1}{2Z_o} \frac{b^2 |E_c|^2}{(2\pi r)^2} \int_0^{2\pi} \int_0^\pi \frac{(k_o^2 - k_x^2)}{k_x^2} \left[ \sin k_x a/2 \right]^2 \times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 \frac{1}{k_x^2 k_o^2} \sin \theta \cos \theta dk_x dk_y
\]

which is (2.44)

\[
P_r = \frac{1}{2Z_o} \frac{b^2 |E_c|^2}{\pi^2 k_o} \int_0^\pi \int_0^\pi \frac{(k_o^2 - k_x^2)}{k_x^2} \left[ \sin k_x a/2 \right]^2 \times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 \frac{1}{k_x^2} dk_x dk_y
\]

which is (2.45)

For \(k_x\) we can also write \(k_x = \sqrt{k_o^2 - k_x^2 - k_y^2}\). This means that \(k_x\) is real for \(k_x^2 + k_y^2 \leq k_o^2\). Thus the radiated power \(P_r\) is real when \(-\sqrt{k_o^2 - k_x^2} \leq k_y \leq \sqrt{k_o^2 - k_x^2}\). For \(k_x\) we can write \(k_x = k_o \sin \theta \cos \phi\). When \(\theta\) varies between \([0, \pi]\) and \(\phi\) between \([0, 2\pi]\), the integration variable \(k_x\) is between \([-k_o, k_o]\).
2.4 Calculation of the model parameters

Hence the limits of integration become

\[
P_r = \frac{1}{2Z_0} \frac{b^2|E_c|^2}{\pi^2 k_0} \int_{-k_o}^{k_o} \int_{-k_o - \sqrt{k_o^2 - k_x^2}}^{k_o} \frac{(k_o^2 - k_z^2)}{k_z^2} [\sin k_x a/2]^2 \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 \frac{d k_x d k_y}{k_z}
\]

Finally, using Eq.(2.22) the radiation conductance \( G_s \) is computed as,

\[
G_s = \frac{1}{\pi^2 Z_0 k_0} \int_{-k_o}^{k_o} \int_{-k_o - \sqrt{k_o^2 - k_x^2}}^{k_o} \frac{(k_o^2 - k_z^2)}{k_z^2} [\sin k_x a/2]^2 \\
\times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 \frac{d k_x d k_y}{k_z}
\]

\[
= \frac{4}{\pi^2 Z_0 k_0} \int_{0}^{k_o} \int_{0}^{\sqrt{k_o^2 - k_x^2}} \frac{(k_o^2 - k_z^2)}{k_z^2} [\sin k_x a/2]^2 \\
\times \left[ \frac{\sin k_y b/2}{k_y b/2} \right]^2 \frac{d k_x d k_y}{k_z}.
\] (2.46)

The integral in Eq.(2.46) is computed in [2, p. 543] as,

\[
G_s \approx \frac{1}{\pi Z_0} \left\{ a_n S_i(a_n) + \frac{\sin a_n}{a_n} + \cos a_n - 2 \right\} \\
\times \left( 1 - \frac{k_n^2}{24} \right) + b_n^2 \left( \frac{1}{3} + \frac{\cos a_n}{a_n^2} - \frac{\sin a_n}{a_n^3} \right) 
\] (2.47)

where \( a_n = k_o a \) is the normalized slot length, \( b_n = k_o b \) is the normalized slot width and

\[
S_i(x) = \int_{0}^{x} \frac{\sin u}{u} du.
\] (2.48)
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROWAVE ANTENNA

2.4.2 The self-susceptance $B_s$

The idea of the self-susceptance $B_s$ comes from the open-end effect at the edge of the microstrip antenna. The fields under the patch are not entirely contained between the patch and the ground plane, but they rather spread out further into the dielectric material, resulting in the so-called stray fields. It is often said that the stray fields electrically extend the microstrip line [2, p. 533]. Fig. 2.8 shows the electrically extended microstrip.

Fig. 2.8: Extension $\Delta l$ of microstrip and transmission-line model.

Fig. 2.8 also shows the electrically extended microstrip line represented as a transmission-line of length $\Delta l$. The expression for $\Delta l$, found in the literature [10, 11], involves many formulas where curve fitting technique have been used to derive them. In this thesis, the length $\Delta l$ is computed as reported in [11]. When a microstrip antenna with a height-to-width ratio $W/h$ is considered, we then obtain the following expression for the microstrip extension length $\Delta l$:

$$\Delta l = h\xi_1 \xi_3 \xi_5 / \xi_1$$  \hspace{1cm} (2.49)

where
2.4 Calculation of the model parameters

\[ \xi_1 = 0.434907 \frac{\varepsilon_{\text{eff}}^{0.81}}{\varepsilon_{\text{eff}}} + 0.26 \left( \frac{W}{h} \right)^{0.8544} + 0.236 \left( \frac{W}{h} \right)^{0.8544} + 0.87 \] (2.50)

\[ \xi_2 = 1 + \frac{(W/h)^{0.371}}{2.358\varepsilon_r + 1} \] (2.51)

\[ \xi_3 = 1 + \frac{0.5274 \arctan \left( 0.084 \frac{W}{h} \right)^{1.9413/\xi_2}}{\varepsilon_{\text{eff}}^{0.9236}} \] (2.52)

\[ \xi_4 = 1 + 0.0377 \arctan \left( 0.067 \left( \frac{W}{h} \right)^{1.456} \right) \] (2.53)

\[ \xi_5 = 1 + 0.218 \exp(-7.5W/h). \] (2.54)

The transmission-line model of the extension is further characterized by a phase constant \( \beta \) and a characteristic admittance \( Y_c = 1/Z_c \), see Fig. 2.8. In general, the propagation constant is defined as \( \gamma = \alpha + j\beta \). But, since we have assumed that the transmission-line is lossless, the propagation constant \( \gamma \) consists here of only the phase constant \( \beta \). The phase constant \( \beta \) and the admittance \( Y_c \) are further explained in the next section.

When the transmission-line equation in [12, 13] is applied to the transmission-line in Fig. 2.8 we obtain,

\[ Y_{in} = \frac{Y_c Z_c + jZ_L \tan(\beta \Delta l)}{Z_L + jZ_c \tan(\beta \Delta l)}, \] (2.55)

where \( Z_L \) represents the load impedance that terminates the transmission-line. \( Z_L \) is assumed to be a very large value (infinity) due to the open edge of the microstrip antenna structure. Using \( Z_L \to \infty \) in Eq.(5.27) we then obtain,

\[ \lim_{Z_L \to \infty} Y_{in} = jY_c \tan(\beta \Delta l). \] (2.56)

The self-susceptance \( B_s \) is finally determined from Eq.(2.56) as,

\[ B_s = Y_c \tan(\beta \Delta l). \] (2.57)
2.4.3 The characteristic impedance $Z_c$ and the propagation constant $\gamma$ of a microstrip line

To obtain the characteristic impedance $Z_c$ and the propagation constant $\gamma = \alpha + j\beta$, the microstrip antenna model is replaced with an equivalent planar-waveguide model [14].

In the microstrip antenna model, the electric field lines originate on the edge and top of the microstrip and partially extend into the free space above the dielectric substrate $\varepsilon_r$, see Fig. 2.9(a).

In the equivalent planar-waveguide model, an effective permittivity $\varepsilon_{eff}$ is used to replace both the air and the dielectric substrate $\varepsilon_r$ with one homogeneous material permittivity $1 < \varepsilon_{eff} < \varepsilon_r$. The equivalent planar-waveguide model is shown in Fig. 2.9(b).

![Fig. 2.9: (a) Cross-section of a microstrip antenna model, (b) Cross-section of the equivalent planar-waveguide model of the antenna.](image)

A frequency-dependent effective width $W_{eff}$ is introduced to model the width of the planar-waveguide model. $W_{eff}$ is larger than the width of the microstrip antenna model, $W_{eff} > W$. The height of the planar-waveguide model is equal to the height of the microstrip line model.

Using the planar-waveguide model in Fig. 2.9(b), we can now write the
2.4 Calculation of the model parameters

characteristic impedance $Z_c$ and the phase constant $\beta$ as,

$$Z_c = \frac{\eta_0}{\sqrt{\varepsilon_{eff}(f) W_{eff}(f)}}$$  \hspace{1cm} (2.58)

$$\beta = \frac{k_0}{\sqrt{\varepsilon_{eff}(f)}},$$  \hspace{1cm} (2.59)

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance in free space and $k_0 = \omega \sqrt{\mu_0/\varepsilon_0}$ is the phase constant in free space.

For a microstrip with a width $W$, a height $h$ and a dielectric permittivity $\varepsilon_r$, the frequency-dependent effective width $W_{eff}$ and the effective permittivity $\varepsilon_{eff}$ are computed as [2],

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}(0)}{1 + P}$$  \hspace{1cm} (2.60)

$$P = P_1 P_2 \{0.1844 + P_3 P_4 f_n\}^{1.5763}$$  \hspace{1cm} (2.61)

$$P_1 = 0.27488 + \{0.6315 + 0.525/(1 + 0.0157 f_n)^{20}\} u - 0.065683 \exp(-8.7513 u)$$  \hspace{1cm} (2.62)

$$P_2 = 0.33622 \{1 - \exp(-0.03442 \varepsilon_r)\}$$  \hspace{1cm} (2.63)

$$P_3 = 0.0363 \exp(-4.6 u) \{1 - \exp[-(f_n/38.7)^{1.97}]\}$$  \hspace{1cm} (2.64)

$$P_4 = 1 + 2.751\{1 - \exp[-(\varepsilon_r/15.916)^6]\}$$  \hspace{1cm} (2.65)

$$f_n = 47.713 k_0 h$$  \hspace{1cm} (2.66)

$$u = \left\{W + (W' - W)/\varepsilon_r\right\}/h$$  \hspace{1cm} (2.67)

and,

$$W_{eff}(f) = \frac{W}{3} + (R_w + P_w)^{1/3} - (R_w - P_w)^{1/3}$$  \hspace{1cm} (2.68)

$$S_w = \frac{c_o^2}{4 f^2 [\varepsilon_{eff}(f) - 1]}$$  \hspace{1cm} (2.69)

$$Q_w = \frac{S_w}{3} - \left(\frac{W}{3}\right)^2$$  \hspace{1cm} (2.70)

$$P_w = \left(\frac{W}{3}\right)^3 + \frac{S_w}{2} \left[W_{eff}(0) - \frac{W}{3}\right]$$  \hspace{1cm} (2.71)

$$R_w = (P_w^2 + Q_w^3)^{1/2}$$  \hspace{1cm} (2.72)

where $c_o$ is here the velocity of light in free-space.
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA

The attenuation constant $\alpha$ consists of dielectric losses in the substrate $\alpha_d$, conducting losses in the strip $\alpha_{cs}$ and in the ground plane $\alpha_{cg}$,

$$\alpha = \alpha_d + \alpha_{cs} + \alpha_{cg}. \tag{2.73}$$

In this thesis, the dielectric losses $\alpha_d$ are computed as \[2\],

$$\alpha_d = 0.5\beta \frac{\varepsilon_r - 1}{\varepsilon_{eff}(f) - 1} \tan \delta, \tag{2.74}$$

while the conducting losses $\alpha_{cs}$ and $\alpha_{cg}$ are calculated as,

$$\alpha_{cs} = \alpha_n R_{ss} F_{\Delta_s} F_s \tag{2.75}$$
$$\alpha_{cg} = \alpha_n R_{sg} F_{\Delta_g} \tag{2.76}$$

where,

$$R_{ss} = \sqrt{\pi f \mu_0 / \sigma_s} \tag{2.77}$$
$$R_{sg} = \sqrt{\pi f \mu_0 / \sigma_g} \tag{2.78}$$
$$F_{\Delta_s} = 1 + \frac{2}{\pi} \arctan \left[ 1.4 (R_{ss} \Delta_s \sigma_s)^2 \right] \tag{2.79}$$
$$F_{\Delta_g} = 1 + \frac{2}{\pi} \arctan \left[ 1.4 (R_{sg} \Delta_g \sigma_g)^2 \right] \tag{2.80}$$
$$F_s = 1 + \frac{2h}{W'} \left( 1 - \frac{1}{\pi} \frac{W' - W}{t} \right) \tag{2.81}$$
2.5 Comparison with published results

In the previous sections, a transmission-line model was presented for the analysis of rectangular microstrip antennas. Experimental results obtained from the literature, demonstrated that the model is capable to accurately predict impedance and return loss. Our objective here is to exactly reproduce the results given in literature to verify the accuracy and correctness of the implemented model.

We tried to reproduce the input impedance graph published in [2, p. 555]. In this example, a rectangular microstrip antenna fed by a microstrip line is considered. The antenna is characterized by: \( W = 144 \) mm, \( L = 76 \) mm, \( h = 1.59 \) mm and \( \varepsilon_r = 2.62 \). Dielectric losses in the substrate and conducting losses in the microstrip and in the ground plane are taken into account by: \( \tan \delta = 0.001 \) (loss tangent), \( \sigma_s = \sigma_g = 0.556 \times 10^5 \) S/mm (conductivity of patch and ground plane), \( \Delta_s = \Delta_g = 0.0015 \) mm (rms surface roughness patch and ground plane). Fig. 2.10 shows the input impedance of the rectangular microstrip antenna. A frequency range of 1.157 GHz to 1.227 GHz is used to plot the Smith chart of the input impedance. The input impedance is indicated by the dashed line and triangle.

![Fig. 2.10: Input impedance locus for a microstrip antenna.](image-url)
Our Smith chart corresponds very well with the figure in [2, p. 555]. It may therefore be assumed that the present model and formulas are indeed correctly implemented.

Additional comparison was made with impedance data found in other published literature [15]. However, other than the comparison above it is now focused on probe-fed microstrip antennas. Since results for the probe fed microstrip antenna from the Pues and van de Capelle group could not be found, it was decided to consult other authors and papers.

From [15] we obtained the input impedance Smith chart of the same antenna for three different feed locations.

The dimension for rectangular microstrip patch are: width \( W = 114.3 \, \text{mm} \) and length \( L = 76.2 \, \text{mm} \). The feed points (L1) are located at 7.6 mm, 22.9 mm and 30.5 mm from the radiating edge. Other relevant antenna properties are: \( \tan \delta = 0.001 \), \( \sigma_s = \sigma_y = 0.556 \times 10^6 \, \text{S/mm} \), and \( \Delta_s = \Delta_y = 0.0015 \, \text{mm} \). The feed points are indicated in Fig. 2.11 with the numbers 1, 2 and 3 respectively.

The calculated input impedance for the implemented model is shown in Fig. 2.11 as a function of the operating frequency.

Fig. 2.11: Input impedance locus for a probe-fed microstrip antenna.
2.5 Comparison with published results

The entire impedance locus can change drastically with the feed location, as seen in Fig. 2.11. An good agreement with [15] indicates that the present model is indeed correctly implemented.

It should be pointed out that the results from our model are phase-shifted relative to the results in the reference [15], see Fig. 2.11 and Fig. 2.12. This phase difference is probably caused by the length of connectors which formed part of the measured antenna.

Fig. 2.12: Impedance locus for a microstrip antenna from [15].
CHAPTER 2. TRANSMISSION-LINE MODEL FOR A MICROSTRIP ANTENNA
Chapter 3

Transmission-line model for the volumetric PIFA

3.1 Introduction

In this chapter a new transmission-line model for the volumetric PIFA is developed. This is in fact a modified version of the microstrip antenna model of the previous chapter. This is the second step in developing the transmission-line model for the PIFA.

In chapter 2 of this thesis it was shown that the transmission-line model can accurately estimate the input impedance of a microstrip antenna. The transmission-line model outlined there contains self-admittances for the narrow rectangular radiating slots. Furthermore, comparison with published results confirmed that the model and corresponding formulas were correctly implemented. Now, after some modifications, can the model in chapter 2 be used for the volumetric PIFA.

At the end of the chapter, results obtained with the modified model are compared with those of other studies published in literature.

3.2 A modified transmission-line model

An example of a volumetric PIFA with height h, width W and length L is shown in Fig. 3.1(a). The volumetric PIFA is basically a microstrip patch antenna which is fully short-circuited across the width W [3]. The antenna length L is approximately a quarter of the wavelength λ, L ≈ λ/4.

In the case of a volumetric PIFA with coaxial probe feed, the transmission-line model has to be modified as shown in Fig. 3.1(b). Similarly to the model
in the previous chapter, the new model consists of two transmission-lines along the length of the antenna. One transmission-line with length $L_1$ and the other one with length $L_2 = L - L_1$. The transmission-line with length $L_1$ is short-circuited to take into account the short-circuited wall of the volumetric antenna.

![Diagram](image)

**Fig. 3.1:** (a) An example of a volumetric PIFA and (b) Modified transmission-line model for volumetric PIFA.

The radiation coming from the two side-slots is neglected. The result is a transmission-line model terminated by only an admittance $Y_s$ representing the radiating edge of the antenna.

The key elements in the model are the admittances $Y_s = G_s + jB_s$ and the characteristic impedance $Z_c$ together with the propagation constant $\gamma$ of both transmission-lines. The conductance $G_s$ is obtained by integration over a finite
3.2 A modified transmission-line model

length slot as in the rectangular microstrip antenna, see Section 2.4.1. The
susceptance $B_s$ is equal to the self-susceptance of an open end microstrip, see
Section 2.4.2. A more detailed explanation of the transmission-line parameters
$Z_c$ and $\gamma$ can be found in Section 2.4.3.

When the feed point is placed at a distance $L_1$ from the short-circuit wall,
the input admittance $Y_{in}$ is written as [12, p. 61],

$$Y_{in} = Y_c \frac{e^{\gamma L_1} + e^{-\gamma L_1}}{e^{\gamma L_1} - e^{-\gamma L_1}} + Y_s \frac{e^{\gamma |L-L_1|} - \Gamma e^{-\gamma |L-L_1|}}{e^{\gamma |L-L_1|} + \Gamma e^{-\gamma |L-L_1|}}$$

(3.1)

where $\Gamma$ is the reflection coefficient:

$$\Gamma = \frac{Y_c - Y_s}{Y_c + Y_s}$$

(3.2)

It is clear from Eq.(3.1) that the feed point of the antenna can be chosen
to give a suitable input impedance. Furthermore, in the case of a probe-fed
volumetric PIFA antenna the total input impedance $Z_{in\text{tot}}$ consists of a probe
reactance $X_f$ in series with the antenna impedance, see Fig. 3.2.

Fig. 3.2: Transmission-line model of a volumetric PIFA with a probe reactance.
CHAPTER 3. TRANSMISSION-LINE MODEL FOR THE VOLUMETRIC PIFA

In Fig. 3.2 the total input impedance $Z_{\text{intot}}$ can be written as,

$$Z_{\text{intot}} = 1/Y_{mn} + X_f. \quad (3.3)$$

The coaxial probe model reported in [16] was adopted here, where the reactance $X_f$ is computed as,

$$X_f = \eta \tan(2\pi h/\lambda), \quad (3.4)$$

where $\eta$ is the intrinsic impedance of the substrate.

3.3 Comparison with published results

This section discusses results from experimental investigations carried out on the volumetric PIFA. The objective here is to determine whether the model described in previous section can be used to estimate the input impedance of several antenna configurations.

Firstly, the antenna in [17] was considered. With the parameters defined as in Fig. 3.1(a), we used $L = 19$ mm, $L_1 = L_2 = 9.5$ mm, $W = 10$ mm, $h = 1.5$ mm and $\varepsilon_r = 1$ (air).

The impedance in Fig. 3.3 was calculated using Eq. 3.1. It was compared to the input impedance $Z_{in}$ data in [17] over a frequency range of 3 to 4 GHz. The frequency increases clockwise with steps of 5 MHz.

The Smith chart in Fig. 3.3 reveals the inductive nature (positive reactance) of the impedance. Fig. 3.3 also shows that the present model estimation is in good agreement with the results published in [17], see Fig. 3.4.

Secondly, the antenna in [18] with the dimensions $W = 160$ mm, $L = 80$ mm and $h = 7.2$ mm was analyzed. The feed point location $L_1$ was varied from 0 to 80 mm and the input impedance $Z_{in}$ was calculated.

Impedance results are shown in Fig. 3.5 and the results from [18] are shown in Fig. 3.6. The new model demonstrates that the short-circuited transmission-line is suitable for the estimation of the input impedance of the volumetric PIFA.
3.3 Comparison with published results

Fig. 3.3: Input impedance $Z_{in}$ from for configuration $L = 19$ mm, $L1 = L2 = 9.5$ mm, $W = 10$ mm, $h = 1.5$ mm and $\epsilon_r = 1$.

Fig. 3.4: Input impedance $Z_{in}$ from [17] for configuration $L = 19$ mm, $L1 = L2 = 9.5$ mm, $W = 10$ mm, $h = 1.5$ mm and $\epsilon_r = 1$. 

37
CHAPTER 3. TRANSMISSION-LINE MODEL FOR THE VOLUMETRIC PIFA

Fig. 3.5: Input impedance \( Z_{in} \) for configuration \( W = 160 \) mm, \( L = 80 \) mm, \( h = 7.2 \) mm and \( L_1 = 0 \) to \( 80 \) mm.

Fig. 3.6: Input impedance from [18] for configuration \( W = 160 \) mm, \( L = 80 \) mm, \( h = 7.2 \) mm and \( L_1 = 0 \) to \( 80 \) mm.
Chapter 4

Transmission-line model for a SCS antenna

4.1 Introduction

In this section a transmission-line model for the Short-Circuited Strip (SCS) antenna has been developed. This is the final step before developing the model for the PIFA itself.

We begin this chapter with some explanation about the SCS antenna. Then, we go into details about the modifications of the model of the volumetric antenna so that it can be applied to the SCS antenna. The difference with the volumetric PIFA is that the SCS antenna is much longer and thinner across its width. A model for the inner probe conductor of the SCS antenna is also included. This is due to the fact that the probe inner conductor is very close to the short-circuited strip. The probe inner conductor and the short-circuited strip are here considered as a transmission-line section.

At the end of the chapter, results from the modified model is compared with results from input impedance measurements. The measurements were done on SCS antennas which were already available.

4.2 A modified transmission-line model

A SCS antenna consists of a thin conducting strip above a metal ground plane. One end of the strip is short-circuited to the ground plane, whereas the other end is left open.

To feed the SCS antenna, a coaxial probe can be used. The inner conductor of the probe is passed through the ground plane and is connected to the conducting strip. The outer conductor of the coaxial probe is attached to the
CHAPTER 4. TRANSMISSION-LINE MODEL FOR A SCS ANTENNA

ground plane. Fig. 4.1(a) shows a picture of a SCS antenna and Fig. 4.1(b) shows a schematic diagram of the antenna. As in the previous chapters, the height $h$, the width $W$ and the length $L$ describe the dimensions of the antenna.

To obtain the transmission-line model for SCS antenna, the model developed for the volumetric PIFA is applied to the SCS antenna. This is done without modifying the volumetric model presented in chapter 3. It is reasonable to use the volumetric PIFA model, because the volumetric PIFA and the SCS antenna are almost identical. The difference is that the SCS antenna has a
4.2 A modified transmission-line model

longer and narrow top plate. The volumetric antennas we have studied have a length-width (L/W) ratio of approximately 2. For the SCS antennas considered here the length-width (L/W) is in the order of 5. Fig. 4.2 shows the volumetric model applied to the SCS antenna.

![Diagram of a SCS antenna and corresponding transmission-line model](image)

Fig. 4.2: (a) Diagram of a SCS antenna and (b) corresponding transmission-line model.

As in chapter 3, the model consists of two transmission-line sections. One side of the transmission-line is terminated by the self-admittance $Y_s$, which models the open end of the antenna. The other side of the transmission-line represents the short-circuited part of the antenna. The transmission-line parameters $Z_c$ (characteristic impedance) and $\gamma$ (propagation constant) were already discussed in the chapters 2 and 3 of this thesis.

In order to obtain good input impedance modelling, modification to our model is necessary. In the antennas considered here, the inner conductor of the probes is very close to the short-circuit strip. This means that the probe and the short-circuit strip may be treated as an transmission-line. Furthermore, the antennas are relative high, which makes the inner probe and circuit strip
CHAPTER 4. TRANSMISSION-LINE MODEL FOR A SCS ANTENNA

relative long. Fig. 4.3 gives an illustration of our model when its modified with a transmission-line section representing the inner conductor and the short-circuit strip.

![Diagram of SCS antenna with ground plane, probe inner conductor, and short-circuit strip.](image)

**Fig. 4.3:** (a) An example of a SCS antenna, (b) Transmission-line model for probe inner conductor and short-circuit strip.

It is assumed that the probe inner conductor and the short-circuit strip in Fig. 4.3 form a lossless transmission-line. The propagation constant is therefore computed as,

$$\beta_{probe} = \omega \sqrt{\mu \epsilon}. \quad (4.1)$$

where $\mu$ represents the permeability and $\epsilon$ is the permittivity. The characteristic impedance is chosen as,

$$Z_{cprobe} = 80 \ \Omega. \quad (4.2)$$
4.2 A modified transmission-line model

Eq.(4.2) was found through trials when the model was compared with results from measurements. A final modification is done to add a coax line to the model. This model is shown in Fig. 4.4.

![Diagram of transmission-line model with coax probe](image)

Fig. 4.4: Transmission-line model with coax probe $L_{\text{coax}}$.

The parameters for the coax line are modelled as,

\[
\beta_{\text{coax}} = \omega \sqrt{\mu \varepsilon} \tag{4.3}
\]

and

\[
Z_{\text{coax}} = 50\Omega. \tag{4.5}
\]
4.3 A modified transmission-line model using equivalent LC-circuits

The model given in Fig. 4.4 shows two transmission-lines interconnected with each other. Because of this, we cannot use the standard transmission-line formulas to compute the input impedance. Therefore, it is easier to replace the transmission-lines with equivalent LC-circuits. Here, is the equivalent LC-circuit an electrical circuit which consists of an inductance $L$ and capacitance $C$ and is used to represent a transmission-line section.

The next steps show how the inductance $L$ and capacitance $C$ are determined from the transmission-line parameters. Fig. 4.5 shows a transmission-line section of length $L_t$ with its corresponding equivalent LC-circuit. The transmission-line is further characterized by the phase constant $\beta_t$ and characteristic impedance $Z_{ct}$. The subscript $t$ is used to denote all the transmission-line parameters.

Fig. 4.5: (a) Transmission-line of length $L_t$ and (b) corresponding LC-circuit.

The inductance $L$ and capacitance $C$ can be written in terms of the transmission-line parameters $\beta_t$, $Z_{ct}$ and $L_t$. For this, we use two equations which contain both $L$, $C$ and the parameters $\beta_t$, $Z_{ct}$ and $L_t$. One equation is found by short-circuiting the transmission-line in Fig. 4.5. The other equation is found using the open-circuited transmission-line.

Fig. 4.6 shows the short-circuited transmission-line and its corresponding equivalent LC-circuit. The input impedance $Z_{ab}$ in Fig. 4.6(a) is written as,

$$Z_{ab} = Z_{ct} j \tan(\beta_t l). \quad (4.6)$$
4.3 A modified transmission-line model using equivalent LC-circuits

The input impedance $Z_{ab}$ in Fig. 4.6(b) is written as,

$$Z_{ab} = j\omega L + \frac{1}{j\omega C + 1/j\omega C} \tag{4.7}$$

![Diagram of short-circuited transmission-line and equivalent circuit](image)

Fig. 4.6: (a) Short-circuited transmission-line of length $L_t$ and (b) corresponding LC-circuit.

Since the transmission-line in Fig. 4.6(a) is equivalent to the circuit in Fig. 4.6(b) we can write that,

$$j\omega L + \frac{1}{j\omega C + 1/j\omega C} = Z_{ct} j \tan(\beta l). \tag{4.8}$$

This is the first equation needed to compute the inductance $L$ and capacitance $C$. For the second equation we consider the open-circuited transmission-line, see Fig. 4.7.

![Diagram of open-circuited transmission-line and equivalent circuit](image)

Fig. 4.7: (a) Open-circuited transmission-line of length $L_t$ and (b) corresponding LC-circuit.

The impedance seen at the input of the open-circuited transmission-line is,

$$Z_{ab} = jY_{ct} \tan(\beta l). \tag{4.9}$$
CHAPTER 4. TRANSMISSION-LINE MODEL FOR A SCS ANTENNA

The impedance seen at the input of the equivalent LC-circuit is,

\[ Z_{ab} = j\omega L + 1/j\omega C. \]  

(4.10)

Because Eq. 4.9 and Eq. 4.10 are equivalent we can write down the equality,

\[ j\omega L + 1/j\omega C = jY_{ct} \tan(\beta t) \]  

(4.11)

Using Eq. (4.8) and Eq. (4.11) we can write the inductance \( L \) and capacitance \( C \) in terms of the transmission-line parameters \( \beta_t, Z_{ct}\) and \( L_t \) as,

\[ L = -\left(\omega Z_{ct} \cot(\beta_t L_t) + \sqrt{\omega^2 Z_{ct}^2 \csc^2(\beta_t L_t)}\right)/\omega^2 \]  

(4.12)

\[ C = 1/\left(\omega Z_{ct}^2 \csc(\beta_t L_t)\right) \]  

(4.13)

The final step in this section is to replace the two interconnected transmission-lines with an equivalent LC-circuit. Here, we have replaced the short-circuited section with the inductance \( L_1 \) and capacitance \( C_1 \). Whereas the inner probe section is replaced with an LC-circuit consisting of \( L_2 \) and \( C_2 \). Here is \( L_1, L_2, C_1 \) and \( C_2 \) computed as,

\[ L_1 = -\left(\omega Z_{ct_1} \cot(\beta_{L1} L_1) + \sqrt{\omega^2 Z_{ct_1}^2 \csc^2(\beta_{L1} L_1)}\right)/\omega^2 \]  

(4.14)

\[ C_1 = 1/\left(\omega Z_{ct_1}^2 \csc(\beta_{L1} L_1)\right) \]  

(4.15)

\[ L_2 = -\left(\omega Z_{ch} \cot(\beta_h h) + \sqrt{\omega^2 Z_{ch}^2 \csc^2(\beta_h h)}\right)/\omega^2 \]  

(4.16)

\[ C_2 = 1/\left(\omega Z_{ch}^2 \csc(\beta_h h)\right) \]  

(4.17)

Finally, we show in Fig. 4.8 the complete model of the SCS antenna using the LC-circuits.
4.3 A modified transmission-line model using equivalent LC-circuits

Fig. 4.8: (a) open-circuited transmission-line of length $L_1$ and (b) corresponding LC-circuit.
4.4 Comparison with measurements

To verify the performance of the present model, return loss measurements were performed on several SCS antennas. For all the measurements an Hewlett Packard vector network analyzer was used. Fig. 4.9 shows the measurement equipment. The frequency range is setup from 0.5 GHz to 3 GHz. Calibration was done for the 3.5 mm connectors. The antenna was placed on the measurement table which is 1 m above ground. It should be pointed out that during the measurements a metal plate is placed below the ground plane of the antenna. This is done to make the ground plane larger. The model assumes an infinite ground surface and thus there is a better agreement between model and experiment with larger ground. This seems to have an important effect on the measurements results. The measurements were carried out on three SCS antennas. Fig. 4.10, Fig. 4.11 and Fig. 4.12 show model results compared with measurements. The dimensions of the antennas are given in table 4.1.

Fig. 4.9: Network analyzer

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2</th>
<th>W</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCS0G8</td>
<td>2.7 mm</td>
<td>74.3 mm</td>
<td>4 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>SCS1G5</td>
<td>2.2 mm</td>
<td>40.3 mm</td>
<td>4.3 mm</td>
<td>9 mm</td>
</tr>
<tr>
<td>SCS1G8</td>
<td>2.2 mm</td>
<td>33.3 mm</td>
<td>2.3 mm</td>
<td>9 mm</td>
</tr>
</tbody>
</table>

Table 4.1: Dimension of the measured SCS antennas
4.4 Comparison with measurements

Fig. 4.10: (a) Return loss and (b) input impedance for the SCS0G8 antenna.
Fig. 4.11: (a) Return loss and (b) input impedance for the SCS1G5 antenna.
4.4 Comparison with measurements

Fig. 4.12: (a) Return loss and (b) input impedance for the SCS1G8 antenna.
Chapter 5

Transmission-line model for the PIFA

5.1 Introduction

In this chapter a transmission-line model for the Planar Inverted F-Antenna (PIFA) is developed. The model consists of two transmission-line sections which are connected in parallel. The transmission-line parameters were computed with the well-known asymmetrical coplanar strips model [4]. It will be shown, however, that this coplanar model can not be used without adaptation.

The transmission-line model is used to estimate the input impedance of the PIFA antenna. Input impedance measurements were done in the antenna laboratory at the Eindhoven University of Technology (TU/e). At the end of the chapter, input impedance results from the model are compared with these measurements.

5.2 Modelling approach

PIFA antennas are printed circuit type antennas. They consist of narrow thin strips printed on a homogeneous dielectric substrate. The strips with the lengths $L_1$ and $L_2$ give the length of the antenna. The total length of the antenna $L = L_1 + L_2$, is about a quarter of a wavelength $\lambda$. Fig. 5.1(a) shows a photograph of the PIFA antenna.

A feeding microstripline is used to provide electrical power to the antenna. The feeding microstripline, the strips $L_1$ and $L_2$ are electromagnetically coupled to a ground plane. Fig. 5.1(b) shows the ground plane on the back of the antenna.
Fig. 5.1: Photograph of a PIFA antenna (a) Upper side view and (b) Lower side view
5.2 Modelling approach

The first transmission-line model of the PIFA antenna is the short-circuit model. The antenna can here be represented by two finite-length transmission-lines connected in parallel and characterized by the propagation constant $\beta$ and the characteristic impedance $Z_0$ as shown in Fig. 5.2. In this model it is assumed that the short-circuit of the PIFA is an ideal short-circuit. Fig. 5.3 shows a second model where the short-circuit is modelled as an equivalent LC-circuit.

In both models, the admittance load $Y_L$ located at the end of the PIFA antenna consists of a real and an imaginary part. The real part $G_{rad}$ of the impedance load is used to represent the radiation loss of the antenna. The imaginary part $B_{rad}$ is the end-effect susceptance which is used to characterize the end-effect of the antenna. In view of the transmission-line model considered in this thesis we are interested in the line parameters $\beta$, $Z_0$, $G_{rad}$ and $B_{rad}$. Each of these parameters will be treated separately.
5.3 Asymmetrical coupled and coplanar strips

The strips $L_1$ and $L_2$ in Fig. 5.4(a) are electromagnetically coupled to the ground plane. In the cross-section view of the PIFA, the strips and the ground plane form a coupled strip structure separated by the dielectric material with thickness $h$. The cross-section of the PIFA is shown in Fig. 5.4(b). The strips with widths $w_1$ and $w_2$ are separated by a distance $s$. An offset $s + \frac{w_1 + w_2}{2}$ is defined between the centers of the strips in the transverse cross-section.

To model the coupled strips structure of the PIFA as a transmission-line, we first try to use the well-known asymmetric coplanar model. The idea here is that if the thickness of the dielectric material $h$ is small, then the coupled strips will behave as the asymmetric coplanar strips. Fig. 5.4(c) shows the cross-section of the asymmetric coplanar strip structure.
5.3 Asymmetrical coupled and coplanar strips

For the asymmetrical coplanar structure exists well-known formulas that describe its transmission-line properties. The exact formulas for those parameters have been expressed in terms of elliptic functions. The effective dielectric constant $\varepsilon_{\text{eff}}$ is given by [4],

$$
\varepsilon_{\text{eff}} = 1 + \frac{(\varepsilon_r - 1)K(k_1)K(k)}{2K(k')K(k')}, \tag{5.1}
$$

where $K(k)$ is the complete elliptic integral of the first kind. The arguments $k$ and $k_1$ are dependent on the geometry as,

$$
k = \sqrt{1 - \frac{1}{(1 + s/w_1)(1 + s/w_2)}}, \tag{5.2}
$$

and

$$
k_1 = \sqrt{\frac{(t_1 - t_2)(t_3 - t_2)}{(t_1 + t_2)(t_3 + t_2)}}, \tag{5.3}
$$

where

$$
t_i = \frac{\exp(\lambda_i) - 1}{\exp(\lambda_i) + 1} \quad \text{for} \quad i = 1, 2, 3 \tag{5.4}
$$

$$
\lambda_1 = \frac{\pi}{2} \left[ \frac{2w_1}{h} + \frac{s}{h} \right], \tag{5.5}
$$

$$
\lambda_2 = \frac{\pi s}{2h}, \tag{5.6}
$$

$$
\lambda_3 = \frac{\pi}{2} \left[ \frac{2w_2}{h} + \frac{s}{h} \right]. \tag{5.7}
$$
CHAPTER 5. TRANSMISSION-LINE MODEL FOR THE PIFA

The complementary arguments \( k' \) and \( k'_1 \) are given by,

\[
k' = \sqrt{1 - k^2}, \quad (5.9)
\]
and

\[
k'_1 = \sqrt{1 - k'_1^2}. \quad (5.10)
\]

The characteristic impedance \( Z_0 \) is given by,

\[
Z_0 = \frac{\eta_0 K(k)}{2\sqrt{\varepsilon_{\text{eff}}} K(k')}, \quad (5.11)
\]

Using Eq. 5.1 we now can express the propagation constant \( \beta \) as,

\[
\beta = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_{\text{eff}}}. \quad (5.12)
\]
5.4 Comparison between asymmetrical coupled and coplanar strips

A major concern in our transmission-line model are the parameters $\varepsilon_{\text{eff}}$, $\beta$ and $Z_0$ of the coupled strips structure used in the PIFA. As said before, the idea is to simply use the line parameters of the asymmetrical coplanar strips for the coupled strips structures. The line parameters of the asymmetrical coplanar strips are found in Eqs. 5.1- 5.12.

A comparison between the asymmetrical coupled and the coplanar structures has been done. The coupled strips structure is simulated in the Ansoft HFSS software package and the asymmetrical coplanar structure is analyzed in the Matlab platform.

Fig. 5.5(a) shows the effective permittivity $\varepsilon_{\text{eff}}$ for both the coupled and the coplanar structures. The solid line represents the simulated asymmetrical coupled strips and the dashed line corresponds to the asymmetrical coplanar strips. It must be mentioned that the simulated $\varepsilon_{\text{eff}}$ does not change with the port size in HFSS. This make this parameter a very reliable and accurate quantity for comparing the two structures.

From Fig. 5.5(a) it is obvious that there is a significant difference between the coupled strips and the coplanar strips. We must conclude that Eq. 5.1 does not perform satisfactory for this configuration and frequency range.

From Fig. 5.5(b) can be concluded that the propagation constants $\beta$ of the two structures do not agree. The last parameter which was studied is the characteristic impedance $Z_0$. Again there is a large different between the coupled strips and the coplanar strips, see Fig. 5.5(c).

Because the results presented above are not acceptable, it was necessary to correct the transmission-line model for the asymmetrical coplanar structure.

If the asymmetrical coplanar structure is viewed as a transmission-line, the capacitance per unit length $C_{\text{asym}}$ of the line is given by,

$$C_{\text{asym}} = 2\varepsilon_0 \frac{K(k')}{K(k)} + \frac{(\varepsilon_r - 1)\varepsilon_0 K(k_1)}{K(k'_1)}$$  \hspace{1cm} (5.13)

and the inductance per unit length $L_{\text{asym}}$ is given by,

$$L_{\text{asym}} = \mu_0 \frac{K(k)}{2K(k')}.$$  \hspace{1cm} (5.14)

Fig. 5.6 shows the transmission-line circuit with the corresponding capacitance and inductance components per unit length.
CHAPTER 5. TRANSMISSION-LINE MODEL FOR THE PIFA

(a)

(b)
5.4 Comparison between asymmetrical coupled and coplanar strips

An approximate correction of the model is obtained by adding a shunt capacitance $C_{corr}$ to the capacitance contribution between the strip and the ground plane. This is because in the coupled strips structure the strip, is strongly coupled to the ground plane, while in the coplanar structure the coupling is weaker. The corrected LC-circuit model with the shunt capacitance $C_{corr}$ is shown in Fig. 5.7.
5.5 Correction capacitor $C_{corr}$

In this section the steps necessary to compute the correction capacitor are shown. The correction capacitance per unit length is computed as a parallel-plate capacitor such that,

$$C_{corr} = \varepsilon_r \varepsilon_0 \frac{W_{con}}{d_{con}}, \quad (5.15)$$

where $\varepsilon_r$ is the permittivity of the dielectric, $W_{con}$ is the mean width of the plates $W_{con1}$ and $W_{con2}$ and $d_{con}$ is the distance between the plates $W_{con1}$ and $W_{con2}$ as shown in Fig. 5.8. $w_1$ and $w_2$ represent the width of the strip and the ground plane respectively; $h$ represents the dielectric thickness.
5.5 Correction capacitor $C_{corr}$

Fig. 5.8: Cross-section of a coupled strip structure.

In Fig. 5.9 is an enlarged picture of the strip $w_2$ shown.

Fig. 5.9: An enlarged picture of strip $W_2$.

The triangle formed by the points $BB'C'$ has an internal angle $B'$ such that,

$$
\angle B' = \arctan \frac{h}{w_1/2 + w_2/2 + s'}
$$

and the angle $B$ is computed as,

$$
\angle B = 90^\circ - \angle B'
$$

If the angle $A$ is an internal angle of the $\Delta AGD$ as shown in Fig. 5.10, then
Fig. 5.10: Cross-section coupled strip

the following equation must be valid,

\[
\angle A = \arctan \frac{h}{w_2 + s},
\]

(5.19)

and the angle C is,

\[
\angle C = 180^\circ - \angle A - \angle B.
\]

(5.20)

The next important step is to compute \(W_{con1}\). It is easy to see that for \(W_{con1}\) we can write,

\[
W_{con1} = \frac{w_2}{\sin \angle C} \sin \angle A.
\]

(5.22)

Thus Eq. 5.22 gives the width \(W_{con1}\) of the first plate of the correction capacitor. Below we computed the width of the second plate \(W_{con2}\) using similar steps as before. Thus for \(W_{con2}\) we write,

\[
W_{con2} = \frac{w_1}{\sin \angle E} \sin \angle F
\]

(5.23)

Now we have calculated the width \(W_{con2}\) of the second plate of the correction capacitance.

The final step is to compute the distance \(d_{con}\) between the plates. We start by
5.6 Results correction capacitor

computing the side length $s_1$ in the $\triangle BB'C'$ using the trigonometric property,

$$s_1 = \frac{w_2}{2} \cos \angle B'$$

$$\text{(5.24)}$$

$$\text{(5.25)}$$

Fig. 5.11: Cross-section coupled strip

Using the cosine property we write that,

$$s_2 = \frac{w_1}{2} \cos \angle D'$$

$$\text{(5.26)}$$

If we define the total distance $s_{tot}$ between strip and ground plane as the distance between the middle of the strip to the middle of the ground plane, $s_{tot} = \sqrt{h^2 + (s + w_1/2 + w_2/2)^2}$ then we can write,

$$s_{con} = s_{tot} - s_1 - s_2.$$  \hspace{1cm} \text{(5.27)}

5.6 Results correction capacitor

This section shows results for the transmission-line parameters when the correction capacitor $C_{corr}$ is used. If the same structure as in section 5.4 is considered, then the effective permittivity $\epsilon_{eff}$ is corrected as shown in Fig. 5.12(a).

Fig. 5.12(b) shows the corrected propagation constant $\beta$. The solid line represents the simulated values and the dot-dashed line corresponds the calculated values. Both graphs show a good match between the calculated and the simulated values. For the structure considered here, the error in the computed $\epsilon_{eff}$ is less than 1.2%. The capacitance without correction $C_{asym}$ is
CHAPTER 5. TRANSMISSION-LINE MODEL FOR THE PIFA

(a)

(b)
5.6 Results correction capacitor

Fig. 5.12: (a) The effective permittivity $\varepsilon_{eff}$, (b) the characteristic impedance $Z_0$ and (c) the characteristic impedance $Z_0$ for the corrected model.

16.18 $\mu$F/m, whereas the shunt correction capacitance $C_{corr}$ is 2.82 $\mu$F/m. Fig. 5.12(c) shows the corrected values for the characteristic impedance $Z_0$. The corrected values do not match with the simulated value. This is because the simulated characteristic impedance values depend strongly on the chosen port size in HFSS. Thus the quantity $Z_0$ obtained with HFSS is not reliable.
5.7 The end-effect susceptance $B_{\text{rad}}$

This section discusses the calculation of the end-effect susceptance $B_{\text{rad}}$ which appears in the model in Fig. 5.4(b). The susceptance $B_{\text{rad}}$ is associated with the end-effect capacitor which originates by the abrupt ending of the coupled strip structure. It can also be seen as an equivalent load to represent the end-effect at the edge of the PIFA, see Fig. 5.13.

![Fig. 5.13: End-effect shown in top-view of PIFA antenna](image)

Similar to model of the microstrip antenna, we can express $B_{\text{rad}}$ as,

$$B_{\text{rad}} = Y_c \tan(\beta \Delta l).$$

(5.28)

where $Y_c$ represents the characteristic admittance, $\beta$ is the propagation constant and $\Delta l$ represents the length of the fringing field associated with the end-effect. Eq. 5.28 can be derived from the ideal condition for a transmission-line with length $\Delta l$. For the PIFA antenna, however, the length of the fringing field $\Delta l$ is unknown. Therefore, the end-effect susceptance $B_{\text{rad}}$ cannot be computed in this manner.

Another method is used to compute the susceptance $B_{\text{rad}}$. In order to compute the susceptance $B_{\text{rad}}$ we first have to compute the end-effect inductor $L'$. In the work of Getsinger [19, pp. 668] it is showed that the end-effect inductor $L'$ of a finite length abruptly ended two wire can be considered to be localized at the of the wire line (see Fig. 5.14).

In the inductive model of the two wire line it was shown that the end-effect
5.7 The end-effect susceptance \( B_{\text{rad}} \)

![Diagram of two round conductors with end-effect inductance](image)

**Fig. 5.14**: Two round conductors with end-effect inductance

Inductor \( L' \) for a single wire is given by,

\[
L' = -\left(\frac{\mu}{4\pi}\right)G,
\]

(5.29)

where \( G \) is the distance between the wires. In addition to that, Getsinger showed that in vacuum the corresponding end-effect capacitor is found by,

\[
C' = -\frac{2L'}{(Z_o^2)},
\]

(5.30)

where \( Z_o \) is the characteristic impedance of the transmission-line. To obtain the end-effect susceptance \( B_{\text{rad}} \) itself we then write,

\[
B_{\text{rad}} = \omega C'.
\]

(5.31)

As we want to use this method for the coupled strips configuration some adaptation is necessary. Since Eq. 5.30 holds for any homogeneous dielectric we can state that the end-effect capacitor \( C'' \) for the coupled strips is,

\[
C'' = -\frac{2L'/(Z_o^2 \epsilon_e)}{Z_o^2 \epsilon_e}.
\]

(5.32)

Here \( \epsilon_e \) represents the effective dielectric constant of the coupled strips structure. Similarly to the microstrip, the end-effect capacitor for the coupled strips is found by requiring that,

\[
C' = -\frac{2L' \epsilon_e'}{(Z_o^2 \epsilon_e)}.
\]

(5.33)
where $\varepsilon'_e$ is called the end-effect effective dielectric constant for the coupled strips. This is consistent with definition found in Getsinger’s paper [19].

The difficulty in this method is to find the end-effect effective dielectric constant $\varepsilon'_e$ for the coupled strip configurations. Below we show that $\varepsilon'_e$ can be found by a trial and error method using a number of configurations. Curves for Eq. 5.31 were computed and compared with computer data obtained from Ansoft HFSS software. After some adjustments, the end-effect effective dielectric constant was found to be $\varepsilon'_e = 2$.

Fig. 5.15(a) and Fig. 5.15(b) show results for a configuration A with ground plane $w_1 = 10$ mm, strip $w_2 = 1.2$ mm, separation $s = 2$ mm and dielectric substrate $\varepsilon_r = 4.9$. $B_{rad}$ was computed with the Getsinger's formula using $\varepsilon'_e = 2.0$. The relative error $\varepsilon_{rr}$ of $B_{rad}$ is defined as,

$$\varepsilon_{rr} = \frac{B_{rad} - B_{rad}^{HFSS}}{B_{rad}^{HFSS}} \cdot 100. \quad (5.34)$$

Fig. 5.16(a) and Fig. 5.16(b) show results for a configuration B with ground plane $w_1 = 10$ mm, strip $w_2 = 1.2$ mm, separation $s = 7.5$ mm and dielectric substrate $\varepsilon_r = 4.9$. The end-effect effective dielectric constant is $\varepsilon'_e = 2.12$.

Finally, Fig. 5.17(a) and Fig. 5.17(b) show results for a configuration C with ground plane $w_1 = 10$ mm, strip $w_2 = 1.2$ mm, separation $s = 3$ mm, dielectric constant $\varepsilon_r = 4.9$ and end-effect effective dielectric constant $\varepsilon'_e = 2.2$.

Fig. 5.15: (a) End susceptance $B_{rad}$ and (b) Error $\varepsilon_{rr}$ in the end susceptance for the coupled strips configuration A.
5.7 The end-effect susceptance $B_{rad}$

Fig. 5.16: (a) End susceptance $B_{rad}$ and (b) Error $\epsilon_{rr}$ in the end susceptance for the coupled strips configuration B.

Fig. 5.17: (a) End susceptance $B_{rad}$ and (b) Error $\epsilon_{rr}$ in the end susceptance for the coupled strips configuration C.
CHAPTER 5. TRANSMISSION-LINE MODEL FOR THE PIFA

5.8 The radiation admittance $G_{rad}$

In this section, it is shown that the radiation admittance $G_{rad}$ can be computed by modelling the open-end of the PIFA as an open-circuited two-wire transmission-line. In section 5.3, it was shown that the strip $w_2$ and the ground plane $w_1$ at the edge of the antenna, formed an asymmetrical coupled strips structure. When viewed from the top, the asymmetrical coupled strips are separated by an offset distance $d = s + (w_1 + w_2)/2$ as shown in Fig. 5.18(a).

It is now assumed that the asymmetrical coupled strips can be replaced with an equivalent symmetrical coupled strips structure with two identical strips $w = (w_1 + w_2)/2$ and an offset distance $d = s + w$, see Fig. 5.18(b). If the equivalent symmetrical coupled strips structure is modelled as an open-circuited two-wire transmission-line with radius,

$$a = w/2 \quad (5.35)$$

and center-to-center spacing,

$$d = s + w \quad (5.36)$$

then the radiation conductance $G_{rad}$ is computed as in Green [20],

$$G_{rad} = \frac{(kd)^4(\delta l/d)^2}{12Z_o(\pi Z_o/\eta)} \quad (5.37)$$

where,

$$\frac{\delta l}{d} = \frac{1}{-3.954 + \sqrt{(2.564 \text{arccosh} \frac{d}{2a})^2 + (3.954)^2}} \quad (5.38)$$

$$\eta = \sqrt{\mu_0/(\varepsilon_0 \varepsilon_{reff})} \quad (5.39)$$

$$Z_o = \frac{\eta}{\pi} \text{arccosh} \frac{d}{2a} \quad (5.40)$$

$$k = 2\pi/\lambda. \quad (5.41)$$

72
5.8 The radiation admittance $G_{rad}$

Fig. 5.18: (a) Top view of the open-end PIFA showing asymmetrical coupled strips structure, (b) Top view of the open-end PIFA replaced with an equivalent symmetrical coupled strips and (c) Open-end PIFA modelled as an open-circuited two-wire transmission-line.
CHAPTER 5. TRANSMISSION-LINE MODEL FOR THE PIFA

Using measurement input impedance data and comparing these with our own model we can compute the radiation admittance $G_{rad}$ of the PIFA antenna. The radiation conductance using the Green method is also depicted in Fig. 5.19. For the PIFA in Fig. 5.19 the following dimensions have been used: $w_1 = 14$ mm, $w_2 = 1.2$ mm, $s = 2$ mm. The computed $G_{rad}$ was obtained from Eq. 5.37 with $\alpha = \frac{100}{95} \cdot \frac{w}{2}$. The factor $\frac{100}{95}$ is used to obtain good match.

![Graph showing radiation admittance](image)

Fig. 5.19: $G_{rad}$ from measurements and $G_{rad}$ using Green formula (Eq. 5.37)

### 5.9 Comparison with measurement

Return loss measurements were done in the antenna laboratory at the Eindhoven University of Technology (TU/e). During the measurement the antenna is placed on a microwave absorbing material to minimize the effect of the measurement table. The measure were done on frequency range of 0.5 to 3 GHz. Fig. 5.20 and 5.21 show typical results for two antennas. For the PIFA aV 107, the following dimensions and properties were used: $L_1 = 6.8$ mm, $L_2 = 17$ mm, $s_1 = 7$ mm, $s_2 = 4$ mm, $w_1 = 14$ mm, $w_2 = 1.2$ mm and $\varepsilon_r = 4.9$. A factor $\frac{100}{95}$ was used in Eq. 5.35. In general good match is obtained.

For the PIFA aV 120, the dimensions and properties are, $L_1 = 6.5$ mm, $L_2 = 15$ mm, $s_1 = s_2 = 8$ mm, $w_1 = 10$ mm, $w_2 = 1.2$ mm and $\varepsilon_r = 4.9$. A factor $\frac{100}{70}$ was used in Eq. 5.35 in order to obtain good. This probably due to fact that the spacing $s_2$ between strip $L_2$ and ground plane is large.
5.9 Comparison with measurement

![Graph showing return loss and input impedance for the PIFA aV 107.](image)

**Fig. 5.20:** (a) Return loss and (b) input impedance for the PIFA aV 107.
Fig. 5.21: (a) Return loss and (b) input impedance for the PIFA aV 120.
Chapter 6

Conclusions and recommendations

6.1 Conclusions

In this thesis, a transmission-line model for the Planar Inverted F-Antenna (PIFA) was developed. The PIFA is a printed circuit type antenna which is often used on the main Printed Circuit Board (PCB) of mobile phones, WLAN and Bluetooth. The PIFA is made of a thin narrow F-shaped strip, printed on one side of a homogeneous dielectric substrate. On the other side of the substrate, a conducting ground plane is used. One edge of the stripline is short-circuited to the ground plane. The total length of the antenna $L$, is about a quarter of the wavelength $\lambda/4$.

Before actually developing the model for the PIFA, several other antennas and models were first studied with the purpose of extending the models to PIFA antenna. The rectangular microstrip antenna was studied using the transmission-line model proposed by van de Capelle and Pues. It has been demonstrated that the present model was correctly implemented.

The model of the rectangular microstrip antenna was subsequently used and modified for the volumetric PIFA. The so-obtained model was verified and further used for the SCS antenna. A part of the SCS model, namely the short-circuit, was further used and modified for the transmission-line model of the PIFA.

Finally, a new transmission-line model for the PIFA was developed. The antenna was modelled as two finite-length transmission-lines, characterized by the propagation constant $\beta$ and the characteristic impedance $Z_0$, while the antenna short-circuit was modelled using an equivalent LC-circuit.

The computation of the transmission-line parameters for the asymmetric coplanar strips was an important issue in this model, since the strips and
the ground plane of the PIFA are electromagnetically coupled to each other such that an asymmetric coupled strips structure is formed. The model for the asymmetrical coupled strip is not well documented, so we have started by using a well-known model for the coplanar strips. However, a comparison between the parameters obtained from the coupled strips simulation and the parameters computed with the coplanar model, showed that the parameters do not correspond well for the particular configuration and frequency range.

The coplanar model was then modified by introduction of a correction of the transmission-line parameters by adding a shunt capacitance $C_{corr}$ to the equivalent LC-circuit of the asymmetrical coplanar strips. Comparison between the modified model and EM simulation, showed that the new model agree well with simulation values.

For the model of the PIFA, the calculation of the susceptance $B_{rad}$ and the radiation admittance $G_{rad}$, associated with the end-effect and the radiation power respectively, was a challenge. In order to compute the susceptance $B_{rad}$, the end-effect inductor, which is basically determined by the distance between the wires, had to be computed.

The radiation admittance $G_{rad}$ can be computed by replacing the asymmetrical coupled strips structure with an equivalent two-wire structure.

An overall verification of the transmission-line model for the PIFA was done by comparing the model with Return Loss measurements. A good agreement was found between the measurements and the new transmission-line model for the PIFA.

6.2 Recommendations

The following recommendations are made regarding follow-up research on the transmission-line modelling of the antenna. More accurate results can probably be obtained by improving the susceptance $B_{rad}$ associated with the open-end of the PIFA. It is recommended to computed the susceptance $B_{rad}$ as in the microstrip antenna, using an empirically determined stray field length $\Delta l$. The radiation admittance $G_{rad}$ for PIFA can be improved by better modelling of the asymmetrical coupled strips structures.

Although dielectric and conducting losses are neglected in the present model, it is desirable to include them for a better matching between measurement and model.
References


REFERENCES


